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Meson Radiation from Quark-Gluon Plasma by Fission of Chromoelectric Flux Tubes

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Abstract

The chromoelectric flux tube model is used to obtain a dynamical description of the evaporation of mesons from a quark-gluon plasma. The radiation pressure is computed to assess whether this process is an important mode for the disassembly of a compressed plasma.

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Lattice gauge solutions of QCD provide fairly convincing theoretical support for the notion that hadrons will dissolve into a quark-gluon plasma at a sufficiently high energy density.[1] If such a plasma is formed in a high-energy hadron-hadron or nucleus-nucleus collision, the high internal pressure of the plasma will cause its rapid disassembly. Therefore, evidence for its prior existence must be found in its decay products. To assess whether signals of the plasma survive the evolution and what they are, a dynamical description of the disassembly is needed. One facet has been studied by Bjorken, and by Kajantie et al., who propose a hydrodynamical model for its expansion.[2] As it expands it will cool, and the conditions for condensation back to the hadronic phase will be attained. In this note we address another facet of the disassembly, the formation and radiation of mesons at the surface of the hot plasma. The pre-freezeout radiation, which has also been emphasized in connection with the disassembly of a hadronic fireball,[3] plays two roles. It carries signals of the state of the plasma as it evolves. Second, it is coupled to the hydrodynamical expansion of the plasma, which may be inhibited by the backward radiation pressure. In the extreme, the plasma may evaporate mesons rather than expand collectively as a plasma. Such an extreme scenario was proposed recently by Danos and Rafelsky.[4] However, we shall see that their two-parameter model of evaporation leads to results quite different from those predicted by a confining mechanism.

The plasma of almost free quarks, antiquarks, and gluons may be viewed as a region of perturbative QCD vacuum embedded in the true nonperturbative vacuum that pervades most of space. Because of the thermal motion, a quark or antiquark may cross the boundary between the two vacua, but it will suffer a strong color interaction with the rest of the plasma and cannot escape alone.

However, there are two ways in which such a quark can initiate the evaporation of a meson. As the quark crosses the boundary, a flux tube of chromoelectric field connects it to the plasma, and it will experience an attractive potential that will pull it back unless the tube fissions. The tube can fission through the quantum tunneling of a virtual $q\bar{q}$ pair, created in the uniform color field of the tube, into a real state. The energy and momentum of the meson so formed is then determined by the initial energy and momentum of the leading quark <u>and</u> the space-time position where the $q\bar{q}$ pair creation occurs. This mechanism of hadronization has been commonly used to describe particle production in high-energy e^+e^- annihilation.[5,6] Although in our problem the average momentum of the quarks is not so high ($T \sim 200$ MeV), we expect that the picture will qualitatively describe the fate of the high momentum component of the thermal distribution. The contribution of the low-momentum quarks to this hadronization process is suppressed as will be seen. This supports our idealization of a smooth plasma surface. A second mechanism for hadronization at the plasma surface is the coalescence of a quark antiquark pair. We shall not further describe this process here, since at the expected plasma temperatures the thermal flux of quarks and antiquarks with specified color charge is small.

Consider a quark (or antiquark) of momentum $\underset{\sim}{k}_0$ that is outward directed with respect to the surface. As the quark passes through the surface, a tube of chromoelectric flux is built up behind it, out of its kinetic energy. The energy per unit length that is stored in the tube is $\sigma = \epsilon^2 A/2$, where ϵ and A are the field strength and cross section of the tube. Gauss' law relates the flux ϵA to the quark charge, g/2 through $\epsilon A = g/2$, yielding

$$\sigma = g\epsilon/4 \tag{1}$$

This string constant can also be related to the Regge slope and so is essentially a known parameter, $\sigma = 0.177\ (\mathrm{GeV})^2$. We assume that the flux tube that shields its color will connect it to the plasma by the shortest path. The motion of the quark will be governed by the equations expressing the conservation of energy and of momentum parallel to the surface. Choose the coordinate system so that the z-axis is normal to the surface, the origin is on the surface, and the motion of the quark is in the z-y plane. Then the conservation laws read

$$\left(\underset{\sim}{k}^2 + m^2\right)^{1/2} + E_s = E_o = \left(\underset{\sim}{k}_o^2 + m^2\right)^{1/2} \quad , \quad k_y + k_s = k_{yo} \tag{2}$$

where k is the instantaneous quark momentum, k_y its component parallel to the surface, and m its mass. Its velocity in the y direction is

$$v_y = k_y/E \quad , \qquad E = \left(\underset{\sim}{k}^2 + m^2\right)^{1/2} \tag{3}$$

The "rest mass" of the string is σz where z is the coordinate of the emitted quark. So the energy and momentum of the string are

$$E_s = \sigma z \left(1 - v_y^2\right)^{-1/2} \quad , \qquad k_s = (\sigma z) v_y \left(1 - v_y^2\right)^{-1/2} \tag{4}$$

These equations, (2)-(4), govern the motion of the quark and can be solved analytically. In particular

$$v_x = 0 \quad , \quad v_y = v_{yo} = \text{const} \quad , \quad v_z = k_z/E$$

$$k_z = k_{zo} - (E_{zo}/E_o)\sigma t \quad , \quad E_{zo} = \left(k_{zo}^2 + m^2\right)^{1/2} \tag{5}$$

$$\sigma z = E_{zo} - \left(k_z^2 + m^2\right)^{1/2}$$

and the instantaneous energy of the quark is

$$E = \left(\underset{\sim}{k}^2 + m^2\right)^{1/2} = E_o\left(1 - \sigma z/E_{zo}\right) \tag{6}$$

The motion of the quark normal to the surface is decelerated until it stops at time

$$t_c = (E_o/E_{zo})(k_{zo}/\sigma) \tag{7}$$

Thereafter it is accelerated back into the plasma.

The above equations describe the motion of a typical quark as it penetrates the surface of the plasma. The color flux tube can fission as the result of the creation of a quark-antiquark pair inside the tube. If such a pair is created, say at a distance z' from the surface of the plasma and at a time t, a meson consisting of the original quark together with the antiquark of the created pair can evolve to a physical meson. Its momentum perpendicular to the surface will be that possessed by the leading quark at the time t, given by (5). Its energy will be E_0 less the energy carried back into the plasma by the fragment of string of length z' and the quark contained in it. Thus, the meson momentum and energy are

$$k_z^M = k_{zo} - (E_{zo}/E_o)\sigma t \quad , \quad E^M = E_o - (\sigma z' + m)(E_o/E_{zo}) \tag{8}$$

where $\left(1 - v_y^2\right)^{1/2} = E_{zo}/E_o$.

Pair creation in a constant external field is similar to the QED process solved by Schwinger[7] and employed in several papers[5,6] to describe particle production in high-energy e^+e^- annihilation. However, earlier authors neglect the interaction of the pair with each other. This is a serious neglect in QCD, where the chromoelectric flux created by the pair is of the same strength as the original field in which they were created. In the present study we use a new result[8] for the $q\bar{q}$ pair creation probability per unit four volume:

$$p = \sum_{\text{flavor}} \frac{(g\varepsilon)^2}{64\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{4\pi^2 m_f^2 n}{g\varepsilon}\right) \tag{9}$$

This formula differs from Schwinger's by the effective reduction of the quark charge by a factor 1/2. Through (1), the pair creation rate is determined by the string tension and the quark masses m_f.

Up to now we have not addressed the question of whether current or constituent quark masses should be used. This is a subtle and still controversial problem. We have chosen to use current quark masses ($m_u = m_d = 0$) as in ref. 6. This choice is at least consistent with jet phenomenology and is convenient for the present problem in which the leading quark emerges from the perturbative plasma. For massless quarks (9) reduces to

$$p \simeq \sigma^2/12\pi \qquad (10)$$

where the contribution of heavier quarks is safely neglected.

Having calculated the string dynamics and the probability per unit four-volume that a real pair will be formed and that hence the string will fission, we can now calculate the probability that the string will fission at the time t measured from the time that the outward-moving quark crosses the surface and in the interval dt. This is given by

$$dP(t) = pAz(t)\,dt\,[1 - P(t)] \qquad (11)$$

where z(t) is the coordinate of the leading quark at time t and hence the length of the string. This integrates to

$$R(t) \equiv 1 - P(t) = \exp\left(-pA\int_0^t z(t)dt\right) \qquad (12)$$

$$= \exp\left(-\frac{pA}{\sigma^2}E_o\left\{k_{zo} - k_z + \frac{1}{2E_{zo}}\left[k_zE_z - k_{zo}E_{zo} + m^2\,\ell n\left(\frac{k_z+E_z}{k_{zo}+E_{zo}}\right)\right]\right\}\right)$$

$$\to \exp\left(-\frac{pA}{\sigma^2}\frac{k_o}{2k_{zo}}(k_z - k_{zo})^2\right)\quad , \quad (m \to 0) \qquad (13)$$

where k_z is given by (5).

The above formula indicates that the parameter that characterizes confinement is

$$k_c = \sqrt{\frac{\sigma^2}{pA}} = \sqrt{\frac{24\sigma}{\alpha_c}} \tag{14}$$

where $\alpha_c = g^2/4\pi$. A quark of momentum k_{zo} at the plasma surface escapes with a probability

$$P_{esc} = 1 - e^{-k_{zo}k_o/2k_c^2} \tag{15}$$

(in the massless quark limit). Its momentum, and hence the momentum of the radiated meson, is distributed according to the probability (see Fig. 1)

$$\frac{dP}{dk_z} = \frac{k_o}{k_{zo}} \frac{|k_z|-k_{zo}}{k_c^2} \exp\left[-\frac{1}{2}\frac{k_o}{k_{zo}}\left(\frac{k_z-k_{zo}}{k_c}\right)^2\right] \tag{16}$$

and can lie anywhere between $-k_{zo} < k_z < k_{zo}$. A quark, emerging with k_{zo} suffers a characteristic loss of momentum, k_c, so that the most probable momentum is

$$k_m = k_{zo} - k_c \tag{17}$$

The momentum scale k_c, set by confinement depends on the values of σ and α_c. These parameters must be related through $\mathcal{L}_{QCD}$, but this relationship is unknown. For $\sigma = 0.177\ (\text{GeV})^2$ and for $\alpha_c = 2$ or 0.55, corresponding respectively to the values used by Casher et al,[5] and by the MIT group,[9] we have

$$k_c = \begin{cases} 1.45\ \text{GeV} & , \quad (\alpha_c = 2) \\ 2.78\ \text{GeV} & , \quad (\alpha_c = 0.55) \end{cases}$$

From (15) only quarks from the tail of the thermal distribution with $k \gtrsim k_c$ have an appreciable chance of forming a meson, and the momentum of the meson is shifted downward by k_c from the original quark momentum. Next we calculate the probability that the string will break at the point z' at the time t while the quark is outward moving. This probability is distributed uniformly along the string. Therefore,

$$d^2P(z',t) = pA\, dz'\, dt\, R(t)\, \theta(z' - z(t))\, \theta(-z')\, \theta(t - t_c)\, \theta(-t) \tag{18}$$

where θ is the step function $\theta(+) = 0$, $\theta(-) = 1$. We fold d^2P with the flux of quarks of momentum $\underset{\sim}{k}_o$ and sum over all such momenta to obtain the number* of mesons radiated per unit time per unit surface area of the plasma having normal momentum k_z^M and energy E^M,

$$\frac{d^5N}{dS\, dt\, dk_z^M\, dE^M} = \frac{\gamma}{(2\pi)^3} \int d^3k_o\, e^{-E_o/T} \frac{k_{zo}}{E_o} \frac{pA}{\sigma^2} R(k_z^M)\, \theta(E_z - m - (E_{zo}/E_o)E^M)$$
$$\times\, \theta(m + (E_{zo}/E_o)E^M - E_{zo})\, \theta(-k_z^M)\, \theta(k_z^M - k_{zo}) \tag{19}$$

(We employ (8) to write $dz'dt = dk_z^M\, dE^M/\sigma^2$). The degeneracy factory γ is $\gamma = \gamma_C \times \gamma_S \times \gamma_F \times 2 = 24$ where 2 counts quarks and antiquarks. $R(k_z^M)$ means that (13) is evaluated at the time (8) that yields the momentum k_z^M.

The radiation pressure exerted on the plasma by the radiation of mesons, and the energy flow per unit surface area per unit time carried in the radiation follow immediately:

$$p^M = \frac{d^3k}{dS\, dt} = \int_0 dk_z\, k_z \int_{E_\pi} dE^M \frac{d^5N}{dS\, dt\, dk_z\, dE^M} \tag{20}$$

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*For non-zero chemical potential, multiply our result by cosh μ/T.

$$\frac{d^3E}{dS\ dt} = \int_0 dk_z \int_{E_\pi} dE^M\ E^M \frac{d^5N}{dS\ dt\ dk_z\ dE^M} \tag{21}$$

where $E_\pi = \sqrt{k_z^2 + m_\pi^2}$

The above formulae characterize the hadronization at the surface of a plasma when confinement is described by the chromoelectric flux tube model. We can compare the above with the case of vanishingly small confinement through

$$\frac{d^5N}{dS\ dt\ dk_z^M\ dE^M} = \frac{\gamma}{(2\pi)^3} \int d^3k_o\ e^{-E_o/T} \frac{k_{zo}}{E_o}\ \delta(k_{zo} - k_z^M)\ \delta(E_o - E^M) \tag{22}$$

Our main result is exhibited in Fig. 2, which, as a function of temperature, shows the ratio of radiation pressure and internal quark pressure. Since this ratio is less than 20% up to T = 500 MeV, we conclude that the radiation due to this process will be a minor perturbation on the collective hydrodynamical disassembly of the plasma. If the gluon pressure is included, then the internal pressure is increased by (16 + 24)/24 for the Boltzmann statistics or by (16 + 21)/21 for the Fermi and Bose statistics, which further reduces the role of the radiation pressure.

This conclusion contradicts an assertion made by Danos and Rafelski.[4] Their conclusion, however, is based on a model that does not contain a dynamical description of color confinement but that instead is characterized by two adjustable parameters, a minimum momentum for emission and an efficiency factor defining the fraction of the quark energy carried off by the meson. In Fig. 3 we compare their result for the surface brightness at zero chemical potential with ours. Although one may fit their curves to our results in a limited temperature interval by choosing different values for the parameters, such a parametrization can be hardly justified from the present study. Figure 1 contradicts the basis of their model, since they assert that a quark of initial momentum k_o ($>p_M$) produces a meson of that momentum with unit probability.

In summary, the hadronization at the surface of a quark-gluon plasma has been studied in the framework of a chromoelectric flux tube model. We find that the radiation pressure is sufficiently small compared to the internal pressure that it can be ignored to first approximation in the hydrodynamical expansion of the plasma. Thus our solution for meson radiation can be folded with a solution to the hydrodynamical expansion to obtain the spectrum of radiated mesons emitted over the history of the expansion. Of special interest is the distinction between strange and non-strange mesons, which is explicit in the theory through the dependences on the quark masses and the thermal populations in the plasma.

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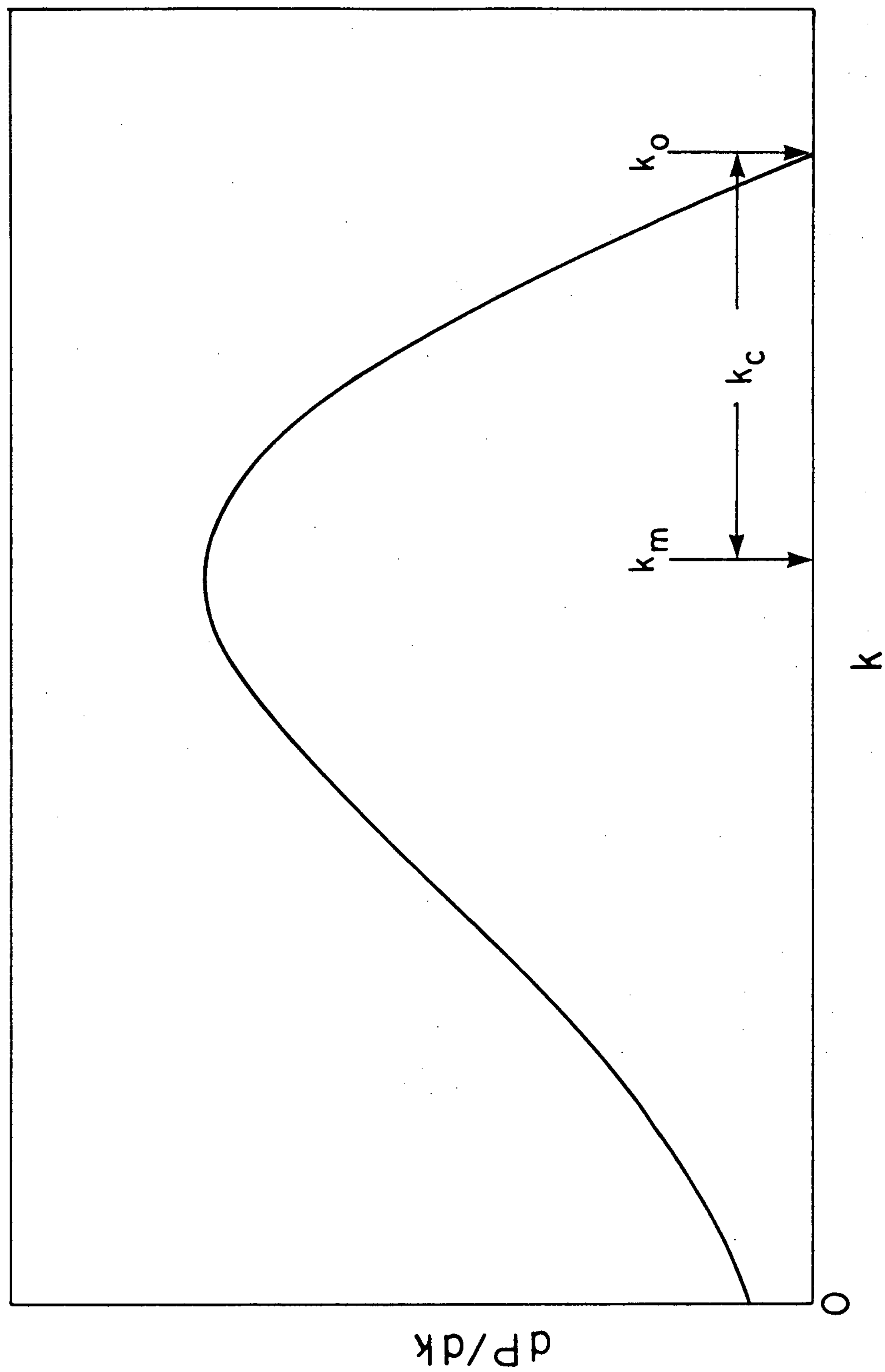
Figure Captions

Fig. 1. Normal momentum distribution of radiated mesons for initial quark momentum, normal to the surface, $k_o > k_c$.

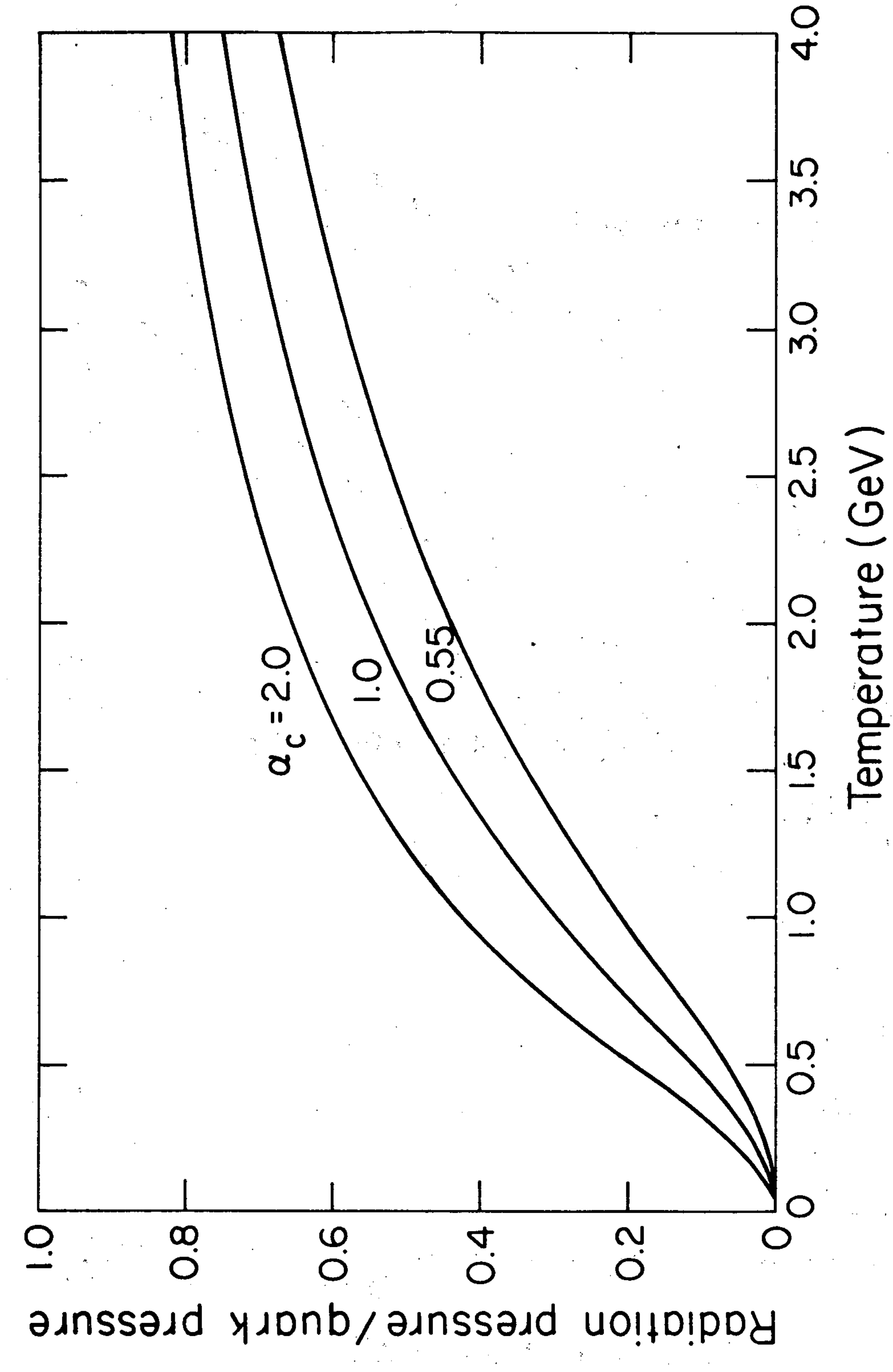
Fig. 2. Pressure acting on the surface of the quark-gluon plasma due to the radiation of mesons as a function of plasma temperature.

Fig. 3. Surface brightness of the quark-gluon plasma due to the meson radiation as a function of plasma temperature. Nonconfinement case (upper solid line) and the results of Danos and Rafelsky (dashed lines) are compared to our results (lower solid lines).



XBL 833-125

Fig. 1



XBL 833-124

Fig. 2

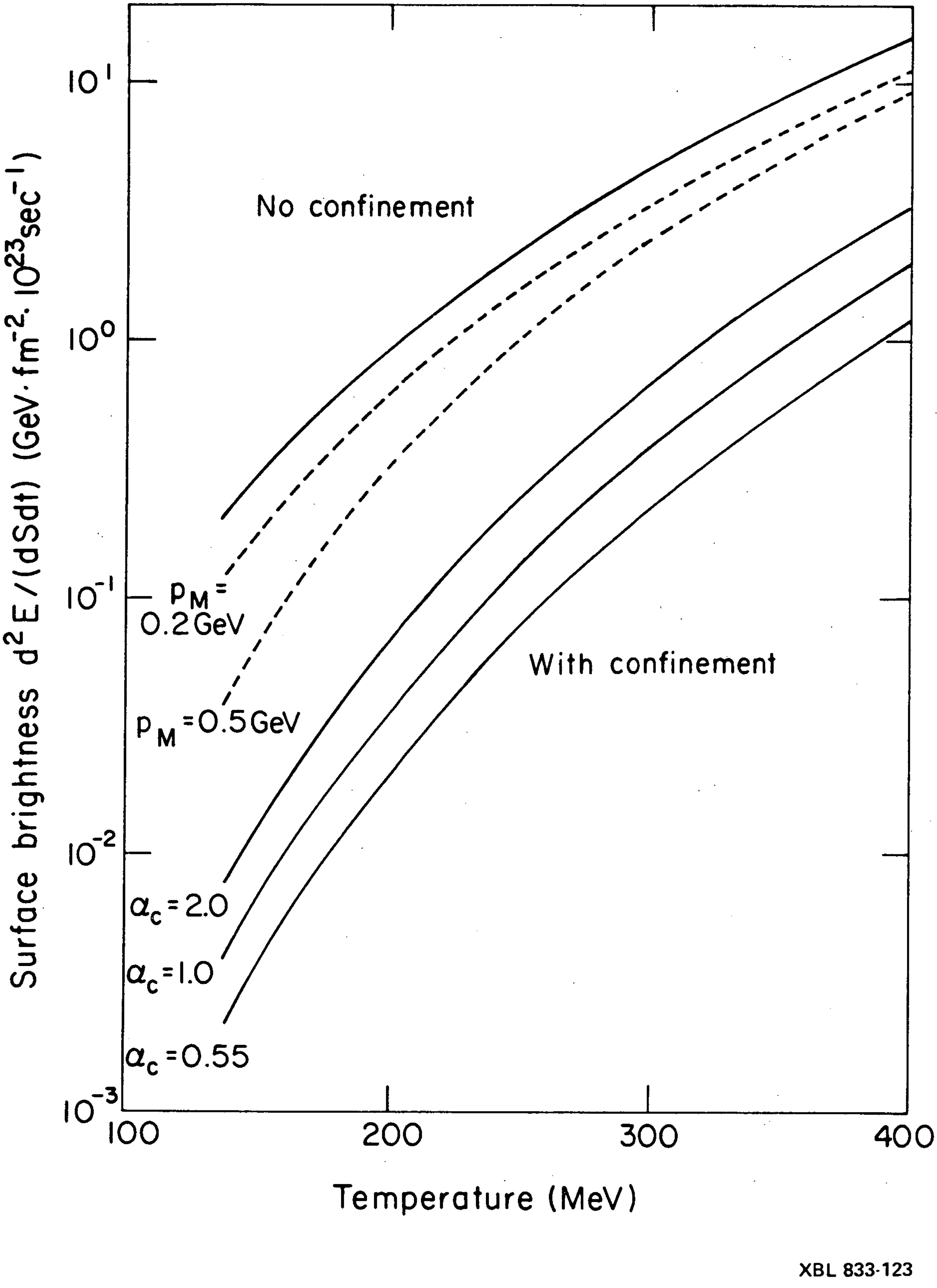

No confinement
With confinement
p_M = 0.2 GeV
p_M = 0.5 GeV
α_c = 2.0
α_c = 1.0
α_c = 0.55
Surface brightness $d^2E/(dSdt)$ (GeV·fm^{-2}·10^{23}sec^{-1})
Temperature (MeV)
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
100
200
300
400
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Fig. 3

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