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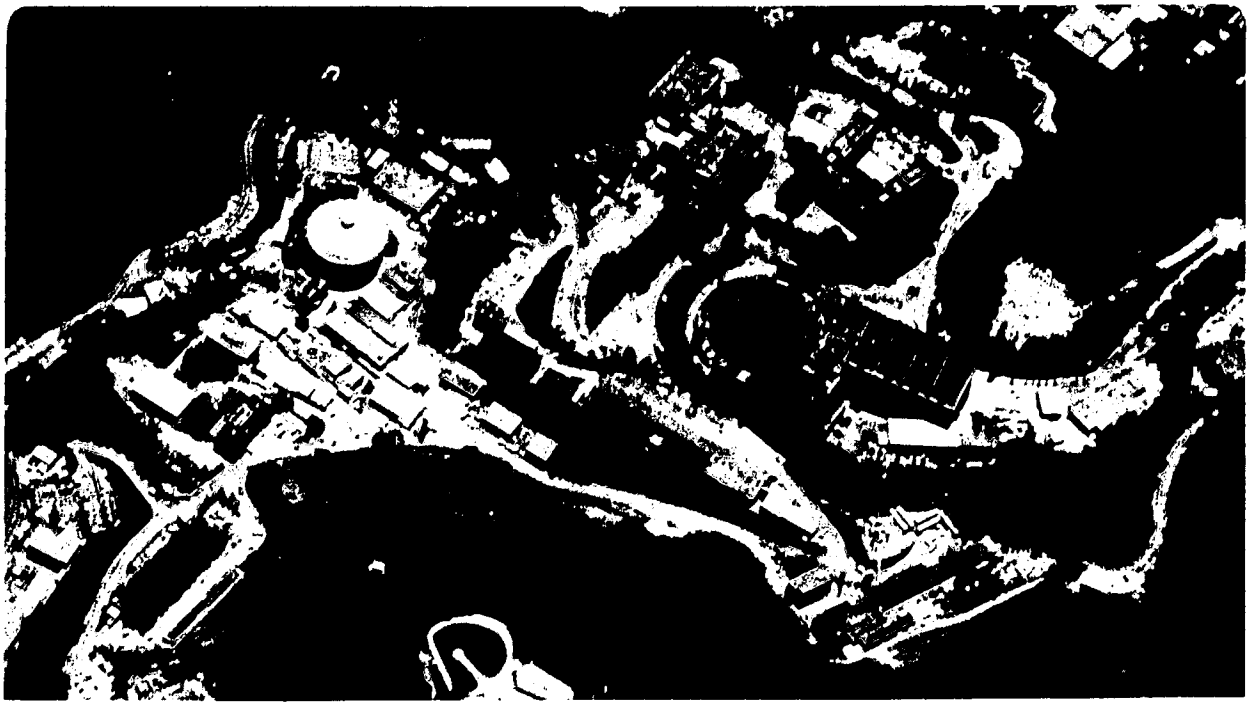

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LOW ENERGY SUPERGRAVITY: FALSE VACUA AND VACUOUS PREDICTIONS[1]

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Abstract

The coupling of supersymmetric gauge theories to N = 1 supergravity leads to a description of physics at the TeV scale in terms of an effective theory in which global supersymmetry is broken by a constrained set of soft operators. In the minimal model of this type, we show that the renormalization group scaling of the Higgs mass-squared operator can induce breaking of the weak interaction gauge symmetry even if all quarks and leptons are lighter than 20 GeV. In this scenario, the vacuum is unstable against decay into a deeper vacuum which violates electromagnetic and color gauge symmetries. However, numerical computations reveal a range of parameters for which the lifetime of the false vacuum is longer than the age of the universe.

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I. INTRODUCTION

The spontaneous breakdown of supersymmetry in supersymmetric gauge theories coupled to N = 1 supergravity[1,2] leads to an effective theory below the Planck scale in which global supersymmetry is explicitly broken by a constrained set of soft operators.[3-13] Although there is no consensus on the best way to incorporate grand unification into such a scheme, a unique minimal low energy model has recently emerged.[14-16] In this model, the physics of the TeV scale is described by an effective SU(3) × SU(2) × U(1) gauge theory in which the breaking of weak interaction gauge symmetry is induced by renormalization group scaling of the Higgs mass-squared operators. The price of this mechanism is either a heavy top quark ($m_t > 55$ GeV) or a fourth heavy generation of quarks and leptons. In this paper, we propose a variation of this mechanism in which the top quark may be as light as 20 GeV without the need for any heavier quarks and leptons. The price of the new mechanism is making the phenomenologically acceptable vacuum unstable against tunnelling into a vacuum in which all gauge symmetries, including color and electromagnetism, are broken. Numerical computations are used to demonstrate the existence of a range of parameters for which the lifetime for this vacuum decay process is greater than the age of the universe. Unfortunately, no firm predictions for new physics at experimentally accessible energies emerge in this allowed range of parameters.

The remainder of this introduction is devoted to the theoretical prejudices which motivate phenomenological supergravity. The soft explicit breaking of supersymmetry[17] provides a simple solution to the problem of giving weak interaction scale masses to the unobserved supersymmetric partners of the known particles.[18,19] Models coupled to N = 1 supergravity possess two advantages when compared with globally supersymmetric models in which soft supersymmetry breaking is inserted by hand. First, the breaking of supersymmetry in supergravity

models is spontaneous as is neccessary in any supersymmetric theory which hopes for the eventual inclusion of gravity. Second, models in which soft supersymmetry breaking is inserted by hand have little predictive power unless *ad hoc* constraints are imposed upon the soft operators. Indeed, even with the minimal set of chiral fields listed in Table I, there is a large number of possible soft operators. In models coupled to N = 1 supergravity, the form of the soft operators is determined in principle by the dynamics. Lacking the ability to deal with strong gravitational corrections at the Planck scale, some additional theoretical assumptions are still needed. When assumptions of this type are included, the number of parameters needed to characterize the soft breaking of supersymmetry can be reduced to four. Moreover, these assumptions are of a sufficiently general nature that the theory is not rendered wholly unappealing.

Globally supersymmetric models in which the breakdown of supersymmetry is spontaneous can be constructed using either the mechanism of O'Raifeartaigh[20,21] or that of Fayet and Iliopoulos.[22] The Fayet-Iliopoulos mechanism requires that the gauge group be extended to contain an additional U(1) factor, and it is difficult to implement this mechanism in realistic models. Models of the O'Raifeartaigh type are usually constructed by dividing the theory into a hidden sector in which supersymmetry is spontaneously broken at a scale Λ_{ss}, and an observable sector which includes the known particles. Supersymmetry breaking in the observable sector can then be generated by radiative corrections involving interactions which communicate between the sectors. When the fields of the hidden sector are integrated out, the result is an effective theory for the observable sector in which supersymmertry is softly broken at a scale $\Lambda_{ss}^{(eff)} < \Lambda_{ss}$. Examples of this with $\Lambda_{ss}^{(eff)} \sim 1$ Tev include early versions[20] with $\Lambda_{ss} \sim 100$ TeV, inverse hierarchy models[23-25] with $\Lambda_{ss} \sim 10^{11}$ GeV and models in which Λ_{ss} can be made arbitrarily large.[21] Realistic models of this type are not particularly appealing, in part

because the method of communication is both complicated and model dependent, and in part because the O'Raifeartaigh mechanism usually requires the presence of a symmetry (R-invariance) which forbids Majorana masses for gauge fermions. In supergravity models, the spontaneous breaking of supersymmetry occurs automatically without the need for any R-invariance. Furthermore, the analysis is simplified by the fact that the communication between sectors is mediated by the auxilliary fields of the supergravity multiplet and takes place at tree level in a model independent way.

In supergravity models, the two sector hypothesis is incorporated by splitting the superpotential of the theory into the sum of two terms: f_H which depends only on the chiral fields ϕ_H of the hidden sector and f_O which depends only on the observable chiral fields ϕ_O (including ordinary matter and Higgs fields). The scalar fields of the hidden sector typically have vevs of order of the Planck mass M_P which cause supersymmetry to be spontaneously broken at a scale Λ_{ss} determined by the parameters in f_H. These parameters may be chosen by hand to make the resulting gravitino mass,[26] $m_{3/2} = \Lambda_{ss}^2/M_P$, be of order of the weak interaction scale. The auxilliary fields of the supergravity multiplet couple to both sectors, and the elimination of these auxilliary fields produces non-renormalizable interactions between the two sectors. The hidden fields ϕ_H are conveniently taken to be neutral under all gauge symmetries; however, there will in general be non-renormalizable interactions between the ϕ_H and gauge fields . When the scalar components of the hidden fields are replaced by their vevs and all superheavy fields are integrated out, the resulting effective theory for the light observable fields and gauge fields will contain both the usual supersymmetric terms and soft operators which break supersymmetry at the effective scale $\Lambda_{ss}^{(eff)} \sim m_{3/2}$.

The soft operators obtained in this way are determined by the couplings of the hidden fields and by radiative corrections at the Planck scale, including those due to

gravity. In spite of this, it is sometimes possible to give the form of these operators without making detailed assumptions about either the hidden sector or the effects of gravitational radiative corrections. The non-renormalizable interactions of the hidden fields with gauge fields will generally lead to a soft Majorana mass term

$$\mathcal{L}_\lambda = -\frac{1}{2} M_0 \lambda_A \lambda_A + h.c. \qquad (1.1)$$

for the light gauge fermions λ_A. Also, the non-renormalizable interactions between the two sectors will produce soft operators in the scalar potential. In some cases, these soft scalar operators may be obtained[13] by dividing the effective superpotential of the low energy theory f_{LE} into pieces f_2 and f_3 which are respectively bilinear and trilinear in the light observable chiral fields. The soft scalar operators are then given by

$$V_{soft} = m_1^2 A_a^* A^a + \left[m_2 f_2(A^a) + m_3 f_3(A^a) + h.c. \right] \qquad (1.2)$$

where the A^a are the scalar components of the light observable fields. The coefficients m_1, m_2, m_3 and M_0 which set the scale $\Lambda_{ss}^{(eff)}$ of soft supersymmetry breaking are all forced to be of order $m_{3/2}$. In fact, when corrections from integrating out superheavy fields are ignored, the coefficient m_1 is actually equal to the gravitino mass $m_{3/2}$.

It is important to note that the simple forms (1.1) and (1.2) are valid only when additional assumptions are made. In writing the gauge fermion mass term (1.1), we imagine that the low energy theory is embedded in a grand unified model at some GUT scale below the Planck mass. This implies that the gaugino masses are equal at the GUT scale so that only the single parameter M_0 is needed to characterize this term. In general, the simple form (1.2) of the soft scalar operators is obtained only if the fundamental supergravity theory is invariant under U(N) rotations of the N chiral superfields ϕ_H and ϕ_O. This symmetry cannot be exact because it forbids all

gauge and superpotential couplings of the chiral fields along with the gaugino mass term (1.1). Naively, one might expect that the simple form (1.2) would be a good approximation if the theory possessed an approximate U(N) symmetry of this type, i.e., if all symmetry breaking terms were small in the fundamental theory at the Planck scale. To leading order in these small couplings the soft operators would then be characterized by the four parameters m_i and M_0. This result might also be expected if the theory possessed an approximate $U(N_O)$ symmetry acting only on the N_O observable fields ϕ_O. In this case, the couplings of the hidden fields ϕ_H would not be constrained, and the gaugino mass term (1.1) would not be restricted to be small. In the absence of any such symmetry, a more general set of soft scalar operators and a corresponding lack of predictive power seem likely to result.

Whether we appeal to an approximate U(N) symmetry or merely to its $U(N_O)$ subgroup, the basic problems with this approach are the same. In order that weak interaction symmetry breaking be induced by renormalization group scaling, it is usually neccessary for some U(N) or $U(N_O)$ symmetry breaking coupling to be relatively large at the Planck scale. This problem is somewhat eased by the fact that renormalization group scaling effects are enhanced by the large logarithm ($\ell n\ M_P/M_W$) of the ratio of mass scales, while corrections to the soft operators from integrating out superheavy fields have no such enhancement. However, we also note that, because grand unified supergravity models contain a large number of fields, it is not always easy to ensure that the GUT gauge coupling, which also violates the approximate symmetry, remains small at M_P. A second problem is associated with the presence of non-renormalizable interactions in supergravity. Although corrections to the soft operators which involve only gravitational interactions respect the approximate symmetry, there are also corrections which involve both small symmetry violating couplings and non-renormalizable interactions. These corrections are small when the divergences of the theory are cut off at the Planck

scale. However, because the theory is non-renormalizable, we are unable to complete the argument by removing the cut off in a sensible way. In spite of this, we will take the soft scalar operators to have the simple form given in Eq. (1.2). This is to be regarded as the minimum assumption needed to retain predictive power.

In the next section, we will describe the minimal model of supergravity at low energies. The presence of a heavy top quark has been shown to induce weak interaction scale symmetry breaking in this model. In the next section, we will also propose a new mechanism for obtaining this symmetry breaking. In this mechanism, the masses of quarks and leptons are not constrained, but the phenomenologically acceptable vacuum is unstable against tunnelling. In Section III, we study a similar vacuum decay problem in a toy model. In Section IV, the parameter space of the minimal model is analyzed in detail, and restrictions following from weak interaction symmetry breaking are imposed. In Section V, we present numerical results for the lifetime of the false vacuum of the minimal model and discuss the particle spectrum obtained in the allowed region of parameter space. Finally, two appendices contain a discussion of the renormalization group equations for the minimal model and a description of the numerical methods used to compute the lifetime of the false vacuum.

II. THE MINIMAL MODEL

A minimal model of low energy supergravity has recently been discussed by several authors.[14-16] The model is based on the standard low energy SU(3) × SU(2) × U(1) gauge group and contains three generations of quarks and leptons, along with two Higgs doublets, as chiral superfields. These fields are listed in Table I together with their SU(3) × SU(2) × U(1) transformation properties. Much of the attractiveness of this model stems from the fact that no gauge symmetries or fields beyond those required in any low energy supersymmetric theory are included. Indeed, there is little motivation for adding further chiral superfields to the low energy theory. For example, the addition of a gauge singlet superfield which couples to H and H′ usually[8,25,27] (but not always)[11] destroys any hope of understanding the hierarchy problem. Also, a light weak triplet coupled to H and H′ would violate the $\Delta I = 1/2$ pattern of weak interaction symmetry breaking.[14]

The low energy effective superpotential of the model consists only of the usual Yukawa couplings of quark and lepton superfields to the Higgs superfields, along with a mass term coupling the two Higgs doublets. Explicitly,

$$f_{LE} = \lambda_u H Q U + \lambda_D H' Q D + \lambda_E H' L E + m H H' \qquad (2.1)$$

in which generations indices, as well as color and SU(2) indices, have been suppressed. With the exceptions of the top quark Yukawa coupling and the mass parameter $m \sim O(m_{3/2})$, the parameters appearing here all contribute to the masses of the observed quarks ands leptons and are known to be small. Neglecting these small couplings, we have

$$f_{LE} \approx \lambda H Q T + m H H' \qquad (2.2)$$

in which Q and T respectively denote the quark doublet and charge - 2/3 antiquark singlet superfields of the third generation, while λ is the top quark Yukawa coupling. The supersymmetric terms of the scalar potential are then given by

$$V_{ss} = |\lambda QT + mH'|^2 + |\lambda HQ|^2 + |\lambda HT|^2 + |mH|^2$$
$$+ \frac{1}{2} g_3^2 \left[Q^\dagger T_\alpha Q - T^\dagger T_\alpha^* T + \cdots \right]^2$$
$$+ \frac{1}{2} g_2^2 \left[Q^\dagger \vec{\tau} Q + H^\dagger \vec{\tau} H + H'^\dagger \vec{\tau} H' + \cdots \right]^2$$
$$+ \frac{1}{2} g'^2 \left[\frac{1}{6}|Q|^2 - \frac{2}{3}|T|^2 + \frac{1}{2}|H|^2 - \frac{1}{2}|H'|^2 + \cdots \right]^2 \qquad (2.3)$$

where the dots represent contributions from matter fields other than Q and T, and in which T_α and τ denote the generators of SU(3) and SU(2), respectively. Also, we use the same symbol to denote a chiral superfield and its scalar component.

The low energy scalar potential also contains soft supersymmetry breaking terms arising from the spontaneous breakdown of supersymmetry in the hidden sector. As discussed in the introduction, these are given by

$$V_{soft} = m_Q^2 |Q|^2 + m_T^2 |T|^2 + m_H^2 |H|^2 + m_{H'}^2 |H'|^2 + \cdots$$
$$+ \left[A\lambda HQT + BmHH' + h.c. \right] \qquad (2.4)$$

in which the mass parameters A, B, m_Q, m_T, m_H and $m_{H'}$ are of order of $m_{3/2}$. The form of the soft scalar operators (1.2) at the Planck scale also provides us with the

boundary condition

$$m_Q^2 = m_T^2 = m_H^2 = m_{H'}^2 = \cdots = m_{3/2}^2 \qquad (2.5)$$

which is to be satisfied at renormalization points of order M_P. The remaining terms of the low energy Lagrangian are the usual supersymmetric terms and the soft gaugino mass terms

$$\mathcal{L}_\lambda = -\frac{1}{2} M_3 g_\alpha g_\alpha - \frac{1}{2} M_2 \vec{w}\cdot\vec{w} - \frac{1}{2} M_1 b b + h.c. \qquad (2.6)$$

involving the SU(3), SU(2) and U(1) gauge fermions g_α, $\vec{w}$ and b, respectively. The hypothesis of grand unification implies the boundary condition

$$M_3 = M_2 = M_1 = M_0 \sim O(m_{3/2}) \qquad (2.7)$$

at the GUT scale. By an R-transformation, we may choose the parameter M_0 to be real . From the form of the one loop renormalization group equations for the M_i, it follows that the gaugino masses will all be real at low energies. By redefining the phases of the chiral superfields, the coefficients, $A\lambda$ and Bm, of the soft scalar operators may also be chosen real. This leaves the phases of m and λ as physical CP-violating phases. However, the existence of these phases is irrelevant for the purposes of this paper, and we will take all of the parameters to be real.

The appearance of the mass parameter m of order of $m_{3/2}$ in the low energy effective superpotential deserves some comment. The quartic terms of the scalar potential (2.3) vanish when the neutral scalar components of H and H' have equal vevs and all other vevs vanish. Lacking a gauge singlet or weak triplet superfield to provide further quartic terms involving H and H', this implies that weak interaction

symmetry breaking with <H> and <H'> non-zero can occur at tree level only if the scalar potential contains a mass term coupling the scalar components of H and H'. Here such a term has been obtained in (2.4) by including mHH' in the superpotential. Although the natural scale for dimensionful parameters appearing in the observable sector superpotential is the superheavy scale, it is possible for the low energy effective superpotential to contain mass parameters of order $m_{3/2}$ in a natural way. Indeed, $m_{3/2}$ sets the natural scale for all effects of the hidden sector upon the observable sector. Such effects could produce the mHH' term in f_{LE} either by causing shifts of order $m_{3/2}$ in the vevs of superheavy scalars[13] or by radiative corrections.[14,27] The mHH' term in f_{LE} is not strictly neccessary provided that radiative corrections produce a soft HH' mass term in the scalar potential. However, we prefer to include it in the superpotential for reasons of generality.

From the soft operators (2.4) and (2.5), it is apparent that supergravity models can easily succeed in giving weak interaction scale masses to squarks, sleptons and gauginos. Unfortunately, the soft operators also give large positive masses-squared to the Higgs doublets, and this makes the breaking of weak interaction gauge symmetry somewhat difficult. Several authors have discussed methods for overcoming this obstacle.[14,15,16,27] A particularly simple solution relies upon the fact that the boundary condition (2.5) need be satisfied only at the Planck scale, and that large renormalization group scaling effects can produce a negative value for m_H^2 at low energies. In Appendix A, we discuss the renormalization group equations for the parameters of the minimal model. Here we concentrate on the scaling of m_H^2 which is given below the GUT scale M_G by

$$\frac{d}{dt}\begin{pmatrix} m_H^2 \\ m_T^2 \\ m_Q^2 \end{pmatrix} = -\frac{2}{\pi}\sum_i \alpha_i T_{ia}^2 M_i^2 + \frac{\lambda^2}{8\pi^2}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}\left[m_H^2 + m_T^2 + m_Q^2 + A^2\right] \tag{2.8}$$

Here we neglect the scaling between M_G and M_P and take $t = \ln(\mu/M_G)$ for a renormalization point μ. Also, α_i and M_i are the gauge coupling and gaugino mass for the ith group factor, and the T_{ia}^2 are the quadratic Casimirs of H, T and Q under this factor. In previous treatments, the $\lambda^2 m^2$ terms of (2.8) were used to obtain a negative value for m_H^2 at low energies. Because this term involes the top quark Yukawa coupling, the use of this mechanism sets a lower bound ($m_t > 55$ GeV) on the top quark mass.[14,15]

The purpose of this paper is to show that weak interaction symmetry breaking can be obtained in the minimal model even if the top quark is as light as 20 GeV. Indeed, because of the presence of the $\lambda^2 A^2$ term in (2.8), the Yukawa coupling λ can be quite small if A is sufficiently large. In Section IV, we will demonstrate that there are sizeable regions of parameter space in which λA is large enough to produce weak interaction symmetry breaking without becoming non-perturbative. The parameter A governs the size of cubic terms in the scalar potential. Consequently, for large values of A, the global minimum of the potential will have non-zero vevs for squark fields.[28] In Section V, we will present numerical results which show that there is a significant portion of the parameter space in which the lifetime for tunnelling into this phenomenologically disastrous vacuum is longer than the age of the universe.

Whether it is large λ or large λA which drives m_H^2 negative, the amount of renormalization group scaling needed depends on the corrections to the boundary condition (2.5). Aside from corrections at the Planck scale, this condition is also corrected by effects at the GUT scale. In globally supersymmetric theories, there are no positive mass-squared terms for H and H' at tree level, and it is possible to obtain weak interaction symmetry breaking from radiative corrections at the GUT scale without the need for renormalization group scaling.[27] However, in supergravity theories, these effects are usually unimportant. For example, in "missing partners"[29] models, the color triplet Higgs of SU(5) are made superheavy, while

keeping the weak doublets light, by adding the terms

$$\lambda H R S + \lambda' H' R S' \qquad (2.9)$$

to the observable sector superpotential. Here R, S and S′ transform as 75, $\overline{50}$ and 50 under SU(5). The contribution of one loop graphs such as those of Fig. 1 to the Higgs mass-squared operators is then

$$\Delta V = \frac{C^2 \lambda^2 m_{3/2}^2}{16\pi^2} |H|^2 + \frac{C'^2 \lambda'^2 m_{3/2}^2}{16\pi^2} |H'|^2 \qquad (2.10)$$

where C and C′ are unknown parameters of order unity from the couplings of the hidden sector. Because such contributions contain no large logarithms, they are formally negligible when compared with the renormalization group scaling of the tree level terms which exist in supergravity theories. As a caveat, we note that it is possible for two loop effects at the GUT scale to be enhanced by ratios of superheavy masses. Lacking a specific unified model in which to address this question, we will assume that corrections at the GUT scale are small compared with those due to renormalization group scaling.

III. A TOY TUNNEL

The scalar potential of the minimal model is rather complicated, and this obfuscates the analysis of the tunnelling problem which exists when the parameter A is large. To gain insight into this problem, we will consider the analogous problem in a toy model. This toy model is obtained by ignoring the mass term in the low energy superpotential of the minimal model and by neglecting the effects of renormalization group scaling. The terms of the scalar potential depending on the fields H, Q and T are then

$$\begin{aligned} V_{toy} = {} & |\lambda Q T|^2 + |\lambda H Q|^2 + |\lambda H T|^2 \\ & + \left[A m_{3/2} \lambda H Q T + h.c. \right] + m_{3/2}^2 \left(|H|^2 + |Q|^2 + |T|^2 \right) \\ & + (\text{D terms from gauge interactions}) \end{aligned} \qquad (3.1)$$

By redefining the fields, all parameters may be made real. The parameters A and $m_{3/2}$ describe the soft supersymmetry breaking obtained from the hidden sector, and, for the purposes of this section, we have chosen to make A dimensionless.

Because $m_{3/2}^2 > 0$, V_{toy} possesses a local minimum at which all fields vanish. For this reason, the toy model is unrealistic. In the minimal model, this is replaced by a local minimum in which only H and H′ have non-zero vevs. For A > 3, there is a deeper minimum in which

$$|Q| = |T| = |H| = v \qquad (3.2)$$

with

$$v = \frac{m_{3/2}}{4\lambda} \left[A + \sqrt{A^2 - 8} \right]$$

The field directions in group space make each D term vanish in this minimum, and

the phases of the fields make the trilinear term real and negative. The potential along this direction is illustrated in Fig. 2. For $A < 3$, the origin is the true vacuum, but, for $A > 3$, it is a false vacuum.

We now take $A > 3$ and consider the lifetime for the decay of the false vacuum at the origin. The tunnelling rate per unit volume is given by [30] $\Gamma/V = M^4 e^{-S}$ in which M is estimated to be of order of 100 GeV. The exponent S is the Euclidean action of the O(4) symmetric field configuration (the bounce) which extremizes the action and which takes the form of a bubble of true vacuum. In the standard cosmology, the probability that a given point will now be in the true vacuum phase is given roughly by

$$P_{True} = 1 - \exp\left(-\frac{\Gamma}{V} T^4\right) \tag{3.3}$$

Taking the age of the universe T as 10^{10} years, we find that $S > 400$ is sufficient to make this probability negligible.

In the toy model, the problem of computing S reduces to a problem involving a single scalar field. Fields other than H, Q and T vanish in both minima and are also irrelevant for the tunnelling problem. It is convenient to make use of this by writing

$$|Q| = |T| = |H| = \frac{m_{3/2}}{\lambda}\phi \tag{3.4}$$

with the field directions and phases chosen as in the true vacuum. After rescaling the Euclidean co-ordinates by a factor of $m_{3/2}$, the action takes the form

$$S = \frac{12\pi^2}{\lambda^2}\int_0^\infty r^3 dr \left\{\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + \frac{1}{2}\phi^2 - \frac{A}{3}\phi^3 + \frac{1}{2}\phi^4\right\} \tag{3.5}$$

in which r is the radial co-ordinate of Euclidean space. We have numerically evaluated S as a function of A using techniques described in Appendix B. The result for $S = \lambda^2\hat{S}$ is displayed in Fig. 3 together with the thinwall approximation,[30]

$$S_{Thinwall} = \frac{27\pi^2}{16\lambda^2}\frac{1}{(A-3)^3} \tag{3.6}$$

which becomes valid as $A \to 3$. Although our results agree with the thinwall approximation in this limit, the approximation is accurate only for A extremely close to 3. For example, for

$A = 3.1$, the approximation gives an action which is only half of the correct result, while for

$A = 4$ it is wrong by an order of magnitude. In Fig. 4, the λ - A parameter space of the toy model has been divided into two regions. The stable region has $S > 400$ and would be phenomenologically acceptable if the Higgs fields had non-zero vevs in the false vacuum. In the unstable region, $S < 400$ and tunnelling occurs over cosmological times.

Equation (3.5) shows that, in the toy model, the action S is proportional to $1/\lambda^2$. Unfortunately, the argument leading to this simple dependence breaks down in the minimal model of Section II. Indeed, in the minimal model, the vevs of H and H′ are non-zero in the false vacuum, and the soft $\lambda AHQT$ term is then important in determining the top squark masses-squared. Also, because of the mHH′ term in the superpotential, the D terms will not vanish in either vacuum. In spite of these complications, there is a simple lesson to be learned from the toy model. When all Yukawa couplings are retained, the potential of the minimal model may possess many deeper local minima into which tunnelling may occur. However, the height of the barrier which inhibits tunnelling into such a minimum is roughly proportional to

$1/\lambda^2_{min}$ where λ_{min} denotes the smallest of the Yukawa couplings for the squark and slepton fields which are non-zero in the deeper minimum. For the deeper minimum in which only the top squarks and Higgs scalars are non-zero, the cubic $\lambda AHQT$ term acts in an important way to lower the barrier by reducing the top squark mass-squared in the false vacuum. This effect does not occur whenever squark or slepton fields other than the top squarks acquire vevs in the deeper minimum. It follows that tunnelling into such a minimum is suppressed by a large barrier and that we need only consider tunnelling into the minimum where H, H′, Q and T are the only non-zero fields. It is for this reason that we may ignore the Yukawa couplings of other quark and lepton superfields in discussing the decay of the false vacuum.

IV. PARAMETERS FOR THE MINIMAL MODEL

In this section, we discuss the effect of renormalization group scaling upon the parameters of the minimal model and impose constraints which follow from the need to obtain weak interaction symmetry breaking at low energies. The minimal model was described in Section II, and the renormalization group equations for the parameters of the model are given in Appendix A. The scaling of the gauge couplings determines the GUT scale to be $M_G = 2 \times 10^{16}$ GeV. We will ignore renormalizaton group scaling effects occurring between this scale and the Planck scale and, therefore, impose the boundary the conditions (2.5) and (2.7) at M_G.

The final results presented in this section and in Section V are based upon numerically integrating the renormalization group equations. However, valuable insights may be obtained from the approximate solutions derived in Appendix A. These approximations are valid in the range of parameters which we find most interesting: $\lambda^2 < g_i^2$, $\lambda^2 A < g_i^2 M_i$ and $m_a^2 < A^2$. We find that the parameters λ and m of the superpotential are increased at low energies by factors of 3.7 and 1.4, respectively, by gauge interaction contributions. A gaugino mass M_0 at the GUT scale similarly increases all scalar masses-squared m_a^2 at low energies by the amounts given in Table II. For example, the mass-squared of a left-handed squark doublet is increased by 7.1 M_0^2, while that of a left-handed slepton doublet is increased by only .53 M_0^2. This is likely to make the average squark somewhat heavier than the average slepton. The gaugino mass also increases the A and B parameters by $4M_0$ and $.6M_0$, respectively. It is clear that, even if A is small at the GUT scale, heavy gauginos can drive A sufficiently large at low energies for the phenomenologically acceptable vacuum to become a false vacuum. The gaugino masses M_i scale in precisely the same way as the corresponding gauge couplings α_i so that, for example, M_3/M_1 is roughly 7 at low energies. A recent cosmological limit[31]

indicates that the photino should be heavier than O(10 GeV), if it is stable[32] and a mass eigenstate. Taking M_1 to characterize the photino mass, we find that the gluino must be heavier than roughly 70 GeV. However, we note that the photino will be a mass eigenstate only if $M_1 = M_2$ while, for the parameters considered here, $M_2 = 2M_1$. Because of this, the actual constraint will be on a complicated function of the masses and mixings of the heavy neutral fermions.

At low energies, the scalar potential must have a local minimum at which only H and H′ have non-zero vevs. This restricts the parameters of the model in two ways. The first set of constraints is found by assuming that the minimum takes the form

$$\langle H \rangle = \begin{pmatrix} 0 \\ h \end{pmatrix} , \qquad \langle H' \rangle = \begin{pmatrix} h' \\ 0 \end{pmatrix} \tag{4.1}$$

with all other vevs vanishing. The potential then reduces to

$$V = \frac{g_2^2 + g'^2}{8} (h^2 - h'^2)^2 + (m_H^2 + m^2) h^2 + (m_{H'}^2 + m^2) h'^2 - 2Bm\, hh' \tag{4.2}$$

in which we have used the fact that the minimum occurs for real h and h′. The potential (4.2) will be unbounded from below in the direction h = h′ unless

$$m_H^2 + m_{H'}^2 + 2m^2 > 2mB \tag{4.3}$$

Requiring that the minimum have h and h′ non-zero leads to the additional constraint

$$(m_H^2 + m^2)(m_{H'}^2 + m^2) < m^2 B^2 \tag{4.4}$$

When these conditions are satisfied, the minimum occurs with h = v cosθ and

h′ = v sin θ where

$$\sin 2\theta = \frac{2mB}{m_H^2 + m_{H'}^2 + 2m^2} \tag{4.5}$$

and

$$v^2 = \frac{2}{g_2^2 + g'^2} \left[\left| \frac{m_H^2 - m_{H'}^2}{\cos 2\theta} \right| - m_H^2 - m_{H'}^2 - 2m^2 \right]$$

These equations determine $m_{3/2}$ when $M_W = g_2 v/\sqrt{2}$ is required to agree the observed value of the W-boson mass.[33]

The regions of parameter space ruled out by the constraints (4.3) and (4.4) are shown in Fig. 5. The dashed lines represent typical renormalization group trajectories which start with $m_H^2 = m_{H'}^2$ at the GUT scale and which evolve towards the left. These starting points lie in region I for which the vevs h and h′ vanish. For a given starting point, there is a minimum amount of scaling needed to reach regions II and III in which these vevs become non-zero. Because the trajectory speed is roughly proportional to $\lambda^2 A^2$, this sets a constraint on λA. If λA is too large, the trajectory will reach the boundary of region IV in which the tree level potential becomes unbounded. In this case, the weak interaction scale is not determined at tree level, but, instead, is determined by the radiative corrections to the potential which prevent it from being unbounded. Although this scenario requires a judicious choice of parameters at the GUT scale to obtain the observed weak interaction scale, it is interesting because it may lead to the supersymmetric partners of the known particles being light.[16]

A second constraint upon the parameters is obtained by requiring that all scalars have positive masses-squared when the vevs of H and H′ are given by Equations (4.1) and (4.5). In practice, the only constraint is due to the top squarks. This is because the trilinear term proportional to A leads to a large splitting between the top squark mass eigenstates. For some, but not all, values of the parameters, the

trajectories of Fig. 5 will enter region III in which one of the top squark masses-squared is negative before the boundary of region IV is reached. Of the regions shown here, only region II possesses an acceptable local minimum in which weak interaction symmetry is broken at tree level by non-zero vevs for H and H′ and in which all scalars have positive masses-squared.

The minimal model of low energy supergravity has five independent parameters at the GUT scale, m_0, λ_0, A_0, B_0 and M_0. These must be chosen so that renormalization group scaling leads to m_H^2 and $m_{H'}^2$ having values in region II at low energies. This constraint has a complicated form which we have analyzed numerically. For initial choices of m_0, B_0 and M_0, we display this constraint in the form of an allowed region in a plot of λ_0 against A_0. Figure 6 shows one such plot for $M_0 = m_{3/2}$, $m_0 = .2\ m_{3/2}$ and $B_0 = 1/2\ m_{3/2}$. The plot is divided into four regions which correspond to the regions I-IV of Fig. 5-- i.e., region I has $h = h' = 0$, region III has a negative top squark mass-squared, and region IV has an unbounded tree level potential. The allowed area (region II) takes the form of a band in which $\lambda_0 A_0$ is roughly constant for large A_0. This is the result of requiring that the hierarchy obtained by scaling the gauge couplings agree with that obtained by scaling the Higgs mass-squared. Note that the product $\lambda_0 A_0$ is perturbative throughout the allowed region. This is also true of the low energy parameter λA.

For larger values of M_0, the scaling of $m_{H'}^2$ becomes more important, and the trajectory will cross the allowed region with a larger slope, as shown by the upper trajectory of Fig.5. This effect produces a pronounced broadening of the allowed region of Fig. 6 for large values of the gaugino mass. Larger values of M_0 also tend to increase m_H^2 at low energies so that larger values of $\lambda_0 A_0$ are needed to induce weak interaction symmetry breaking. The gaugino mass also contributes to the scaling of A, and these effects lead to the low energy value of λA becoming non-perturbative in the allowed region for values of M_0 larger than roughly 3 $m_{3/2}$.

The opposite effect is illustrated by Fig. 7 for $M_0 = 1/3\ m_{3/2}$, $m_0 = .2\ m_{3/2}$ and $B_0 = 2\ m_{3/2}$. The allowed region is quite narrow, and the low energy values of λA in this region are smaller. These effects are enhanced by the larger value of B_0. As B increases, the regions of Fig. 5 move closer to the starting point of the trajectory. This contributes to the narrowing of the allowed region and the decrease in λA shown in Fig. 7.

The effect of larger m_0 is shown in Fig. 8 for $M_0 = m_{3/2}$, $m_0 = m_{3/2}$ and $B_0 = 1/2\ m_{3/2}$. An increase in this parameter moves the starting point of the trajectory in Fig. 5 further from the allowed region. This produces the wider regions shown in Fig. 8 and also results in a need for larger values of $\lambda_0 A_0$ to obtain weak interaction symmetry breaking. Again this can lead to non-perturbative values of λA at low energies. However, in contrast to the case of larger gaugino masses, the increase in m_0 does not enhance the soft squark masses at low energies. As a result the cubic $\lambda AHQT$ term can easily lead to a negative top squark mass-squared in the false vacuum, and region III which is ruled out for this reason becomes much wider.

Figure 9 displays a final example in which $M_0 = m_{3/2}$, $m_0 = .5\ m_{3/2}$ and $B_0 = 2m_{3/2}$. This plot is included because it contains particularly interesting values of the low energy parameters. Also, the effects due to large values of B_0 may be isolated from those due to larger values of M_0 by comparing this plot with Figs. 6 and 7, respectively.

V. VACUUM DECAY IN THE MINIMAL MODEL

In this section, we consider the decay of the phenomenologically acceptable vacuum which exists for parameters in region II of Fig 6-9. To conclude the paper, we remark upon the particle spectrum which is obtained in this region.

In Section III, we argued that only the top quark Yukawa coupling was relevant for the decay of the acceptable vacuum and that this decay could thus be discussed using the potential given in Eq (2.3) and (2.4). For small values of the A parameter, the acceptable vacuum is the global minimum of this potential. This occurs in subregion IIA of Figures 6-9. In this subregion, the breaking of weak interaction gauge symmetry is primarily due to the effect of the top quark Yukawa coupling upon the scaling of the Higgs mass-squared, and it is for this subregion that a lower bound of 55 GeV was derived for the top quark mass.[14,15]

In subregions IIB and C, we must investigate the stability of the acceptable vacuum by evaluating the tunnelling action S defined in Section III. To simplify this evaluation, we retain only the neutral components of H and H′. We also keep only a single color component of T and the same color component of the charge 2/3 scalars in Q. Finally, these four scalar fields are taken to be real. The neglect of all other fields is justified because they vanish in both the acceptable false vacuum and the deeper true vacuum. Also, we find that the fields equations in Appendix B which determine S can be satisfied when the neglected fields vanish. In spite of this reduction of the degrees of freedom, the numerical evaluation of S is far from straightforward, and we have included Appendix B to describe the method used. The results of this evaluation are shown in Figs. 6, 8 and 9 in the form of the boundary between subregions IIB and C. In Subregion IIB, the acceptable vacuum is cosmologically long lived ($S > 400$). subregion IIC is not allowed because $S < 400$ so that tunnelling occurs readily over cosmological times. The corresponding boundary is not shown in Fig. 7 because the

combined region (IIB and C) is already quite thin.

Although the evaluation of S is quite complicated, we may qualitatively interpret the results in a simple way. At the boundary with subregion IIA, the energy difference between the two vacua vanishes, and tunnelling is suppressed as in the thinwall approximation. In region III, the soft $\lambda AHQT$ term of the potential produces a negative top squark mass-squared in the false vacuum. Near the boundary of this region, the acceptable vacuum is barely stable classically, and tunnelling occurs readily. Indeed it is because of the importance of this cubic term that the boundary between subregions IIB and IIC occurs for roughly constant λA and parallels the boundaries of regions II and III.

The neglect of finite temperature effects in these tunnelling calculations deserves a word of comment. At high temperatures, the scalar masses-squared receive additional positive contributions proportional to the temperature squared. These trap the theory in a phase with all low energy gauge symmetries unbroken. As the universe cools below temperatures of order of $m_{3/2}$, there will be a second order phase transition to the most symmetric minimum of the potential to occur. This places the theory in the acceptable vacuum with only H and H′ non-zero. If this vacuum is sufficiently stable, the universe will supercool to quite low temperatures prior to the demise of the false vacuum.

For some values of M_0, m_0 and B_0, the area of subregion IIB in which the acceptable vacuum is cosmologically long lived is comparable to that of subregion IIA in which it is absolutely stable (Figs. 6 and 9). For other choices of these parameters, it is much smaller (Fig.7). Although region IIB constitutes a significant portion of the total allowed region, our interest in it is not entirely due to its size. Instead, we find this region interesting because it illustrates a valuable, if discouraging, point about the vacuous predictive power of supergravity at low energies. Throughout much of region IIB, many of the phenomenologically interesting particles have masses far too

heavy to be experimentally accessible in the near future. Others have masses which vary so widely across the region that little can be said about the possibility of their detection. In no case do we find a clear signal which could be used as evidence for or against the model. To conclude the paper, we discuss examples of this in the form of tabulated spectra (Table III) for points (a-e) of Figs. 7-9.

Point (a) on Fig. 7 is symptomatic of our frustration. For this point, the only new particles to be discovered in the near future are those of the standard model. The top quark mass (33 GeV) at this point shows that the bound obtained in region IIA is violated in the new region. The only supersymmetric partners lighter than 600 GeV are the linear combinations of charged winos and higgsinos (w/hinos) and the linear combinations of the neutral bino, wino and higgsinos (b/w/hinos). Even the lightest of these (at 95 GeV and 87 GeV, respectively) will not be particularly easy to detect, and this point of parameter space will remain experimentally indistinguishable from the standard model for many years. Point (b) of Fig.7 is somewhat more promising - - there is a charged w/hino to be discovered at 33 GeV. Unfortunately, this point is probably ruled out by the presence of a top squark at 3 GeV. Also, although the energy difference between the vacua is small, the false vacuum is not cosmologically long lived. Indeed, we have shown this point to illustrate the effectiveness of the cubic $\lambda AHQT$ term in lowering the barrier which inhibits tunnelling. Even more discouraging is the specrum obtained at point (c) of Fig 8. At this point, the false vacuum is cosmologically long lived, and the only supersymmertric partner lighter than a TeV is a single b/w/hino.

Much more interesting are points (d) and (e) on Fig. 9. These have cosmologically long lived false vacua. At point (d), there is a relatively light top squark (102 GeV) and the slepton masses are all less than 100 GeV. In addition, the lightest charged w/hino (at 22 GeV) lies near the experimental limit. The spectrum varies smoothly between points (d) and (e) and, in particular, there is an

experimentally accessible charged w/hino in most of this region. We also note that the top quark mass is only 18 GeV at point (e). This indicates that the only lower bound on this mass in region IIB is the current experimental limit. On a more positive note, should be remarked that the entire spectrum is determined by only five parameters.

In this paper, we have considered the possiblility that low energy supergravity lives in a false vacuum and have found a significant region of parameter space in which this false vacuum is cosmologically stable. In portions of this region, the low energy parameter λA is non-perturbative; and, in other portions, the neglect of the bottom quark Yukawa coupling is unrealistic. However, the main conclusions of the paper would be unaltered if these portions were excised from the allowed region. Indeed, most of the allowed regions shown in Fig. 6-9, including points (a-e), do not suffer from these objections. The examples of points (a-e) considered above reveal that even the most minimal of supergravity theories leads to no firm predictions at low energies. In this regard, it is significant that region IIB allows for the possibility of a light top quark. The only hopeful signal for supersymmetry in this region is the charged w/hino which is lighter than the W-boson for some, but not all, allowed values of the parameters. This particle could be produced pairwise in e^+e^- annihilation or, possibly, in the decay of a W-boson to a w/hino and a b/w/hino. The specific case of this decay in which the b/w/hino is the photino was shown to have a branching ratio of a few percent in Ref. 34. A very small portion of region IIB is ruled out because this charged w/hino is the lightest supersymmetric partner and is then absolutely stable.

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APPENDIX A. THE RENORMALIZATION GROUP EQUATIONS

The one loop renormalization group (RG) equations for the minimal model of Sections II, IV and V are well known[14,15,19], and the relevant equations are repeated in this appendix. We are primarily interested in regions of parameter space in which the gaugino masses M_i and the A parameter are large, while the top quark Yukawa coupling λ is quite small. In this limit ($\lambda^2 < g_i^2$, $\lambda^2 A < g_i^2 M_i$ and $m_a^2 < A^2$ where g_i denotes the gauge couplings and m_a^2 are the coefficients of the scalar mass-squared operators), we may give analytic solutions to the RG equations. These solutions are extremely helpful in understanding the evolution of the parameters as we scale down to low energies. Although the analytic approximations given here are often quite accurate, the results discussed in the text are based upon numerical integration of the RG equations.

For three matter generations, the three gauge couplings α_i scale according to

$$\frac{d\alpha_i}{dt} = \frac{b_i}{2\pi}\alpha_i^2 \qquad (A.1)$$

where $b_1 = 33/5$, $b_2 = 1$ and $b_3 = -3$ for U(1), SU(2) and SU(3), respectively. For a renormalization point μ, we define $t = \ell n(\mu/M_G)$ where M_G is the unification scale. Finally, we warn the reader that $\alpha_1 = g_1^2/4\pi = 5/3\, g'^2/4\pi$ is normalized so that $\alpha_i(M_G) = \alpha_G$. Inputting the values $\alpha_S = .12$ and $\alpha = 1/128$ for the strong and electromagnetic couplings at the weak scale $M_W = 85$ GeV, we find that $\sin^2\theta_W = .23$ at M_W, a unification scale $M_G = 2 \times 10^{16}$ GeV and $\alpha_i(M_G) \doteq \alpha_G = 1/24$. We will ignore the scaling of paramerters from M_P to M_G because this would require using a particular unified model and because the effects are expected to be small.

The gaugino masses evolve in a manner identical to that for the gauge couplings:

$$\frac{d}{dt}\left(\frac{M_i}{\alpha_i}\right) = 0 \qquad \text{with} \qquad M_i(M_G) = M_0 \qquad (A.2)$$

The solution is $M_i/M_0 = \alpha_i/\alpha_G = .41$, .82, and 2.9 for U(1), SU(2) and SU(3), respectively.

There is one other set of RG equations which can be solved exactly, those for the soft mass terms of scalars other than H, Q and T. This is because the Yukawa couplings of these scalars can be neglected so that these RG equations are

$$\frac{dm_a^2}{dt} = -\frac{2}{\pi}\sum_i \alpha_i T_{ia}^2 M_i^2 \qquad (A.3)$$

where $T_{ia}{}^2 = \Sigma\, T_{ia}{}^A T_{ia}{}^A$ is the quadratic Casimir of the ath scalar with respect to the ith group factor. It is significant that there are no $\alpha_i m_a{}^2$ contributions. These gauge contributions also cancel in the RG equations for A and B. Using the boundary condition $m_a{}^2(M_G) = m_{3/2}{}^2$, the solution to (A3) is

$$\begin{aligned} m_a^2 - m_{3/2}^2 &= -\sum_i \frac{2}{b_i} T_{ia}^2 (M_i^2 - M_0^2) \\ &= \sum_i \Delta_{ia} M_0^2 \end{aligned} \qquad (A.4)$$

The amounts Δ_{ia} by which these parameters are increased are given in Table II. It is clear that even a moderate gaugino mass is likely to make all squarks somewhat heavier than the sleptons.

The RG scaling of the top quark Yukawas coupling is given by

$$\frac{d\lambda}{dt} = \frac{\lambda}{4\pi}\left[-\frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{5}\alpha_1 + \frac{6\lambda^2}{4\pi}\right] \qquad (A.5)$$

Neglecting λ^2 compared with $g_i{}^2$, we find that λ is increased from its value λ_0 at M_G by

the factor

$$\frac{\lambda}{\lambda_0} = \left(\frac{\alpha_3}{\alpha_G}\right)^{8/9}\left(\frac{\alpha_G}{\alpha_2}\right)^{3/2}\left(\frac{\alpha_G}{\alpha_1}\right)^{13/198} = 3.7 \qquad (A.6)$$

Of course this result may not be used when λ becomes large since the λ^2 term will then become important. Indeed, for large λ, there is a fixed point at which the right hand side of (A5) vanishes. This gives an upper bound of $m_t < 200$ GeV.

For the other superpotential parameter m, we have

$$\frac{dm}{dt} = \frac{m}{4\pi}\left[-3\alpha_2 - \frac{3}{5}\alpha_1 + \frac{3\lambda^2}{4\pi}\right] \qquad (A.7)$$

This has a similar approximate solution

$$\frac{m}{m_0} = \left(\frac{\alpha_G}{\alpha_2}\right)^{3/2}\left(\frac{\alpha_G}{\alpha_1}\right)^{1/22} = 1.4 \qquad (A.8)$$

The A and B parameters scale according to

$$\frac{dA}{dt} = \frac{1}{4\pi}\left[-\frac{32}{3}\alpha_3 M_3 - 6\alpha_2 M_2 - \frac{26}{15}\alpha_1 M_1 + \frac{12\lambda^2 A}{4\pi}\right] \qquad (A.9)$$

and

$$\frac{dB}{dt} = \frac{1}{4\pi}\left[-6\alpha_2 M_2 - \frac{6}{5}\alpha_1 M_1 + \frac{6\lambda^2 A}{4\pi}\right] \qquad (A.10)$$

(The last term of Eq. (A9) differs from that given in Ref. [19]).Neglecting $\lambda^2 A$ compared to $g_i{}^2 M_i$, we find that these parameters increase by the amounts

$$A - A_0 = \frac{16}{9}(M_3 - M_0) - 3(M_2 - M_0) - \frac{13}{99}(M_1 - M_0)$$

and

$$= 4 M_0 \qquad (A.11)$$

$$B - B_0 = -3(M_2 - M_0) - \frac{1}{11}(M_1 - M_0) = .6\, M_0 \qquad (A.12)$$

For large values of M_0, the A parameter can become important at low energies even if it is small at the superheavy scale.

The only remaining equation is for the soft masses of H, T and Q. In this equation,

$$\frac{d}{dt}\begin{pmatrix} m_H^2 \\ m_T^2 \\ m_Q^2 \end{pmatrix} = -\frac{2}{\pi}\sum_i \alpha_i T_{ia}^2 M_i^2 + \frac{\lambda^2}{8\pi^2}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}\left[m_H^2 + m_T^2 + m_Q^2 + A^2\right] \qquad (A.13)$$

we neglect m_a^2 with respect to A^2. The equations then decouple, and the contribution of the first term is given in Eq. (A4). In the last term, we retain only the effects of the SU(3) gauge interactions upon λ and A. This produces an additional contributon to $m_a^2 - m_{3/2}^2$ of

$$-\frac{3\lambda_0^2 A_0^2}{4\pi\alpha_G}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}\left\{\frac{(1-\xi)^2}{7}\left[\left(\frac{\alpha_3}{\alpha_G}\right)^{7/9} - 1\right] + \frac{\xi(1-\xi)}{8}\left[\left(\frac{\alpha_3}{\alpha_G}\right)^{\frac{16}{9}} - 1\right] + \frac{\xi^2}{25}\left[\left(\frac{\alpha_3}{\alpha_G}\right)^{\frac{25}{9}} - 1\right]\right\} \qquad (A.14)$$

in which $\xi = 16\, M_0/9\, A_0$. It is this contribution which is responsible for the breaking of weak interactions at low energies. Collecting the contributions, we find that numerically

$$m_H^2 = m_{3/2}^2 + .53\, M_0^2 - 3\lambda_0^2 A_0^2\left[1.0 + 1.9\xi + 1.1\xi^2\right]$$

$$m_T^2 = m_{3/2}^2 + 6.7\, M_0^2 - 2\lambda_0^2 A_0^2\left[1.0 + 1.9\xi + 1.1\xi^2\right]$$

$$m_Q^2 = m_{3/2}^2 + 7.1\, M_0^2 - \lambda_0^2 A_0^2\left[1.0 + 1.9\xi + 1.1\xi^2\right] \qquad (A.15)$$

The approximate solutions of the RG equations given in this appendix are reasonably accurate over much of the interesting region of parameter space, and they provide a useful guide in understanding the numerical results. The least accurate of the approximate solutions are Eq. (A15) which consistently underestimates the contribution of the $\lambda^2 A^2$ term. For large values of the gaugino mass, further underestimation occurs because the squark masses grow rapidly so that the $\lambda^2 m_a^2$ terms are not longer negligible.

APPENDIX B. FINDING THE BOUNCE

In this appendix, we discuss the numerical methods used to compute the lifetime of the false vacuum. For convenience, we work in a theory containing a set of real scalar fields ϕ_a with a scalar potential $V(\phi_a)$. We assume that this potential possesses a local minimum (the false vacuum) at which $\phi_a = \phi_a^{(f)}$, and we choose to make the potential vanish at this point. We also assume that V possesses at least one deeper minimum with $\phi_a = \phi_a^{(t)}$ and $V(\phi_a^{(t)}) < 0$.

Coleman[30] has shown that, at zero temperature, the rate per unit volume for tunnelling is proportional to $\exp(-S[\bar{\phi}])$ in which

$$S[\bar{\Phi}] = 2\pi^2 \int_0^\infty r^3 dr \left[\frac{1}{2}\left(\frac{d\bar{\Phi}_a}{dr}\right)^2 + V(\bar{\Phi}_a)\right] \qquad (B.1)$$

is the Euclidean action of an O(4) symmetric field configuration ϕ_a known as the bounce. This field configuration is a solution of the Euclidean field equations

$$-\frac{d^2\bar{\Phi}_a}{dr^2} - \frac{3}{r}\frac{d\bar{\Phi}_a}{dr} + \left.\frac{\partial V}{\partial \Phi_a}\right|_{\Phi=\bar{\Phi}} = 0 \qquad (B.2)$$

which satisfies the boundary conditions

$$\left.\frac{d\bar{\Phi}_a}{dr}\right|_{r=0} = 0 \quad , \quad \bar{\Phi}_a\Big|_{r\to\infty} \longrightarrow \Phi_a^{(f)} \qquad (B.3)$$

The solution must also be non-trivial in the sense that we cannot use $\bar{\phi}_a \equiv \phi_a^{(f)}$. If more than one such solution exists, the leading contribution is from the one with the smallest action.

It is trivial to compute $S[\bar{\phi}]$ numerically whenever the tunnelling problem can be reduced to a problem involving a single field. A simple method is to numerically

integrate the field equation (B2) using as initial conditions $\bar{\phi}(r = 0) = \phi_0$ and the first of equations (B3). One then searches for the value of ϕ_0 which leads to the second of equations (B3) being satisfied at a large value of r. The solution $\bar{\phi}$ may then be used to compute $S[\bar{\phi}]$. This is the method which was used to produce Fig. 3 and 4 for the toy model. Unfortunately, this method is difficult to use when more than a single field participates in the tunnelling process. This is because a multi-dimensional space must be searched for the correct initial point ϕ_0. The large r behavior of solutions to equation (B2) depends critically on the choice of the initial point. There is no good criterion to decide whether a given initial point is close to the correct choice unless it is extremely nearby.

An alternative method makes use of the fact that the field Eq. (B2) implies that ϕ_a is a stationary point of the action. For some large value of R, we divide the region [0, R] into N intervals of size $\Delta = R/N$. On such a lattice, the action becomes

$$S[\Phi] = 2\pi^2 \Delta^4 \sum_{n=0}^{N-1} n^3 \left\{ \frac{[\Phi_a^{(n+1)} - \Phi_a^{(n)}]^2}{2\Delta^2} + V(\Phi_a^{(n)}) \right\} \qquad (B.4)$$

If the bounce were a minimum of the action, we could successively vary the values of $\phi_a^{(n)}$ in (B4) to reduce the action. With the endpoints constrained by

$$\Phi_a^{(0)} = \Phi_a^{(1)} \quad , \quad \Phi_a^{(N)} = \Phi_a^{(f)} \qquad (B.5)$$

the action would eventually converge to the correct value. Of course, the bounce is not really a minimum of the action, but is rather a saddle point. Indeed, the false vacuum decays because its energy has an imaginary part, and the configuration ϕ_a can contribute to this imaginary part only if the inverse propagator

$$\delta_{ab}\left[-\frac{d^2}{dr^2} - \frac{3}{r}\frac{d}{dr}\right] + \left.\frac{\partial^2 V}{\partial\Phi_a \partial\Phi_b}\right|_{\Phi=\bar{\Phi}} \qquad (B.6)$$

has a negative eigenvalue. If the above method were actually used, the fields would converge either to $\phi_a(r) = \phi_a^{(f)}$ with zero action or to $\phi_a(r) = \phi_a^{(t)}$, except near $r = R$, with a large negative action.

To overcome this problem, we use one further property of the bounce. Typically, the operator (B6) has only a single negative eigenvalue. While we cannot find the corresponding eigenfunction without first finding the bounce, we can construct a sufficiently good approximation. The action (B1) is the sum of a kinetic term

$$S_T[\Phi] = 2\pi^2 \int_0^\infty r^3 dr \, \frac{1}{2} \left(\frac{d\Phi_a}{dr} \right)^2 \qquad (B.7)$$

and a potential term

$$S_V[\Phi] = 2\pi^2 \int_0^\infty r^3 dr \, V(\Phi_a) \qquad (B.8)$$

Under a scale transformation with $\phi_a(r) \rightarrow \phi_a(r/\lambda)$, we have

$$S[\Phi] \rightarrow \lambda^2 S_T[\Phi] + \lambda^4 S_V[\Phi] \qquad (B.9)$$

Because the bounce is a stationary point of the action,

$$S_V[\bar{\Phi}] = -\frac{1}{2} S_T[\bar{\Phi}] < 0 \qquad (B.10)$$

This implies that the bounce is a maximum of the action with respect to scale transformations.

We will use this information in the following way. The initial values of the $\phi_a^{(n)}$

in equation (B4) are chosen to take the form of a step function with $\phi_a^{(n)} = \phi_a^{(t)}$ for $n \leq N/2$and $\phi_a^{(n)} = \phi_a^{(f)}$ for $n > N/2$. We then vary the values of the $\phi_a^{(n)}$ subject to the constraints (B5) and seeking to reduce the action (B4). After each pass through the lattice, we perform a scale transformation to maximize the action. This is most easily done by choosing a new value for the lattice spacing Δ according to

$$\Delta^2 = -\frac{S_T[\Phi]}{2 S_V[\Phi]} \qquad (B.11)$$

in which $S_T[\phi]$ and $S_V[\phi]$ are the lattice approximations to (B7) and (B8). These procedures are iterated until convergence is obtained. Applying this algorithm to the toy model reproduces the curve in Fig. 3 within errors of roughly 5-10%. The results for the minimal model which are discussed in Section V were obtained using this method.

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Table I. SU(3) × SU(2) × U(1) transformation properties of the chiral fields appearing in the minimal low energy model of supergravity.

	SU(3)	SU(2)	U(1)
Q	3	2	1/6
U	3	1	-2/3
D	3	1	1/3
L	1	2	-1/2
E	1	1	1
H	1	2	1/2
H'	1	2	-1/2

Table II. Contributions to the low energy coefficients m_a^2 of the scalar mass-squared operators due to the gaugino masses M_i. These are derived in Appendix A.

a	Q	U	D	L,H',H	E
Δ_{a1}	0.005	0.064	0.016	0.036	0.128
Δ_{a2}	0.49	0.0	0.0	0.49	0.0
Δ_{a3}	6.6	6.6	6.6	0.0	0.0

Table III. Spectra for the points (a-e) on Figures 7-9. The masses are in GeV units, while the tunnelling action is dimensionless.

	(a)	(b)	(c)	(d)	(e)
Gravitino	630	145	1430	67	130
Top Squarks	830,610	254,3	3820,3360	241,102	405,274
Up Squarks	837,827	190,187	4060,3950	187,184	367,358
Down Squarks	840,826	200,192	4060,3840	197,187	373,359
Charged Sleptons	646,633	154,151	1760,1530	90,79	165,144
Sneutrinos	643	140	1760	68	151
Top Quark	33	84	47	61	18
Charged Higgs	750	183	2870	117	193
Neutral Higgs	750,750,56	182,162,58	2780,2780,78	122,81,32	191,173,54
Gluino	610	140	4150	195	377
W/Hino	255,95	128,33	1980,1170	131,22	173,54
B/W/Hino	208,190,160,87	102,101,55,4	1970,1970,1170 ,580	116,84,53,2.5	150,85,73,21
Action	480	140	707	30800	1030

FIGURE CAPTIONS

Figure 1. One loop supergraphs involving the superheavy fields of the missing partners model [29] which contribute to the mass-squared operators of H and H′. Here Y denotes a spurion field which represents the effect of supersymmetry breaking in the hidden sector, while the fields R, S and S′ are defined in the text.

Figure 2. Potential of the toy model of section III plotted for H = Q = T.

Figure 3. The reduced action ($\hat{S} = \lambda^2 S$) for tunnelling in the toy model. Both the exact result and the thin wall approximation are shown.

Figure 4. The parameter space of the toy model. The stable region has $S > 400$, while the unstable region has $S < 400$.

Figure 5. Behavior of the potential as a function of the Higgs mass-squared parameters m_H^2 and $m_{H'}^2$. In region I, $(m_H^2 + m^2)(m_{H'}^2 + m^2) > m^2B^2$ so that the vevs of H and H′ vanish. In region IV, $m_H^2 + m_{H'}^2 + 2m^2 < 2mB$ and the potential is unbounded below. Region III has no minimum with vanishing squark vevs, while region II has an acceptable local minimum with only $\langle H \rangle$ and $\langle H' \rangle$ non-zero. The dashed lines represent renormalization group trajectories which start with $m_H^2 = m_{H'}^2$.

Figure 6. Parameter space of λ_0 and A_0 for given values of M_0, m_0 and B_0. The parameters are renormalized at the GUT scale and masses are in units of the gravitino mass. Regions I, III and IV have no acceptable local minimun (see Figure 5). In region IIA, the acceptable vacuum is the global minimum and is absolutely stable. In regions IIB and C, there is a deeper minimum in which all gauge symmetries are broken, and the acceptable vacuum is unstable against tunnelling. Region IIB is acceptable because tunnelling is suppressed
($S > 400$). Region IIC has $S < 400$ and is unacceptable.

Figures.7,8,9. As for Figure 6, but with different values of M_0, m_0 and B_0. The spectra for points (a-e) are given in Table III.

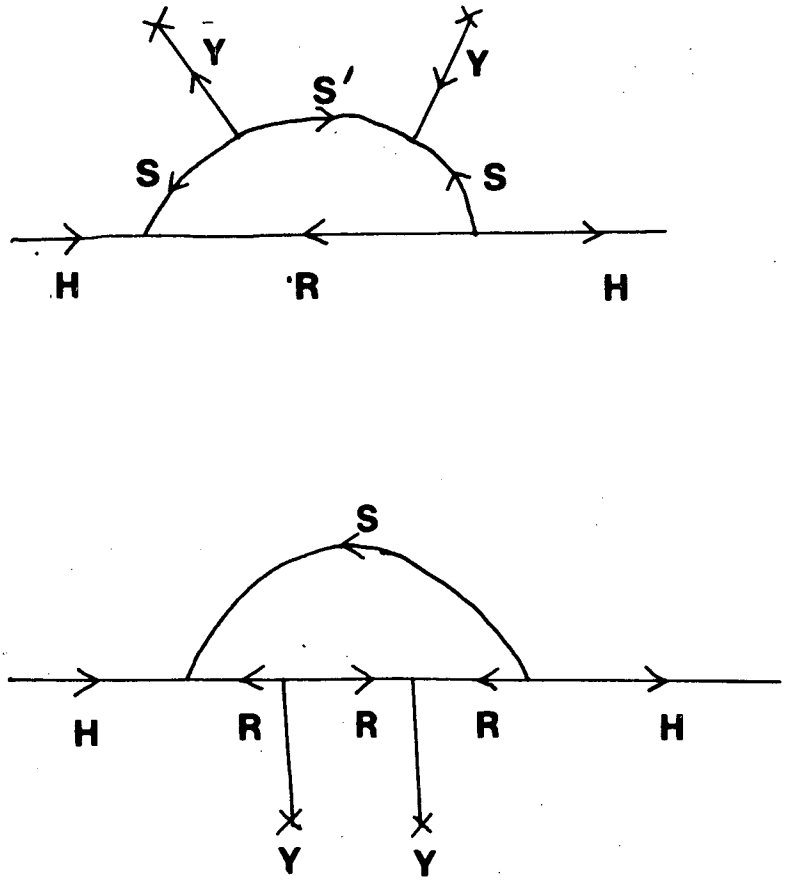


FIG 1

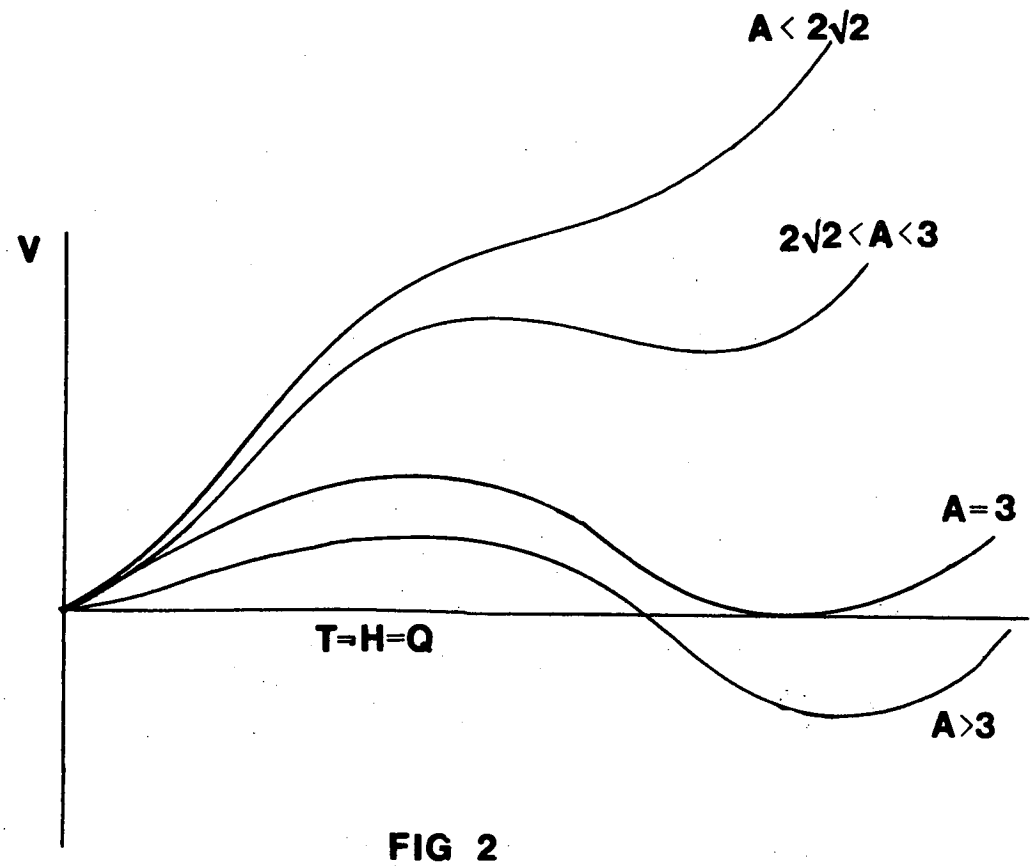
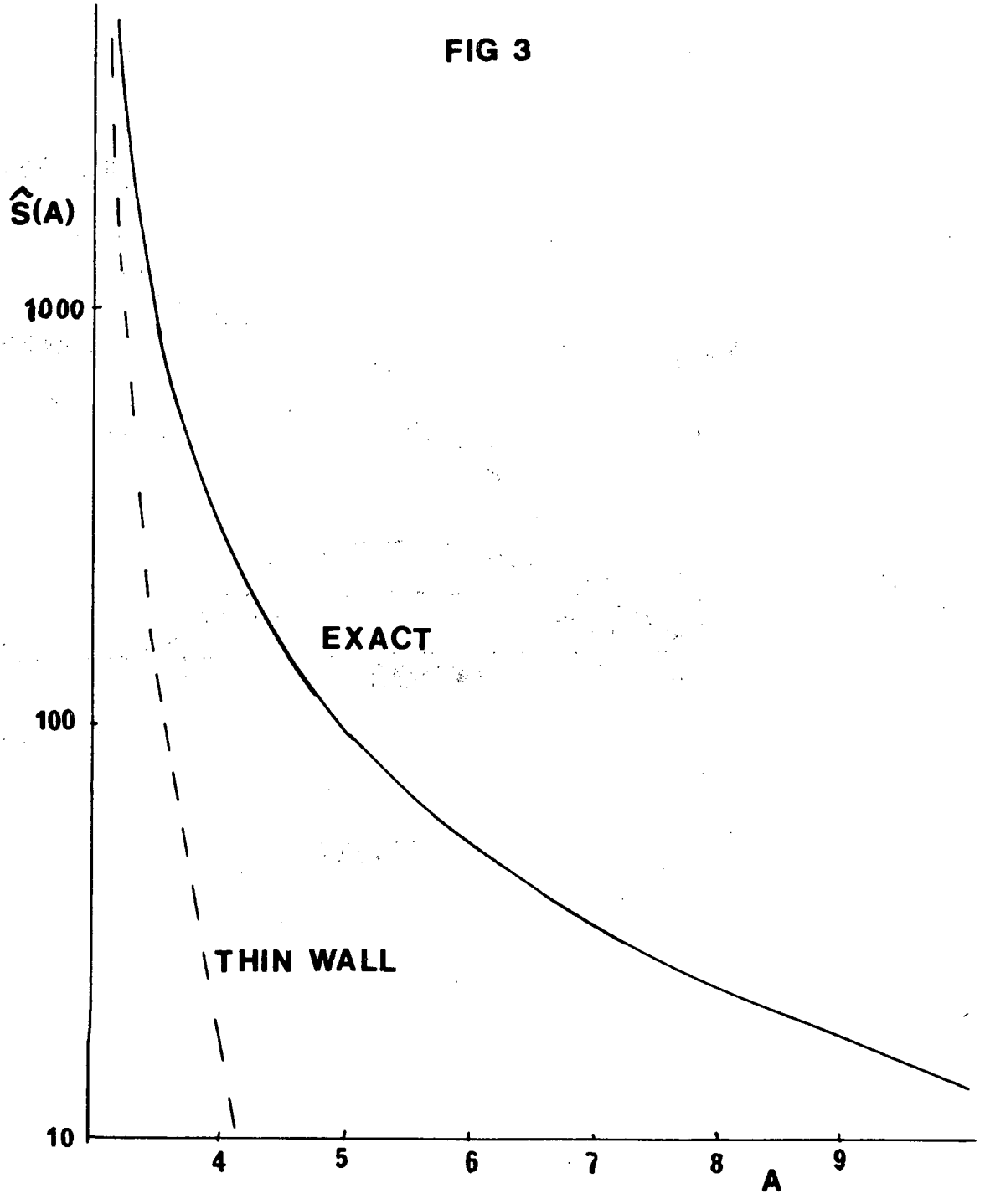


FIG 2

FIG 3



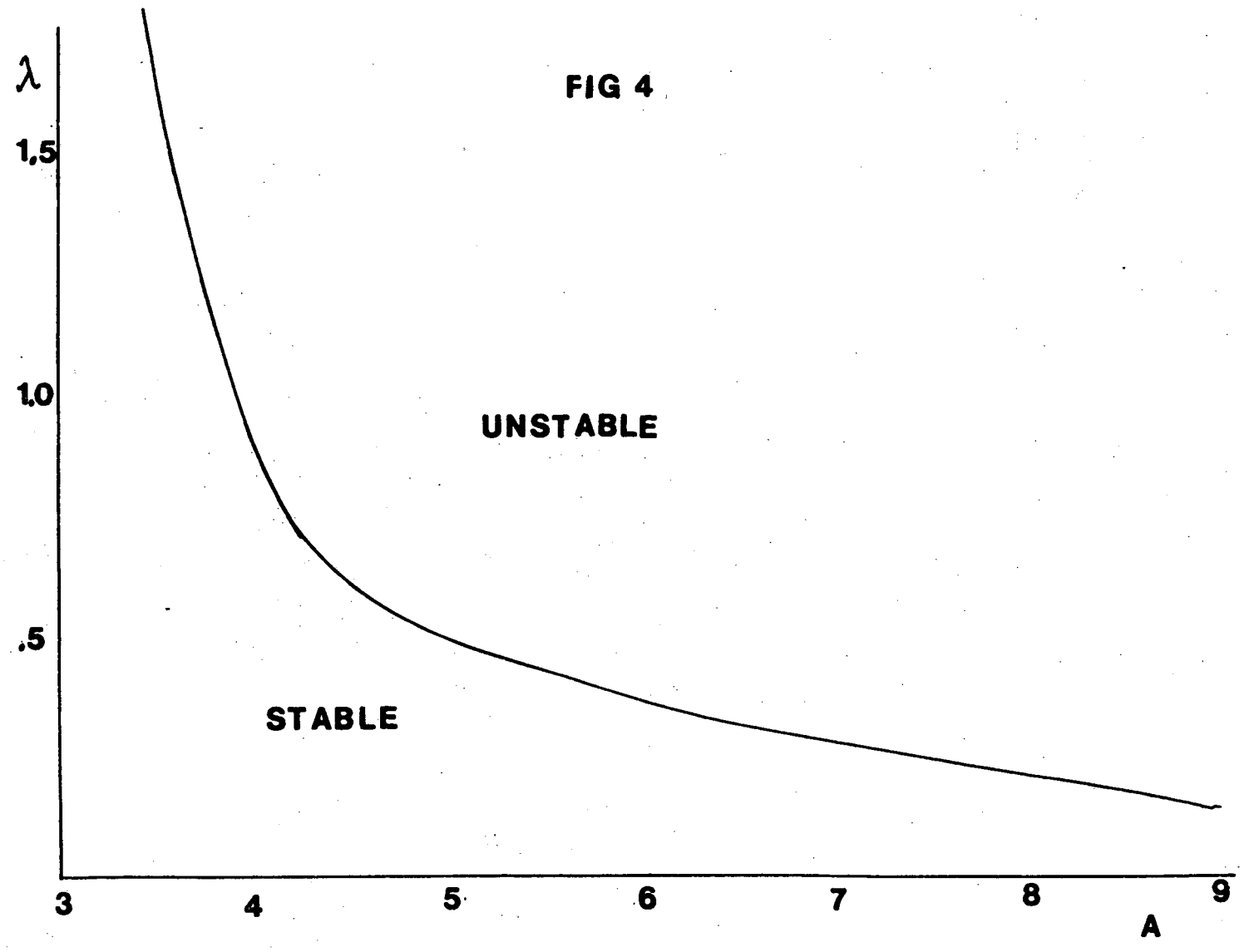


FIG 4

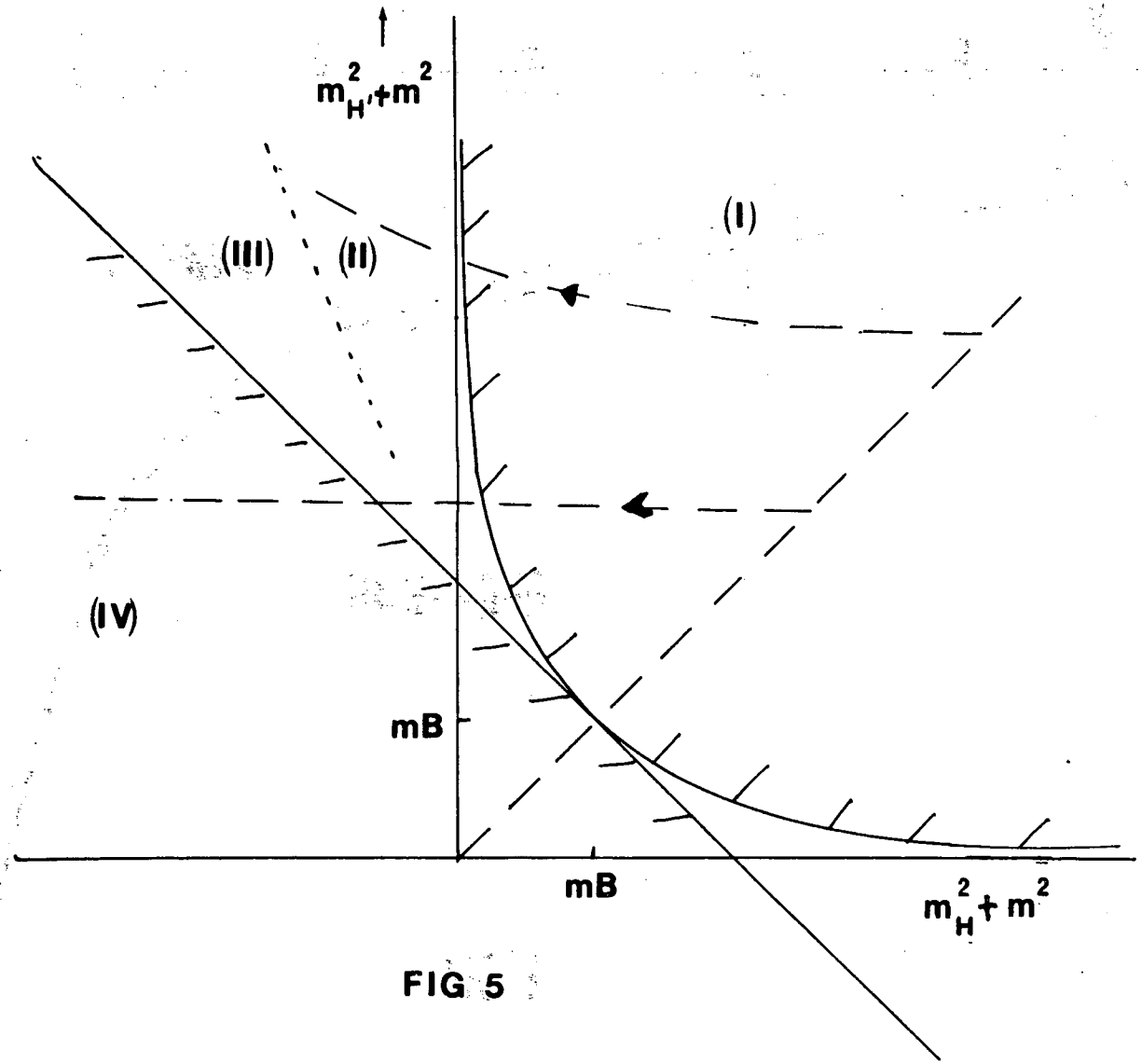


FIG 5

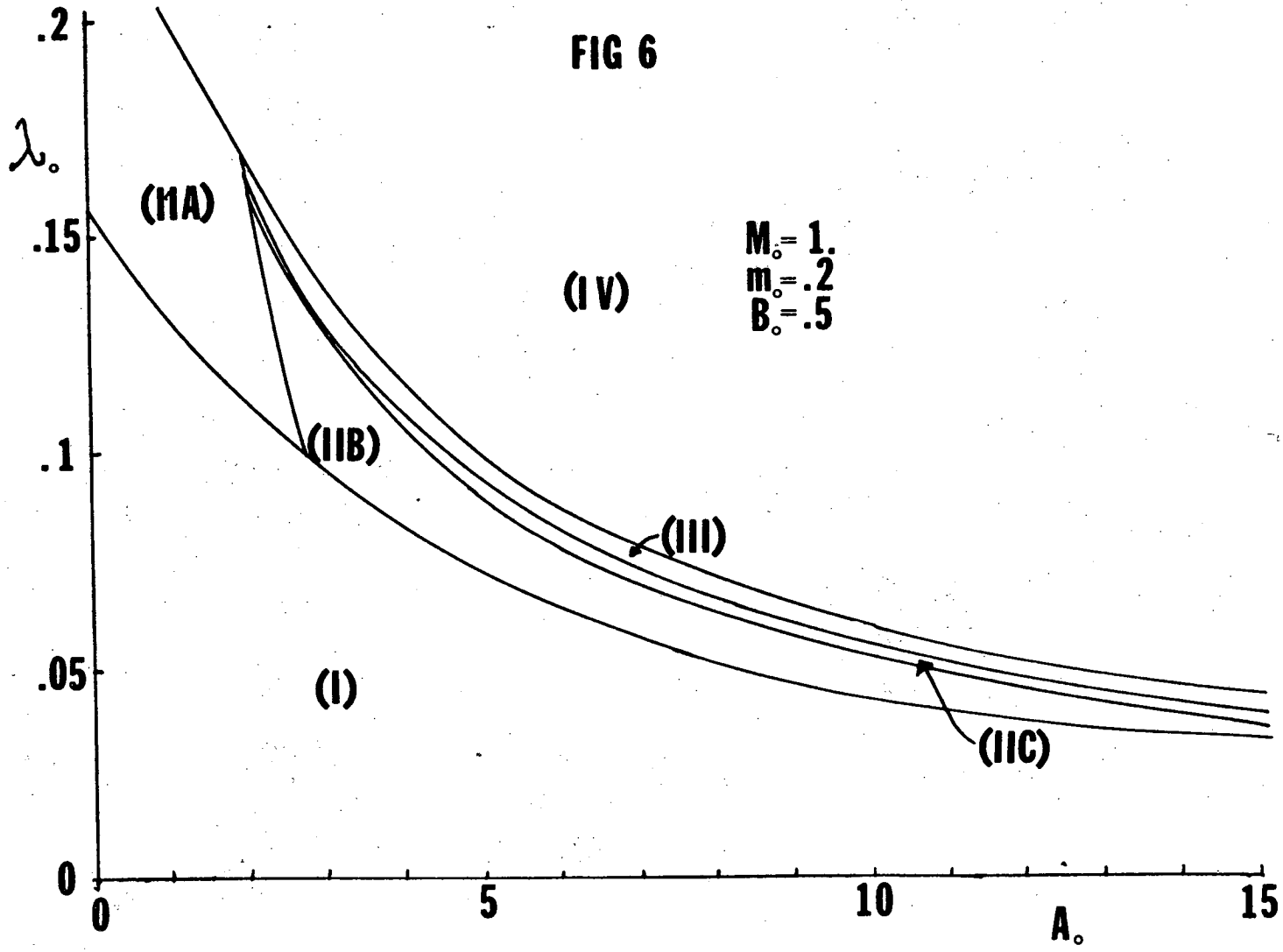


FIG 6
λ_o
.2
.15
.1
.05
0
0
5
10
15
A_o
(IIA)
(IIB)
(IV)
(III)
(I)
(IIC)
$M_o = 1.$
$m_o = .2$
$B_o = .5$

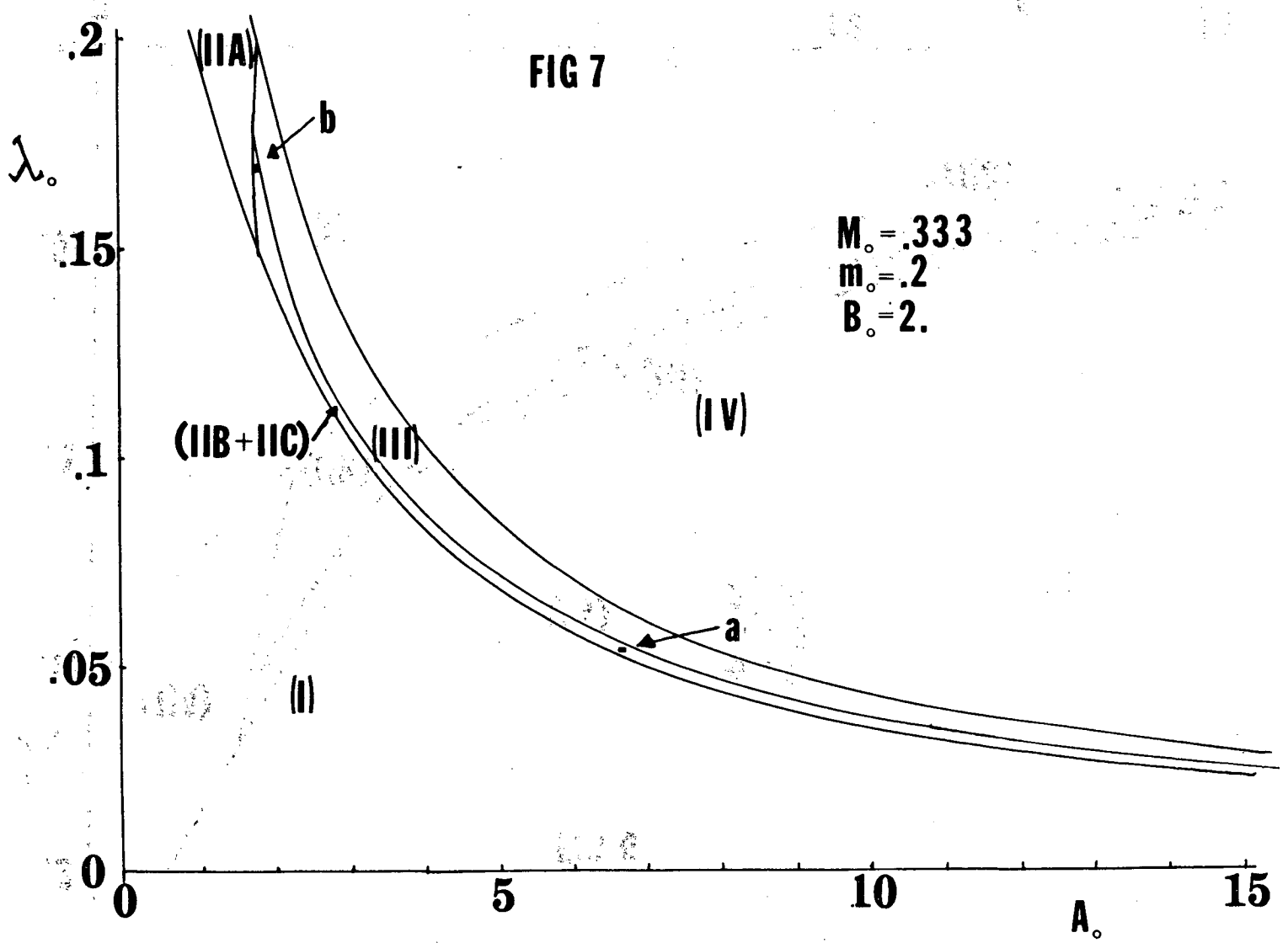


FIG 7
$M_o = .333$
$m_o = .2$
$B_o = 2.$
(IIA)
b
(IIB+IIC)
(III)
(IV)
a
(I)
λ_o
.2
.15
.1
.05
0
0
5
10
15
A_o

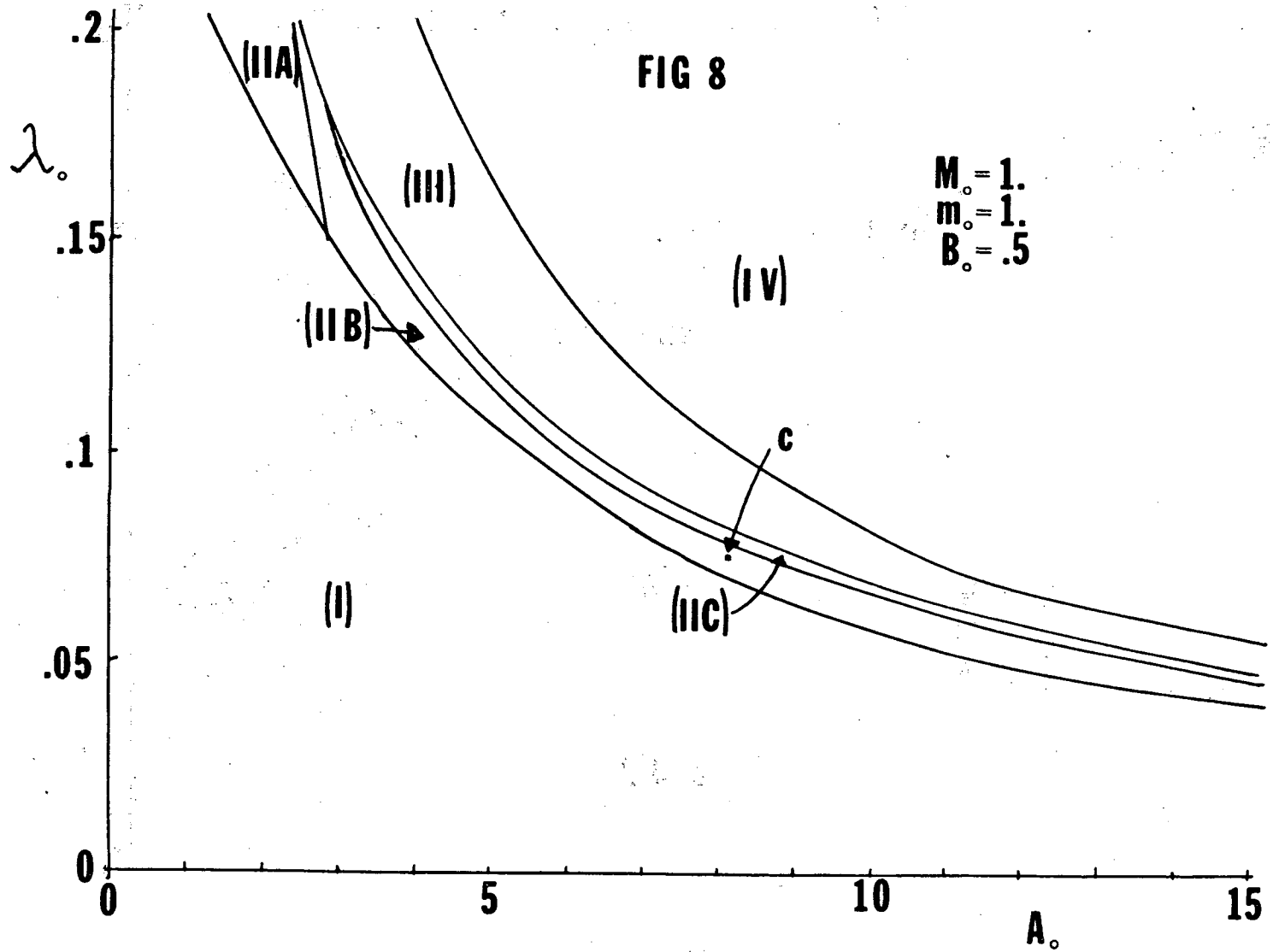


FIG 8
$M_o = 1.$
$m_o = 1.$
$B_o = .5$
λ_o
A_o
.2
.15
.1
.05
0
0
5
10
15
(IIA)
(III)
(IV)
(IIB)
(I)
(IIC)
c

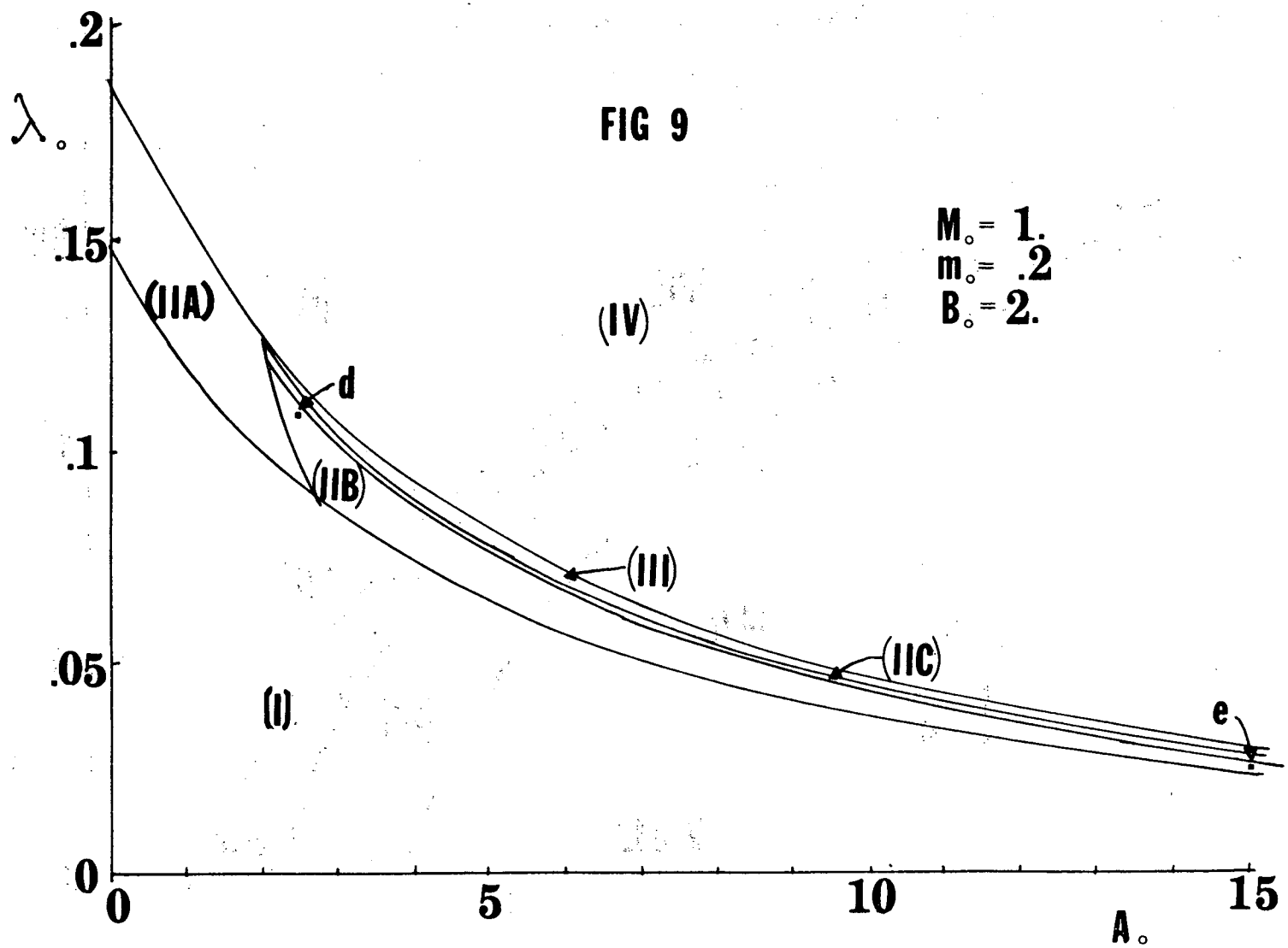


FIG 9

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