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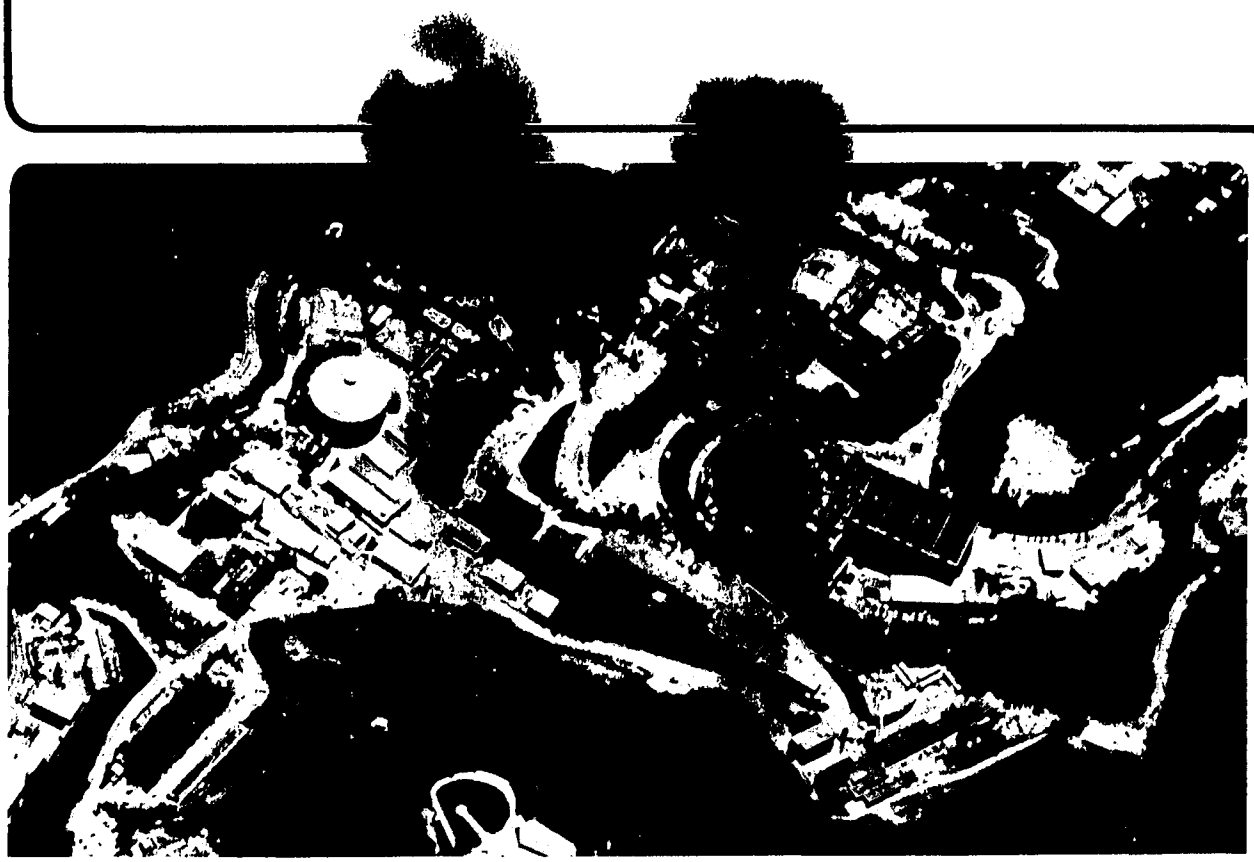

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BOOTSTRAP THEORY OF PARTICLES*

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ABSTRACT

Recent developments in S-matrix theory are reviewed, which have resulted in a topological bootstrap theory of particles. Each component of an S-matrix topological expansion is associated with a pair of intersecting surfaces: a "classical" surface describing space-time aspects of particle interactions, and a "quantum" surface carrying the internal quantum numbers. The theory implies a definite and restricted set of elementary particles, all of which are composite structures built from two kinds of topological constituents that are not themselves particles. One of the two constituents in identified with topological quarks, which appear in 3 colors, 8 flavors, and with integral charges. The lowest level of the topological expansion exhibits a topological supersymmetry involving a single dimensionless coupling constant that has been identified tentatively with the fine-structure constant.

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I. INTRODUCTION

According to the bootstrap hypothesis, proposed by Geoffrey Chew in 1959, nature cannot be reduced to fundamental entities, like fundamental building blocks of matter, but has to be understood entirely through self-consistency. Things exist by virtue of their mutually consistent relationships, and all of physics has to follow uniquely from the requirement that its components be consistent with one another and with themselves. The purpose of this paper is to review some recent developments in S-matrix theory, the theoretical foundation of bootstrap physics. Over the last few years, Chew and his collaborators have succeeded in formulating a bootstrap theory of particles that is extremely promising but is not yet widely known with in the physics community. An earlier version of this theory was reviewed in Ref. 1.

In the early sixties, the bootstrap approach was quite successful, but then it became bogged down in the mathematical complexities of S-matrix theory. In the bootstrap view, every particle is related to every other particle, including itself, which makes the mathematical formalism highly nonlinear, and this nonlinearity was impenetrable until recently. In the mid-sixties, therefore, the bootstrap approach went through a crisis of faith, and the support dwindled to a handful of physicists. At the same time, the quark idea, based on the notion of fundamental building blocks and formalized mathematically within quantum chromodynamics (QCD),[2] gained momentum and its adherents presented the bootstrappers with the challenge to explain the observed quark structure.

In 1974, topology came to the rescue. The combination of the S-matrix framework with combinatorial topology made it possible to handle the mathematical nonlinearity and has now resulted in a bootstrap theory of particles that can account for the observed quark structure without assuming the existence of quarks as physical building blocks of particles. Moreover, the new bootstrap theory illuminates a number of questions not previously understood.[3] These results have generated great enthusiasm among S-matrix theorists and are likely to prompt the physics community to re-evaluate its attitudes toward the bootstrap approach.

In the bootstrap theory of particles, there is no continuous space-time; there is no equation of motion, and there are no fields. Physical reality is described in terms of isolated events that are causally connected but are not embedded in continuous space-time. Space-time is introduced macroscopically, in terms of the experimental apparatus; there is no implication of a local space-time continuum. This description, together with quantum superposition, forms the basic framework of S-matrix theory. The question of why the space-time continuum is such a good approximation will eventually have to be answered. From the S-matrix point of view, space and time are not primary notions but represent approximations.

The starting point, then, is the assumption that there are discrete events, such as radioactive decay or particle collisions. Each such event is associated with a probability amplitude and represented by a diagram. For example, a typical event would be a scattering process where two particles A and B collide and emerge from the collision as particles C and D (see Fig. 1). The S matrix is the collection of

probability amplitudes for all possible physical events. The amplitudes are analytic functions of particle momenta with certain isolated singularities, which correspond to all the possible ways in which energy and momentum (plus other information) can be transferred over macroscopic distances. The singularities are, in turn, represented by diagrams. For example, for a reaction involving six particles the singularities of the corresponding amplitude will include the ones represented in Fig. 2. One special type of singularity that is of particular interest is the type shown in Fig. 2(a), in which a single particle transmits information over the macroscopic distance. The singularity represented by this diagram is a pole.[4]

The corner stone of S-matrix theory is the unitarity equation,[5]

$$\mathrm{Im}\, T_{fi} = \frac{1}{2} \sum_n T^{\dagger}_{fn} T_{ni}, \tag{1.1}$$

where T_{fi} denotes the probability amplitude for an event in which a collection i of ingoing particles leads to a collection f of outgoing particles. Equation (1.1) can be represented graphically as shown in Fig. 3, where we have taken a $2 \to 2$ amplitude as an example. Through unitarity, this amplitude is related to an infinite sum over products of $2 \to n$ amplitudes. These products determine all the singularities of the amplitude; in other words, the amplitude is built from its singularities. Physically, this means that each particle is a composite "bound state" of other particles -- a central feature of the hadron bootstrap.

The unitarity equation imposes a very restrictive, nonlinear condition on S-matrix amplitudes, and the aim of S-matrix theory is

to construct mathematical models of the S matrix that satisfy this condition. Only the full physical S matrix will satisfy unitarity completely, but there may be approximations to the physical S matrix, which satisfy the unitarity relations to some extent and bear sufficient resemblance to the observed physical phenomena.

The key discovery, which led to the recent breakthroughs in bootstrap physics, was made by Veneziano[6] in 1973. Veneziano recognized that the complexity of the interconnections of subatomic processes, and thus the complexity of the corresponding graphs, can be given a precise mathematical formulation. To do so, graphs are embedded in two-dimensional surfaces, whose complexity can be determined with the help of topology.[7] The topology of two-dimensional surfaces is completely understood by mathematicians and seems to be ideally suited for S-matrix theory, as these surfaces allow the formation of connected sums corresponding to matrix multiplication and the contraction of surface areas corresponding to the contraction of matrix indices. Surfaces can be orientable or nonorientable, and both kinds are used in the topological bootstrap theory.

With the help of topology, unitarity can be systematically approached by expanding the physical amplitude T_{fi} in Eq. (1.1) into an infinite series of topological amplitudes,

$$T_{fi} = \sum_{\gamma} T^{\gamma}_{fi}. \qquad (1.2)$$

Each value of the index γ in the topological expansion (1.2) is associated with a graph embedded in a two-dimensional surface of a particular topology. In other words, a physical event is represented

as a superposition of event patterns, each of which is associated with a precise complexity. Related to this is the notion of entropy, which means that, when individual event patterns are tied together, the complexity of the combined pattern can never diminish. The concept of entropy is used in analogy with thermodynamics where it is also a quantity that can never decrease.

The usefulness of expanding the scattering amplitude into an infinite series of terms, represented by graphs of increasing complexity, was already appreciated by Feynman in 1947. Feynman introduced his celebrated graphs, whose lines carry energy-momentum, in the context of quantum electrodynamics, but they were later recognized, especially by Landau, to have more general significance. Energy-momentum carrying graphs are also an important part of the new topological formalism. However, as defined originally, Feynman graphs are not adequate for the topological theory, because they lack a crucial property, that of graph contraction. We shall see below that the contraction property, discovered independently by Harari and Rosner[8] in 1969 and often characterized as "duality", is essential to the topological bootstrap.

The topological approach initiated by Veneziano was developed by Chew and Rosenzweig[9] into a topological theory of hadrons, which is now called "classical DTU", the initials DTU standing for "dual topological unitarization". In this theory, each graph representing a term in the topological expansion is embedded in a two-dimensional, oriented, bounded surface, now called the classical surface because it carries classical variables -- energy, momentum, spin and electric

charge. These are accepted as part of the basic framework of S-matrix theory and are not deduced from self-consistency. Classical DTU, as reviewed in Ref. 1, provided an impressive understanding of meson interactions and, in particular, made it possible to derive the quark structure of strong interactions from self-consistency considerations.

However, classical DTU was still incomplete and deficient in several ways. It was restricted, essentially, to mesons, ignored spin and did not allow to derive the internal quantum numbers of hadrons from self-consistency. These deficiencies have been overcome in the new bootstrap theory with the help of a more elaborate topological formalism. The principal new feature of the topological bootstrap is an additional surface, called the quantum surface, describing the internal quantum numbers (observable only indirectly, through selection rules), which are derived from self-consistency. Quantum surfaces are closed surfaces, and for each graph the classical and quantum surfaces intersect in such a way that their line of intersection coincides with the boundary of the classical surface. On the classical surface, particles are represented by boundary segments, on the quantum surface by surface areas.

At the lowest level of complexity, called the zero-entropy level, the two surfaces are given by a plane (classical surface) "inside" a sphere (quantum surface). The zero-entropy level plays a very special role, because the sphere is the only pattern of complexity that can reproduce itself. By gluing together spheres to form connected sums, representing the right-hand side of the unitarity equation, one may again obtain spheres. At any higher order, the complexity will always

increase. This means that the only place where the system is nonlinear, i.e. where patterns reproduce themselves, is at the zero-entropy level, the level of spheres.[10] Higher complexities are built up in successive steps in a linear process. As a consequence, the internal quantum numbers of hadrons are determined by zero-entropy self-consistency. The confinement of the nonlinearity in the hadron bootstrap to the spherical, zero-entropy level was the essential breakthough brought about by the use of topology.

The results of the topological boostrap for hadrons are now well-established. They include an understanding of baryon number, color, flavor, and broken symmetries. More recently, the topological formalism has been extended to the electroweak interactions, where it has led to a natural explanation for the quantization of electric charge and for the observed charges of leptons and Hadrons. The topological theory of electroweak interactions involves a quartet of vector bosons (indentified with the photon, the $W^{\pm}$, and the Z_0) and a quartet of scalar bosons (associated with the Higgs particles).

The topological bootstrap results in a definite and restricted set of elementary particles. On the quantum surface, these particles are represented by areas (orientable for hadrons and nonorientable for electroweak particles) that are built from two types of triangles, one of which is identified with topological quarks. Quarks, however, do not carry momentum and do not play the role of particles, even though they carry color, flavor, and electric charge. Quark confinement emerges as a natural consequence of S-matrix self-consistency. The topological quarks have 3 colors, 8 flavors, and integral charges. The reason why quarks appear to have

fractional charges in QCD is readily understood.

Finally, a topological supersymmetry, involving a single dimensionless coupling constant for all elementary hadrons, has emerged. This coupling constant turns out to be closely associated with the fine-structure constant.

To review the formalism and results of the topological bootstrap, we have organized our paper as follows. In Sections II and III, we review the topological theory of strong interactions. These sections are based, principally, on the work of Chew and Poenaru.[11] Section II deals with the topology of the classical surface and is based, besides Ref. 11, on parallel work by Stapp.[12,13] It consists of several parts. In the first part (Sec. II-A), we present the formalism of classical DTU in a form differing slightly from that of Ref. 1, which allows further extensions to include spin, parity, and hadrons other than mesons. These extensions are reviewed in the subsequent parts. The incorporation of spin and parity (Sec. II-B) follows the approach proposed by Stapp[12] but takes into account subsequent modifications by Chew and Finkelstein[14] which provide a more consistent and more economical scheme. The discussion of the topological representation of electric charge, which has been elaborated by a group of authors but principally by Finkelstein,[15,16] requires several sections in our review (II-C, III-D, and V-B). In Section II-C we show how electric charge is represented on the classical surface, and to conclude Section II we review the extension of classical-surface topology to baryons (Sec. II-D).

In Section III, which again consists of several parts, we review the topology of the quantum surface. After presenting the basic triangulation pattern implied by the zero-entropy bootstrap (Sec. III-A), we discuss the resulting set of elementary hadrons, the basic triangles of which these are composed, and the identification of one kind of triangles with quarks (Sec. III-B). In Section III-C we illustrate the topology of the quantum surface with several examples, and in Section III-D we review the topological representation of charge and flavor on the quantum surface. In Section III-E we discuss the concept of topological color, and we conclude Section III with an example of a scattering amplitude for which we have drawn the classical and quantum surfaces in such a way that all topological features discussed in the preceding sections are exhibited (Sec. III-F).

In Section IV we review the calculations of elementary hadron coupling constants in the zero-entropy approximation by Chew, Finkelstein, and Levinson,[20] the notion of topological supersymmetry proposed by several authors,[21,22] and the tentative identification of the zero-entropy coupling constant with the fine-structure constant suggested by Chew.[22] The subsequent Section V gives an outline of the topologcial theory of electroweak interactions developed recently by Chew, Finkelstein, McMurray, and Poenaru,[15-19] with special emphasis on the topological representations of electroweak bosons and leptons.

In Sec. VI we summarize the key results of the topological bootstrap theory by presenting the quantum areas of the complete set of elementary particles, emphasizing once more the striking fact that <u>all</u> particles are composed of two basic topological constituents and reviewing the quantum numbers associated with these constituents. This section is

based on the work of Chew, Finkelstein, Nicolescu, and Poĕnaru[23] who coined the term "topological grand unification" to describe the remarkable regularities of the composite particle picture that has emerged from the topological bootstrap. To conclude our review, in Section VII, we discuss the relationship between the topological bootstrap and the orthodox framework of particle physics, especially that of quantum chromodynamics. We argue that the bootstrap approach is more fundamental and we anticipate that the successful features of QCD will eventually be derived from the topological bootstrap theory.

II. CLASSICAL SURFACE

A. Classical DTU

The topology of classical DTU is that of classical surfaces; two-dimensional, oriented, bounded surfaces housing the graphs of topological amplitudes. The zero-entropy terms of the topological expansion ("planar" terms in the language of classical DTU) are represented by single-vertex Feynman graphs characterized by a particular sequential order of their external lines, each of which carries energy-momentum. For example, the topological expansion of the amplitude for the scattering process depicted in Fig. 1, where we shall assume all four particles to be mesons, contains the zero-entropy terms shown in Fig. 4. The classical surface for each zero-entropy component is a disk and the Feynman graph is embedded in that disk in such a way that the end of each line touches the boundary of the surface (see Fig. 5). The particles are then associated with segments of the boundary dual to the external lines of the graph and separated from each other by demarcations called trivial vertices.

The orientation of the surface, called global orientation and depicted by a circular arrow, indicates that the sequential order of the particles is understood, by convention, to be "clockwise". All statements about orientations in the topological bootstrap theory refer to orientations relative to the global surface orientation, and thus it does not matter from which side of the surface one looks at the graphs embedded in it.

In classical DTU, as formulated originally, the Feynman graph was a redundant feature of the topology and was therefore not exhibited. The diagram shown in Fig. 5, for example, was drawn as a so-called ring diagram, as shown in Fig. 6. In the new theory, the Feynman graph is exhibited explicitly, because it is no longer redundant in the more elaborate topological formalism to be introduced below.

The boundary of the classical surface inherits the surface orientation. By cutting it open at the points where it is touched by the Feynamn graph, we obtain the familiar Harari-Rosner (HR) graph (see Fig. 7). In other words, the HR graph represents the boundary of the classical surface, and hence the global surface orientation is also called HR orientation. Equivalently, the HR arcs may be seen as a "thickening" of the Feynman graph, which defines the classical surface.

When the topological expansion is inserted into the unitarity Eq. (1.1), the right-hand side of the equation consists of a sum over products of topological amplitudes. These products are represented by connected sums of the corresponding Feynman graphs. To form a connected sum of two ordered Feynman graphs, the boundary segments corresponding to intermediate particles are identified and erased, as shown in Fig. 8, and the orientations of the two surfaces are matched, so that the resulting two-vertex graph on the new surface has a coherent orientation. The corresponding HR graphs are shown in Fig. 9.

Higher-order terms in the unitarity equation will involve connected sums of graphs that can no longer be embedded on planes. For example, Fig. 10 shows a connected sum in which the resulting surface has two

distinct boundary components. Particles A and B belong to one boundary component, C and D to the other. The topology of this surface is that of a cylinder or, equivalently, of a sphere with two holes punched out.[24] An even more complicated connected sum is shown in Fig. 11, where the resulting surface is a torus with a single boundary component.[25]

The operation of forming connected sums of classical surfaces has the important property that it can never result in a decrease of surface complexity, a property referred to as "entropy". As a consequence of the entropy property, there is a minimal level of complexity -- the zero-entropy level -- at which zero-entropy amplitudes are built, through unitarity, from zero-entropy components.

The complexity of a surface is determined uniquely by a set of entropy indices. In classical DTU, there are two (non-decreasing) entropy indices, g_1 and g_2. The index g_1 specifies the genus of the surface (a topological parameter equal to twice the number of handles),[26] while g_2 is related to the number of its boundary components by the formula

$$g_2 = b + g_1 - 1. \tag{2.1}$$

The zero-entropy level is uniquely characterized by $g_1 = g_2 = 0$. All higher-order hadron terms are built from zero-entropy components, and the full topological expansion contains components belonging to all possible values of g_1 and g_2. In any connected sum, $\Sigma = \Sigma' \# \Sigma''$, the entropy index g_1 satisfies the "strong-entropy" condition

$$g_1 \geqslant g_1' + g_1'', \tag{2.2}$$

while g_2 satisfies the "weak-entropy" condition

$$g_2 \geqslant \max\,(g_2', g_2'')\,. \qquad (2.3)$$

The fact that every hadron is a bound state of other hadrons, which corresponds to the mathematical statement that each amplitude is built from its singularities, implies a set of topological contraction rules. For mesons, in classical DTU, there are two such rules. Two "parallel" internal Feynman arcs may be contracted to a single arc (see Fig. 12a), and any internal arc connecting two vertices may be shrunk to a point, so that the two vertices become one vertex (see Fig. 12b). These contractions do not change the surface complexity, and thus no change occurs in the entropy indices. The non-contracted and contracted graphs are topological equivalent. Full contraction of a Feynman graph will always lead to a single vertex, even though certain internal arcs may survive. For example, the graph of Fig. 10 can be contracted to the form shown in Fig. 13 in which the two distinct boundaries characteristic of the cylinder topology are clearly visible. In terms of HR graphs, the contraction rules correspond to the well-known duality transformations, as illustrated in Fig. 14. The explanation of the quark structure of mesons, which allows to describe HR arcs as "quark lines", has been one of the main achievements of classical DTU.[27]

B. Spin and parity.

In this section, we shall review the first extension of classical DTU -- the incorporation of spin and parity into the topology of the classical surface. To describe spin and parity, a patchwise orientation of the classical surface is introduced in such a way that adjacent

patches always have opposite orientation. At zero entropy, all patch boundaries are given by Feynman arcs and all combinations of patch orientations occur symmetrically, but at higher orders, patch boundaries not coinciding with Feynman arcs will appear. These patch boundaries are called transition arcs and can be associated with the concept of "topological gluons".[14]

If the patch orientation agrees with the HR orientation, the topology is labeled "ortho", if the two orientations disagree, the label "para" is employed. Stapp[12] has shown how the ortho-para distinction can be associated unambiguously with a two-component spin formalism, in which the spin of each meson is carried by a pair of spin indices. At zero entropy, this amounts to attaching a single 2-valued spin index to each end of every HR arc. Stapp's spin formalism applies consistently to the formation of connected sums at the zero-entropy level, and it retains its consistency for all higher-order terms of the topological expansion.

The parity operation is associated with the reversal of patch-orientation, i.e. it turns ortho components into para components and vice versa. Since all patch orientations occur symmetrically at zero-entropy, and since all higher-order hadron terms are built from zero-entropy components, it follows that parity is a symmetry of the strong interactions. Another strong-interaction symmetry is charge conjugation (the transformation of aprticles into antiparticles), which corresponds to the simultaneous reversal of both patch and HR orientations. These symmetries under C and P are maintained for the strong interactions when hadrons other than mesons are introduced.

The patchwise orientation of the classical surface is such that each quark line is attached unambiguously either to an ortho or to a para patch, for example as shown in Fig. 15. At zero entropy, then, each quark appears in two forms, ortho and para. Accordingly, each meson appears four times at the zero-entropy level, but it turns out that only "ortho plus para" states with intrinsic negative parity survive after a sum over all zero-entropy amplitudes is performed. The resulting spin-parity content of mesons is identical to that of the ground states in the constituent-quark model.

Stapp has also shown how the two-component spin formalism can be translated into the more familiar four-component spin notation, in which each quark line carries, in addition to its two spin indices, a factor $(1 + \gamma_5)$ if it is ortho and $(1 - \gamma_5)$ if it is para. Because of these factors, which are usually associated with handedness in particle physics, the patchwise orientation of the classical surface is said to represent the "chirality" of zero-entropy quark lines.

When connected sums are formed, it may happen that a boundary segment attached to an ortho patch is identified with one attached to a para patch. In such a case, a transition arc will be created which does not coincide with a Feynman arc. In Fig. 16 this has been illustrated by redrawing the connected sum shown in Fig. 8 with a certain patchwise orientation. The resulting graph exhibits a transition arc in the loop, and consistency requirements imply that such loops cannot be contracted. Contraction of Feynman graphs is possible only within individual patches. The number of points where a transition arc touches a Feynman arc constitutes a new entropy index, g_3. Like the entropy index g_1, g_3 satisfies the strong-entropy condition

$$g_3 \geqslant g_3' + g_3''. \tag{2.4}$$

C. Electric charge

In addition to energy, momentum, and spin the classical surface carries one more classical variable, that of electric charge. The full topological representation of electric charge will become apparent only after we have introduced the quantum surface (Sec. III-D) and the topological theory of electromagnetism (Sec. V) but, being a classical variable, some of its features are reflected in classical-surface topology. Electric charge is represented on the classical surface by a set of non-intersecting directed lines, called charge arcs, each of which connects two different boundary segments (associated with different particles) without crossing a Feynman arc. This is illustrated in Fig. 17 by adding four charge arcs to Fig. 5. Charge quantization and charge conservation are guaranteed by associating a charge arc with charge ±1 (zero) when its direction is equal (opposite) to the HR orientation. It will be seen that this rule reproduces the known charge patterns of hadrons and leptons.

D. Baryons

After the initial successes of classical DTU, it soon became apparent that the classical-surface topology described in Secs. II-A and II-B is inadequate for the representation of baryons, and several authors[28] suggested independently that multi-sheeted surfaces might be appropriate. Over the past years, this suggestion gradually evolved into a consistent topology of "feathered" surfaces, a term inherited from an image proposed by Stapp,[28] according to which a baryon could be pictured as a set of three surfaces arranged like the

feathers of an arrow, with each outer edge a quark line and all three inner edges placed in close proximity to a single line called the junction line.

In the present topological theory, a feathered classical surface is a two-dimensional, multi-sheeted, bounded surface with a finite number of junction lines, along which three sheets of the surface meet (see Fig. 18). The reasons for the number three will emerge in Sec. III-A from consistency requirements on the quantum surface. It is associated with the three colors of baryons, but three-feathered surfaces can also accomodate hadrons with more, or with less, than three quarks. We shall see below, in Sec. III-E, that higher-order topological-expansion components may involve bridges between different sheets, which increase the entropy index g_1.

The boundary of the full, multi-sheeted surface is a closed graph, called the belt, consisting of boundary segments (separated by trivial vertices) and junction lines. Consistency requirements on the quantum surface, discussed in Sec. III-B, will imply the existence of three kinds of elementary hadrons: mesons, baryons (together with antibaryons), and baryoniums (composed of two quarks and two antiquarks). On the classical surface these elementary hadrons are represented by definite boundary segments (belt pieces), as shown in Fig. 19. The entire belt is made up from such pieces. For example, the belt corresponding to Fig. 18 is shown in Fig. 20, where it has been divided into segments associated with five hadrons (2 baryons, 2 antibaryons, 1 meson). The Feynman graph has also been included.

Notice that in Fig. 20 the entire Feynman graph lies on one sheet of the feathered classical surface. This is a general feature of the

hadron topology, valid to all orders in the topological expansion. It amounts to picturing baryons as "quark-diquark" structures. At zero entropy only one of the baryon's three quarks interacts with mesons, the other two playing the role of "spectator quarks", but at higher orders more quarks than one may interact. We shall see in Sec. III-E that such components will involve "switches" corresponding to color exchange between quarks, each color being separately conserved but movable from one quark to another. This quark-color switching will be given a smooth topological representation, where the entire Feynman graph still resides on a single sheet of the classical surface.[29]

As in classical DTU, the belt can be opened up to obtain the corresponding HR graph. To do so, we must take into account a topological feature that will become apparent when the quantum surface is introduced: not all segments of the belt can be identified with quark lines (see Sec. III- B). In Fig. 21a those belt segments of Fig. 20 that do not correspond to quarks lines have been marked by dotted circles. Notice that these circles cut the belt at three points, one of them coinciding with the point where it is cut by the Feynman arc. Removing the belt segments marked by the circles, we are left with the HR graph shown in Fig. 21b.

Each sheet of the feathered classical surface exhibits patchwise orientation (associated with chirality) and HR orientation. Attaching HR orientations to the sheets also orients the junction lines, and overall consistency of orientation requires that all three sheets meeting along a junction line must give that junction line the same orientation. (See Fig. 20.)

The patchwise orientation of the feathered surface differs from that of the meson case, where only one sheet is present, in that the

feathered surface contains patches that are not oriented.[14] These are the areas adjacent to junction lines, which have no trivial vertices and are not identified with quark lines. This is illustrated in Fig. 22 with the principal sheet of the surface shown in Fig. 20 (i.e. the sheet containing the Feynman graph).

On the secondary sheets, which do not contain any Feynman arcs, oriented and nonoriented patches are separated by lines that occupy positions analogous to the Feynman arcs but do not carry momentum. These lines are not only necessary for consistent patchwise orientation but also play a crucial role in the "switches" corresponding to color exchange between quarks (see Sec. III-E). Because of their important connection with color, we shall call them color arcs. As an example, Fig. 23 shows a secondary sheet of the feathered surface pictured in Fig. 20, the oriented patch having been chosen arbitrarily as a para patch. The color arcs touch the belt at the end points of quark lines, i.e. at the points marked in Fig. 21a by the dotted circles.

The notion of connected sums is readily extended to feathered classical surfaces. As in classical DTU, the boundary segments corresponding to intermediate particles are identified and erased. Coherent HR orientation of the new surface will be ensured if the pairs of matched belt segments have opposite HR orientations. An example of a connected sum involving two intermediate particles (one baryon and one meson) is shown in Fig. 24, where the loop in the Feynman graph resulting from the connected sum is eliminated by subsequent contraction. The corresponding HR graphs are shown in Fig. 25.

The properties of the feathered classical surface discussed in this section restrict the zero-entropy belt graph to an extremely simple form -- a single-strand "necklace with beads" (see Fig. 26).

III. QUANTUM SURFACE

A. triangulation pattern

The topology of the classical surface (Σ_C) alone is not sufficient to derive the internal quantum numbers of hadrons from self-consistency requirements. To do so, the topological bootstrap theory features an additional surface called the quantum surface (Σ_Q) -- a closed surface intersected by Σ_C in such a way that the line of intersection coincides with the belt. Like Σ_C, Σ_Q carries a number of topological structures, and all points of topological significance along the belt are also points of topological significance on Σ_Q. For strong interactions, Σ_Q is completely covered by polygons, representing particle areas (also called particle disks), which cut the belt into the particle pieces shown in Fig. 20. Like Σ_C, Σ_Q exhibits a global orientation and is also patchwise oriented. We shall see that these orientations are associated with the particle/antiparticle distinction.

At zero entropy, Σ_Q is a shpere, while Σ_C is the multi-sheeted, planar, bounded surface described in the previous section. At each order of the topological expansion (1.2), the index γ corresponds to a surface pair $\Sigma = (\Sigma_Q, \Sigma_C)$, and all higher-entropy strong-interaction components are built up from zero-entropy surface pairs by connected sums in such a way that each connected sum of classical surfaces is accompanied by a connected sum of the corresponding quantum surfaces. At zero entropy, the complete topology of Σ_C may be inferred from Σ_Q, while the converse is not true, so that the zero-entropy bootstrap can be worked out entirely on the quantum surface. On the other hand Σ_C is absolutely necessary at higher orders of the topological expansion,

not only as a carrier of classical variables but also for a precise definition of entropy, because it turns out that the genus of Σ_Q may decrease in certain connected sums.

As in classical DTU, the starting point of the zero-entropy bootstrap is the distinction between amplitude graphs, picturing the whole amplitude, and channel graphs containing only the ingoing or outgoing particles.[30] Accordingly, the quantum sphere is divided into an ingoing channel disk and an outgoing channel disk, and this division has to be carried out for all communicating channels in such a way that the two channel disks uniquely define the amplitude when they are joined along their perimeters. This means that all the topologcial features of their perimeters must match in appropriate ways.

A further key element of the topological hadron bootstrap is a system of mutually consistent contraction rules regarding surface areas on Σ_Q, which turn out to constrain the pattern of internal quantum numbers and result in quark confinement. As on the classical surface, these contraction rules reflect the fact that every hadron is a bound state of other hadrons. Specifically, every channel disk, representing a multi-particle combination, must be uniquely contractible to a single-particle disk that is determined completely by its perimeter. This implies that any zero-entropy quantum surface must be completely contractible to a propagator, represented by a single-particle disk plus the corresponding antiparticle disk, which, because of crossing,[31] corresponds to the "in" and "out" states of the same single-particle channel. An extended search over several years has now resulted in a specific topological pattern for quantum spheres that is consistent with the aforementioned bootstrap requirements and with the zero-entropy

classical surface described in Sec. II. It is not known at present whether this solution is unique, but no satisfactory alternatives have emerged, in spite of many proposed variations. At the same time, the adopted pattern is in agreement with a variety of established experimental facts and is not known to conflict with any.

The pattern to be described involves a specific triangulation of quantum spheres, i.e. the decomposition of Σ_Q into specific oriented triangular patches. The role of the triangle as a basic topological unit is closely connected with the use of two-dimensional surfaces, which, in turn, is related to the use of graphs. Ultimately, it goes back to the description of reality in terms of isolated, causally connected events -- the starting point of S-matrix theory. The basic role of the triangle will also turn out to be connected with "triality", the fact that a baryon is built of three quarks or, equivalently, that quarks have three colors.

To triangulate the quantum sphere appropriately, one divides it first into two triangular patches of opposite orientations, as shown in Fig. 27a. Then, any of the edges is replaced by a pattern called "lunar insertion pattern", or briefly "lune", which has been drawn in Fig. 28 in three equivalent ways. The lunar insertion pattern consists of two triangles of opposite orientations with one trivial vertex (i.e. a vertex at which two edges meet), which has been drawn as a small circle in Fig. 28a. The result of replacing one of the edges in Fig. 27a by a lune is shown in Fig. 27b. The process of inserting lunes can be continued indefinitely and results in a triangulation characterized by the feature that all triangles occur in "mated" pairs of opposite orientations. A pair of mates is uniquely identifiable by

the fact that the two triangles share all three vertices. Thus, the entire quantum sphere is covered with triangles of alternating orientations.

The triangulation of Σ_Q in terms of mated pairs of triangles assures that the contraction rules are satisfied, because the creation of a mated pair of triangles by insertion of a lune is just the opposite of a contraction. In other words, the operation of contraction consists in removing pairs of mated triangles.

Trivial vertices are always shared by a pair of mated triangles in position to be contracted, and since any fully contracted particle disk cannot contain both members of a mated pair, it follows that all trivial vertices lie on the perimeters of particle disks, so that the two members of a mated pair always belong to different particle areas. In fact, the trivial vertices on the quantum surface uniquely define the particle areas and will be seen to correspond to the trivial vertices on the classical surface that divide the belt into particle pieces.

B. Basic triangles, quarks, elementary hadrons.

Once the basic triangulation pattern has been established, one can proceed to identify the possible particle areas on the quantum surface. This yields the surprising result that only three kinds of hadron disks are consistent with the contraction requirements. None of them can be contracted any further and, if combined into multi-hadron channel disks, these will uniquely contract back to one of the three basic forms. Thus, the zero-entropy bootstrap results in a definite and restricted set of elementary hadrons. Although each of these elementary particles is equivalent to a bound state of other elementary particles and none of their properties is fundamental, all of them

being determined by S-matrix consistency requirements, Chew's original idea of a "nuclear democracy"[32] (in which the proton and the deuteron, for example, would have equal status) is invalidated by the topological bootstrap.

The polygons representing the three types of elementary hadrons -- mesons, baryons (together with antibaryons), and baryoniums -- are pictured in Fig. 29. Their belt segments (dotted lines) can readily be identified with those pictured in Fig. 19. Note that the belt crosses particle boundaries always at a trivial vertex, each trivial vertex on Σ_Q coinciding with a trivial vertex on Σ_C. The cross (x) on the belt segment of each hadron marks the point where the Feynman arc representing that hadron on the classical surface touches the belt. Because Feynman arcs carry energy-momentum they are also called momentum lines. We shall see that the precise location of the momentum line within each hadron polygon has several important implications.

It is apparent from Fig. 29 that all hadron disks are composed of two basic triangles, which are pictured in Fig. 30. Each of the two triangles encloses a segment of the belt, and because of the different shapes of the two belt segments we shall call the triangles "I-triangle" and "Y-triangle". I-triangles intersect a single sheet of the classical surface, while Y-triangles intersect three sheets. The extension of the topological bootstrap theory to electroweak interactions in Sec. V will show that also the leptons and electroweak bosons are represented by topological structures composed of the same two triangles. Thus, the topological bootstrap yields the remarkable result that all particles can be represented as composites

of two basic topological constituents.

It is amusing to note that the picutre of matter emerging from the topological bootstrap theory has a strong Platonic flavor. The elementary particles in Plato's Timaeus are represented by regular solids that consist of two kinds of basic triangles. "When the greater bodies are broken up", wrote Plato, "many small bodies will spring up out of them and take their own proper figures; or, again, when many small bodies are dissolved into their triangles, if they become one, they will form one large mass of another kind".

In the hadron polygons, the I-triangles are also called "peripheral" triangles, because they contribute two edges to the particle perimeter, and the Y-triangles, which do not contribute any edges to the particle perimeter, are called "core" triangles. However, we shall see that this terminology loses its meaning in the topological structures representing the vector and scalar bosons.

Comparing the elementary hadron polygons in Fig. 29 with orthodox quark models, one is immediately led to associate the I-triangles with quarks. Indeed, their full topological features, to be discussed below, justify the name "topological quarks" for I-triangles. However, it must be remembered that topological quarks are not particles. A single triangle cannot be identified with a particle, because a channel disk composed of two such particles could not be contracted to a single particle, contraction being defined as the removal of a pair of triangles. Thus, the contraction rules explain quark confinement. Another way of realizing that quarks cannot be particles is to note from Fig. 29 that they do not carry momentum. The I-triangles either share their momentum line with another triangle or do not carry any

momentum line at all. Hence, they cannot represent particles.

The identification of I-triangles with quarks makes it clear that only those sections of the belt that are cut by I-triangles can be called quark lines. The belt sections cut by Y-triangles do not correspond to quark lines. These are the sections marked by dotted circles in Fig. 21a.

All triangles are oriented "clockwise" or "counterclockwise", i.e. they carry an orientation equal or opposite to the global surface orientation. These two orientations are associated with the distinction between particles and antiparticles, so that I-traingles with clockwise orientation represent quarks, those with counterclockwise orientation antiquarks. Because of crossing,[31] the distinction between particles and antiparticles is related to the distinction between "in" and "out" states, ingoing particles being equivalent to outgoing antiparticles. We shall adopt the convention that clockwise I-triangles represent outgoing quarks or ingoing antiquarks, while counterclockwise triangles represent outgoing antiquarks or ingoing quarks.

Stapp's spin formalism, described in Sec. II-B, effectively associates a 2-valued spin index with each end of a quark line. Since quark lines are attached to I-triangles, the Stapp formalism is equivalent to attaching one 2-valued spin index to each I-triangle.[33] Thus mesons, baryons, and baryoniums carry 2, 3, and 4 spin indices, respectively.

The existence of baryonium, composed of two quarks and two antiquarks, as an elementary hadron is a prediction of the topological bootstrap that has not yet been verified. Section IV will show that baryonium particles are expected to have an extremely short lifetime, and thus it is not surprising that they have so far eluded observation.

Hadron states composed of more than four quarks are not allowed in the topological bootstrap. The reason is connected with the asymmetry between the three belt components in the Y-triangles, which is introduced by the position of the momentum line and could not be maintained in hadrons represented by more than two Y-triangles, since each particle has just one momentum line. We shall see in Sec. III-E that this asymmetry is related to color asymmetry and is crucial to the consistent formation of connected sums in the unitarity products of amplitudes. Thus, the restriction of elementary hadrons to the pattern shown in Fig. 29 emerges as a direct consequence of unitarity and S-matrix self-consistency.

Y-triangles have no counterpart in orthodox quark models. In the topological theory they are the source of baryon number, which is defined as the number of counter-clockwise Y-triangles minus that of clockwise Y-triangles. Since all triangles on the quantum sphere occur in mated pairs of opposite orientations, baryon number is automatically conserved. There is a three-to-one ratio between peripheral triangles and core triangles, representing the "triality" relation between quark number and baryon number. Defining N_I as the number of clockwise I-triangles minus counterclockwise I-triangles, with the corresponding definition of N_Y for Y-triangles, these relations take the form

$$B = - N_Y = \frac{1}{3} N_I, \tag{3.1}$$

which can be combined to

$$B = \frac{1}{4} (N_I - N_Y). \tag{3.2}$$

We shall see that relation (3.2) is valid for all particles, while (3.1) holds only for hadrons.

C. Quantum-sphere topologies and connected sums

Any strong-interaction quantum surface is completely covered by a combination of the three elementary hadron polygons, with every triangle being cut by the belt, so that every piece of Σ_Q is in contact with Σ_C. To illustrate this topology with a few simple examples we have redrawn the elementary hadron polygons in the topologically equivalent fashion shown in Fig. 31, which makes it somewhat easier to represent fully covered quantum spheres. Note that all trivial vertices in Fig. 31 have been drawn as small circles and all particle perimeters as heavy lines. Furthermore, we have adopted the notation of shading all triangles with counter-clockwise orientation, while unshaded triangles are understood to have clockwise orientation.

Our first examples of fully triangulated quantum spheres are the three elementary hadron propagators shown in Fig. 32. In each of these examples, one particle disk plus the corresponding antiparticle disk cover the entire sphere. The corresponding HR graphs are also exhibited in Fig. 32, and the basic lunar insertion patterns, as depicted in Fig. 28a are clearly visible.

The following four examples (Fig., 33-36) show quantum spheres representing scattering amplitudes together with the corresponding HR graphs. Figure 33 pictures a four-meson interaction, and Fig. 34 shows a reaction involving a baryon-antibaryon pair and two mesons. Notice that both mesons in Fig. 34 interact with the same quark and thus share their nontrivial vertices with the same peripheral triangles. Figure 35 pictures the quantum sphere corresponding to

Figure 20, and to conclude our examples we have drawn, in Fig. 36, the triangulation for an amplitude involving one baryonium, one baryon-antibaryon pair, and one meson. In all these examples it is apparent that the trivial vertices (drawn as small circles) provide a unique delineation of the particle perimeters (heavy lines). Notice also that the particle boundaries always separate mated pairs of peripheral triangles, each mated pair representing a lunar insertion pattern, and that all vertices (trivial and nontrivial) lie on these boundaries.

Figures 32-36 exhibit another topological feature, the significance of which remains somewhat mysterious at present. Each particle disk has one special edge that carries the momentum line (marked by x in Fig. 29), and at zero entropy all those special edges on the quantum surface share the same two vertices. Thus, two special points are defined on the quantum sphere, which are appropriately called the "north pole" and the "south pole".

The zero-entropy level has the unique property that connected sums of two quantum surfaces result in a new quantum surface of the same complexity. Thus, the zero-entropy bootstrap is confined to the level of quantum spheres. To form a conneced sum of two quantum spheres, the disks representing the intermediate particles are glued together in such a way that their orientations mismatch, and then the two joined disks are erased. The result is a new quantum sphere with unique and coherent topological features. This operation assures that the connected sum of the two quantum spheres, $\Sigma_Q' \# \Sigma_Q''$, is accompanied by the connected sum of the corresponding classical surfaces, $\Sigma_C' \# \Sigma_C''$, as discussed in Sec. II-A.

This procedure is illustrated in Fig. 37 with the connected sum pictured in Fig. 25 in terms of HR graphs. The quantum spheres corresponding to the two intitial graphs are shown in Figs. 37a,b where the particle disks have been labeled according to Fig. 25. To carry out the connected sum of these two quantum spheres, the spheres have to be plugged by gluing together the disks representing particles E and F. To illustrate this operation, which is somewhat awkward on a plane sheet of paper, we shall proceed in two steps. First, we erase the two particle areas E and F in both triangulations, as shown in Figs. 37c,d. The remaining patterns, together, represent the triangulation of the new quantum sphere, which is obtained by fitting Fig. 37d into Fig. 37c. The result, shown in Fig. 37e, corresponds to the connected sum shown in Fig. 25 after contraction of the meson loop.

In addition to the "single-plugged" connected sums discussed above, the products of topological amplitudes in the unitarity equation (1.1) will also involve "multi-plugged" connected sums resulting in quantum surfaces of higher complexity.[34]

D. Electric charge and flavor

The topological theory described so far in this review features several surface orientations. The surfaces Σ_Q and Σ_C are both globally oriented and patchwise oriented. On Σ_Q, the patchwise orientation is associated with the particle/antiparticle distinction, on Σ_C with chirality. The global orientation of Σ_C, also called Harari-Rosner orientation, is coupled to the patchwise orientation of Σ_Q through HR arcs, or "quark lines". These are the sections of the belt cut by I-triangles. They are directed arcs connecting mated pairs of I-triangles, the directions being chosen, by convention, in such a way

that each quark line runs from a counterclockwise triangle (antiquark) toward a clockwise triangle (quark).

There are two additional kinds of orientations that can be exhibited on the two surfaces, and both of them are associated with quantum numbers -- one with electric charge, the other with flavor. Because the triangles on Σ_Q constitute part of the boundary of Σ_C, we may associate a direction <u>into</u> or <u>out of</u> Σ_C with each triangle. This direction will turn out to control electric charge (see Sec. V-B) and is therefore called the charge direction. It is identified with the direction of the charge arcs introduced in Sec. II-C. Each charge arc touches the quantum surface at the center of a triangle and, in strong interactions, always connects a pair of mated triangles.

The charge arcs connecting I-triangles can be directed either way, but the direction of the Y-charge arcs, which run along the junction lines, must be fixed because of the requirement that all topological features representing variable particle properties must reside on the perimeter of the corresponding quantum areas (see Sec. III-A). The direction of the Y-charge arcs may be fixed arbitrarily, either equal or opposite that of the adjacent junction lines. Whichever way it is chosen, it will turn out to correspond to charged Y-triangles. It has been chosen, by convention, to coincide with the direction of the adjacent junction lines, and thus with the global orientation of Σ_C, as illustrated in Fig. 38 with the principal sheet of the topology of Fig. 36.

In Fig. 39 we have completed the topology of Fig. 38 by drawing the full featherered surface and by exhibiting all charge arcs. Note that Y-charge arcs touch the junction lines at their end points and that I-charge arcs begin and end next to a trivial vertex. The directions of the Y-charge arcs are fixed by the global orientation of Σ_C, but the I-charge arcs can be directed either way (arrows on I-charge arcs have been omitted in Fig. 39 for the sake of clarity). The topological representation of electromagnetism in Sec. V-B will make it apparent that the charges of I-triangles (quarks) can be identified by comparing their orientations with the directions of their charge arcs. The results are shown in Fig. 40, where "in" and "out" stand for charge dicrections into and out of Σ_C, respectively. Since each charge arc is accompanied by a HR arc running from a counterclockwise triangle to a clockwise triangle, Fig. 40 can be summarized by saying that an I-triangle (quark) will be charged if the charge direction agrees with the HR direction and neutral if the two directions are opposite. In this scheme, quarks have integral charges, but the apparent fractional charges in orthodox quark models are readily understood when the Y-triangles are taken into account.

The direction of the Y-charge arcs is fixed and equal to the HR direction, which means that Y-triangles are always charged. The Y-triangle in a baryon has counterclockwise orientation and hence charge -1, while the Y-triangle in an antibaryon has charge +1. Defining N_c as the number of charged clockwise triangles minus charged counterclockwise triangles, with corresponding definitions of N_{Ic}, N_{Io} and N_Y for charged I-triangles, neutral I-triangles, and Y-triangles, respectively, the total electric charge of a hadron disk can be written.

$$Q \equiv N_c = N_{Ic} + N_Y. \tag{3.3}$$

In strong interactions, N_{Ic} and N_Y are separately conserved, but these conservation laws do not extend to electroweak interactions. However, N_c turns out to be conserved in all interactions; thus charge conservation is guaranteed. Because of (3.1), the relation (3.3) can also be written

$$\begin{aligned} Q &= N_{Ic} - B \\ &= 2/3\ N_{Ic} - 1/3\ N_{Io}, \end{aligned} \tag{3.4}$$

which makes it evident that quarks appear to have charges 2/3 and - 1/3 if the Y-triangles are ignored, as they are in orthodox quark models. By including charged Y-triangles the topological theory produces the observed hadron charges with integrally charged quarks.

Flavor is represented on the quantum surface by giving directions to the edges making up the perimeters of particle disks. These edges

all belong to I-triangles represneting quarks, and each I-triangle can accomodate four "edge flavors", as shown in Fig. 41. Notice that the third edge in each triangle (the one cut by the belt) is not oriented because it does not lie on the particle-disk perimeter. This means the Y-triangles do not carry flavor. Since each peripheral triangle on Σ_Q is placed against its mate, the edge flavors automatically match, and such flavor matching has to occur also in all connected sums.

For each value of edge flavor the quark may be either charged or neutral, and hence the theory predicts a total number of 8 quark flavors, appearing in 4 charge doublets with charges (1, 0). The edge flavors, usually called "generations", will be separately conserved on any strong-interaction quantum surface and, because of charge conservation, the quark flavors will be conserved as well. The identification of the different topological flavors with the observed quark flavors[35] -- u, d, s, c, etc. -- remains to be worked out.

E. Topological color

A baryon disk may appear on the quantum surface as any one of 6 distinct permutations of three I-triangles around one Y-triangle, each I-triangle (quark) carrying definite values of flavor and spin. Since the specific permutation is important in the formation of connected sums, one is led to labeling the three positions of the quarks in a baryon. This results in the concept of topological color, with each quark carrying one of 3 distinct colors. The position of the momentum line singles out one of the three I-triangles, which, by convention, is given color #1. Colors #2 and #3 are then assigned in clockwise order, following the orientation of the quark triangles, as shown in Fig. 42a.

Topological color may also be introduced on the classical surface by assigning a distinct color to each of the three sheets making up a baryon, and the three quark lines may be labeled as shown in Fig. 42b, where the momentum line has been drawn as a heavy line. According to the convention adopted in Fig. 42a, the single quark lying on the principal sheet of Σ_C carries color #1, while the "diquark" (see Sect. II-D) carries colors #2 and #3. Mesons carry only color #1, baryoniums only colors #2 and #3. Hency, baryonium does not couple to mesons at zero entropy, although baryonium-meson transitions will be present at higher orders. These higher-order topologies involve the color switches described below.

Topological color is a physically inaccessible degree of freedom, since one has to sum over all color permutations in the topological expansion when constructing physical amplitudes. This means that, when connected sums are formed, any color permutation representing a hadron must be joinable ("pluggable") to any other permutation representing the same hadron. Plugs between different permutations, involving color exchanges between quarks, are called "switches". Although colors move from one quark to the other in these switches, each topological color is separately conserved.

The precise rules for quark-color switching have been given by Finkelstein.[36] In a baryon-baryon plug, the two Y-triangles are plugged first so as to match the ends of the momentum arcs, and then the quark triangles are plugged so as to match flavors. In doing so, two kinds of switches, corresponding to "even" and "odd" quark permutations, may occur. These two cases are illustrated in Fig. 43 with "in" and "out" baryon disks containing quarks with flavors u, d,

and s. In Fig.43a, the two baryon disks differ by an "even" (cyclic) permutation of their quarks, and thus the connected sum will result in a cyclic color permutation. In Fig. 43b, the two baryon disks differ by an "odd" quark permutation, corresponding to an interchange of colors #2 and #3. Such an odd permutation may also arise in baryonium.

All switch plugs are unique and may be presented as combinations of the two cases illustrated in Fig. 43. Any odd-permutation switch increases g_1 (the genus of the classical surface) by one unit, while any even-permutation switch increases g_1 by two units. These increases in classical-surface complexity arise because switch plugs create bridges between sheets that start out with distinct colors at zero entropy. In order to keep track of color as it moves from one quark to another, Finkelstein introduced color arcs (originally called "momentum-copy arcs") as a new topological feature of the secondary sheets of Σ_C, as shown in Fig. 23. Color arcs are always associated with diquarks and carry the colors #2 and #3.

F. Summary of topological features in strong interactions

To conclude our presentation of strong-interaction topology we have drawn, in Fig. 44, the classical and quantum srufaces representing a four-point amplitude in such a way that all the topological features discussed in the preceding sections (except edge flavor) are exhibited. The corresponding HR graph is shown in Fig. 45. We have chosen an amplitude describing the scattering process $p + \pi^- \to n + \pi^0$ and have attached the particle labels to the momentum arcs in Fig. 44a and to the particle disks in Fig. 44b. Note that ingoing particles have been labeled as outgoing antiparticles, according to the convention

adopted in Sec. III-D.

As in Figs. 32-36, triangles with counterclockwise orientation have been shaded in Fig. 44b. Edge flavors have not been included because their association with the observed flavor generations has not yet been established. In our example, the quark lines are identified simply by their electric charges, the d quark being neutral and the u quark carrying charge +1. The patchwise orientation in Fig. 44a has been assigned arbitrarily with the understanding that amplitudes with all possible patch orientations have to be summed in order to obtain the correct spin dependence of the physical amplitude. Finally, we notice from Fig. 44b that all points where the belt crosses a line in the triangulation of Σ_Q are points of topological significance -- trivial vertices, momentum arcs, or color arcs.

IV. ZERO-ENTROPY BOOTSTRAP, TOPOLOGICAL SUPERSYMMETRY, AND THE FINE-STRUCTURE CONSTANT

In the topological theory of hadrons the nonlinear bootstrap problem is confined to the lowest level of complexity, the zero-entropy level. The spin formalism developed by Stapp (see Sec. II-B) implies that spin dependence at zero entropy factors away completely from momentum dependence. Since flavor and color dependencies also factor away at this level, the bootstrap problem is vastly simplified, being reduced to the calculation of planar discontinuities of spinless, flavorless connected parts.

In other words, the zero-entropy amplitudes depend only on the particle momenta, associated with Feynman graphs, and are independent of the quark lines. For example, the three amplitudes represented in Fig. 46 are described by a single momentum function (M-function) independent of the quark spins and flavors. This effectively places bosons and fermions into a single supermultiplet and is therefore called topological supersymmetry.[21,22]

At zero entropy, then, all elementary mesons, baryons, and baryoniums share a single mass m_o and couple to each other through a single, dimensionless coupling constant g_o. Any physical coupling constant is determined, in the zero-entropy approximation, by counting how many different zero-entropy topologies are associated with that particular vertex, and then multiplying, with appropriate Clebsch-Gordan coefficients, by g_o. Assuming that coupling-constant corrections from higher-order terms of the topological expansion will be small even though mass shifts will be large, which remains

to be confirmed, Chew, Finkelstein and Levison [20] have calculated elementary three-hadron coupling constants in the zero-entropy approximation and have found $SU(6)_w$ ratios to emerge. Moreover, their calculations predict enormous baryonium coupling constants and corresponding large widths, which explains the experimental failure to find narrow baryonium states.

The value of the most accurately measured hadronic coupling constant, $(g^2_{N\bar{N}}/4\pi) = 14.3$, implies that g_o must be a small number, close to the fine-structure constant.[20] The surprising smallness of g_o is a consequence of quark multiplicities within closed hadron loops. At zero entropy, any closed loop may carry either one or two quark lines, and each quark appears in 32 varieties (2 spins, 2 chiralities, and 8 flavors). This results in an effective multiplicity

$$f = (-32) + (-32)^2 = 32 \times 31 \approx 10^3 \tag{4.1}$$

for each closed hadron loop (a minus sign being required for each closed fermion loop, as in standard Feynman theory), which leads to a "renormalized" coupling constant

$$g_R = f^{1/2} g_o. \tag{4.2}$$

The quark multiplicities also imply that, when all spins and flavors are counted, there are 256 elementary mesons, 816 elementary baryons, and 18496 elementary baryoniums.

The exact value of g_o is expected to emerge from the solution of the zero-entropy bootstrap equations which, although greatly simplified by the factorization of spin, flavor, and color, are still nonlinear and have not yet been solved. The approximate calculations of Chew et al.,[20] indicate that $g_o \approx e$ to within 6%. This result has led Chew[22] to conjecture that S-matrix unitarity will require $g_o = e$. Although this conjecture had to be modified in view of the discovery of the "naked cylinder" topology (see Sec. V-A), there remains the exciting possibility of calculating the value of the fine-structure constant from the zero-entropy hadron bootstrap.

V. ELECTROWEAK INTERACTIONS

The extension of the topological bootstrap theory to electroweak interactions is relatively new and on much less firm a basis than the theory of hadrons reviewed in the previous sections. Whereas the hadron theory is a true bootstrap theory, derived from S-matrix self-consistency, the topological features representing electroweak particles and their interactions have been motivated by certain attractive features of Lagrangian field theory and, so far, are supported only in part by consistency considerations. Nevertheless, it is expected that electroweak interactions, too, will be "bootstrapped" eventually, so that their raison d'être and all their characteristics -- zero photon mass, left-handed currents, the value of the fine-structure constant, etc. -- will be understood from S-matrix self-consistency. This expectation is encouraged by the observation that almost all the ingredients generated by the hadron bootstrap can be adapted to describe electroweak topologies motivated by Lagrangian field theory.[19]

In view of the fact that the topological theory of electroweak interactions is still in a state of flux and does not yet rest on a bootstrap basis, we shall limit ourselves, in the following sections, to describing its broad outlines without discussing the full set of known consistency requirements.

A. Electroweak bosons

The topological bootstrap theory explains most of the essential features of hadrons and strong interactions as necessary consequences of S-matrix self-consistency. In view of this success of the bootstrap approach, the question naturally arises: why should there be anything

in nature beyond strong interactions? In other words, does the existence of electroweak bosons, leptons, and of electroweak interactions also follow from the requirement of a self-consistent S matrix? This question has not yet been answered for leptons, but the electroweak bosons and their interactions are indeed generated by the topological bootstrap.

The topological object representing electroweak particles appears at a level of complexity close to zero entropy. As described in Secs. II-A and II-B, each topology γ in the topological expansion (1.2) is characterized by three entropy indices: g_1 (the genus of Σ_c), g_2 (counting the number of boundary components), and g_3 (counting the number of "topological gluons"). Poles in the S-matrix amplitudes, corresponding to elementary particles, are generated by nonlinear discontinuity formulas represented graphically by connected sums in which patterns of complexity reproduce themselves (see Sec. I).

While classical DTU was being developed, the thinking about self-reproducing patterns of complexity and their associated S-matrix poles was rather fuzzy, but after spin had been incorporated into the topological theory, which led to the entropy index g_3, it became clear that there are exactly two levels that can reproduce themselves. One is the level characterized by $g_1 = g_2 = g_3 = 0$, representing the zero-entropy topologies that generate all elementary hadrons. The other level is called the "naked cylinder", C_o, and is characterized by $g_1 = g_3 = 0$, $g_2 = 1$.[18] Even though one of its entropy indices is non-zero, the C_o topology reproduces itself because g_2 satisfies only the weak entropy condition (2.3). All higher-order topologies will satisfy linear equations that do not generate any new poles. In particular, the cylinder topology of classical DTU[9] is obtained from

C_o by taking an infinite sum over g_3, i.e. by "dressing" the "naked" cylinder with topological gluons.

Careful investigation of the C_o pole has shown that it has vacuum quantum numbers, $J^P = 0^+$, and that it is a totally symmetric singlet in all topological degrees of freedom, coupling equally to all hadrons.[18] Such a state cannot be identified with any of the elementary hadrons and has to be given a new representation on the quantum sphere. A natural choice for representing C_o on Σ_Q is a closed surface, which has no perimeter and couples to the rest of Σ_Q with vacuum quantum numbers. The first choice for such a closed surface was a sphere composed of two triangles,[15] but this was later found to lead to inconsistencies. The correct representation of C_o, which emerged eventually from the large number of consistency conditions that this new topology must satisfy,[23] is depicted in Fig. 47. It consists of two I-triangles of opposite orientations whose edges a-a and b-b have to be "glued together", i.e. mathematically identified, in such a way that the arrows match. The topology of this structure is that of a Klein bottle. It is a globally nonorientable surface that cannot be constructed in 3-dimensional space but nevertheless has well-defined topological properties. It is a closed surface, and yet has no "inside" but is one-sided, like a Möbius band. In fact, the Klein bottle can be represented by two Möbius bands glued together along their edges.

When all the spin and charge orientations of the I-triangles are taken into account, Fig. 47 is seen to represent a quartet of vector bosons plus a quartet of scalar bosons, both decomposable into an isosinglet plus an isotriplet. Several models have predicted a

mass lower than the zero-entropy hadron mass and a coupling constant approximately equal to g_o for the C_o state.[18] Since the zero-entropy bootstrap suggests $g_o \approx e$,[22] and since zero mass for the C_o ground state is a possibility compatible with model estimates, it is natural to identify the two quartets with the electroweak vector and scalar bosons. This places the topological theory strikingly close to the starting point of the standard Weinberg-Salam theory of electroweak interactions.[37] A mechanism analogous to the Higgs mechanism is expected to emerge at higher levels of the topological expansion, leading by the usual route to the massless photon, 3 massive vector bosons ($W^{\pm}$, Z^o), and one residual scalar boson. The latter particle, known as the Higgs boson, may be regarded as the physical representative of the naked-cylinder ground state.

B. Minimal electroweak vertices

On the classical surface, the photon is associated with a cylinder topology, i.e. a sphere with two holes. This is illustrated in Fig. 48 with a two-meson-photon vertex, where the photon Feynman arc has been drawn as a wavy line. The corresponding quantum surface has two separate components and can be pictured as two concentric closed surfaces, the outer one being a sphere and the inner a Klein bottle. Accordingly, the outer boundary component in Fig. 48 belongs entirely to the hadron pair, while the inner belongs to the photon.

The topology of Fig. 48 is similar to that of the meson cylinder shown in Fig. 13, but there is an important difference. Strong-interaction cylinders always contain a closed Feynman loop which separates the two boundary components. Because of the absence of this loop in Fig. 48, the HR arcs are not sufficient to define a

thickened Feynman graph, which is necessary for the topological representation of chirality (see Sec. II-B). They have to be supplemented by appropriately oriented charge arcs when photons are present in order to assure an unambiguous global surface orientation. Figure 49(a) shows the topology of Fig. 48, supplemented by charge arcs, and Fig. 49(b) shows the corresponding thickened Feynman graph.

The charge arcs exhibited in Fig. 49(a) are of two kinds. Those into which the photon boundary is "inserted" must be directed in such a way that their orientation agrees with the HR orientation, while the other two charge arcs may have either direction. In other words, a photon can be coupled to a charge arc if and only if its orientation is the same as the HR orientation, and hence it is justified to associate this charge-arc direction with electric charge. Charge arcs coupling to photons are called "active", those connecting mated triangles are called "passive".

The amplitude pictured in Fig. 49 is an example of a class of amplitudes called "minimal electroweak vertices",[17] describing the interaction of an electroweak vector or scalar boson with a pair of elementary particles. The coupling constant associated with these amplitudes is expected to be identified with e, the elementary electric charge. If the two particles are leptons and the electroweak boson is a photon, the minimal electroweak vertex represents the basic Feynman vertex of quantum electrodynamics. A minimal vertex cannot be built from other components in the topological expansion through connected sums; it is the basic unit of electroweak interactions from which all higher-order electroweak topologies are built. This means that the contraction rules characteristic of strong-interaction topologies

(see Sec. II-A) do not hold for electroweak topologies, so that intermediate electroweak particles never disappear from the topology. As a consequence, all graphs manifesting electroweak topologies may immediately be associated with Feynman amplitude graphs, and a precise correspondence between the non-hadronic components of the topological expansion and standard quantum electrodynamics is expected to emerge.[16]

C. Leptons

The striking new feature exhibited by the topology of electroweak bosons is the nonorientable quantum surface. This new element also appears in the topological representation of leptons, so that global orientability of Σ_Q emerges as a special property of strong interactions. The quantum area for leptons, which resulted from a long process of trial and error,[19] is depicted in Fig. 50. It consists of an I-triangle and a Y-triangle whose edges c-c have to be identified so that the arrows match (see Fig. 50(a)). The resulting topological object is a Möbius band, as shown in Fig. 50(b), where identification of the edges d-d in the sense of the arrows is understood. The lepton Möbius band contains two vertices. One of them (labeled 1) results from the contraction of the 3 vertices of the Y-triangle in the process of identifying the edges c-c, the other (labeled 2) is the trivial vertex of the I-triangle. The two edges of the I-triangle adjacent to the trivial vertex make up the edge of the Möbius band, while the third edge, which carries the end of the momentum line ("x"), appears on its surface as a closed loop.

The identification of edges in the quantum areas of electroweak particles restricts the degrees of freedom available to these particles,

and it is this effect that makes electroweak interactions weak. In other words, strong interactions are strong because the perimeters of hadron disks can carry many quantum numbers. At this stage, then, the topological bootstrap theory offers a qualitative understanding of relative interaction strength as a function of complexity, the nonorientable surfaces of electroweak particles exhibiting higher complexity than the orientable disks of strongly interacting particles.

All lepton quantum numbers are carried by the I-triangle, which may be charged or neutral, while the Y triangle is always neutral. The peripheral I-triangle plays a role very similar to that of the I-triangles in baryon disks. In fact, from the topological point of view, we are justified in saying that leptons, like baryons, consist of quarks (I-triangles) plus Y-triangles. However, there are several important differences between the lepton and baryon topologies. The I-triangle in the lepton quantum area is oriented counterclockwise following the conventional definition of leptons as carrying negative charge, while quarks carry positive charge. Consistency considerations require the Y-triangle to be oriented counterclockwise as well, so that the quantum area of a lepton is built from two triangles of the same orientation.

VI. TOPOLOGICAL GRAND UNIFICATION

Perhaps the most remarkable and aesthetically beautiful result of the topological bootstrap theory is the fact that all elementary particles are represented as composites of two basic topological constituents, the I-triangle and the Y-triangle. These constituents do not carry energy-momentum and cannot be interpreted as particles, but they do carry charge, spin, and flavor, and their orientation distinguishes triangles from "antitriangles". As far as quantum numbers are concerned, therefore, all particles are composed of I- and Y-constituents.

The constituent picture of the complete set of elementary particles is shown in Fig. 51. The shapes of the hadron disks ($I\bar{I}$, $III\bar{Y}$, or $II\bar{Y}Y\bar{I}\bar{I}$) derive from the hadron bootstrap; the quantum area of the electroweak bosons ($I\bar{I}$) is suggested by consistency requirements and model estimates, and the lepton quantum area ($\bar{I}\bar{Y}$) has been obtained partly through comparison with Lagrangian field theory. Given the topologies of leptons and electroweak bosons, it is natural to expect that a $Y\bar{Y}$ Klein bottle will also represent an elementary particle. This topology has been included in Fig. 51, although it is not at present required by S-matrix self-consistency. The corresponding particle, labeled "H boson" would be a neutral scalar and may be related to the Higgs boson. Which of the neutral scalars ($I\bar{I}$ or $Y\bar{Y}$) can actually be identified with the Higgs particle is not clear at this stage.

In spite of the different derivations of the composite particle representations, the resulting picture exhibits a strikingly simple unified pattern, which has appropriately been named "Topological

Grand Unification" (TGU).[23] The entire set of elementary particles falls into two broad groups -- hadrons and electroweak particles. The former are represented by orientable disks, the latter by non-orientable surfaces. All particle quantum areas can be seen as consisting of two pieces, sewn together along the single special edge that carries the end of the momentum line (marked by x in Fig. 51). The two triangles adjacent to this special edge are of the same kinds for the three types of hadrons and the three types of electroweak particles. Thus TGU implies a precise parallelism between baryons and leptons, mesons and electroweak bosons, and between baryoniums and H bosons.

The picture of elementary particles emerging from TGU alters the usual relationship between quarks and particles. In the orthodox theory, quarks and leptons are on the same footing, representing two types of basic structureless building blocks. In the topological theory, the I- and Y-constituents are the basic building blocks but neither of them is a physical particle, and <u>all</u> elementary particles are composite structures. Topological quarks may be identified with the peripheral I-triangles appearing in hadrons <u>and</u> leptons. In vector bosons, I-triangles are Möbius bands, while Y-triangles are uniformly disks in hadrons and Möbius bands in electroweak particles. The precise definition of universal constituents, together with the proper evaluation of higher-order terms in the topological expansion is expected to yield a rich array of phenomenological predictions.

We shall now review the quantum numbers carried by the I- and Y-constituents. As described in Sec. III-D, each triangle is a charge doublet, being touched by a charge arc at its center and carrying charge ±1 or 0 according to the relative orientation of charge arc and triangle. For Y-triangles, this orientation must be fixed in such a way that hadron Y-triangles are charged, while electroweak Y-triangels are neutral. Electric charge is conserved on any quantum surface because the total number of triangles equals the total number of antitriangles.

Baryon number, B, and lepton number, L, are given by the relations[23]

$$B = 1/4\ (N_I - N_Y) \tag{6.1}$$

$$L = -\ 1/4\ (N_I + 3N_Y), \tag{6.2}$$

where N_I is the number of I triangles minus $\bar{I}$-antitriangles, and N_Y the number of Y-triangles minus $\bar{Y}$-antitriangles. It follows from (6.1) and (6.2) that N, the total number of triangles minus antitriangles, which is always conserved, is given by

$$N = 2\ (B - L). \tag{6.3}$$

The quantities N_I and N_Y, and thus B and L, are individually conserved for those components of the topological expansion where every triangle is mated to a triangle of opposite orientation, in the sense discussed in Sec. III-A. This rule is broken by a mechanism proposed by Chew and Poénaru[19] for a possible accomodation of lepton-baryon mixing, which involves the mating of triangles of the same orientation.

Flavor is carried by all peripheral I-triangles (quarks) and is defined as the combination of electric charge with a generation index, represented by the orientation of two triangle edges (see Sec. III-D). Y-triangles and non-peripheral I-triangles (in electroweak bosons) are always flavorless.

The spin content of hadrons resides in their I-triangles, which inherit a 2-valued spin index, corresponding to spin 1/2, from the HR arcs (see Sec. III-B). A recently developed topology of electroweak currents[17] implies that I-triangles in electroweak particles, too, are associated with spin 1/2, while all Y-triangles are spinless.

The quantum numbers associated with the I- and Y-constituents are summarized in Fig. 52, where S, Q, B, and L denote spin, charge, baryon number, and lepton number, respectively, and $Y_{st}(Y_{ew})$ denote Y-constituents in strongly interacting (electroweak) particles. All these quantum numbers, together with edge flavor and energy-momentum (carried by the Feynman graph in Σ_c), represent degrees of freedom that are accessible to experiment, either directly or indirectly. In addition, the topological theory also features inaccessible degrees of freedom, expressed in terms of entropy indices, which measure complexity. In the topological expansion (1.2) the sum runs over all inaccessible degrees of freedom. They include the cyclical ordering of Feynman arcs (affecting the entropy indices g_1 and g_2), the patch orientation of Σ_c (determining chirality and the entropy index g_3), and topological color (affecting classical-surface complexity, and thus g_1, through quark-color switches).

The notion of inaccessible degrees of freedom and the associated notion of complexity are crucial to the topological bootstrap theory, and it is this aspect of the theory that "orthodox" physicists find most difficult to appreciate. However, as Chew has pointed out,[3] one should remember that quantum mechanics, too, depends crucially on a mathematical feature that is inaccessible to experiment -- the phase of the complex vector ψ in Hilbert space. Thus the inaccessible topological features of the bootstrap theory of particles may be seen as an extension of the complex numbers of quantum mechanics to a broader domain of mathematical structures.

VII. RELATION TO ORTHODOX PARTICLE PHYSICS

Most physicists today are still unaware of the recent developments in S-matrix theory and believe that the bootstrap idea, although philosophically attractive, could not be cast into a proper scientific theory of subatomic particles. The purpose of this review has been to dispel this belief by showing that a comprehensive bootstrap theory of particles is now being developed. The topological bootstrap can account for a wide variety of established experimental facts, is not known to conflict with any, and has illuminated a number of questions not previously understood. However, it is formulated within a framework that is quite uncommon and, perhaps, uncomfortable for most particle physicists who are accustomed to dealing with continuous space-time, local fields, and Lagrangians exhibiting continuous symmetries. The topological bootstrap theory contains none of these features. To conclude this review, it will therefore be useful to discuss the relationship between the topological theory and the orthodox framework.

According to Chew,[3] continuous space-time, as we perceive it through our senses, together with the classical objects embedded in it, are approximations that acquire validity through event patterns of high complexity, not unlike the continuous thermodynamic notion of temperature. The properties of continuous space-time are expected to emerge, eventually, from the topological expansion, and with them an understanding of gravitation as an aspect of space-time. Gravitons are not expected to appear in such a framework. The present state of the topological theory, dealing with zero entropy and a few levels of low complexity, is very far removed from the high-complexity level of gravitation and classical

physics. A high degree of complexity, of course, can end up averaging out in such a way that it produces effective simplicity. This effect makes classical physics possible.

Electromagnetism seems to play a crucial role in the relation between the quantum world of isolated events and the classical world of continuous space-time.[3] Electromagnetic events have the unique characteristic that they may involve an arbitrary number of emissions and absorptions of soft photons. Successions of such "gentle" events will build up the continuous trajectories associated with classical objects. From the topological point of view, any photon event, no matter how soft, is noncontractible and will increase the overall complexity. Hence large numbers of soft photons, characteristic of classical reality, will be associated with high topological complexity.

Like classical physics, quantum mechanics was formulated within a continuous space-time in spite of its emphasis on discrete phenomena, and Chew believes that this contrast lies at the root of the celebrated paradoxes surrounding quantum mechanics.[3] Quantum field theory, from this point of view, seems to represent a semi-classical domain. Fields can be defined only near the classical region, at high complexity, because only there can local space-time be given a meaning. Thus quantum field theory appears as a perturbative correction to the classical limit.

When S-matrix theory and quantum chromodynamics are put in this perspective, they seem to be complementary approaches that do not overlap for the time being. The topological bootstrap works well in the domain of small transverse momentum, where it can be used to derive the quark structure of hadrons and their quantum numbers, while QCD

works well only at large transverse momentum where, for reasons not well understood, particles behave somewhat like classical objects.

Because of the lack of overlap between the two theories it is difficult, at present, to arrange an experimental confrontation.[11] However, we do believe that the bootstrap approach is more fundamental than QCD in the same sense that quantum mechanics is more fundamental than classical mechanics. Thus it is anticipated that the physical content of perturbative QCD will eventually be derived, together with local space-time, in the high-complexity limit of the topological bootstrap theory.

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FIGURE CAPTIONS

1. Diagram picturing a scattering process involving four particles.
2. Examples of singularities for a reaction involving six particles.
3. Graphic representation of the unitarity equation for a $2 \to 2$ amplitude.
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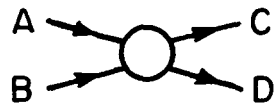
50. The lepton Möbius band.
51. Quantum areas for the complete set of elementary particles.
52. Quantum numbers of I- and Y-constituents.



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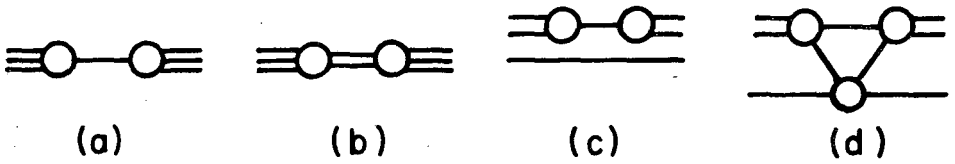

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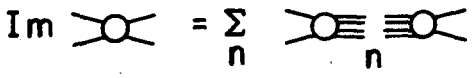

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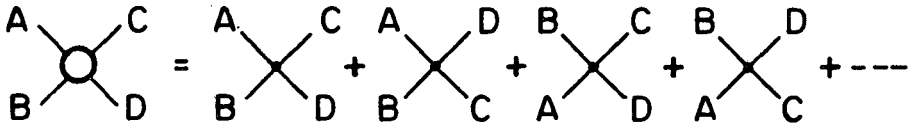

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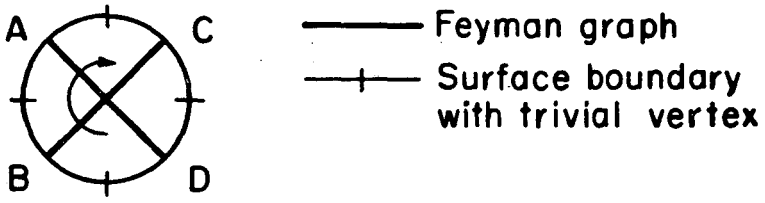

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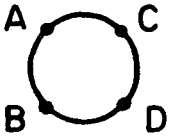

Fig. 6

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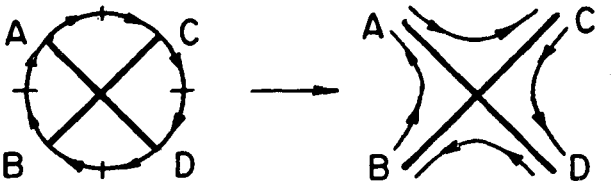
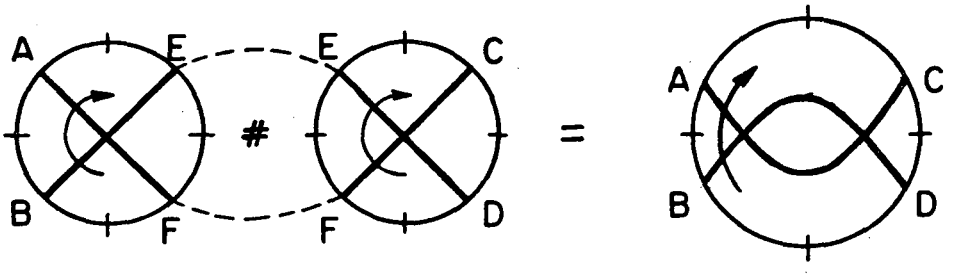


Fig.7



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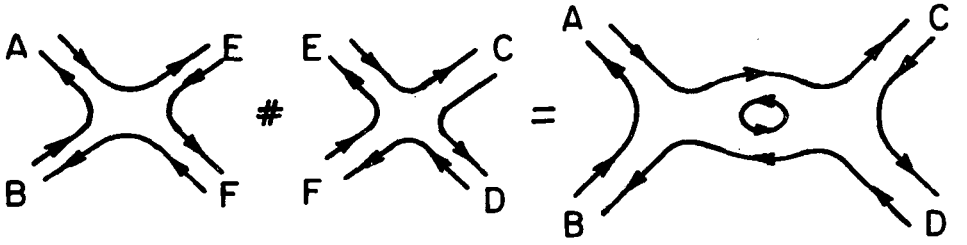

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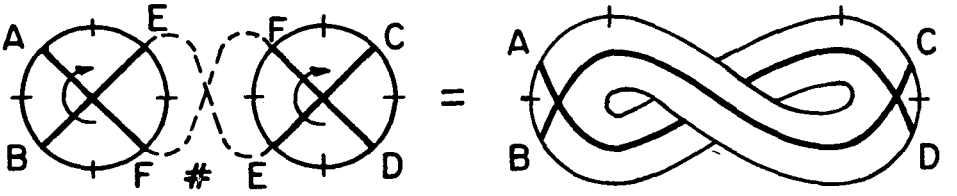

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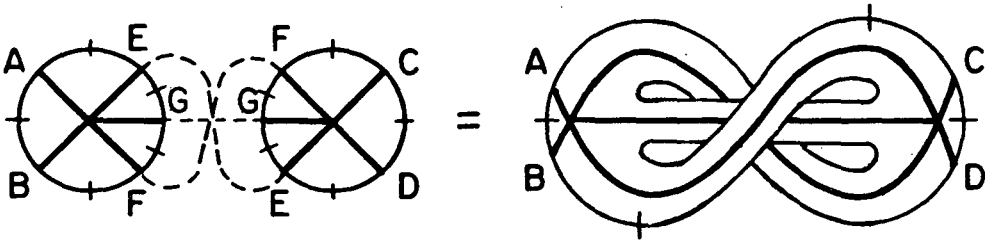

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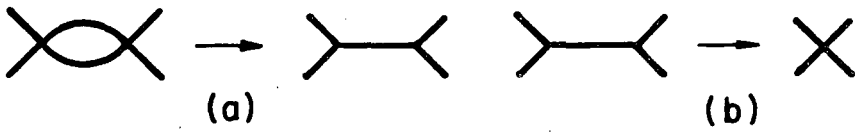


Fig. 12

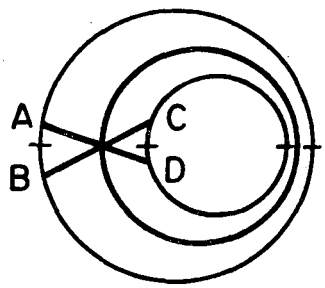
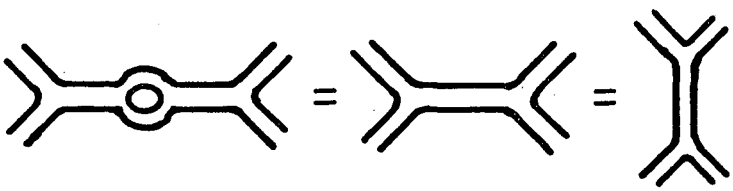


Fig. 13



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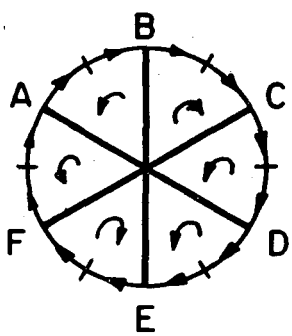

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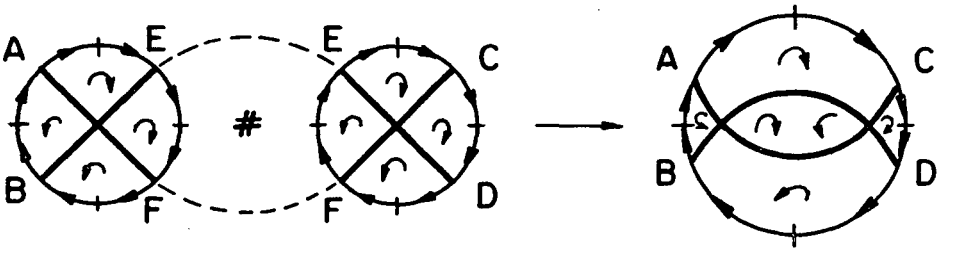

Fig.16

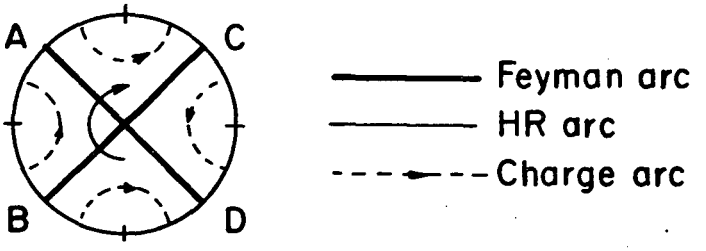

Fig.17

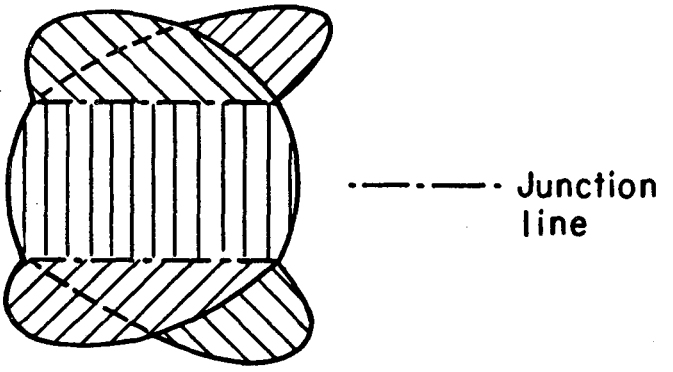

Fig. 18

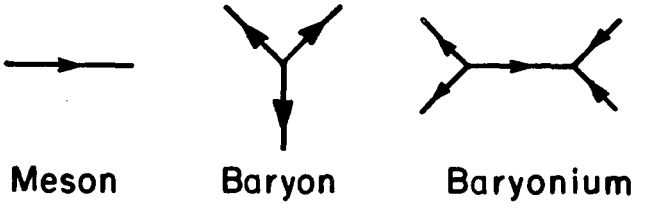

Fig.19

XBL829-4095

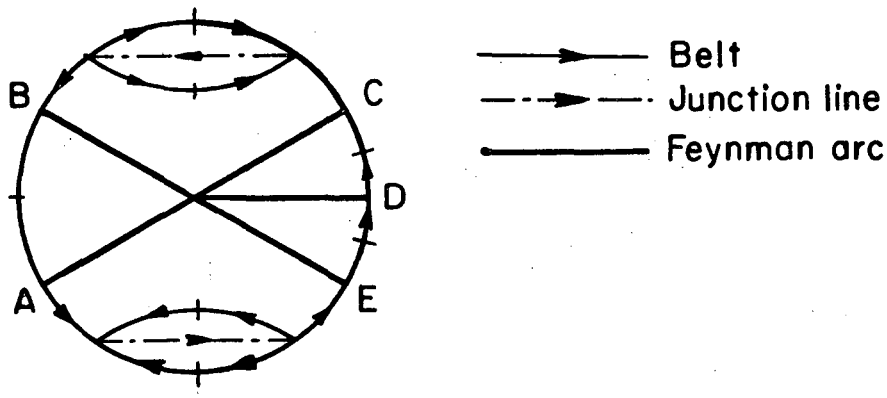

Fig.20

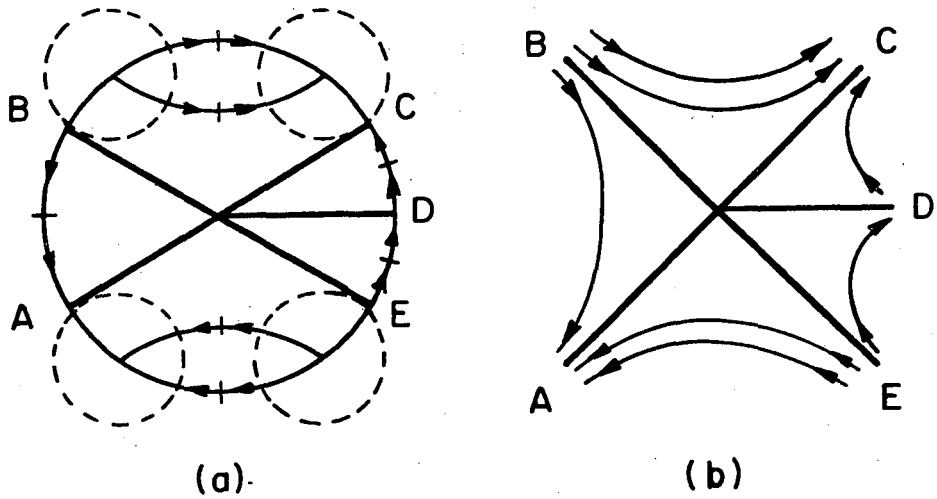

Fig.21

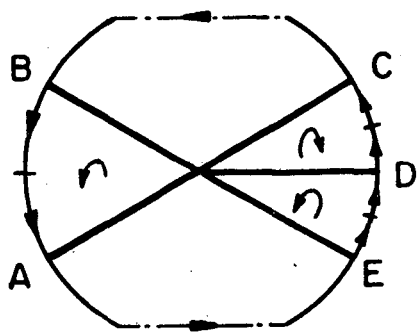

Fig. 22

XBL829-4096

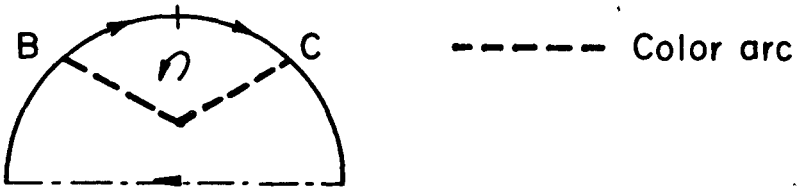

Fig. 23

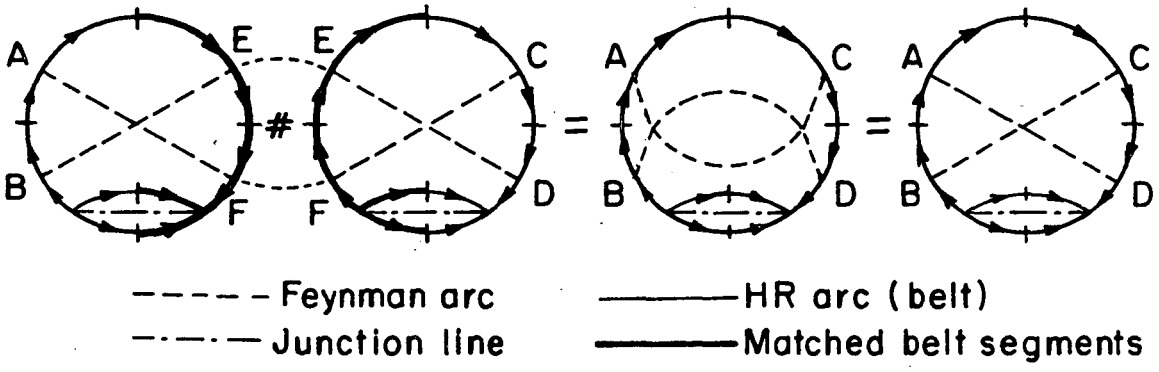

Fig. 24

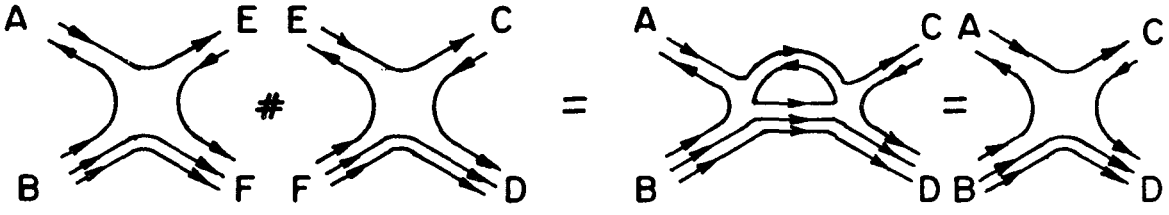

Fig. 25

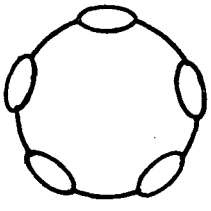

Fig. 26

XBL829-4097

Fig.27

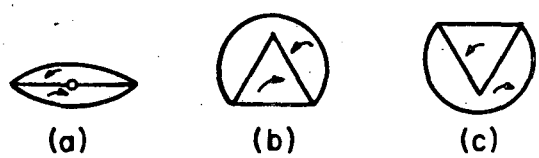

Fig.28

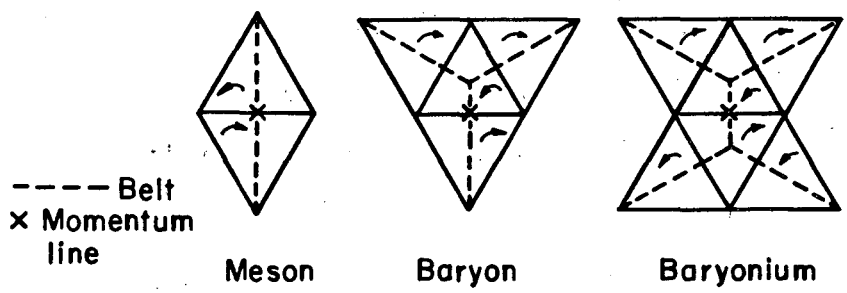

Fig.29

Fig.30

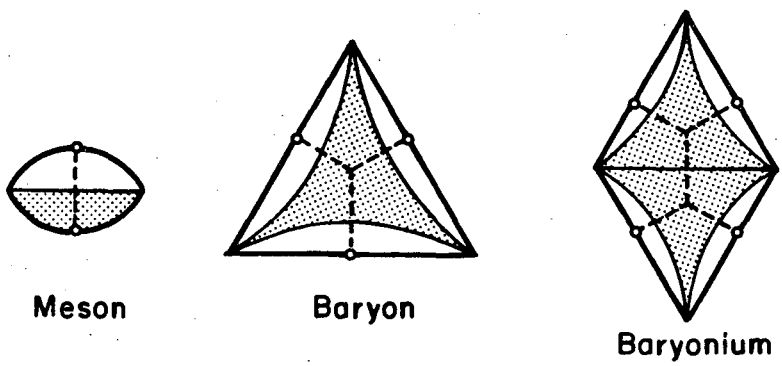

Fig.31

XBL829-4098

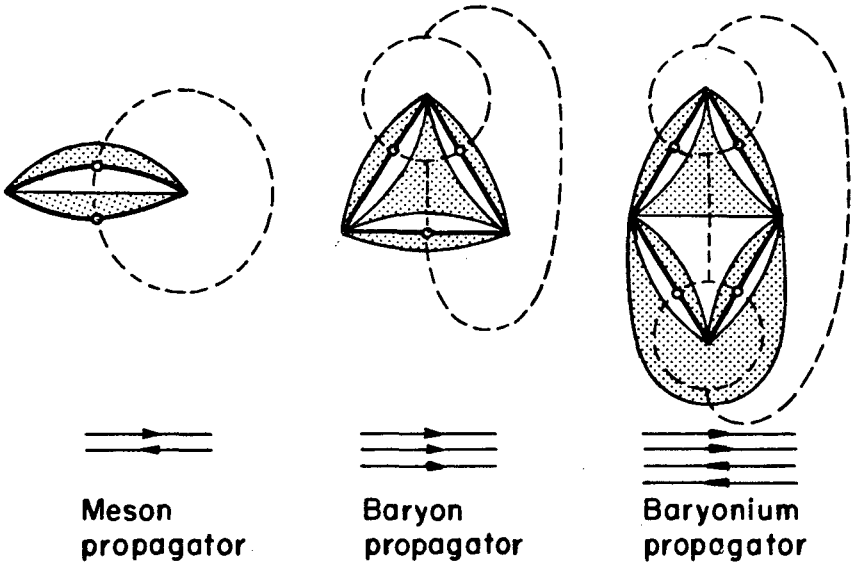

Fig. 32

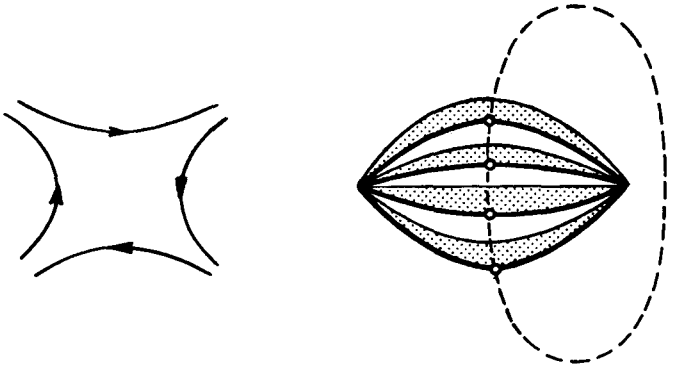

Fig. 33

XBL829-4099

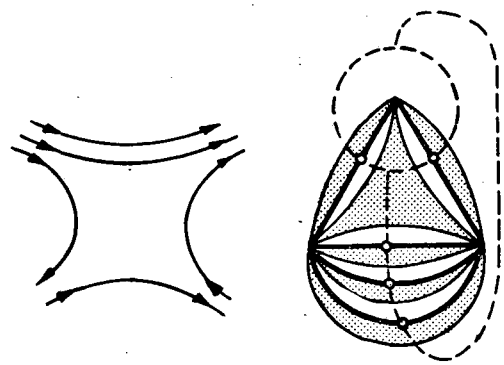

Fig. 34

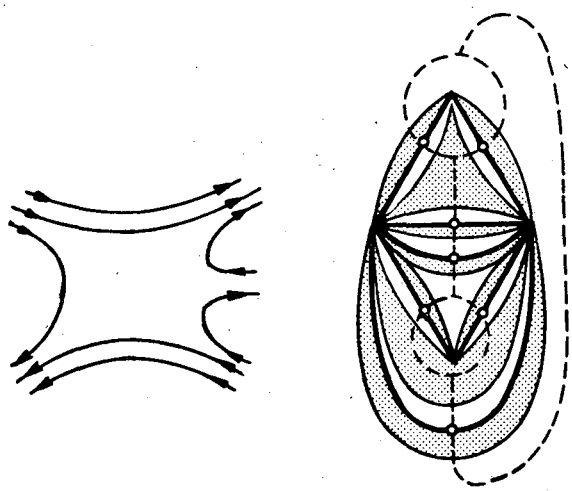

Fig. 35

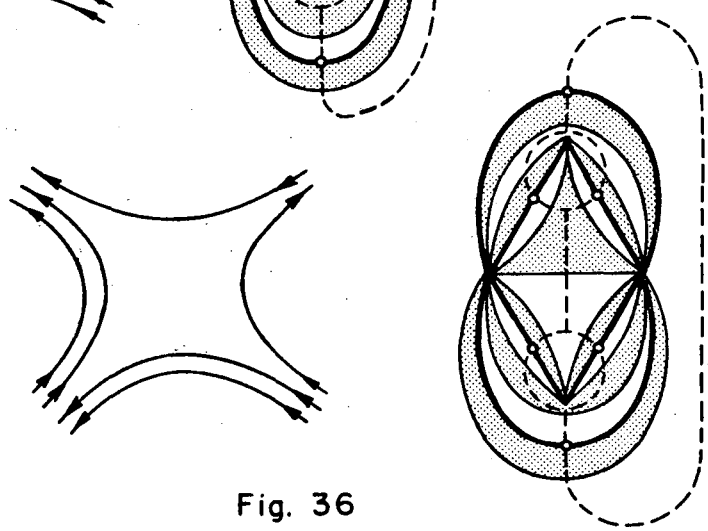

Fig. 36

XBL 829-4100

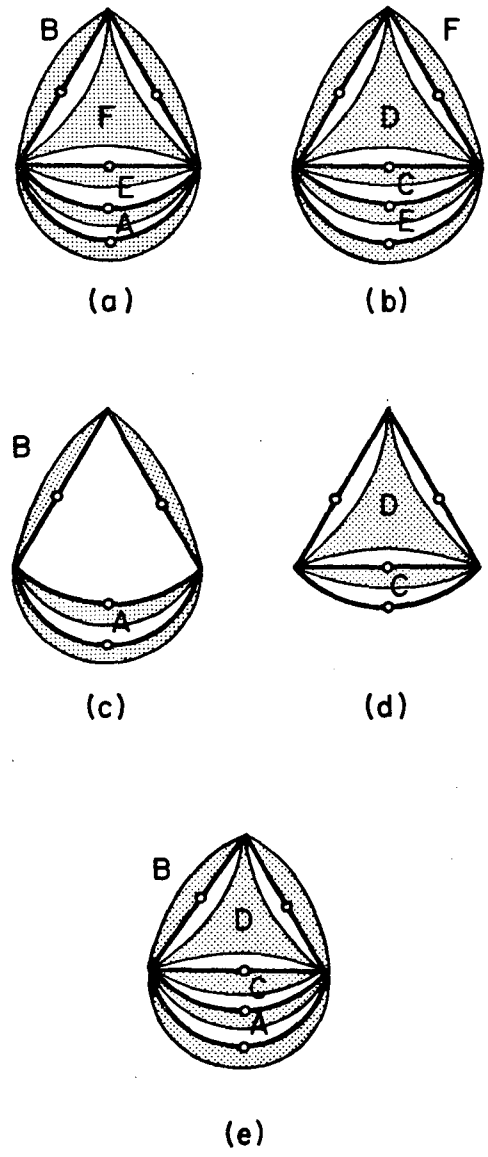

Fig. 37

XBL829-4101

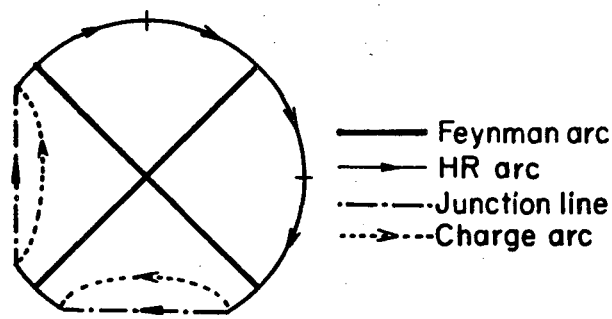

Fig. 38

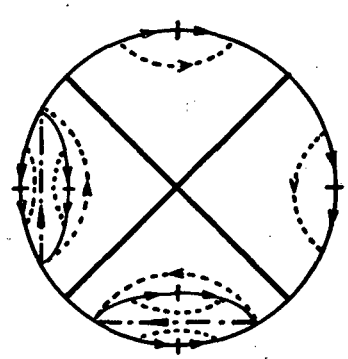

Fig. 39

Orientation of I-triangle	Direction of charge arc	Charge of quark or antiquark
Clockwise	Out	+1
Clockwise	In	0
Counterclockwise	Out	0
Counterclockwise	In	-1

Fig. 40

XBL829-4102

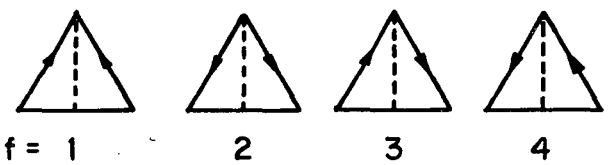

Fig. 41

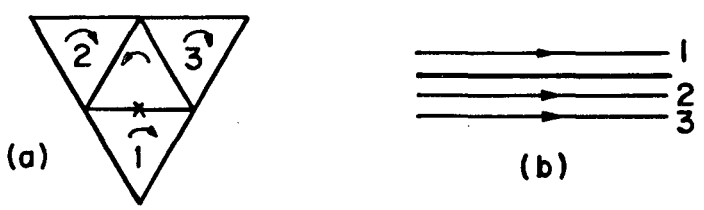

Fig. 42

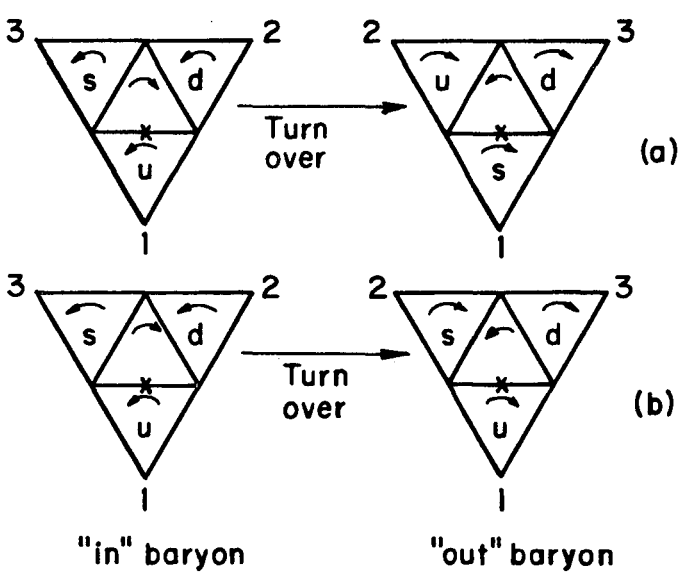

Fig. 43

XBL829-4103

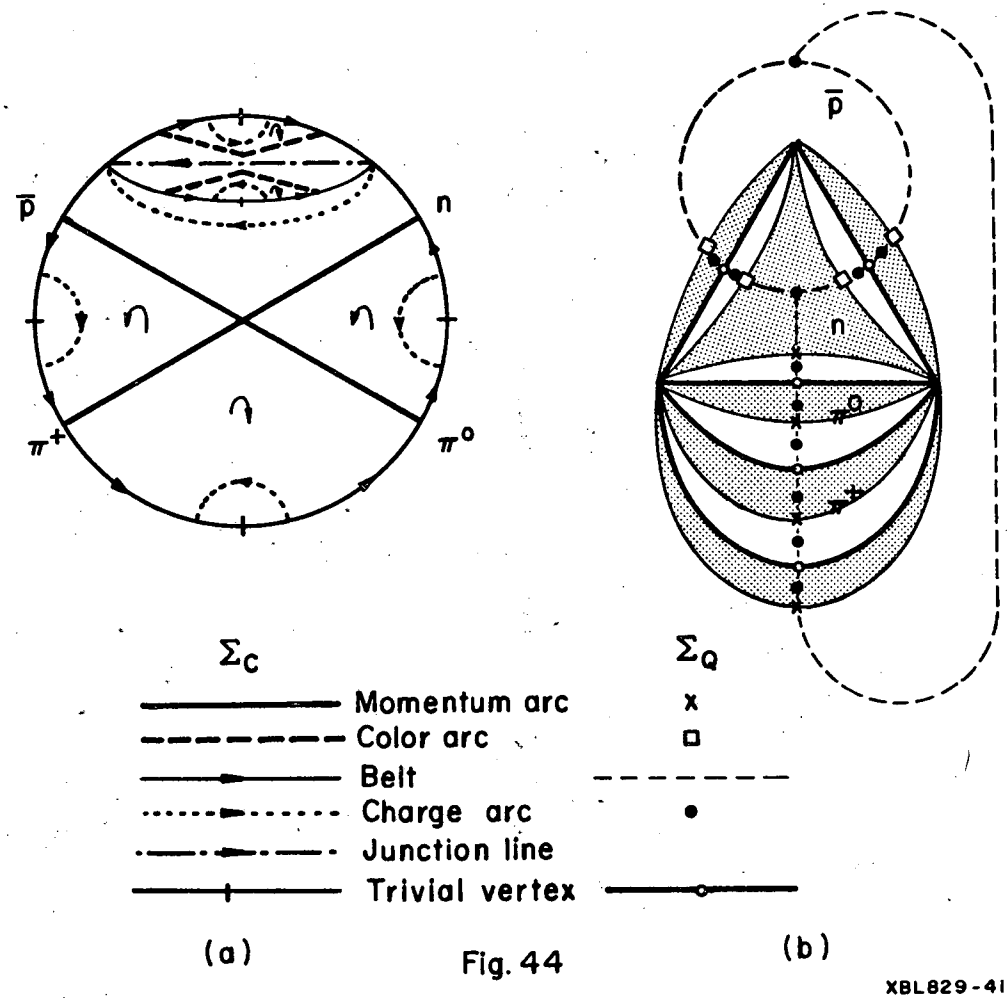

Fig. 44

XBL829-4104

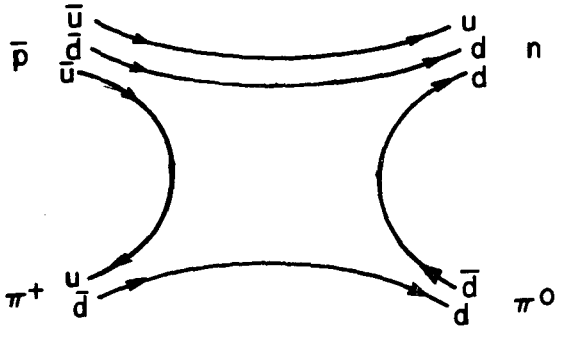

Fig. 45

Fig. 46

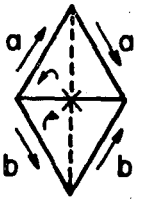

Fig. 47

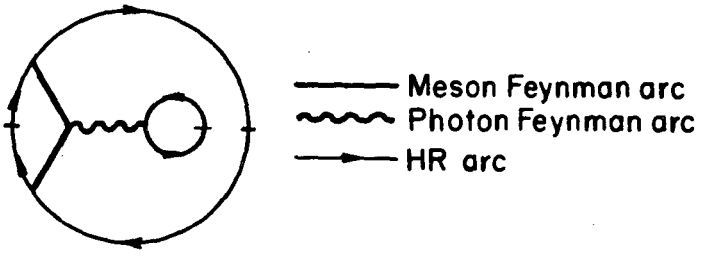

Fig. 48

XBL829-4105

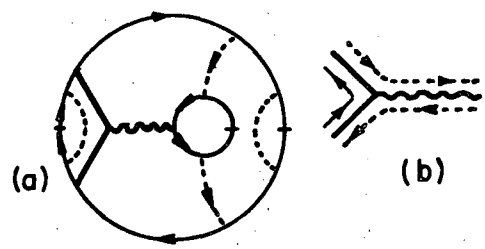

Fig. 49

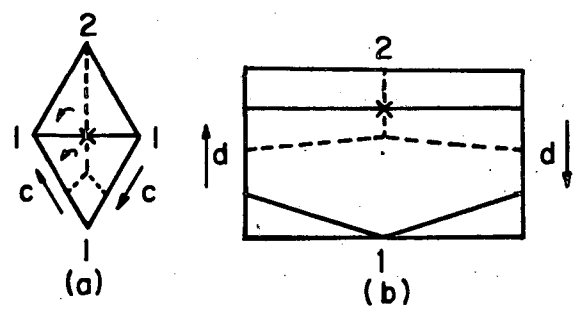

Fig. 50

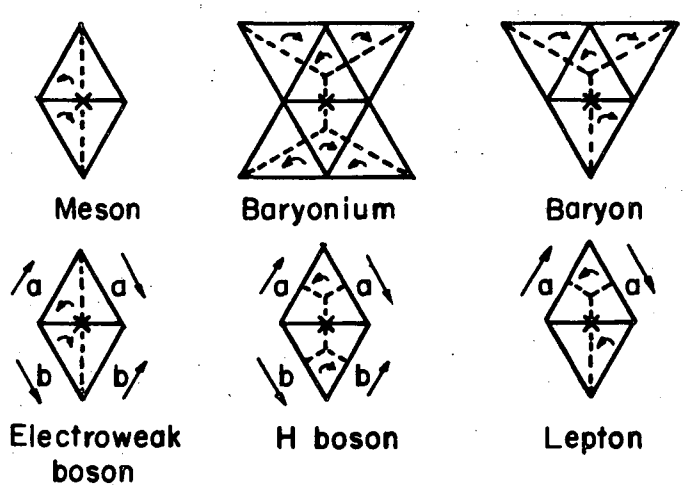

Fig. 51

XBL829-4106

	S	Q	B	L
I	1/2	+1,0	1/4	- 1/4
Y_{st}	0	+1	-1/4	- 3/4
Y_{ew}	0	0	-1/4	-3/4

Fig. 52

XBL829-4107

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