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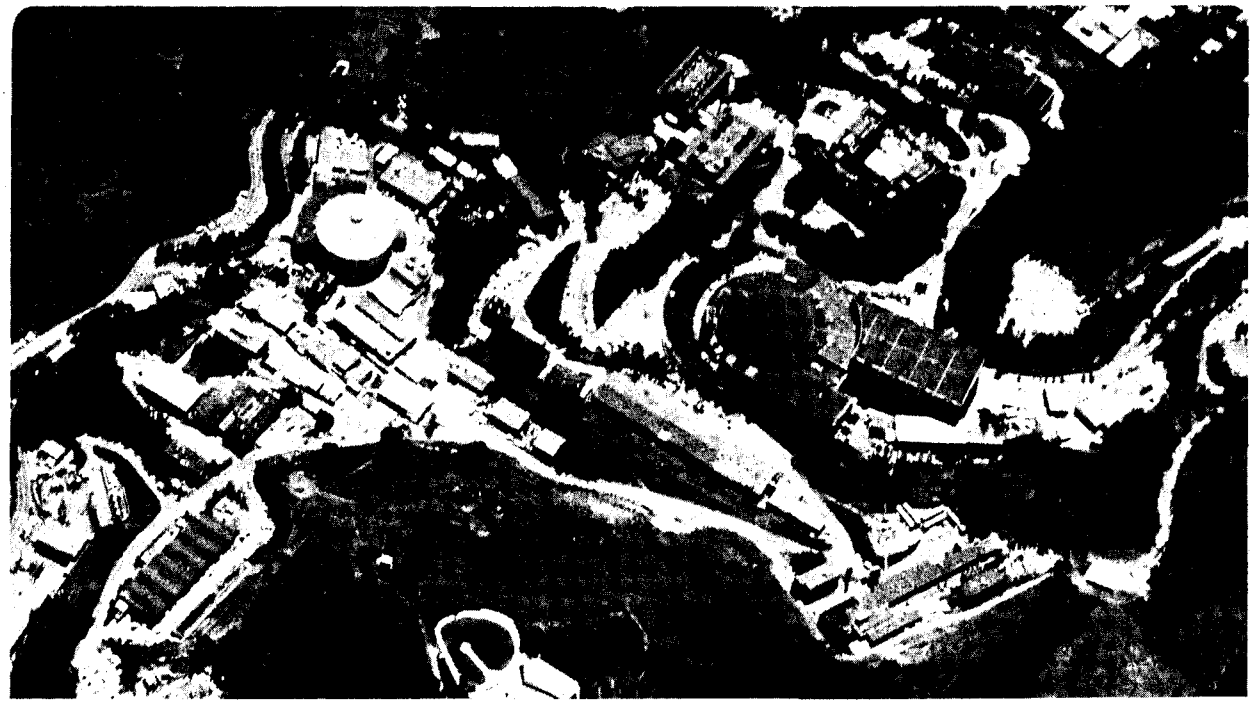

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EXPLICIT R PARITY BREAKING IN SUPERSYMMETRIC MODELS[1]

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ABSTRACT

Supersymmetric models generally invoke R parity to ensure that baryon and lepton numbers are symmetries of the renormalizable operators of the low energy effective theory. The phenomenology of lepton number violaton is analyzed in low energy models in which R parity is explicitly broken by superrenormalizable operators. Constraints on lepton number violating parameters are found to be mild. The photino is able to decay, avoiding a stringent cosmological lower bound on its mass. Alternatives to R parity are considered in the context of an SU(5) grand unified model coupled to N = 1 supergravity. One possibility, θ parity, leads to new mechanisms for baryon number violation in addition to lepton number violation.

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I. INTRODUCTION

The recent interest in supersymmetric extensions of the standard model has occured without the slightest experimental prompting. The clearest experimental vindication of these models would be the observation of an R odd particle, a supersymmetric partner of one of the known particles, such as a gluino or selectron. Much effort has been spent in understanding production mechanisms and signatures for such sparticles[1]. Unfortunately, their non-observation can be accommodated all too easily in most theories, as the spectrum usually involves several unknown parameters. In this paper we will be concerned primarily with models of low energy supersymmetry which are the remnants of a grand unified theory coupled to N = 1 supergravity,[2,3] with supersymmetry broken in a hidden sector of the theory.[4-16] While supersymmetry violating operators are responsible for weak interaction breaking, it may be that all sparticles have a mass of a TeV or more;[16] sparticle searches would be a disheartening occupation for at least a decade.

In this paper we consider other possible observable manifestations of an underlying supersymmetric theory. These effects concern abnormal interactions among the presently observed particles, and detection usually requires a high accuracy measurement at low energies rather than the attainment of new particle thresholds at high energies. We will concentrate on lepton number violating interactions which may have a very rich structure in supersymmetric theories, but which are usually removed by imposing discrete symmetries, such as R parity,[17] on the theory. Without such symmetries it is difficult to understand why lepton and baryon number violations have not yet been observed. In this paper we show that there are alternatives to R parity which explain why the proton lives much longer than the neutron and why the neutrinos are much lighter than the

charged leptons, but which allow proton decay, neutrino masses and oscillations and other interesting effects to occur at levels accessible in the near future.

R parity[17] (R_p) is defined to be +1 for every particle of the standard model. and -1 for their supersymmetric partners. This multiplicative parity may remain unbroken even after breaking the continuous global R invariance,[18] with which R_p should not be confused. On component fields a convenient assignment is

$$R_p = (-1)^{3B+L+2S} \tag{1.1}$$

where S is the particle spin, while B and L are baryon and lepton numbers. If B and L were accidental symmetries of the renormalizable terms of the effective $SU(3) \times SU(2) \times U(1)$ Lagrangian, it is clear that R_p would also be an accidental invariance. This has led to an almost universal acceptance of R_p as a good symmetry. In the standard non-supersymmetric $SU(3) \times SU(2) \times U(1)$ theory, gauge invariance and representation content were sufficient to ensure B and L invariance of all renormalizable interactions. Even in the minimal supersymmetric extensions of the theory, this understanding of the longevity of the proton and of the near absence of L violating processes is lost. Neither B or L is an accidental symmetry, and therefore neither is R_p. One is forced to impose something like R_p by hand. There is no reason, other than simplicity, why it should be R_p rather than some other symmetry which suppresses B and L violation to an acceptable level. With a view to grand unification we do not want to impose the anomalous B or L symmetries themselves, so that some alternative symmetry must be imposed. There is considerable freedom in the choice of this symmetry and in the structure of the resulting B and L violation. A few, but certainly not all, such possible structures are analyzed in this paper. We stress that L violation is explicit, we will not be plagued by a Majoron.[19]

In the context of locally supersymmetric theories there is an additional motivation for the breaking of R_p: the problem of the cosmological photino abundance. Recently it has been pointed out that the known upper limit for the present mass density of the universe plays an important role in constraining the lower bound on the mass of an essentially stable photino.[20,21] Since photinos are Majorana particles and annihilate predominantly through p wave, considerable suppression of the annihilation rate occurs because at temperatures below their mass they are non-relativistic. Since the annihilation rate is proportional to the square of the photino mass, they must be rather heavy to annihilate sufficiently before freezeout. For squark masses of 40-50 GeV, the lower limit on the photino mass is about 20 GeV, an order of magnitude more stringent than for a heavy Dirac neutrino.[22] Of course, this point is only relevant when the photino lifetime is greater than or comparable to the age of the universe. This is commonly expected to be true when the photino is the lightest R_p odd particle, as in most supergravity theories. A lower bound on the photino mass of 20 GeV is very discouraging; in supergravity theories one then expects the gluinos to have a mass of 200 GeV, and the majority of squarks and sleptons to have a mass of order 500 GeV. Breaking R_p and allowing the photino to decay rapidly to $bb\nu$ or $\nu\gamma$ removes this cosmological mass constraint.

In the next section we consider the form of R_p breaking operators in an $SU(3) \times SU(2) \times U(1)$ supersymmetric theory. A general analysis of R_p breaking is far too complicated even with a minimal set of fields, and we restrict the operators of the theory in two ways. Firstly, the structure of the tree level supersymmetry breaking operators is assumed to have the form given by the simplest supergravity theories. Secondly, only those R_p violating operators are included which result from grand unified theories of the sort considered in Section IV. In Section III we study the phenomenology of this $SU(3) \times SU(2) \times U(1)$

model and derive constraints on the R_p violating parameters from a consideration of neutrino masses, and rare meson and lepton decay modes. In Section IV we give an example of an SU(5) model which has an alternative discrete symmetry to R_p, and which yields the lepton number violation at low energy studied in Sections II and III. Although rather unappealing, this model has interesting new sources of baryon number violation as well as lepton number violation. A summary of our conclusions is given in Section V.

II. SU(3) × SU(2) × U(1) MODELS WITH R_p VIOLATON

We restrict our attention to renormalizable operators in SU(3) × SU(2) × U(1) theories, and will consider in Section IV the additional restrictions and non-renormalizable operators arising in grand unified theories. With the minimal set of chiral superfields (Table 1) the superpotential may contain not only the usual terms

$$f_1 = L\bar{E}H' + Q\bar{D}H' + Q\bar{u}H + H'H \qquad (2.1)$$

(here and elsewhere we will suppress generation indices and coupling constants) but also the terms

$$f_2 = \bar{u}\bar{D}\bar{D} + Q\bar{D}L + L\bar{E}L + LH \qquad (2.2)$$

For our purposes the most convenient definition of R_p is in terms of superfields:

$$\begin{aligned} M &\to -M \\ V &\to V \\ X &\to X \\ \theta &\to -\theta \\ \bar{\theta} &\to -\bar{\theta} \end{aligned} \qquad (2.3)$$

[Footnote 1] where θ and $\bar{\theta}$ are the two component anticommuting coordinates of superspace, V are vector superfields, M are matter superfields and X denote all other chiral superfields (H and H′ in the minimal case). Imposing R_p forbids f_2 while allowing f_1, because these sets of operators have odd or even numbers of

matter superfields respectively. However, there are many other interesting possibilities for discrete symmetries. For example $Q \to -Q$, $\bar{U} \to -\bar{U}$, $\bar{D} \to -\bar{D}$ forbids the $\Delta B = 1$ operator $\bar{U}\bar{D}\bar{D}$ while allowing f_1 and the lepton number violating terms $Q\bar{D}L$, $L\bar{E}L$ and LH. Alternatively lepton number may be preserved by $L \to -L$ and $\bar{E} \to -\bar{E}$, which would allow only $\bar{U}\bar{D}\bar{D}$ from f_2. Since L is unbroken, the proton is unable to decay unless it is heavier than the photino, however the B violation would allow neutron oscillations. The appropriate setting for a detailed discussion of possible discrete symmetries is grand unified theories, and we discuss these later. In fact neither of the above possibilities will follow from grand unification, rather we will find that R_p is an almost exact symmetry of the dimension 4 operators of the low energy theory so that the coefficients of $\bar{U}\bar{D}\bar{D}$, $Q\bar{D}L$ and $L\bar{E}L$ are extremely small, while R_p may be substantially broken in superrenormalizable terms such as LH. It is this latter possibility which is explored in this section.

We will assume that supersymmetry breaking in the low energy effective $SU(3) \times SU(2) \times U(1)$ theory occurs only in soft operators.[23] For simiplicity we will assume the highly constrained set of soft operators which can result when the theory is coupled to $N = 1$ supergravity and supersymmetry is broken in a hidden sector of the theory.[9,12] Much of the phenomenology of this paper would be unchanged if the soft supersymmetry breaking operators had a different form, for example, from radiative corrections in models of global supersymmetry with supersymmetry breaking somewhat isolated from the known particles. We therefore expect that much of our analysis will be applicable to a wider class of theories. Dividing the superpotential into quadratic and cubic terms $f = f^{(2)} + f^{(3)}$, the soft operators are

$$V_{SOFT} = A_3 m_{3/2} f^{(3)} + h.c. + A_2 m_{3/2} f^{(2)} + h.c. + \sum_i m_i^2 A_i^* A_i \qquad (2.4)$$

where A_2 and A_3 are unknown numerical parameters of O(1), $m_{3/2}$ is the gravitino mass which sets the scale of the supersymmetry breaking and is typically O(100 GeV) to O(1 TeV), and m_i is a soft mass for scalar A_i, which is very close to $m_{3/2}$ for all scalars except those which occur in operators with large Yukawa couplings such as the top squark. With the minimal set of chiral superfields, the most general model of this sort with arbitrary R_p breaking in superrenormalizable terms is described by the superpotential

$$f = m H'H + m_a L_a H + \lambda_u Q\bar{U}H + \lambda_D Q\bar{D}H' + \lambda_E L\bar{E}H'. \qquad (2.5)$$

We will allow one further complication to the theory: small scalar mass terms of the form $\mu_{ij}^2 A_i A_j$ + h.c. Such terms invariably result from radiative corrections in grand unified theories when superheavy particles are integrated out. The potential for our model is

$$V = V_{SUSY} + V_{SOFT} + V_{RAD} \qquad (2.6)$$

where V_{susy} contains the usual F^2 and D^2 terms, V_{soft} is given by inserting (2.5) in (2.4), and

$$V_{RAD} = \mu_R^2 H'H + \mu_{Ra}^2 L_a H + h.c. \qquad (2.7)$$

At first sight the lepton number violation caused by m_a and μ_{Ra} would appear to require these parameters to be very small. This is not the case, because first one must decide which linear combinations of L_a and H are to be interpreted as

leptons and which as a Higgs. We define the leptons to be $\tilde{L}_a$, such that under an SU(4) rotation on superfields

$$\begin{pmatrix} H' \\ L_a \end{pmatrix} = U \begin{pmatrix} \tilde{H}' \\ \tilde{L}_a \end{pmatrix} \tag{2.8}$$

$f^{(2)}$ does not contain $\tilde{L}_a$. Working to first order in $\delta_a \equiv m_a/m$, which we assume to be less than unity, $U = \begin{pmatrix} 1 & -\delta_a \\ \delta_a & I \end{pmatrix}$ and, after dropping tildes, the theory is described by [Footnote 2]:

$$\begin{aligned} f_{LE} &= mH'H + \lambda_U Q\bar{U}H + \lambda_D Q\bar{D}H' + \lambda_E L\bar{E}H' \\ &\quad + (\lambda_D Q\bar{D})\delta_a L_a + (\lambda_E L\bar{E})\delta_a L_a \end{aligned} \tag{2.9}$$

$$V_{LE} = V_{susy} + V_{SOFT} + V_{RAD}$$

where V_{susy} and V_{soft} are obtained from f_{LE} in the usual way, and

$$V_{RAD} = \mu^2 HH' + \mu_a^2 L_a H + h.c. \tag{2.10}$$

with $\mu^2 = \mu_R^2 + \delta_a\mu_{Ra}^2$ and $\mu_a^2 = \mu_{Ra}^2 - \delta_a\mu_R^2$. Generation indices have been omitted from the Yukawa couplings; we will work in a basis where the Higgs vacuum expectation values (vevs) give diagonal contributions to the fermion mass matrices.

There are two sources of lepton number violation: trilinear terms in f_{LE} which are suppressed by Yukawa couplings, and small scalar mass parameters μ_a^2 which arise radiatively in grand unified theories. We note that the special case $\mu_a^2 = 0$ has particularly simple lepton number violation as the sneutrinos have no vevs and the neutrinos are massless at tree level. This special case

requires $\delta_a \equiv m_a/m = \mu_{Ra}^2/\mu_R^2 \equiv \theta_a$; that is, the ratio of H'H and L_aH terms is the same in f and V_{RAD}. This does not occur in the grand unified theories which we have investigated. In Section III we study the phenomenological constraints on the parameters δ_a and μ_a^2. In the remainder of this section we consider the vevs resulting from the potential V_{LE}.

For μ_a^2 small compared with $m_{3/2}^2$ the usual mechanism for weak interaction breaking through vevs of H and H' is not upset.[13,14] A large top quark Yukawa coupling causes m_H^2 to become negative under renormalization group scaling from the Planck mass to low energies. At low energies we assume that the potential is bounded from below

$$m_H^2 + m_{H'}^2 + 2m^2 > 2(\mu^2 + A_2 m_{3/2} m) \tag{2.11}$$

and that the Higgs boson mass squared matrix has a negative eigenvalue

$$(m_H^2 + m^2)(m_{H'}^2 + m^2) < (\mu^2 + A_2 m_{3/2} m)^2 \tag{2.12}$$

so that both H and H' acquire vevs: $H = \begin{pmatrix} 0 \\ h \end{pmatrix}$, $H' = \begin{pmatrix} h' \\ 0 \end{pmatrix}$ with $h = (v/\sqrt{2})\cos\beta$ and $h' = (v/\sqrt{2})\sin\beta$ where

$$v^2 = \frac{4}{g^2+g'^2}\left\{-(m_H^2 + m_{H'}^2 + 2m^2) + \left|\frac{m_H^2 - m_{H'}^2}{\cos 2\beta}\right|\right\}$$

$$\sin 2\beta = \frac{2(\mu^2 + A_2 m_{3/2} m)}{m_H^2 + m_{H'}^2 + 2m^2} \tag{2.13}$$

The gravitino mass is chosen such that v = 250 GeV, yielding the correct W mass.

Inserting the H vev into the $\mu_a^2 L_a H$ term yields a term in V_{LE} linear in the sneutrino field N_a giving sneutrino vevs

$$n_a \sim \frac{\mu_a^2}{m_{\nu_2}^2} v \cos\beta \sim \left(\frac{\mu_a^2}{250\ \text{GeV}}\right)\left(\frac{250\ \text{GeV}}{m_{\nu_2}}\right)^2 \text{GeV}. \tag{2.14}$$

The approximations which we use here and elsewhere are: $\mu_a^2 << m_{3/2}^2$ so that $n_a < v$, $A_2, A_3 \sim O(1)$, $\mu^2 < m m_{3/2}$ and $m < m_{3/2}$ so that $\sin\beta \sim m/m_{3/2}$. We will quote all our results for the central values of m = 25 GeV and $m_{3/2}$ = 250 GeV, and will also display the dependence on m and $m_{3/2}$. There are large uncertainties in the values of m and $m_{3/2}$, but it should be noted that if $m \leq 10^{-2} m_{3/2}$ the bottom quark Yukawa coupling becomes non-perturbative.

III. PHENOMENOLOGICAL CONSTRAINTS ON L VIOLATING PARAMETERS

In this section we study the lepton number violation of the theory given by Equations 2.9 and 2.10, and derive constraints on δ_a and μ_a^2.

There are at least three contributions to neutrino masses. The sneutrino vevs lead to a tree level mass, while the parameters δ_a give rise to a radiative neutrino mass at one loop. There are further one loop diagrams which result both from δ_a and from fermion mass mixing via sneutrino vevs. The neutral fermion mass matrix involving the fields $\tilde{\gamma}$, $\tilde{z}$, H^0, H'^0 and ν_a has off diagonal elements mixing the neutrinos with the zino $\tilde{z}$. On diagonalization, this results in one linear combination of the neutrinos $\nu_1 = \hat{n}_a \nu_a$ acquiring a Majorana mass

$$\frac{m_{\nu_1}}{2} \sim \frac{\sum_a \mu_a^4}{m_{\nu_1}^4} m \quad \text{, where} \quad \hat{n}_a = \frac{n_a}{\sqrt{\sum_a n_a^2}} .$$

The trilinear L violating interactions of f_{LE} allow the diagrams of Figure 2 to give a Majorana mass to the linear combination $\nu_2 = \hat{\delta}_a \nu_a$ where $\hat{\delta}_a = \frac{\delta_a}{\sqrt{\sum_a \delta_a^2}}$

The eigenvalue is given by

$$m_{\nu_2} = \frac{1}{2\sqrt{2}\pi^2} \frac{G_F}{\sin^2\beta} \frac{A_3}{m_{3/2}} \left(3 \sum_b m_{D_b}^4 + \sum_b m_{E_b}^4\right) \sum_a \delta_a^2 \tag{3.1}$$

where m_{Db} and m_{Eb} are the down quark and charged lepton masses of generation b and G_F is the Fermi constant. The contribution from the diagrams of Figure 3 is smaller than the leading terms of Equation 3.1 by a factor of 5.10^4 and is ignored.

Inserting sneutrino vevs into the gauge Yukawa couplings mixes charged leptons (E_b and $\bar{E}_b$) with charged winos ($\tilde{w}^-$ and $\tilde{w}^+$), as well as neutrinos with the zino. These mass mixings allow radiative neutrino masses from the one loop diagrams of Figure 4 in which one vertex is F type and the other D type, and mass terms are shown as crosses. These diagrams involve two powers less of the light

fermion masses than do the purely F type diagrams of Figure 2. However, the lepton number violating mass mixing introduces a suppression factor of n_b divided by the gaugino mass which we take to be of order $m_{3/2}$. Thus these diagrams give leading contributions of order

$$\frac{g^2}{8\pi^2} \frac{(3 m_{D_3}^2, m_{E_3}^2)}{m_{3/2}\, v \sin\beta} \delta_a n_b \quad \nu_a \nu_b \,. \qquad (3.2)$$

The bottom squark/quark loop gives masses comparable to m_{ν_1} and m_{ν_2} and consequently leads to similar limits on δ_a and μ_a^2. The lepton loop contribution is over an order of magnitude smaller and will again be dropped. Notice that the 1PI part of Figure 4(b) is a radiative contribution to the operator $H^*(\nu_a \tilde{z})$, which is absent at tree level. The divegent piece is canceled by a counter term, which is non diagonal by wavefunction mixing, while the finite remainder corrects the tree level mixing of order $(g^2 + g'^2)^{1/2} n_a$ by an amount of order δ_a GeV.

We have argued that the largest contributions to the neutrino mass matrix have the form:

$$\frac{\sum_a \mu_a^4}{m_{3/2}^4} m \quad (\nu_1 \nu_1) + \frac{G_F}{2\pi^2 \sin^2\beta} \frac{m_{D_3}^4}{m_{3/2}} \sum_a \delta_a^2 \quad (\nu_2 \nu_2)$$

$$+ \frac{g^2}{2\pi^2} \cot\beta \frac{m_{D_3}^2}{m_{3/2}^3} \sqrt{\sum_a \mu_a^4} \sqrt{\sum_b \delta_b^2} \quad (\nu_1 \nu_2) \qquad (3.3)$$

where the numerical coefficients are approximate. The general analysis for constraints on δ_a and μ_a resulting from these masses is quite complicated, especially from neutrino oscillations. We will discuss only the simplified case in

which only a single neutrino has an appreciable mass. It would seem that this is likely to occur only if either δ_a or μ_a vanish. In fact it also occurs when $\hat{n}_a = \hat{\delta}_a$ so that $\nu_1 = \nu_2$; this very special case will be precisely the one which results from the grand unified model considered in Section IV. Even in this special case the sub-dominant terms which we neglect will give small masses to other linear combinations of neutrinos.

If the heavy neutrino is almost entirely ν_e, ν_μ or ν_τ then the mass limits on these neutrinos of 46eV, 520 keV and 250 MeV lead to constraints on μ_a and δ_a. If the tree level terms of Equation 3.3 dominate, then the respective limits are

$$\mu_e \lesssim 2 \cdot \left(\frac{m_{3/2}}{250\,\text{GeV}}\right)\left(\frac{25\,\text{GeV}}{m}\right)^{1/4} \text{GeV} \quad \text{or} \quad n_e \lesssim 20 \sqrt{\frac{25\,\text{GeV}}{m}}\,\text{MeV}$$

$$\mu_\mu \lesssim 20 \cdot \left(\frac{m_{3/2}}{250\,\text{GeV}}\right)\left(\frac{25\,\text{GeV}}{m}\right)^{1/4} \text{GeV} \quad \text{or} \quad n_\mu \lesssim 2 \sqrt{\frac{25\,\text{GeV}}{m}}\,\text{GeV}$$

$$\mu_\tau \lesssim 100 \cdot \left(\frac{m_{3/2}}{250\,\text{GeV}}\right)\left(\frac{25\,\text{GeV}}{m}\right)^{1/4} \text{GeV} \quad \text{or} \quad n_\tau \lesssim 40 \sqrt{\frac{25\,\text{GeV}}{m}}\,\text{GeV}. \qquad (3.4)$$

Since μ_a are expected to result radiatively from a grand unified theory, these restrictions are not very severe. Infact the leading contributions to the neutrino masses may well come from the radiative term of Equation 3.1. For the case when ν_2 is ν_μ or ν_τ, δ_μ or δ_τ of O(1) would be acceptable. However if ν_2 is ν_e then we find a limit $\delta_e \leq 1/50\ (m/25\ \text{GeV})\ (250\ \text{GeV}/m_{3/2})^{1/2}$.

Searches for neutrinoless double beta decay have imposed a strong upper limit on the coefficient of the Majorana mass operator $\nu_e \nu_e$ of 5.6 eV.[24] This leads to a constraint on δ_e and μ_e which is independent of δ_μ, δ_τ, μ_μ and μ_τ:

$$\left| \frac{2\mu_e^4}{m_{3/2}^4} m + \frac{G_F}{\pi^2 \sin^2\beta} \frac{m_{D_3}^4}{m_{3/2}} \delta_e^2 + \frac{g^2}{\pi^2} \cot\beta \frac{m_{D_3}^2}{m_{3/2}^2} \mu_e^2 \delta_e \right| < 5.6\,\text{eV} \qquad (3.5)$$

where numerical factors of O(1) and relative signs have been dropped. Assuming no cancellations, this improves the limits to $\mu_e \leq 1$ GeV, $\delta_e \leq 1/150$. A fourth

generation with $m_{D_4} \sim 50$ GeV would tighten the limits on δ_a by two orders of magnitude.

We note that the cosmological bound on stable neutrinos masses gives $m_\nu < 100$ eV.[25] This would decrease considerably the upper bounds on δ_μ, δ_τ, μ_μ and μ_τ

$$\sqrt{\delta_\mu^2 + \delta_\tau^2} \lesssim \frac{1}{35} \quad \text{and} \quad \sqrt[4]{\mu_\mu^4 + \mu_\tau^4} \lesssim 2\,\text{GeV}. \qquad (3.6)$$

More stringent limits result when there is substantial mixing among the three neutrino flavors causing neutrino oscillations. For a single heavy Majorana neutrino $\nu_1 = \nu_2 = \hat{\delta}_a \nu_a = \hat{n}_a \nu_a$ with mass m_ν given by Equation 3.3 and energy E, the probability for oscillation from weak eigenstate ν_a to $\nu_b \neq \nu_a$ over a length L is

$$P_{ab} = 4\,\hat{\delta}_a^2\,\hat{\delta}_b^2 \sin^2\frac{\Delta}{2} \qquad (3.7)$$

and the probability to remain ν_a is

$$P_{aa} = 1 - 4\,\hat{\delta}_a^2\left(1 - \hat{\delta}_a^2\right)\sin^2\frac{\Delta}{2} \qquad (3.8)$$

where $\Delta = m_\nu^2\, L/2E$. It is interesting that the minimum value for the asymptotic value of P_{ee} is 1/2, occuring for $\hat{\delta}_e = 1/\sqrt{2}$. There are large uncertainties associated with values of P_{ee} obtained from solar neutrino experiments, and we do not use these results to impose constraints on $\hat{\delta}_a$. Constraints on the mixing parameters $\hat{\delta}_a$ from accelerator and reactor experiments[26] depend on the value of m_ν. If $m_\nu \leq 0.3$ eV then all terrestrial experiments have $\sin^2 \Delta/2$ sufficiently small that there are no constraints on $\hat{\delta}_a$

[Footnote 3]. For 1 eV $\leq m_\nu \leq$ 25 eV the constraints are rather complicated, however for $m_\nu \geq$ 25eV, $\sin^2\Delta/2$ can be replaced by its asymptotic value of 1/2 and simple constraints result. Reactor experiments yield values of P_{ee} within 10% of unity, and the Baksan experiment gives $P_{\mu\mu}$ within 20% of unity, hence $\hat{\delta}_e$ and $\hat{\delta}_\mu$ are either close to zero or one. For example, if $\hat{\delta}_\mu$ is close to one then it can only differ from one by about 5%; $1 - \hat{\delta}_\mu \leq 1/20$; and in this case $P_{\mu e} \leq 10^{-3}$ requires $\hat{\delta}_e < 1/45$. If it is $\hat{\delta}_\tau$ that is near unity then the limits are very mild; for example $\hat{\delta}_\tau \geq 0.9$, $\hat{\delta}_e \leq 0.2$ and $\hat{\delta}_\mu \leq 0.1$ are acceptable values. This is both the least restricted and most natural case.

The lepton number violating trilinear interactions of f_{LE} will allow a variety of rare processes. In a mass basis λ_D will be a diagonal matrix, so that $\delta_a L_a(\lambda_D Q\overline{D})$ has flavor changing quark interactions only for the charged component of L_a and not for the neutral component. Thus the diagrams of Figures 5 and 6 for $K_L \to \mu^+e^-$ (also μ^-e^+, e^+e^-, $\mu^+\mu^-$) and $K^o\overline{K^o}$ mixing are absent in this theory. They would be present in a theory in which the R_p violating operators are not restricted to be superrenormalizable in the original basis. The $\delta_a\lambda_D$ vertex involving charged sleptons gives rise to new box diagram contributions to $K^o\overline{K^o}$ mixing as shown in Figure.7. Although not GIM suppressed, these diagrams are suppressed by powers of the Yukawa couplings and are

$$O\left(\frac{m_d m_s}{m_c^2}\;\frac{\sum_a \hat{\delta}_a^2}{\sin^2\beta}\;\frac{m_K^2}{(m_d+m_s)^2}\right)$$

compared to the usual amplitude. For $\sin\beta \geq 1/15$ the quark mass suppression is sufficient for δ_a to be unconstrained. If $\sin\beta$ is near its miminum acceptable value of 10^{-2} then an important new constraint emerges: $\delta_\mu, \delta_\tau \leq O(1/5)$. The example of the box diagram illustrates a fairly general point: our definition of lepton number is such that all lepton number violating operators in f_{LE} are suppressed by a small Yukawa coupling. In any process these operator will always lead to a

squark or slepton propagator, so that the effect tends to have a magnitude comparable to second order weak effects.

In $\pi^{\mp}(K^{\mp}) \to \ell^{\mp}\overset{(-)}{\nu}$ decay the first order weak amplitude is inhibited by a helicity suppression factor of $m_\ell(m_u + m_d)/m_\pi^2$ or $m_\ell(m_u + m_s)/m_K^2$ compared with the squark exchange diagram of Figure 8 which induces $\pi^{\mp}(K^{\pm}) \to \ell^{\mp}\tilde{\gamma}$ for a sufficiently light photino. In this case the small Yukawa couplings of the lepton number violating process are offset by the absence of any helicity suppression, so that a limit on δ_ℓ is required to ensure that the $\ell^{\mp}\tilde{\gamma}$ final state does not mimic a $\ell^{\mp}\overset{(-)}{\nu}$ final state. This would upset the agreement between experiments and the standard model predictions for $\pi^{\mp}(K^{\mp}) \to \ell^{\mp}X$, with X unobserved neutrals. The decay rate for $M \to \ell\tilde{\gamma}$ is

$$\Gamma(M \to \ell\tilde{\gamma}) = \frac{\alpha G_F}{2\sqrt{2}\sin^2\beta} \frac{f_M^2 m_M^2 \delta_\ell^2}{m_{3/2}^4 \left(1 + \frac{m_u}{m_{d,s}}\right)^2} \left(1 - \frac{m_\ell^2 + m_{\tilde{\gamma}}^2}{m_M^2}\right) p_{\tilde{\gamma}} \qquad (3.9)$$

where f_M is $f_\pi \cos\theta_c$, $f_K \sin\theta_c$ for $M = \pi, K$. The improved accuracy of the $\pi^+ \to e^+X$ decay measurements[27] restricts the decay rate to $e^+\tilde{\gamma}$ to be less than 1% of the rate to $e^+\nu_e$ as calculated in the standard model. This gives

$$\delta_e \lesssim 1.0 \left(\frac{m}{25\,\text{GeV}}\right)\left(\frac{m_{3/2}}{250\,\text{GeV}}\right)\sqrt{\frac{p_\nu}{p_{\tilde{\gamma}}}}\left(1 - \frac{m_{\tilde{\gamma}}^2}{m_\pi^2}\right)^{-1/2} . \qquad (3.10)$$

For K decay the corresponding limit

$$\delta_e \lesssim \frac{1}{6}\left(\frac{m}{25\,\text{GeV}}\right)\left(\frac{m_{3/2}}{250\,\text{GeV}}\right)\sqrt{\frac{p_\nu}{p_{\tilde{\gamma}}}}\left(1 - \frac{m_{\tilde{\gamma}}^2}{m_K^2}\right)^{-1/2} \qquad (3.11)$$

results from requiring $\Gamma(K^+ \to e^+\tilde{\gamma}) < (1/10)\,\Gamma(K^+ \to e^+\nu_e)$. For most values of m and $m_{3/2}$ these limits are less stringent than that from double beta decay; the limits on δ_μ are more interesting. The recent efforts to search for components of

massive neutrinos in $\pi_{\mu 2}$ and $K_{\mu 2}$ decays set upper bounds on $\pi \to \mu\tilde{\gamma}$ and $K \to \mu\tilde{\gamma}$,[28] and therefore on δ_μ. Since the experimental upper bounds depend sensitively on values of $m_{\tilde{\gamma}}$, the resulting bounds on δ_μ are also $m_{\tilde{\gamma}}$-dependent. They cover a range of $m_{\tilde{\gamma}}$ from 4 to 34 MeV for $\pi_{\mu 2}$ and 60 to 320 MeV for $K_{\mu 2}$. The most stringent bound is obtained from $K_{\mu 2}$ for $m_{\tilde{\gamma}}$ between 200 and 300 MeV[29] giving

$$\delta_\mu \lesssim \frac{1}{3}\left(\frac{m}{25\,\text{GeV}}\right)\left(\frac{m_{3/2}}{250\,\text{GeV}}\right)\sqrt{\frac{p_\nu}{p_{\tilde{\gamma}}}} . \qquad (3.12)$$

The diagrams of Figure 9 lead to lepton number violating semileptonic decays of B mesons. Such branching ratios are of order 10^{-4} and would not give rise to a measurable anomalous value for the total fraction of semileptonic events.

The rare decay modes of muons give further constraints on δ_a and μ_a. The diagrams of Figure 10 lead to an amplitude proportional to $\delta_e\delta_\mu$ for $\mu \to e\tilde{\gamma}$. However, the structure of the L violating cubic interactions are such that no diagram exists with internal quark/squark as tao/stao loops. The remaining diagrams have suppression factors of $(m_e^2 m_{\nu_\mu}/m_{3/2}^3)$ and $(m_e m_{\nu_e}/m_{3/2}^2)$ and are consequently unimportant. For non-zero μ_a, $\mu \to e\tilde{\gamma}$ does lead to a constraint on μ_a and δ_a from the diagrams of Figure 11 which have no $(m_\nu/m_{3/2})$ suppression. From Figure 11-a the constraint is

$$\delta_e \mu_\mu \lesssim \frac{1}{40}\,\text{GeV}\left(\frac{m}{25\,\text{GeV}}\right)\left(\frac{m_{3/2}}{250\,\text{GeV}}\right) \qquad (3.13)$$

and from Figure 11b the constraint is

$$\delta_\mu \mu_e \lesssim 5\,\text{GeV}\left(\frac{m}{25\,\text{GeV}}\right)\left(\frac{m_{3/2}}{250\,\text{GeV}}\right) . \qquad (3.14)$$

The latter constraint is less powerful by a factor m_μ/m_e and since we already know $n_e \leq 10^{-2}$ GeV it provides no constraint on δ_μ. For small $m_{3/2}$, the constraint on $\delta_e n_\mu$ may be more powerful than the combined limit on δ_e from double beta decay and n_μ from tree level neutrino masses

$$\delta_e n_\mu \lesssim \frac{1}{150}\ \mathrm{GeV}\left(\frac{m}{25\ \mathrm{GeV}}\right)^{1/2}\left(\frac{250\ \mathrm{GeV}}{m_{3/2}}\right)^{1/2} . \qquad (3.15)$$

If $m_{3/2}$ is larger than 250 GeV (for example above the 10 TeV cosmological bound)[30] then these constraints from K, π and μ decay become less stringent, while those from neutrino masses are more restrictive. The converse holds for a lighter gravitino.

The decay $\mu \to e\bar{e}e$ also requires the breaking of both electron and muon numbers. The diagrams of Figure 12 are proportional to $\delta_e\delta_\mu$ and do not require small chirality changing masses, leading to a constraint

$$\delta_e \delta_\mu \lesssim \frac{1}{300}\left(\frac{m}{25\ \mathrm{GeV}}\right)^2 . \qquad (3.16)$$

Accepting the double beta decay limit of $\delta_e \leq 1/150$ this only provides a more stringent limit for small m or $m_{3/2}$. Similarly the decay $\tau \to \mu\bar{\mu}\mu$ has a branching ratio less than 5.10^{-4}, and this narrowly misses constraining $\delta_\mu\delta_\tau$. In fact we are essentially unable to constrain δ_μ and δ_τ except by Equation (3.12) and from certain coupled limits with δ_e from neutrino oscillations.

We have demonstrated that μ_a and δ_a are constrained from a variety of sources. However, the most interesting result is that the constraints are relatively mild. For example, with m = 25 GeV and $m_{3/2}$ = 250 GeV present terrestrial experiments do not rule out $\delta_e \sim 1/150$, $\delta_\mu \sim 1/10$, $\delta_\tau \sim 1$, $\mu_e \sim 1$ GeV,

$\mu_\mu \sim 5$ GeV and $\mu_\tau \sim 50$ GeV. A theory with these parameters could lead to the observation of neutrino masses and oscillation, $\mu \to e\gamma$, $\mu \to e\bar{e}e$, $\tau \to \mu\bar{\mu}\mu$ and an anomalous result for $K \to \mu X$ in the near future.

In models with R_p unbroken the lightest R_p odd particle is absolutely stable. For models considered in this paper, those coupled to N = 1 supergravity, this particle is usually a linear combination of the neutral fermions. The gaugino Majorana masses are essentally free parameters in this theory but are plausibly of order $m_{3/2}$. However, it often occurs that the lightest neutral R_p odd fermion has a mass of only a GeV or so.[13,15,16,31] If the wino and bino have a common mass at low energies, this lightest eigenstate is the photino [Footnote 4]. The dominant photino decay mode depends on its mass. The decay into $b\bar{b}\nu_a$ via the diagram of Figure 13 is dominant for $m_{\tilde{\gamma}} > 2m_b$, with a decay rate

$$\Gamma_{b\bar{b}\nu+\bar{b}b\bar{\nu}} = \frac{\alpha}{96\sqrt{2}\pi^2}\frac{G_F}{\sin^2\beta}\frac{m_b^2 m_{\tilde{\gamma}}^5}{m_{3/2}^4}\left(\delta_e^2+\delta_\mu^2+\delta_\tau^2\right). \qquad (3.17)$$

The lifetime therefore scales as

$$\tau \sim 2\cdot 10^{-13}\frac{1}{\delta_e^2+\delta_\mu^2+\delta_\tau^2}\left(\frac{10\ \mathrm{GeV}}{m_{\tilde{\gamma}}}\right)^5\left(\frac{m}{25\ \mathrm{GeV}}\right)^2\left(\frac{m_{3/2}}{250\ \mathrm{GeV}}\right)^2 \text{ seconds.} \qquad (3.18)$$

Actually, if the photino were sufficiently heavy, a similar diagram would allow a decay to $t\bar{b}\ell_a$ and $\bar{t}b\bar{\ell}_a$. Below the $b\bar{b}\nu$ threshold, the photino decays predominately to $\nu_a\gamma$ and $\bar{\nu}_a\gamma$ via the diagrams of Figure 4(a), with a rate

$$\Gamma_{\nu_a\gamma+\bar{\nu}_a\gamma} = \frac{\alpha^2 G_F}{144\sqrt{2}\pi^3\sin^2\beta}\frac{m_b^4 m_{\tilde{\gamma}}^3}{m_{3/2}^4}\left(\ln\frac{m_{3/2}^2}{m_b^2}-\frac{3}{2}\right)^2\delta_a^2 \qquad (3.19)$$

leading to a lifetime

$$\tau \sim 10^{-8}\ \text{seconds}\ \frac{1}{\delta_e^2+\delta_\mu^2+\delta_\tau^2}\left(\frac{1\,\text{GeV}}{m_{\tilde\gamma}}\right)^3\left(\frac{m}{25\,\text{GeV}}\right)^2\left(\frac{m_{3/2}}{25\,\text{GeV}}\right)^2 \tag{3.20}$$

which is valid providing the photino mass is large compared to the neutrino masses. The cosmological abundance of photinos will not conflict with the standard picture of nucleosynthesis providing $m_{\tilde\gamma} \geq 1/(\delta_e^2 + \delta_\mu^2 + \delta_\tau^2)^{1/3}$ MeV, a lower bound which is certainly expected to be satisfied in these models. We have discussed photino decay when diagrams involving δ_a are larger than those with sneutrino vevs. If the reverse is true, the details will change but not the overall picture. For example, the diagrams of Figure 4(b) lead to a lifetime

$$\tau \sim 10^{-8}\ \text{seconds}\left(\frac{100\ \text{GeV}^2}{n_e^2+n_\mu^2+n_\tau^2}\right)\left(\frac{1\,\text{GeV}}{m_{\tilde\gamma}}\right)^3\left(\frac{m}{25\,\text{GeV}}\right)^2\left(\frac{m_{3/2}}{250\,\text{GeV}}\right)^2 \tag{3.21}$$

which may compete with (3.20), depending on the relative importance of n_a and δ_a.

In the next section we demonstrate that this rich structure of lepton number violation could result from a grand unified theory.

IV. A GRAND UNIFIED THEORY WITH R_p BREAKING

The first part of this paper has been concerned with setting restrictions on the R_p violating interactions of the SU(3) × SU(2) × U(1) theory described by Equations 2.4-2.7. Our analysis is relevant whenever this form for the low energy effective theory results from a more unified model. We now consider a particular example of such a model: a supersymmetric SU(5) grand unified theory coupled to N = 1 supergravity,[2,3] with supersymmetry breaking arising from a hidden sector.[4-16] A general analysis of how such models can lead at low energies to the theory of Equations 2.9, 10 will not be given; rather we will give a specific example of such a model and explain the reasons for our choice. It is both interesting and surprising that without any fine tuning of parameters we can write down models having no R_p invariance which nonetheless lead to acceptable and observable B and L violating operators in the low energy superpotential.

Many SU(5) models require a fine tuning of parameters of the superpotential to keep the Higgs doublets much lighter than their color triplet partners. We will avoid this by making use of the missing partners method,[32] so that the minimal chiral superfield content of the theory is $\Sigma(75)$, , $H(5)$, $\bar H(\bar 5)$, $S(50)$, $\bar S(\overline{50})$, $T_a(10)$ and $F_a(\bar 5)$, where the SU(5) representations are in parenthesis. The grand unified theory is the effective theory describing interaction of particles, which are much lighter than the Planck mass (M_p), at momenta much less than M_p, and we expect it to contain non-renormalizable operators scaled by powers of $(1/M_p)$. Hence it is not sufficient to explain the absence of the R_p violating terms $\bar F_a H$ and $T_a \bar F_b \bar F_c$ from the superpotential; the absence of $\bar F_a \Sigma^2 H$ and $T_a \bar F_b \bar F_c \Sigma^n$ must be explained, at least for $n < O(6)$. Instead of forbidding these terms by R_p, which would make R_p an exact symmetry of the theory, we forbid them by a new discrete symmetry, θ parity, which allows R_p violating interactions among T, $\bar F$ and superheavy fields. Under θ parity $\theta \to i\theta$, so that each term in the superpotential must change sign, and fields rotate by

$\exp(i\theta_p \pi/5)$ where the θ_p charges are $\Sigma(2)$, $T(4)$, $\bar{H}(6)$, $\bar{S}(6)$, $H(-3)$, $S(-3)$ and $\bar{F}(-5)$. All allowed terms in the superpotential must have θ_p equal to an odd multiple of 5, which is always odd. The operators $\bar{F}H$, $T\bar{F}\bar{F}$ and Σ all have even θ_p so that the dangerous R_p violating terms are forbidden.

Missing partner models work only if $\bar{H}\Sigma^n H$ can be forbidden for $n < O(6)$. For renormalizable terms this can be arranged by using a U(l) global symmetry.[32] θ parity is a very powerful and helpful tool in this respect, since it allows only $\bar{H}\Sigma H\Sigma^{5n}$ for integral n. The lowest dimension term which is both θ_p invariant and gauge invariant is $\bar{H}\Sigma^6 H$, which will lead to a desired Higgs mass at low energies. Ours is an "all orders" missing partners model.

It is very simple to write down all the terms of the superpotential of dimension less than eight. The fields T_a, $\bar{H}$, Σ and $\bar{S}$ have θ_p even (E fields). In fact θ_p is equal to F + 2 modulo 5, where F is the fivality of the representation. The remaining fields $\bar{F}$, H and S have odd θ_p(O fields), and are in a different fivality class having θ_p = F - 4 modulo 5. Hence up to dimension seven the only allowed operators in the superpotential have the form E^2O, EO^3 and O^5, giving [Footnote 5]

$$f_1 = \lambda_{U\,ab} T_a T_b H + \lambda_{Daa} T_a \bar{F}_a \bar{H} + \lambda_1 \bar{S}\Sigma S + \lambda_2 \bar{S}\Sigma H + \lambda_3 S\Sigma\bar{H} + TTS + \bar{H}H\bar{F}H + \bar{F}H\bar{S}S + \bar{H}\bar{F}SS + SS\bar{S}\bar{F} + S^5 + \lambda_8 \bar{H}\Sigma^6 H + \dots \qquad (4.1)$$

We label only those Yukawa's which will be of interest to us later. Notice that we have gone to a basis of the T_a and $\bar{F}_a$ for which λ_D is a real and diagonal matrix. Thus at tree level these fields are in the mass eigenstate basis for down quarks although not, of course, for the up quarks. The unitary matrix needed to diagonalize the symmetric matrix λ_U is the Kobayashi Maskawa matrix. The only R_p violating operators to this order have a single F field and are of dimension four. All R_p

violating terms induced radiatively in the low energy theory will therefore be suppressed by at least one power of M_G/M_p, where M_G is the grand unification mass.

The SU(5) symmetry breaking is a weak aspect of this model: nothing forces Σ to acquire a vev at the grand unified scale, and nothing at this scale picks out the direction in which the unbroken subgroup is SU(3) × SU(2) × U(1). However, nothing prevents such a vev either, so that we would not view this as a disaster. The disaster is that even if Σ acquired such a vev, its 63 uneaten components are not superheavy, ruining any hope of perturbative unification of gauge couplings. This is rectified by quadratically coupling Σ in f to a new field $\Sigma'(75)$ which has $\theta_p = 3$. The terms in the superpotential involving Σ' are:

$$f_2 = M_\Sigma \Sigma\Sigma' + \lambda_{4a} \Sigma' S \bar{F}_a + \Sigma'^5 + \Sigma^3\Sigma'^3 + \Sigma'^3 T\bar{H}S + \Sigma'^3 T\bar{S}\bar{S} + \Sigma'^3 T\bar{H}\bar{H} + \Sigma'^4 \bar{H}H + \Sigma'^4 \bar{S}S + \Sigma\Sigma'^6 + \dots \qquad (4.2)$$

We have not attempted to search for the deepest minimum of the tree level effective potential obtained by coupling $f_1 + f_2$ to N = 1 supergravity, with an additional hidden sector superpotential designed to break supersymmetry [Footnote 6]. We simply assume that there is a choice of parameters for which Σ acquires a vev breaking SU(5) to SU(3) × SU(2) × U(1), while all other fields have zero vev. In particular, if Σ' acquired a vev the hierarchy would be ruined; in θ parity models of this sort, only fields with even θ_p should acquire superheavy vevs. The addition of Σ' has promoted R_p violation to the renormalizable interactions of $\bar{F}_a$ with superheavy fields. This is important in inducing lepton number violating parameters m_a and μ_a^2 of sufficient size to be interesting.

We will derive the low energy form of the theory [Footnote 7].

$$\mathcal{L} = \Big[\Phi_i^\dagger e^{gV} \Phi_i + \bar\theta^2 (f_1 + f_2) + \theta^2 (\bar f_1 + \bar f_2) \tag{4.3}$$
$$+ \bar\theta^2 (f_1 + f_2) Y + \theta^2 (\bar f_1 + \bar f_2) Y^\dagger + m_i^2 \Phi_i^\dagger \Phi_i Y^\dagger Y \Big]_D$$

where $i = 1 \ldots 8$ runs over the 8 chiral superfields. The last three terms represent the soft supersymmetry breaking operators which result from supergravity auxiliary field couplings and are assumed to have the minimal form [Footnote 8]. The field Y is a dimensionless spurion field given by $Y = Am_{3/2}\,\theta^2$ and is assigned zero θ_p. Any hidden sector which breaks supersymmetry also necessarily spontaneously breaks θ_p, and we are not surprised that these soft operators are θ_p violating. The parameter A is a constraint of O(1) which can depend on the dimensionality of the term in f which it multiples, and $m_{3/2}$ is the gravitino mass which sets the scale of the soft operators. The parameters m_i are of $O(m_{3/2})$ as are the gaugino Majorana masses which have been omitted. Below the grand unified scale $M_G \sim 10^{16}$GeV, the renormalization group scaling of the Higgs mass squared operator m_H^2 is sufficient to break weak interactions providing the top quark is heavier than 50 GeV,[13-15] or providing the A parameter is large and we live in a long lived false vacuum.[16]

To see that this model does lead to a low energy theory of the form given by Equations 2.4-2.7 we consider the radiative generation of operators involving the light fields $H_2 \subset H$, $H_2' \subset \bar H$, $\bar F$ and T, which we generically call Λ. The doublet and triplet components of H and $\bar H$ are subscripted 2 and 3. It is convenient to work in the unbroken theory and insert Σ vevs later, since this allows the use of θ parity to enumerate allowed operators. All radiative operators generated by loop diagrams have the form $[O]_D$ where O has θ_p equal to zero modulo 10 and can involve Y and $Y^\dagger$ fields. It is clear that none of the operators appearing in f are radiatively induced; in fact there are very few radiative operators of interest. Diagonal wavefunction renormalization operators $[\Lambda^\dagger \Lambda]_D$ are of no interest, although an off-diagonal mixing wavefunction renormalization $[H^\dagger \bar F \Sigma^n]_D$ would be very dangerous but is fortunately

forbidden by θ_p. Operators of the form $[\Lambda^\dagger \Lambda Y]_D$ occur with logarithmically divergent coefficients. On eliminating auxiliary fields by their equations of motion, these operators are found to renormalization group scale the soft scalar operators. Similarly $[\Lambda^\dagger \Lambda Y^\dagger Y]_D$ scales the m_i^2 coefficients. We have stressed that such scalings have important applications, but not for the leading R_p violating interactions at low energy which is our main concern. The lowest dimension operators which give R_p violation are $[\bar F H \Sigma^\dagger \Sigma^\dagger \Sigma Y^\dagger]_D$ and $[\bar F H \Sigma^\dagger \Sigma^\dagger \Sigma Y^\dagger Y]_D$ which are induced by diagrams of Figure 16 giving the low energy terms:

$$\frac{\lambda_1 \lambda_2}{8\pi^2} \left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 \right) A_3 m_{3/2} \lambda_{4a} \left\{ [L_a H_2]_F + A_3 m_{3/2} [L_a H_2]_A \right\} \tag{4.4}$$

There are no $[\bar H H \Sigma^n Y^\dagger]_D$ or $[\bar H H \Sigma^n Y^\dagger Y]_D$ radiative terms as they violate θ_p, however there is a tree level term $\lambda_8 M_G^6 / M_p^5 [H_2 H_2']_F$.

All other radiative D terms involve three or more light Λ fields and are suppressed by powers of $1/M_G$, and are therefore unimportant unless they are B violating. Operators $[TT\bar F^\dagger \Sigma^n]_D$ and $[T\bar H H^\dagger \Sigma^n]_D$ are forbidden by θ_p, as are the same operators with Y and $Y^\dagger Y$ insertions. This leaves a single dangerous B violating operator $[T_a \bar F_b \bar F_c \Sigma^\dagger \Sigma^\dagger Y^\dagger]_D$ which contains $C_{abc}[\bar U_a \bar D_b \bar D_c]_F$, where C_{abc} involves elements of the Kobayashi Maskawa matrix needed to ensure that $\bar U_a$ are mass eigenstate fields. The experimental upper limit on the coefficients $C_{112}, C_{113}, C_{123}$ is derived from the diagrams of Figure 17, and found to be 10^{-22} for values of δ_a near unity. Note that for $\delta_\tau > \delta_{e,\mu}$ the dominant decay mode for the proton is the Cabibbo allowed $K^+ \bar\nu_\tau$ mode. One might expect the coefficients to be $\lambda^2/8n^2\,(Am_{3/2})/M$ which would require Yukawa couplings λ to be less than 10^{-3}. It is straightforward to demonstrate that any 1PI diagram generating this operator via the R_p violating $\bar F S \Sigma'$ operator must have the structure of the diagram of Figure 18 and will involve at least seven trilinear vertices of which one must be a fermion mass Yukawa vertex, and

must be of at least two loops. An example is given in Figure 19. Thus the 1PI generation of this operator is not troublesome unless all the coupings involved are made rather large $\lambda \geq O(1/3)$.

The operator $C_{abc}\bar{U}_a\bar{D}_b\bar{D}_c \subset T_a\bar{F}_b\bar{F}_c$ can also arise from the mass mixing of $\bar{F}_3$ with the superheavy color triplets S_3, $\bar{S}_3$, H_3 and $\bar{H}_3$. The diagrams of Figure 20 give radiative $[\bar{F}_3H_3]_F$ and $[\bar{F}_3S_3]_F$ terms of order $m_{3/2}$ and cause the triplet eigenstates of zero mass to become

$$\bar{D}_a = \bar{F}_{3a} + \frac{Am_{3/2}}{M}\frac{\lambda_{4a}}{8\pi^2}\left\{\left(\frac{\lambda_1^2(\lambda_1^2+\lambda_2^2+\lambda_3^2)-\lambda_3^2}{\lambda_3}\right)\bar{H}_3 - \lambda_1^2(\lambda_1^2+\lambda_2^2+\lambda_3^2)\bar{S}_3\right\} + O\left(\frac{m_{3/2}^2}{M^2}\right) \qquad (4.5)$$

where M is roughly the largest mass in the diagram. This means that the $\bar{H}_3$ triplet is not a mass eigenstate, but contains a small component of the massless $\bar{D}_a$ states. Hence in terms of light mass eigenstate fields the second term of f_1 actually contains $C_{abc}\bar{U}_a\bar{D}_b\bar{D}_c$ where:

$$C_{abc} \sim \left(m_{D_b}U_{ba}\lambda_{4c}\right)\frac{A}{8\pi^2}\frac{m_{3/2}}{Mv}\left(\frac{\lambda_1^2(\lambda_1^2+\lambda_2^2+\lambda_3^2)-\lambda_3^2}{\lambda_3}\right) \qquad (4.6)$$

where U_{ba} is the Kobayaski Maskawa matrix, and b is not summed. These contributions to C_{abc} can easily be made $\leq O(10^{-22})$. However, both C_{abc} and δ_a arise from similar radiative mass mixing (compare diagrams of Figures 16 and 20) and the question is whether either, or both, can lead to interesting phenomenology. They are related by:

$$C_{abc} \sim \left(m_{D_b}U_{ba}\,\delta_c\right)\frac{m}{Mv}\frac{1}{\lambda_1\lambda_2\lambda_3}\left(\lambda_1^2 - \frac{\lambda_3^2}{\lambda_1^2+\lambda_2^2+\lambda_3^2}\right) \qquad (4.7)$$

$$C_{1bc} \sim 10^{-19}\,\delta_c\left(\frac{10^{16}\,\mathrm{GeV}}{M}\right)\left(\frac{m}{25\,\mathrm{GeV}}\right)\left(\frac{1}{\lambda_1\lambda_2\lambda_3}\right)\left(\lambda_1^2 - \frac{\lambda_3^2}{\lambda_1^2+\lambda_2^2+\lambda_3^2}\right)$$

where we have taken $m_d \sim m_sU_{21} \sim m_bU_{31} \sim 10$ MeV. This relationship is very interesting, and is typical of models where R_p violation occurs predominately through a single operator. The parameters governing proton decay and neutrino masses and oscillations are intimately related. In the present model we see that if δ_c is large enough to cause observable effects in the neutrino mass matrix, then the $\bar{U}_a\bar{D}_b\bar{D}_c$ operators are almost certainly also of physical interest. For both C_{1bc} and δ_c to be phenomenologically relevant we must arrange for the product of the last four factors of Equation 4.7 to give $O(10^{-2})$.

After a thorough analysis of the operators induced by radiative corrections, we have found that this model does yield a low energy effective theory given by Equations (2.4)-(2.7) with $m_a = \lambda_1\lambda_2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)\,A_3m_{3/2}\,\lambda_{4a}/8\pi^2$,

$$m = \lambda_8\frac{\langle\Sigma\rangle^6}{M_p^5}, \quad \mu_R^2 = 0 \quad \text{and} \quad \mu_{Ra}^2 = A_3m_{3/2}m_a$$

where A_2 and A_3 are the A parameters for bilinear and trilinear interactions. This model clearly predict $\delta_a \neq \theta_a = \mu_{Ra}^2/\mu_R^2$ [Footnote 9] although the size of δ_a and θ_a are essentially free parameters. This is because m_a and m have different origins: m_a occurs radiatively from renormalizable interactions and is much less than M_G because of supersymmetric cancellations, while m occurs at tree level and is much less than M_G because it arises only in an operator of high dimension.

In Section III our phenomenological analysis allowed the independent R_p violating parameters m_a and μ_a^2. In this SU(5) theory both parameters have a common dominant source: λ_{4a}, and are related by $\mu_a^2 \sim m_am_{3/2}$, so that $\delta_a = \mu_a^2/mm_{3/2}$. For large m and $m_{3/2}$, the δ_a parameters will be irrelevant even for μ_a^2 taking their largest values. For small m and $m_{3/2}$, both parameters may be important. In this theory $n_a \propto \mu_a^2 \propto \delta_a$ so that $\hat{n}_a = \hat{\delta}_a$; the tree level and radiative

contributions to the neutrino mass matrix give a mass to the same linear combination of ν_a. This relationship between μ_a^2 and δ_a has occured because there is only a single R_p violating operator of dimension 3 in the superpotential. In more complicated models such a simple relationship will be lost.

This model demonstrates the feasibility of building grand unified models with the R_p violation at low energy discussed in this paper. It is interesting that this can be done without introducing huge B violation, although there are new sources of B violation from $[\overline{U}_a\overline{D}_b\overline{D}_c]_F$. For missing partner models the Weinberg-Sakai-Yanagida[34] proton decay diagrams are suppressed, and when the superheavy gauge vectors weigh $\geq O(10^{16})$ GeV, these new operators will be the dominant sources of B violation. It is rather interesting that if $\delta_\tau >> \delta_{e,\mu}$ then the only appreciable decay mode of the proton (neutron)is $K^+\overline{\nu}_\tau (K^o\overline{\nu}_\tau)$.

V. CONCLUSIONS

In this paper we have studied the possibility that the near absence of B and L violation from dimension 4 operators of the low energy theory is a consequence of a discrete symmetry other than R_p in the underlying grand unified theory .[10] We have considered an SU(5) theory invariant under a sign change of the superpotential. This theory is neither simpler nor more appealing than SU(5) models which have invariance under matter parity, however it preserves a gauge hierarchy without fine tuning even in the presence of operators of arbitrary dimension. Our main reasons for studying this theory are: firstly that the origin of R parity is not understood and therefore alternatives should be considered. Secondly, we find the resulting phenomenology of B and L violation very interesting. Three parameters of the theory, λ_{4a}, may lead to a variety of lepton number violating processes as well as extraordinary baryon decay. If $\lambda_{4\tau}$ is the largest of these parameters, then one might hope to see a mass for ν_τ in the near future, as well as such decays as $\tau \rightarrow \mu\overline{\mu}\mu$. Furthermore the decay of the proton (neutron) to final states other than $K^+\overline{\nu}_\tau(K^o\overline{\nu}_\tau)$ is highly suppressed.

ACKNOWLEDGMENTS

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FOOTNOTES

[1]. This is similar, but not identical to matter parity: $M \rightarrow -M$. They differ only by a sign change of θ and $\bar{\theta}$ which is always irrelevant since every vertex involves an even number of fermions. Operationally R_p and matter parity are interchangeable.

[2]. The reader may object that this SU(4) rotation introduces more confusion than it removes. For example, the lepton number violating effects have now infected the dimension 4 interactions and will therefore cause radiative wavefunction mixing between the L_a and H (for example from the diagrams of Figure 1) which would have been absent in the original basis. This is not a severe problem since counter terms are chosen to ensure that $L_a \rightarrow L_b$ $(a \neq b)$ and $L_a \rightarrow H'$ vanish on shell. We argue that it is the original basis which is cumbersome, since in this basis the L_a fields are not the lepton fields, even at tree level.

[3]. If the result of the Baksan experiment is included then m_ν must be less than .006 eV for $\hat{\delta}_a$ to be unconstrained.

[4]. The photino need not be a mass eigenstate,[13,15,16,31] in which case our observations apply to the lightest neutral fermion mass eigenstate (other than the neutrinos) which will contain an appreciable photino component. Our results will then be slightly modified by mixing angles.

[5]. The non-renormalizable operators in f mix under renormalization with various D terms. For example $[\bar{F}H\bar{S}S]_F$ mixes via Figure 15 into the operator $[\bar{F}S\Sigma^\dagger]_D$. We will ignore such D terms, even though they will be present at tree level, as their R_p violating effects are small.

[6]. θ parity forbids the appearance of a constant in the hidden sector superpotential. The cosmological constant is made to vanish by fine tuning of parameters of the Kähler potential.

[7]. Further gymnastics are necessary to yield a realistic unification. The problem is that the (3,2, 5/6) + (3,2, - 5/6) components of Σ' are light because the equivalent components of Σ are eaten by the supersymmetric extension of the Higgs mechanism. The presence of these fields in the low energy theory results in a unification scale which is too low. One way around this problem is to add an adjoint field A, and allow an interaction $\Sigma\Sigma'A$ so that now it is the (8, 1, 0) + (1, 3, 0) + (1, 1, 0) of A which are light. The unification scale is now too high, but we can arrange for $M_G \sim 10^{16}$ GeV and $\sin^2\theta \sim 0.2$ by having two sets of 50/5 missing partner mechanisms, and two sets of 40/10 missing partner mechanisms. The two mechanisms are related in SU(6).[33]

[8]. A common problem with these types of theories is that the SU(5) gauge coupling is not asymptotically free. This is particularly worrisome in models which are coupled to a hidden sector via supergravity, because a large gauge coupling at the Planck scale invalidates the approximate symmetry which guarantees the minimal structure of the soft operators.

[9]. It is very hard to use θ_p to give a model with $\theta_a = \delta_a$. Such a model would have to forbid radiative $[\bar{F}H\Sigma^n]_D$ terms as well as radiative $[\bar{H}H\Sigma^n]_D$ terms. In this case we would have $V_{RAD} = 0$, with $[L_aH]_F$ and $[H'H]_F$ occuring only at tree level: $f = \bar{H}\Sigma^6H + \bar{F}\Sigma^{6+n}H$ where only n = 0, 1 or 2 would be of interest. The problem with any scheme of this sort is that $\bar{H}$ and $\bar{F}\Sigma^n$ must be given the same θ_p which means that $[T\bar{F}\bar{H}]_F$ would be accompanied by $[T\bar{F}\bar{F}\Sigma^n]_F$ which is not acceptable for n = 0, 1 or 2.

[10]. For grand unified theories based on SO(10) the operators of (2.2) are forbidden at tree level by gauge invariance. It is probable that interesting SO(10) models can be constructed in which these operators occur radiatively. However, supersymmetric SO(10) models have their own problems. We do not know how to extend the missing partners mechanism to SO(10). The simplest SO(10)

models do not work when they are made supersymmetric because the radiative right handed neutrino masses[35] are suppressed by supersymmetric cancellations.[36]

TABLE 1

Chiral superfields of the minimal SU(3) × SU(2) × U(1) theory.

	$L_a = \begin{pmatrix} N \\ E \end{pmatrix}_a$	$\bar{E}_a$	$Q_a = \begin{pmatrix} U \\ D \end{pmatrix}_a$	$\bar{U}_a$	$\bar{D}_a$	H	H′
SU(3)	1	1	3	3	3	1	1
SU(2)	2	1	2	1	1	2	2
U(1)	-1/2	1	1/6	-2/3	1/3	+1/2	-1/2

FIGURE CAPTIONS

[1] Superfield diagrams which lead to off diagonal wavefunction renormalization amongst L_a and H′ fields.

[2]. Component diagrams for radiative neutrino masses which are proportional to $\delta_a \delta_b$. Tildes denote R_p odd members of a superfield.

[3]. Diagrams that contribute only to off-diagonal radiative neutrino masses, proportional to $\delta_a \delta_a (a \neq b)$.

[4]. Radiative contributions to neutrino masses which vanish if $n_a = 0$.

(a). Results from $\bar{E}_a$ mixing with $\tilde{w}^+$.

(b). Results from a renormalization of the $\nu_a \to \tilde{z}$ transition.

[5]. Diagram for $K_L \to \mu^+ e^-$ using

(a). Superfields

(b). Component fields.

[6]. Diagram for $K^o \overline{K^o}$ mixing using

(a). Superfields

(b). Component fields.

[7]. Box diagrams for $K^o \overline{K^o}$ mixing.

[8]. Squark exchange diagrams for the decay $\pi^- \to \ell_a^- \tilde{\gamma}$.

[9]. Slepton exchange diagrams for semileptonic decay of a b quark.

[10]. Diagrams for $\mu \to e\gamma$ which are proportional to $\delta_e \delta_\mu$. The photon can be attached to any charged line.

[11]. Diagrams for $\mu \to e\gamma$ which involve neutrino vevs. The photon can be attached to any charged line.

[12]. Diagram for $\mu^- \to e^- e^+ e^-$.

[13]. Diagrams for $\tilde{\gamma} \to \tau^+ \nu_\tau e^-$ or $\tau^+ \nu_\tau \mu^-$.

[14]. Diagrams for $\tilde{\gamma} \to \nu_a \gamma$.

[15]. Logarithmically divergent diagram which mixes $[\bar{F}H\bar{S}S]_F$ with $[\bar{F}S\Sigma^\dagger]_D$ under renormalization.

[16]. (a). Diagrams which generate $[\bar{F}H\Sigma^\dagger\Sigma^\dagger\Sigma Y^\dagger]_D$.

(b). Diagrams which generate $[\bar{F}H\Sigma^\dagger\Sigma^\dagger \Sigma Y^\dagger Y]_D$.

[17]. Diagram for proton decay to a neutral meson and charged antilepton via the vertex $[\bar{U}\bar{D}_b\bar{D}_c]_F$.

[18]. Structure of 1PI diagram for generating $[T\bar{F}\bar{F}\Sigma^\dagger\Sigma^\dagger]_D$.

[19]. Example of a three loop 1PI diagram for generating $[T\bar{F}\bar{F}\Sigma^\dagger\Sigma^\dagger]_D$.

[20]. Diagrams which cause mass mixing between the triplets in $\bar{F}_a$ and those in H and S.

FIGURE 1

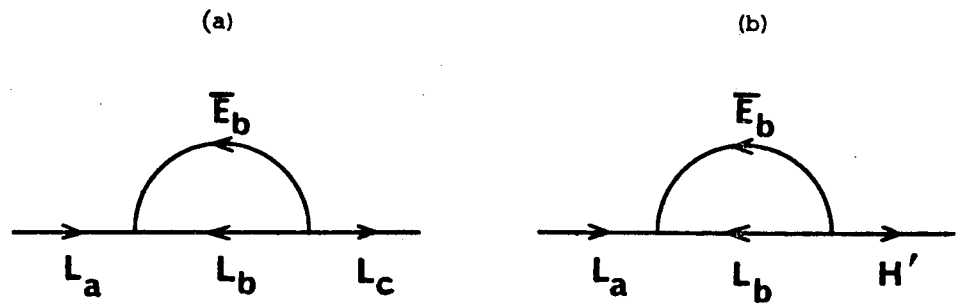


FIGURE 2

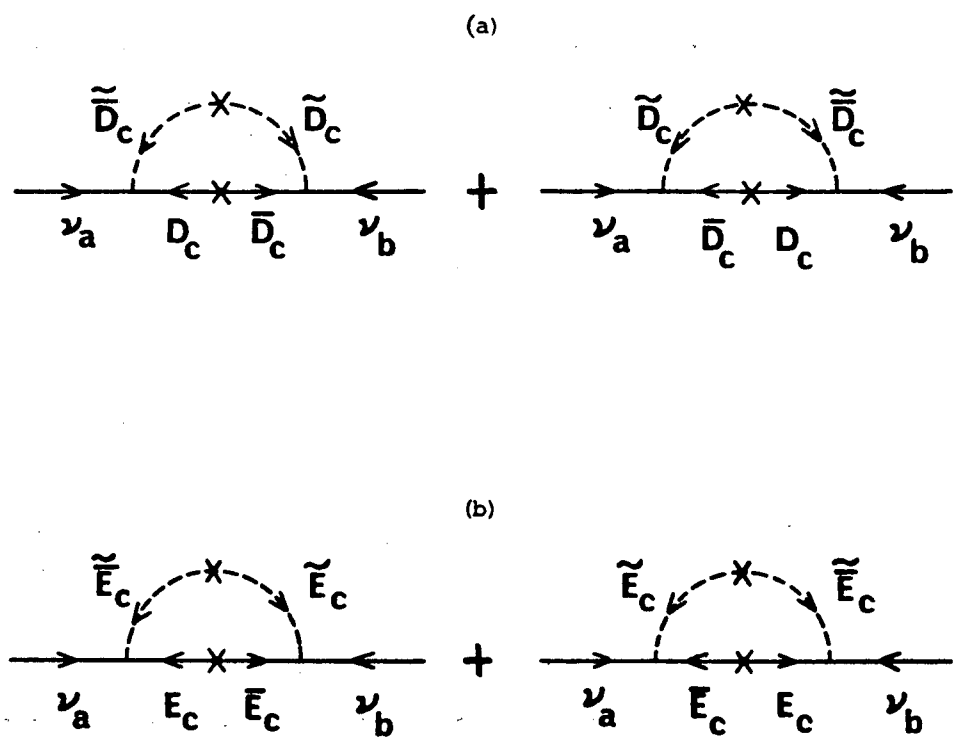


FIGURE 3

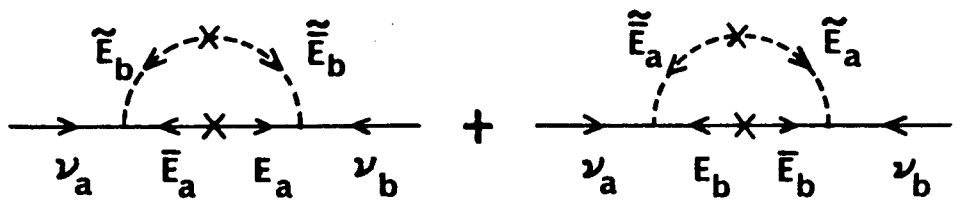
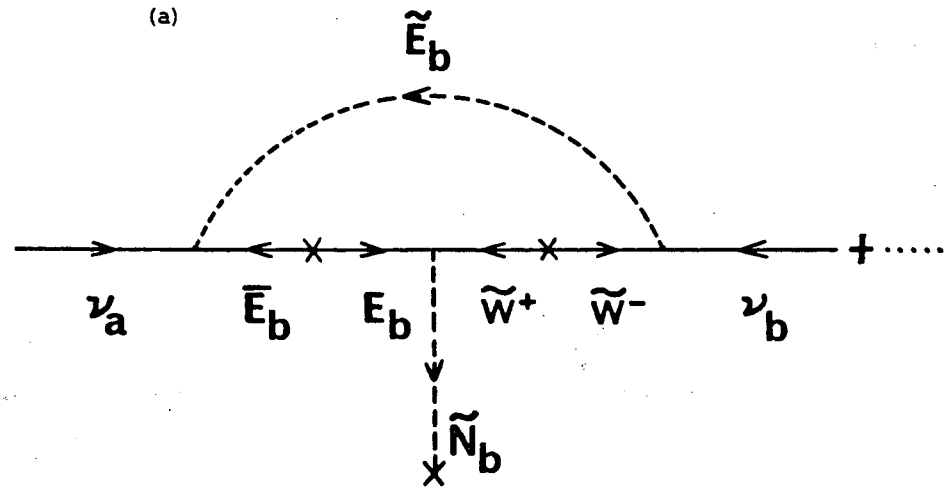


FIGURE 4

(a)



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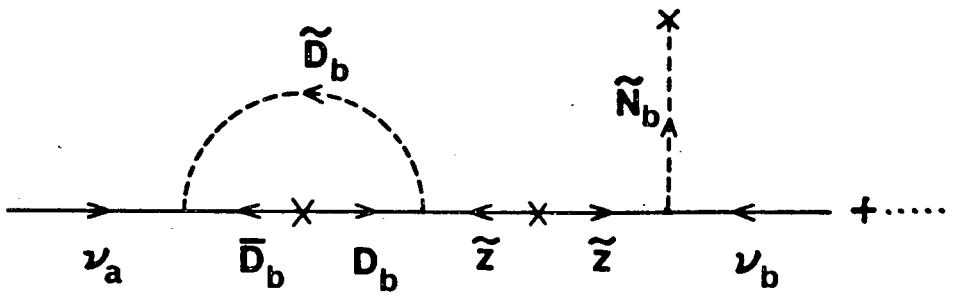


FIGURE 5

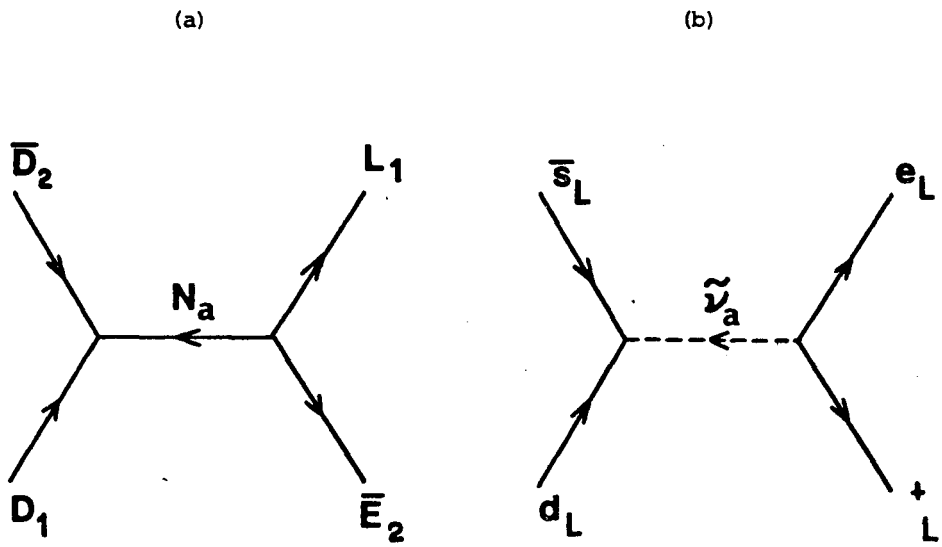


FIGURE 6

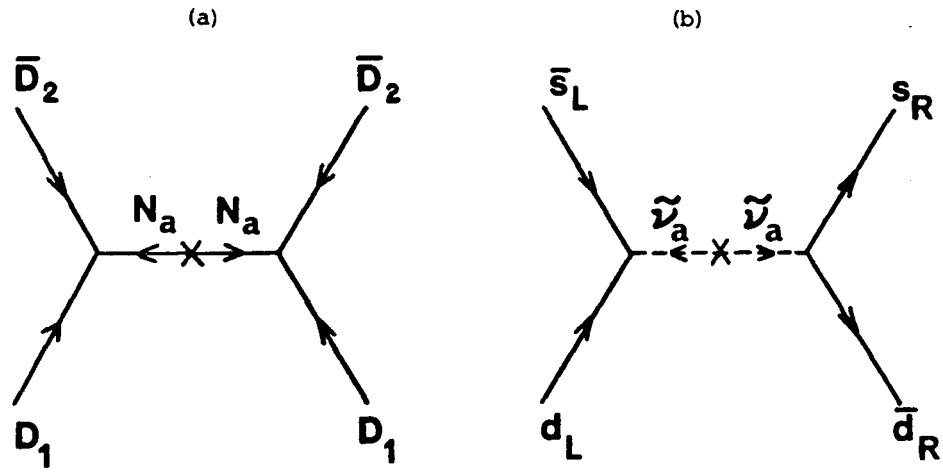


FIGURE 7

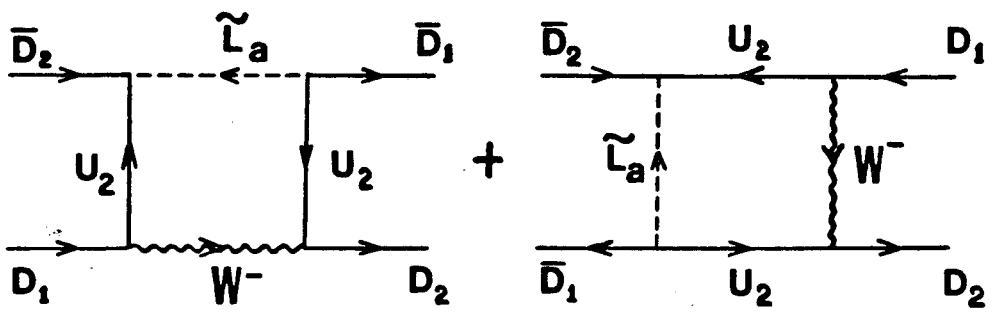


FIGURE 8

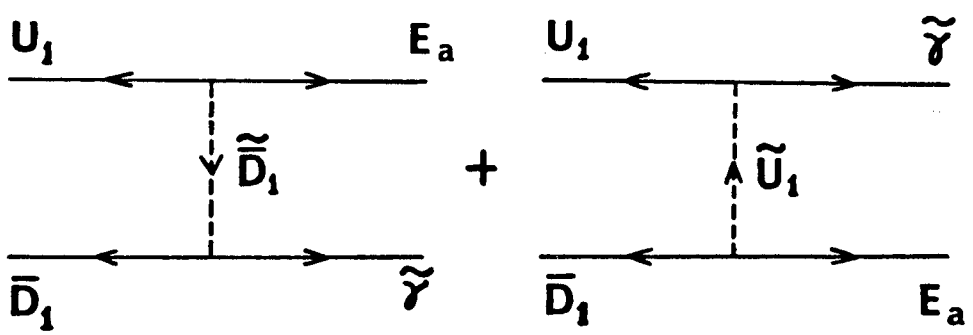


FIGURE 9

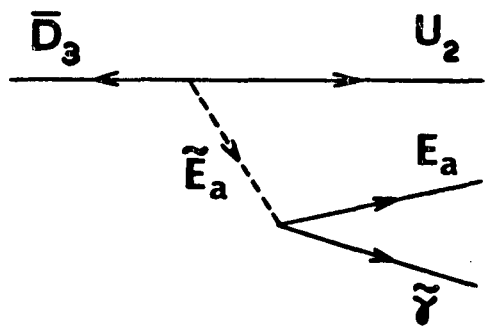


FIGURE 10

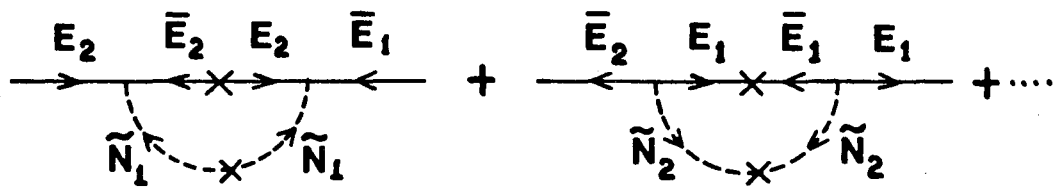


FIGURE 11

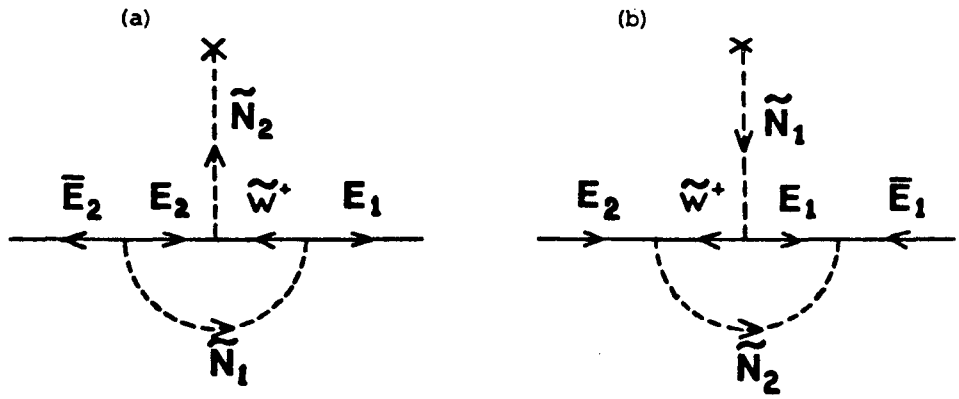


FIGURE 12

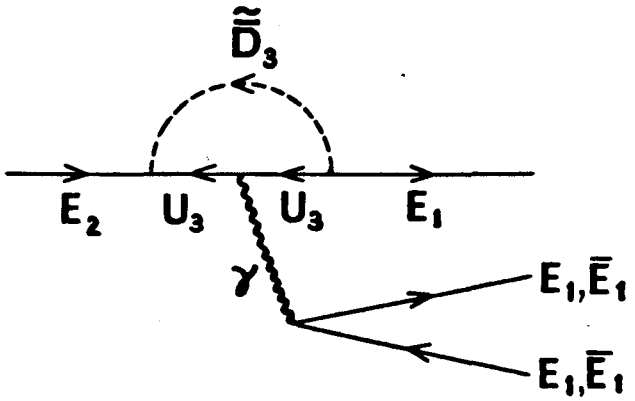


FIGURE 13

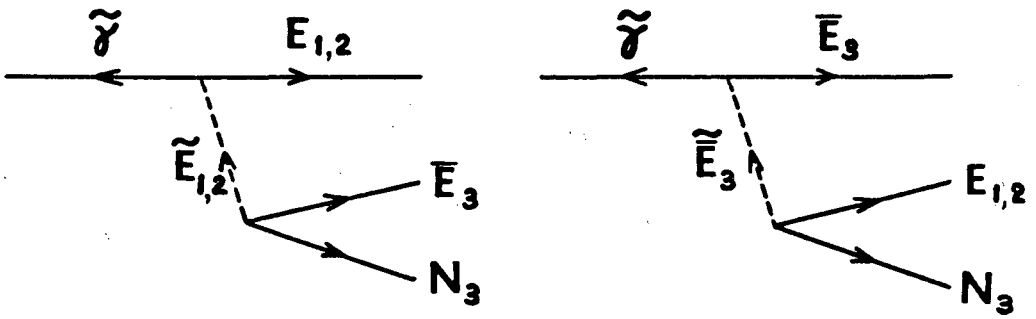


FIGURE 14

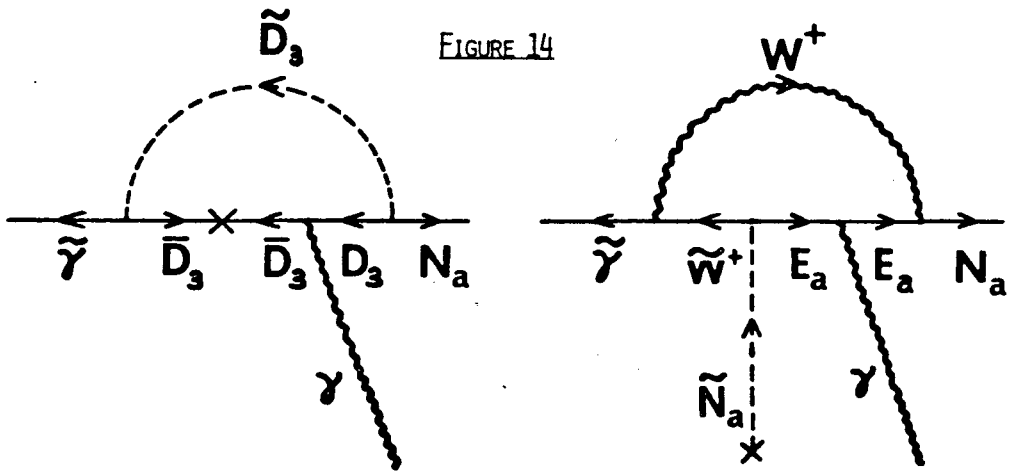


FIGURE 15

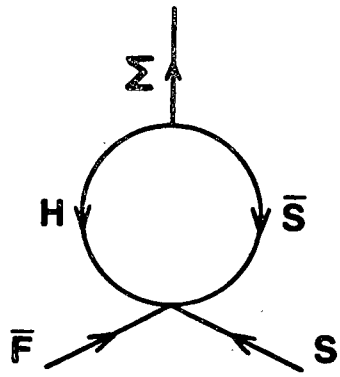
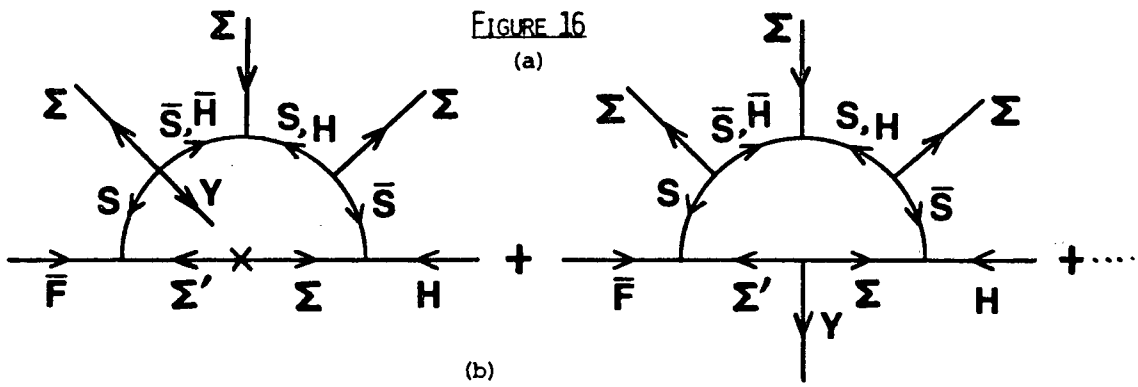


FIGURE 16

(a)



(b)

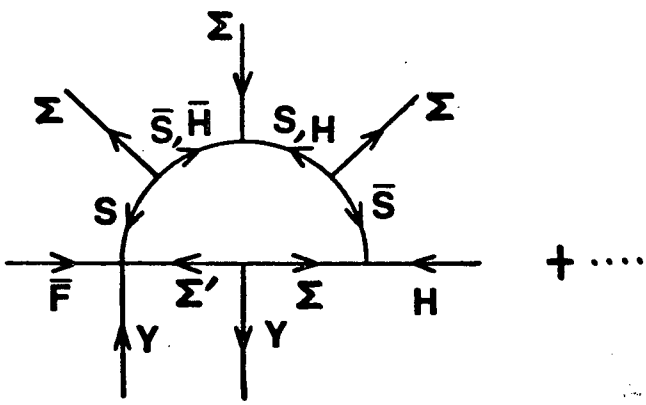


FIGURE 17

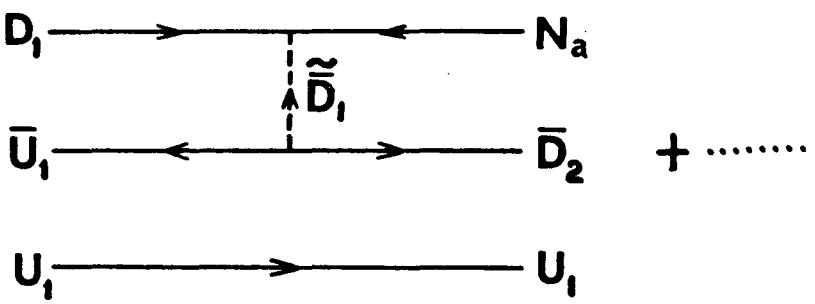


FIGURE 18

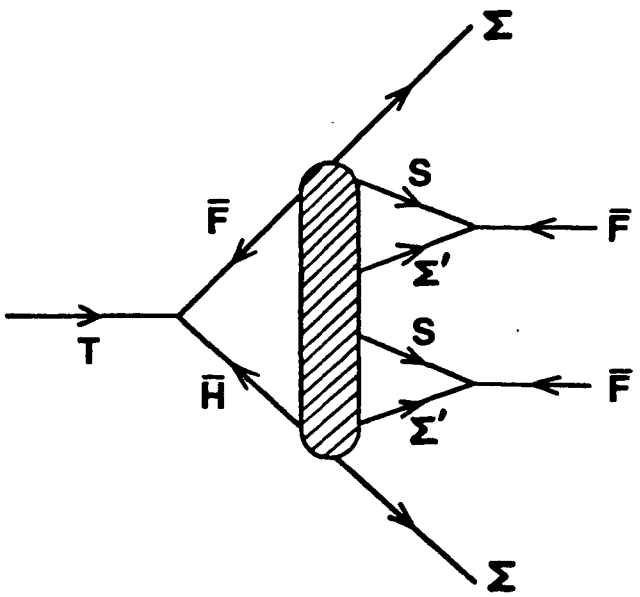


FIGURE 19

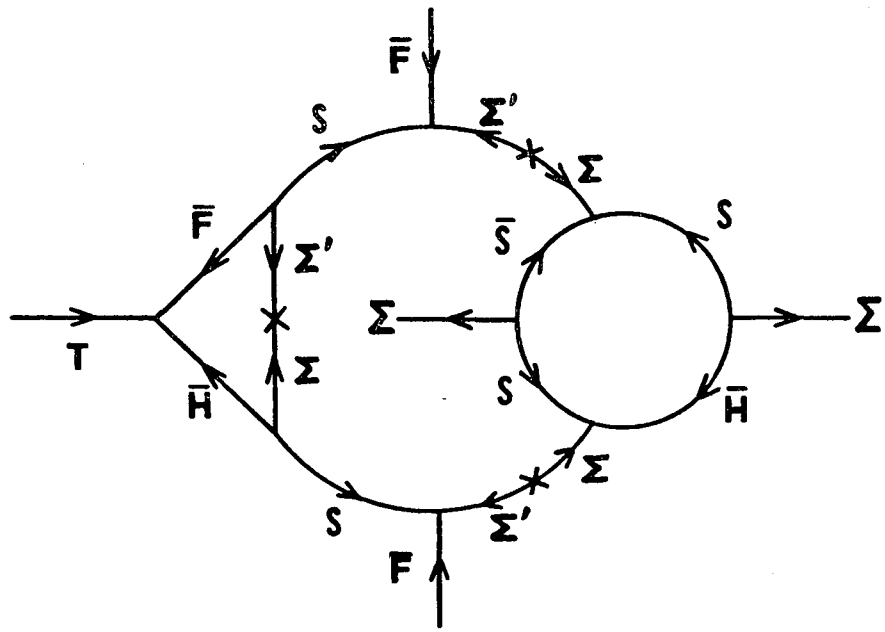
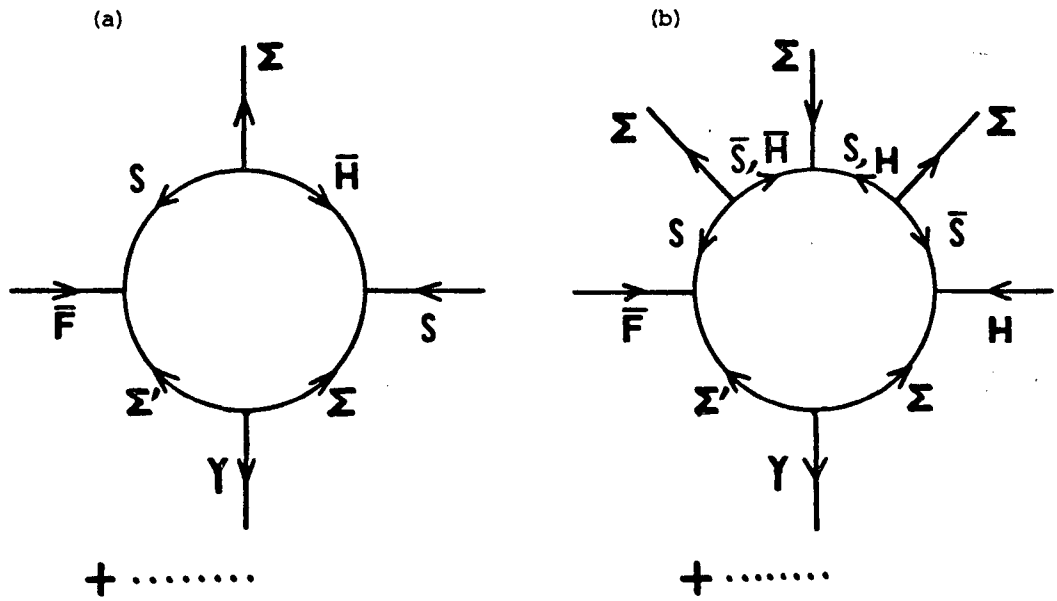


FIGURE 20



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