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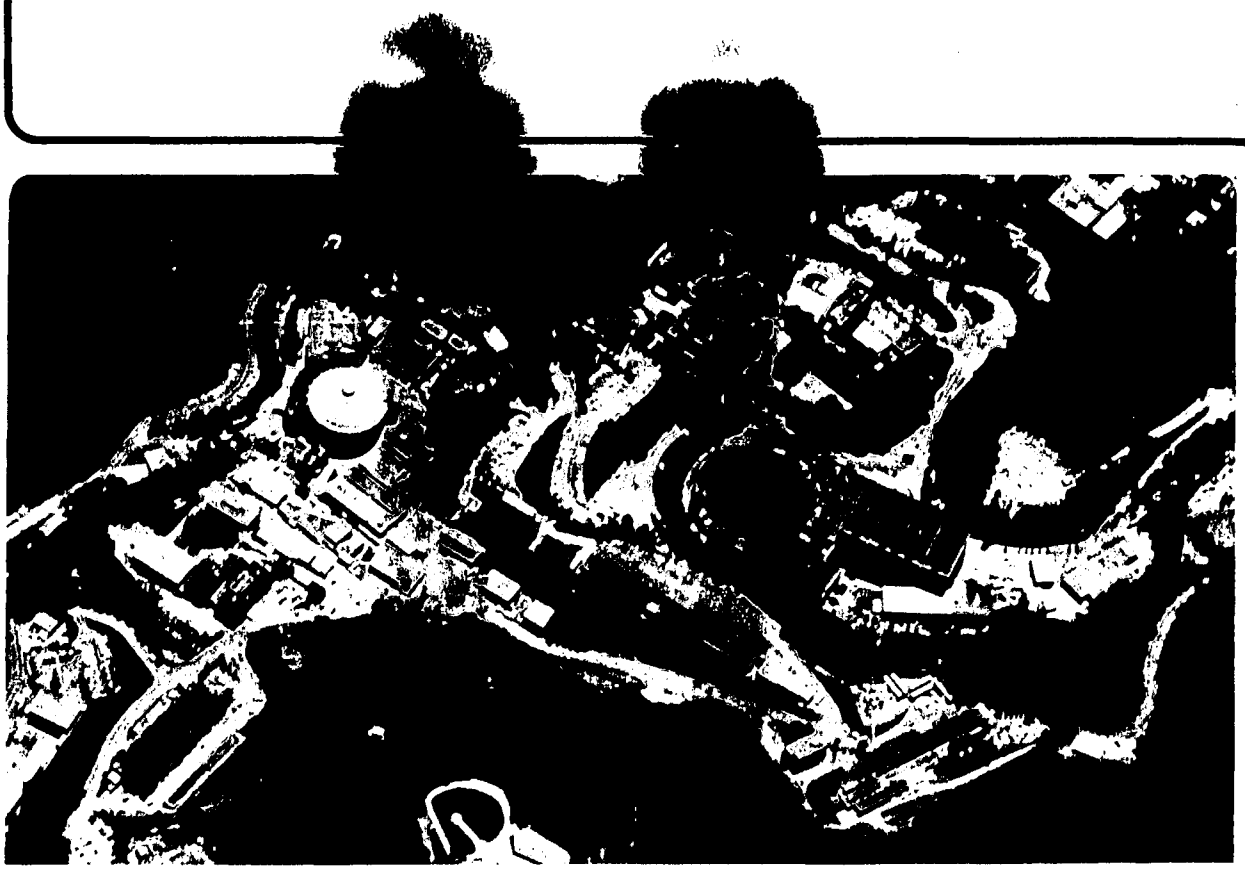

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AXIOMATIC SET THEORY AND PROPOSED RESOLUTION OF THE EINSTEIN-PODOLSKY-ROSEN AND BELL PARADOXES*

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ABSTRACT

A recent proposal for using axiomatic set theory to resolve the Einstein-Podolsky-Rosen and Bell paradoxes is examined and found to violate the critical locality requirement.

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Pitowsky[1] has claimed to have resolved the Einstein-Podolsky-Rosen and Bell paradoxes by means of a local model of spin correlations based on axiomatic set theory. Mermin[2] and Macdonald[3] have apparently refuted this claim: they have demonstrated that one key property of Pitowsky's model, namely that it assigns to each electron a definite spin-value of + 1/2 or - 1/2 in each direction, is by itself incompatible with the validity of the statistical predictions of quantum theory. In particular, Mermin showed, essentially, that no combination of finite sequences of spin values + 1/2 or - 1/2 in three specified direction can yield values of the associated spin-correlation average values that lie simultaneously within certain finite distances of the limiting values predicted by quantum theory: at least one of the examined spin-correlation average values cannot approach the large-n limit predicted by quantum theory. Since experimental physics deals with large yet finite numbers of experiments it seems apparent that no argument based on abstract postulates concerning transfinite numbers and operations can overturn the elementary numerical argument of Mermin. Yet Pitowsky does not retract his claim.[4]

To evaluate this situation two remarks about Pitowsky's arguments may prove helpful.

Pitowsky's arguments are based on the axiom of choice and the continuum hypothesis (or the weaker Martin's axiom). The deep results of Gödel and Cohen showed that one can independently add either one, or both, of these two axioms to the remaining standard axioms of set theory and obtain alternative set theories that are self-consistent if the original one is. The question thus arises: which of these alternatives set theories, if any, is a suitable ingredient of physical theory?

Important light is shed on this question by the fact that if one includes the two extra axioms then it is possible, by using the methods employed by Pitowsky, to prove (nonconstructively) the existence of a bivalent function $f(x, y) = \pm 1/2$ defined on the unit square $S = \{(x, y);\ 0 \leqslant x \leqslant 1;\ 0 \leqslant y \leqslant 1\}$ that enjoys the following properties: $\int_0^1 f(x, y)\, dx = 1/2$ for every $y (0 \leqslant y \leqslant 1)$; $\int_0^1 f(x, y)\, dy = -1/2$ for every $x (0 \leqslant x \leqslant 1)$; $f(a, b) = -f(b, a)$ for every $a \neq b$. Let $(x_1, x_2, \ldots)$ and $(y_1, y_2, \ldots)$ be two infinite sequences satisfying $0 \leqslant x_i \leqslant 1$, $0 \leqslant y_i \leqslant 1$ (all i), and define $f_n = n^{-1}(f(x_1, y_1) + \ldots + f(x_n, y_n))$. Then the law of large numbers entails the following two properties: (1), for any fixed sequence $(y_1, y_2, \ldots)$ the sequence $(f_1, f_2, \ldots)$ tends to 1/2 on a set of sequences $(x_1, x_2, \ldots)$ of measure one, where this measure is calculated by, for each i, restricting (x_i, y_i) to the line of fixed y_i; (2), for any fixed sequence $(x_1, x_2, \ldots)$ the sequence $(f_1, f_2, \ldots)$ tends to $-1/2$ on a set of sequences $(y_1, y_2, \ldots)$ of measure one, where the measure is calculated by, for each i, restricting (x_i, y_i) to the line of fixed x_i. If measure one corresponds to probability one then one concludes that the sequence $(f_1, f_2, \ldots)$ tends to $+1/2$ with probability one if the sequence $(y_1, y_2, \ldots)$ is considered fixed, but tends to $-1/2$ with probability one if the sequence $(x_1, x_2, \ldots)$ is considered fixed. Since the predicted limit of the sequence $(f_1, f_2, \ldots)$ should not depend on whether one imagines the x_i's or y_i's to be fixed one can conclude that the version of set theory that includes the two extra axioms does not always lead to sensible physical theories.

This conclusion does not necessarily entail that the particular theory developed by Pitowsky is not sensible. But Pitowsky attempts to circumvent Mermin's argument by using his model not as a literal model of the sequence of electrons in a particular experiment but rather as a basis for claiming that certain averages converge to their quantum-theoretical values with probability one. However, the example of the function $f(x, y)$ discussed above shows that if the two extra axioms are accepted then the value to which an average converges with probability one can depend critically upon what restrictions are placed on the set of points that are used to calculate this probability.

In Pitowsky's model one considers first a sequence of spin functions, and then the subsequence $(s_1, s_2, \ldots)$ in which the spin is "up" in some direction y. If w represents some second direction then the average value $A_n(w, y) = n^{-1}(s_1(w) + \ldots + s_n(w))$ is shown to tend, in the large-n limit, to the quantum-theoretical value $y \cdot w/2$ on a set of directions w of measure one, <u>where this measure is calculated by restricting the directions w to the circle $c(y, \arccos y \cdot w)$</u>.

The quantity $A_n(w, y)$ depends on y exclusively through the fact that the sequence $(s_1, s_2, \ldots)$ is defined relative to y. But the quantity that equals $y \cdot w/2$ in Pitowsky's theory depends on y in a second way: the explicitly specified measure is with respect to the set of directions w confined to the circle $c(y, \arccos y \cdot w)$. In the context of the EPR-Bell experiment this explicit second dependence on y is a dependence on the far-away choice of measurement. Thus the procedures used to calculate the value

$y \cdot w/2$ from the underlying local spin model introduce an explicit nonlocal element. It is the occurrence of this explicit nonlocal element in Pitowsky's calculational procedure that reconciles his theory with Bell's result that no local theory can reproduce the statistical predictions of quantum theory.

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