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FLAVOR CHANGING Z^0 DECAY

Alan Axelrod
(Ph.D. thesis)

October 1982

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FLAVOR CHANGING Z^0 DECAY*

(Ph.D. Thesis)

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ABSTRACT

The discovery of the Z^0, the particle mediating the weak neutral interaction of the $SU(2)_L \times U(1)$ electroweak theory, is anxiously awaited and is expected to occur at the next generation of accelerators. Large projected Z^0 production rates will make the study of rare decay modes possible. The predicted sixth quark flavor, or top, has also not been discovered and may be too heavy to produce by $Z \to t\bar{t}$. Therefore it is natural to study the feasibility of producing the top quark via a flavor changing neutral current decay process such as $Z \to t\bar{c}$. Flavor changing neutral currents are also of interest for the constraints on theories that they give.

For three generations, the branching ratios are found to be no larger than about 10^{-10}, thus essentially ruling out discovery of the top quark by this process. If there is a fourth generation, however, a supermassive b' quark can greatly increase the rates. As the b' mass is varied from 25 GeV to 1 TeV, and for reasonable choices of the other parameters, the branching ratios can be as large as about 10^{-8} to about 10^{-3}. A potential form of CP violation is also considered in this latter case, but is small.

In Chapter I I review the phenomenological situation. I also provide simple estimates of various rates in order to convey the physical

intuition necessary to guide one through the equations and numbers that follow. Chapter II presents technical aspects of the general flavor changing Z^0 decay calculation, with emphasis on the integration scheme used. In Chapter III I describe a number of nontrivial checks on the calculation that were performed. Chapter IV contains the entire general algebraic result for the decay rate. Chapter V describes numerical aspects of the computer evaluation, and discusses the parameter values used and the results for the three generation case. A similar presentation for the four generation case is given in Chapter VI. Chapter VII describes what experimentalists should look for in a semiquantitative way. Some possibilities for rate enhancement, and some related processes are mentioned in Chapter VIII.

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I. Introduction

The standard $SU(2)_L \times U(1)$ theory of electroweak interactions[1] appears thus far to have passed its experimental tests. Confidence in the theory has been growing since the crucial discovery of the predicted weak neutral current in neutrino scattering in 1973.[2] Since then, more and more refined data on various processes has made possible detailed and satisfactory comparisons with the theory.[3] Important recent confirmations of theoretical predictions have included the observation of parity violation in polarized electron-deuteron scattering[4] and in atomic physics,[5] and the detection of forward-backward asymmetry in e^+e^- annihilation into leptons.[6]

An essential untested ingredient of the theory is the existence of a massive neutral vector particle, the Z^0, that communicates the weak neutral force. Although this particle has not been discovered yet, its mass, total decay rate, and branching ratios are tightly constrained by fitting the theory's parameters to other already measured data.[7] One of the theory's main attractions is its renormalizability,[8] which ensures that there is a consistent framework within which finite radiative corrections to physical processes may be computed. This has led several groups to calculate radiative corrections to current energy processes and thereby obtain highly constrained predictions for the mass of the Z^0, M_Z.[9,10] As M_Z is expected to be nearly 100 GeV, it should be within the range of the next generation of accelerators.[11]

In the theory, charge +2/3 quarks appear in lefthanded doublets with their charge -1/3 partners. All known fermions are consistent with

this isodoublet assignment. However, the charge +2/3 partner of the b quark, called the top quark, has not been seen. It has been searched for extensively in e^+e^- annihilation at PETRA, resulting in a lower bound for the top quark mass M_T of roughly 20 GeV.[12] Attempts to derive phenomenological bounds from K^0-$\bar{K}^0$ mixing and the $K_L \to \mu^+\mu^-$ decay rate indicate that a mass of less than 100 GeV is likely.[13,14,15] Various other theoretical arguments point toward a top quark mass less than a few hundred GeV, and with very high probability no more than 1 TeV.[16] These include arguments based on partial wave unitarity,[17] radiative corrections to $\rho = M_W/(M_Z \cos\theta_W)$,[17,18] radiative corrections to the effective Higgs potential,[19] cancellation of quadratic divergences,[20] and the preservation of good predictions of grand unification and/or the study of infrared fixed points in coupling constant evolutions.[21]

We can obtain insight into the scale of these bounds by deriving a simple naive bound ourselves. The Yukawa coupling of a "superheavy" quark q to the physical Higgs field H is

$$\mathcal{L}_{Yuk} = -\lambda \bar{q} q H = -\frac{M_q}{v} \bar{q} q H \ , \tag{1.1}$$

where v is the Higgs vacuum expectation value (VEV). A bound for M_q is now obtained by demanding that the $\bar{q}qH$ coupling remains perturbatively small; i.e., that the Yukawa sector does not become strongly coupled. Of course this is not a fundamental requirement, but merely reflects our ignorance of how to calculate in strongly coupled quantum field theory. For the expansion parameter I assume α_{Yuk}/π; I take as my guide for this the anomalous magnetic moment of the electron, which

to eighth order can be expressed as a series in α_{QED}/π with coefficients of O(1).[22] Therefore, for the validity of perturbation theory I require that

$$\frac{\alpha_{Yuk}}{\pi} = \frac{M_q^2}{4\pi^2 v^2} < L, \tag{1.2}$$

where L is some number < 1. Noting that the Higgs VEV is related to the Fermi coupling constant by

$$v^2 = \frac{1}{G_F\sqrt{2}}, \tag{1.3}$$

we obtain a bound of

$$M_q < \left(\frac{4\pi^2}{G_F\sqrt{2}}\right)^{\frac{1}{2}} L^{\frac{1}{2}} = 1.55 L^{\frac{1}{2}} \text{ TeV}. \tag{1.4}$$

For $L = 0.5$, $M_q < 1.1$ TeV. For $L = 0.1$, $M_q < 500$ GeV. Thus the order of magnitude of these bounds on M_T may be easily understood.

It should be noted that there has been some discussion of models that omit a top quark, as alternatives to the above scenario.[23] B meson decay data from CESR on $B \to \ell^+\ell^- + X$ via flavor changing neutral currents, and $B \to$ baryons, have made this option quite unlikely.[24]

Therefore, it is certainly not unreasonable to assume that there is a lefthanded isodoublet top quark with mass lying somewhere between about 20 GeV and the Z^0 mass. If $M_T < M_Z/2$, the Z^0 can decay into a $t\bar{t}$ pair via a simple tree graph. In this case, this will almost certainly dominate the top quark production. If however $M_T > M_Z/2$, this

decay is energetically disallowed. In this event the decay $Z \to t\bar{c}$ or $t\bar{u}$ can occur as a weak radiative effect at one loop order and could lead to the discovery of the top quark.

The process $Z \to t\bar{c}$ (and the related $\bar{t}c$, $t\bar{u}$, $\bar{t}u$, $c\bar{u}$, and $\bar{c}u$) is an example of a flavor changing neutral current interaction. These processes have played an important role in recent particle physics due to their high degree of suppression. For example, they made possible an accurate prediction for the mass of the charmed quark before it was discovered.[25] The strong constraints implied by these data must be confronted by every new class of models, whether it be weak interactions,[26] grand unification,[27] technicolor,[28] or supersymmetry.[29]

In the $SU(2)_L \times U(1)$ theory, the suppression of these processes occurs through the GIM mechanism.[30] Not only are neutral current couplings connecting quarks of different generations absent in the tree Lagrangian, but they are suppressed even beyond the $O(g^3)$ one would expect from their appearance at one-loop order. This may be qualitatively understood as follows, using the $Z \to t\bar{c}$ one-loop amplitude (Figure 1) as a prototype. Its general form will be

$$\text{Ampl} = \sum_J U^{KM}_{TJ} (U^{KM})^{\dagger}_{Jc} f(\text{masses})_J, \tag{1.5}$$

where $(U^{KM})_{TJ}$ is the (top quark, jth bottom quark) matrix element of the unitary quark mixing matrix,[31] and $f(\text{masses})_J$ is some function of the masses, including M_J, and external momenta and spins. Now suppose all the M_J's were equal, eg. $M_d = M_s = M_b$ for three generations. Then $f(\text{masses})_J$ becomes independent of J, so we may remove its subscript

and write

$$\text{Ampl} = f(\text{masses}) \sum_J U^{KM}_{TJ}(U^{KM})^{\dagger}_{Jc} = f(\text{masses})\, \mathbb{1}_{Tc} = 0, \qquad (1.6)$$

since any off-diagonal element of the identity matrix is 0. The amplitude will be nonvanishing only if there are nonzero quark mass differences, and so is naively proportional to $\Delta(M_J^2)$. As it is natural for the scale to be otherwise set by M_W, this will result in a huge suppresion for ordinary quark mass differences.

I adopt the convention that ($Z \to$ top) means ($Z \to t$ or $\bar{t} + X$), where X is not a t or $\bar{t}$.* Experimentally we know that three generations of quarks and leptons exist. Since for three generations ΔM_J^2 is dominated by M_b^2, we are led to the estimate for the branching ratio of

$$\text{B.R.} = \frac{\Gamma(Z\to\text{top})}{\Gamma(Z\to\text{all})} \sim \left(\frac{g^3 \frac{M_b^2}{M_W^2} 10^{-\frac{1}{2}}}{g}\right)^2 \frac{1}{10}$$

$$\sim 10^{-2} G_F^2 M_b^4 \sim 10^{-9}, \qquad (1.7)$$

where $10^{-\frac{1}{2}}$ is from KM angles, and the extra 1/10 takes rough account of the number of open channels for Z decay.

This branching ratio would be very small and probably not measurable, even if the projected electron-positron collider at CERN, LEP, were to produce 10^8 Z's per year.[32] However, an exact calculation is

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* Until Chapter VIII I consider only the case where X is another quark.

required to see if the rate lies in a measurable range. This is the primary subject of this thesis.

A possibility for enhancement of this branching ratio would be the existence of more generations of quarks.* A fourth generation of quarks would consist of a massive charge 2/3 quark and a massive charge -1/3 quark, which I call the t' and b', respectively. There is little reason to believe there are more than three generations, but I know of no reason to rule out this possibility. Probably the most persuasive argument in its favor is that the minimal SU(5) GUT needs (more Higgs or) at least one more fermion generation in order to reproduce the cosmological baryon asymmetry[33]; however, this must be weighed against SU(5) calculations of M_b/M_τ that imply no more than three generations.[79] There are other astrophysical arguments suggestive of more generations.[34]

e^+e^- data indicates that either a t' or b' mass below about 20 GeV is very unlikely.[12] Analysis of the K system gave correlated lower bounds on M_T and $M_{T'}$ that allow both to get large together.[15] Other arguments and speculations regarding fourth generation masses and mass bounds are almost the same as those mentioned earlier for the top quark and will not be repeated here. I will consider about 1 TeV to be an optimistic upper bound on $M_{b'}$, based on my earlier naive estimate regarding the breakdown of perturbation theory.

In order to estimate the effect of a b' on BR(Z → top), I will assume that $Z \to t\bar{t}'$ is energetically forbidden, and that M_q^4 behavior

---

* In order to cancel anomalies, there should be an equal number of generations of quarks and leptons.

of the decay rate persists out to very large values of M_q, i.e. for $M_q \gtrsim M_W$ or $M_q \gg M_W$. This is not obvious, and must be checked by calculation. Additionally, for the purpose of an estimate it is natural to suppose that the b' contribution is doubly Cabibbo suppressed relative to the b. This is because $t \to b \to c$ contains one cross-generation jump whereas $t \to b' \to c$ contains 3, i.e.

$$\begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \quad \text{vs.} \quad \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{pmatrix} t' \\ b' \end{pmatrix} . \tag{1.8}$$

One therefore estimates

$$\begin{aligned} BR(Z\to top,\text{with } b') &\sim (\sin\theta_c)^4 \left(\frac{M_{b'}}{M_b}\right)^4 BR(Z\to top,\text{no } b') \\ &\sim 10^{-15} M_{b'}^4, \text{ (in GeV)}. \end{aligned} \tag{1.9}$$

For $M_{b'}$ = 100 GeV, the right hand side is 10^{-7}. For $M_{b'}$ = 1 TeV, it is 10^{-3}. Therefore, if these estimates are correct, a massive b' quark could make the rates increase significantly. GIM suppression is "restored" only if intergenerational mixing is very small.

Another possibility in the four generation case is that of observable CP violation. At this point, CP violation has been observed only in the K^0-$\bar{K}^0$ system.[7] It is a small effect, and in the $SU(2)_L \times U(1)$ theory with three generations it can be explained by the inclusion of a small complex phase in the 3×3 quark mixing matrix.[31,35]

In the case of flavor changing Z decay, it will be pointed out that $\Gamma(Z \to A\bar{B}) = \Gamma(Z \to \bar{A}B)$, where A and B are external quarks, is a consequence

of CP invariance. In the present framework, this occurs if the quark mixing matrix is real. For three generations this is an excellent approximation, and as BR(Z → top) is expected to be very small anyway, any CP violation should be a tiny correction to a small effect and thus unmeasurable and negligible.

In the four generation case, however, two factors could conspire to produce a measurable CP violation. Firstly, if $M_{b'}$ is large the overall decay rate is expected to be bigger. Secondly, the 4 X 4 mixing matrix contains three arbitrary complex phases instead of one.[36] Hence, there is enough freedom that small CP violation in the K^0 system could be compatible with large CP violation in another process. K^0-$\bar{K}^0$ mixing involves the KM couplings to d and s quarks only; for flavor changing Z → top, the couplings to b and b' are expected to dominate.

Drawing our attention to figure 1, we see that the $Z \to t\bar{c}$ (for example) amplitude is a sum of contributions from four internal J quarks, which is then squared to get the rate. Let the interference in the squared sum between an internal quark I amplitude, and the complex conjugate of an internal quark J amplitude, be denoted by IJ^*. Then one may check that any JJ^* does not produce CP violation. This is because essentially

$$\Gamma(Z\to t\bar{c})_{IJ^*} - \Gamma(Z\to \bar{t}c)_{IJ^*} \sim \mathrm{Im}(U_{tI}U^*_{cI}U^*_{tJ}U_{cJ}). \tag{1.10}$$

For I=J, the indicated product of KM matrix elements is real. However, for I ≠ J, there is no reason for this product to be real, and in general it is not. For four generations, the leading contribu-

tion should therefore, in most cases, be given by

$$\Gamma(Z\to t\bar{c}) - \Gamma(Z\to \bar{t}c) \sim \mathrm{Im}(U_{tb'}U^*_{cb'}U^*_{tb}U_{cb}) \ . \tag{1.11}$$

Define the width for CP violation as

$$\Gamma(Z \to \mathrm{top,CPV}) \equiv \Gamma(Z \to t + X) - \Gamma(Z \to \bar{t} + X) \ , \tag{1.12}$$

where X is a quark other than t or $\bar{t}$. Then, since we have assumed that the b' quark dominates the total rates which are $\sim M^4_{b'}$, we estimate that (assuming all other relevant factors are O(1))

$$\begin{aligned} \mathrm{BR}(Z\to \mathrm{top,CPV}) &\sim \left(\frac{M_b}{M_{b'}}\right)^2 \frac{\mathrm{BR}(Z\to \mathrm{top,with\ b'})}{\sin^2\theta_c} \\ &\sim (\sin\theta_c)^2 \left(\frac{M_{b'}}{M_b}\right)^2 \mathrm{BR}(Z\to \mathrm{top,\ no\ b'}) \\ &\sim 10^{-3} M^2_{b'} \mathrm{BR}(Z\to \mathrm{top,\ no\ b'}), \quad (M_{b'} \text{ in GeV}) \end{aligned} \tag{1.13}$$

where 2 factors of $\sin\theta_c$ were gained due to the CP violation being less Cabibbo suppressed than the total rate.

Therefore, for $M_{b'}$ = 100 GeV, one estimates CP violation at 10 times the total three generation rate, or $\sim 10^{-8}$ branching ratio. For $M_{b'}$ = 1 TeV, the estimate is CP violation at 1000 times the total three generation branching ratio, or $\sim 10^{-6}$. Thus it is not out of the question that CP violation could be measurable in the flavor changing decays for four generations.

Finally, due to the smallness of the branching ratios above, particularly in the less speculative three generation case, it has been recognized that the four fermion final state $Z \rightarrow t f_1 \bar{f}_2 \bar{f}_3$ may be a better way to discover the top quark if $M_T > M_Z/2$ than the one that is the major subject of this thesis. This will be estimated later.

The purpose of this chapter was to provide physical intuition and insight into the sorts of numbers involved here. Later chapters discuss primarily technical details, results, and measurement aspects.

II. Calculational Scheme

In this chapter the methods and techniques of calculation are discussed. For most things, the number of generations need not be fixed or considered until the concrete numerical results are discussed in Chapters V and VI. Any exceptions to this will be clear from explicit comments I will make.

The first order of business is to choose a gauge to work in. I choose to work in the unitary gauge. As compared to 't Hooft-Feynman[8] or the generalized R_ξ gauge,[37] it has the advantage of fewer diagrams since there are no Higgs ghosts. The four diagrams that need to be calculated for $Z \to t\bar{c}$ are in Figure 1; the ones for $\bar{t}c$, $t\bar{u}$, $\bar{t}u$, $c\bar{u}$, $\bar{c}u$, etc., follow from these by trivial substitution. The main disadvantage of unitary gauge is that the vector boson propagator is[38]

$$\frac{-i(g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2})}{k^2 - M^2} \tag{2.1}$$

and therefore more complicated than 't Hooft-Feynman gauge where it is

$$\frac{-ig_{\mu\nu}}{k^2 - M^2} \, . \tag{2.2}$$

This ensures that the individual diagrams are more complicated in unitary gauge. Therefore it is not clear which gauge is preferable. The most difficult choice would be one of the other R_ξ gauges (eg. Landau gauge) which have the Higgs ghosts as well as the complicated vector boson propagator

$$-i\,\frac{\left\{ g_{\mu\nu} + \dfrac{(1-\frac{1}{\xi})k_\mu k_\nu}{(\frac{k^2}{\xi}-M^2)} \right\}}{k^2-M^2}\,, \tag{2.3}$$

and therefore combines the difficulties of the unitary and t'Hooft-Feynman gauges.

I present here the Lagrangian for unitary gauge for later reference:

$$\begin{aligned}
\mathcal{L} = \sum_i \{ & \bar{t}_i(i\not\partial - m_{t_i} - \frac{m_{t_i}}{v}H)t_i + \bar{b}_i(i\not\partial - m_{b_i} - \frac{m_{b_i}}{v}H)b_i \\
& + eA_\mu[\frac{2}{3}\bar{t}_i\gamma^\mu t_i - \frac{1}{3}\bar{b}_i\gamma^\mu b_i] \\
& + \frac{g}{4c_W} Z_\mu[\bar{t}_i\gamma^\mu(-1+\frac{8}{3}s_W^2+\gamma_5)t_i + \bar{b}_i\gamma^\mu(1-\frac{4}{3}s_W^2-\gamma_5)b_i]\} \\
& + \frac{g}{2\sqrt{2}} \sum_{ij} \{W_\mu^+\bar{t}_i\gamma^\mu(1-\gamma_5)U_{ij}b_j + W_\mu^-\bar{b}_j\gamma^\mu(1-\gamma_5)(U^\dagger)_{ji}t_i\} \\
& - \frac{1}{4}|\partial_\mu A_\nu - \partial_\nu A_\mu - ie(W_\mu^+W_\nu^- - W_\nu^+W_\mu^-)|^2 \\
& - \frac{1}{4}|\partial_\mu Z_\nu - \partial_\nu Z_\mu + igc_W(W_\mu^+W_\nu^- - W_\mu^-W_\nu^+)|^2 \\
& - \frac{1}{2}|\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+A_\nu - W_\nu^+A_\mu) - igc_W(W_\mu^+Z_\nu - W_\nu^+Z_\mu)|^2 \\
& + \frac{1}{2}\partial_\mu H\partial^\mu H - \frac{\mu^2}{2}(v+H)^2 - \frac{\lambda}{4}(v+H)^4 + \frac{1}{2}M_Z^2 Z_\mu Z^\mu + M_W^2 W_\mu^+ W^{\mu-} \\
& + \frac{1}{8}(2vH+H^2)[(\frac{g^2}{c_W^2})Z_\mu Z^\mu + 2g^2 W_\mu^+ W^{\mu-}] \;.
\end{aligned} \tag{2.4}$$

The notation is:

W_μ = charged weak boson; Z_μ = neutral weak boson;

A_μ = photon; H = physical Higgs;

t_i = charge +2/3 quarks; b_i = charge -1/3 quarks;

g = $SU(2)_L$ coupling; e = $U(1)_{QED}$ coupling;

s_W, c_W = sin and cos of Weinberg angle; U = KM quark mixing matrix;

μ, λ = quadratic and quartic couplings of Higgs potential; and

v = Higgs vacuum expectation value.

Leptons and the question of gauge fixing in the QED sector do not concern us here so I ignore them.

Dimensional regularization[39] is used to regulate the ultraviolet loop integral devergences that exist before all internal quarks and diagrams are summed over. It is sometimes said that the divergences in unitary gauge are more severe than in 't Hooft-Feynman or generalized R_ξ gauges. A glance at the vector boson propagators above indicates that this is true by naive power counting. However, this is somewhat irrelevant in the dimensional scheme when one performs Laurent expansion in N about physical 4 dimensions, since any divergence then appears as a pole in N-4 irrespective of its (even) degree.

As alluded to above, the sum over all internal quarks and diagrams is ultraviolet finite. In the dimensional scheme this means that the result for the amplitude is free of 1/(N-4) pole terms. This is expected in analogy with previous work on $K_L \to \mu^+\mu^-$,[40,41,42] where the effective one-loop Zds vertex was found to be finite.

An immediate obstacle is the choice of what to use for γ_5 in N dimensions. Although it is now ten years since the introduction of

dimensional regularization, there is still a lot of discussion and at times acerbic controversy in the recent literature regarding this point.[43,44] The two main competitors seem to be the non-anticommuting prescription[39]

$$\gamma_5^2 = 1;$$

$$\{\gamma_5,\gamma_\mu\} = 0,\ \mu = 0,1,2,3;$$

$$[\gamma_5,\gamma_\mu] = 0,\ \mu = 4,\ldots,N-1, \tag{2.5}$$

and the anticommuting prescription[45,43]

$$\gamma_5^2 = 1;$$

$$\{\gamma_5,\gamma_\mu\} = 0 \text{ for all } \mu. \tag{2.6}$$

Without getting bogged down in the technical controversies, one may easily understand why there is a potentially tricky decision to be made here. The two prescriptions obviously agree at N=4 dimensions, and therefore disagree at O(N-4) only. In the course of doing the calculation, divergent loop integrals produce poles that can produce finite parts when multiplied by O(N-4) pieces. Although the coefficient of the poles in the whole calculation vanishes when summed over internal quarks and diagrams, I don't know any argument to guarantee that the subset of pole coefficients that multiply the O(N-4) difference

between the two prescriptions must also sum to 0. Thus one could conceivably end up with a spurious finite part in the result that spoils the whole calculation if one is not careful. Similar considerations led Ref. [43] to conclude that prescription (2.5) introduced spurious anomalies into canonical Ward identities.

I adopt the anticommuting γ_5 prescription (2.6). The authors of Refs. [43,45] determined that this prescription preserved the Ward identities, at least for one-loop graphs, whereas prescription (2.5) did not. Here, I rely on the checks described in Chapter III, in particular the Ward identities described there, to verify that I am not in error.

The question arises of whether there is a legitimate approximation scheme. Previous calculations of flavor changing neutral current processes have often neglected internal masses, or external masses or momenta either completely or with respect to some large scale such as M_W. This cannot be justified for the large masses and momenta involved in $Z \to t\bar{c}$, for example, as compared to the low energy processes that were studied previously, such as K^0-$\bar{K}^0$ mixing or $K_L \to \mu^+\mu^-$ decay. The on-shell decay rate, summed over fermion spins and averaged over boson polarizations, is a function of M_Z, M_W, M_A, M_B, and M_J, where A and B are the external quarks and J are the internal quarks. M_Z and M_W set the large scale and if, say, $M_A = M_T$ then it is of the same order and cannot be neglected. If M_J is ignored completely, the decay rate is exactly zero by the GIM mechanism. In the three generation case, it is possible to argue to neglect $M_B = M_c$, M_u and/or high powers of M_J. If one ignores M_B one also loses the possibility of computing $Z \to t\bar{c}$ and $\bar{t}c$ independently, and therefore loses the check of Chapter III that

these should be equal. In the four generation case, if one computes $Z \to t\bar{t}'$ with an internal b' quark, then all five masses are often of the same order and clearly none can be neglected. I chose the safest and most general route and kept all masses and momenta without approximation.

Keeping all five masses variable and without approximation produces an algebraic explosion. A computer algebraic manipulator, MACSYMA, [53] was essential in handling the algebra. The calculation would have taken far longer without it, assuming one would ever get the right answer. At times the expressions become so long that various strategies to break up or simplify the calculation had to be devised in order not to exceed the computer's capacity.

The gamma matrix algebra is done by first moving γ_5's to the outside of γ - matrix products using the anticommuting prescription (2.6). The resulting products of γ matrices and slash vectors can then be reduced using the standard N dimensional forms

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}; \; g_{\mu\nu}g^{\mu\nu} = N. \tag{2.7}$$

Care is taken to bring external quark momentum slash vectors adjacent to external quark spinors. Then, Dirac equation can be used repeatedly to eliminate external 4-momenta in favor of masses. Additionally, since we have a physical external Z in one of three polarization states characterized by the polarization vector ε^μ, we have $\varepsilon \cdot p_Z = 0$. Therefore, if p_A and p_B are the external quark momenta, the relation $\varepsilon \cdot p_A = -\varepsilon \cdot p_B$ may be used to simplify.

It is useful to throw out M_J-independent pieces at the earliest

opportunity. These contribute 0 due to GIM cancellation. Also, one can simplify things by expanding in (N-4), keeping only pole and finite parts, wherever convenient. Higher orders in (N-4) can merely be set to zero and dropped.

In general, it is found that if a symmetry or constraining equation exists, it should be used to simplify or check. For example, I used the relation $M_W = M_Z \cos\theta_W$ to eliminate $\cos\theta_W$ as an additional variable. The more independent variables there are, the harder it is to handle the algebra.

To close this chapter, I wish to describe the integration scheme in some detail. It was desired to do as much of the work algebraically as possible; one reason for this was to try to handle analytically certain nasty cancellations between terms before they cropped up later numerically, as alluded to in [46]. By using the following scheme, one can reduce all relevant loop integrals to algebraic sums of integrals with loop momenta in the denominators only ("scalar loop integrals"). These can then be written analytically as sums of constants, logarithms, and Spence functions, combined together algebraically, and numerically evaluated. The advantage of this method is that it avoids the need to carry around Feynman parameters in the numerators, and the later need to perform such messy Feynman integrations. One never gets Feynman parameters in the numerators of integrals that have loop momenta in denominators only.

The scheme is similar to the one presented in Appendices D and E of Ref. [47]. The major differences are that

(i) the formulas are valid for arbitrary N (assuming N is such

that the integrals converge), allowing algebraic reduction of all integrals to linear combinations of scalar loop integrals only;

(ii) the metric of Ref. [38] is used; and

(iii) some extra formulas are given, including some useful in unitary gauge.

Let the integration "measure" be

$$d^N\tilde{k} = \mu^{3(4-N)/2} \frac{d^N k}{(2\pi)^N} , \tag{2.8}$$

where $g\mu^{(4-N)/2}$ is the dimensionful coupling constant in N dimensions. Let

$$I = (k^2-m_1^2); \; II = ((k+\ell)^2-m_2^2); \; III = ((k+\ell+s)^2-m_3^2) \tag{2.9}$$

where ℓ and s are some external momenta and m_i are masses.

It is desired to express all relevant loop integrals as algebraic linear combinations of

$$A_0(m_1) = \int d^N\tilde{k} \left(\frac{1}{I}\right); \; B_0(\ell,m_1,m_2) = \int d^N\tilde{k} \left(\frac{1}{I\ II}\right) ;$$

$$C_0(\ell,s,m_1,m_2,m_3) = \int d^N\tilde{k} \left(\frac{1}{I\ II\ III}\right) . \tag{2.10}$$

For the current calculation, for example, the only scalar loop integrals that will appear are (recall $Z \to A\bar{B}$ with p_A, p_B the quark external momenta, and J stands for internal quarks)

$A_0(M_W)$, $A_0(M_J)$;

$B_0(p_A,M_W,M_J)$, $B_0(p_B,M_W,M_J)$, $B_0(p_A+p_B,M_W,M_W)$, $B_0(p_A+p_B,M_J,M_J)$;

$$C_0(p_A,p_B,M_W,M_J,M_W),\ C_0(p_A,-(p_A+p_B),M_W,M_J,M_J). \tag{2.11}$$

The following formulas are derived using Lorentz invariance and index symmetry, and utilizing vector momenta and $g^{\mu\nu}$ as "projectors" until enough equations are derivedto solve for all the scalar factors. For later use define

$$s_1 = \ell^2+m_1^2-m_2^2$$

$$s_2 = s^2+2\ell s+m_2^2-m_3^2 \ . \tag{2.12}$$

Then, for example, "projecting" B_{μ_1} below with $\ell^{\mu 1}$ and using $\ell k = \frac{1}{2}(II-I-s_1)$ gives the result for B_1. The formulas below then define an iterative ladder in which the ones on top are defined in terms of the ones on the rung below, etc., with A_0, B_0, and C_0 at the bottom.

$$\tilde{B}_0;\ B_{\mu_1};\ \tilde{B}_{\mu_1};\ B_{\mu_1\mu_2}(\ell,m_1,m_2) = \int d^N\tilde{k}\ \frac{k^2;\ k_{\mu_1};\ k^2k_{\mu_1};\ k_{\mu_1}k_{\mu_2}}{I\ II} \tag{2.13}$$

$$B_{\mu_1} = \ell_{\mu_1}B_1(\ell,1,2)$$

$$\tilde{B}_{\mu_1} = \ell_{\mu_1}\tilde{B}_1$$

$$B_{\mu_1\mu_2} = \ell_{\mu_1}\ell_{\mu_2}B_{21} + g_{\mu_1\mu_2}B_{22} \ , \tag{2.14}$$

where

$$\widetilde{B}_0 = A_0(m_2) + m_1^2 B_0$$

$$B_1 = \frac{1}{2\ell^2}[A_0(m_1) - A_0(m_2) - s_1 B_0]$$

$$\widetilde{B}_1 = m_1^2 B_1 - A_0(m_2)$$

$$B_{21} = \frac{1}{2\ell^2(N-1)}[(N-2)A_0(m_2) - 2m_1^2 B_0 - Ns_1 B_1] \ .$$

$$B_{22} = \frac{1}{2(N-1)}[A_0(m_2) + 2m_1^2 B_0 + s_1 B_1]$$

$$\widetilde{C}_0;\ \widetilde{C}'_0;\ C_{\mu_1};\ \widetilde{C}_{\mu_1};\ C_{\mu_1\mu_2};\ \widetilde{C}_{\mu_1\mu_2};\ C_{\mu_1\mu_2\mu_3}(\ell,s,m_1,m_2,m_3) =$$

$$\int d^N\widetilde{k}\ \frac{k^2;\ (k^2)^2;\ k_{\mu_1};\ k^2k_{\mu_1};\ k_{\mu_1}k_{\mu_2};\ k^2k_{\mu_1}k_{\mu_2};\ k_{\mu_1}k_{\mu_2}k_{\mu_3}}{\mathrm{I\ II\ III}} \tag{2.15}$$

$$C_{\mu_1} = \ell_{\mu_1}C_{11}(\ell,s,1,2,3) + s_{\mu_1}C_{12}(\ell,s,1,2,3)$$

$$\widetilde{C}_{\mu_1} = \ell_{\mu_1}\widetilde{C}_{11} + s_{\mu_1}\widetilde{C}_{12}$$

$$C_{\mu_1\mu_2} = \ell_{\mu_1}\ell_{\mu_2}C_{21} + s_{\mu_1}s_{\mu_2}C_{22} + \{\ell s\}_{\mu_1\mu_2}C_{23} + g_{\mu_1\mu_2}C_{24}$$

$$\widetilde{C}_{\mu_1\mu_2} = \ell_{\mu_1}\ell_{\mu_2}\widetilde{C}_{21} + s_{\mu_1}s_{\mu_2}\widetilde{C}_{22} + \{\ell s\}_{\mu_1\mu_2}\widetilde{C}_{23} + g_{\mu_1\mu_2}\widetilde{C}_{24}$$

$$C_{\mu_1\mu_2\mu_3} = \ell_{\mu_1}\ell_{\mu_2}\ell_{\mu_3}C_{31} + s_{\mu_1}s_{\mu_2}s_{\mu_3}C_{32} + \{\ell\ell s\}_{\mu_1\mu_2\mu_3}C_{33}$$

$$+ \{\ell ss\}_{\mu_1\mu_2\mu_3}C_{34} + \{\ell g\}_{\mu_1\mu_2\mu_3}C_{35} + \{sg\}_{\mu_1\mu_2\mu_3}C_{36} \ , \tag{2.16}$$

where $\{\ \}_{\mu_1\cdots\mu_n}$ means the product of all tensors inside, totally symmetrized over all indices shown.

Let $X^{-1} = \begin{pmatrix} \ell^2 & \ell s \\ \ell s & s^2 \end{pmatrix}^{-1} = \dfrac{1}{\ell^2 s^2 - (\ell s)^2}\begin{pmatrix} s^2 & -\ell s \\ -\ell s & \ell^2 \end{pmatrix}$. Then

$$\widetilde{C}_0 = B_0(s,2,3) + m_1^2 C_0$$

$$\widetilde{C}'_0 = \widetilde{B}_0(s,2,3) - 2(\ell s)B_1(s,2,3) + \ell^2 B_0(s,2,3) + m_1^2\widetilde{C}_0$$

$$\begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} = \frac{1}{2}X^{-1}\begin{pmatrix} B_0(\ell+s,1,3) - B_0(s,2,3) - s_1C_0 \\ B_0(\ell,1,2) - B_0(\ell+s,1,3) - s_2C_0 \end{pmatrix}$$

$$\widetilde{C}_{11} = -B_0(s,2,3) + m_1^2 C_{11}$$

$$\widetilde{C}_{12} = B_1(s,2,3) + m_1^2 C_{12}$$

$$\begin{pmatrix} C_{21} \\ C_{23} \end{pmatrix} = \frac{1}{2}X^{-1}\begin{pmatrix} B_1(\ell+s,1,3) + B_0(s,2,3) - s_1C_{11} - 2C_{24} \\ B_1(\ell,1,2) - B_1(\ell+s,1,3) - s_2C_{11} \end{pmatrix}$$

$$C_{22} = \frac{1}{2[\ell^2 s^2 - (\ell s)^2]} \{-\ell s [B_1(\ell+s,1,3) - B_1(s,2,3) - s_1 C_{12}]$$

$$+ \ell^2 [-B_1(\ell+s,1,3) - s_2 C_{12} - 2C_{24}] \}$$

$$C_{24} = \frac{1}{2(N-2)} [B_0(s,2,3) + 2m_1^2 C_0 + s_1 C_{11} + s_2 C_{12}]$$

$$\tilde{C}_{21} = B_0(s,2,3) + m_1^2 C_{21}$$

$$\tilde{C}_{22} = B_{21}(s,2,3) + m_1^2 C_{22}$$

$$\tilde{C}_{23} = -B_1(s,2,3) + m_1^2 C_{23}$$

$$\tilde{C}_{24} = B_{22}(s,2,3) + m_1^2 C_{24}$$

$$\begin{pmatrix} C_{31} \\ C_{33} \end{pmatrix} = \frac{1}{2} X^{-1} \begin{pmatrix} B_{21}(\ell+s,1,3) - B_0(s,2,3) - s_1 C_{21} - 4C_{35} \\ B_{21}(\ell,1,2) - B_{21}(\ell+s,1,3) - s_2 C_{21} \end{pmatrix}$$

$$\begin{pmatrix} C_{34} \\ C_{32} \end{pmatrix} = \frac{1}{2} X^{-1} \begin{pmatrix} B_{21}(\ell+s,1,3) - B_{21}(s,2,3) - s_1 C_{22} \\ -B_{21}(\ell+s,1,3) - s_2 C_{22} - 4C_{36} \end{pmatrix}$$

$$\begin{pmatrix} C_{35} \\ C_{36} \end{pmatrix} = \frac{1}{2} X^{-1} \begin{pmatrix} B_{22}(\ell+s,1,3) - B_{22}(s,2,3) - s_1 C_{24} \\ B_{22}(\ell,1,2) - B_{22}(\ell+s,1,3) - s_2 C_{24} \end{pmatrix} .$$

Other equations, which naively look different from those in the set above, were derived. However, these were always found to be equi-

valent to one of the above when both were expressed in terms of A_0, B_0, and C_0 functions. I.e., they are just rewritings of the same fundamental equations. Redundant equations were included in Ref. [47].

A $k_\mu k_\nu$ numerator factor is more difficult to work out than if its indices are known to be contracted into k^2, because the number of available Lorentz tensors increases rapidly with the number of tensor indices. If one looks at the unitary gauge expressions appearing in this calculation naively, it appears that one needs to compute objects as bad as

$$C_{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6} = \int d^N\tilde{k} \frac{k_{\mu_1} k_{\mu_2} k_{\mu_3} k_{\mu_4} k_{\mu_5} k_{\mu_6}}{\mathrm{I\ II\ III}} , \tag{2.17}$$

which would be very tedious due to the large number of available tensors. However, if one first multiplies terms out and does Dirac algebra, simplifications occur and only the integrals evaluated above are seen to be needed.

As no box diagrams appear here, integrals with four denominator factors are not necessary. Such objects were considered in Appendix F of Ref. [47]. Clearly this method can be extended to arbitrary numbers of denominator factors; the only obstacles are sufficient patience to do all the algebra, and the difficulty in performing the general n-factor scalar loop integrals.

Scalar loop integrals through four denominator factors were studied in Ref. [48]. Assuming the timelike momenta and nonvanishing real masses of the current calculation, I rederived A_0, B_0, and C_0 and present the results here (keeping pole and finite parts only):

$$A_0(m_1) = \mu^{(4-N)/2} \frac{im_1^2}{16\pi^2} [\frac{2}{4-N} + \log 4\pi + 1 - \gamma_E - \log\frac{m_1^2}{\mu^2}] , \qquad (2.18)$$

where γ_E is the Euler number = 0.5772...;

$$B_0(\ell,m_1,m_2) = \mu^{(4-N)/2} \frac{i}{16\pi^2} [\frac{2}{4-N} + \log 4\pi - \gamma_E - \log\frac{\ell^2}{\mu^2}$$

$$- \sum_{j=1}^{2} \{(1-x_j)\log(1-x_j) + x_j\log(-x_j) - 1\}] , \qquad (2.19)$$

where

$$x_{1,2} = \frac{(\ell^2+m_1^2-m_2^2) \pm \sqrt{(\ell^2)^2+m_1^4+m_2^4-2\ell^2m_1^2-2\ell^2m_2^2-2m_1^2m_2^2+4i\ell^2\varepsilon}}{2\ell^2} ;$$

$$C_0(\ell,s,m_1,m_2,m_3) = \mu^{(4-N)/2} (\frac{-i}{16\pi^2}) (\frac{1}{C + 2\alpha B}) \Sigma , \qquad (2.20)$$

where

$$\Sigma = [Sp(\frac{y_1}{y_1-z_1^i}) - Sp(\frac{y_1-1}{y_1-z_1^i}) + Sp(\frac{y_1}{y_1-z_2^i}) - Sp(\frac{y_1-1}{y_1-z_2^i})]$$

$$- [Sp(\frac{y_2}{y_2-z_1^{ii}}) - Sp(\frac{y_2-1}{y_2-z_1^{ii}}) + Sp(\frac{y_2}{y_2-z_2^{ii}}) - Sp(\frac{y_2-1}{y_2-z_2^{ii}})]$$

$$+ [Sp(\frac{y_3}{y_3-z_1^{iii}}) - Sp(\frac{y_3-1}{y_3-z_1^{iii}}) + Sp(\frac{y_3}{y_3-z_2^{iii}}) - Sp(\frac{y_3-1}{y_3-z_2^{iii}})]$$

$$z_{1,2}^{i} = \frac{(\ell^2+m_2^2-m_1^2) \pm \sqrt{(\ell^2)^2+m_2^4+m_1^4-2\ell^2m_2^2-2\ell^2m_1^2-2m_1^2m_2^2+4i\ell^2\varepsilon}}{2\ell^2}$$

$$z_{1,2}^{ii} = \frac{((\ell+s)^2+m_3^2-m_1^2) \pm \sqrt{((\ell+s)^2)^2+m_1^4+m_3^4-2(\ell+s)^2m_1^2-2(\ell+s)^2m_3^2 -2m_1^2m_3^2+4i(\ell+s)^2\varepsilon}}{2(\ell+s)^2}$$

$$z^{iii}_{1,2} = \frac{(s^2+m_3^2-m_2^2) \pm \sqrt{(s^2)^2+m_3^4+m_2^4-2s^2m_2^2-2s^2m_3^2-2m_2^2m_3^2+4is^2\varepsilon}}{2s^2}$$

$$y_1 = y_0 + \alpha \ , \ y_2 = \frac{y_0}{1-\alpha} \ , \ y_3 = -\frac{y_0}{\alpha}$$

$$y_0 = -\frac{D + \alpha E}{C + 2\alpha B}$$

and α is a root of

$$B\alpha^2 + C\alpha + A = 0$$

with

$$A = s^2, \ B = \ell^2, \ C = 2\ell s,$$

$$D = -(s^2 + m_3^2 - m_2^2), \ E = -(\ell^2 + 2\ell s + m_2^2 - m_1^2)$$

and the Spence function is

$$Sp(z) = -\int_0^z \frac{\log(1-t)}{t}\, dt \ .$$

Notice that in doing the integrations I have retained the $i\varepsilon$'s in the denominators, since these originate from the usual Feynman prescription for propagators inside the diagrams. As a practical matter this is crucial in order to specify correctly which side of the cut of the complex logarithm or Spence function one is on.

III. Checks on Calculation

The calculation is very lengthy so that checks are indispensable to guard against errors. They are also useful in verifying the validity of certain calculational procedures.

Many checks suggest themselves at different stages of the work. These are too numerous to be discussed here. The checks presented in this chapter are the most global checks in the sense that they verify large amounts of work and/or important assumptions. In most cases, however, the algebraic derivations are far too long to be presented here.

Firstly, there is the question of pole cancellation. As noted earlier, the calculation is expected to be finite; all $1/(N-4)$ terms are expected to cancel. Vanishing of the poles with M_J (internal quark mass) -independent coefficients is rather trivial due to GIM. It is less trivial that the M_J - dependent pole coefficients sum to 0. These can come about from Dirac algebra as well as integrals such as

$$\int d^N k \left(\frac{1}{k^2-M_J^2+i\varepsilon}\right) \sim \frac{M_J^2}{N-4} + O(N-4)^0 \ . \tag{3.1}$$

Exact pole cancellation was verified algebraically.

It is interesting to note that certain pole parts require the tree relation $M_W = M_Z \cos\theta_W$ in order to vanish. Any $O(g^2)$ correction to this relation is only relevant for the $O(g^5)$, i.e. two loop, $Z \to t\bar{c}$, etc., amplitudes and therefore irrelevant here.* This proportionality is

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* It should be noted that the relation $M_W = M_Z \cos\theta_W$ can be preserved by the choice of renormalization scheme anyway.

probably indicative of the dangers of calculating in unitary gauge off the mass shell, since M_Z^2 appears as the square of the 4-momentum of the Z external leg.

Secondly, the behavior of the $SU(2)_L \times U(1)$ theory under CP may be utilized as a check. In the three generation case, I will choose the KM matrix to be real. In the four generation case, real matrices will be included in the variation. In these cases, the calculation is performed in a completely CP-invariant theory. This requires that $\Gamma(Z \to t\bar{c}) = \Gamma(Z \to \bar{t}c)$, etc. This is because

$$\begin{aligned} &\langle t(\vec{p},h_1)\ \bar{c}(-\vec{p},h_2) \mid S \mid Z^0(\vec{0},s)\rangle \\ &= \langle t(\vec{p},h_1)\ \bar{c}(-\vec{p},h_2) \mid (CP)^{-1}(CP)S(CP)^{-1}(CP) \mid Z^0(\vec{0},s)\rangle \\ &\sim \langle \bar{t}(-\vec{p},-h_1)c(\vec{p},-h_2) \mid S \mid Z^0(\vec{0},s)\rangle , \end{aligned} \tag{3.2}$$

where S is the Z^0 spin and h_i are the fermion helicities. Since $Z \to t\bar{c}$ and $Z \to \bar{t}c$, etc., are computed independently from each other, this provided a numerical check that was verified. If the KM matrix has non-vanishing phases then the CP related widths need not be the same; this will be discussed further in Chapter VI.

As a third check, I will discuss some Ward identities. It turns out that the relevant Z couplings to the order of interest are given in unitary gauge by the sum of a conserved vector current, and a partially conserved axial vector current. This leads to Ward identities that should be satisfied by the $Z \to t\bar{c}$, etc., decay amplitudes.

The subsequent discussion is in reference to the unitary gauge Lagrangian given in (2.4), and notation is as described immediately following it. Of course, this Lagrangian is invariant under the local U(1) gauge symmetry of QED, and therefore under global U(1) QED. Only the global U(1) is required to show electric charge conservation. Now although the local $SU(2)_L \times U(1)_Y$ gauge symmetry of the original Lagrangian has been destroyed by fixing the unitary gauge, there are residual continuous symmetries left associated with the Z couplings.

Consider the global U(1) transformations, with real parameters α and β, given by

	$U^V(1)$	$U^A(1)$
$t_i \rightarrow$	$e^{i\alpha r}\, t_i$	$e^{i\beta u\gamma_5}\, t_i$
$b_i \rightarrow$	$e^{i\alpha s}\, b_i$	$e^{-i\beta u\gamma_5}\, b_i$
$W_\mu^+ \rightarrow$	$e^{i\alpha(r-s)}\, W_\mu^+$	$e^{-2i\beta u}\, W_\mu^+$, (3.3)

where $r = -\frac{g}{4c_W}(-1 + \frac{8}{3}s_W^2)$, $s = -\frac{g}{4c_W}(1 - \frac{4}{3}s_W^2)$, and $u = -\frac{g}{4c_W}$. $\mathcal{L}$ is invariant under $U_V(1)$; $U_A(1)$ is broken only by quark masses and Yukawa couplings to Higgs. The associated currents are conserved in the case of $U_V(1)$ and partially conserved in the case of $U^A(1)$, and are given by Noether's theorem:

$$J_V^\mu = \sum_i \left(\frac{g}{4c_W}\right) [\,(-1+\frac{8}{3}s_W^2)\bar{t}_i\gamma^\mu t_i + (1-\frac{4}{3}s_W^2)\bar{b}_i\gamma^\mu b_i]$$

$$+ \{i(\frac{g}{2c_W} - gc_W)W_\nu^+ [\,(\partial^\mu W^{\nu-} - \partial^\nu W^{\mu-}) - ie(W^{\mu-}A^\nu - W^{\nu-}A^\mu)$$

$$+ igc_W(W^{\mu-}Z^\nu - W^{\nu-}Z^\mu)]\} + \{\text{herm. conj. of vector term}\} \quad (3.4)$$

$$\partial_\mu J_V^\mu = 0 \quad (3.5)$$

$$J_A^\mu = \sum_i \left(\frac{g}{4c_W}\right) [\,\bar{t}_i\gamma^\mu\gamma_5 t_i - \bar{b}_i\gamma^\mu\gamma_5 b_i]$$

$$+ \{\text{same vector term + h.c. as } J_V^\mu \text{ except } (\frac{g}{2c_W} - gc_W) \to (-\frac{g}{2c_W})\}$$

$$(3.6)$$

$$\partial_\mu J_A^\mu = \frac{ig}{2c_W} \sum_i \{\bar{t}_i(m_{t_i} + \frac{m_{t_i}}{v}H)\gamma_5 t_i - \bar{b}_i(m_{b_i} + \frac{m_{b_i}}{v}H)\gamma_5 b_i\} \equiv iP^5 \,. \quad (3.7)$$

The sum of these two currents gives almost the full current coupling to the Z, but not quite. For one thing, it misses the triple vector term $\sim \partial Z(WW-WW)$. However, integration by parts, dropping surface terms, produces an equivalent Lagrangian and gives terms $\sim Z\partial(WW-WW)$, leading one to define

$$J^\mu_{\text{deriv}} = igc_W\partial_\nu(W^{\nu+}W^{\mu-} - W^{\mu+}W^{\nu-}) \quad (3.8)$$

$$\partial_\mu J^\mu_{\text{deriv}} = 0 \,, \quad (3.9)$$

where (3.9) follows from index symmetry. Now one can define

$$\tilde{J}_V^\mu = J_V^\mu + (1 - \frac{1}{2c_W^2}) J^\mu_{deriv}$$

$$\tilde{J}_A^\mu = J_A^\mu + \frac{1}{2c_W^2} J^\mu_{deriv}$$

$$J_Z^\mu = \tilde{J}_V^\mu + \tilde{J}_A^\mu \ , \tag{3.10}$$

then

$$\partial_\mu \tilde{J}_V^\mu = 0; \quad \partial_\mu \tilde{J}_A^\mu = iP^5 \ . \tag{3.11}$$

J_Z^μ is now the entire source current for the Z except for the ZZH and ZZHH couplings. The Euler-Lagrange equations for the evolution of the Z field have the standard form

$$\partial_\xi \frac{\partial \mathcal{L}}{\partial(\partial_\xi Z_\mu)} - \frac{\partial \mathcal{L}}{\partial Z_\mu} = 0 \ . \tag{3.12}$$

This leads straightforwardly to the equation of motion for the Z field

$$(\Box + M_Z^2) Z^\mu - \partial^\mu(\partial \cdot Z) = - J_Z^\mu - \frac{1}{4}(2vH + H^2) \frac{g^2}{c_W^2} Z^\mu \equiv \mathcal{S}^\mu \ . \tag{3.13}$$

The above equations imply Ward identities obeyed by the $Z \to t\bar{c}$, etc., decay amplitudes, generically Z (momentum p, polarization ε) $\to$ A (p_A, spin s_A) + $\bar{B}$ (p_B, s_B). The decay amplitude is

$$S_{fi} \sim \int d^4x d^4y_A d^4y_B e^{-i(px-p_Ay_A-p_By_B)} \bar{u}_A(p_A,s_A) \overrightarrow{(i\not{\partial}_{y_A} - M_A)}$$

$$<0 \mid T(\varepsilon_\mu(p)\, \mathcal{S}^\mu(x)\bar{\Psi}_B(y_B)\Psi_A(y_A)) \mid 0> \overleftarrow{(-i\not{\partial}_{y_B} - M_B)}\ v_B(p_B,s_B)\ . \tag{3.14}$$

Now ignore the Higgs term in $\mathcal{S}^\mu$; to one loop order it is irrelevant. Ward identities can then be derived by replacing ε_μ by p_μ in (3.14), integrating by parts, noting the vanishing of the equal time commutator terms (resulting from time derivatives of Θ functions in the time ordered product) since on-shell $A \to B$ propagation is not allowed, and using (3.10) and (3.11):

<u>Vector Ward Identity</u>

$$\{S_{fi} \text{ with } \varepsilon_\mu \mathcal{S}^\mu \to p_\mu(-\tilde{J}^\mu_V)\} = 0 \tag{3.15}$$

<u>Axial Ward Identity</u>

$$\{S_{fi} \text{ with } \varepsilon_\mu \mathcal{S}^\mu \to p_\mu(-\tilde{J}^\mu_A)\} = \{S_{fi} \text{ with } \varepsilon_\mu \mathcal{S}^\mu \to (-P^5)\}\ , \tag{3.16}$$

where P^5 has been defined in (3.7).

These identities can be represented diagrammatically by figure 2. Notice that checking the axial vector Ward identity requires computing an additional set of diagrams giving the "decay of a fictitious pseudoscalar" into $t\bar{c}$. Note also that the coupling of the density P^5 is reminiscent of the coupling of the neutral Higgs ghost in 't Hooft-

Feynman gauge. I do not believe this to be accidental. In unitary gauge the polarization sum for the Z is given by

$$\sum_{\text{pol}} \varepsilon_\mu(p)\varepsilon_\nu^*(p) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{M_Z^2} . \tag{3.17}$$

In the Z^0 rest frame $p^\mu = (M_Z, \vec{0})$ so that

$$\sum_{\text{pol}} \varepsilon_\mu(p)\varepsilon_\nu^*(p) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \tag{3.18}$$

I.e., the $p_\mu p_\nu$ part has eliminated the unphysical timelike polarization implicit in the $g_{\mu\nu}$ part. When $p_\mu p_\nu$ hits the axial current coupling to Z, $\tilde{J}_A^\mu$, the Ward identity indicates that this is equivalent to inserting the P^5 density.

In 't Hooft-Feynman gauge, however, the Z propagator and polarization sum are $\sim g_{\mu\nu}$ only. Put another way, relative to the unitary gauge they contain an anomalous part $\sim (-p_\mu p_\nu)$, which cancels the corresponding unitary gauge bit. Something else needs to be done to cancel the unphysical timelike polarization in the on-shell Z rest frame. This is the function performed by the Higgs ghost in 't Hooft-Feynman gauge. The physical Z is really a five-component object (Z_μ, Φ) where Φ is the Higgs ghost. Since the Higgs ghost decay diagrams play the same role in 't Hooft-Feynman gauge as the $p_\mu p_\nu$ part of the polarization sum

in unitary gauge (i.e. cancelling g_{00} in the rest frame), it is not surprising that their couplings are similar. In this sense the $p_\mu p_\nu$ part of the polarization sum in unitary gauge can be regarded as the "ghost of the Higgs ghost."[50]

Checking that these Ward identities are satisfied is an important verification of the results; in particular it is helpful for checking the choice of γ_5 of (2.6). In practice,since one straightaway attacks the diagrams of Figure 1, a strategy is needed to keep track of the vector and axial vector couplings separately if one wishes to individually check each Ward identity and not just the sum of the two. I did this by "tagging" the respective couplings with arbitrary parameters that were set to 1 after the checking was done. Both the vector and axial vector Ward identities were found to be exactly satisfied algebraically.

The considerations discussed in this section on Ward identities are sufficient to one-loop order for the decay amplitude, the subject of this thesis. As we have seen, to one-loop order no renormalization was required. An all orders discussion may be more problematical and is uninteresting from a phenomenological point of view. Here I will just briefly comment on a couple of aspects of higher orders. One should keep separate track of the Higgs term in $\mathcal{S}^\mu$. Also, eventually the leptons should be included, since their couplings will be of the same order as the quarks and they are required to cancel anomalies. Their charges are deduced from their couplings to Z:

	$U^V(1)$ charge	$U^A(1)$ charge
$\ell_i(e,\mu,\tau,\ldots)$	$\frac{g}{c_W}(s_W^2 - \frac{1}{4})$	$\frac{g}{4c_W}$
$\nu_i(\nu_e,\nu_\mu,\nu_\tau,\ldots)$	$\frac{g}{4c_W}$	$-\frac{g}{4c_W}$,

(3.19)

and the fermion sectors of the currents expand to include the leptons.

The fourth and final check that I will discuss in this chapter is that of a separate computation of the imaginary part of the decay amplitude. As indicated earlier in this chapter, the KM matrix is chosen to be real in the three generation case, and real part of the time in the four generation case. When the KM matrix is real, the theory is invariant under time-reversal (with the appropriate choice of time-reverse phases for the fields[51]). This means that the imaginary part of the $Z \to t\bar{c}$ decay amplitude (and related amplitudes) is related to the absorptive part via

$$2\mathrm{Im}\langle t\bar{c} \mid T \mid Z^0\rangle = \sum_n \langle n \mid T \mid t\bar{c}\rangle^* \langle n \mid T \mid Z^0\rangle, \quad (3.20)$$

where the S matrix is $S = 1 + iT$. (3.20) is derived by using the unitarity of the S matrix, $S^\dagger S = 1$.

This check was confined to the simplest case of $M_T < M_W + M_J$, where the only cut that contributes is shown in Figure 3A. For M_T above the W-J threshold, several other cuts such as the one in Figure 3C will contribute, and I have not attempted to compute them. Of course,

for $M_J = M_{b'}$, if $M_{b'} > M_Z/2$ the cut of Figure 3A does not exist either, and that quark's contribution to the imaginary part should be consistent with 0. What has been calculated is the sum over the intermediate phase space of the product of the two diagrams shown in Figure 3B (shown for the $t\bar{c}$ case). All masses are kept without approximation, as before. As this obviously requires no regularization, it is another good check on our dimensional scheme, as well as on the algebra and numerics.

Even here the algebra is long, due to so many independent and variable masses. The methods of integration of Chapter II are used to do the angular integrals, only using O(3) tensors and δ_{ij} as a projector, instead of Lorentz tensors and $g_{\mu\nu}$. Although the integrals occurring here are trivial, I expect that this method would be useful in computing very complicated phase space integrals.

I will now quote the result, which is reduced to a similar form to that given for the amplitude $i\mathcal{T}$ in Chapter IV:

$$\mathrm{Im}\,\mathcal{T} = \sum_J U^{KM}_{AJ}(U^{KM})^{\dagger}_{JB}\bar{u}_A(p_A,s_A)\ \{G_1\varepsilon\cdot p_A + G_2\not{\varepsilon} + G_3\varepsilon\cdot p_A\gamma_5 + G_4\not{\varepsilon}\gamma_5\}$$

$$v_B(p_B,s_B) \quad , \qquad (3.21)$$

where Z(momentum $p_A + p_B$, polarization ε) $\rightarrow$ A(p_A, spin s_A) + $\bar{B}$(p_B, s_B), and the G_i are functions of the masses. $M_W = M_Z\cos\theta_W$ was used to eliminate $\cos\theta_W$. Of course U is real here so $U^{\dagger} = U^t$. In practice one performs the check numerically by comparing the G_i to the imaginary parts of the corresponding functions for the full amplitude. One might worry that the right hand side of (3.21) is not real. In usual Dirac repre-

sentation, choosing $\vec{p}_A = -\vec{p}_B$ along the z-axis and taking an irreducible spherical tensor basis for the Z polarizations eliminates this question.

Define four functions A_i by

$$G_i = \left(- \frac{g^3}{384\pi^2 M_W^3} \right) A_i \ , \ i = 1\text{-}4. \tag{3.22}$$

Some care needs to be taken to obtain the correct overall sign of the fermion rescattering diagram by studying the Wick contractions. The A_i are as follows:

$$A_1 = \left(\frac{M_B+M_A}{M_Z^4} \right) \{ (M_Z^2+2M_W^2) [(2C_Q+2B)M_W^2M_Z^2+(C_Q-B)M_AM_BM_Z^2-2C_QM_B^2M_W^2$$

$$+4C_QM_AM_BM_W^2-2C_QM_A^2M_W^2-C_QM_AM_B^3+2C_QM_A^2M_B^2-C_QM_A^3M_B] -M_J^2 [(2C_Q-3B+A)M_Z^4$$

$$+(6B-2C_Q-4A)M_W^2M_Z^2+(3B-2C_Q)M_B^2M_Z^2+(4C_Q-6B)M_AM_BM_Z^2+(3B-2C_Q)M_A^2M_Z^2$$

$$+2C_QM_B^2M_W^2-4C_QM_AM_BM_W^2+2C_QM_A^2M_W^2] \}$$

$$A_2 = \left(\frac{1}{2M_Z^4} \right) \{ (M_Z^2+2M_W^2) \tfrac{1}{2} [(2C_Q+4B+2A)M_W^2M_Z^4+(C_Q-2B+A)M_AM_BM_Z^4$$

$$-(4C_Q+4B)M_B^2M_W^2M_Z^2-8BM_AM_BM_W^2M_Z^2-(4C_Q+4B)M_A^2M_W^2M_Z^2+8C_DM_W^2M_Z^2$$

$$+(2B-2C_Q)M_AM_B^3M_Z^2+4BM_A^2M_B^2M_Z^2+(2B-2C_Q)M_A^3M_BM_Z^2+4C_DM_AM_BM_Z^2+2C_QM_B^4M_W^2$$

$$-4C_QM_A^2M_B^2M_W^2+2C_QM_A^4M_W^2+C_QM_AM_B^5-2C_QM_A^3M_B^3+C_QM_A^5M_B] - M_J^2 [(C_Q-2B+A)M_Z^6$$

$$+(2B+7A-C_Q)M_W^2M_Z^4-2AM_J^2M_Z^4+(5B-A-2C_Q)M_B^2M_Z^4+(2A-2B)M_AM_BM_Z^4$$

$$+(5B-2C_Q-A)M_A^2M_Z^4+4C_DM_Z^4-8AM_W^4M_Z^2-4AM_J^2M_W^2M_Z^2+(2C_Q-2B+4A)M_B^2M_W^2M_Z^2$$

$$+(4A-4B)M_AM_BM_W^2M_Z^2+(2C_Q-2B+4A)M_A^2M_W^2M_Z^2-4C_DM_W^2M_Z^2+(C_Q-3B)M_B^4M_Z^2$$

$$+(6B-2C_Q)M_A^2M_B^2M_Z^2+(C_Q-3B)M_A^4M_Z^2-C_QM_B^4M_W^2+2C_QM_A^2M_B^2M_W^2-C_QM_A^4M_W^2\ ]\ \}$$

$$A_3 = (A_1 \text{ with } M_A \to -M_A)$$

$$A_4 = (-A_2 \text{ with } M_A \to -M_A)\ . \tag{3.23}$$

A, B, C_D, and C_Q are functions arising from the phase space integral; they are

$$A = \frac{\pi\xi}{x}\log\left(\frac{1+x}{1-x}\right)$$

$$B = \frac{4\pi\ell\xi^2}{x^2}\left[\,1 - \frac{1}{2x}\log\left(\frac{1+x}{1-x}\right)\right]$$

$$C_D = \frac{\pi\ell^2\xi}{x^2}\left[\,1 - \frac{1}{2x}\log\left(\frac{1+x}{1-x}\right) + \frac{x}{2}\log\left(\frac{1+x}{1-x}\right)\right]$$

$$C_Q = -\frac{12\pi\ell^2\xi^3}{x^4}\left[\,1 - \frac{1}{2x}\log\left(\frac{1+x}{1-x}\right) + \frac{x}{6}\log\left(\frac{1+x}{1-x}\right)\right]\ , \tag{3.24}$$

where

ℓ = magnitude of J-quark 3-momentum

$$= \frac{1}{2}(M_Z^2 - 4M_J^2)^{\frac{1}{2}}$$

$$\xi = \frac{2\ell}{(M_A^2 + M_B^2 + 2M_J^2 - M_Z^2 - 2M_W^2)}$$

$$x = 2\xi\,|\vec{p}_A| = 2\xi\,|\vec{p}_B|$$

$$= \left(\frac{\xi}{M_Z}\right)\left(M_Z^4 + M_A^4 + M_B^4 - 2M_A^2M_Z^2 - 2M_B^2M_Z^2 - 2M_A^2M_B^2\right)^{\frac{1}{2}} .$$

The term under the square root in x is a phase space factor that frequently appears. It factors into an often convenient form

$$(M_Z + M_A + M_B)\ (M_Z + M_A - M_B)\ (M_Z - M_A + M_B)\ (M_Z - M_A - M_B) .$$

For $x \ll 1$ strong cancellations can occur. To retain numerical precision, it is preferable to expand the logarithms in (3.24):

$$A = 2\pi\xi \sum_{n=0}^{\infty} \frac{x^{2n}}{2n+1}$$

$$B = -4\pi\ell\xi^2 \sum_{n=1}^{\infty} \frac{x^{2n-2}}{2n+1}$$

$$C_D = 2\pi\ell^2\xi \sum_{n=1}^{\infty} \frac{x^{2n-2}}{4n^2-1}$$

$$C_Q = 16\pi\ell^2\xi^3 \sum_{n=2}^{\infty} \left(\frac{n-1}{4n^2-1}\right) x^{2n-4} . \qquad (3.25)$$

Notice that all these start out O(1); i.e., what look naively like singularities in x have all cancelled.

Having computed the right-hand side of (3.20), this can be compared to the left-hand side which is computed using the final expression

for the total amplitude given in Chapter IV. This provides a numerical check which was verified. When M_T rises above the W-J threshold, the agreement disintegrates as expected, indicating that other uncalculated cuts are contributing.

IV. Algebraic Results

In this chapter I present the full algebraic results for the decay amplitude and width. This may be discussed independently of the number of generations. The sum of diagrams in Figure 1 for Z(momentum $p_A + p_B$, polarization ε) $\to$ A(p_A, spin s_A) + $\bar{B}$(p_B, s_B) is

$$i\mathcal{T} = \sum_J U^{KM}_{AJ}(U^{KM})^{\dagger}_{JB}\bar{u}_A(p_A,s_A)\{F_1\varepsilon\cdot p_A + F_2\not{\varepsilon} + F_3\varepsilon\cdot p_A\gamma_5 + F_4\not{\varepsilon}\gamma_5\}$$

$$v_B(p_B,s_B) \ , \qquad (4.1)$$

where the F_k are functions of the five masses M_J (internal quarks), M_A, M_B, M_Z, and M_W. Define E_k functions by

$$F_k = \frac{ig^3M_Z}{768\pi^2M_W^3}E_k \ , \quad k=1-4, \qquad (4.2)$$

where g is the $SU(2)_L$ coupling $g = e/\sin\theta_W$. I am at all times replacing $\cos\theta_W$ by its tree value M_W/M_Z. The E_k are written in turn as

$$E_k = \sum_{\ell=1}^{12} \text{FINITE}(k,\ell) \times \text{TERM}(\ell) \ , \qquad (4.3)$$

where FINITE(k,ℓ) is a 4 $\times$ 12 = 48 element array that is presented at the end of this chapter, and TERM(ℓ) is a 12 element array given below:

TERM(1) = 1 ;

$$\mathrm{TERM}(1+\alpha) = -\log\left(\frac{M_\alpha^2}{\mu^2}\right) \quad , \quad \alpha = 1\text{-}5;$$

$$\mathrm{TERM}(6+\beta) = -\sum_{n=\pm} \left[(1-x_{\beta,n})\log(1-x_{\beta,n}) + x_{\beta,n}\log(-x_{\beta,n}) \right] \quad , \quad \beta = 1\text{-}4;$$

$$\mathrm{TERM}(10+\gamma) = \left\{ \sum_{r=1}^{3} \sum_{n=\pm} (-1)^r \left[\mathrm{Sp}\left(\frac{y_{\gamma,r} - 1}{y_{\gamma,r} - z_{\gamma,r,n}}\right) - \mathrm{Sp}\left(\frac{y_{\gamma,r}}{y_{\gamma,r} - z_{\gamma,r,n}}\right) \right] \right\} / P(A,B)$$

$$, \quad \gamma = 1\text{-}2, \qquad (4.4)$$

where μ = mass scale for the N-dimensional coupling constant;

$$M_1 = M_J$$

$$M_2 = M_W$$

$$M_3 = M_A$$

$$M_4 = M_B$$

$$M_5 = M_Z \ ;$$

$$x_{1,n} = \frac{(M_W^2 + M_A^2 - M_J^2) \pm \sqrt{(M_W+M_A+M_J)\ (M_W+M_A-M_J)\ (M_W-M_A+M_J)\ (M_W-M_A-M_J) + 4iM_A^2\varepsilon}}{2M_A^2}$$

$$x_{2,n} = x_{1,n} \text{ except } M_A \to M_B$$

$$x_{3,n} = \frac{M_Z \pm \sqrt{(M_Z+2M_W)\ (M_Z-2M_W) + 4i\varepsilon}}{2M_Z}$$

$$x_{4,n} = x_{3,n} \text{ except } M_W \to M_J \ .$$

ε is an infinitesimal, with dimensions of mass squared, originating from the $i\varepsilon$'s in the propagator denominators. P(A,B) is given below.

Now define

$$C_1(A,B) = M_Z^2 + M_A^2 - M_B^2$$

$$C_2(A,B) = M_Z^2 - M_A^2 - M_B^2$$

$$P(A,B) = \sqrt{(M_Z+M_A+M_B)\ (M_Z+M_A-M_B)\ (M_Z-M_A+M_B)\ (M_Z-M_A-M_B)} \quad . \quad (4.5)$$

Then

$$y_{1,1} = \frac{2M_A^2M_Z^2 + (M_A^2+M_J^2-M_W^2)\ (P(A,B) - C_1(A,B))}{2M_A^2\ P(A,B)}$$

$$y_{1,2} = \frac{-2M_A^2M_Z^2 + (M_B^2+M_J^2-M_W^2-M_Z^2)\ (P(A,B) - C_1(A,B))}{P(A,B)\ (P(A,B) - C_2(A,B))}$$

$$y_{1,3} = \frac{-2M_A^2M_Z^2 + (M_B^2+M_J^2-M_W^2-M_Z^2)\ (P(A,B) - C_1(A,B))}{P(A,B)\ (P(A,B) - C_1(A,B))}$$

$$y_{2,1} = \frac{2M_A^2\ (M_B^2+M_J^2-M_W^2) + (M_A^2+M_J^2-M_W^2)\ (P(A,B) + C_2(A,B))}{2M_A^2\ P(A,B)}$$

$$y_{2,2} = \frac{2M_A^2\ (M_J^2-M_B^2-M_W^2) + (M_Z^2+M_J^2-M_B^2-M_W^2)\ (P(A,B) + C_2(A,B))}{P(A,B)\ (P(A,B) + C_1(A,B))}$$

$$y_{2,3} = \frac{2M_A^2\ (M_J^2-M_B^2-M_W^2) + (M_Z^2+M_J^2-M_B^2-M_W^2)\ (P(A,B) + C_2(A,B))}{P(A,B)\ (P(A,B) + C_2(A,B))}$$

$$z_{1,1,n} = 1 - x_{1,-n}$$

$$z_{1,2,n} = 1 - x_{2,-n}$$

$$z_{1,3,n} = x_{4,n}$$

$$z_{2,1,n} = z_{1,1,n}$$

$$z_{2,2,n} = x_{3,n}$$

$$z_{2,3,n} = x_{2,n}\ .$$

μ is a carry-over from the dimensional regularization; as no renormalization was performed, the final result should be independent of it. This is reflected in the vanishing of the sum of the coefficients of the $\log(M_\alpha^2/\mu^2)$ terms, which was checked. The ε pieces are important for determining which side of the cuts of the complex logarithm or Spence function one is on.

I now discuss the formula for the width. The total decay width, averaged over Z polarizations and summed over A, B spins and momenta, is

$$\Gamma(Z \to A\bar{B}) = \left(\frac{P(A,B)}{16\pi M_Z^5}\right) \tau\ , \tag{4.6}$$

where τ is proportional to the trace, and P(A,B) was defined in (4.5). The role of P(A,B) as a phase space suppression factor may be inferred from the formula. The quarks are the usual color triplets; this requires an extra factor of 3 which has been included in (4.6).

Introduce additional notation:

$$w_{1,2,3,4} = M_Z + M_B + M_A \ , + - \ , - + \ , - -$$

$$u_{1,2} = M_B + M_A \ , \ M_B - M_A \quad .$$

$$F_{kR} \ , \ F_{kI} = \text{Re} \ , \ \text{Im of} \ (\sum_J U_{AJ} U^{\dagger}_{JB} F_k) \ , \quad k = 1\text{-}4. \tag{4.7}$$

Then

$$\begin{aligned} \tau = \ & \frac{1}{2} \{ P^2(A,B) [\ w_1 w_4 (F_{1R}^2 + F_{1I}^2) + w_2 w_3 (F_{3R}^2 + F_{3I}^2) \\ & - 4u_1 (F_{1R} F_{2R} + F_{1I} F_{2I}) + 4u_2 (F_{3R} F_{4R} + F_{3I} F_{4I})] \\ & + w_2 w_3 (8M_Z^2 + 4u_1^2)(F_{2R}^2 + F_{2I}^2) + w_1 w_4 (8M_Z^2 + 4u_2^2)(F_{4R}^2 + F_{4I}^2) \} \ . \end{aligned} \tag{4.8}$$

One might worry that I have averaged over Z polarization states, whereas not all Z polarization states will be democratically produced in e^+e^- annihilation due to the helicity conserving nature of V and A type interactions. However, this is of no consequence for the total decay widths which must be independent of the Z polarization due to rotational invariance. One could just as well fix the Z polarization

arbitrarily and get the same answer.

Finally, I return to the FINITE(k,ℓ) array. The formulas below are reproduced from computer output created using MACSYMA,[53] a computer algebraic manipulation program. The notation is as follows:

$$\mathrm{FINITEk}_{\ell} = \mathrm{FINITE}(k,\ell) \ ;$$

$$\mathrm{MA} = M_A,\ \mathrm{MB} = M_B,\ \mathrm{MZ} = M_Z,\ \mathrm{MW} = M_W,\ \mathrm{MJ} = M_J \ . \tag{4.9}$$

I now present FINITE(k,ℓ):

$$\mathrm{FINITE1}_1 = (((2\,MB + 2\,MA)\,MJ^2\,MW^2 + (2\,MB + 2\,MA)\,MJ^4$$

$$+ (18\,MA\,MB^2 + 18\,MA^2\,MB)\,MJ^2)\,MZ^2 + (-8\,MB - 8\,MA)\,MJ^2\,MW^4$$

$$+ ((-8\,MB - 8\,MA)\,MJ^4 + (-24\,MA\,MB^2 - 24\,MA^2\,MB)\,MJ^2)\,MW^2)$$

$$/(MA\,MB\,MZ^4 + (-MA\,MB^3 - 2\,MA^2\,MB^2 - MA^3\,MB)\,MZ^2)$$

$$\mathrm{FINITE1}_2 = -(((4\,MB + 4\,MA)\,MJ^2\,MW^2 + (2\,MB + 2\,MA)\,MJ^4$$

$$+ (8\,MA\,MB^2 + 8\,MA^2\,MB)\,MJ^2)\,MZ^2 + (-16\,MB - 16\,MA)\,MJ^2\,MW^4$$

$$+ ((-8\,MB - 8\,MA)\,MJ^4 + (-8\,MA\,MB^2 - 8\,MA^2\,MB)\,MJ^2)\,MW^2)$$

$$/(MA\,MB\,MZ^4 + (-MA\,MB^3 - 2\,MA^2\,MB^2 - MA^3\,MB)\,MZ^2)$$

$$\mathrm{FINITE1}_3 = \frac{(2\,MB + 2\,MA)\,MJ^2\,MW^2\,MZ^2 + (-8\,MB - 8\,MA)\,MJ^2\,MW^4}{MA\,MB\,MZ^4 + (-MA\,MB^3 - 2\,MA^2\,MB^2 - MA^3\,MB)\,MZ^2}$$

$$\mathrm{FINITE1}_4 = ((2\,MJ^2\,MW^2 + 2\,MJ^4 + (-8\,MA\,MB - 14\,MA^2)\,MJ^2)\,MZ^6$$

$$+ (-8\,MJ^2\,MW^4 + ((-4\,MB^2 + 2\,MA\,MB + 22\,MA^2)\,MJ^2 - 8\,MJ^4)\,MW^2$$

$$+ (-4\,MB^2 - 6\,MA\,MB - 10\,MA^2)\,MJ^4 + (16\,MA\,MB^3 + 16\,MA^2\,MB^2 - 38\,MA^3\,MB$$

$$- 14\,MA^4)\,MJ^2)\,MZ^4 + ((16\,MB^2 + 24\,MA\,MB + 40\,MA^2)\,MJ^2\,MW^4$$

$$+ ((16\,MB^2 + 24\,MA\,MB + 40\,MA^2)\,MJ^4 + (2\,MB^4 - 10\,MA\,MB^3 - 38\,MA^2\,MB^2$$

$$+ 50\,MA^3\,MB - 20\,MA^4)\,MJ^2)\,MW^2 + (2\,MB^4 + 6\,MA\,MB^3 + 2\,MA^2\,MB^2 - 6\,MA^3\,MB$$

$$- 4\,MA^4)\,MJ^4 + (-8\,MA\,MB^5 - 2\,MA^2\,MB^4 - 14\,MA^3\,MB^3 - 26\,MA^4\,MB^2 + 22\,MA^5\,MB$$

$$+ 28\,MA^6)\,MJ^2)\,MZ^2 + (-8\,MB^4 - 24\,MA\,MB^3 - 8\,MA^2\,MB^2 + 24\,MA^3\,MB + 16\,MA^4)$$

$$MJ^2\,MW^4 + ((-8\,MB^4 - 24\,MA\,MB^3 - 8\,MA^2\,MB^2 + 24\,MA^3\,MB + 16\,MA^4)\,MJ^4$$

$$+ (8\,MA\,MB^5 + 8\,MA^2\,MB^4 + 8\,MA^3\,MB^3 + 8\,MA^4\,MB^2 - 16\,MA^5\,MB - 16\,MA^6)\,MJ^2)$$

$$MW^2)/(MA\,MZ^8 + (-3\,MA\,MB^2 - 2\,MA^2\,MB - 3\,MA^3)\,MZ^6$$

$$+ (3\ MA\ MB^{4} + 4\ MA^{2}\ MB^{3} + 2\ MA^{3}\ MB^{2} + 4\ MA^{4}\ MB + 3\ MA^{5})\ MZ^{4}$$

$$+ (-\ MA\ MB^{6} - 2\ MA^{2}\ MB^{5} + MA^{3}\ MB^{4} + 4\ MA^{4}\ MB^{3} + MA^{5}\ MB^{2} - 2\ MA^{6}\ MB - MA^{7})$$

$$MZ^{2})$$

$$FINITE1_{5} = ((2\ MJ^{2}\ MW^{2} + 2\ MJ^{4} + (-\ 14\ MB^{2} - 8\ MA\ MB)\ MJ^{2})\ MZ^{6}$$

$$+ (-\ 8\ MJ^{2}\ MW^{4} + ((22\ MB^{2} + 2\ MA\ MB - 4\ MA^{2})\ MJ^{2} - 8\ MJ^{4})\ MW^{2}$$

$$+ (-\ 10\ MB^{2} - 6\ MA\ MB - 4\ MA^{2})\ MJ^{4} + (-\ 14\ MB^{4} - 38\ MA\ MB^{3} + 16\ MA^{2}\ MB^{2}$$

$$+ 16\ MA^{3}\ MB)\ MJ^{2})\ MZ^{4} + ((40\ MB^{2} + 24\ MA\ MB + 16\ MA^{2})\ MJ^{2}\ MW^{4}$$

$$+ ((40\ MB^{2} + 24\ MA\ MB + 16\ MA^{2})\ MJ^{4} + (-\ 20\ MB^{4} + 50\ MA\ MB^{3} - 38\ MA^{2}\ MB^{2}$$

$$- 10\ MA^{3}\ MB + 2\ MA^{4})\ MJ^{2})\ MW^{2} + (-\ 4\ MB^{4} - 6\ MA\ MB^{3} + 2\ MA^{2}\ MB^{2} + 6\ MA^{3}\ MB$$

$$+ 2\ MA^{4})\ MJ^{4} + (28\ MB^{6} + 22\ MA\ MB^{5} - 26\ MA^{2}\ MB^{4} - 14\ MA^{3}\ MB^{3} - 2\ MA^{4}\ MB^{2}$$

$$- 8\ MA^{5}\ MB)\ MJ^{2})\ MZ^{2} + (16\ MB^{4} + 24\ MA\ MB^{3} - 8\ MA^{2}\ MB^{2} - 24\ MA^{3}\ MB - 8\ MA^{4})$$

$$MJ^{2}\ MW^{4} + ((16\ MB^{4} + 24\ MA\ MB^{3} - 8\ MA^{2}\ MB^{2} - 24\ MA^{3}\ MB - 8\ MA^{4})\ MJ^{4}$$

$$+ (-\ 16\ MB^{6} - 16\ MA\ MB^{5} + 8\ MA^{2}\ MB^{4} + 8\ MA^{3}\ MB^{3} + 8\ MA^{4}\ MB^{2} + 8\ MA^{5}\ MB)\ MJ^{2})$$

$$MW^{2})/(MB\ MZ^{8} + (-\ 3\ MB^{3} - 2\ MA\ MB^{2} - 3\ MA^{2}\ MB)\ MZ^{6}$$

$$+ (3\ MB^{5} + 4\ MA\ MB^{4} + 2\ MA^{2}\ MB^{3} + 4\ MA^{3}\ MB^{2} + 3\ MA^{4}\ MB)\ MZ^{4}$$

$$+ (-\ MB^{7} - 2\ MA\ MB^{6} + MA^{2}\ MB^{5} + 4\ MA^{3}\ MB^{4} + MA^{4}\ MB^{3} - 2\ MA^{5}\ MB^{2} - MA^{6}\ MB)$$

$$MZ^{2})$$

$$FINITE1_{6} = ((30\ MB + 30\ MA)\ MJ^{2}\ MZ^{4}$$

$$+ ((-\ 36\ MB - 36\ MA)\ MJ^{2}\ MW^{2} + (12\ MB + 12\ MA)\ MJ^{4}$$

$$+ (-\ 18\ MB^{3} + 6\ MA\ MB^{2} + 6\ MA^{2}\ MB - 18\ MA^{3})\ MJ^{2})\ MZ^{2}$$

$$+ (-\ 48\ MB - 48\ MA)\ MJ^{2}\ MW^{4} + ((-\ 48\ MB - 48\ MA)\ MJ^{4} + (48\ MB^{3} + 48\ MA^{3})\ MJ^{2})$$

$$MW^2 + (-12\,MB^5 - 12\,MA\,MB^4 + 24\,MA^2\,MB^3 + 24\,MA^3\,MB^2 - 12\,MA^4\,MB - 12\,MA^5)$$

$$MJ^2)/(MZ^6 + (-3\,MB^2 - 2\,MA\,MB - 3\,MA^2)\,MZ^4$$

$$+ (3\,MB^4 + 4\,MA\,MB^3 + 2\,MA^2\,MB^2 + 4\,MA^3\,MB + 3\,MA^4)\,MZ^2 - MB^6 - 2\,MA\,MB^5$$

$$+ MA^2\,MB^4 + 4\,MA^3\,MB^3 + MA^4\,MB^2 - 2\,MA^5\,MB - MA^6)$$

$$FINITE1_7 = -((4\,MW^4 + (-2\,MJ^2 - 24\,MA\,MB - 12\,MA^2)\,MW^2 - 2\,MJ^4$$

$$+ (8\,MA\,MB + 14\,MA^2)\,MJ^2 + 4\,MA^3\,MB)\,MZ^6$$

$$+ (-16\,MW^6 + (8\,MJ^2 - 8\,MB^2 + 12\,MA\,MB - 20\,MA^2)\,MW^4$$

$$+ (8\,MJ^4 + (4\,MB^2 - 2\,MA\,MB - 22\,MA^2)\,MJ^2 + 48\,MA\,MB^3 + 56\,MA^2\,MB^2 - 4\,MA^3\,MB$$

$$- 44\,MA^4)\,MW^2 + (4\,MB^2 + 6\,MA\,MB + 10\,MA^2)\,MJ^4$$

$$+ (-16\,MA\,MB^3 - 16\,MA^2\,MB^2 + 38\,MA^3\,MB + 14\,MA^4)\,MJ^2 + 12\,MA^4\,MB^2$$

$$- 4\,MA^5\,MB)\,MZ^4 + ((32\,MB^2 + 48\,MA\,MB + 80\,MA^2)\,MW^6$$

$$+ ((-16\,MB^2 - 24\,MA\,MB - 40\,MA^2)\,MJ^2 + 4\,MB^4 - 36\,MA\,MB^3 - 100\,MA^2\,MB^2$$

$$- 132\,MA^3\,MB + 24\,MA^4)\,MW^4 + ((-16\,MB^2 - 24\,MA\,MB - 40\,MA^2)\,MJ^4$$

$$+ (-2\,MB^4 + 10\,MA\,MB^3 + 38\,MA^2\,MB^2 - 50\,MA^3\,MB + 20\,MA^4)\,MJ^2 - 24\,MA\,MB^5$$

$$- 44\,MA^2\,MB^4 - 52\,MA^3\,MB^3 - 60\,MA^4\,MB^2 + 92\,MA^5\,MB + 56\,MA^6)\,MW^2$$

$$+ (-2\,MB^4 - 6\,MA\,MB^3 - 2\,MA^2\,MB^2 + 6\,MA^3\,MB + 4\,MA^4)\,MJ^4$$

$$+ (8\,MA\,MB^5 + 2\,MA^2\,MB^4 + 14\,MA^3\,MB^3 + 26\,MA^4\,MB^2 - 22\,MA^5\,MB - 28\,MA^6)\,MJ^2$$

$$- 4\,MA^3\,MB^5 - 4\,MA^4\,MB^4 + 4\,MA^5\,MB^3 + 4\,MA^6\,MB^2)\,MZ^2$$

$$+ (-16\,MB^4 - 48\,MA\,MB^3 - 16\,MA^2\,MB^2 + 48\,MA^3\,MB + 32\,MA^4)\,MW^6$$

$$+ ((8\,MB^4 + 24\,MA\,MB^3 + 8\,MA^2\,MB^2 - 24\,MA^3\,MB - 16\,MA^4)\,MJ^2 + 24\,MA\,MB^5$$

$$+ 104\,MA^2\,MB^4 + 88\,MA^3\,MB^3 - 72\,MA^4\,MB^2 - 112\,MA^5\,MB - 32\,MA^6)\,MW^4$$

$$+ ((8\,MB^4 + 24\,MA\,MB^3 + 8\,MA^2\,MB^2 - 24\,MA^3\,MB - 16\,MA^4)\,MJ^4$$

$$+ (- 8\ MA\ MB^{5} - 8\ MA^{2}\ MB^{4} - 8\ MA^{3}\ MB^{3} - 8\ MA^{4}\ MB^{2} + 16\ MA^{5}\ MB + 16\ MA^{6})\ MJ^{2}$$

$$+ 16\ MA^{3}\ MB^{5} + 16\ MA^{4}\ MB^{4} - 16\ MA^{5}\ MB^{3} - 16\ MA^{6}\ MB^{2})\ MW^{2})$$

$$/(MA\ MZ^{8} + (- 3\ MA\ MB^{2} - 2\ MA^{2}\ MB - 3\ MA^{3})\ MZ^{6}$$

$$+ (3\ MA\ MB^{4} + 4\ MA^{2}\ MB^{3} + 2\ MA^{3}\ MB^{2} + 4\ MA^{4}\ MB + 3\ MA^{5})\ MZ^{4}$$

$$+ (- MA\ MB^{6} - 2\ MA^{2}\ MB^{5} + MA^{3}\ MB^{4} + 4\ MA^{4}\ MB^{3} + MA^{5}\ MB^{2} - 2\ MA^{6}\ MB - MA^{7})$$

$$MZ^{2})$$

$$FINITE1_{8} = - ((4\ MW^{4} + (- 2\ MJ^{2} - 12\ MB^{2} - 24\ MA\ MB)\ MW^{2} - 2\ MJ^{4}$$

$$+ (14\ MB^{2} + 8\ MA\ MB)\ MJ^{2} + 4\ MA\ MB^{3})\ MZ^{6}$$

$$+ (- 16\ MW^{6} + (8\ MJ^{2} - 20\ MB^{2} + 12\ MA\ MB - 8\ MA^{2})\ MW^{4}$$

$$+ (8\ MJ^{4} + (- 22\ MB^{2} - 2\ MA\ MB + 4\ MA^{2})\ MJ^{2} - 44\ MB^{4} - 4\ MA\ MB^{3} + 56\ MA^{2}\ MB^{2}$$

$$+ 48\ MA^{3}\ MB)\ MW^{2} + (10\ MB^{2} + 6\ MA\ MB + 4\ MA^{2})\ MJ^{4}$$

$$+ (14\ MB^{4} + 38\ MA\ MB^{3} - 16\ MA^{2}\ MB^{2} - 16\ MA^{3}\ MB)\ MJ^{2} - 4\ MA\ MB^{5} + 12\ MA^{2}\ MB^{4})$$

$$MZ^{4} + ((80\ MB^{2} + 48\ MA\ MB + 32\ MA^{2})\ MW^{6}$$

$$+ ((- 40\ MB^{2} - 24\ MA\ MB - 16\ MA^{2})\ MJ^{2} + 24\ MB^{4} - 132\ MA\ MB^{3} - 100\ MA^{2}\ MB^{2}$$

$$- 36\ MA^{3}\ MB + 4\ MA^{4})\ MW^{4} + ((- 40\ MB^{2} - 24\ MA\ MB - 16\ MA^{2})\ MJ^{4}$$

$$+ (20\ MB^{4} - 50\ MA\ MB^{3} + 38\ MA^{2}\ MB^{2} + 10\ MA^{3}\ MB - 2\ MA^{4})\ MJ^{2} + 56\ MB^{6}$$

$$+ 92\ MA\ MB^{5} - 60\ MA^{2}\ MB^{4} - 52\ MA^{3}\ MB^{3} - 44\ MA^{4}\ MB^{2} - 24\ MA^{5}\ MB)\ MW^{2}$$

$$+ (4\ MB^{4} + 6\ MA\ MB^{3} - 2\ MA^{2}\ MB^{2} - 6\ MA^{3}\ MB - 2\ MA^{4})\ MJ^{4}$$

$$+ (- 28\ MB^{6} - 22\ MA\ MB^{5} + 26\ MA^{2}\ MB^{4} + 14\ MA^{3}\ MB^{3} + 2\ MA^{4}\ MB^{2} + 8\ MA^{5}\ MB)\ MJ^{2}$$

$$+ 4\ MA^{2}\ MB^{6} + 4\ MA^{3}\ MB^{5} - 4\ MA^{4}\ MB^{4} - 4\ MA^{5}\ MB^{3})\ MZ^{2}$$

$$+ (32\ MB^{4} + 48\ MA\ MB^{3} - 16\ MA^{2}\ MB^{2} - 48\ MA^{3}\ MB - 16\ MA^{4})\ MW^{6}$$

$$+ ((- 16\ MB^{4} - 24\ MA\ MB^{3} + 8\ MA^{2}\ MB^{2} + 24\ MA^{3}\ MB + 8\ MA^{4})\ MJ^{2} - 32\ MB^{6}$$

$$- 112\ MA\ MB^5 - 72\ MA^2\ MB^4 + 88\ MA^3\ MB^3 + 104\ MA^4\ MB^2 + 24\ MA^5\ MB)\ MW^4$$

$$+ ((- 16\ MB^4 - 24\ MA\ MB^3 + 8\ MA^2\ MB^2 + 24\ MA^3\ MB + 8\ MA^4)\ MJ^4$$

$$+ (16\ MB^6 + 16\ MA\ MB^5 - 8\ MA^2\ MB^4 - 8\ MA^3\ MB^3 - 8\ MA^4\ MB^2 - 8\ MA^5\ MB)\ MJ^2$$

$$- 16\ MA^2\ MB^6 - 16\ MA^3\ MB^5 + 16\ MA^4\ MB^4 + 16\ MA^5\ MB^3)\ MW^2)$$

$$/(MB\ MZ^8 + (- 3\ MB^3 - 2\ MA\ MB^2 - 3\ MA^2\ MB)\ MZ^6$$

$$+ (3\ MB^5 + 4\ MA\ MB^4 + 2\ MA^2\ MB^3 + 4\ MA^3\ MB^2 + 3\ MA^4\ MB)\ MZ^4$$

$$+ (- MB^7 - 2\ MA\ MB^6 + MA^2\ MB^5 + 4\ MA^3\ MB^4 + MA^4\ MB^3 - 2\ MA^5\ MB^2 - MA^6\ MB)$$

$$MZ^2)$$

$$FINITE1_9 = ((30\ MB + 30\ MA)\ MJ^2\ MZ^4$$

$$+ ((- 96\ MB - 96\ MA)\ MJ^2\ MW^2 + (36\ MB + 36\ MA)\ MJ^4$$

$$+ (- 30\ MB^3 - 18\ MA\ MB^2 - 18\ MA^2\ MB - 30\ MA^3)\ MJ^2)\ MZ^2$$

$$+ (- 72\ MB - 72\ MA)\ MJ^2\ MW^4 + ((- 72\ MB - 72\ MA)\ MJ^4$$

$$+ (60\ MB^3 + 36\ MA\ MB^2 + 36\ MA^2\ MB + 60\ MA^3)\ MJ^2)\ MW^2)$$

$$/(MZ^6 + (- 3\ MB^2 - 2\ MA\ MB - 3\ MA^2)\ MZ^4$$

$$+ (3\ MB^4 + 4\ MA\ MB^3 + 2\ MA^2\ MB^2 + 4\ MA^3\ MB + 3\ MA^4)\ MZ^2 - MB^6 - 2\ MA\ MB^5$$

$$+ MA^2\ MB^4 + 4\ MA^3\ MB^3 + MA^4\ MB^2 - 2\ MA^5\ MB - MA^6)$$

$$FINITE1_{10} = - (((20\ MB + 20\ MA)\ MW^2 + 2\ MA\ MB^2 + 2\ MA^2\ MB)\ MZ^4$$

$$+ ((64\ MB + 64\ MA)\ MW^4 + ((- 60\ MB - 60\ MA)\ MJ^2 - 20\ MB^3 - 20\ MA\ MB^2$$

$$- 20\ MA^2\ MB - 20\ MA^3)\ MW^2 + (24\ MB + 24\ MA)\ MJ^4$$

$$+ (- 12\ MB^3 - 24\ MA\ MB^2 - 24\ MA^2\ MB - 12\ MA^3)\ MJ^2 - 2\ MA\ MB^4 + 6\ MA^2\ MB^3$$

$$+ 6\ MA^3\ MB^2 - 2\ MA^4\ MB)\ MZ^2 + (48\ MB + 48\ MA)\ MW^6$$

$$+ ((-24\,MB - 24\,MA)\,MJ^2 - 40\,MB^3 - 48\,MA\,MB^2 - 48\,MA^2\,MB - 40\,MA^3)\,MW^4$$

$$+ ((-24\,MB - 24\,MA)\,MJ^4 + (12\,MB^3 + 36\,MA\,MB^2 + 36\,MA^2\,MB + 12\,MA^3)\,MJ^2$$

$$- 4\,MA\,MB^4 + 12\,MA^2\,MB^3 + 12\,MA^3\,MB^2 - 4\,MA^4\,MB)\,MW^2$$

$$+ (12\,MB^5 + 12\,MA\,MB^4 - 24\,MA^2\,MB^3 - 24\,MA^3\,MB^2 + 12\,MA^4\,MB + 12\,MA^5)\,MJ^2)$$

$$/(MZ^6 + (-3\,MB^2 - 2\,MA\,MB - 3\,MA^2)\,MZ^4$$

$$+ (3\,MB^4 + 4\,MA\,MB^3 + 2\,MA^2\,MB^2 + 4\,MA^3\,MB + 3\,MA^4)\,MZ^2 - MB^6 - 2\,MA\,MB^5$$

$$+ MA^2\,MB^4 + 4\,MA^3\,MB^3 + MA^4\,MB^2 - 2\,MA^5\,MB - MA^6)$$

$$FINITE1_{11} = -((8\,MB + 8\,MA)\,MW^2\,MZ^8$$

$$+ ((48\,MB + 48\,MA)\,MW^4 + ((-36\,MB - 36\,MA)\,MJ^2 - 16\,MB^3 - 16\,MA\,MB^2$$

$$- 16\,MA^2\,MB - 16\,MA^3)\,MW^2 + (4\,MB + 4\,MA)\,MJ^4$$

$$+ (-4\,MB^3 - 4\,MA\,MB^2 - 4\,MA^2\,MB - 4\,MA^3)\,MJ^2 + 4\,MA^2\,MB^3 + 4\,MA^3\,MB^2)\,MZ^6$$

$$+ ((88\,MB + 88\,MA)\,MW^6 + ((-120\,MB - 120\,MA)\,MJ^2 - 64\,MB^3 - 68\,MA\,MB^2$$

$$- 68\,MA^2\,MB - 64\,MA^3)\,MW^4 + ((80\,MB + 80\,MA)\,MJ^4 + (8\,MB^3 + 8\,MA^3)\,MJ^2$$

$$+ 8\,MB^5 + 8\,MA\,MB^4 + 40\,MA^2\,MB^3 + 40\,MA^3\,MB^2 + 8\,MA^4\,MB + 8\,MA^5)\,MW^2$$

$$+ (-24\,MB - 24\,MA)\,MJ^6 + (16\,MB^3 + 28\,MA\,MB^2 + 28\,MA^2\,MB + 16\,MA^3)\,MJ^4$$

$$+ (8\,MB^5 - 24\,MA^2\,MB^3 - 24\,MA^3\,MB^2 + 8\,MA^5)\,MJ^2 - 4\,MA^2\,MB^5 - 4\,MA^5\,MB^2)\,MZ^4$$

$$+ ((48\,MB + 48\,MA)\,MW^8 + ((-72\,MB - 72\,MA)\,MJ^2 - 64\,MB^3 - 72\,MA\,MB^2$$

$$- 72\,MA^2\,MB - 64\,MA^3)\,MW^6 + ((32\,MB^3 + 64\,MA\,MB^2 + 64\,MA^2\,MB + 32\,MA^3)\,MJ^2$$

$$+ 16\,MB^5 + 16\,MA\,MB^4 + 64\,MA^2\,MB^3 + 64\,MA^3\,MB^2 + 16\,MA^4\,MB + 16\,MA^5)\,MW^4$$

$$+ ((24\,MB + 24\,MA)\,MJ^6 + (-16\,MB^3 - 40\,MA\,MB^2 - 40\,MA^2\,MB - 16\,MA^3)\,MJ^4$$

$$+ (36\,MB^5 + 44\,MA\,MB^4 - 40\,MA^2\,MB^3 - 40\,MA^3\,MB^2 + 44\,MA^4\,MB + 36\,MA^5)\,MJ^2$$

$$- 8\ MA^{2}\ MB^{5} - 8\ MA^{5}\ MB^{2}\ )\ MW^{2} + (- 20\ MB^{5} - 20\ MA\ MB^{4} + 40\ MA^{2}\ MB^{3}$$

$$+ 40\ MA^{3}\ MB^{2} - 20\ MA^{4}\ MB - 20\ MA^{5}\ )\ MJ^{4}$$

$$+ (- 4\ MB^{7} + 4\ MA\ MB^{6} + 12\ MA^{2}\ MB^{5} - 12\ MA^{3}\ MB^{4} - 12\ MA^{4}\ MB^{3} + 12\ MA^{5}\ MB^{2}$$

$$+ 4\ MA^{6}\ MB - 4\ MA^{7}\ )\ MJ^{2}\ )\ MZ^{2} + (16\ MB^{5} + 16\ MA\ MB^{4} - 32\ MA^{2}\ MB^{3} - 32\ MA^{3}\ MB^{2}$$

$$+ 16\ MA^{4}\ MB + 16\ MA^{5}\ )\ MJ^{2}\ MW^{4} + ((8\ MB^{5} + 8\ MA\ MB^{4} - 16\ MA^{2}\ MB^{3} - 16\ MA^{3}\ MB^{2}$$

$$+ 8\ MA^{4}\ MB + 8\ MA^{5}\ )\ MJ^{4} + (- 8\ MB^{7} - 16\ MA\ MB^{6} + 24\ MA^{3}\ MB^{4} + 24\ MA^{4}\ MB^{3}$$

$$- 16\ MA^{6}\ MB - 8\ MA^{7}\ )\ MJ^{2}\ )\ MW^{2}\ )/(MZ^{8} + (- 3\ MB^{2} - 2\ MA\ MB - 3\ MA^{2}\ )\ MZ^{6}$$

$$+ (3\ MB^{4} + 4\ MA\ MB^{3} + 2\ MA^{2}\ MB^{2} + 4\ MA^{3}\ MB + 3\ MA^{4}\ )\ MZ^{4}$$

$$+ (- MB^{6} - 2\ MA\ MB^{5} + MA^{2}\ MB^{4} + 4\ MA^{3}\ MB^{3} + MA^{4}\ MB^{2} - 2\ MA^{5}\ MB - MA^{6}\ )\ MZ^{2}\ )$$

$$FINITE1_{12} = ((12\ MB + 12\ MA)\ MJ^{2}\ MZ^{8}$$

$$+ ((24\ MB + 24\ MA)\ MW^{4} + ((- 84\ MB - 84\ MA)\ MJ^{2} - 24\ MB^{3} - 12\ MA\ MB^{2}$$

$$- 12\ MA^{2}\ MB - 24\ MA^{3}\ )\ MW^{2} + (48\ MB + 48\ MA)\ MJ^{4}$$

$$+ (- 24\ MB^{3} - 12\ MA\ MB^{2} - 12\ MA^{2}\ MB - 24\ MA^{3}\ )\ MJ^{2} + 12\ MA^{2}\ MB^{3} + 12\ MA^{3}\ MB^{2}\ )$$

$$MZ^{6} + ((- 48\ MB - 48\ MA)\ MW^{6} + ((108\ MB + 108\ MA)\ MJ^{2} + 48\ MB^{3} - 36\ MA\ MB^{2}$$

$$- 36\ MA^{2}\ MB + 48\ MA^{3}\ )\ MW^{4} + ((- 168\ MB - 168\ MA)\ MJ^{4}$$

$$+ (72\ MB^{3} + 96\ MA\ MB^{2} + 96\ MA^{2}\ MB + 72\ MA^{3}\ )\ MJ^{2} + 48\ MB^{5} + 48\ MA\ MB^{4}$$

$$- 48\ MA^{2}\ MB^{3} - 48\ MA^{3}\ MB^{2} + 48\ MA^{4}\ MB + 48\ MA^{5}\ )\ MW^{2} + (36\ MB + 36\ MA)\ MJ^{6}$$

$$+ (- 48\ MB^{3} - 36\ MA\ MB^{2} - 36\ MA^{2}\ MB - 48\ MA^{3}\ )\ MJ^{4}$$

$$+ (12\ MB^{5} + 48\ MA^{2}\ MB^{3} + 48\ MA^{3}\ MB^{2} + 12\ MA^{5}\ )\ MJ^{2} - 12\ MA^{2}\ MB^{5} - 12\ MA^{5}\ MB^{2}\ )$$

$$MZ^{4} + ((- 144\ MB - 144\ MA)\ MW^{8} + ((216\ MB + 216\ MA)\ MJ^{2} + 96\ MB^{3} + 312\ MA\ MB^{2}$$

$$+ 312\ MA^{2}\ MB + 96\ MA^{3}\ )\ MW^{6} + ((- 48\ MB^{3} - 240\ MA\ MB^{2} - 240\ MA^{2}\ MB - 48\ MA^{3}\ )$$

$$MJ^{2} - 120\ MB^{5} - 168\ MA\ MB^{4} - 168\ MA^{4}\ MB - 120\ MA^{5}\ )\ MW^{4}$$

$$+ ((-72\,MB - 72\,MA)\,MJ^{6} + (96\,MB^{3} + 72\,MA\,MB^{2} + 72\,MA^{2}\,MB + 96\,MA^{3})\,MJ^{4}$$

$$+ (12\,MB^{5} + 36\,MA\,MB^{4} - 168\,MA^{2}\,MB^{3} - 168\,MA^{3}\,MB^{2} + 36\,MA^{4}\,MB + 12\,MA^{5})\,MJ^{2}$$

$$- 24\,MB^{7} - 36\,MA\,MB^{6} + 36\,MA^{2}\,MB^{5} + 48\,MA^{3}\,MB^{4} + 48\,MA^{4}\,MB^{3} + 36\,MA^{5}\,MB^{2}$$

$$- 36\,MA^{6}\,MB - 24\,MA^{7})\,MW^{2})\,MZ^{2} + (-48\,MB^{5} - 48\,MA\,MB^{4} + 96\,MA^{2}\,MB^{3}$$

$$+ 96\,MA^{3}\,MB^{2} - 48\,MA^{4}\,MB - 48\,MA^{5})\,MW^{6}$$

$$+ ((-24\,MB^{5} - 24\,MA\,MB^{4} + 48\,MA^{2}\,MB^{3} + 48\,MA^{3}\,MB^{2} - 24\,MA^{4}\,MB - 24\,MA^{5})\,MJ^{2}$$

$$+ 48\,MB^{7} + 120\,MA\,MB^{6} + 24\,MA^{2}\,MB^{5} - 192\,MA^{3}\,MB^{4} - 192\,MA^{4}\,MB^{3} + 24\,MA^{5}\,MB^{2}$$

$$+ 120\,MA^{6}\,MB + 48\,MA^{7})\,MW^{4})/(MZ^{8} + (-3\,MB^{2} - 2\,MA\,MB - 3\,MA^{2})\,MZ^{6}$$

$$+ (3\,MB^{4} + 4\,MA\,MB^{3} + 2\,MA^{2}\,MB^{2} + 4\,MA^{3}\,MB + 3\,MA^{4})\,MZ^{4}$$

$$+ (-MB^{6} - 2\,MA\,MB^{5} + MA^{2}\,MB^{4} + 4\,MA^{3}\,MB^{3} + MA^{4}\,MB^{2} - 2\,MA^{5}\,MB - MA^{6})\,MZ^{2})$$

$$\mathrm{FINITE2}_1 = ((4\,MJ^2\,MW^2 + 4\,MJ^4 + 6\,MA\,MB\,MJ^2)\,MZ^2 - 4\,MJ^2\,MW^4$$

$$+ (-4\,MJ^4 - 12\,MA\,MB\,MJ^2)\,MW^2)/(MA\,MB\,MZ^2)$$

$$\mathrm{FINITE2}_2 = -((8\,MJ^2\,MW^2 + 4\,MJ^4 + MA\,MB\,MJ^2)\,MZ^2 - 8\,MJ^2\,MW^4$$

$$+ (-4\,MJ^4 - 4\,MA\,MB\,MJ^2)\,MW^2)/(MA\,MB\,MZ^2)$$

$$\mathrm{FINITE2}_3 = \frac{4\,MJ^2\,MW^2\,MZ^2 - 4\,MJ^2\,MW^4}{MA\,MB\,MZ^2}$$

$$\mathrm{FINITE2}_4 = (((4\,MB + MA)\,MJ^2\,MW^2 + (4\,MB + MA)\,MJ^4$$

$$+ (-MA\,MB^2 - 8\,MA^2\,MB - MA^3)\,MJ^2)\,MZ^6$$

$$+ ((-4\,MB - 4\,MA)\,MJ^2\,MW^4 + ((-4\,MB - 4\,MA)\,MJ^4$$

$$+ (-8\,MB^3 + MA\,MB^2 + 3\,MA^3)\,MJ^2)\,MW^2 + (-8\,MB^3 - 3\,MA\,MB^2 - 8\,MA^2\,MB - MA^3)$$

$$MJ^4 + (2\,MA\,MB^4 + 14\,MA^2\,MB^3 - 3\,MA^3\,MB^2 + 18\,MA^4\,MB + 9\,MA^5)\,MJ^2)\,MZ^4$$

$$+ ((8\,MB^3 + 12\,MA\,MB^2 + 8\,MA^2\,MB + 4\,MA^3)\,MJ^2\,MW^4$$

$$+ ((8\,MB^3 + 12\,MA\,MB^2 + 8\,MA^2\,MB + 4\,MA^3)\,MJ^4$$

$$+ (4\,MB^5 - 6\,MA\,MB^4 - 16\,MA^2\,MB^3 - 16\,MA^3\,MB^2 - 20\,MA^4\,MB - 10\,MA^5)\,MJ^2)\,MW^2$$

$$+ (4\,MB^5 + 2\,MA\,MB^4 - 8\,MA^2\,MB^3 - 4\,MA^3\,MB^2 + 4\,MA^4\,MB + 2\,MA^5)\,MJ^4$$

$$+ (-MA\,MB^6 - 6\,MA^2\,MB^5 - 6\,MA^3\,MB^4 + 12\,MA^4\,MB^3 + 15\,MA^5\,MB^2 - 6\,MA^6\,MB$$

$$- 8\,MA^7)\,MJ^2)\,MZ^2 + (-4\,MB^5 - 8\,MA\,MB^4 + 8\,MA^2\,MB^3 + 16\,MA^3\,MB^2 - 4\,MA^4\,MB$$

$$- 8\,MA^5)\,MJ^2\,MW^4 + ((-4\,MB^5 - 8\,MA\,MB^4 + 8\,MA^2\,MB^3 + 16\,MA^3\,MB^2 - 4\,MA^4\,MB$$

$$- 8\,MA^5)\,MJ^4 + (4\,MA\,MB^6 - 12\,MA^5\,MB^2 + 8\,MA^7)\,MJ^2)\,MW^2)$$

$$/((MA\,MB^2 - MA^3)\,MZ^6 + (2\,MA^5 - 2\,MA\,MB^4)\,MZ^4$$

$$+ (MA\,MB^6 - 3\,MA^3\,MB^4 + 3\,MA^5\,MB^2 - MA^7)\,MZ^2)$$

$$
\begin{aligned}
\mathrm{FINITE2}_5 = -\,(((MB + 4\,MA)\,MJ^2\,MW^2 + (MB + 4\,MA)\,MJ^4 \\
+ (-\,MB^3 - 8\,MA\,MB^2 - MA^2\,MB)\,MJ^2)\,MZ^6 \\
+ ((-\,4\,MB - 4\,MA)\,MJ^2\,MW^4 + ((-\,4\,MB - 4\,MA)\,MJ^4 \\
+ (3\,MB^3 + MA^2\,MB - 8\,MA^3)\,MJ^2)\,MW^2 + (-\,MB^3 - 8\,MA\,MB^2 - 3\,MA^2\,MB - 8\,MA^3) \\
MJ^4 + (9\,MB^5 + 18\,MA\,MB^4 - 3\,MA^2\,MB^3 + 14\,MA^3\,MB^2 + 2\,MA^4\,MB)\,MJ^2)\,MZ^4 \\
+ ((4\,MB^3 + 8\,MA\,MB^2 + 12\,MA^2\,MB + 8\,MA^3)\,MJ^2\,MW^4 \\
+ ((4\,MB^3 + 8\,MA\,MB^2 + 12\,MA^2\,MB + 8\,MA^3)\,MJ^4 \\
+ (-\,10\,MB^5 - 20\,MA\,MB^4 - 16\,MA^2\,MB^3 - 16\,MA^3\,MB^2 - 6\,MA^4\,MB + 4\,MA^5)\,MJ^2) \\
MW^2 + (2\,MB^5 + 4\,MA\,MB^4 - 4\,MA^2\,MB^3 - 8\,MA^3\,MB^2 + 2\,MA^4\,MB + 4\,MA^5)\,MJ^4 \\
+ (-\,8\,MB^7 - 6\,MA\,MB^6 + 15\,MA^2\,MB^5 + 12\,MA^3\,MB^4 - 6\,MA^4\,MB^3 - 6\,MA^5\,MB^2 \\
- MA^6\,MB)\,MJ^2)\,MZ^2 + (-\,8\,MB^5 - 4\,MA\,MB^4 + 16\,MA^2\,MB^3 + 8\,MA^3\,MB^2 - 8\,MA^4\,MB \\
- 4\,MA^5)\,MJ^2\,MW^4 + ((-\,8\,MB^5 - 4\,MA\,MB^4 + 16\,MA^2\,MB^3 + 8\,MA^3\,MB^2 - 8\,MA^4\,MB \\
- 4\,MA^5)\,MJ^4 + (8\,MB^7 - 12\,MA^2\,MB^5 + 4\,MA^6\,MB)\,MJ^2)\,MW^2) \\
/((MB^3 - MA^2\,MB)\,MZ^6 + (2\,MA^4\,MB - 2\,MB^5)\,MZ^4 \\
+ (MB^7 - 3\,MA^2\,MB^5 + 3\,MA^4\,MB^3 - MA^6\,MB)\,MZ^2)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{FINITE2}_6 = (MJ^2\,MZ^4 + (-\,2\,MJ^2\,MW^2 + 2\,MJ^4 \\
+ (5\,MB^2 + 4\,MA\,MB + 5\,MA^2)\,MJ^2)\,MZ^2 - 8\,MJ^2\,MW^4 \\
+ ((4\,MB^2 - 16\,MA\,MB + 4\,MA^2)\,MJ^2 - 8\,MJ^4)\,MW^2 \\
+ (-\,6\,MB^4 + 12\,MA^2\,MB^2 - 6\,MA^4)\,MJ^2)/(MZ^4 + (-\,2\,MB^2 - 2\,MA^2)\,MZ^2 + MB^4 \\
- 2\,MA^2\,MB^2 + MA^4)
\end{aligned}
$$

$$
\mathrm{FINITE2}_7 = -\,(((8\,MB + 2\,MA)\,MW^4 + ((-\,4\,MB - MA)\,MJ^2 - 3\,MA\,MB^2
$$

$$- 4\,MA^{2}\,MB + 2\,MA^{3})\,MW^{2} + (-4\,MB - MA)\,MJ^{4} + (MA\,MB^{2} + 8\,MA^{2}\,MB + MA^{3})\,MJ^{2}$$

$$- MA^{3}\,MB^{2} - 4\,MA^{4}\,MB)\,MZ^{6} + ((-8\,MB - 8\,MA)\,MW^{6}$$

$$+ ((4\,MB + 4\,MA)\,MJ^{2} - 16\,MB^{3} + 6\,MA\,MB^{2} - 12\,MA^{2}\,MB - 10\,MA^{3})\,MW^{4}$$

$$+ ((4\,MB + 4\,MA)\,MJ^{4} + (8\,MB^{3} - MA\,MB^{2} - 3\,MA^{3})\,MJ^{2} + 6\,MA\,MB^{4} + 20\,MA^{2}\,MB^{3}$$

$$- 12\,MA^{3}\,MB^{2} + 14\,MA^{5})\,MW^{2} + (8\,MB^{3} + 3\,MA\,MB^{2} + 8\,MA^{2}\,MB + MA^{3})\,MJ^{4}$$

$$+ (-2\,MA\,MB^{4} - 14\,MA^{2}\,MB^{3} + 3\,MA^{3}\,MB^{2} - 18\,MA^{4}\,MB - 9\,MA^{5})\,MJ^{2} + 12\,MA^{4}\,MB^{3}$$

$$+ 4\,MA^{5}\,MB^{2} + 4\,MA^{6}\,MB)\,MZ^{4} + ((16\,MB^{3} + 24\,MA\,MB^{2} + 16\,MA^{2}\,MB + 8\,MA^{3})\,MW^{6}$$

$$+ ((-8\,MB^{3} - 12\,MA\,MB^{2} - 8\,MA^{2}\,MB - 4\,MA^{3})\,MJ^{2} + 8\,MB^{5} - 20\,MA\,MB^{4}$$

$$- 60\,MA^{2}\,MB^{3} + 8\,MA^{3}\,MB^{2} + 36\,MA^{4}\,MB - 4\,MA^{5})\,MW^{4}$$

$$+ ((-8\,MB^{3} - 12\,MA\,MB^{2} - 8\,MA^{2}\,MB - 4\,MA^{3})\,MJ^{4}$$

$$+ (-4\,MB^{5} + 6\,MA\,MB^{4} + 16\,MA^{2}\,MB^{3} + 16\,MA^{3}\,MB^{2} + 20\,MA^{4}\,MB + 10\,MA^{5})\,MJ^{2}$$

$$- 3\,MA\,MB^{6} - 16\,MA^{2}\,MB^{5} - 22\,MA^{3}\,MB^{4} + 20\,MA^{4}\,MB^{3} + 25\,MA^{5}\,MB^{2} - 20\,MA^{6}\,MB$$

$$- 16\,MA^{7})\,MW^{2} + (-4\,MB^{5} - 2\,MA\,MB^{4} + 8\,MA^{2}\,MB^{3} + 4\,MA^{3}\,MB^{2} - 4\,MA^{4}\,MB$$

$$- 2\,MA^{5})\,MJ^{4} + (MA\,MB^{6} + 6\,MA^{2}\,MB^{5} + 6\,MA^{3}\,MB^{4} - 12\,MA^{4}\,MB^{3} - 15\,MA^{5}\,MB^{2}$$

$$+ 6\,MA^{6}\,MB + 8\,MA^{7})\,MJ^{2} + MA^{3}\,MB^{6} - 2\,MA^{5}\,MB^{4} + MA^{7}\,MB^{2})\,MZ^{2}$$

$$+ (-8\,MB^{5} - 16\,MA\,MB^{4} + 16\,MA^{2}\,MB^{3} + 32\,MA^{3}\,MB^{2} - 8\,MA^{4}\,MB - 16\,MA^{5})\,MW^{6}$$

$$+ ((4\,MB^{5} + 8\,MA\,MB^{4} - 8\,MA^{2}\,MB^{3} - 16\,MA^{3}\,MB^{2} + 4\,MA^{4}\,MB + 8\,MA^{5})\,MJ^{2}$$

$$+ 12\,MA\,MB^{6} + 40\,MA^{2}\,MB^{5} - 8\,MA^{3}\,MB^{4} - 80\,MA^{4}\,MB^{3} - 20\,MA^{5}\,MB^{2} + 40\,MA^{6}\,MB$$

$$+ 16\,MA^{7})\,MW^{4} + ((4\,MB^{5} + 8\,MA\,MB^{4} - 8\,MA^{2}\,MB^{3} - 16\,MA^{3}\,MB^{2} + 4\,MA^{4}\,MB$$

$$+ 8\,MA^{5})\,MJ^{4} + (-4\,MA\,MB^{6} + 12\,MA^{5}\,MB^{2} - 8\,MA^{7})\,MJ^{2} + 8\,MA^{3}\,MB^{6} - 16\,MA^{5}\,MB^{4}$$

$$+ 8\,MA^{7}\,MB^{2})\,MW^{2})/((MA\,MB^{2} - MA^{3})\,MZ^{6} + (2\,MA^{5} - 2\,MA\,MB^{4})\,MZ^{4}$$

$$+ (MA\,MB^{6} - 3\,MA^{3}\,MB^{4} + 3\,MA^{5}\,MB^{2} - MA^{7})\,MZ^{2})$$

$$
\begin{aligned}
\mathrm{FINITE2}_{8} = {} & (((2\,MB + 8\,MA)\,MW^{4} + ((-\,MB - 4\,MA)\,MJ^{2} + 2\,MB^{3} - 4\,MA\,MB^{2} \\
& - 3\,MA^{2}\,MB)\,MW^{2} + (-\,MB - 4\,MA)\,MJ^{4} + (MB^{3} + 8\,MA\,MB^{2} + MA^{2}\,MB)\,MJ^{2} \\
& - 4\,MA\,MB^{4} - MA^{2}\,MB^{3})\,MZ^{6} + ((-\,8\,MB - 8\,MA)\,MW^{6} \\
& + ((4\,MB + 4\,MA)\,MJ^{2} - 10\,MB^{3} - 12\,MA\,MB^{2} + 6\,MA^{2}\,MB - 16\,MA^{3})\,MW^{4} \\
& + ((4\,MB + 4\,MA)\,MJ^{4} + (-\,3\,MB^{3} - MA^{2}\,MB + 8\,MA^{3})\,MJ^{2} + 14\,MB^{5} - 12\,MA^{2}\,MB^{3} \\
& + 20\,MA^{3}\,MB^{2} + 6\,MA^{4}\,MB)\,MW^{2} + (MB^{3} + 8\,MA\,MB^{2} + 3\,MA^{2}\,MB + 8\,MA^{3})\,MJ^{4} \\
& + (-\,9\,MB^{5} - 18\,MA\,MB^{4} + 3\,MA^{2}\,MB^{3} - 14\,MA^{3}\,MB^{2} - 2\,MA^{4}\,MB)\,MJ^{2} + 4\,MA\,MB^{6} \\
& + 4\,MA^{2}\,MB^{5} + 12\,MA^{3}\,MB^{4})\,MZ^{4} + ((8\,MB^{3} + 16\,MA\,MB^{2} + 24\,MA^{2}\,MB + 16\,MA^{3})\,MW^{6} \\
& + ((-\,4\,MB^{3} - 8\,MA\,MB^{2} - 12\,MA^{2}\,MB - 8\,MA^{3})\,MJ^{2} - 4\,MB^{5} + 36\,MA\,MB^{4} \\
& + 8\,MA^{2}\,MB^{3} - 60\,MA^{3}\,MB^{2} - 20\,MA^{4}\,MB + 8\,MA^{5})\,MW^{4} \\
& + ((-\,4\,MB^{3} - 8\,MA\,MB^{2} - 12\,MA^{2}\,MB - 8\,MA^{3})\,MJ^{4} \\
& + (10\,MB^{5} + 20\,MA\,MB^{4} + 16\,MA^{2}\,MB^{3} + 16\,MA^{3}\,MB^{2} + 6\,MA^{4}\,MB - 4\,MA^{5})\,MJ^{2} \\
& - 16\,MB^{7} - 20\,MA\,MB^{6} + 25\,MA^{2}\,MB^{5} + 20\,MA^{3}\,MB^{4} - 22\,MA^{4}\,MB^{3} - 16\,MA^{5}\,MB^{2} \\
& - 3\,MA^{6}\,MB)\,MW^{2} + (-\,2\,MB^{5} - 4\,MA\,MB^{4} + 4\,MA^{2}\,MB^{3} + 8\,MA^{3}\,MB^{2} - 2\,MA^{4}\,MB \\
& - 4\,MA^{5})\,MJ^{4} + (8\,MB^{7} + 6\,MA\,MB^{6} - 15\,MA^{2}\,MB^{5} - 12\,MA^{3}\,MB^{4} + 6\,MA^{4}\,MB^{3} \\
& + 6\,MA^{5}\,MB^{2} + MA^{6}\,MB)\,MJ^{2} + MA^{2}\,MB^{7} - 2\,MA^{4}\,MB^{5} + MA^{6}\,MB^{3})\,MZ^{2} \\
& + (-\,16\,MB^{5} - 8\,MA\,MB^{4} + 32\,MA^{2}\,MB^{3} + 16\,MA^{3}\,MB^{2} - 16\,MA^{4}\,MB - 8\,MA^{5})\,MW^{6} \\
& + ((8\,MB^{5} + 4\,MA\,MB^{4} - 16\,MA^{2}\,MB^{3} - 8\,MA^{3}\,MB^{2} + 8\,MA^{4}\,MB + 4\,MA^{5})\,MJ^{2} \\
& + 16\,MB^{7} + 40\,MA\,MB^{6} - 20\,MA^{2}\,MB^{5} - 80\,MA^{3}\,MB^{4} - 8\,MA^{4}\,MB^{3} + 40\,MA^{5}\,MB^{2} \\
& + 12\,MA^{6}\,MB)\,MW^{4} + ((8\,MB^{5} + 4\,MA\,MB^{4} - 16\,MA^{2}\,MB^{3} - 8\,MA^{3}\,MB^{2} + 8\,MA^{4}\,MB \\
& + 4\,MA^{5})\,MJ^{4} + (-\,8\,MB^{7} + 12\,MA^{2}\,MB^{5} - 4\,MA^{6}\,MB)\,MJ^{2} + 8\,MA^{2}\,MB^{7} - 16\,MA^{4}\,MB^{5}
\end{aligned}
$$

```
      6   3    2       3      2        6          4            5    4
+ 8 MA  MB ) MW )/((MB  - MA  MB) MZ  + (2 MA  MB - 2 MB ) MZ

     7        2   5        4   3      6        2
+ (MB  - 3 MA  MB  + 3 MA  MB  - MA  MB) MZ )

                          2   4            2   2        4
     FINITE2  = (3 MJ  MZ  + (- 12 MJ  MW  + 6 MJ
            9

          2                    2   2    2        2   4
+ (- 3 MB  + 6 MA MB - 3 MA ) MJ ) MZ  - 12 MJ  MW

          2                    2   2        4    2
+ ((6 MB  - 12 MA MB + 6 MA ) MJ  - 12 MJ ) MW )

    4          2        2    2     4        2   2     4
/(MZ  + (- 2 MB  - 2 MA ) MZ  + MB  - 2 MA  MB  + MA )

                          2        2            4
     FINITE2   = - ((6 MW  + 2 MJ  - MA MB) MZ
            10

          4            2        2                    2    2        4
+ (16 MW  + (- 10 MJ  - 6 MB  - 8 MA MB - 6 MA ) MW  + 4 MJ

          2                    2   2        3        2   2     3        2        6
+ (- 8 MB  + 2 MA MB - 8 MA ) MJ  + MA MB  + 4 MA  MB  + MA  MB) MZ  + 8 MW

          2        2                    2    4
+ (- 4 MJ  - 12 MB  - 12 MA MB - 12 MA ) MW

          4        2                 2    2          3        2   2        3
+ (- 4 MJ  + (2 MB  + 4 MA MB + 2 MA ) MJ  + 2 MA MB  + 8 MA  MB  + 2 MA  MB)

   2        4        2   2        4    2
 MW  + (6 MB  - 12 MA  MB  + 6 MA ) MJ )

    4          2        2    2     4        2   2     4
/(MZ  + (- 2 MB  - 2 MA ) MZ  + MB  - 2 MA  MB  + MA )

                          2   8          4
     FINITE2   = - (4 MW  MZ  + (16 MW
            11

          2        2                 2    2        4          2        2    2
+ (- 16 MJ  - 8 MB  - 4 MA MB - 8 MA ) MW  + 2 MJ  + (- 2 MB  - 2 MA ) MJ

      2   2    6          6          2        2                    2    4
+ 2 MA  MB ) MZ  + (20 MW  + (- 20 MJ  - 24 MB  - 14 MA MB - 24 MA ) MW

          4        2                 2    2        4          3        2   2
+ (16 MJ  + (14 MB  + 8 MA MB + 14 MA ) MJ  + 4 MB  + 4 MA MB  + 12 MA  MB

      3          4    2        6        2                 2    4
+ 4 MA  MB + 4 MA ) MW  - 4 MJ  + (2 MB  - 2 MA MB + 2 MA ) MJ

      4          3        2   2        3          4    2        2   4
+ (4 MB  + 2 MA MB  - 4 MA  MB  + 2 MA  MB + 4 MA ) MJ  - 2 MA  MB

      3   3        4   2    4        8          2        2
- 2 MA  MB  - 2 MA  MB ) MZ  + (8 MW  + (- 12 MJ  - 16 MB  - 12 MA MB

      2    6                 2        4          3        2   2        3
- 16 MA ) MW  + (16 MA MB MJ  + 8 MB  + 8 MA MB  + 16 MA  MB  + 8 MA  MB
```

```
+ 8 MA^4 ) MW^4 + (4 MJ^6 + (- 8 MB^2 - 4 MA MB - 8 MA^2 ) MJ^4

+ (6 MB^4 + 4 MA MB^3 + 8 MA^2 MB^2 + 4 MA^3 MB + 6 MA^4 ) MJ^2 - 4 MA^2 MB^4

- 4 MA^3 MB^3 - 4 MA^4 MB^2 ) MW^2 + (- 4 MB^4 + 8 MA^2 MB^2 - 4 MA^4 ) MJ^4

+ (- 2 MB^6 - 2 MA MB^5 + 2 MA^2 MB^4 + 4 MA^3 MB^3 + 2 MA^4 MB^2 - 2 MA^5 MB - 2 MA^6 )

MJ^2 ) MZ^2 + (8 MB^4 - 16 MA^2 MB^2 + 8 MA^4 ) MJ^2 MW^4

+ ((4 MB^4 - 8 MA^2 MB^2 + 4 MA^4 ) MJ^4 + (- 4 MB^6 - 4 MA MB^5 + 4 MA^2 MB^4

+ 8 MA^3 MB^3 + 4 MA^4 MB^2 - 4 MA^5 MB - 4 MA^6 ) MJ^2 ) MW^2 )

/(MZ^6 + (- 2 MB^2 - 2 MA^2 ) MZ^4 + (MB^4 - 2 MA^2 MB^2 + MA^4 ) MZ^2 )

      FINITE2_12 = - ((24 MJ^2 MW^2 - 6 MJ^4 - 6 MA MB MJ^2 ) MZ^6

+ (48 MW^6 + (- 30 MJ^2 - 36 MB^2 + 18 MA MB - 36 MA^2 ) MW^4

+ (24 MJ^4 + (- 30 MB^2 - 30 MA^2 ) MJ^2 - 6 MA MB^3 + 24 MA^2 MB^2 - 6 MA^3 MB) MW^2

- 6 MJ^6 + (6 MB^2 - 6 MA MB + 6 MA^2 ) MJ^4

+ (6 MA MB^3 - 6 MA^2 MB^2 + 6 MA^3 MB) MJ^2 - 6 MA^3 MB^3 ) MZ^4

+ (24 MW^8 + (- 36 MJ^2 - 72 MB^2 - 84 MA MB - 72 MA^2 ) MW^6

+ ((12 MB^2 + 72 MA MB + 12 MA^2 ) MJ^2 + 60 MB^4 + 36 MA MB^3 + 36 MA^3 MB

+ 60 MA^4 ) MW^4 + (12 MJ^6 + (- 12 MB^2 + 12 MA MB - 12 MA^2 ) MJ^4

+ (6 MB^4 - 12 MA MB^3 - 12 MA^3 MB + 6 MA^4 ) MJ^2 + 6 MA MB^5 + 6 MA^5 MB) MW^2 ) MZ^2

+ (24 MB^4 - 48 MA^2 MB^2 + 24 MA^4 ) MW^6 + ((12 MB^4 - 24 MA^2 MB^2 + 12 MA^4 ) MJ^2

- 24 MB^6 - 36 MA MB^5 + 24 MA^2 MB^4 + 72 MA^3 MB^3 + 24 MA^4 MB^2 - 36 MA^5 MB

- 24 MA^6 ) MW^4 )/(MZ^6 + (- 2 MB^2 - 2 MA^2 ) MZ^4 + (MB^4 - 2 MA^2 MB^2 + MA^4 ) MZ^2 )
```

$$
\begin{aligned}
\mathrm{FINITE3}_1 = -\,(((&2\,MB - 2\,MA)\,MJ^2\,MW^2 + (2\,MB - 2\,MA)\,MJ^4 \\
&+ (18\,MA^2\,MB - 18\,MA\,MB^2)\,MJ^2)\,MZ^2 + (8\,MA - 8\,MB)\,MJ^2\,MW^4 \\
&+ ((8\,MA - 8\,MB)\,MJ^4 + (24\,MA\,MB^2 - 24\,MA^2\,MB)\,MJ^2)\,MW^2) \\
&/(MA\,MB\,MZ^4 + (-\,MA\,MB^3 + 2\,MA^2\,MB^2 - MA^3\,MB)\,MZ^2)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{FINITE3}_2 = ((((&4\,MB - 4\,MA)\,MJ^2\,MW^2 + (2\,MB - 2\,MA)\,MJ^4 \\
&+ (8\,MA^2\,MB - 8\,MA\,MB^2)\,MJ^2)\,MZ^2 + (16\,MA - 16\,MB)\,MJ^2\,MW^4 \\
&+ ((8\,MA - 8\,MB)\,MJ^4 + (8\,MA\,MB^2 - 8\,MA^2\,MB)\,MJ^2)\,MW^2) \\
&/(MA\,MB\,MZ^4 + (-\,MA\,MB^3 + 2\,MA^2\,MB^2 - MA^3\,MB)\,MZ^2)
\end{aligned}
$$

$$
\mathrm{FINITE3}_3 = -\,\frac{(2\,MB - 2\,MA)\,MJ^2\,MW^2\,MZ^2 + (8\,MA - 8\,MB)\,MJ^2\,MW^4}{MA\,MB\,MZ^4 + (-\,MA\,MB^3 + 2\,MA^2\,MB^2 - MA^3\,MB)\,MZ^2}
$$

$$
\begin{aligned}
\mathrm{FINITE3}_4 = -\,((&2\,MJ^2\,MW^2 + 2\,MJ^4 + (8\,MA\,MB - 14\,MA^2)\,MJ^2)\,MZ^6 \\
&+ (-\,8\,MJ^2\,MW^4 + ((-\,4\,MB^2 - 2\,MA\,MB + 22\,MA^2)\,MJ^2 - 8\,MJ^4)\,MW^2 \\
&+ (-\,4\,MB^2 + 6\,MA\,MB - 10\,MA^2)\,MJ^4 + (-\,16\,MA\,MB^3 + 16\,MA^2\,MB^2 + 38\,MA^3\,MB \\
&- 14\,MA^4)\,MJ^2)\,MZ^4 + ((16\,MB^2 - 24\,MA\,MB + 40\,MA^2)\,MJ^2\,MW^4 \\
&+ ((16\,MB^2 - 24\,MA\,MB + 40\,MA^2)\,MJ^4 + (2\,MB^4 + 10\,MA\,MB^3 - 38\,MA^2\,MB^2 \\
&- 50\,MA^3\,MB - 20\,MA^4)\,MJ^2)\,MW^2 + (2\,MB^4 - 6\,MA\,MB^3 + 2\,MA^2\,MB^2 + 6\,MA^3\,MB \\
&- 4\,MA^4)\,MJ^4 + (8\,MA\,MB^5 - 2\,MA^2\,MB^4 + 14\,MA^3\,MB^3 - 26\,MA^4\,MB^2 - 22\,MA^5\,MB \\
&+ 28\,MA^6)\,MJ^2)\,MZ^2 + (-\,8\,MB^4 + 24\,MA\,MB^3 - 8\,MA^2\,MB^2 - 24\,MA^3\,MB + 16\,MA^4) \\
&MJ^2\,MW^4 + ((-\,8\,MB^4 + 24\,MA\,MB^3 - 8\,MA^2\,MB^2 - 24\,MA^3\,MB + 16\,MA^4)\,MJ^4 \\
&+ (-\,8\,MA\,MB^5 + 8\,MA^2\,MB^4 - 8\,MA^3\,MB^3 + 8\,MA^4\,MB^2 + 16\,MA^5\,MB - 16\,MA^6)\,MJ^2) \\
&MW^2)/(MA\,MZ^8 + (-\,3\,MA\,MB^2 + 2\,MA^2\,MB - 3\,MA^3)\,MZ^6
\end{aligned}
$$

```
                                                                    4
+ (3 MA MB^4 - 4 MA^2 MB^3 + 2 MA^3 MB^2 - 4 MA^4 MB + 3 MA^5 ) MZ^4

                                                                    6
+ (- MA MB^6 + 2 MA^2 MB^5 + MA^3 MB^4 - 4 MA^4 MB^3 + MA^5 MB^2 + 2 MA^6 MB - MA^7 )

  2
MZ^2 )

                                                                    2   2
    FINITE3  = ((2 MJ^2 MW^2 + 2 MJ^4 + (8 MA MB - 14 MB^2 ) MJ^2 ) MZ^6
         5

                                                                    2   2
+ (- 8 MJ^2 MW^4 + ((22 MB^2 - 2 MA MB - 4 MA^2 ) MJ^2 - 8 MJ^4 ) MW^2

                                                                    4
+ (- 10 MB^2 + 6 MA MB - 4 MA^2 ) MJ^4 + (- 14 MB^4 + 38 MA MB^3 + 16 MA^2 MB^2

                                                                    2   2   4
- 16 MA^3 MB) MJ^4 ) MZ^2 + ((40 MB^2 - 24 MA MB + 16 MA^2 ) MJ^2 MW^4

                                                                    2   2
+ ((40 MB^2 - 24 MA MB + 16 MA^2 ) MJ^4 + (- 20 MB^4 - 50 MA MB^3 - 38 MA^2 MB^2

                                                                    3
+ 10 MA^3 MB + 2 MA^4 ) MJ^2 ) MW^2 + (- 4 MB^4 + 6 MA MB^3 + 2 MA^2 MB^2 - 6 MA^3 MB

                                                                    4   2
+ 2 MA^4 ) MJ^4 + (28 MB^6 - 22 MA MB^5 - 26 MA^2 MB^4 + 14 MA^3 MB^3 - 2 MA^4 MB^2

                                                                    3
+ 8 MA^5 MB) MJ^2 ) MZ^2 + (16 MB^4 - 24 MA MB^3 - 8 MA^2 MB^2 + 24 MA^3 MB - 8 MA^4 )

  2   4                                                                 4
MJ^2 MW^4 + ((16 MB^4 - 24 MA MB^3 - 8 MA^2 MB^2 + 24 MA^3 MB - 8 MA^4 ) MJ^4

                                                                    2
+ (- 16 MB^6 + 16 MA MB^5 + 8 MA^2 MB^4 - 8 MA^3 MB^3 + 8 MA^4 MB^2 - 8 MA^5 MB) MJ^2 )

  2                                                                 6
MW^2 )/(MB MZ^8 + (- 3 MB^3 + 2 MA MB^2 - 3 MA^2 MB) MZ^6

                                                                    4
+ (3 MB^5 - 4 MA MB^4 + 2 MA^2 MB^3 - 4 MA^3 MB^2 + 3 MA^4 MB) MZ^4

                                                                    5
+ (- MB^7 + 2 MA MB^6 + MA^2 MB^5 - 4 MA^3 MB^4 + MA^4 MB^3 + 2 MA^5 MB^2 - MA^6 MB)

  2
MZ^2 )

                                                                    2   4
    FINITE3  = ((30 MB - 30 MA) MJ^2 MZ^4
         6

                                                                    4
+ ((36 MA - 36 MB) MJ^2 MW^2 + (12 MB - 12 MA) MJ^4

                                                                    2   2
+ (- 18 MB^3 - 6 MA MB^2 + 6 MA^2 MB + 18 MA^3 ) MJ^2 ) MZ^2

                                                                    4
+ (48 MA - 48 MB) MJ^2 MW^4 + ((48 MA - 48 MB) MJ^4 + (48 MB^3 - 48 MA^3 ) MJ^2 ) MW^2
```

```
                5            4          2   3          3   2          4            5    2
+ (- 12 MB  + 12 MA MB  + 24 MA  MB  - 24 MA  MB  - 12 MA  MB + 12 MA ) MJ )

  6            2                    2    4
/(MZ  + (- 3 MB  + 2 MA MB - 3 MA ) MZ

          4            3          2   2          3            4    2      6            5
+ (3 MB  - 4 MA MB  + 2 MA  MB  - 4 MA  MB + 3 MA ) MZ  - MB  + 2 MA MB

     2   4          3   3          4   2          5          6
+ MA  MB  - 4 MA  MB  + MA  MB  + 2 MA  MB - MA )

                                4            2                    2    2          4
    FINITE3  = ((4 MW  + (- 2 MJ  + 24 MA MB - 12 MA ) MW  - 2 MJ
          7

          2                2          3          6
+ (14 MA  - 8 MA MB) MJ  - 4 MA  MB) MZ

            6          2          2                    2    4
+ (- 16 MW  + (8 MJ  - 8 MB  - 12 MA MB - 20 MA ) MW

          4          2                    2    2            3          2   2          3
+ (8 MJ  + (4 MB  + 2 MA MB - 22 MA ) MJ  - 48 MA MB  + 56 MA  MB  + 4 MA  MB

          4    2          2                    2    4
- 44 MA ) MW  + (4 MB  - 6 MA MB + 10 MA ) MJ

            3          2   2          3            4    2          4   2          5
+ (16 MA MB  - 16 MA  MB  - 38 MA  MB + 14 MA ) MJ  + 12 MA  MB  + 4 MA  MB)

  4            2                    2    6
MZ  + ((32 MB  - 48 MA MB + 80 MA ) MW

            2                    2    2          4            3          2   2
+ ((- 16 MB  + 24 MA MB - 40 MA ) MJ  + 4 MB  + 36 MA MB  - 100 MA  MB

          3            4    4            2                    2    4
+ 132 MA  MB + 24 MA ) MW  + ((- 16 MB  + 24 MA MB - 40 MA ) MJ

          4            3          2   2          3            4    2            5
+ (- 2 MB  - 10 MA MB  + 38 MA  MB  + 50 MA  MB + 20 MA ) MJ  + 24 MA MB

          2   4          3   3          4   2          5            6    2
- 44 MA  MB  + 52 MA  MB  - 60 MA  MB  - 92 MA  MB + 56 MA ) MW

          4            3          2   2          3            4    4
+ (- 2 MB  + 6 MA MB  - 2 MA  MB  - 6 MA  MB + 4 MA ) MJ

            5          2   4          3   3          4   2          5            6    2
+ (- 8 MA MB  + 2 MA  MB  - 14 MA  MB  + 26 MA  MB  + 22 MA  MB - 28 MA ) MJ

        3   5          4   4          5   3          6   2    2
+ 4 MA  MB  - 4 MA  MB  - 4 MA  MB  + 4 MA  MB ) MZ

            4            3          2   2          3            4    6
+ (- 16 MB  + 48 MA MB  - 16 MA  MB  - 48 MA  MB + 32 MA ) MW

          4            3          2   2          3            4    2            5
+ ((8 MB  - 24 MA MB  + 8 MA  MB  + 24 MA  MB - 16 MA ) MJ  - 24 MA MB

          2   4          3   3          4   2          5            6    4
+ 104 MA  MB  - 88 MA  MB  - 72 MA  MB  + 112 MA  MB - 32 MA ) MW

          4            3          2   2          3            4    4
+ ((8 MB  - 24 MA MB  + 8 MA  MB  + 24 MA  MB - 16 MA ) MJ
```

$$+ (8\ MA\ MB^{5} - 8\ MA^{2}\ MB^{4} + 8\ MA^{3}\ MB^{3} - 8\ MA^{4}\ MB^{2} - 16\ MA^{5}\ MB + 16\ MA^{6})\ MJ^{2}$$

$$- 16\ MA^{3}\ MB^{5} + 16\ MA^{4}\ MB^{4} + 16\ MA^{5}\ MB^{3} - 16\ MA^{6}\ MB^{2})\ MW^{2})$$

$$/(MA\ MZ^{8} + (- 3\ MA\ MB^{2} + 2\ MA^{2}\ MB - 3\ MA^{3})\ MZ^{6}$$

$$+ (3\ MA\ MB^{4} - 4\ MA^{2}\ MB^{3} + 2\ MA^{3}\ MB^{2} - 4\ MA^{4}\ MB + 3\ MA^{5})\ MZ^{4}$$

$$+ (- MA\ MB^{6} + 2\ MA^{2}\ MB^{5} + MA^{3}\ MB^{4} - 4\ MA^{4}\ MB^{3} + MA^{5}\ MB^{2} + 2\ MA^{6}\ MB - MA^{7})$$

$$MZ^{2})$$

$$FINITE3 = - ((4\ MW^{4} + (- 2\ MJ^{2} - 12\ MB^{2} + 24\ MA\ MB)\ MW^{2} - 2\ MJ^{4}$$

$$+ (14\ MB^{2} - 8\ MA\ MB)\ MJ^{2} - 4\ MA\ MB^{3})\ MZ^{6}$$

$$+ (- 16\ MW^{6} + (8\ MJ^{2} - 20\ MB^{2} - 12\ MA\ MB - 8\ MA^{2})\ MW^{4}$$

$$+ (8\ MJ^{4} + (- 22\ MB^{2} + 2\ MA\ MB + 4\ MA^{2})\ MJ^{2} - 44\ MB^{4} + 4\ MA\ MB^{3} + 56\ MA^{2}\ MB^{2}$$

$$- 48\ MA^{3}\ MB)\ MW^{2} + (10\ MB^{2} - 6\ MA\ MB + 4\ MA^{2})\ MJ^{4}$$

$$+ (14\ MB^{4} - 38\ MA\ MB^{3} - 16\ MA^{2}\ MB^{2} + 16\ MA^{3}\ MB)\ MJ^{2} + 4\ MA\ MB^{5} + 12\ MA^{2}\ MB^{4})$$

$$MZ^{4} + ((80\ MB^{2} - 48\ MA\ MB + 32\ MA^{2})\ MW^{6}$$

$$+ ((- 40\ MB^{2} + 24\ MA\ MB - 16\ MA^{2})\ MJ^{2} + 24\ MB^{4} + 132\ MA\ MB^{3} - 100\ MA^{2}\ MB^{2}$$

$$+ 36\ MA^{3}\ MB + 4\ MA^{4})\ MW^{4} + ((- 40\ MB^{2} + 24\ MA\ MB - 16\ MA^{2})\ MJ^{4}$$

$$+ (20\ MB^{4} + 50\ MA\ MB^{3} + 38\ MA^{2}\ MB^{2} - 10\ MA^{3}\ MB - 2\ MA^{4})\ MJ^{2} + 56\ MB^{6}$$

$$- 92\ MA\ MB^{5} - 60\ MA^{2}\ MB^{4} + 52\ MA^{3}\ MB^{3} - 44\ MA^{4}\ MB^{2} + 24\ MA^{5}\ MB)\ MW^{2}$$

$$+ (4\ MB^{4} - 6\ MA\ MB^{3} - 2\ MA^{2}\ MB^{2} + 6\ MA^{3}\ MB - 2\ MA^{4})\ MJ^{4}$$

$$+ (- 28\ MB^{6} + 22\ MA\ MB^{5} + 26\ MA^{2}\ MB^{4} - 14\ MA^{3}\ MB^{3} + 2\ MA^{4}\ MB^{2} - 8\ MA^{5}\ MB)\ MJ^{2}$$

$$+ 4\ MA^{2}\ MB^{6} - 4\ MA^{3}\ MB^{5} - 4\ MA^{4}\ MB^{4} + 4\ MA^{5}\ MB^{3})\ MZ^{2}$$

$$+ (32\ MB^{4} - 48\ MA\ MB^{3} - 16\ MA^{2}\ MB^{2} + 48\ MA^{3}\ MB - 16\ MA^{4})\ MW^{6}$$

$$+ ((- 16\ MB^{4} + 24\ MA\ MB^{3} + 8\ MA^{2}\ MB^{2} - 24\ MA^{3}\ MB + 8\ MA^{4})\ MJ^{2} - 32\ MB^{6}$$

```
                5          2   4          3   3          4   2          5          4
+ 112 MA MB  - 72 MA  MB  - 88 MA  MB  + 104 MA  MB  - 24 MA  MB) MW

                4            3          2   2          3           4    4
+ ((- 16 MB  + 24 MA MB  + 8 MA  MB  - 24 MA  MB + 8 MA ) MJ

          6            5          2   4          3   3          4   2          5          2
+ (16 MB  - 16 MA MB  - 8 MA  MB  + 8 MA  MB  - 8 MA  MB  + 8 MA  MB) MJ

        2   6          3   5          4   4          5   3    2
- 16 MA  MB  + 16 MA  MB  + 16 MA  MB  - 16 MA  MB ) MW )

       8           3           2          2          6
/(MB MZ  + (- 3 MB  + 2 MA MB  - 3 MA  MB) MZ

          5           4          2   3          3   2          4          4
+ (3 MB  - 4 MA MB  + 2 MA  MB  - 4 MA  MB  + 3 MA  MB) MZ

          7           6      2   5          3   4      4   3          5   2      6
+ (- MB  + 2 MA MB  + MA  MB  - 4 MA  MB  + MA  MB  + 2 MA  MB  - MA  MB)

  2
MZ )

      FINITE3   = ((30 MB - 30 MA) MJ  MZ
              9                          2   4

+ ((96 MA - 96 MB) MJ  MW  + (36 MB - 36 MA) MJ
                     2   2                      4

          3           2          2           3    2    2
+ (- 30 MB  + 18 MA MB  - 18 MA  MB + 30 MA ) MJ ) MZ

                     2   4                      4
+ (72 MA - 72 MB) MJ  MW  + ((72 MA - 72 MB) MJ

          3           2          2           3    2    2
+ (60 MB  - 36 MA MB  + 36 MA  MB - 60 MA ) MJ ) MW )

    6          2                    2    4
/(MZ  + (- 3 MB  + 2 MA MB - 3 MA ) MZ

          4           3          2   2          3           4    2    6           5
+ (3 MB  - 4 MA MB  + 2 MA  MB  - 4 MA  MB + 3 MA ) MZ  - MB  + 2 MA MB

    2   4          3   3          4   2          5          6
+ MA  MB  - 4 MA  MB  + MA  MB  + 2 MA  MB - MA )

      FINITE3   = - (((20 MB - 20 MA) MW  - 2 MA MB  + 2 MA  MB) MZ
              10                                 2          2          2          4

+ ((64 MB - 64 MA) MW  + ((60 MA - 60 MB) MJ  - 20 MB  + 20 MA MB
                     4                                 2          3           2

- 20 MA  MB + 20 MA ) MW  + (24 MB - 24 MA) MJ
          2           3    2                      4

          3           2          2           3    2           4          2   3
+ (- 12 MB  + 24 MA MB  - 24 MA  MB + 12 MA ) MJ  + 2 MA MB  + 6 MA  MB

        3   2          4          2                      6
- 6 MA  MB  - 2 MA  MB) MZ  + (48 MB - 48 MA) MW
```

$$+ ((24\ MA - 24\ MB)\ MJ^{2} - 40\ MB^{3} + 48\ MA\ MB^{2} - 48\ MA^{2}\ MB + 40\ MA^{3})\ MW^{4}$$

$$+ ((24\ MA - 24\ MB)\ MJ^{4} + (12\ MB^{3} - 36\ MA\ MB^{2} + 36\ MA^{2}\ MB - 12\ MA^{3})\ MJ^{2}$$

$$+ 4\ MA\ MB^{4} + 12\ MA^{2}\ MB^{3} - 12\ MA^{3}\ MB^{2} - 4\ MA^{4}\ MB)\ MW^{2}$$

$$+ (12\ MB^{5} - 12\ MA\ MB^{4} - 24\ MA^{2}\ MB^{3} + 24\ MA^{3}\ MB^{2} + 12\ MA^{4}\ MB - 12\ MA^{5})\ MJ^{2})$$

$$/(MZ^{6} + (-3\ MB^{2} + 2\ MA\ MB - 3\ MA^{2})\ MZ^{4}$$

$$+ (3\ MB^{4} - 4\ MA\ MB^{3} + 2\ MA^{2}\ MB^{2} - 4\ MA^{3}\ MB + 3\ MA^{4})\ MZ^{2} - MB^{6} + 2\ MA\ MB^{5}$$

$$+ MA^{2}\ MB^{4} - 4\ MA^{3}\ MB^{3} + MA^{4}\ MB^{2} + 2\ MA^{5}\ MB - MA^{6})$$

$$FINITE3_{11} = -((8\ MB - 8\ MA)\ MW^{2}\ MZ^{8}$$

$$+ ((48\ MB - 48\ MA)\ MW^{4} + ((36\ MA - 36\ MB)\ MJ^{2} - 16\ MB^{3} + 16\ MA\ MB^{2}$$

$$- 16\ MA^{2}\ MB + 16\ MA^{3})\ MW^{2} + (4\ MB - 4\ MA)\ MJ^{4}$$

$$+ (-4\ MB^{3} + 4\ MA\ MB^{2} - 4\ MA^{2}\ MB + 4\ MA^{3})\ MJ^{2} + 4\ MA^{2}\ MB^{3} - 4\ MA^{3}\ MB^{2})\ MZ^{6}$$

$$+ ((88\ MB - 88\ MA)\ MW^{6} + ((120\ MA - 120\ MB)\ MJ^{2} - 64\ MB^{3} + 68\ MA\ MB^{2}$$

$$- 68\ MA^{2}\ MB + 64\ MA^{3})\ MW^{4} + ((80\ MB - 80\ MA)\ MJ^{4} + (8\ MB^{3} - 8\ MA^{3})\ MJ^{2}$$

$$+ 8\ MB^{5} - 8\ MA\ MB^{4} + 40\ MA^{2}\ MB^{3} - 40\ MA^{3}\ MB^{2} + 8\ MA^{4}\ MB - 8\ MA^{5})\ MW^{2}$$

$$+ (24\ MA - 24\ MB)\ MJ^{6} + (16\ MB^{3} - 28\ MA\ MB^{2} + 28\ MA^{2}\ MB - 16\ MA^{3})\ MJ^{4}$$

$$+ (8\ MB^{5} - 24\ MA^{2}\ MB^{3} + 24\ MA^{3}\ MB^{2} - 8\ MA^{5})\ MJ^{2} - 4\ MA^{2}\ MB^{5} + 4\ MA^{5}\ MB^{2})\ MZ^{4}$$

$$+ ((48\ MB - 48\ MA)\ MW^{8} + ((72\ MA - 72\ MB)\ MJ^{2} - 64\ MB^{3} + 72\ MA\ MB^{2}$$

$$- 72\ MA^{2}\ MB + 64\ MA^{3})\ MW^{6} + ((32\ MB^{3} - 64\ MA\ MB^{2} + 64\ MA^{2}\ MB - 32\ MA^{3})\ MJ^{2}$$

$$+ 16\ MB^{5} - 16\ MA\ MB^{4} + 64\ MA^{2}\ MB^{3} - 64\ MA^{3}\ MB^{2} + 16\ MA^{4}\ MB - 16\ MA^{5})\ MW^{4}$$

$$+ ((24\ MB - 24\ MA)\ MJ^{6} + (-16\ MB^{3} + 40\ MA\ MB^{2} - 40\ MA^{2}\ MB + 16\ MA^{3})\ MJ^{4}$$

$$+ (36\ MB^{5} - 44\ MA\ MB^{4} - 40\ MA^{2}\ MB^{3} + 40\ MA^{3}\ MB^{2} + 44\ MA^{4}\ MB - 36\ MA^{5})\ MJ^{2}$$

$$- 8\ MA^{2}\ MB^{5} + 8\ MA^{5}\ MB^{2})\ MW^{2} + (-20\ MB^{5} + 20\ MA\ MB^{4} + 40\ MA^{2}\ MB^{3}$$

```
      3   2          4            5    4
- 40 MA  MB  - 20 MA  MB + 20 MA ) MJ

        7           6          2   5          3   4          4   3          5   2
+ (- 4 MB  - 4 MA MB  + 12 MA  MB  + 12 MA  MB  - 12 MA  MB  - 12 MA  MB

      6           7    2    2          5            4          2   3          3   2
+ 4 MA  MB + 4 MA ) MJ ) MZ  + (16 MB  - 16 MA MB  - 32 MA  MB  + 32 MA  MB

      4           5    2   4          5            4          2   3          3   2
+ 16 MA  MB - 16 MA ) MJ  MW  + ((8 MB  - 8 MA MB  - 16 MA  MB  + 16 MA  MB

      4           5    4          7            6          3   4          4   3
+ 8 MA  MB - 8 MA ) MJ  + (- 8 MB  + 16 MA MB  - 24 MA  MB  + 24 MA  MB

      6           7    2    2     8           2                  2    6
- 16 MA  MB + 8 MA ) MJ ) MW )/(MZ  + (- 3 MB  + 2 MA MB - 3 MA ) MZ

      4           3        2   2        3           4    4
+ (3 MB  - 4 MA MB  + 2 MA  MB  - 4 MA  MB + 3 MA ) MZ

      6           5     2   4        3   3     4   2        5          6    2
+ (- MB  + 2 MA MB  + MA  MB  - 4 MA  MB  + MA  MB  + 2 MA  MB - MA ) MZ )

                                                   2   8
     FINITE3    = ((12 MB - 12 MA) MJ  MZ
            12

                                 4                          2          3           2
+ ((24 MB - 24 MA) MW  + ((84 MA - 84 MB) MJ  - 24 MB  + 12 MA MB

      2           3    2                          4
- 12 MA  MB + 24 MA ) MW  + (48 MB - 48 MA) MJ

          3           2          2           3    2          2   3          3   2
+ (- 24 MB  + 12 MA MB  - 12 MA  MB + 24 MA ) MJ  + 12 MA  MB  - 12 MA  MB )

  6                          6                          2          3           2
MZ  + ((48 MA - 48 MB) MW  + ((108 MB - 108 MA) MJ  + 48 MB  + 36 MA MB

      2           3    4                          4
- 36 MA  MB - 48 MA ) MW  + ((168 MA - 168 MB) MJ

        3           2          2           3    2          5           4
+ (72 MB  - 96 MA MB  + 96 MA  MB - 72 MA ) MJ  + 48 MB  - 48 MA MB

      2   3         3   2         4           5    2                          6
- 48 MA  MB  + 48 MA  MB  + 48 MA  MB - 48 MA ) MW  + (36 MB - 36 MA) MJ

          3           2          2           3    4
+ (- 48 MB  + 36 MA MB  - 36 MA  MB + 48 MA ) MJ

        5         2   3         3   2         5    2         2   5         5   2
+ (12 MB  + 48 MA  MB  - 48 MA  MB  - 12 MA ) MJ  - 12 MA  MB  + 12 MA  MB )

  4                          8                          2          3           2
MZ  + ((144 MA - 144 MB) MW  + ((216 MB - 216 MA) MJ  + 96 MB  - 312 MA MB

      2           3    6           3           2          2           3
+ 312 MA  MB - 96 MA ) MW  + ((- 48 MB  + 240 MA MB  - 240 MA  MB + 48 MA )

  2         5           4          4           5    4
MJ  - 120 MB  + 168 MA MB  - 168 MA  MB + 120 MA ) MW
```

$$+ ((72\,MA - 72\,MB)\,MJ^{6} + (96\,MB^{3} - 72\,MA\,MB^{2} + 72\,MA^{2}\,MB - 96\,MA^{3})\,MJ^{4}$$

$$+ (12\,MB^{5} - 36\,MA\,MB^{4} - 168\,MA^{2}\,MB^{3} + 168\,MA^{3}\,MB^{2} + 36\,MA^{4}\,MB - 12\,MA^{5})\,MJ^{2}$$

$$- 24\,MB^{7} + 36\,MA\,MB^{6} + 36\,MA^{2}\,MB^{5} - 48\,MA^{3}\,MB^{4} + 48\,MA^{4}\,MB^{3} - 36\,MA^{5}\,MB^{2}$$

$$- 36\,MA^{6}\,MB + 24\,MA^{7})\,MW^{2})\,MZ^{2} + (- 48\,MB^{5} + 48\,MA\,MB^{4} + 96\,MA^{2}\,MB^{3}$$

$$- 96\,MA^{3}\,MB^{2} - 48\,MA^{4}\,MB + 48\,MA^{5})\,MW^{6}$$

$$+ ((- 24\,MB^{5} + 24\,MA\,MB^{4} + 48\,MA^{2}\,MB^{3} - 48\,MA^{3}\,MB^{2} - 24\,MA^{4}\,MB + 24\,MA^{5})\,MJ^{2}$$

$$+ 48\,MB^{7} - 120\,MA\,MB^{6} + 24\,MA^{2}\,MB^{5} + 192\,MA^{3}\,MB^{4} - 192\,MA^{4}\,MB^{3} - 24\,MA^{5}\,MB^{2}$$

$$+ 120\,MA^{6}\,MB - 48\,MA^{7})\,MW^{4})/(MZ^{8} + (- 3\,MB^{2} + 2\,MA\,MB - 3\,MA^{2})\,MZ^{6}$$

$$+ (3\,MB^{4} - 4\,MA\,MB^{3} + 2\,MA^{2}\,MB^{2} - 4\,MA^{3}\,MB + 3\,MA^{4})\,MZ^{4}$$

$$+ (- MB^{6} + 2\,MA\,MB^{5} + MA^{2}\,MB^{4} - 4\,MA^{3}\,MB^{3} + MA^{4}\,MB^{2} + 2\,MA^{5}\,MB - MA^{6})\,MZ^{2})$$

$$\mathrm{FINITE4}_1 = ((4\,MJ^2\,MW^2 + 4\,MJ^4 - 6\,MA\,MB\,MJ^2)\,MZ^2 - 4\,MJ^2\,MW^4 + (12\,MA\,MB\,MJ^2 - 4\,MJ^4)\,MW^2)/(MA\,MB\,MZ^2)$$

$$\mathrm{FINITE4}_2 = -\frac{(8\,MJ^2\,MW^2 + 4\,MJ^4 - MA\,MB\,MJ^2)\,MZ^2 - 8\,MJ^2\,MW^4 + (4\,MA\,MB\,MJ^2 - 4\,MJ^4)\,MW^2}{MA\,MB\,MZ^2}$$

$$\mathrm{FINITE4}_3 = \frac{4\,MJ^2\,MW^2\,MZ^2 - 4\,MJ^2\,MW^4}{MA\,MB\,MZ^2}$$

$$\begin{aligned}
\mathrm{FINITE4}_4 = {} & (((4\,MB - MA)\,MJ^2\,MW^2 + (4\,MB - MA)\,MJ^4 \\
& + (MA\,MB^2 - 8\,MA^2\,MB + MA^3)\,MJ^2)\,MZ^6 + ((4\,MA - 4\,MB)\,MJ^2\,MW^4 \\
& + ((4\,MA - 4\,MB)\,MJ^4 + (-8\,MB^3 - MA\,MB^2 - 3\,MA^3)\,MJ^2)\,MW^2 \\
& + (-8\,MB^3 + 3\,MA\,MB^2 - 8\,MA^2\,MB + MA^3)\,MJ^4 \\
& + (-2\,MA\,MB^4 + 14\,MA^2\,MB^3 + 3\,MA^3\,MB^2 + 18\,MA^4\,MB - 9\,MA^5)\,MJ^2)\,MZ^4 \\
& + ((8\,MB^3 - 12\,MA\,MB^2 + 8\,MA^2\,MB - 4\,MA^3)\,MJ^2\,MW^4 \\
& + ((8\,MB^3 - 12\,MA\,MB^2 + 8\,MA^2\,MB - 4\,MA^3)\,MJ^4 \\
& + (4\,MB^5 + 6\,MA\,MB^4 - 16\,MA^2\,MB^3 + 16\,MA^3\,MB^2 - 20\,MA^4\,MB + 10\,MA^5)\,MJ^2)\,MW^2 \\
& + (4\,MB^5 - 2\,MA\,MB^4 - 8\,MA^2\,MB^3 + 4\,MA^3\,MB^2 + 4\,MA^4\,MB - 2\,MA^5)\,MJ^4 \\
& + (MA\,MB^6 - 6\,MA^2\,MB^5 + 6\,MA^3\,MB^4 + 12\,MA^4\,MB^3 - 15\,MA^5\,MB^2 - 6\,MA^6\,MB \\
& + 8\,MA^7)\,MJ^2)\,MZ^2 + (-4\,MB^5 + 8\,MA\,MB^4 + 8\,MA^2\,MB^3 - 16\,MA^3\,MB^2 - 4\,MA^4\,MB \\
& + 8\,MA^5)\,MJ^2\,MW^4 + ((-4\,MB^5 + 8\,MA\,MB^4 + 8\,MA^2\,MB^3 - 16\,MA^3\,MB^2 - 4\,MA^4\,MB \\
& + 8\,MA^5)\,MJ^4 + (-4\,MA\,MB^6 + 12\,MA^2\,MB^5 - 8\,MA^7)\,MJ^2)\,MW^2) \\
& /((MA\,MB^2 - MA^3)\,MZ^6 + (2\,MA^5 - 2\,MA\,MB^4)\,MZ^4
\end{aligned}$$

```
        6       3   4       5   2     7    2
+ (MA MB  - 3 MA  MB  + 3 MA  MB  - MA ) MZ )

                                   2   2                   4
    FINITE4  = (((MB - 4 MA) MJ  MW  + (MB - 4 MA) MJ
          5

        3          2     2       2    6
+ (- MB  + 8 MA MB  - MA  MB) MJ ) MZ

                     2   4                      4        3     2          3   2
+ ((4 MA - 4 MB) MJ  MW  + ((4 MA - 4 MB) MJ  + (3 MB  + MA  MB + 8 MA ) MJ )

  2        3          2       2          3    4
MW  + (- MB  + 8 MA MB  - 3 MA  MB + 8 MA ) MJ

        5           4       2   3        3   2       4       2    4
+ (9 MB  - 18 MA MB  - 3 MA  MB  - 14 MA  MB  + 2 MA  MB) MJ ) MZ

        3          2        2          3    2   4
+ ((4 MB  - 8 MA MB  + 12 MA  MB - 8 MA ) MJ  MW

        3          2        2          3    4
+ ((4 MB  - 8 MA MB  + 12 MA  MB - 8 MA ) MJ

          5           4       2   3        3   2       4          5    2
+ (- 10 MB  + 20 MA MB  - 16 MA  MB  + 16 MA  MB  - 6 MA  MB - 4 MA ) MJ )

  2        5          4       2   3       3   2       4          5    4
MW  + (2 MB  - 4 MA MB  - 4 MA  MB  + 8 MA  MB  + 2 MA  MB - 4 MA ) MJ

          7          6        2   5        3   4       4   3       5   2
+ (- 8 MB  + 6 MA MB  + 15 MA  MB  - 12 MA  MB  - 6 MA  MB  + 6 MA  MB

     6       2    2          5          4        2   3       3   2       4
- MA  MB) MJ ) MZ  + (- 8 MB  + 4 MA MB  + 16 MA  MB  - 8 MA  MB  - 8 MA  MB

       5    2   4           5          4        2   3       3   2       4
+ 4 MA ) MJ  MW  + ((- 8 MB  + 4 MA MB  + 16 MA  MB  - 8 MA  MB  - 8 MA  MB

       5    4        7        2   5       6       2    2
+ 4 MA ) MJ  + (8 MB  - 12 MA  MB  + 4 MA  MB) MJ ) MW )

     3     2       6        4          5    4
/((MB  - MA  MB) MZ  + (2 MA  MB - 2 MB ) MZ

      7       2   5       4   3     6       2
+ (MB  - 3 MA  MB  + 3 MA  MB  - MA  MB) MZ )

                        2   4          2   2       4
    FINITE4  = - (MJ  MZ  + (- 2 MJ  MW  + 2 MJ
          6

        2                 2    2     2       2   4
+ (5 MB  - 4 MA MB + 5 MA ) MJ ) MZ  - 8 MJ  MW

        2                 2    2       4    2
+ ((4 MB  + 16 MA MB + 4 MA ) MJ  - 8 MJ ) MW

          4        2   2       4    2     4          2       2    2     4
+ (- 6 MB  + 12 MA  MB  - 6 MA ) MJ )/(MZ  + (- 2 MB  - 2 MA ) MZ  + MB

       2   2     4
- 2 MA  MB  + MA )
```

$$
\begin{aligned}
\mathrm{FINITE4} = {} & - (((8\,MB - 2\,MA)\,MW^4 + ((MA - 4\,MB)\,MJ^2 + 3\,MA\,MB^2 - 4\,MA^2\,MB^7 \\
& - 2\,MA^3)\,MW^2 + (MA - 4\,MB)\,MJ^4 + (-\,MA\,MB^2 + 8\,MA^2\,MB - MA^3)\,MJ^2 + MA^3\,MB^2 \\
& - 4\,MA^4\,MB)\,MZ^6 + ((8\,MA - 8\,MB)\,MW^6 + ((4\,MB - 4\,MA)\,MJ^2 - 16\,MB^3 - 6\,MA\,MB^2 \\
& - 12\,MA^2\,MB + 10\,MA^3)\,MW^4 + ((4\,MB - 4\,MA)\,MJ^4 + (8\,MB^3 + MA\,MB^2 + 3\,MA^3)\,MJ^2 \\
& - 6\,MA\,MB^4 + 20\,MA^2\,MB^3 + 12\,MA^3\,MB^2 - 14\,MA^5)\,MW^2 \\
& + (8\,MB^3 - 3\,MA\,MB^2 + 8\,MA^2\,MB^3 - MA^3)\,MJ^4 \\
& + (2\,MA\,MB^4 - 14\,MA^2\,MB^3 - 3\,MA^3\,MB^2 - 18\,MA^4\,MB + 9\,MA^5)\,MJ^2 + 12\,MA^4\,MB^3 \\
& - 4\,MA^5\,MB^2 + 4\,MA^6\,MB)\,MZ^4 + ((16\,MB^3 - 24\,MA\,MB^2 + 16\,MA^2\,MB^3 - 8\,MA^3)\,MW^6 \\
& + ((-\,8\,MB^3 + 12\,MA\,MB^2 - 8\,MA^2\,MB^3 + 4\,MA^3)\,MJ^2 + 8\,MB^5 + 20\,MA\,MB^4 \\
& - 60\,MA^2\,MB^3 - 8\,MA^3\,MB^2 + 36\,MA^4\,MB^5 + 4\,MA^5)\,MW^4 \\
& + ((-\,8\,MB^3 + 12\,MA\,MB^2 - 8\,MA^2\,MB^3 + 4\,MA^3)\,MJ^4 \\
& + (-\,4\,MB^5 - 6\,MA\,MB^4 + 16\,MA^2\,MB^3 - 16\,MA^3\,MB^2 + 20\,MA^4\,MB^5 - 10\,MA^5)\,MJ^2 \\
& + 3\,MA\,MB^6 - 16\,MA^2\,MB^5 + 22\,MA^3\,MB^4 + 20\,MA^4\,MB^3 - 25\,MA^5\,MB^2 - 20\,MA^6\,MB^7 \\
& + 16\,MA^7)\,MW^2 + (-\,4\,MB^5 + 2\,MA\,MB^4 + 8\,MA^2\,MB^3 - 4\,MA^3\,MB^2 - 4\,MA^4\,MB^5 \\
& + 2\,MA^4)\,MJ^4 + (-\,MA\,MB^6 + 6\,MA^2\,MB^5 - 6\,MA^3\,MB^4 - 12\,MA^4\,MB^3 + 15\,MA^5\,MB^2 \\
& + 6\,MA^6\,MB - 8\,MA^7)\,MJ^2 - MA^3\,MB^6 + 2\,MA^5\,MB^4 - MA^7\,MB^2)\,MZ^2 \\
& + (-\,8\,MB^5 + 16\,MA\,MB^4 + 16\,MA^2\,MB^3 - 32\,MA^3\,MB^2 - 8\,MA^4\,MB + 16\,MA^5)\,MW^6 \\
& + ((4\,MB^5 - 8\,MA\,MB^4 - 8\,MA^2\,MB^3 + 16\,MA^3\,MB^2 + 4\,MA^4\,MB - 8\,MA^5)\,MJ^2 \\
& - 12\,MA\,MB^6 + 40\,MA^2\,MB^5 + 8\,MA^3\,MB^4 - 80\,MA^4\,MB^3 + 20\,MA^5\,MB^2 + 40\,MA^6\,MB \\
& - 16\,MA^7)\,MW^4 + ((4\,MB^5 - 8\,MA\,MB^4 - 8\,MA^2\,MB^3 + 16\,MA^3\,MB^2 + 4\,MA^4\,MB \\
& - 8\,MA^5)\,MJ^4 + (4\,MA\,MB^6 - 12\,MA^5\,MB^2 + 8\,MA^7)\,MJ^2 - 8\,MA^3\,MB^6 + 16\,MA^5\,MB^4 \\
& - 8\,MA^7\,MB^2)\,MW^2)/((MA\,MB^2 - MA^3)\,MZ^6 + (2\,MA^5 - 2\,MA\,MB^4)\,MZ^4
\end{aligned}
$$

$$+ (MA\ MB^{6} - 3\ MA^{3}\ MB^{4} + 3\ MA^{5}\ MB^{2} - MA^{7})\ MZ^{2})$$

$$FINITE4_{8} = - (((2\ MB - 8\ MA)\ MW^{4} + ((4\ MA - MB)\ MJ^{2} + 2\ MB^{3} + 4\ MA\ MB^{2}$$

$$- 3\ MA^{2}\ MB)\ MW^{2} + (4\ MA - MB)\ MJ^{4} + (MB^{3} - 8\ MA\ MB^{2} + MA^{2}\ MB)\ MJ^{2} + 4\ MA\ MB^{4}$$

$$- MA^{2}\ MB^{3})\ MZ^{6} + ((8\ MA - 8\ MB)\ MW^{6} + ((4\ MB - 4\ MA)\ MJ^{2} - 10\ MB^{3} + 12\ MA\ MB^{2}$$

$$+ 6\ MA^{2}\ MB + 16\ MA^{3})\ MW^{4} + ((4\ MB - 4\ MA)\ MJ^{4}$$

$$+ (- 3\ MB^{3} - MA^{2}\ MB - 8\ MA^{3})\ MJ^{2} + 14\ MB^{5} - 12\ MA^{2}\ MB^{3} - 20\ MA^{3}\ MB^{2}$$

$$+ 6\ MA^{4}\ MB)\ MW^{2} + (MB^{3} - 8\ MA\ MB^{2} + 3\ MA^{2}\ MB - 8\ MA^{3})\ MJ^{4}$$

$$+ (- 9\ MB^{5} + 18\ MA\ MB^{4} + 3\ MA^{2}\ MB^{3} + 14\ MA^{3}\ MB^{2} - 2\ MA^{4}\ MB)\ MJ^{2} - 4\ MA\ MB^{6}$$

$$+ 4\ MA^{2}\ MB^{5} - 12\ MA^{3}\ MB^{4})\ MZ^{4} + ((8\ MB^{3} - 16\ MA\ MB^{2} + 24\ MA^{2}\ MB - 16\ MA^{3})\ MW^{6}$$

$$+ ((- 4\ MB^{3} + 8\ MA\ MB^{2} - 12\ MA^{2}\ MB + 8\ MA^{3})\ MJ^{2} - 4\ MB^{5} - 36\ MA\ MB^{4}$$

$$+ 8\ MA^{2}\ MB^{3} + 60\ MA^{3}\ MB^{2} - 20\ MA^{4}\ MB - 8\ MA^{5})\ MW^{4}$$

$$+ ((- 4\ MB^{3} + 8\ MA\ MB^{2} - 12\ MA^{2}\ MB + 8\ MA^{3})\ MJ^{4}$$

$$+ (10\ MB^{5} - 20\ MA\ MB^{4} + 16\ MA^{2}\ MB^{3} - 16\ MA^{3}\ MB^{2} + 6\ MA^{4}\ MB + 4\ MA^{5})\ MJ^{2}$$

$$- 16\ MB^{7} + 20\ MA\ MB^{6} + 25\ MA^{2}\ MB^{5} - 20\ MA^{3}\ MB^{4} - 22\ MA^{4}\ MB^{3} + 16\ MA^{5}\ MB^{2}$$

$$- 3\ MA^{6}\ MB)\ MW^{2} + (- 2\ MB^{5} + 4\ MA\ MB^{4} + 4\ MA^{2}\ MB^{3} - 8\ MA^{3}\ MB^{2} - 2\ MA^{4}\ MB$$

$$+ 4\ MA^{5})\ MJ^{4} + (8\ MB^{7} - 6\ MA\ MB^{6} - 15\ MA^{2}\ MB^{5} + 12\ MA^{3}\ MB^{4} + 6\ MA^{4}\ MB^{3}$$

$$- 6\ MA^{5}\ MB^{2} + MA^{6}\ MB)\ MJ^{2} + MA^{2}\ MB^{7} - 2\ MA^{4}\ MB^{5} + MA^{6}\ MB^{3})\ MZ^{2}$$

$$+ (- 16\ MB^{5} + 8\ MA\ MB^{4} + 32\ MA^{2}\ MB^{3} - 16\ MA^{3}\ MB^{2} - 16\ MA^{4}\ MB + 8\ MA^{5})\ MW^{6}$$

$$+ ((8\ MB^{5} - 4\ MA\ MB^{4} - 16\ MA^{2}\ MB^{3} + 8\ MA^{3}\ MB^{2} + 8\ MA^{4}\ MB - 4\ MA^{5})\ MJ^{2}$$

$$+ 16\ MB^{7} - 40\ MA\ MB^{6} - 20\ MA^{2}\ MB^{5} + 80\ MA^{3}\ MB^{4} - 8\ MA^{4}\ MB^{3} - 40\ MA^{5}\ MB^{2}$$

$$+ 12\ MA^{6}\ MB)\ MW^{4} + ((8\ MB^{5} - 4\ MA\ MB^{4} - 16\ MA^{2}\ MB^{3} + 8\ MA^{3}\ MB^{2} + 8\ MA^{4}\ MB$$

$$- 4\ MA^{5})\ MJ^{4} + (- 8\ MB^{7} + 12\ MA^{2}\ MB^{5} - 4\ MA^{6}\ MB)\ MJ^{2} + 8\ MA^{2}\ MB^{7} - 16\ MA^{4}\ MB^{5}$$

```
      6   3     2       3      2        6          4          5    4
+ 8 MA  MB ) MW )/((MB  - MA  MB) MZ  + (2 MA  MB - 2 MB ) MZ

      7        2   5        4   3      6        2
+ (MB  - 3 MA  MB  + 3 MA  MB  - MA  MB) MZ )

                                2   4            2   2        4
     FINITE4  = - (3 MJ  MZ  + (- 12 MJ  MW  + 6 MJ
         9

          2                    2    2    2         2   4
+ (- 3 MB  - 6 MA MB - 3 MA ) MJ ) MZ  - 12 MJ  MW

          2                    2    2         4    2
+ ((6 MB  + 12 MA MB + 6 MA ) MJ  - 12 MJ ) MW )

   4           2        2    2      4        2   2      4
/(MZ  + (- 2 MB  - 2 MA ) MZ  + MB  - 2 MA  MB  + MA )

                                2        2            4
     FINITE4   = ((6 MW  + 2 MJ  + MA MB) MZ
            10

          4            2        2                    2    2        4
+ (16 MW  + (- 10 MJ  - 6 MB  + 8 MA MB - 6 MA ) MW  + 4 MJ

          2                    2    2         3        2   2      3         2        6
+ (- 8 MB  - 2 MA MB - 8 MA ) MJ  - MA MB  + 4 MA  MB  - MA  MB) MZ  + 8 MW

          2         2                    2    4
+ (- 4 MJ  - 12 MB  + 12 MA MB - 12 MA ) MW

          4         2                    2    2          3        2   2        3
+ (- 4 MJ  + (2 MB  - 4 MA MB + 2 MA ) MJ  - 2 MA MB  + 8 MA  MB  - 2 MA  MB)

   2         4         2   2        4    2
MW  + (6 MB  - 12 MA  MB  + 6 MA ) MJ )

   4           2        2    2      4        2   2      4
/(MZ  + (- 2 MB  - 2 MA ) MZ  + MB  - 2 MA  MB  + MA )

                                2   8          4            2        2                    2
     FINITE4   = (4 MW  MZ  + (16 MW  + (- 16 MJ  - 8 MB  + 4 MA MB - 8 MA )
            11

   2        4           2        2    2        2   2    6
MW  + 2 MJ  + (- 2 MB  - 2 MA ) MJ  + 2 MA  MB ) MZ

          6           2        2                    2    4
+ (20 MW  + (- 20 MJ  - 24 MB  + 14 MA MB - 24 MA ) MW

          4          2                    2    2        4           3         2   2
+ (16 MJ  + (14 MB  - 8 MA MB + 14 MA ) MJ  + 4 MB  - 4 MA MB  + 12 MA  MB

       3          4    2        6        2                    2    4
- 4 MA  MB + 4 MA ) MW  - 4 MJ  + (2 MB  + 2 MA MB + 2 MA ) MJ

          4          3        2   2        3          4    2        2   4
+ (4 MB  - 2 MA MB  - 4 MA  MB  - 2 MA  MB + 4 MA ) MJ  - 2 MA  MB

       3   3        4   2    4         8            2        2
+ 2 MA  MB  - 2 MA  MB ) MZ  + (8 MW  + (- 12 MJ  - 16 MB  + 12 MA MB

       2    6                    2        4           3        2   2        3
- 16 MA ) MW  + (- 16 MA MB MJ  + 8 MB  - 8 MA MB  + 16 MA  MB  - 8 MA  MB
```

```
      4    4          6           2                    2    4
+ 8 MA ) MW  + (4 MJ  + (- 8 MB  + 4 MA MB - 8 MA ) MJ

      4           3        2   2        3           4    2        2   4
+ (6 MB  - 4 MA MB  + 8 MA  MB  - 4 MA  MB + 6 MA ) MJ  - 4 MA  MB

      3   3        4   2    2           4        2   2        4    4
+ 4 MA  MB  - 4 MA  MB ) MW  + (- 4 MB  + 8 MA  MB  - 4 MA ) MJ

          6           5        2   4        3   3        4   2        5          6
+ (- 2 MB  + 2 MA MB  + 2 MA  MB  - 4 MA  MB  + 2 MA  MB  + 2 MA  MB - 2 MA )

  2    2        4        2   2        4    2   4
MJ ) MZ  + (8 MB  - 16 MA  MB  + 8 MA ) MJ  MW

        4        2   2        4    4          6          5        2   4
+ ((4 MB  - 8 MA  MB  + 4 MA ) MJ  + (- 4 MB  + 4 MA MB  + 4 MA  MB

      3   3        4   2        5          6    2    2
- 8 MA  MB  + 4 MA  MB  + 4 MA  MB - 4 MA ) MJ ) MW )

   6          2        2    4        4        2   2        4    2
/(MZ  + (- 2 MB  - 2 MA ) MZ  + (MB  - 2 MA  MB  + MA ) MZ )

                          2   2        4             2    6
     FINITE4   = ((24 MJ  MW  - 6 MJ  + 6 MA MB MJ ) MZ
            12

        6           2        2                    2    4
+ (48 MW  + (- 30 MJ  - 36 MB  - 18 MA MB - 36 MA ) MW

        4           2        2    2          3        2   2        3        2
+ (24 MJ  + (- 30 MB  - 30 MA ) MJ  + 6 MA MB  + 24 MA  MB  + 6 MA  MB) MW

      6          2                    2    4
- 6 MJ  + (6 MB  + 6 MA MB + 6 MA ) MJ

            3        2   2        3        2        3   3    4
+ (- 6 MA MB  - 6 MA  MB  - 6 MA  MB) MJ  + 6 MA  MB ) MZ

        8           2        2                    2    6
+ (24 MW  + (- 36 MJ  - 72 MB  + 84 MA MB - 72 MA ) MW

          2                    2    2        4           3        3
+ ((12 MB  - 72 MA MB + 12 MA ) MJ  + 60 MB  - 36 MA MB  - 36 MA  MB

        4    4          6           2                    2    4
+ 60 MA ) MW  + (12 MJ  + (- 12 MB  - 12 MA MB - 12 MA ) MJ

        4           3        3           4    2          5        5        2    2
+ (6 MB  + 12 MA MB  + 12 MA  MB + 6 MA ) MJ  - 6 MA MB  - 6 MA  MB) MW ) MZ

        4        2   2        4    6          4        2   2        4    2
+ (24 MB  - 48 MA  MB  + 24 MA ) MW  + ((12 MB  - 24 MA  MB  + 12 MA ) MJ

        6           5        2   4        3   3        4   2        5
- 24 MB  + 36 MA MB  + 24 MA  MB  - 72 MA  MB  + 24 MA  MB  + 36 MA  MB

        6    4    6          2        2    4        4        2   2        4    2
- 24 MA ) MW )/(MZ  + (- 2 MB  - 2 MA ) MZ  + (MB  - 2 MA  MB  + MA ) MZ )
```

V. Numerical Results - Three Generations

Given the algebraic results of the previous section, one needs to insert values for the parameters and evaluate the widths. A program designed for this task was written for and run on the CDC 7600 at Lawrence Berkeley Laboratory.[52] Because of worries about loss of numerical precision due to strong cancellations between terms, it was decided to use double precision complex arithmetic. Double precision arithmetic on the CDC 7600 is in principle accurate to approximately 29 decimal digits; however, only single precision complex arithmetic was available and so subroutines to perform double precision complex arithmetic had to be written.

Routines for the complex logarithm and Spence function require some care. It is important to program the $i\varepsilon$ parts so that one stays on the correct side of the cuts of these functions.

As for the log, in most cases it is sufficient to use the usual

$$\log z = \log|z| + i\tan^{-1}\left(\frac{\mathrm{Im}\ z}{\mathrm{Re}\ z}\right) , \tag{5.1}$$

where the branch of the log is such that the imaginary part varies between $-\pi$ and π (the cut goes from 0 to $-\infty$ along the real axis). However, for z near 1, say $|1-z| \leqslant 0.1$ for example, it is safer to use instead the power series expansion

$$\log z = \log[\,1-(1-z)] = -\sum_{n=1}^{\infty} \frac{(1-z)^n}{n} \tag{5.2}$$

to guard against loss of numerical precision. The optimal thing in this case is if 1-z has already been computed naturally, as opposed to z. It is then destructive to compute $z = 1 - (1-z)$ and then call the log, because digits are lost in the subtraction from 1. Instead, one directly calls the series (5.2) with the argument of (1-z).

In general, the TERM($6+\beta$), $\beta = 1-4$ functions of the previous chapter deserve a thoughtful evaluation to retain precision. An excellent discussion is given in Ref. [46], although due to my maximization of the algebraic work not all the tricks were required. Here I will briefly reiterate a couple of remarks concerning roots of quadratic expressions. The $i\varepsilon$ is also necessary to fix the sign of the imaginary part of square roots. Also, if a quadratic equation is

$$ax^2 + bx + c = a(x-x_1)(x-x_2) = 0, \tag{5.3}$$

then it is often beneficial to first evaluate the larger root x_1 and then obtain x_2 by

$$x_2 = \frac{c}{ax_1}, \tag{5.4}$$

in order to retain numerical precision.

For the Spence function, I used a modified version of the algorithm given in Ref. [48]. First I review their method, including those parts of it that I used. Recall that the Spence function, or dilogarithm, is defined by

$$Sp(z) = -\int_0^z \frac{\log(1-t)}{t}\, dt \ . \tag{5.5}$$

This is cut along the positive real axis starting at z = 1. Given z, one first checks that it lies in the unit disk with real part less than ½. If not, one maps the argument of Sp there using one or both of the following identities if necessary:

$$Sp(z) = -Sp\left(\frac{1}{z}\right) - \frac{\pi^2}{6} - \frac{1}{2}[\log(-z)]^2 \tag{5.6}$$

$$Sp(z) = -Sp(1-z) + \frac{\pi^2}{6} - \log z \ \log(1-z) \ . \tag{5.7}$$

Once z is in the indicated region, Sp(z) is evaluated by using the series

$$Sp(z) = \sum_{n=0}^{\infty} B_n \frac{[-\log(1-z)]^{n+1}}{(n+1)!} \ , \tag{5.8}$$

,where B_n are the Bernoulli numbers.

I modified this procedure by replacing the series (5.8) by an equivalent series obtained by rewriting the Bernoulli numbers in terms of the Riemann zeta function:

$$Sp(z) = -\log(1-z) - \frac{1}{4}[\log(1-z)]^2 + 4\pi \sum_{m=1}^{\infty} \frac{(-1)^m \zeta(2m)}{(2m+1)} \left[\frac{\log(1-z)}{2\pi}\right]^{2m+1} , \tag{5.9}$$

where the Riemann zeta function is given by

$$\zeta(n) = \sum_{k=1}^{\infty} \frac{1}{k^n} \quad . \tag{5.10}$$

Series (5.9) has two main advantages. Firstly, $\zeta(2m)$ are easy to compute. Additionally, beyond a certain m, a computer will not be able to distinguish it from 1, whereupon it can be set equal to 1 for the remainder of the series. These properties are useful for iterative high precision computation.

Secondly, it is obvious what the expansion parameter is, since the 2π denominator has been made explicit ($\zeta(2m)/(2m+1)$ is monotonic decreasing as m increases). This is as compared to the series (5.8), where both B_n and $(n+1)!$ are getting large and it is not clear just what is going on. In fact, in the region of interest

$$|\,\mathrm{Re}\,\log(1-z)| \leqslant \log 2$$

$$|\,\mathrm{Im}\,\log(1-z)| < \frac{\pi}{3}$$

$$\Longrightarrow \left|\frac{\log(1-z)}{2\pi}\right| < \frac{\sqrt{(\log 2)^2 + \pi^2/9}}{2\pi} \cong 0.2 \quad . \tag{5.11}$$

Both my complex log and Spence function routines were extensively checked against other available routines, methods of calculation, and/or tables or other known values to verify their accuracy. For example, my Sp(z) was successfully compared with a CERN library routine that returns a single precision real part of the Spence function for z real,

using a Chebyshev approximation.[54]

The consistency of the final results, the insensitivity to variation of parameters, and the high degree of verification of the numerical imaginary part and CP invariance checks make it clear that the convergence of the calculation was excellent and that the numerical niceties were well understood.

Numerical results for three generations are presented in Figures 4 through 7 and discussed below. Parameter choice is as follows:

(i) Quark masses: The (unknown) top quark mass is varied from 20 up to M_Z-1 GeV. For the other quarks, current[55] internal masses and constituent external ones were taken: M_d = 7.5 MeV, M_s = 150 MeV, M_b = 4.75 GeV, M_u = 330 MeV, and M_c = 1.5 GeV. Varying M_b by ±0.25 GeV causes the rates to change by ±~ 20-25%. Small variations in the others make negligible difference. One could even take current external masses with the same effect, and constituent d and s quark masses tend to decrease the widths by only $\lesssim$ 2%. These statements are true except when the KM couplings to b are too small or one is right at a $Z \to A\bar{B}$ decay threshold, in which case the rates are completely insignificant anyway.

(ii) M_Z, M_W, θ_W : A lot of work has been done on obtaining precise estimates of these values based on calculating electroweak radiative corrections to "low" energy processes.[9,10] I took my numbers to be the central values of Ref. [9] , which were deduced from an analysis of charged and neutral current ν - hadron scattering: M_W = 83.0 GeV, M_Z = 93.8 GeV, $\sin^2\theta_W$ = 0.217 in the renormalization scheme where $\cos\theta_W = M_W/M_Z$. Varying these in the $\pm 2\sigma$ region of [9] , i.e.

$89.8 < M_Z < 98.7$, $0.245 > \sin^2\theta_W > 0.189$, produces little ($\lesssim$ 30% through M_T = 80 GeV) change, except near the endpoints since M_Z controls the position of the thresholds and of the zero in the phase space cutoff factor P(A,B). But in this region the rates are dropping anyway and my conclusions are unaffected.

The different groups evaluating M_W and M_Z do not appear to differ significantly. Also, it should be realized that Ref. [9] assumes M_T = 18 GeV in its analysis, which of course is not being assumed here. However, using formulas given in Ref. [56] it was determined that it was sufficient for the purposes of this calculation to ignore corrections from larger $M_T < M_Z$.

(iii) Z total decay width: This varies with the top quark mass. The leading order approximation for the total width includes the tree decays into three generations of fermion-antifermion pairs, only (three quark and three lepton doublets). For $20 < M_T < M_Z/2$ the Born amplitude into $t\bar{t}$ is phase space suppressed relative to the other fermions, and for $M_T > M_Z/2$ it drops out altogether. The formulas used were taken from Ref. [57] . For $M_T > M_Z/2$ we have (ignoring all other fermion masses)

$$\Gamma(Z\to\text{all}) = \frac{g^2 M_Z}{\pi c_W^2}\left[\frac{1}{12}(3-6s_W^2+8s_W^4) - \frac{1}{32}\left(1-\frac{8}{3}s_W^2+\frac{32}{9}s_W^4\right)\right] \qquad (5.12)$$

whereas for $M_T \leqslant M_Z/2$ we have

$$\Gamma(Z\to\text{all}) = \Gamma(Z\to\text{all})\Big|_{M_T > M_Z/2} + \frac{g^2 M_Z}{32\pi c_W^2}\left[1-4\left(\frac{M_T}{M_Z}\right)^2\right]^{\frac{1}{2}}$$

$$\times\left[\left(1-\frac{8}{3}s_W^2+\frac{32}{9}s_W^4\right) - \left(\frac{M_T}{M_Z}\right)^2\left(1+\frac{16}{3}s_W^2-\frac{64}{9}s_W^4\right)\right], \qquad (5.13)$$

with $g = e/s_W$.

For the current parameters $\Gamma(Z \to \text{all}) \approx 2.7$ GeV for $M_T = 20$ GeV, and ≈ 2.5 GeV for $M_T > M_Z/2$. Thus the dependence of $\Gamma(Z \to \text{all})$ on M_T will be part of the dependence of the various branching ratios on M_T. It should also be noted that, for fixed luminosity L, the number of Z's per year produced at an e^+e^- machine (needed, for example, if one is interested in the number of top quarks produced per year by Z decay) also depends on $\Gamma(Z \to \text{all})$ through (M_Z in GeV, L in $cm^{-2}sec^{-1}$)

$$(\#Z\text{'s/year}) = \frac{4.6 \times 10^{-19}}{M_Z^2} \frac{\Gamma(Z \to e^+e^-)}{\Gamma(Z \to \text{all})} L , \tag{5.14}$$

where

$$\Gamma(Z \to e^+e^-) = \frac{g^2 M_Z}{96\pi c_W^2} (1-4s_W^2+8s_W^4) . \tag{5.15}$$

This is because[7]

$$\frac{\sigma(e^+e^- \to Z^0 \to \text{all})}{\sigma_{pt}} = \frac{9}{\alpha^2} \frac{\Gamma(Z \to e^+e^-)}{\Gamma(Z \to \text{all})} , \tag{5.16}$$

where

$$\sigma_{pt} = \sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} . \tag{5.17}$$

For the canonical $L = 10^{32}/cm^2$-sec at LEP, say, one gets $(\#Z\text{'s/year}) \approx 10^8$.

(iv) KM matrix: My conventions for U^{KM} are the same as in Ref.

[58], which has become the more or less standard way to express U^{KM} for three generations:

$$U^{KM} = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3+s_2s_3e^{i\delta} & c_1c_2s_3-s_2c_3e^{i\delta} \\ -s_1s_2 & c_1s_2c_3-c_2s_3e^{i\delta} & c_1s_2s_3+c_2c_3e^{i\delta} \end{pmatrix}, \tag{5.18}$$

where s_i, c_i = sin, cos(θ_i), and δ is the CP-violating phase. This matrix comes about in the charged current because the quark mass eigenstates and weak eigenstates are not identical. One may, without loss of generality, take $0 \leqslant \theta_i \leqslant \pi/2$ and $-\pi < \delta \leqslant \pi$.[59,*]

In order to observe the effects of varying the mixing angles, a set of matrices consistent with the physics of u, d, and s quarks was generated by utilizing the analysis of Refs. [14,35,60]. I chose to ignore the small CP-violation by taking the KM matrix to be real, i.e. $\cos\delta = \pm 1$. Next, $\cos\theta_1$ (θ_1 is often called the Cabibbo angle) was fixed at 0.9737 since this value is known with high accuracy from comparison of μ decay with superallowed $0^+ \to 0^+$ nuclear Fermi β decays. $\sin\theta_3$ is varied between 0 and 0.5, the range allowed from hyperon and K_{e3} decays. Finally, $\sin\theta_2$ was generated using the constraining Equation (12) of Ref.[14], obtained from the study of $K_L \to \mu^+\mu^-$ and K^0-$\bar{K}^0$

* $-\pi$ and π are equivalent values for δ since they differ by 2π. This is sometimes overlooked.

mixing:

$$\tan\theta_3 = \frac{\cos\theta_1(\sin^2\theta_2 - a^2)}{(\sin\theta_2)(\cos\theta_2)(\cos\delta)} \quad . \tag{5.19}$$

The allowed values of a depend on M_T. For fixed M_T, a was varied by reading values off Figure 2 of [14] that agree with the $K_L \to \mu^+\mu^-$ data (upper band) and/or K^0-$\bar{K}^0$ mixing for $0.2 < B < 0.6$ (B is a bag parameter). Widths were computed for all the matrices generated in this manner.

In Figure 4 I have plotted the high, low, and average ($\equiv$(high+low)/2) values varied over the KM matrices, of $\Gamma(Z \to \text{top}) \equiv \Gamma(Z \to t\bar{c}) + \Gamma(Z \to \bar{t}c) + \Gamma(Z \to t\bar{u}) + \Gamma(Z \to \bar{t}u)$ and the corresponding branching ratios $BR(Z \to \text{top})$ as functions of the top quark mass. In Figure 4, it was demanded that high or low values be generated by KM matrices satisfying, in addition to the above requirements from u, d, and s quark physics, the following constraints from b and c quark physics[7,61]:

a) $\left|\frac{U_{ub}}{U_{cb}}\right| \leqslant 0.3$ from $B \to K$'s, leptons;

b) $|U_{cb}| \geqslant 0.04$ from B lifetime[62];

c) $0.192 \leqslant |U_{cd}| \leqslant 0.34$ from $\nu + d \to c + \mu$;

d) $|U_{cs}| \geqslant 0.6$ from $D \to Ke\nu$. (5.20)

These constraints were not applied in (iv) above because they are

somewhat crude and model-dependent at this time. For example, a and b were derived using the spectator model for B decays, which is in some doubt. However, they seem reasonable and contain important information about the KM matrix. In any case, for the parameter set used in Figure 4 the value of $\Gamma(Z \to \text{top, high})$ was found, for all values of M_T checked, to be independent of whether the constraints (5.20) were applied or not, provided only that $Z \to t\bar{c}$ was energetically allowed (i.e., for $M_T < M_Z - M_c$). For M_T larger than this the rate is vanishingly small anyway. (5.20) does tend to exclude extremely low values, but the average values are also relatively unaffected since there is usually about an order of magnitude's disparity between high and low values, implying that average $\cong$ high/2.

The main feature of Figure 4 is how small the branching ratios are. The maximum over all top quark masses and KM matrices used is $\sim 7 \times 10^{-11}$, almost certainly unmeasurable with projected luminosities. The average value for potentially interesting values of M_T ($\gtrsim$ 50 GeV) is $\lesssim 3 \times 10^{-11}$. There is, for the most part, a steady drop in all curves as M_T increases, and a sharp drop near the kinematic boundary (especially after the $t\bar{c}$ threshold is crossed, although this is not obvious from the graph). The exceptions to this steady drop are a shoulder at around 85 GeV that will be explained later, and a dip in $\Gamma(Z \to \text{top, low})$ at about 30 GeV which is due to fluctuations in which KM matrices agree with experimental constraints as M_T varies in that region.

The simple physics estimate of (1.7) was not too far off, and even a bit high. That estimate assumed that the coupling to the b quark

dominated the amplitude. Now, $Z \to t\bar{c}$ is generally Cabibbo enhanced relative to $Z \to t\bar{u}$. Therefore, it is expected that

$$\Gamma(Z \to \text{top}) \sim \Gamma(Z \to t\bar{c}) + \Gamma(Z \to \bar{t}c)$$

$$\sim U_{Tb}^2 U_{cb}^2 = (c_1 s_2 s_3 + c_\delta c_2 c_3)^2 \, (c_1 c_2 s_3 - c_\delta s_2 c_3)^2$$

$$= \frac{1}{4} \sin^2(2\theta_2 - 2c_\delta \theta_3) + O(1-c_1) \tag{5.21}$$

(recall $c_\delta = \pm 1$ here).

To check this, $\Gamma(Z \to \text{top})$ is plotted in Figure 5 for all the KM matrices used (ignoring the constraints (5.20)), for four values of M_T. The solid lines represent what was actually computed, but the dashed lines are only their extension into the origin. In the neighborhood of the dashed lines, the computation produced a splatter of points with values generally higher than the line. As a further check, note that if b quark dominance holds,

$$\frac{\Gamma(Z \to t\bar{u})}{\Gamma(Z \to t\bar{c})} \cong \frac{U_{ub}^2}{U_{cb}^2} = \left(\frac{s_1 s_3}{c_1 c_2 s_3 - c_\delta s_2 c_3}\right)^2$$

$$= \frac{s_1^2 s_3^2}{\sin^2(\theta_2 - c_\delta \theta_3)} \left[1 + O(1-c_1)\right]. \tag{5.22}$$

$\Gamma(Z \to t\bar{u})/\Gamma(Z \to t\bar{c})$ was plotted vs. $(U_{ub}/U_{cb})^2$ for all the same values as Figure 5 (not shown here), and all points were observed to lie on a straight line intersecting the origin, with slope 1. Thus, in the dashed line region of Figure 5, the coupling to b is still dominant but

$t\bar{u}$ has become important relative to $t\bar{c}$.

Therefore, the KM couplings to b are the ones probed by the decay rates. The only exceptions to this noticed were the unusual cases when one of the KM matrix elements needed was ≈ 0. For example, if $\sin\theta_3 = 0$ then $U_{ub} = 0$, and $Z \to t\bar{u}$ must go through the s or d quark. But in these cases the widths were seen to be negligibly small anyway.

As an additional check on the physics of (1.7), M_b was varied in the region around the physical b quark mass. As expected, the rates were observed to be roughly proportional to M_b^4, with the above-mentioned exceptions.

Variation of $\Gamma(Z \to \text{top, high})$ with M_T can be traced to three factors, two of which can be inferred from Figure 5. Note that the high endpoints of the solid lines (the highest computed points for each mass) get lower with increasing M_T both because the slopes of the lines decrease, and the allowed maximum value of $(U_{Tb}U_{cb})^2$ tends to decrease. The former can be attributed to the drop in the phase space factor $P(t,c)$ of (4.5) as M_T increases.

To verify this, I have plotted $\dfrac{\Gamma(Z \to \text{top, high})}{(U_{Tb}U_{cb})^2 P(t,c)}$ vs. M_T in Figure 6. Notice that this quantity drops by only a factor of 4 from $M_T = 20$ to 91.8 GeV, while $\Gamma(Z \to \text{top, high})$ falls by about two orders of magnitude in the same range. Aside from this, Figure 6 is dominated by an enhancement peaked at around 86 GeV, that corresponds to the shoulder seen in Figure 4. This is due to the onset of the $t \leftrightarrow W\text{-}J$ thresholds; i.e., the W and J quark can propagate closer and closer to mass shell as M_T grows.

The remaining small variation with M_T not attributable to these

factors is then buried in the formula for the width.

Thus, one is led to the conclusion that, for three generations, it is very unlikely that a massive top quark will be discovered by this mechanism.

I do not believe my conclusions about unmeasurability are restricted by my choice of KM matrix. For example, if one takes the KM matrix giving the largest branching ratio into top ($\sim 7 \times 10^{-11}$) and for the sake of argument sets $U_{cb} = U_{Tb} = 1$ (the unitarity bound), the width would increase by only a factor of 4, raising the branching ratio to only 3×10^{-10}.

Thus BR($Z \to$ top) is at most $\sim 10^{-10}$, and for many of the allowed values of M_T and the quark mixing matrix is much worse. If LEP produces 10^8 Z's per year, then the prospects are for at most one top quark every 100 years by this process, making it a dismal experimental proposition.

$\Gamma(Z \to u\bar{c}) + \Gamma(Z \to \bar{u}c)$ was also calculated; its average values, defined as for $\Gamma(Z \to \text{top})$, are plotted in Figure 7. These are very close to ½ the high values because the low values are down by five orders of magnitude. The variation seen with M_T is due to the variation of the allowed KM matrices with M_T. The couplings through b are strongly Cabibbo suppressed, and so the high value for the branching ratio is never more than 2×10^{-12}, making this experimentally irrelevant.

VI. Numerical Results - Four Generations

The computer calculation here is very similar to the one described in Chapter V for three generations. The main difference is that the number of free parameters increases greatly for four generations, particularly in the KM matrix. However, reasonable assessments of the different widths can be made under various simplifying assumptions that make the calculation more manageable.

I now discuss the parameter choice for the four generation case:

(i) Quark masses: In Chapter V we saw that small variations in the light quark masses make little difference. Here I fix their values to the ones used before: M_d = 7.5 MeV, M_s = 150 MeV, M_u = 330 MeV, and M_c = 1.5 GeV; the b quark is now also "light" and I fix M_b = 4.75 GeV. $M_{b'}$ is varied from 25 GeV to 10 TeV.

For M_T and $M_{T'}$, I first note that I am free to choose $M_T \leqslant M_{T'}$. That is, if there are two new massive charge +2/3 quarks, I assign the lighter one to the third generation and call it the top, and I assign the heavier one to the fourth generation and call it the top prime. This is completely general because it merely corresponds to a unitary reordering of the charge +2/3 mass eigenbasis.

Next, two cases are distinguished. In the first case, I assume that $M_{T'} > M_Z$, so that $Z \to t' + X$ is not allowed. In addition, I fix M_T to the interesting value of 50 GeV. In this chapter, the KM matrices used will not depend on M_T, so the main variation of the decay rates with M_T is expected to be due to the phase space factor discussed in Chapter V. For the second case I assume that $Z \to t' + X$ is allowed.

In particular I focus on the topologically new possibility of $Z \to t\bar{t}'$. Therefore, as a representative I choose $M_T = 25$ GeV and $M_{T'} = 50$ GeV.

(ii) M_Z, M_W, and θ_W: I again fixed $M_W = 83.0$ GeV, $M_Z = 93.8$ GeV, and $\sin^2\theta_W = 0.217$. It should be kept in mind that superheavy quarks or leptons may have non-negligible effect on these numbers through radiative corrections.[56] However, these effects appear to depend sensitively on the actual values of the masses and are in principal higher order in perturbation theory, so I have ignored them.

(iii) Z total decay width: If the Born decays into $t'\bar{t}'$ or $b'\bar{b}'$ are energetically allowed, they should be added to the $\Gamma(Z \to \text{all})$ formulas of (5.12) or (5.13). We have[57]

$$\Gamma(Z \to t'\bar{t}') = \frac{g^2 M_Z}{32\pi c_W^2}\left[1-4\left(\frac{M_{T'}}{M_Z}\right)^2\right]^{\frac{1}{2}} \times \left[\left(1-\frac{8}{3}s_W^2+\frac{32}{9}s_W^4\right) - \left(\frac{M_{T'}}{M_Z}\right)^2\left(1+\frac{16}{3}s_W^2-\frac{64}{9}s_W^4\right)\right] \tag{6.1}$$

$$\Gamma(Z \to b'\bar{b}') = \frac{g^2 M_Z}{32\pi c_W^2}\left[1-4\left(\frac{M_{b'}}{M_Z}\right)^2\right]^{\frac{1}{2}} \times \left[\left(1-\frac{4}{3}s_W^2+\frac{8}{9}s_W^4\right) - \left(\frac{M_{b'}}{M_Z}\right)^2\left(1+\frac{8}{3}s_W^2-\frac{16}{9}s_W^4\right)\right] . \tag{6.2}$$

Furthermore, in order to cancel anomalies a fourth generation of leptons is also needed. I assume that the charged lepton has mass $> M_Z/2$, but take the fourth neutrino, $\nu_{\tau'}$, to be approximately massless by analogy with the other known neutrinos. This adds to the Born width a piece given by

$$\Gamma(Z \to \nu_{\tau'}\bar{\nu}_{\tau'}) = \frac{g^2 M_Z}{96\pi c_W^2} \quad . \tag{6.3}$$

For case I where $M_T = 50$ GeV, this gives $\Gamma(Z \to \text{all}) = 2.9$ GeV for $M_{b'} = 25$ GeV, and $= 2.7$ GeV for $M_{b'} > M_Z/2$. For case II where $M_T = 25$ GeV and $M_{T'} = 50$ GeV, one has $\Gamma(Z \to \text{all}) = 3.1$ GeV for $M_{b'} = 25$ GeV, and $= 2.9$ GeV for $M_{b'} > M_Z/2$.

(iv) KM matrix: In the four generation case, the KM matrix becomes immensely more complicated. The reason is that the $N \times N$ unitary mixing matrix has in general $(N-1)^2$ degrees of freedom. For four generations this means 9 degrees of freedom - 6 angles and 3 phases. I adopt a convention[15,80] for $U^{KM}_{4\times 4}$ that is a simple generalization of $U^{KM}_{3\times 3}$ of (5.18), and reduces to the latter when all the extra angles and phases are 0:

$$
U^{KM} = \begin{vmatrix}
c_1 & s_1c_3 & s_1s_3c_5 & s_1s_3s_5 \\
-s_1c_2 & c_1c_2c_3 & c_1c_2s_3c_5 & c_1c_2s_3s_5 \\
 & +s_2s_3c_6e^{i\delta_1} & -s_2c_3c_5c_6e^{i\delta_1} & -s_2c_3s_5c_6e^{i\delta_1} \\
 & & +s_2s_5s_6e^{i(\delta_1+\delta_3)} & -s_2c_5s_6e^{i(\delta_1+\delta_3)} \\
-s_1s_2c_4 & c_1s_2c_3c_4 & c_1s_2s_3c_4c_5 & c_1s_2s_3c_4s_5 \\
 & -c_2s_3c_4c_6e^{i\delta_1} & +c_2c_3c_4c_5c_6e^{i\delta_1} & +c_2c_3c_4s_5c_6e^{i\delta_1} \\
 & -s_3s_4s_6e^{i\delta_2} & -c_2c_4s_5s_6e^{i(\delta_1+\delta_3)} & +c_2c_4c_5s_6e^{i(\delta_1+\delta_3)} \\
 & & +c_3s_4c_5s_6e^{i\delta_2} & +c_3s_4s_5s_6e^{i\delta_2} \\
 & & +s_4s_5c_6e^{i(\delta_2+\delta_3)} & -s_4c_5c_6e^{i(\delta_2+\delta_3)} \\
-s_1s_2s_4 & c_1s_2c_3s_4 & c_1s_2s_3s_4c_5 & c_1s_2s_3s_4s_5 \\
 & -c_2s_3s_4c_6e^{i\delta_1} & +c_2c_3s_4c_5c_6e^{i\delta_1} & +c_2c_3s_4s_5c_6e^{i\delta_1} \\
 & +s_3c_4s_6e^{i\delta_2} & -c_2s_4s_5s_6e^{i(\delta_1+\delta_3)} & +c_2s_4c_5s_6e^{i(\delta_1+\delta_3)} \\
 & & -c_3c_4c_5s_6e^{i\delta_2} & -c_3c_4s_5s_6e^{i\delta_2} \\
 & & -c_4s_5c_6e^{i(\delta_2+\delta_3)} & +c_4c_5c_6e^{i(\delta_2+\delta_3)}
\end{vmatrix}
, \quad (6.4)
$$

where s_i, c_i = sin, cos(θ_i), and δ_i are CP-violating phases. Again, by convention one can take $0 \leqslant \theta_i \leqslant \pi/2$ and $-\pi < \delta_i \leqslant \pi$.

By varying these angles and phases, I want to observe the maximum values, and variations, in the interesting decay rates. The usual set-up is to order the quark mass eigenbasis from lightest to heaviest and write the charged current (symbolically) as

$$J_\mu^{\text{charged}} \sim (u\ c\ t\ t')\ (U^{KM}) \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix} . \tag{6.5}$$

However, the couplings to b', which are expected to dominate the decay rates, and to b, which are expected to contribute to CP violation, are very complicated in the parametrization (6.4). A dependable variation of the angles and phases would take an enormous amount of computer time; even then it is not obvious, in a finite variation, whether important regions of parameter space have been missed.[81] On the other hand, the irrelevant couplings to d and s are comparatively simple. I chose to reverse this situation by writing the charged current (symbolically) as

$$J_\mu^{charged} \sim (u\ c\ t\ t')\ (U^{KM}) \begin{pmatrix} b' \\ b \\ s \\ d \end{pmatrix} , \tag{6.6}$$

with U^{KM} still as in (6.4). Now the simpler couplings in the first and second columns of U^{KM} are the most important; the more complicated couplings of the third and fourth columns have an insignificant role.

By leaving the charge+2/3 quarks in their usual order, however, some simple constraints on angles can be derived. For this I used the updated charged current analysis of Ref.[82]. In particular, I used the constraints

$$|U_{ud}| = s_1 s_3 s_5 = 0.9730 \pm 0.0024 \tag{6.7}$$

$$|U_{us}| = s_1 s_3 c_5 = 0.227 \pm 0.016. \tag{6.8}$$

θ_5 is an unimportant angle because it does not appear in the b' or b couplings; I fixed it by dividing (6.7) by (6.8):

$$s_5 = 0.974 . \tag{6.9}$$

θ_1 and θ_3 obey a joint constraint because

$$|U_{ud}|^2 + |U_{us}|^2 = s_1^2 s_3^2 \quad \Longrightarrow$$

$$0.993 \leqslant s_1 s_3 \leqslant 1 . \tag{6.10}$$

Other potential phenomenological constraints on the remaining three angles and three phases were ignored. In particular, it seems reasonable to assume that δ_1 and δ_2 are constrained little by the small CP violation in the K system, which involves couplings to d and s quarks only. Since δ_3 does not appear in b' or b couplings, I fix it by whim:

$$\delta_3 = 0 \ . \tag{6.11}$$

At this point we see that we still have 5 unconstrained parameters - 3 angles (θ_2, θ_4, θ_6) and 2 phases (δ_1, δ_2). In order to impose further restrictions, I consider quark mass cases 1 and 2 above separately.

In case 1, I fixed M_T = 50 GeV and $M_{T'} > M_Z$. Therefore I am interested primarily in studying flavor changing top production such as $Z \to t\bar{c}$, $t\bar{u}$. Therefore, I maximize the t-b' coupling by fixing $c_4 = 1$. Furthermore, since s_4 is now 0 and the t' couplings are of no interest, δ_2 becomes irrelevant and I fix it to be 0. The remaining parameters are varied in their allowed ranges.

On the other hand, in case 2 I fixed M_T = 25 GeV, $M_{T'}$ = 50 GeV, and I am interested primarily in studying $Z \to t\bar{t}'$. Now I maximize the t-b' and t'-b' couplings together by fixing $s_2 = 1$. This makes δ_1 unimportant, so I fix it to be 0. Again, the remaining parameters are varied in their ranges.

The CP invariance and imaginary part tests of Chapter III were checked to high precision and observed to work for real KM matrices.

In the tables below I quote results for some interesting branching ratios, for a variety of values of $M_{b'}$:

Case 1 : M_T = 50 GeV, $M_{T'} > M_Z$

$M_{b'}$ (GeV)	(a)BR($Z \to u\bar{c}+\bar{u}c$,max)	(b)BR($Z \to$ top,max)	(c)BR(CPV top,max)
25	2.4×10^{-9}	3.2×10^{-8}	$\pm 6 \times 10^{-17}$
50	3.2×10^{-8}	3.9×10^{-7}	$\pm 2 \times 10^{-14}$
100	4.1×10^{-8}	5.7×10^{-7}	$\pm 8 \times 10^{-15}$
250	1.1×10^{-6}	1.3×10^{-5}	$\pm 2 \times 10^{-13}$
500	1.5×10^{-5}	1.7×10^{-4}	$\pm 8 \times 10^{-13}$
1000	2.2×10^{-4}	2.3×10^{-3}	$\pm 3 \times 10^{-12}$
10000	2.0×10^{0}	2.1×10^{1}	$\pm 3 \times 10^{-10}$

Case 2 : M_T = 25 GeV, $M_{T'}$ = 50 GeV

$M_{b'}$ (GeV)	(a)BR($Z \to t\bar{t}'+t'\bar{t}$,max)	(b)BR(CPV tt',max)
25	2.7×10^{-8}	$\pm 3 \times 10^{-12}$
50	3.1×10^{-7}	$\pm 5 \times 10^{-11}$
100	4.7×10^{-7}	$\pm 2 \times 10^{-11}$
250	1.0×10^{-5}	$\pm 3 \times 10^{-10}$
500	1.3×10^{-4}	$\pm 1 \times 10^{-9}$
1000	1.7×10^{-3}	$\pm 4 \times 10^{-9}$
10000	1.5×10^{1}	$\pm 4 \times 10^{-7}$

Definitions are as follows (max means maximum value under variation) :

$Z \to$ top is defined as in Chapter V.

$\Gamma(\text{CPV top}) \equiv \Gamma(Z \to t + X) - \Gamma(Z \to \bar{t} + X)$.

$\Gamma(\text{CPV tt}') \equiv \Gamma(Z \to t\bar{t}') - \Gamma(Z \to \bar{t}t')$.

BR = branching ratio (actually just Γ/Γ_{Born}. For $M_{b'}$ = 10000 GeV, for example, this is not really the branching ratio).

For the ordinary ($\equiv$ non-CPV) branching ratios above, the minimum is 0, so that the average $\equiv$ (high + low)/2 is ½ the maximum value. The ordinary branching ratio maximum values are achieved, or nearly achieved, when $|U_{Tb'}| = |U_{cb'}| = .71$ in case 1(b), when $|U_{Tb'}| = |U_{T'b'}| = .71$ in case 2(a), and when $|U_{ub'}| = .12$, $|U_{cb'}| = .99$ in case 1(a), so that if one prefers more Cabibbo suppression a reasonable estimate may be made by rescaling the above. For the CPV branching ratios, the average values defined in the above described way are evidently 0.

For the ordinary branching ratios, one sees a steady increase with $M_{b'}$ and the behavior seems to be asymptotically $\sim M_{b'}^4$, as suggested in Chapter I. However, there is significant threshold behavior for $M_{b'} \lesssim M_W$, and $M_{b'} \gtrsim M_W$. The rates begin increasing rapidly for $M_{b'} >$ 100 GeV, but true $M_{b'}^4$ is approached only for $M_{b'} \gtrsim 250$ GeV, and particularly for $M_{b'} \gtrsim 1$ TeV. Evidently a large $M_{b'}$ can bring the top quark production rates into a measurable range. For $M_{b'}$ = 10 TeV, the flavor changing decay rates can dominate the Born terms; however, as discussed in Chapter I this is an unrealistically large value for $M_{b'}$, and the enormous flavor changing rates probably reflect the breakdown of perturbation theory.

The CP violating rates are small and unmeasurable. The expectation of asymptotic $M_{b'}^2$ behavior discussed in Chapter I is confirmed. Also, for $M_{b'}$ = 500 GeV in case 1(c), doubling M_b increased the CPV rate by a factor of 4, supporting also the M_b^2 expectation. Unfortunately, the CPV rates are suppressed by more than $M_b^2/M_{b'}^2$ relative to their non-CPV

partners (at least for the quoted maximum values). This may be Cabibbo-related, or due to some uncertain dynamics controlling the difference of the two widths.

VII. Experimental Measurement

Experimenters looking for flavor changing Z^0 decays will, unfortunately, almost certainly not be measuring free flavored quarks in their detectors. They will be faced with the task of deciding whether the collection of hadrons, leptons, and photons that they will measure represents a flavor changing decay. It is therefore a conscientious theorist's duty to give them some idea of what to look for.

If we knew for sure that the three generation $SU(2)_L \times U(1)$ world of Chapter V was the real world (and there is no convincing reason to believe otherwise at this time), then this chapter could be deleted and the advice to experimentalists would be "don't bother." However, it is easy to imagine the rates being larger than in Chapter V; one such model was discussed in Chapter VI, and others will be mentioned in Chapter VIII. Therefore, experimentalists should look for these events, if only to place upper bounds on the rates; any deviation from Chapter V would be most exciting. And if they are going to look, then they need to know what to look for.

Therefore, in the following I offer some suggestions for experimental detection of the flavor changing Z^0 decays calculated in this thesis. I should preface these remarks by saying that I have not made the careful study of rates, backgrounds, competing processes, experimental feasibility based on Monte Carlo simulations, etc., that are necessary to make definitive statements. Such an approach is even more difficult than usual here since the rates are unknown over orders of magnitude if one ignores Chapter V, and additionally the spectrum of

particles with masses greater than about 20 GeV (i.e., those that would not have been produced in e^+e^- by now; in particular, the all-important top quark) is not known. Thus what follows should be regarded as educated speculations only.

First of all I restrict my attention to Z^0 production at e^+e^- machines. In these machines, the energy can be tuned so that they sit on the Z pole and produce Z's en masse; almost every (annihilation) event produces a Z and nothing else. Additionally, the center of mass frame of the produced Z and the lab frame are identical, and the initial state disappears completely so that there is no background junk. Z's can be produced at e-p, p-p, or $p-\bar{p}$ colliders too, but these do not share the above advantages of e^+e^- so that measuring rare Z^0 decays there should be more difficult. The following discussion will apply to these other machines to the extent that a produced Z, decaying in its rest frame, does not care how it was produced. For a comparison of Z production at various machines see Ref.[11] .

Secondly, I focus on events where at least one massive top quark is produced. The $u\bar{c}$ and $\bar{u}c$ channels will be practically indistinguishable from the lowest order Born decays since both the u and c are massless on the scale of M_Z, so one will see two thin jets (to lowest order in QCD) in either case. This mode is likely to be Cabibbo suppressed anyway.

Therefore, consider $t\bar{u}$, $\bar{t}u$, $t\bar{c}$, and $\bar{t}c$ back-to-back production by Z decay. Let E_T be the energy of the t or $\bar{t}$, E_q be the energy of the remaining quark or antiquark, and ignore M_q. According to our usual ideas from QCD, the two quarks will first fragment into hadronic jets;

I ignore $O(\alpha_s)$ corrections from hard gluon bremsstrahlung. From simple decay kinematics one has

$$E_q = \frac{M_Z}{2}[1 - (\frac{M_T}{M_Z})^2]$$

$$E_T = \frac{M_Z}{2}[1 + (\frac{M_T}{M_Z})^2] \quad . \tag{7.1}$$

Thus on the opposite side of the top quark one normally finds a "healthy" jet. Eg., even if $M_T = 0.9M_Z$, one still has $E_q \sim 0.1M_Z \sim 10$ GeV.

As for the t or $\bar{t}$, it is a nontrivial question to ask whether it really does fragment or not. The point is that the top quark's weak decay rate is generally a rapidly increasing function of M_T in the mass range of interest (growing roughly like M_T^5),[63] and eventually the "weak" time scale for the decay may become comparable to or less than the strong interaction time scale, making it difficult for the top to have enough time to fragment. According to Ref. [83], this is a potential issue only for $M_T > M_W$, in particular when the $t \to bW$ decay channel opens and that decay rate starts shooting up. For now I assume $M_T < M_W$ (+ M of any down quark that it has significant KM coupling to other than b') and briefly comment on the other case later.

Now the t or $\bar{t}$ fragments into a "jet." Due to the flavor-conserving nature of the gluon interaction, the "jet" must contain at least one top meson, say T or T^*, and perhaps several other ordinary hadrons. According to current ideas about how heavy quarks fragment into jets, the top meson will on the average carry a fragmentation parameter z of near 1; i.e., it will carry away most of the energy of the fragmenting

top quark.[64] This picture is qualitatively consistent with the observed hard D^* fragmentation distribution at CESR.[65] For example, suppose $z = 0.95$ and $M_T = M_Z/2$. Then the amount of energy E_{frag} that is available to fragment into ordinary hadrons other than the top meson is

$$E_{frag} = 0.05E_T = 0.03M_Z \sim 3 \text{ GeV}, \tag{7.2}$$

so one might still expect an accompanying smallish jet of ordinary hadrons with limited transverse momentum, flying along the direction of the emitted t quark.

The final step is the decay of the top meson. The quark masses of interest are so large that the spectator decay model should be reliable.[66] I.e., the t quark inside the top meson decays by emitting a virtual W and turning into a d, s, or b quark; the virtual W then materializes into a quark-antiquark pair (nonleptonic decay) or a lepton-neutrino combination (semileptonic). Neglecting the masses of the three fermions resulting from the decay one sees that, in the t rest frame, each has about $M_T/3$ of energy on average. Again ignoring hard gluon bremsstrahlung, the produced quarks will fragment into jets with limited transverse momentum. The e's and μ's will be detected as is, τ's decay into a few particles, and ν's pass undetected through the detector. The velocity of the initial top quark's rest frame with respect to the Z rest frame is

$$\beta = \frac{1 - \left(\frac{M_T}{M_Z}\right)^2}{1 + \left(\frac{M_T}{M_Z}\right)^2} \quad . \tag{7.3}$$

For $M_T \geqslant M_Z/2$, one has $\beta \leqslant 0.6$. Therefore, a lightlike particle emitted at 120^0 to the top quark direction of flight in the top quark's rest frame, is emitted at no less than 82^0 with respect to the same direction of flight in the Z rest frame, for this range of M_T. Therefore the emitted particles and jets from the top meson decay should not be collimated so much by the Lorentz boost to the center of mass frame that they cannot be readily distinguished in that frame too. As a final comment I assume that the light spectator quark in the top meson decay does not have enough energy or transverse momentum to fly off and fragment into a unique jet of its own, and instead blends into one of the jets discussed above without changing the arguments.

Therefore, for the nonleptonic decay one sees a signature of four energetic jets (and possibly one smaller one along the direction of motion of the original top quark). Unfortunately there is an obvious background from 4- and 5- jet events due to ordinary QCD gluon emission. The QCD n-jet branching ratio will be of order $(\alpha_s(M_Z)/\pi)^{n-2}$. For $\alpha_s(M_Z) \sim 0.15$, this is $\sim (5 \times 10^{-2})^{n-2}$. Thus the four QCD jet branching ratio is $\sim 3 \times 10^{-3}$, and the five jet is $\sim 10^{-4}$. Unless the flavor changing branching ratio is very large, the nonleptonic channel is likely to be obliterated by the QCD background. One hope seems to be to look for a dynamical bump in the invariant mass spectrum of jets taken three at a time, using $\geqslant 4$ jet events, indicating (possibly) the existence of a

superheavy meson. Also, one might try to exploit the fact that one of the t decay jets is likely to be a b quark jet, if this can be reliably determined experimentally. If $t\bar{q}$ is very large then another possibility is a deviation from expected distributions of jet measures, such as thrust or sphericity of events.

The semileptonic top meson decay seems to offer a more promising signature. One expects two energetic jets (and perhaps a small one from the t fragmentation), and an energetic electron or muon. The lepton, having come from the decay of a slowly moving massive quark, will often be by itself away from the jets and have a large transverse momentum of order $M_T/3$ relative to the thrust axis. Additionally, there will be of order $M_T/3$ missing (often transverse) momentum due to the unmeasured hard neutrino from the decay. Cuts on jet variables may also be helpful in isolating t semileptonic decays from all events with an energetic lepton. Again, one might try to exploit the fact that the hadronic jet resulting from the t's virtual W emission is likely to be a b quark jet. Ideas similar to these have already been discussed in the b quark production case[67] and successfully applied to b jets at PETRA.[12,62,68] Unfortunately, one pays a price for this since the top semileptonic branching ratio should only be of order 20-30%.[69]

This type of event also offers a signature for CP violation; i.e., the number of t's produced by such Z^0 decays vs. the number of $\bar{t}$'s. One compares the number of the preceding paragraph's events with a hard e^+ or μ^+ to the number of events with one of opposite charge. A difference between + and - greater than that given by statistics would be a signal for CP violation.

There are other possibilities if the top production is large enough.

Weak annihilation diagrams of the top mesons should be suppressed relative to the spectator decays discussed above, but when they occur they should provide interesting wide angle two body signatures such as $\ell\nu$ or $\bar{q}q$.[66] One could also search for and measure properties of multilepton cascade decays, such as $t \to b\ell\nu \to c\ell\nu\ell\nu \to s\ell\nu\ell\nu\ell\nu$, if one doesn't mind paying the price of several semileptonic branching ratios.[69,70]

As mentioned earlier, for M_T large enough $t \to Wq$ can be important. The W decays into a fermion-antifermion pair in a similar manner to the way the virtual W materialized into a fermion-antifermion pair previously, so the event topology will be very similar. However, the t may not have time to fragment into a top hadron plus jet, so that accompanying jet is more likely to be absent than before.

The discussion of t' production (recall I have by definition taken $M_{T'} > M_T$) is more or less identical to the discussion of t production above. One difference occurs if $M_{b'}$ is conveniently located, i.e. $M_T < M_{b'} < M_{T'}$. In this case, one might get a spectacular multilepton or multijet cascade by $t' \to b' \to t \to b \to c \to s$ chains. A similar cascade could also occur for the t alone if $M_{b'} < M_T$, i.e. $t \to b' \to c \to s$. Such events could be very spherical and have a lot of missing momentum and several hard leptons. If $Z \to t\bar{t}'$ occurs, then one gets the top quark decay scenarios described above, only with back-to-back emission. This will look similar to $t\bar{t}$, but with differing invariant masses from the two fragmentations and decays. One expects generally spherical events, but also hard leptons and missing momentum on one or both sides if semileptonic decay occurs.

I now add some warnings about some other potential backgrounds.

Examples include the decay of the Z into some superheavy particle not considered here, and one or more light particles. One example that must be present is $Z \to W$ + anything, which was calculated to have branching ratio $\sim 10^{-7}$.[71] $Z \to t$ (or t', b') + 3 other fermions could be relevant - this will be briefly discussed in the next chapter. So could $Z \to b'\bar{b}$, etc., if $M_{b'}$ is in an interesting range. Also the Born decays $Z \to q\bar{q}$ for any superheavy quarks needs to be cut out experimentally. Finally, the backgrounds due to any other superheavy, lurking in the spectrum for us to find, that can be produced singly by Z decay, will need to be understood. This includes possibly superheavy Higgs by $Z \to Hf\bar{f}$ [72] or $H\gamma$, [73] new heavy lepton τ', or other particles that could exist if the minimal $SU(2)_L \times U(1)$ theory is not the whole story in the mass range up to M_Z. The topologies of the decays listed above might not be so much different from those described in this chapter, and thus should be carefully considered in a complete study.

To close, for flavor changing rates so large that they are comparable to the Born rates, flavor changing decays could be measured by measuring the deviation of the total Z width from the Born width. Of course, in this case an effort should be made to understand whether perturbation theory has broken down.

VIII. Other Possibilities

In this chapter I comment on a couple of other possible enhancements for the rates computed in this thesis, and a couple of closely related processes.

In Chapter V it was found that the standard scenario with three generations provided little optimism for the observation of the decays studied. One enhancement mechanism - a fourth generation of quarks - was studied in Chapter VI. Other ways of altering or adding to the minimal model could result in increased rates. Additional Higgs doublets can induce larger flavor changing neutral currents because of off-flavor diagonal couplings in the tree Lagrangian.[26,74] Models of dynamical symmetry breaking, or technicolor,[75] are known to have difficulties suppressing flavor changing neutral current rates.[28] Rates for the currently popular supersymmetric theories[76] could certainly be checked. CP violation by means other than the KM matrix could also increase this particular effect.

One obvious similar calculation to the one done here is flavor changing Z decay into two quarks of weak isospin $-\frac{1}{2}$, eg. $Z \to b\bar{s}$. In the three generation case, this has the advantage that the top quark, which was external before, now runs inside the loop and therefore is the participant in the GIM cancellation. Therefore, whereas $Z \to t\bar{c}$ was controlled by M_b, one expects $Z \to b\bar{s}$ to be controlled by M_T, with $M_T \gg M_b$. A quick estimate is

$$BR(Z \to b\bar{s}) \sim \left(\frac{M_T}{M_b}\right)^4 BR(Z \to t\bar{c}) \quad . \tag{8.1}$$

For M_T = 50 GeV, the enhancement factor is 10^4, which brings the branching ratios into the domain of respectability. In analogy with Chapter VI, $M_T > M_Z$ is expected to provide a very great enhancement. Therefore this process could in principal be used to put a bound on (or establish a prediction for) the top quark mass.

Unfortunately, it is not obvious whether decays such as $Z \to d\bar{s}$, $b\bar{d}$, or $b\bar{s}$ have a striking enough signature to be experimentally measurable. The main hope seems to be if one could apply cuts to ascertain whether a jet was initiated by a fragmenting b or $\bar{b}$ quark, or not. As noted in Chapter VII, there seems to be some success in identifying b jets at PETRA, but for Z decays the jet energies are much larger and the particles in the jet will be more collimated. I have not studied this issue further.

The four generation case is again of potential interest here. A superheavy t' could provide additional enhancement, given a favorable KM matrix. If $M_{b'} < M_Z$, $Z \to b'\bar{b}$ could be a way to discover the b' and is potentially measurable (by methods similar to those of Chapter VII) if the rate is large enough.

As discussed earlier in this thesis, there is no convincing reason to think that there are more than three generations. This leaves us with the miserably pessimistic results of Chapter V, which just about rule out the discovery of the sought-for top quark by this mechanism. But if $M_T > M_Z/2$, one would not want to just give up on discovering this important particle in the near future at worst, or at best hope that problems of low rates and large backgrounds at hadronic colliders could be overcome and the top will be discovered there.[77] It would be better

if there was another way to produce the top at the e^+e^- Z^0 factories.

There does exist another possibility; it is $Z \to t + 3$ other fermions.[78] This could occur at $O(g^3)$ through tree diagrams with branches of fermions and W's. This is actually the same order in weak interactions as the loop diagrams considered in this paper; however, they have the advantage that there does not appear to be an associated GIM cancellation suppression mechanism. However reduced phase space due to the four body final state is a potential disadvantage. Higgs branches cannot produce a single top quark in the final state to lowest order since the Higgs couplings are flavor diagonal at tree level.

Potential final states are $Z \to t\bar{b}_i \ell \bar{\nu}_\ell$, $\bar{t} b_i \bar{\ell} \nu_\ell$, $t\bar{b}_i \bar{u}_k b_j$, or $\bar{t} b_i u_k \bar{b}_j$, where t = top quark, u_i = u or c quark, b_i = d, s, or b quark, and ℓ and ν_ℓ are a lepton and its neutrino. The lack of a GIM mechanism is most obvious for the final states that include a lepton. Suppose we take the KM matrix to be the identity matrix, so that t couples only to b, with matrix element 1. Then, in the first of the four final state classifications above, only $t\bar{b}\ell\bar{\nu}_\ell$ can occur and is proportional to the KM matrix element 1. Then as usual we ignore the neutrino masses and set the leptonic mixing matrix to the identity. One concludes that there does not seem to be a way to induce a GIM cancellation in this class of diagrams from summing over fermion species.

Taking $M_T \sim M_Z/2$, I am led to the following guess for the branching ratio:

$$BR(Z \to t \text{ or } \bar{t} + f^3) \sim \left(\frac{\alpha_W}{8\pi}\right)^2 \left[\frac{(M_Z/2)^3}{M_W^2(M_Z/2)}\right]^2 \left(\frac{M_Z/2}{M_Z}\right)$$

$$= \frac{1}{2} \left(\frac{\alpha}{8\pi s_W^2}\right)^2 \left(\frac{M_Z}{2M_W}\right)^4 \sim 10^{-7} \quad . \tag{8.1}$$

In this estimate I have taken all fermions other than t to be massless, and have assumed $M_Z - M_T \sim M_Z/2$ controls the phase space and sets dimensions where necessary. I gave $1/(M_W^2)$ for one W propagator, and $1/(M_Z/2)$ for one fermion propagator.* The extra factor $(M_Z/2)/M_Z$ was inserted because M_Z gives dimensions to $\Gamma(Z \to \text{all})$. Taking U^{KM} to be the identity, one has no KM angles entering; also the number of open channels is approximately the same as for the Born decays, so there is no extra suppression from that. I took (α_{Weak}/π) as the expansion parameter, with an extra $(1/2\sqrt{2})^2 = 1/8$ from the charged current couplings.

Therefore this seems to merit a more detailed calculation as a potential way to discover the top quark; the above estimate is only a rough guess. It should be noted that destructive interference between amplitudes could kill this rate. If I try to estimate $Z \to Wf_1\bar{f}_2$ (which has been calculated) in a similar manner, my guess is too large by a factor of 30; part of this may be attributed to destructive interference.[71] Also, as M_T increases, the phase space will cut this off more and more.

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* The diagram with a ZWW vertex gives a similar contribution because of the momentum factor that comes with that vertex.

IX. Conclusions

I calculated flavor changing Z^0 decay to charge 2/3 quarks in the $SU(2)_L \times U(1)$ theory of electroweak interactions. In particular I considered the possibility of discovering the top quark by this mechanism. The exact expression for the decay rate is obtained as a function of the five independent masses, to one-loop order. Ward identities for the unitary gauge are derived to help check the result.

I computed numerical results in the three generation model for realistic values of the parameters involved. The branching ratios such as $BR(Z \to top)$ are at most $\sim 10^{-10}$, and for many of the allowed values of M_T and the quark mixing matrix are much worse. If LEP produces 10^8 Z's per year, then the prospects are for at most one top quark every 100 years by this process, making it a dismal experimental proposition.

Numerical results for four generations are also computed, assuming a supermassive internal b' quark. The rates are observed to grow asymptotically in $M_{b'}$ as roughly $M_{b'}^4$. For $M_{b'}$ large enough, the branching ratios can be significant and potentially measurable. I also considered a new form of CP violation, but find it to be small despite potentially large new phases in the 4×4 KM matrix.

Finally, I considered experimental signatures, and estimated some other, related, processes.

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Figure Captions

Figure 1. Diagrams giving the one-loop $Z \to t\bar{c}$ amplitude in unitary gauge.

Figure 2. Diagrammatic representation of the Ward identities of Chapter III. (A) Vector Ward identity; (B) Axial Ward identity.

Figure 3. (A) Cut giving the absorptive part of the one-loop $Z \to t\bar{c}$ amplitude for $M_T < M_W + M_J$. (B) Product of diagrams computed for the imaginary part check. (C) An additional cut contributing for $M_T > M_W + M_J$.

Figure 4. High, low, and average values of $\Gamma(Z \to \text{top})$ and $BR(Z \to \text{top})$, as defined in the text, vs. top quark mass (three generation case).

Figure 5. $\Gamma(Z \to \text{top})$ vs. KM matrix elements $(U_{Tb}U_{cb})^2$, for four top quark masses (three generation case). Computed points follow the solid lines, but deviate from the dashed lines which are only the extensions of the solid lines to the origin. The high endpoint of each solid line gives the highest computed Γ for that M_T.

Figure 6. High value of $\Gamma(Z \to \text{top})$ (as in Fig. 4), divided by product of KM matrix elements $(U_{Tb}U_{cb})^2$ and phase space factor $P(t,c)$ of Eq. (4.5) (three generation case).

Figure 7. Average values of $\Gamma(Z \to u\bar{c} + \bar{u}c)$ and $BR(Z \to u\bar{c} + \bar{u}c)$, as defined in the text (three generation case).

Figure 1

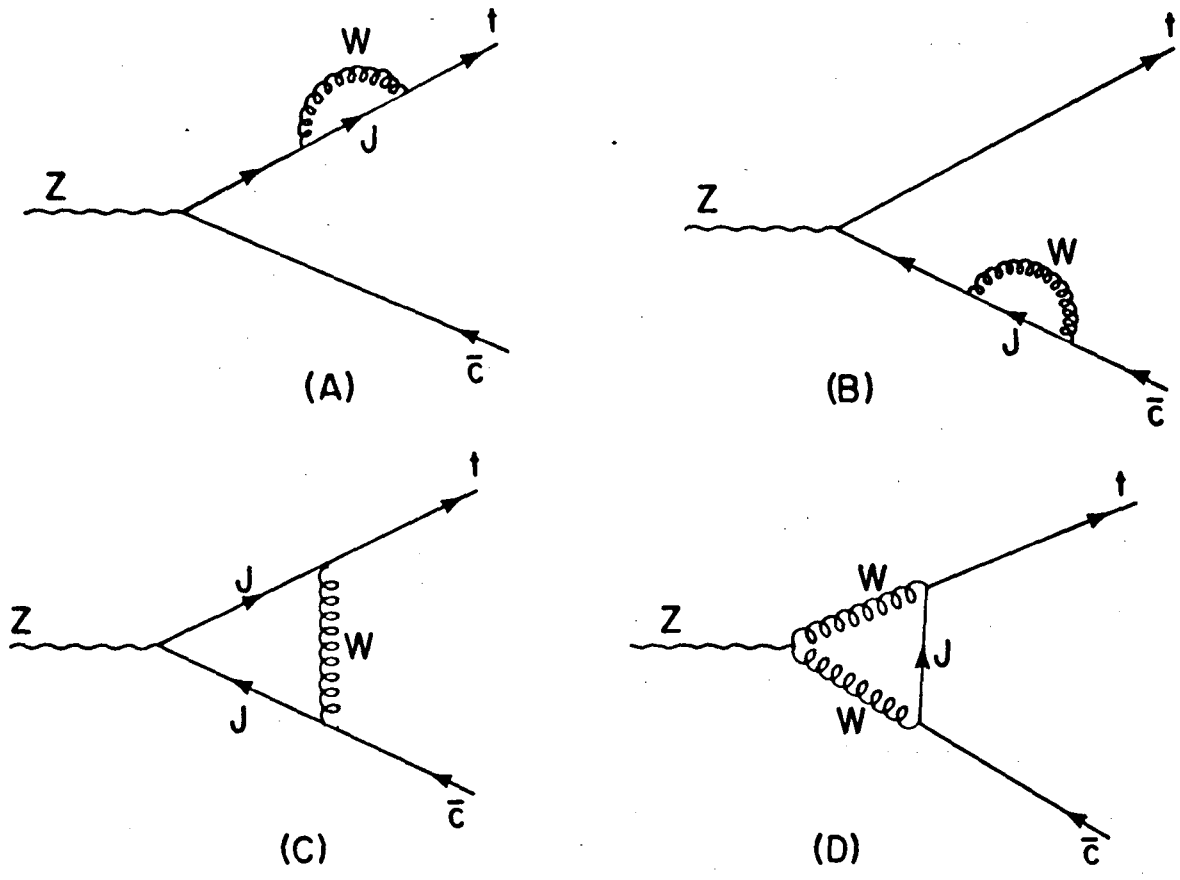
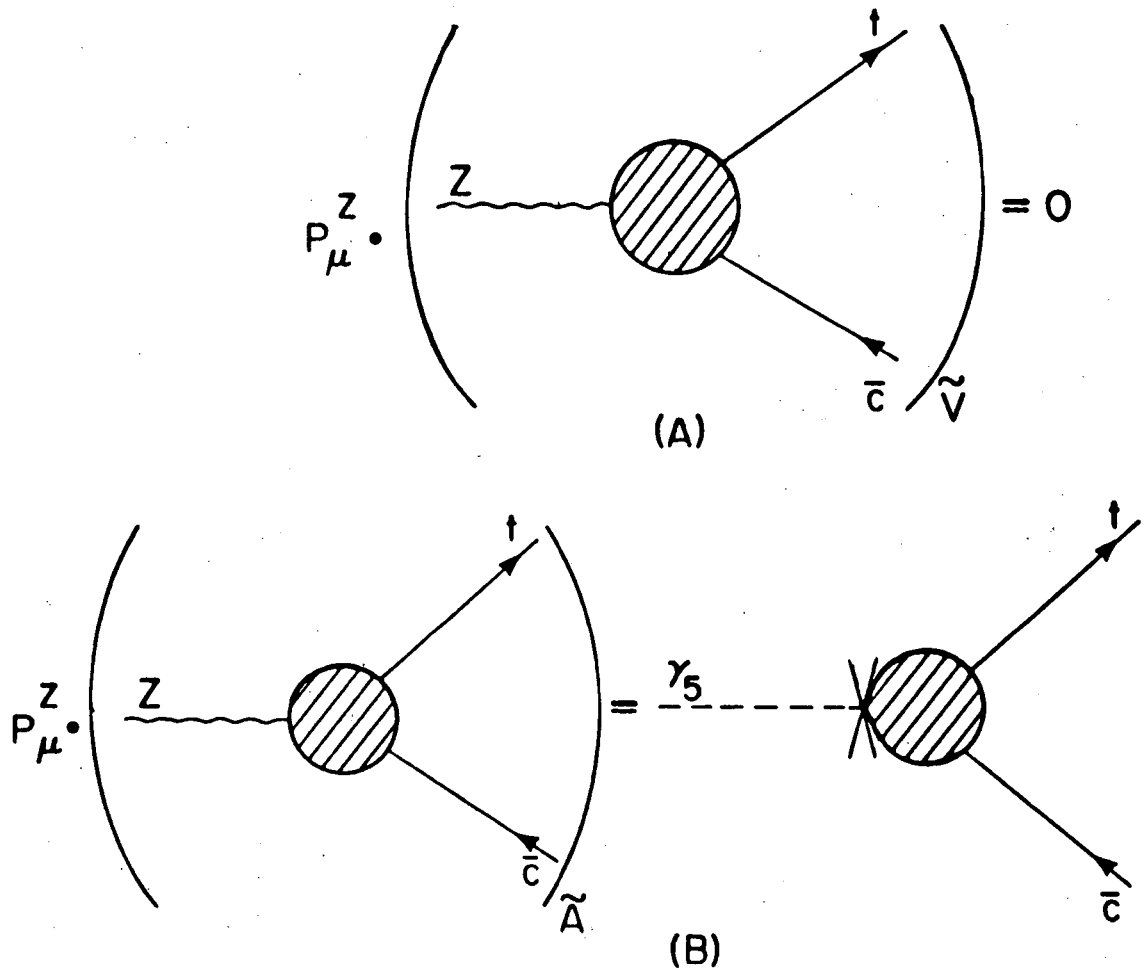


Figure 2


$P^Z_\mu \cdot$
Z
t
$\bar{c}$
$\tilde{V}$
= 0
(A)
$P^Z_\mu \cdot$
Z
t
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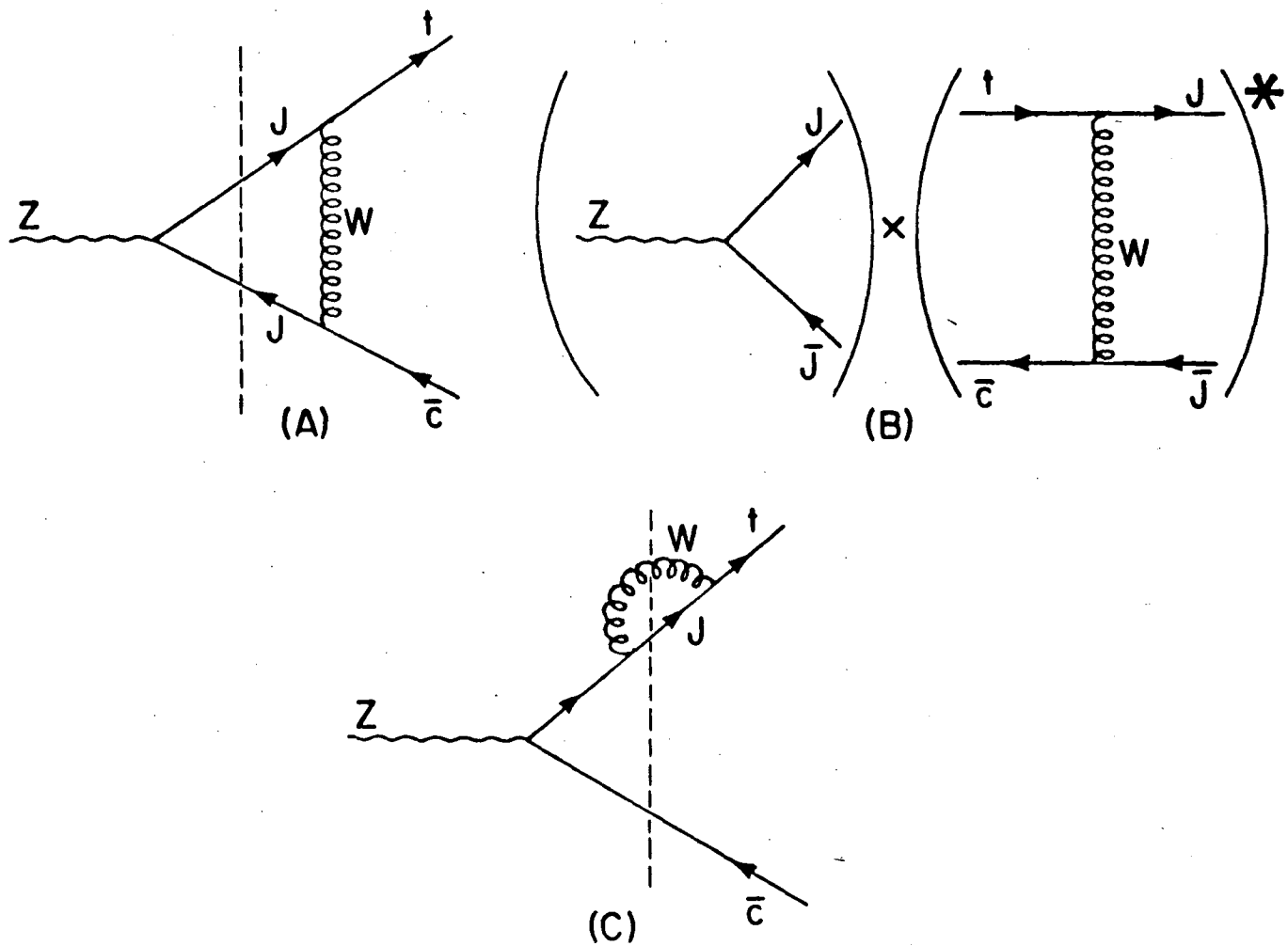
=
γ_5
t
$\bar{c}$
(B)

Figure 3



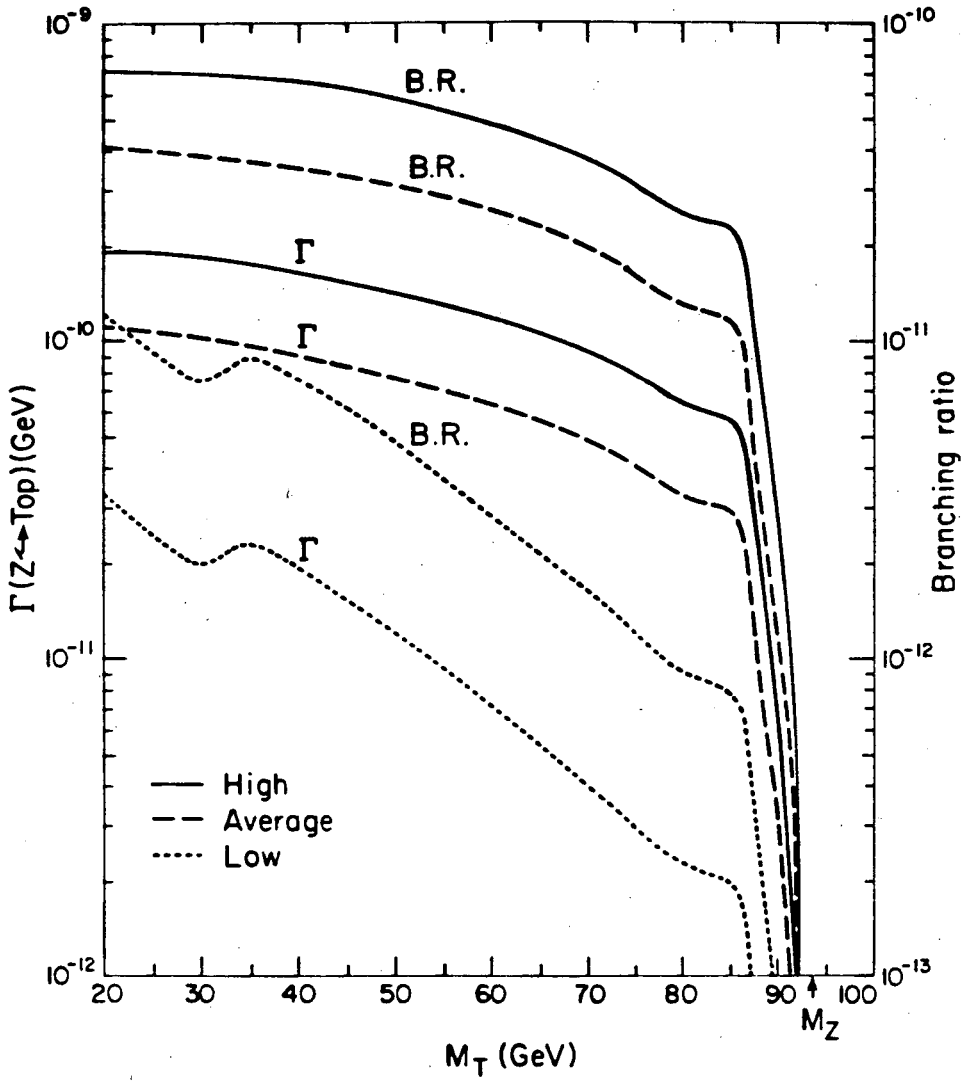

B.R.
B.R.
Γ
Γ
B.R.
Γ
High
Average
Low
Γ(Z↔Top)(GeV)
Branching ratio
M_T (GeV)
M_Z
10^-9
10^-10
10^-11
10^-12
10^-10
10^-11
10^-12
10^-13
20
30
40
50
60
70
80
90
100

Figure 4

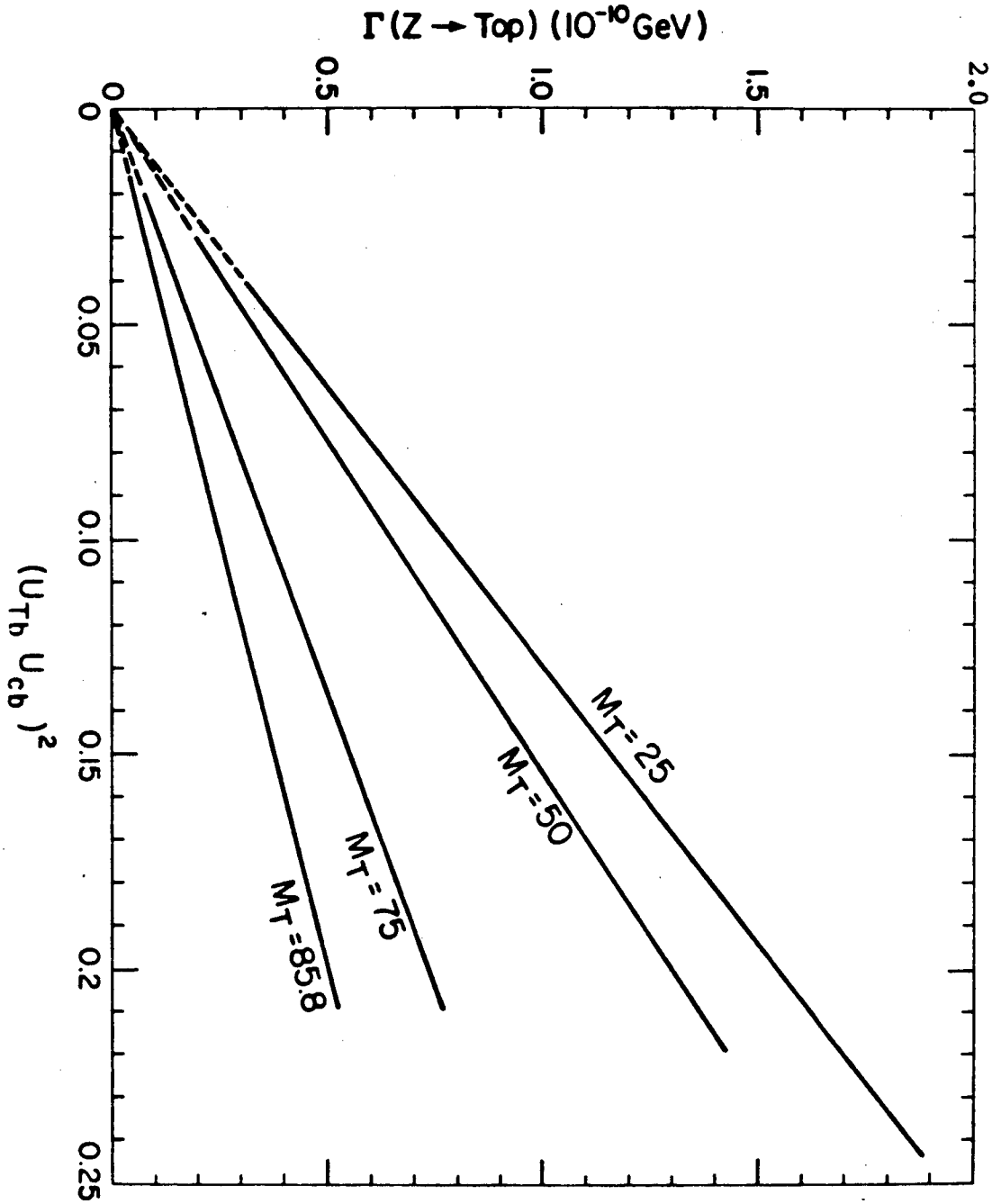

Γ(Z → Top) (10⁻¹⁰GeV)
2.0
1.5
1.0
0.5
0
0
0.05
0.10
0.15
0.2
0.25
(U_Tb U_cb)²
M_T=25
M_T=50
M_T=75
M_T=85.8

Figure 5

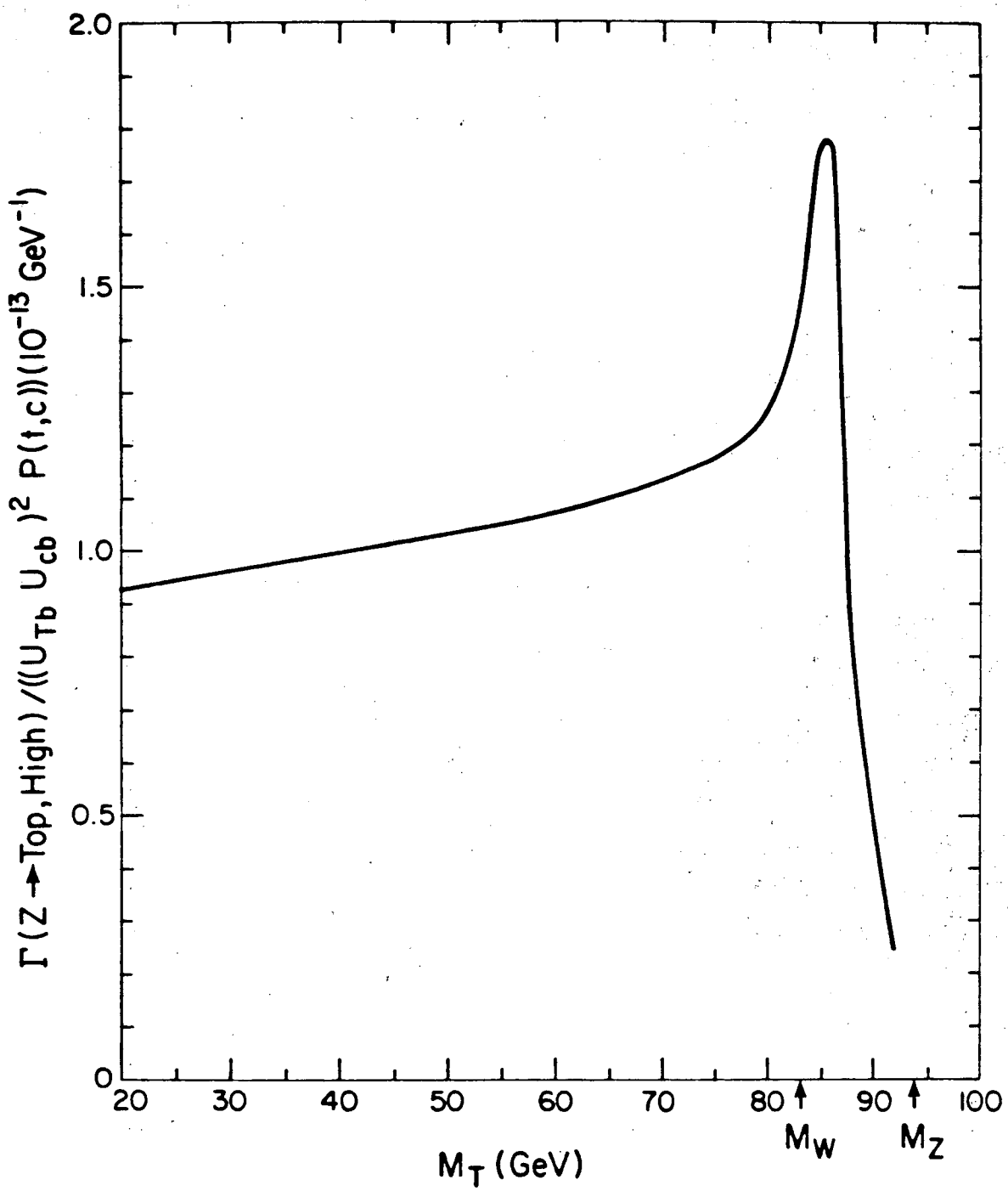

2.0
1.5
1.0
0.5
0
20
30
40
50
60
70
80
90
100
M_W
M_Z
M_T (GeV)
$\Gamma(Z \to Top, High)/((U_{Tb}\, U_{cb})^2\, P(t,c))(10^{-13}\, GeV^{-1})$

Figure 6

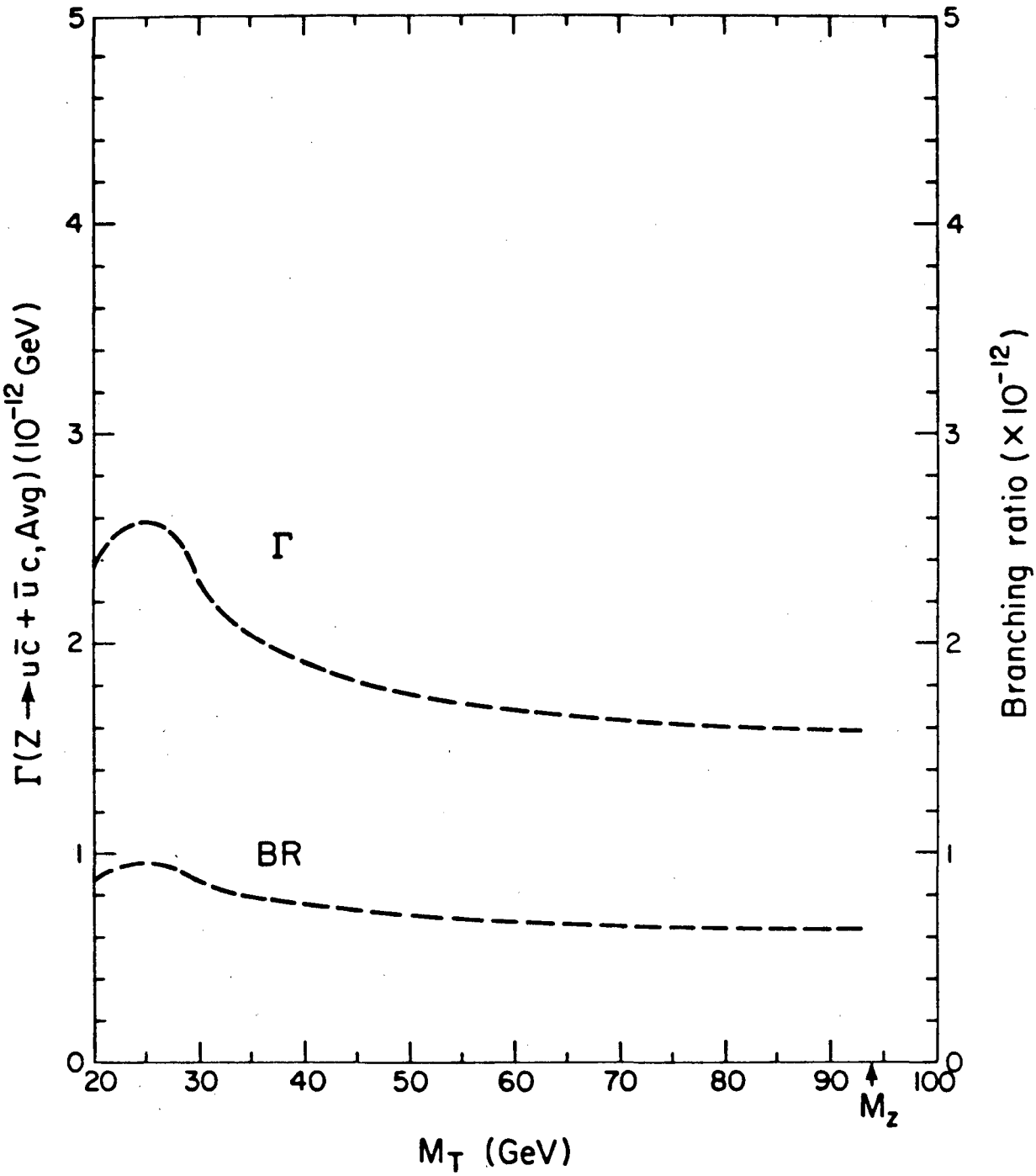

5
4
3
2
1
0
20
30
40
50
60
70
80
90
100
M_Z
M_T (GeV)
$\Gamma(Z \to u\bar{c} + \bar{u}c, \text{Avg})$ (10^{-12} GeV)
Branching ratio ($\times 10^{-12}$)
Γ
BR

Figure 7

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