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November 1982

NUCLEAR SCIENCE DIVISION

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

LBL-15285 c.2

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LBL-15285

DEUTERON FORMATION IN NUCLEAR COLLISIONS

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[Nuclear Reactions Ne + U, Ar + Ca, E = 400 MeV/A deuteron production calculated]

PACS 25.70Np

Deuteron production cross sections are calculated from an intranuclear cascade model for Ne + U and Ar + Ca at 400 MeV/A. In effect, there are no adjustable parameters in this calculation. The parameters of the cascade code are fixed by sum charged inclusive data. Furthermore, the high momentum deuteron inclusive distribution is found to be insensitive to the details of the deuteron wavefunction. This latter observation leads to a justification of a generalized coalescence formula involving both momentum and spatial coordinates. Satisfactory agreement is found between our calculations and both the untriggered and high multiplicity triggered inclusive spectra of deuterons with energies $\geq$ 50 MeV/A.

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and by National Science Foundation Grant THY8006210 and PHY77-27804.

I. INTRODUCTION

One of the striking features of relativistic nuclear collisions is copious production of light composite (d,t,...) fragments. For Ne + U at 400 MeV/nucleon, for example, up to two thirds of the protons emerge bound in light fragments in certain kinematical regions. Up to now, simplified analytical models have been used to account for composite formation.

The coalescence model[1] is based on the intuitive assumption that if two or more nucleons emerge from the reaction region with relative momenta less than a coalescence radius $\sim$ 250 MeV/c, then through final state interactions they will form a bound nuclear fragment. This model predicts that the momentum distribution, of a nuclear fragment with A nucleons should be therefore proportional to the Ath power of the proton distribution. This power law has been verified experimentally for a very large class of reactions.[2] The absolute normalization of the fragment distributions cannot be calculated in this model.

Models based on assumptions of chemical and thermal equilibrium have also been proposed.[3] These have had some difficulty in correctly predicting the shape of deuteron spectra.[2]

Similar models[4] have recently been used to attempt to relate the ratio of total deuteron to proton yields to properties of nuclear matter during the high temperature and density phase of the collision. An experimentally quoted [2b] ratio of $\sim$ 0.3 has been interpreted as evidence of higher entropy than could be accounted for by conventional nuclear degrees of freedom alone.

Many questions however have been raised as to whether the assumptions used in this interpretation are valid.

The normalization ambiguities in the coalescence model, difficulties in the chemical model and questions in respect to deuteron/proton yeilds have motivated us to treat composite production using more detailed and exact methods. The intranuclear cascade model is used to generate an approximation to the full density operator of the colliding system as a function of time. The relation between this classical stochastic model and the quantum density operator is via the Wigner representation of the density as first outlined in Ref. 6. Many versions of the cascade model exist and have been on the whole rather successful in accounting for the summed charge inclusive cross section. We have used a version due to Cugnon.[5] The relation between the time dependent density in the Wigner representation and inclusive fragment cross-sections is essentially kinematic and can be found exactly.[6,7]

In the next section we review the derivation of the necessary formulas found in Refs. 6,7. In addition we show that the cross section is composed of two parts. The first by itself is a generalization of the coalescence model expression. For fast deuteron production ($E \gtrsim 50$ MeV/nucleon) we show that the second part, which is simple but more time consuming to calculate, serves as a small correction. It is ignored in the present study.

Section IV contains our numerical results compared to the data. In the last section we briefly discuss (1) total deuteron/proton yields and (2) suppression of forward protons.

Our main conclusions are as follows: (1) The cascade model plus the generalized coalescence formula agree well with fast deuteron production

data to within theoretical and experimental statistical uncertainties of order 50%. In effect, the calculation has no free parameters. (2) The correction to the generalized coalescence term in the exact formula is small primarily because the cascade model generated no significant two body correlations on the order of the size of the deuteron in phase space. One corollary of this is the insensitivity of our results to approximations of the deuteron wave function. Another corollary is that it suggests the adequacy of kinetic theory approximations to the dynamics which also ignore two body correlations. (3) For slower deuterons we expect the cascade code as currently implemented as well as the generalized coalescence term by itself to become inadequate. This agrees with the fact that our agreement with data deteriorated below 50 MeV/nucleon. (4) Evidence for both high entropy and flow effects is currently inconclusive because of possibly important contributions from production of nuclear fragments heavier than the deuteron. These have not been measured adequately in experiment nor have they been accounted for adequately by previous theoretical discussions.

II. THEORY

A. Primordial Differential Yields

All information about a system can be extracted from its A-body density $\rho(t)$ which satisfies Schroedinger's equation ($\hbar = 1$)

$$\partial\rho/\partial t = -i[H,\rho] \quad , \tag{2.1}$$

where H is the A-body Hamiltonian

$$H = \sum_i K_i + \sum_{i>j} V_{ij} \ , \tag{2.2}$$

K_i, the kinetic energy of nucleon i and V_{ij}, the interaction between i and j. Deuteron production information can be extracted with the aid of a two body density

$$\rho_D = |\psi_D\rangle\langle\psi_D| \quad , \tag{2.3}$$

where we take ψ_D to be a momentum eigenstate of a deuteron formed from nucleons 1 and 2. Thus $tr(\rho_D\rho(t))$, where the trace is taken over all A body coordinates, measures the inclusive probability for finding nucleons 1 and 2 in the subspace defined by ρ_D at time t. Since the trace sums over <u>all</u> states of the remaining particles then in particular, as $t \to \infty$, this expression includes contributions from final state fragments larger than the deuteron,[7] for example from <u>bound</u> states of 1, 2 <u>and</u> 3. As usually understood, the inclusive probability does not include such possibilities. Thus this probability is inclusive in an extended sense of the word. To distinguish it we call it primordial. When $\rho(0)$ is normalized to describe one incident beam nucleus per unit area then the probability for 1 and 2 to be detected as a deuteron in the final state

equals the cross-section, σ_D. Thus, assuming this normalization, we write

$$"\sigma_D" = \lim_{t\to\infty} \mathrm{tr}\ \rho_D \rho(t) \tag{2.4}$$

and call this the primordial (differential with respect to 3-momentum) cross section for nucleons 1 and 2 to emerge as a deuteron. The primordial cross section "σ_D" is always greater than the inclusive σ_D and is obtained by adding to it contributions from the measured inclusive H^3, He^3, ... production in a manner to be discussed. The above must finally be summed over contributions from all A(A-1)/2 pairs of nucleons to be compared with experiment.

It is difficult to use Eq. (2.4) directly because it requires that $\rho(t)$ provide a good approximation to 2-nucleon correlations as $t \to \infty$. However, the intranuclear cascade asymptotically describes only free streaming. The intranuclear cascade model can provide, at best, an approximation to $\rho(t)$ only during the reaction time when many large momentum transfer collisions occur. To overcome this we turn instead to an expression for a transition rate $\Gamma_D(t)$ which is related to the primordial cross-section via

$$\begin{aligned} "\sigma_D" - \mathrm{tr}(\rho_D \rho(0)) &= \int_0^\infty \Gamma_D(t)\ dt \\ &= \int_0^\infty \mathrm{tr}(\rho_D \partial\rho/\partial t)\ dt \\ &= -i \int_0^\infty \mathrm{tr}(\rho_D [H,\rho])\ dt \quad . \end{aligned} \tag{2.5}$$

A compact expression for Γ_D can now be derived using the following decomposition of H,

$$H = H_{12} + H_{\overline{12}} + U_{12} \quad , \tag{2.6}$$

where $H_{12} = K_1 + K_2 + U_{12}$ is the two body Hamiltonian of particles 1 and 2,

$H_{\overline{12}} = \Sigma K_i + V_{ij}$ for $j > i \geq 3$ is the A-2 body Hamiltonian of particles 3...A and

$$U_{12} = \sum_{j \geq 3} (V_{1j} + V_{2j}) \tag{2.7}$$

is the interaction of particles 1 and 2 with 3...A. Equation 2.6 into 2.5 yields

$$\Gamma_D(t) = -i \text{ tr } \rho_D[U_{12}, \rho(t)] \tag{2.8}$$

using the cyclic property of traces. Only the commutator of U_{12} with ρ survives because (a) ψ_D is an eigenstate of H_{12} and hence $[\rho_D, H_{12}] = 0$ and, (b) $[\rho_D, H_{\overline{12}}] = 0$ since $H_{\overline{12}}$ does not act on the coordinates of particles 1 and 2. Equation 2.8 can be considered to be the analog of an expression for the transition matrix. It coincides with our expectation that the probability of finding 1 and 2 in a bound state can be changed only by their interaction with a third body.

The finite range of the nuclear potentials V_{ij} causes $\Gamma_D(t)$ to cut off in time. The reaction time is thus dynamically defined and no "freeze out" time need be inserted ad hoc in a calculation based on Eq. 2.8. It is now possible to employ approximations for $\rho(t)$ based on dynamical models such as intranuclear cascade.

B. Application to Intranuclear Cascade

In this model, trajectories $\{\bar{\underset{\sim}{x}}_i(t), \bar{\underset{\sim}{p}}_i(t); i = 1 \ldots A, 0 \leq t < \infty\}$ are calculated as a function of time. Interactions are taken into account by allowing the $\bar{\underset{\sim}{p}}_i$ to change at discrete scattering times $\{t_{ij}(\nu)\}$ with momentum transfers $\Delta p_{ij}(\nu)$ chosen at random from experimental NN cross sections. Typically the times by prescription are chosen to be at the distance of closest approach of i with j subject to the restriction that $|\bar{\underset{\sim}{x}}_i - \bar{\underset{\sim}{x}}_j| \leq (\sigma_{NN}/\pi)^{1/2}$ at

that time. Thus in this notation, $t_{ij}(\nu)$ is the time of the νth collision between i and j and the momentum of i at t can be written as

$$\bar{\underset{\sim}{p}}_i(t) = \underset{\sim}{p}_i(0) + \sum_{j\neq i} \sum_{\nu} \theta(t - t_{ij}(\nu))\, \Delta\underset{\sim}{p}_{ij}(\nu) \tag{2.9}$$

where $\Delta\underset{\sim}{p}_{ij} = -\Delta\underset{\sim}{p}_{ji}$ and $\underset{\sim}{p}_i(0)$ is selected from a boosted Fermi momentum distribution corresponding to the initial state of the nucleus that nucleon i started in. These models typically equate the canonical with the kinematic ($m\, d\underset{\sim}{x}/dt$) momentum so that Eq. 2.9 also implies straight line trajectories between collisions.

In order that the intranuclear cascade model be used to calculate composite particle production along the lines indicated, these trajectories must be related to the density operator $\rho(t)$. Ideally this would be done by deriving the model from the Schroedinger equation. A completely satisfying derivation is still lacking but a first step[6] suggests that the relation goes via the Wigner representation $\rho_\omega(t)$ of the density as follows. Recall that the Wigner representation of any operator e.g. A(t) is related to is coordinate space representation by

$$\begin{aligned} A_\omega(t) &\equiv A_\omega(\underset{\sim}{x}_1, \underset{\sim}{p}_1, \ldots, \underset{\sim}{x}_A, \underset{\sim}{p}_A; t) \\ &= \int d\underset{\sim}{y}_1 \ldots d\underset{\sim}{y}_A e^{-i\underset{\sim}{p}_1\cdot\underset{\sim}{y}_1} \ldots e^{-i\underset{\sim}{p}_A\cdot\underset{\sim}{y}_A} \\ &\quad \cdot \langle \underset{\sim}{x}_1 + \tfrac{1}{2}\underset{\sim}{y}_1, \ldots, \underset{\sim}{x}_A + \tfrac{1}{2}\underset{\sim}{y}_A | A(t) | \underset{\sim}{x}_1 - \tfrac{1}{2}\underset{\sim}{y}_1, \ldots, \underset{\sim}{x}_A - \tfrac{1}{2}\underset{\sim}{y}_A \rangle \ . \end{aligned} \tag{2.10}$$

Each cascade run provides the following approximation for the density matrix:

$$\rho_\omega^c(t) = \prod_{i=1}^{A} h^3 \delta(\underset{\sim}{x}_i - \bar{\underset{\sim}{x}}_i(t))\, \delta(\underset{\sim}{p}_i - \bar{\underset{\sim}{p}}_i(t)) \ . \tag{2.11}$$

Averaging ρ_ω^c over many cascade runs corresponds to sampling the allowed trajectories. Therefore we approximate the exact density matrix in the Wigner

representation by

$$\rho_\omega(t) = \langle \rho_\omega^c(t) \rangle \quad , \tag{2.12}$$

where $\langle \cdots \rangle$ denotes the ensemble average (over all trajectories and/or initial starting configurations).

This approximation however is not such as to allow one simply to insert Eqs. 2.11 and 2.12 into Eq. 2.8 to calculate $\Gamma_D(t)$ directly since it is clearly in gross error in the region of small internucleon separation where V_{ij} is strongest. This problem can be neatly avoided by noting that the required commutator in Eq. 2.8 is already done for us in the cascade model's approximation as follows. According to eq. (2.12), the rate of change of $\rho_\omega(t)$ is, using Eq. 2.9,

$$\partial\rho_\omega(t)/\partial t = \langle \sum_{i=1}^{A} \bar{\underset{\sim}{v}}_i \cdot \bar{\partial}_{\underset{\sim}{x}_i} \rho_\omega^c(t) \rangle$$

$$+ \langle \sum_{j>i} \sum_{\nu} \delta(t - t_{ij}(\nu))(\rho_\omega^c(t+\varepsilon) - \rho_\omega^c(t-\varepsilon)) \rangle \quad , \tag{2.13}$$

where $\bar{\underset{\sim}{v}}_i = \bar{\underset{\sim}{p}}_i(t)/m$. The first term arises from the free translation of particles between collisions while the second is due (in the context of the classical stochastic model) to the impulses received at times $t_{ij}(\nu)$. The limit $\varepsilon \to 0^+$ is implicit. The exact equation of motion is, on the other hand,

$$\partial\rho_\omega(t)/\partial t = -i \sum_{i=1}^{A} [K_i, \rho(t)]_\omega - i \sum_{i>j} [V_{ij}, \rho(t)]_\omega \quad . \tag{2.14}$$

But it is straightforward to show that

$$-i[K_i, \rho(t)]_\omega = \underset{\sim}{v}_i \cdot \partial_{\underset{\sim}{x}_i} \rho_\omega(t) \tag{2.15}$$

so that comparing Eq. 2.13 and 2.14 we find that

$$-i[V_{ij}, \rho(t)]_\omega \cong \langle \sum_\nu \delta(t - t_{ij}(\nu))(\rho^c_\omega(t + \varepsilon) - \rho^c_\omega(t - \varepsilon))\rangle \tag{2.16}$$

is an approximation to the commutator which is already included in the model. Equation 2.16 into Eq. 2.8 yields

$$\Gamma_D(t) = \langle \sum_{i=1,2} \sum_{j\geq 3} \sum_\nu \delta(t - t_{ij}(\nu)) \quad \int d\underset{\sim}{x}_1 d\underset{\sim}{p}_1 h^{-3} \cdots d\underset{\sim}{x}_A d\underset{\sim}{p}_A h^{-3}$$

$$\cdot\ \rho_{D,\omega}(\underset{\sim}{x}_1, \underset{\sim}{p}_1; \underset{\sim}{x}_2, \underset{\sim}{p}_2)[\rho^c_\omega(t + \varepsilon) - \rho^c_\omega(t - \varepsilon)]\rangle$$

$$= \langle \sum_{i=1,2} \sum_{j\geq 3} \sum_\nu \delta(t - t_{ij}(\nu))(\rho_{D,\omega}(t + \varepsilon) - \rho_{D,\omega}(t - \varepsilon))\rangle \quad , \tag{2.17}$$

using the notation

$$\rho_{D,\omega}(t) \equiv \rho_{D,\omega}(\bar{\underset{\sim}{x}}_1(t), \bar{\underset{\sim}{p}}_1(t); \bar{\underset{\sim}{x}}_2(t), \bar{\underset{\sim}{p}}_2(t))$$

$$= \delta(\underset{\sim}{p} - \underset{\sim}{p}(t))\ D(\underset{\sim}{r}(t), \underset{\sim}{q}(t)) \ , \tag{2.18}$$

$$\langle \underset{\sim}{x}_1 \underset{\sim}{x}_2 | \psi_D\rangle = h^{-3/2} \exp(-i\ \underset{\sim}{P}\cdot(\underset{\sim}{x}_1 + \underset{\sim}{x}_2)/2)\langle \underset{\sim}{r} | D\rangle \ , \tag{2.19}$$

$$D(\underset{\sim}{r}, \underset{\sim}{q}) = \int d\underset{\sim}{y}\ e^{-i\underset{\sim}{q}\cdot\underset{\sim}{y}} \langle \underset{\sim}{r} + \frac{1}{2}\underset{\sim}{y} | D\rangle\langle D | \underset{\sim}{r} - \frac{1}{2}\underset{\sim}{y}\rangle \quad . \tag{2.20}$$

Thus $\underset{\sim}{P}$ is the deuteron total momentum, $|D\rangle$ its internal wave function unit normalized and, D, the Wigner representative of $|D\rangle\langle D|$. The phase space integral in Eq. 2.17 is trivial because of the delta function form of $\rho^c_\omega(t)$ given by

Eq. 2.11 and the variables in Eq. 2.18 are

$$\underset{\sim}{p}(t) = \bar{p}_1(t) + \bar{p}_2(t)$$
$$\underset{\sim}{q}(t) = (\bar{\underset{\sim}{p}}_1(t) - \bar{\underset{\sim}{p}}_2(t))/2$$
$$\underset{\sim}{r}(t) = (\bar{\underset{\sim}{x}}_1(t) - \bar{\underset{\sim}{x}}_2(t)) \quad . \tag{2.21}$$

Equations 2.17 and 2.18 into 2.5 yield

$$"\sigma_D" - \mathrm{Tr}(\rho_D\rho(0)) = \langle \sum_{n=1}^{N} (\rho_{D,\omega}(t_n + \varepsilon) - \rho_{D,\omega}(t_n - \varepsilon)) \rangle \quad , \tag{2.22}$$

where for convenience we have ordered the scattering times $t_{ij}(\nu)$ sequentially and called them t_n;

$$\{t_n | t_{n+1} > t_n\} = \{t_{ij}(\nu) | i=1,2; j \geq 3; \nu\} \quad . \tag{2.23}$$

The 6A dimensional phase space point $\{\bar{\underset{\sim}{x}}_i(0), \bar{\underset{\sim}{p}}_i(0), i = 1 \ldots A\}$, the initial condition for a cascade trajectory, is normally chosen from a distribution representing one beam per unit area just prior to collision with a single localized target nucleus. Internal consistency requires that this distribution in principle be $\rho(0)_\omega$. This is however impractical because $\rho(0)_\omega$ can become negative at some points. Since it is most probably unnecessary to keep each detailed oscillation in $\rho(0)_\omega$ for relativistic heavy ion collisions, a simple positive approximation is used (as noted following Eq. 2.9). Once this is done we then have

$$\mathrm{Tr}(\rho_D\rho(0)) \cong \langle \rho_{D,\omega}(0) \rangle \ , \tag{2.24}$$

the right hand side being the Monte-Carlo approximation to the phase space integral on the left. We can then rewrite Eq. 2.22 as

$$"\sigma_D" = < \sum_{n=0}^{N} (\rho_{D,\omega}(t_n + \varepsilon) - \rho_{D,\omega}(t_n - \varepsilon)) > \quad , \tag{2.25}$$

defining $\rho_{D,\omega}(t) = 0$ for $t < 0$.

The right hand side of Eq. 2.25 is a sum of terms each proportional to a delta function in total deuteron momentum. Clearly we are really interested in a smoothed version of this function. Assume this is done by collecting contributions in bins of volume $\Delta\underset{\sim}{P}$ to form the cascade models approximation to $"\sigma_D" \Delta\underset{\sim}{P}$. Thus each $\underset{\sim}{P}(t_n \pm \varepsilon)$ lies in some bin and one adds $\pm D(\underset{\sim}{r}(t_n), \underset{\sim}{q}(t_n \pm \varepsilon))$ to accumulate $"\sigma_D" \Delta\underset{\sim}{P}$ for that bin. Generally $\underset{\sim}{P}(t_n + \varepsilon)$ will not be in the same bin as $\underset{\sim}{P}(t_n - \varepsilon)$ but, because of momentum conservation between collisions, it will be in the same bin as $\underset{\sim}{P}(t_{n+1} - \varepsilon)$. Since D is generally positive this leads to important cancellations and it is useful to rewrite the series to reflect these facts as follows,

$$\begin{aligned} "\sigma_D" &= <\delta(\underset{\sim}{P} - \underset{\sim}{P}_N)D_N> + < \sum_{0}^{N-1} \delta(\underset{\sim}{P} - \underset{\sim}{P}_n)\delta D_n> \\ &\equiv <\rho_N> + <\delta\rho> \quad , \end{aligned} \tag{2.26}$$

$$\begin{aligned} \underset{\sim}{P}_n &= \underset{\sim}{P}(t_n + \varepsilon) = \underset{\sim}{P}(t_{n+1} - \varepsilon) \\ \underset{\sim}{q}_n &= \underset{\sim}{q}(t_n + \varepsilon) = \underset{\sim}{q}(t_{n+1} - \varepsilon) \\ \underset{\sim}{r}_n &= \underset{\sim}{r}(t_n + \varepsilon) = \underset{\sim}{r}(t_n - \varepsilon) \quad , \end{aligned} \tag{2.27}$$

$$\begin{aligned} D_N &= D(\underset{\sim}{r}_N, \underset{\sim}{q}_N) \\ \delta D_n &= D(\underset{\sim}{r}_n, \underset{\sim}{q}_n) - D(\underset{\sim}{r}_{n+1}, \underset{\sim}{q}_n) \quad . \end{aligned} \tag{2.28}$$

Note that in all the foregoing N, as well as $\{\underset{\sim}{P}_n, \underset{\sim}{q}_n, \underset{\sim}{r}_n;\ n = 0 \ldots N\}$, vary from one cascade to another.

Before proceeding we must correct a slight discrepancy that has been allowed for simplicity to creep into the notation. Equations 2.27 are inconsistent with the restriction $j \geq 3$ in the definition of t_n given by Eq. 2.23 since a self-interaction of the pair, a collision between particles 1 and 2 which could occur between a t_n and t_{n+1}, would cause $\underset{\sim}{q}(t_n + \varepsilon) \neq \underset{\sim}{q}(t_{n+1} - \varepsilon)$. However note that D is a function only of $|\underset{\sim}{r}|$, $\underset{\sim}{r}\cdot\underset{\sim}{q}$ and, $|\underset{\sim}{q}|$; the first two are zero at a self collision and the last is conserved in elastic scattering. Thus inclusion of self collisions in Eq. 2.22 will not change the result since they all give zero contributions. If we do this by removing the restriction $j \geq 3$ in Eq. 2.23, it becomes consistent with Eq. 2.27, and Eq. 2.26 is still correct. It is interesting to note that this result can be viewed as a reflection of the orthogonality between bound and scattering wave functions which is preserved in this model.

We have seen that "σ_D" is made up of two parts. The first, $\langle\rho_N\rangle$ is calculated using last collision coordinates only, the second, $\langle\delta\rho\rangle$ involves all of them. Since typically there may be $\sim$ 5 collisions/nucleon/cascade, $\langle\delta\rho\rangle$ could take 10 times longer to calculate than $\langle\rho_N\rangle$, and it is often impractical to do this. However a previous preliminary numerical investigation of fast $((\frac{1}{2}P)^2/2m \gtrsim 50$ MeV/nucleon) deuteron production from 600 MeV protons on gold showed that $\langle\delta\rho\rangle/\langle\rho_N\rangle \sim 0.01$. This suggests that cancellations in the second part of "σ_D" are large. In the next section we study under what conditions such cancellations in $\langle\delta\rho\rangle$ can be expected to occur. We show that when $\langle\rho_N\rangle$ is insensitive to the deuteron wavefunction, then

$$\langle\delta\rho\rangle/\langle\rho_N\rangle \ll 1. \tag{2.29}$$

In that case "σ_D" can be calculated from the generalized coalescence formula, $\langle\rho_N\rangle$.

C. The Generalized Coalescence Formula

The approximate deuteron Wigner density used in this investigation is that of a ground state harmonic oscillator wave-function,

$$D(\underset{\sim}{r},\underset{\sim}{q}) = 8e^{-r^2/d^2-q^2d^2} \tag{2.29}$$

A value of d = 1.7 fm corresponds to a r.m.s. radius of 2.1 fm. We found that varying d^2 by a factor of two had no detectable effect on the calculated value of $\langle\rho_N\rangle$. This result has significant consequences as shown below.

Let us define source distribution functions[6]

$$S_+(\underset{\sim}{P};\underset{\sim}{q},\underset{\sim}{r})\Delta\underset{\sim}{P} = \int_{\Delta\underset{\sim}{P}} d\underset{\sim}{P} \left\langle \sum_{n=0}^{N-1} \delta(\underset{\sim}{P} - \underset{\sim}{P}_n)\delta(\underset{\sim}{q} - \underset{\sim}{q}_n)\delta(\underset{\sim}{r} - \underset{\sim}{r}_n)\right\rangle$$

$$S_-(\underset{\sim}{P};\underset{\sim}{q},\underset{\sim}{r})\Delta P = \int_{\Delta\underset{\sim}{P}} d\underset{\sim}{P} \left\langle \sum_{n=0}^{N-1} \delta(\underset{\sim}{P} - \underset{\sim}{P}_n)\delta(\underset{\sim}{q} - \underset{\sim}{q}_n)\delta(\underset{\sim}{r} - \underset{\sim}{r}_{n+1})\right\rangle$$

$$S_f(\underset{\sim}{P};\underset{\sim}{q},\underset{\sim}{r})\Delta P = \int_{\Delta\underset{\sim}{P}} d\underset{\sim}{P} \langle\delta(\underset{\sim}{P} - \underset{\sim}{P}_N)\delta(q - q_N)\delta(r - r_N)\rangle \tag{2.30}$$

In terms of these, the cross section into $\Delta\underset{\sim}{P}$ can be written as

$$"\sigma_D" = \int d\underset{\sim}{q}d\underset{\sim}{r}[D(\underset{\sim}{r},\underset{\sim}{q})(S_+(\underset{\sim}{P};\underset{\sim}{q},r) - S_-(\underset{\sim}{P};\underset{\sim}{q},\underset{\sim}{r})) + D(\underset{\sim}{r},\underset{\sim}{q})\ S_f(\underset{\sim}{P};\underset{\sim}{q},\underset{\sim}{r})] \quad . \tag{2.31}$$

S_+ gives the number of pairs produced at $\underset{\sim}{P},\underset{\sim}{q},\underset{\sim}{r}$ due to all collisions before the last, S_- gives the number annihilated and, S_f the number produced due to all last

collisions. A physically reasonable S_f must possess a minimum momentum scale for each $\underset{\sim}{P}$ say $q_{min}(P)$ such that for $q < q_{min}(P)$, S_f varies negligibly from its value at $\underset{\sim}{q} = 0$. Our observation that $\int d\underset{\sim}{q} d\underset{\sim}{r}\ D(\underset{\sim}{q},\underset{\sim}{r})\ S_f(\underset{\sim}{P};\underset{\sim}{q},\underset{\sim}{r})$ did not vary with changes in d^{-1}, the momentum scale in D, shows that $q_{min}(P) >> d^{-1}$. But $q_{min}(P)$ tells us about the range in relative momentum between nucleons of momentum 1/2 P and other nucleons in the medium in which they travel. The corresponding momentum scales in $S_{\pm}$ cannot be less than $q_{min}(P)$ because they refer to earlier (and therefore "hotter") times during the reaction. We therefore conclude that we may set $\underset{\sim}{q} = 0$ in $S_{\pm}$ under the integral in Eq. 2.31 in the same limit. This causes $\langle\delta\rho\rangle$ to vanish.

An order of magnitude estimate of $\langle\delta\rho\rangle$ can be obtained as follows. In Eq. 2.30 set

$$\underset{\sim}{r}_{n+1} = \underset{\sim}{r}_n + 2\underset{\sim}{q}_n/m\ (t_{n+1} - t_n) \quad , \tag{2.32}$$

$$t_{n+1} - t_n \sim \lambda_d/v_d \sim m\lambda_N/P_n \ , \tag{2.33}$$

where the deuteron mean free path, λ_d, is approximately one half the nucleon mean free path $\lambda_N \sim d$. This into Eq. 2.30 yields

$$S_-(\underset{\sim}{P};\underset{\sim}{q},\underset{\sim}{r}) \cong S_+(\underset{\sim}{P},\underset{\sim}{q},\underset{\sim}{r} - \underset{\sim}{\delta r}) \ , \tag{2.34}$$

$$\underset{\sim}{\delta r} = 2\lambda\underset{\sim}{q}/P \quad . \tag{2.35}$$

This into Eq. 2.31 yields

$$\begin{aligned}\langle\delta\rho\rangle &\cong \int d\underset{\sim}{q} d\underset{\sim}{r}\ D(r,q)\ (S_+(\underset{\sim}{P};\underset{\sim}{q},\underset{\sim}{r}) - S_+(\underset{\sim}{P};\underset{\sim}{q},\underset{\sim}{r} - \underset{\sim}{\delta r}))\\ &\cong \int d\underset{\sim}{q} d\underset{\sim}{r}\ D(r,\underset{\sim}{q})\ \ q_i(r - \delta r)_j\ S_{ij}(\underset{\sim}{P})\end{aligned} \tag{2.36}$$

where the leading contribution from the Taylor series expansion of S_+ about

$\underset{\sim}{q} = \underset{\sim}{r} = 0$ has been kept using the definition

$$S_{ij}(\underset{\sim}{P}) = \partial^2 S_+(\underset{\sim}{P};q,r)/\partial q_i \partial r_j|_{\underset{\sim}{q}=\underset{\sim}{r}=0} \tag{2.37}$$

and implied summation over repeated indices. Note also that the arguments concerning the momentum scale in $S_{\pm}$ can be repeated for its length scale, $r_{min}(\underset{\sim}{P}) >> d$, which justifies Eq. 2.36 as the leading contribution. The only natural scale parameter for r_{min} which is much larger than d is the nucleus radius R.

A very gross estimate of $S_{ij}(\underset{\sim}{P})$ is possible;

$$S_{ij}(\underset{\sim}{P}) \sim S_+(\underset{\sim}{P};\underset{\sim}{0},\underset{\sim}{0})/(q_{min}(P)r_{min}(P))\ \delta_{ij} \quad . \tag{2.38}$$

This into Eq. 2.36 yields

$$<\delta\rho> \sim (\lambda/r_{min})(d^2Pq_{min})^{-1} \int d\underset{\sim}{q}d\underset{\sim}{r}D(r,q)\ S_+(\underset{\sim}{P};\underset{\sim}{0},\underset{\sim}{0}) \quad , \tag{2.39}$$

dropping constants of order unity. Estimating

$$S_+(\underset{\sim}{P};\underset{\sim}{0},\underset{\sim}{0}) \sim <N>_p\ S_f(\underset{\sim}{P};\underset{\sim}{0},\underset{\sim}{0}) \quad , \tag{2.40}$$

where $<N>_p$ is the average total number of collisions made by a fast pair of nucleons,

$$<N_p> \sim R/\lambda \sim r_{min}/\lambda \tag{2.41}$$

gives

$$<\delta\rho>/<\rho_N> \sim <N>_p\ (\lambda/r_{min})(d^2Pq_{min})^{-1} \sim (dP)^{-1}\ (dq_{min})^{-1} \tag{2.42}$$

Since we find $dq_{min} >> 1$, empirically for high momentum deuterons ($dP >> 1$), eq. (2.29) is satisfied in our case.

We derive next a practical criterion to test if $\langle\delta\rho\rangle/\langle\rho_N\rangle$ is indeed small. An order of magnitude estimate of q_{min} can be obtained from the sensitivity of $\langle\rho_N\rangle$ to the momentum parameter appearing in D as follows. Note that we could have more generally used a model such as

$$D(\underset{\sim}{r},\underset{\sim}{q}) = 8(d_q/d_r)^3\, e^{-r^2/d_r^2 - q^2 d_q^2} \tag{2.43}$$

instead of Eq. 2.29. In this case

$$\partial/\partial d_q^{\,2}\langle\rho_N\rangle \cong \int d\underset{\sim}{r}\, d\underset{\sim}{q}\; \partial/\partial d_q^{\,2}\; D(r,q)\; q^2(\partial S_f(\underset{\sim}{P};\underset{\sim}{q},\underset{\sim}{r})/\partial q^2)_{q=0}$$

$$\sim d_q^{\,-4}\; q_{min}^{\,-2}\; \langle\rho_N\rangle \tag{2.44}$$

using the same sort of estimate as in Eq. 2.38. Equation 2.44 into Eq. 2.42 yields

$$\langle\delta\rho\rangle/\langle\rho_N\rangle \sim [\partial\ell n\langle\rho_N\rangle/\partial\ell n\; d_q^{\,2}]^{1/2} \tag{2.45}$$

Thus the error involved in neglecting $\langle\delta\rho\rangle$ is linked to the sensitivity of the calculation of $\langle\rho_N\rangle$ to the model deuteron Wigner distribution used.

We conclude this discussion of the validity of Eq. 2.29 by cautioning that is based on the smallness of $\partial\ell n\langle\rho_N\rangle/\partial\ell n d_q^{\,2}$ which has been verified only for fast deuterons. This could very well break down for slower deuterons as evident from the factor $(dP)^{-1}$ in Eq. 2.42.

With eq. 2.29 satisfied, we obtain finally the generalized coalescence formula

$$"\sigma_D" = 3/4 \sum_{i_1 i_2} \langle\delta(\underset{\sim}{P} - \underset{\sim}{P}_N) D_N\rangle\ , \tag{2.46}$$

where 3/4 is the statistical spin factor for the deuteron now inserted, i_1 and i_2 are indices respectively for protons and neutrons now summed over, N is the last scattering index which now varies with i_1, i_2 as well as over cascades. $\underset{\sim}{P}_N = \underset{\sim}{P}_{i_1 i_2, N}$, is the total momentum of the pair i_1, i_2 after the last collision of either with a third particle and,

$D_N = D(\underset{\sim}{r}_{i_1 i_2, N}, \underset{\sim}{q}_{i_1 i_2, N})$ is the Wigner function evaluated at the final $\underset{\sim}{r}$, $\underset{\sim}{q}$ of i_1, i_2. The assignment of neutron and proton labels in the Cugnon Code is done statistically according to the N/Z ratio of projectile and target nuclei.

The physical interpretation of Eq. 2.46 is that the probability for n-p pairs to form deuterons is determined by their relative separation and momentum when both cease interacting with other nucleons. In this sense it is a final state interaction approximation. Last collision times t_N vary over the entire history of the reaction. The familiar coalescence model[1] in which a pair is assumed to form a deuteron if $q < d^{-1}$ is corrected by the inclusion of spatial correlation effects. This and other such models are also corrected by the realization that the cross section "σ_D" being calculated is the primordial and not simply the observed one. Also note that there is no arbitrary freezeout time in eq. 2.46. All quantities in eq. 2.46 are determined from the dynamical cascade calculation. Finally, we emphasize that even this generalization of the coalescence formula fails in kinematic regions for which $(d^2 P q_{min})^{-1} \gtrsim 1$, as seen from eq. (2.42). A significant result of this section is the derivation of a practical test eq. (2.45) for the validity of the generalized coalescence formula.

III. NUMERICAL RESULTS

We have performed calculations for the reactions Ne(400 MeV/A) + U and Ar(400 MeV/A) + Ca. Inclusive differential cross sections for ^1H, ^2H, ^3H, ^3He, ^4He were reported for these reactions over a wide kinematic range in Ref. 8.

The sum charge inclusive yield is just the primordial proton cross-section and in the cascade model is given by

$$"\sigma_{^1H}" = \langle \sum_i \delta(\underset{\sim}{P} - \underset{\sim}{P}_N) \rangle \tag{3.1}$$

where $\underset{\sim}{P}_N = \underset{\sim}{P}_{N,i}$ is the momentum of proton i after its last collision and the sum is over all protons. There is no term corresponding to $\langle \delta\rho \rangle$ here - it vanishes identically. This term exists only for composite particles. The primordial proton cross section is related to the experimental inclusive cross sections for composites $\gamma = {}^1H, {}^2H, {}^3H, {}^3He, {}^4He, \ldots$ by

$$"\sigma_{^1H}" \cong \sum_\gamma Z_\gamma \, \sigma_\gamma \tag{3.2}$$

where Z_γ is number of protons in γ. The approximation indicated in Eq. 3.2 refers to neglect of effects of Fermi momentum of the proton in γ.[7]

In Fig. 1.A,C, the cascade model prediction of $"\sigma_{^1H}"$ is compared to the data calculated via Eq. 3.2 where the sum over γ extends to ^4He. For each impact parameter $b_n = n/10\ b_{max}$, 100 cascades were generated and averaged over. Thus we calculated

$$"\sigma_{^1H}" = \sum_n 2\pi b_n \, \Delta b \, \langle \delta(\underset{\sim}{P} - \underset{\sim}{P}_N) \rangle_{b_n} , \tag{3.3}$$

where $\Delta b = b_{max}/10$.

An important momentum shift correction, however, was applied to the direct output of the Cugnon code[5]. Nucleons before and during the reaction find themselves in an average optical potential ~ -50 MeV. They must overcome a potential on this order of magnitude to emerge unbound as a lightly bound fragments. This effect is not included in the code. To take this final state interaction into account in a simple rough way, we have distorted each final nucleon momentum P to

$$P' = (P^2 + 2mV_0)^{1/2} \tag{3.4}$$

where V_0 was varied to fit the sum charged data. A best value $V_0 \cong -46$ MeV was found. Without this distortion, cascade results are systematically shifted to higher energies and clearly disagree with the data. Of course at high energies $E \gg V_0$ the shift has no effect but in the kinematic domain $E < 100$ MeV it is important. It is interesting that one reasonable value of V_0 leads to such an accurate reproduction of the data at all energies and angles in two rather different reactions. It should be kept clearly in mind that this procedure is a correction to the code and not to the theory discussed in the previous section. The theory in section II presupposes the accuracy of the underlying cascade model.

Having used the sum charge data to determine V_0, we proceed to calculate the primordial deuteron cross section. As was first pointed out in Ref. 7, this is related to the experimental cross sections for composites by the direct generalization of Eq. 3.2,

$$"\sigma_D" \cong \sum_\gamma \mathcal{P}(^2H/\gamma)\ \sigma_\gamma \tag{3.5}$$

where $\mathcal{P}(^2H/\gamma)$ is an apriori probability for finding a deuteron like configuration of an n-p pair in a fragment γ. An exact expression for $\mathcal{P}$ is given in Ref. 7 which involves overlaps of wave functions. Equation 3.5 again neglects Fermi motion of the n-p system in γ. The sum clearly begins with $\gamma = D = {}^2H$ for which $\mathcal{P}(^2H/^2H) = 1$. A rough estimate of higher probabilities based on simple spin-isospin considerations was given in Ref. 9; $\mathcal{P}(^2H/^1H) \cong \mathcal{P}(^2H/^3He) \cong 1.5$, $\mathcal{P}(^2H/^4He) \cong 3.0$. Keeping only these terms we have therefore used the expression

$$"\sigma_D" \cong \sigma_D + 1.5(\sigma_{^3H} + \sigma_{^3He}) + 3.0\ \sigma_{^4He} \tag{3.6}$$

all cross sections being evaluated at the same energy per nucleon.

The large triangles in Figs. 1B,C show the "σ_D" data obtained using Eq. 3.6. The large size of the triangles reflects an estimated 50% uncertainty in the $\mathcal{P}(^2H/\gamma)$ values used. The dots show σ_D. Note that in the kinematic region considered there are substantial contributions to "σ_D" from higher mass fragments γ. The calculated curve is based on Eqs. 2.46. To within the 50% uncertainties in the experimental "σ_D" values and the statistical uncertainties in the cascade output, the calculated "σ_D" is in excellent agreement with the data with respect to both normalization and shape over a 3 decade fall of the cross sections in both reactions.

We emphasize that in comparison to thermal models where large normalization corrections must be introduced, there are no adjustable parameters in our calculation of "σ_D". Once the cascade model is adjusted to reproduce the sum charge data by adjusting V_0, the deuteron calculation depends only on the deuteron wavefunction.

However, as mentioned before, the results were insensitive to factor of two variations in the d^2 parameter in the deuteron Wigner density D. This implies that the cascade model generates no two body correlations on the scale of the deuteron. Essentially, there are only long range ~ 4 fm correlations due to finite size effects but on the scale of $d \sim 2 \pm 1$ fm the two particle density shows no significant structure. A corollary of this is that insofar as deuteron production is concerned, the two nucleon density can be well approximated by the product of single particle densities. This in turn accounts for the success of the coalescence model.

We have also calculated primordial proton and deuteron yields from high multiplicity selected reactions of Ne(400 MeV/A) on U. Results are shown in Fig. 2. Theoretically, the maximum impact parameter in Eq. 3.3 was cut to 2.1 fm in order to fit the primordial proton (i.e. sum charge) data. As Fig. 2 shows, this can be done only for $E \gtrsim 50$ MeV protons. The discrepancy at lower energies had led to the speculation[10] that collective flow effects, absent in present cascade codes, may be important at these low energies and impact parameters. Generally we expect deficiencies in present day cascade codes to intensify at lower energies and during denser intermediate states. This is especially clear because of the necessity to introduce binding corrections such as eq. 3.4. The domain of applicability of this code seems to be $E \gtrsim 50$ MeV for central collisions and there it agrees remarkably well with the data.

Using the same impact parameter cutoff, "σ_D" was calculated and is shown in Fig. 2. The experimental [11] "σ_D" was calculated from Eq. 3.5 summing only for $\gamma = {}^2H, {}^3H$ since only these were measured for central trigger events. Nevertheless the overall agreements again leads us to conclude that to within 50% accuracy the cascade model can account for composite spectra even for central collisions.

IV. DISCUSSION

We briefly discuss here the relation between our results and two topics of recent interest.

First, there is the topic of the ratio of the total primordial deuteron yield to the total primordial proton yield. Our value of this ratio, in agreement with that obtained in Ref. 9, is too high ("D"/"P" $\cong 5.2/6.6 \sim 1$) as compared to the experimentally quoted[2b] value of 0.3. However, most primordial deuterons are at low energies where the number of heavier composites becomes largest. Experimentally these are not directly observed and the extrapolation procedure from the high energy light composite spectra used to obtain the quoted value of 0.3 tends to underestimate their number. It also neglects some lighter fragments associated with the nuclear fragmentation region. Both effects imply that the experimentally quoted ratio should be considered a lower bound. Theory also increasingly tends to be in error for low energy primordial deuterons. As mentioned already this is due both to defects in the cascade model and to neglect of the $\langle\delta\rho\rangle$ term in the formula for "σ_D". Therefore we are not confident in assigning significance to the present discrepancy between theory and experiment.

Second there is the question of the suppression of forward protons in central collisions as evidence of flow effects.[10] The phase space density in the forward direction is generally large in cascade calculations thereby increasing the probability of composite formation. If it is large

enough free protons will be suppressed only because primordial protons will emerge bound in composites. That is, the free proton depletion could be caused by an enhanced composite production. Preliminary ^{2}H and ^{3}H data also seem to show forward suppression for central collisions but heavier fragment data are not yet available. If heavier fragments also are found to be suppressed in the forward direction then a clear case for collective flow effects can be made. Otherwise, abundant composite formation may be a viable explanation.

ACKNOWLEDGMENTS

We are most grateful to J. Cugnon for allowing us to utilize his code to generate the cascade "data". Stimulating discussions with G. Bertsch and P. Danielewicz are gratefully acknowledged. We also thank the Institute for Theoretical Physics (UCSB) for early support and encouragement of this work.

Part of this work was supported by the Office of Energy Research, Division of Nuclear Physics, U.S. Department of Energy under Contract DE-AC03-76SF00098, and by National Science Foundation Grant THY8006210 and PHY77-27804.

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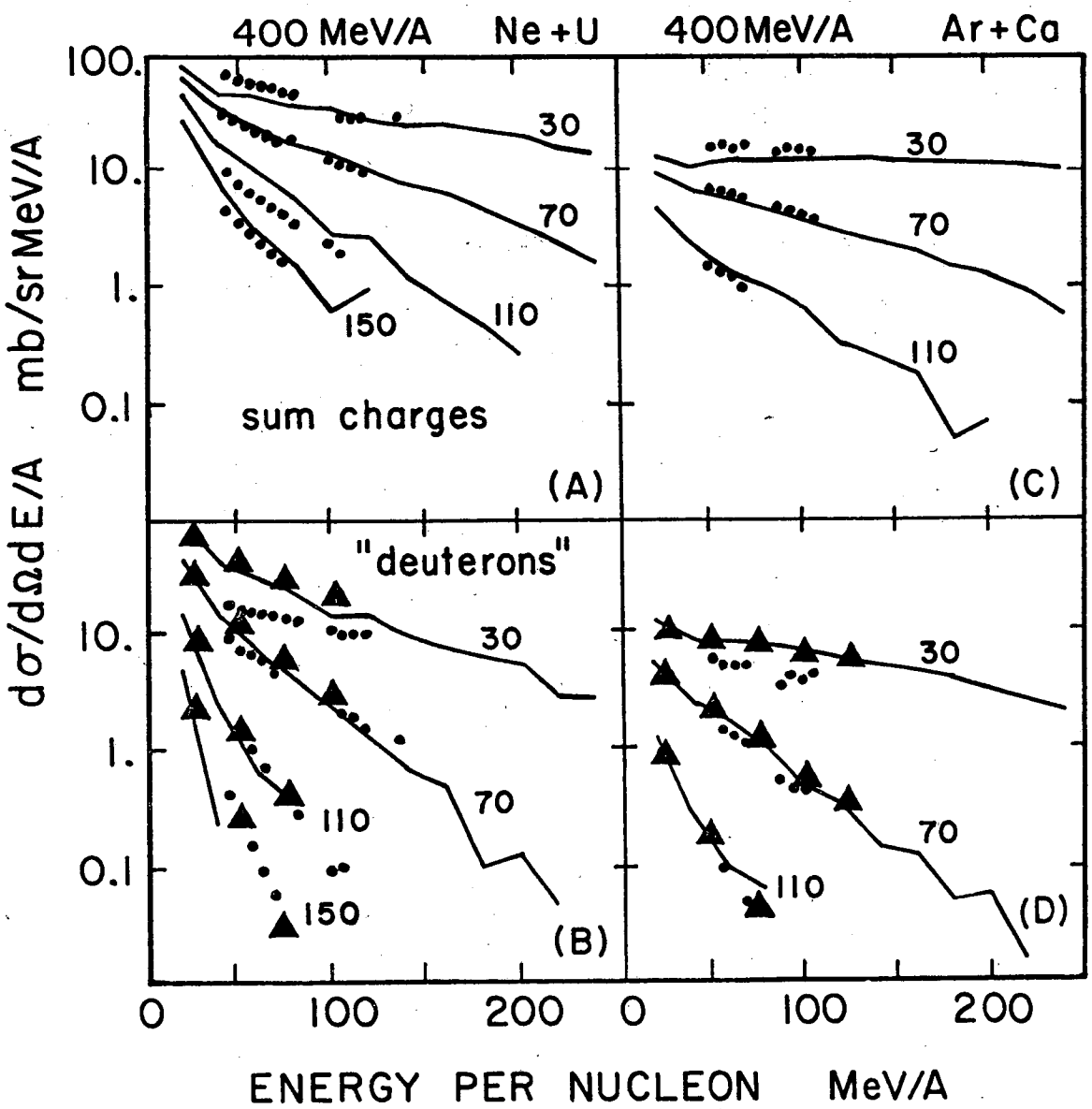
FIGURE CAPTIONS

Fig. 1. Comparison of sum charge inclusive data[8] (a,c) with cascade predictions. In (b,d) the primordial deuteron distribution as calculated (solid lines) using Eq. 2.46 is compared to data (solid triangles) using Eq. 3.6. The free deuteron data are indicated by dots.

Fig. 2. Sum charge and primordial deuteron (including ^{1}H and ^{2}H only) data[11] from high multiplicity triggered Ne + U at 400 MeV/A. The lines are our calculations with an impact parameter cut off of 2.1 fm.



XBL 822-7955

Fig. 1

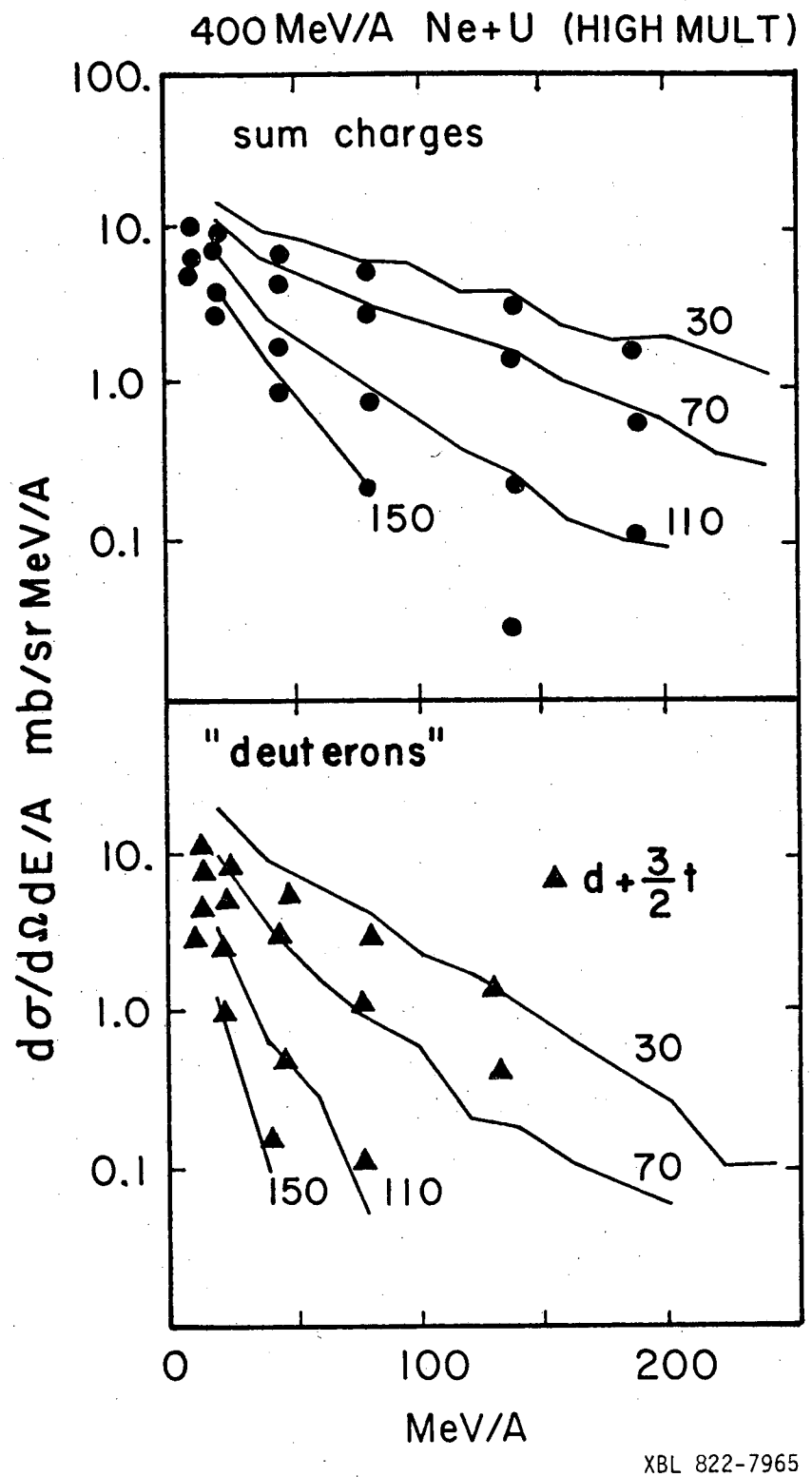

400 MeV/A Ne+U (HIGH MULT)
sum charges
30
70
110
150
"deuterons"
d + 3/2 t
30
70
110
150
100.
10.
1.0
0.1
dσ/dΩdE/A mb/sr MeV/A
0
100
200
MeV/A
XBL 822-7965

Fig. 2

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

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