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To be presented at the 7th Society of Petroleum Engineers Symposium on Reservoir Simulation, San Francisco, CA, November 15-18, 1983

A SEVEN-POINT FINITE DIFFERENCE METHOD FOR IMPROVED GRID ORIENTATION PERFORMANCE IN PATTERN STEAM FLOODS

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August 1983

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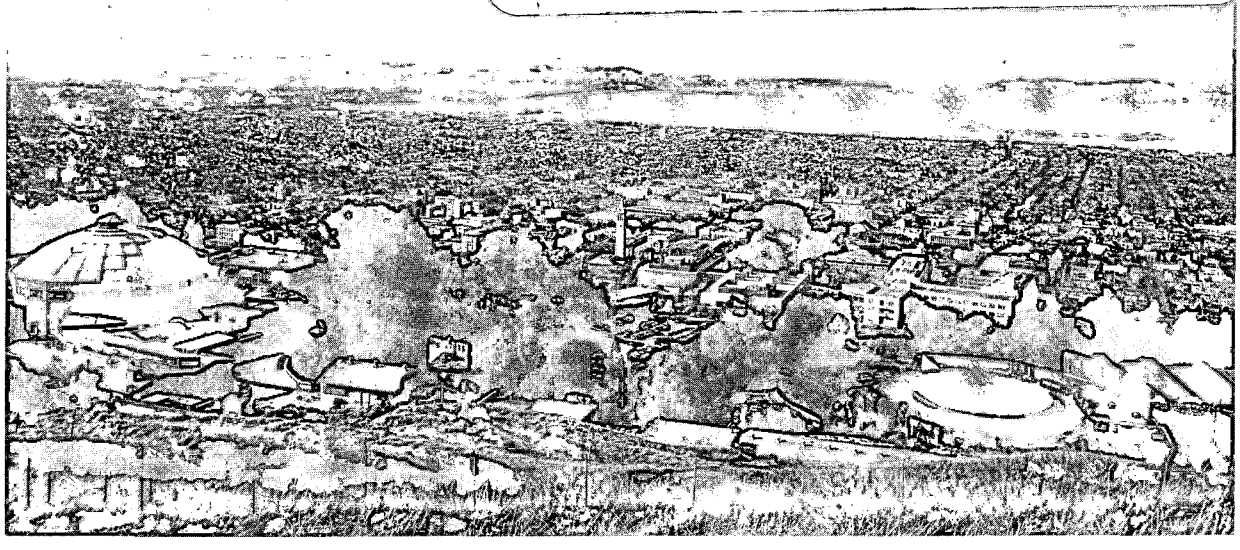

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A SEVEN-POINT FINITE DIFFERENCE METHOD FOR IMPROVED GRID ORIENTATION PERFORMANCE IN PATTERN STEAM FLOODS

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Abstract

This paper presents a novel seven-point finite difference approximation for simulations of adverse mobility ratio displacements. The method is based on partitioning of a two-dimensional flow domain into regular or nearly regular hexagons. The accuracy of the method for pattern steam floods of heavy oil reservoirs is compared to five- and nine-point approximations. For five-spot floods, the accuracy of the seven-point method is good and comparable to that of the nine-point scheme. For seven-spot floods, the seven-point method provides good numerical accuracy at substantially less computational work than five- or nine-point methods. For nine-spot floods, only the nine-point method is found to give accurate results.

Introduction

It has been demonstrated by several authors that two-dimensional simulations of pattern floods with adverse mobility ratio exhibit grid orientation effects (e.g. Todd et al., 1972; Coats et al., 1974; Coats, 1982). The conventional five-point finite difference approximation gives erroneous results, which depend strongly upon the orientation of the computational grid relative to the lines connecting production and injection wells. Generally speaking, breakthrough of the displacing fluid occurs earlier, and sweep efficiency is reduced, when producers and injectors are connected by a "parallel" string of grid blocks, with grid lines parallel (and perpendicular) to the main direction of flow. The opposite effects occur for "diagonal" grids, which are rotated by 45° in comparison to parallel grids. The grid orientation errors increase dramatically with mobility ratio. In steam floods of highly viscous crudes, the mobility of the displacing gas phase can exceed the mobility of the hydrocarbon phase ahead of the displacement front by factors of several hundred. For this type of problem, steam breakthrough times in five-spot floods were reported to differ by more than a factor of 3 between parallel and diagonal grids, when five-point differencing was used (Coats and Ramesh, 1982).

As shown by Yanosik and McCracken (1979), grid orientation effects can be alleviated by a nine-point finite difference approximation. The nine-point scheme employs a combination of interblock transmissibilities for "parallel" and "diagonal" rectangular grids. The "parallel" five-point grid considers flow betwen a nodal point P and its four neighbors N, E, S, W (Figure 1). The corresponding "diagonal" grid involves flow between P and the diagonal neighbors NE, SE, SW, NW. The nine-point method employs a linear combination of the four parallel and four diagonal flow terms, to obtain an improved rotation invariance for the finite difference representations of differential operators.

A detailed study of grid orientation effects in pattern steam floods was presented by Coats and Ramesh (1982). These authors found that in many cases grid orientation errors are reduced to acceptable levels by the nine-point approximation. However, strong grid orientation effects persist in seven-spot floods, and generally in simulations with non-square grid blocks.

Potempa (1983) developed a discretization technique which uses a mix of finite difference and finite element methods. Grid orientation errors for five- and nine-spot patterns were shown to be small. Results for a seven-spot flood were presented for only one grid orientation, so that no assessment of accuracy was possible in this case.

In this paper we present a novel "seven-point" method, which offers certain advantages over both five- or nine-point approximations.

The Seven-point Approximation

In the seven-point method, a two-dimensional flow domain is partitioned into identical regular hexagons (see Figure 2). Interblock flow can occur between each nodal point and its six neighbors, all located at equal distance h, and at angles $n \cdot 60°$ ($n = 1, \ldots, 6$). For the two-dimensional Laplacian, the seven-point finite difference approximation is written:

$$\Delta^{(7)}U = \frac{2}{3h^2}\left(U_{NE} + U_E + U_{SE} + U_{SW} + U_W + U_{NW} - 6\,U_P\right) \qquad (1)$$

To obtain an error estimate, Taylor series expansions about P are inserted into (1). This gives

$$\Delta^{(7)}U = \Delta U + \frac{3h^2}{48}\Delta^2 U + \frac{11h^4}{8.6!}\left\{\Delta^3 U - \frac{18}{11}U^{(4,2)} + \frac{12}{11}U^{(2,4)} - \frac{2}{11}U^{(6,0)}\right\}$$

$$+ h^6\{\ \ldots\ \} \qquad (2)$$

For harmonic functions ($\Delta U = 0$), the leading error term is

$$\Delta^{(7)}U - \Delta U = \frac{h^4}{4.6!}\left(-9U^{(4,2)} + 6U^{(2,4)} - U^{(6,0)}\right) \qquad (3)$$

Equation (3) shows that, for Laplace's equation, the accuracy of the seven-point method is of order h^4. This is intermediate in comparison to an accuracy for Laplace's equation of order h^2 and h^6, respectively, for five- and nine-point methods. It should be emphasized that these error estimates can only serve as a rough guide, because the operators appearing in steam flood equations differ considerably from Laplace's operator. In general, it would be expected that the accuracy of the seven-point method should be intermediate between the five- and nine-point schemes. The computational effort associated with the seven-point method will also in general be intermediate, because it considers six flow connections per grid block, compared to four and eight connections, respectively, for five- and nine-point methods. However the seven-point method offers special advantages for problems with a favorable symmetry, e.g. seven-spot floods (see below).

Integral Finite Differences

Implementation of the seven-point method is most easily made with an integral finite difference (IFD) formulation, which avoids any reference to a global system of coordinates (Edwards, 1972; Narasimhan and Witherspoon, 1976). In the IFD-formulation, the complete geometric specifications of a simulation problem are provided as input, in the form of a list of grid block volumes V_n ($n = 1, \ldots, N$), interface areas A_{nm}, and nodal distances D_{nm}. (For three-dimensional problems, we also require the angle γ_{nm} between the line connecing nodes n and m, and the vertical.) This provides for great flexibility and convenience, because one-, two-, or three-dimensional systems with arbitrary coordinates (e.g., Cartesian, cylindrical, spherical) can be treated. Also, five-, seven-, or nine-point finite difference approximations are contained within the general IFD-framework as special cases. The various

coordinate systems and differencing schemes can be implemented simply by providing the appropriate geometrical parameters (V_n, A_{nm}, D_{nm}) as input, without making any programming changes in the simulator itself.

The IFD-method differs from conventional finite differences in that spatial discretization of mass- and energy-balance equations is made directly from their integral form, rather than going through an intermediate step of writing down partial differential equations. A typical mass balance equation is written:

$$\frac{d}{dt} \int_{V_n} M^{(\kappa)} \, dv = \int_{\Gamma_n} \underset{\sim}{F}^{(\kappa)} \cdot \underset{\sim}{n} \, d\Gamma + \int_{V_n} q^{(\kappa)} \, dv \tag{4}$$

where

$$M^{(\kappa)} = \phi \sum_{\alpha=1}^{NPH} S_\alpha \, \rho_\alpha \, X_\alpha^{(\kappa)} \tag{5}$$

is the mass accumulation term for component κ, and

$$\underset{\sim}{F}^{(\kappa)} = - k \sum_{\alpha=1}^{NPH} X_\alpha^{(\kappa)} \frac{k_\alpha \rho_\alpha}{\mu_\alpha} (\nabla P_\alpha - \rho_\alpha \underset{\sim}{g}) \tag{6}$$

is the mass flux vector for component κ. Analogous equations can be written down for the energy balance.

Introducing volumetric and areal averages,

$$\int_{V_n} M^{(\kappa)} \, dv = V_n \, M_n^{(\kappa)} \tag{7}$$

$$\int_{\Gamma_n} \underset{\sim}{F}^{(\kappa)} \cdot \underset{\sim}{n} \, d\Gamma = \sum_m A_{nm} \, F_{nm}^{(\kappa)}, \tag{8}$$

Equation (4) is discretized spatially as

$$\frac{d}{dt} V_n M_n^{(\kappa)} = \sum_m A_{nm} F_{nm}^{(\kappa)} + V_n q_n^{(\kappa)} \tag{9}$$

The sum over m in equations (8) and (9) extends over all grid blocks V_m which have a flow connection with V_n. The discretized mass flux is

$$F_{nm}^{(\kappa)} = k_{nm} \sum_{\alpha=1}^{NPH} \left(X_\alpha^{(\kappa)}\right)_{nm} \left(\frac{k_\alpha \rho_\alpha}{\mu_\alpha}\right)_{nm} \left[\frac{P_{\alpha,m} - P_{\alpha,n}}{D_{nm}} - (\rho_\alpha)_{nm}\, g_{nm}\right] \tag{10}$$

Depending upon the problem at hand, one can employ different weighting procedures (harmonic weighting, spatial interpolation, upstream weighting) for the various "interface quantities", labeled with subscripts "nm".

Equations (9) and (10) are in a form which permits immediate implementation of the seven-point method. All that is required is a list of geometric parameters (V_n, A_{nm}, D_{mn}) appropriate for a partitioning into regular hexagons, as shown in Figure 2.

The nine-point approximation to Eqs. (9, 10) is obtained by forming a suitable linear combination of five-point "parallel" and "diagonal" interblock flow expressions. The five-point parallel and diagonal Laplacians are, respectively (see Figure 1)

$$\Delta_{par}^{(5)} U = \frac{1}{h^2} (U_N + U_E + U_S + U_W - 4U_P) \tag{11}$$

$$\Delta_{dia}^{(5)} U = \frac{1}{2h^2} (U_{NE} + U_{SE} + U_{SW} + U_{NW} - 4U_P) \tag{12}$$

The nine-point formula is (Forsythe and Wasow, 1960)

$$\Delta^{(9)} U = \frac{2}{3} \Delta_{par}^{(5)} U + \frac{1}{3} \Delta_{dia}^{(5)} U \tag{13}$$

To obtain the proper linear combination of the "parallel" and "diagonal" form of Equation (9), corresponding to the nine-point formula Equation (13), note that the volume of the "diagonal" grid blocks is twice that of the "parallel" grid blocks, $V_{n,dia} = 2 \cdot V_{n,par}$. Therefore, the proper linear combination, which will yield the nine-point approximation to the Laplacian (Equation 13) is 2/3 the "parallel" version of Equation (9) plus 1/6 the "diagonal" version. This can be shown in detail by considering idealized flux terms corresponding to the full Laplace operator. Ignoring gravity effects and spatial variations in mobility, the simplified flux terms are

$$F_{nm} = C \frac{P_m - P_n}{D_{nm}} \qquad (14).$$

Noting that $A_{nm,par}/V_{n,par} = 1/h$; $A_{nm,dia}/V_{n,dia} = 1/h\sqrt{2}$, we obtain

$$\frac{d}{dt}\bar{M}_n^{(\kappa)} = C\frac{1}{6h^2}\{4[P_N + P_E + P_S + P_W] + P_{NE} + P_{SE} + P_{SW} + P_{NW} - 20\,P_n\} = C\,\Delta^{(9)}P \qquad (15)$$

where the nine point accumulation term is given by

$$\bar{M}_n^{(\kappa)} = \frac{1}{3}(2M_{n,par}^{(\kappa)} + M_{n,dia}^{(\kappa)}) \qquad (16)$$

To summarize this discussion, the implementation of the nine-point approximation in the IFD-method is made as follows:

(i) generate a list of geometric parameters V_n, A_{nm}, D_{nm} appropriate for the conventional five-point method (solid flow lines in Figure 1);

(ii) reduce all interface areas in the "parallel" grid by a factor 2/3;

(iii) append a list of "diagonal" connections (A_{nm}, D_{nm}), with all interface areas reduced by a factor 1/6 from their actual values.

The same procedure is followed in the general case, when the flux expressions on the right hand side of Equation (4) do not add up to a full Laplacian.

Simulation of Pattern Steam Floods

We have simulated five-, seven-, and nine-spot steam floods of heavy oil reservoirs with and without volatile hydrocarbons. In each case, runs were made using five-, seven-, and nine-point finite difference approximations, with "parallel" and "diagonal" grids, so that accuracy and efficiency of the different methods could be compared. A brief technical description of our steam flood simulator is given in the appendix. Here we summarize the specification of reservoir parameters and computational grids.

The fluid and rock properties are identical to those adopted by Coats (1978). Table 1 gives the more important parameters; for full details the reader is referred to Table 5 of Coats' original paper. Specifications of the steam flood problems simulated in the present work are given in Table 2. Problem parameters are similar to those employed by Coats and Ramesh (1982). All rates are reported for the full patterns, even when only part of a pattern needs to be simulated due to symmetry (see Table 2). In the five- and seven-spot problems the oil contains a 30% mole fraction of distillable components, whereas in the nine-spot problem the oil is assumed non-volatile. In all calculations we employ grids consisting of equilateral or approximately equilateral blocks (see Figures 3 through 6, and Table 2). Slightly irregular blocks were necessary for modeling five- or nine-spot patterns with hexagonal grid blocks, and for modeling a seven-spot pattern with rectangular grid blocks (see aspect ratios in Table 2). Computational grids were chosen such as to minimize deviations from equilateral blocks, and to obtain approximately equal block volumes for different grids used on the same problem.

Figure 3 shows the 8 x 8 "diagonal" and 11 x 6 "parallel" grid used for modeling 1/8 of a five- or nine-spot, with either five- or nine-point differencing. (Five- and nine-point calculations always use the same basic grid.) The symbols I, P, N denote injector, (far) producer, and near producer, respectively. In the seven-point method the flow domain is discretized into (approximately) regular hexagons. The minimum symmetry element which can be modeled is 1/4 pattern (Figure 4).

For seven-spot floods the seven-point grid has the proper symmetry, so that perfectly regular hexagons can be used, while five-and nine-point grids require slightly non-square grid blocks. Also, the favorable symmetry makes it possible to model a 6 times smaller symmetry element in the seven-point method than in the five- or nine-point methods, thereby giving substantial computational savings. Figure 5 shows diagonal and parallel grids for 1/12 of a seven-spot. The rectangular five- or nine-point grid for 1/2 of the seven-spot is shown in Figure 6.

For each of the grids chosen, the detailed geometric specifications (V_n, A_{nm}, D_{nm}) needed as input to the simulator were generated by computer.

Results

Simulation results are given in Figures 7 through 10, and Table 3. The following observations can be made.

Five-Spot

Strong grid orientation effects are noted for the five-point method. Steam breakthrough in the "parallel" mesh occurs almost twice as rapidly as in the "diagonal" mesh (Table 3), and the two meshes produce rather different production rates and oil recovery curves (Figures 7 and 8). In the nine-point method, grid orientation effects are negligibly small. These findings agree with the results previously obtained by Coats and Ramesh (1982), and they

demonstrate that our implementation of the nine-point method by means of integral finite differences is correct (see the discussion following Eq. 16, above). The seven-point method produces excellent results. Steam breakthrough is slightly slower than in the nine-point method, but the discrepancy is not larger than the discrepancy among the "parallel" and "diagonal" nine-point predictions themselves. Overall agreement of seven- and nine-point recovery curves is excellent.

Seven-spot

Steam breakthrough in the "parallel" seven-point grid occurs somewhat more rapidly than in the "diagonal" seven-point grid (Figure 9). This indicates that the seven-point method tends to overpredict transmissibility in the "parallel" direction, while underpredicting in the "diagonal" direction. However, the differences are not large. Except for a rather brief period at times close to steam breakthrough, the "parallel" and "diagonal" recovery curves agree well (Figure 9). By comparison, Coats and Ramesh reported large differences (factors of 3 or more) for steam breakthrough time and cumulative oil recovery in seven-spot floods when comparing "parallel" and "diagonal" nine-point grids. It appears that these strong grid orientation effects are caused by the non-square grid blocks used by these authors. Our nine-point calculation for the seven-spot flood employs nearly square grid blocks, and the oil recovery predictions are always intermediate between those for the "parallel" and "diagonal" seven-point grids (Figure 9). Actually, the nine-point results are almost identical to those obtained with the "parallel" seven-point grid. This provides evidence that the nine-point results are very accurate (rather free of grid orientation errors) even for seven-spot floods. It should be noted that, due to the symmetry of the problem, the seven-point method offers substantial savings in computing time. If nearly equilateral grid blocks are

to be used, the nine-point method requires 1/2 of the full pattern to be simulated, whereas the seven-point method can simulate 1/12 of the pattern. Therefore, if grid blocks of equal volume are employed, the seven-point method requires only 1/6 the number of blocks needed in five- or nine-point grids. Figure 9 shows that the five-point method gives large errors for steam breakthrough and oil recovery in seven-spot floods.

Nine-spot

In agreement with the results of Coats and Ramesh (1982), we find that for nine-spot floods, five-point grid orientation errors are very large (Figure 10). Steam breakthrough times differ by factors in excess of 5 between parallel and diagonal grids (see Table 3), and in the "parallel" five-point mesh, steam breaks through at the far well earlier than at the near well. The nine-point method gives good results. For the near well, steam breakthrough is slightly faster in the diagonal grid. This diminishes somewhat the sweep towards the far well, so that a later breakthrough is predicted than in the parallel grid. The sensitivity of the problem can be seen by noting that a difference of less than 5 days in predicted breakthrough time for the near well impacts on the sweep process in such a way that far well breakthrough times differ by almost 90 days (see Table 3). However, none of these discrepancies are "large", and predicted cumulative oil recoveries agree well for "parallel" and "diagonal" nine-point grids (Figure 10). The seven-point method performs much better than the five-point method, but not nearly as well as the nine-point method in this case. Due to the 60° symmetry of the hexagonal grid blocks, the two kinds of near wells respond differently. Well N' is connected with the injector by a "parallel" flow path, giving much more rapid steam breakthrough at that well than for the "diagonally" connected well N (Figure 4 and Table 3). In this particular problem it was assumed that there were no volatile

hydrocarbons present, so that oil viscosity, mobility ratios, and grid orientation effects are larger than in the case with volatile hydrocarbons. Cumulative oil recovery is generally lower, due to the absence of a distillable component.

Conclusions

The results of this study can be summarized as follows.

(1) A novel seven-point method has been developed, which for five- and nine-spot floods is intermediate between five- and nine-point methods both in grid orientation accuracy and computational effort.

(2) For five-spot patterns, the seven-point method is virtually as accurate as the nine-point method, while the latter has a distinct advantage for nine-spots.

(3) Seven-spot floods offer a particularly favorable geometry for application of the seven-point method, so that good numerical accuracy can be obtained simultaneously with a substantial reduction in computational work.

(4) The nine-point method gives accurate results for seven-spot floods when nearly square grid blocks are used. Large grid orientation errors reported in the literature for nine-point simulations of seven-spot patterns appear to be caused by the use of non-square grid blocks.

(5) Five-, seven-, and nine-point differencing methods can be conveniently implemented by means of an integral finite difference formulation (IFD). All geometrical parameters of the grid are provided as input data in the IFD-method, and no programming changes are necessary when changing the differencing method.

Nomenclature

A_{nm}	interface area between grid blocks n and m (m^2)
C	proportionality constant in Eq. 14 (kg/m·Pa·s)
D_{nm}	nodal distance between grid blocks n and m (m)
F	mass or energy flux ($kg/m^2 \cdot s$ or W/m^2)
g	gravitational acceleration (m/s^2)
h	grid spacing (m)
I	injection well
k	permeability (m^2); also: time step index
k_α	relative permeability for phase α
M	accumulation term in mass or energy balance equations (kg/m^3 or J/m^3)
M_n	average of accumulation term in grid block V_n
N,N'	"near" production well
NPH	number of phases
$\underset{\sim}{n}$	inward normal vector
P	pressure (Pa); also: production well
P_α	pressure in phase α (Pa)
q	sink or source rate ($kg/m^3 \cdot s$ or W/m^3)
$R_n^{(\kappa)}$	residual for component κ-equation in grid block n
S	saturation
t	time (s)
U	general solution to partial differential equation
$U^{(\ell,k)}$	$= (\partial^\ell/\partial x^\ell)(\partial^k/\partial y^k)U$
V	volume (m^3)
V_n	volume of grid block n (m^3)
X	mass fraction

Subscripts

dia	diagonal
E	east
g	gas
N	north
NE	northeast
NW	northwest
n,m	index of grid blocks
o	oil
P	nodal point
par	parallel
S	south
SE	southeast
SW	southwest
W	west
w	water

Greek

α	phase (o: oil, g: gas, w: water)
Δ	Laplace operator
Γ	surface area (m^2)
κ	component index
μ	viscosity (Pa $\cdot$ s)
ρ	density (kg/m^3)
ϕ	porosity

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Appendix: Steamflood Simulator

The calculations reported in this paper were carried out with our fully implicit numerical model STMFLD1*, which simultaneously solves for the transport of three phases (gaseous, aqueous, hydrocarbon), and heat. The mathematical and numerical methods used in our steam flood simulator are similar to the geothermal reservoir simulators SHAFT79 and MULKOM (Pruess and Schroeder 1980; Pruess and Narasimhan, 1982; Pruess, 1983). The reservoir fluid is approximated as having 3 (pseudo-)components: (1) "light" (volatile) hydrocarbons, (2) "heavy" (non-volatile) hydrocarbons, and (3) water. Space discretization of mass- and energy-balance equations is made with the integral finite difference method, resulting in equations of the form (9) and (10) (analogous for heat balance). Time is discretized fully implicitly as a first order finite difference. The treatment of PVT-properties is the same as that of Coats (1976, 1978). All thermophysical properties of the multiphase fluids are generated as functions of a set of four independent primary variables in an "equation-of-state module" (EOS). The EOS contains logic to recognize the appearance and disappearance of phases, and to properly "switch" primary variables in phase transitions. Constraints such as $S_g + S_w + S_o = 1$ are embedded in the EOS-module. The discretized balance equations are written in residual form (compare Equation (9))

$$R_n^{(\kappa),k+1} \equiv M_n^{(\kappa),k+1} - M_n^{(\kappa),k} - \frac{\Delta t}{V_n} \left\{ \sum_m A_{nm} F_{nm}^{(\kappa),k+1} + V_n q_n^{(\kappa),k+1} \right\} = 0$$

$$(\kappa = 1, \ldots, 4;\ n = 1, \ldots, N) \qquad \text{(A-1)}$$

where k is the time step index. The 4·N coupled non-linear equations (A-1) for a system with N grid blocks are solved completely simultaneously, using Newton-Raphson iteration. The derivatives with respect to primary

*STMFLD1 is an improved version of a code which was jointly developed by K. Pruess, G.S. Bodvarsson, and Energy Resources Co.

variables needed for the Jacobian matrix are directly evaluated for the residuals $R_n^{(\kappa)}$, using numerical differentiation, with all required incremented thermophysical parameters provided by the EOS module. Iteration is continued until the residuals are reduced to less than a specified fraction ε of the accumulation terms. The linear equations arising at each iteration step are solved directly, using Gaussian elimination and sparse storage techniques. The linear algebra is performed with the Harwell subroutine package "MA28", which efficiently handles non-symmetric matrices with random sparsity structure (Duff, 1977). This feature permits exploitation of the geometric flexibility of the integral finite difference method to the fullest extent. Flow connections between grid blocks can be specified in any way desired, without any limitations as to band width or dimensionality of the problem. The simulator permits a flexible treatment of wells, with constant or time-dependent rates, and provides a deliverability option with gravity correction for flowing wellbore pressure. Automatic time-stepping can be specified, with time steps increasing or decreasing in dependence upon the number of iterations needed for convergence. Boundary conditions are treated by introducing appropriate boundary elements (Dirichlet type) and/or appropriate sinks or sources (Neumann type). Heat loss to cap and base rocks can be modeled either numerically, or with the semi-analytical method of Vinsome and Westerveld (1980).

Table 1. Fluid and rock properties

Oil Properties			
viscosity	1.38 Pa·s at 37.8°C		(= 1380 cp at 100°F)
	.0025 Pa·s at 260°C		(= 2.5 cp at 500°F)
compressibility	7.25×10^{-10} Pa^{-1}		(= 5×10^{-6} psi^{-1})
expansivity	7.38×10^{-4}/°C		(= 4.1×10^{-4}/°F)
heat capacity	2093.4 J/kg°C		(=0.5 Btu/lb°F)
molecular weight	volatile oil	120	
	non-volatile oil	300	
Formation Properties			
permeability	4×10^{-12} m^2		(=4000 md)
porosity	38%		
compressibility	2.9×10^{-10} Pa^{-1}		(= 2×10^{-6} psi^{-1})
heat capacity	2.35×10^{6} J/m^3°C		(= 35 Btu/cu ft -°F)

Table 2. Parameters of steam flood problems

	five- or nine-spot	seven-spot
pattern area	9604 m^2(=2.37 acres)	11317 m^2 (= 2.80 acres)
reservoir thickness	15 m (=49.2 ft)	10.9728 m (=36.0 ft)
bottomhole flowing pressure	0.517 MPa (=75 psi)	0.4137 MPa (=60 psi)
injection rate	0.4 kg/s (= 217.8 BPD-CWE)	0.24 kg/s (= 130.7 BPD-CWE)
steam quality	80% at 205°C (=401°F)	70% at 195°C (= 383°F)
productivity index	1.6×10^{-10} m^3 (=599.5 RB·cp/D/psi)	2.4×10^{-10} m^3 (=899.2 RB·cp/D/psi)
initial reservoir temperature	38°C (= 100.4°F)	32.2°C (= 90.0°F)
initial reservoir pressure	0.7 MPa (=101.5 psi)	0.5171 MPa (= 75 psi)
initial water saturation	30%	30%
initial mole fraction of volatile hydrocarbons	30% (0 for nine-spot)	30%

	five- or nine point	seven-point	five- or nine point	seven-point
minimal symmetry element[a]	1/8	1/4	1/2	1/12
number of grid blocks	36	68	104	27
grid block volume "parallel"	720.29 m^3	643.1 m^3	739.2 m^3	511.1 m^3
grid block volume "diagonal"	735.0 m^3			551.9 m^3
aspect ratio[b]	1.0	1.023	1.010	1.0

[a]for nearly equilateral grid blocks

[b]ratio of longer to shorter side of grid block

Table 3. Steam breakthrough times (in days)

MESH	PATTERN	five-spot[a]	nine-spot[b] P	nine-spot[b] N	nine-spot[b] N'	seven-spot[a]
five-point	parallel	267.8	313.7	822.5	822.5	326.2
	diagonal	515.3	2178.0	153.8	153.8	
nine-point	parallel	386.8	1784.5	317.0	317.0	533.0
	diagonal	395.6	1872.7	312.2	312.2	
seven-point	parallel	403.6		289.7	697.1	512.5
	diagonal					594.3

[a] with volatile hydrocarbons

[b] no volatile hydrocarbons

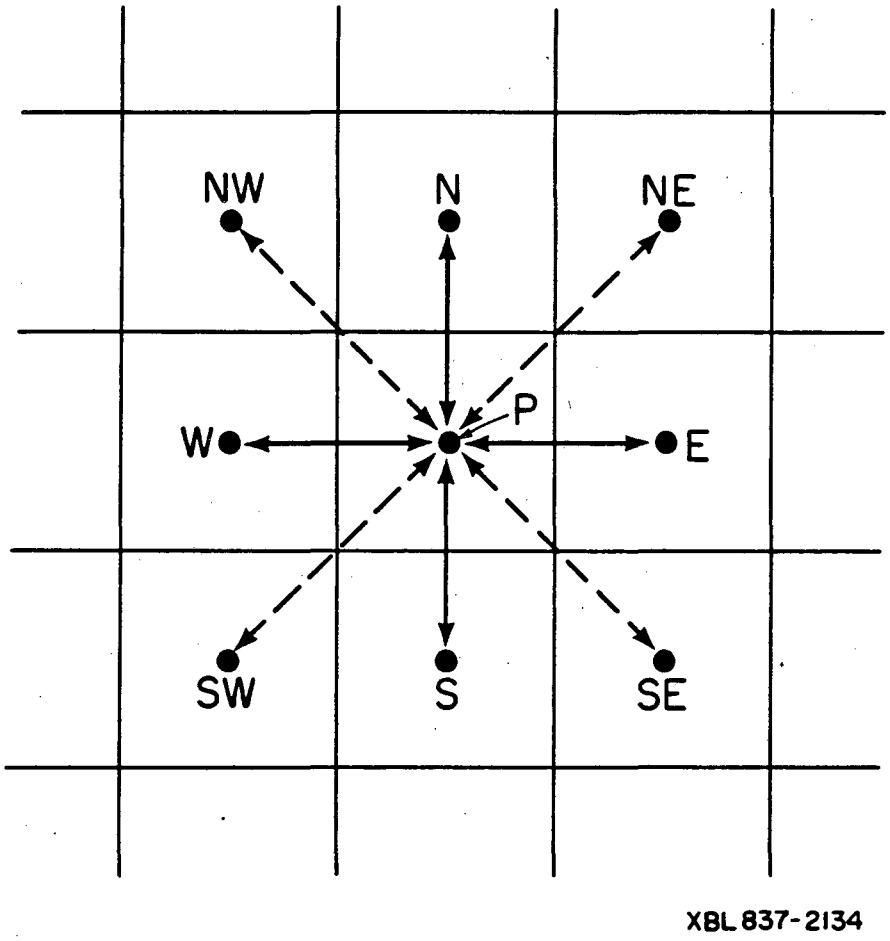


Figure 1: Five- and nine-point finite difference approximations.

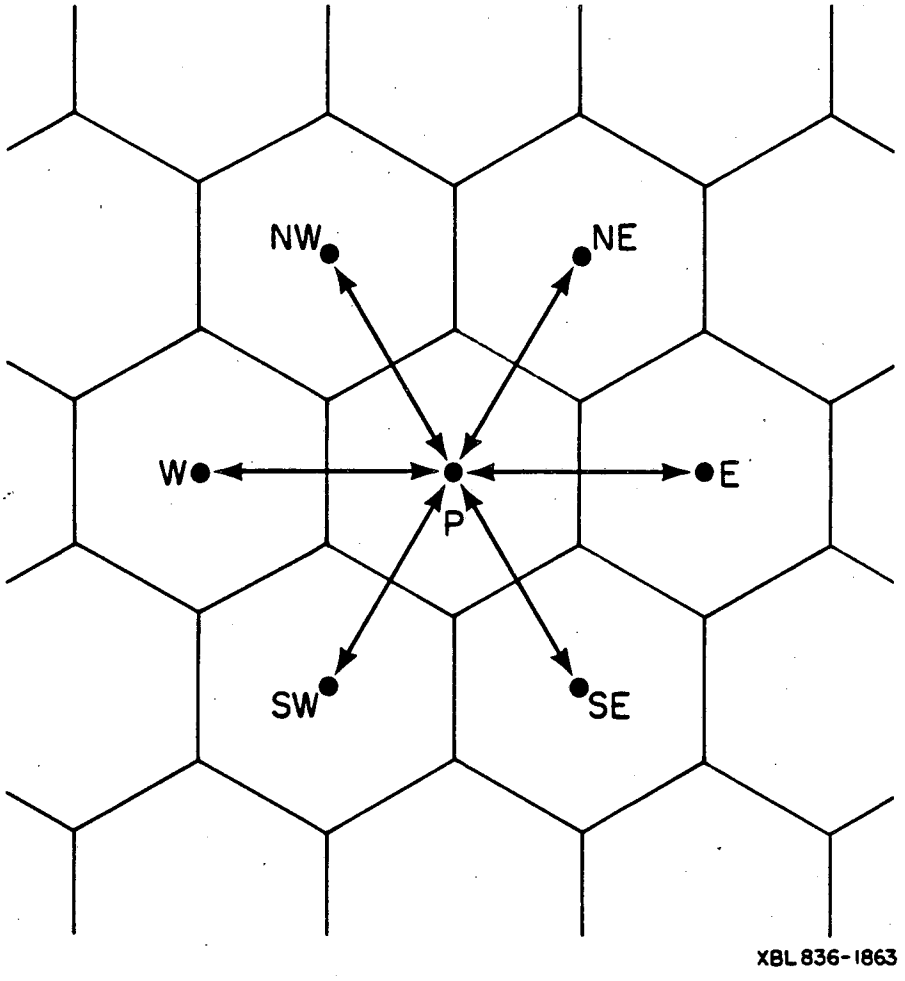
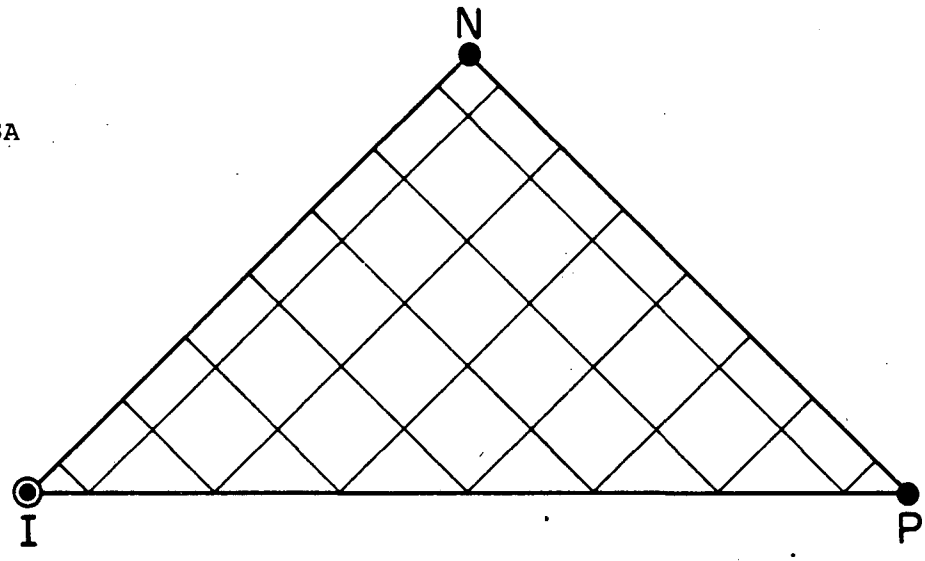


Figure 2: Computational grid of regular hexagons for seven-point finite differences.

3A



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3B

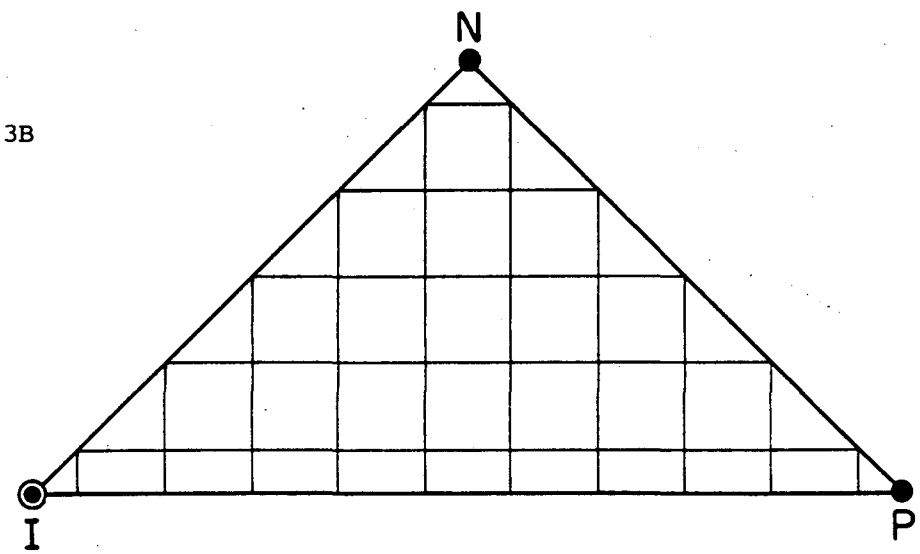

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Figure 3: Grids with square blocks for modeling 1/8 of a five- or nine-spot. (a - diagonal, b- parallel).

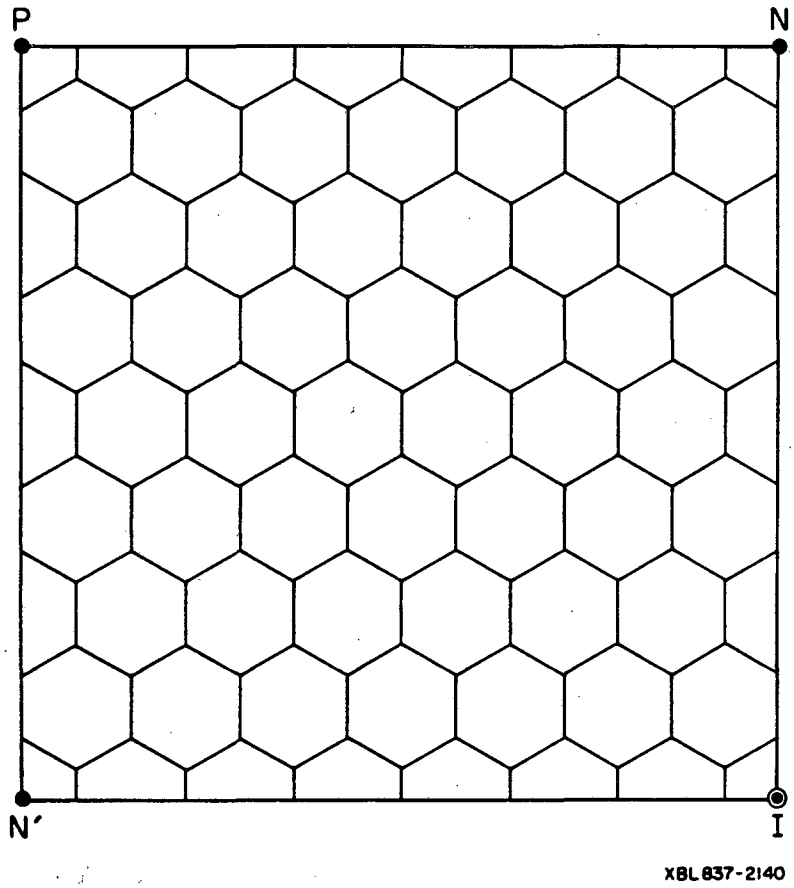
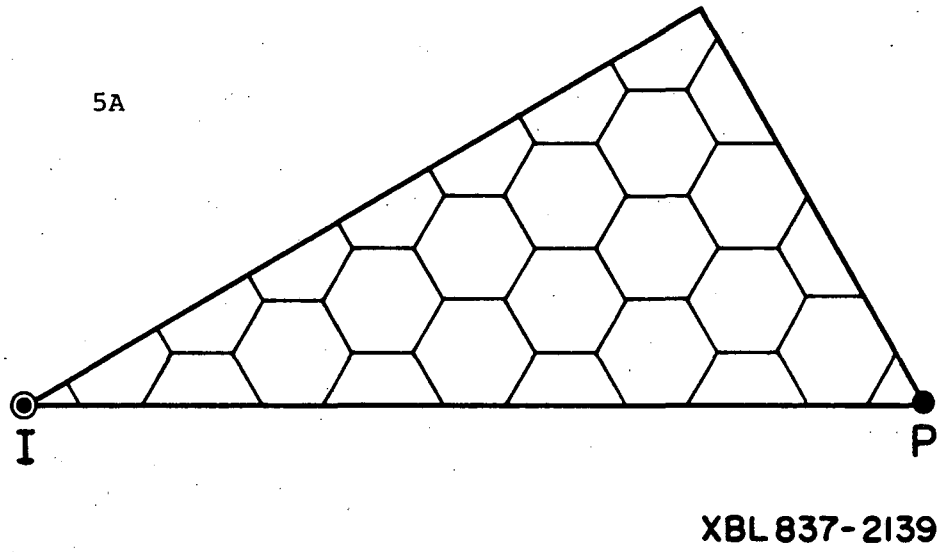


Figure 4: Hexagonal grid for 1/4 of a five- or nine-spot.



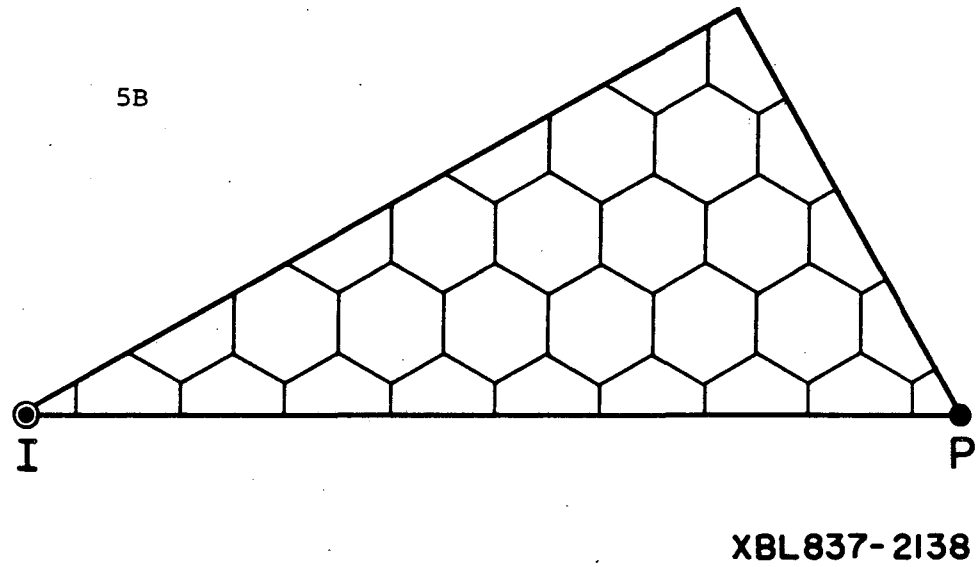


Figure 5: Hexagonal grids for 1/12 of a seven-spot (a - diagonal, b - parallel).

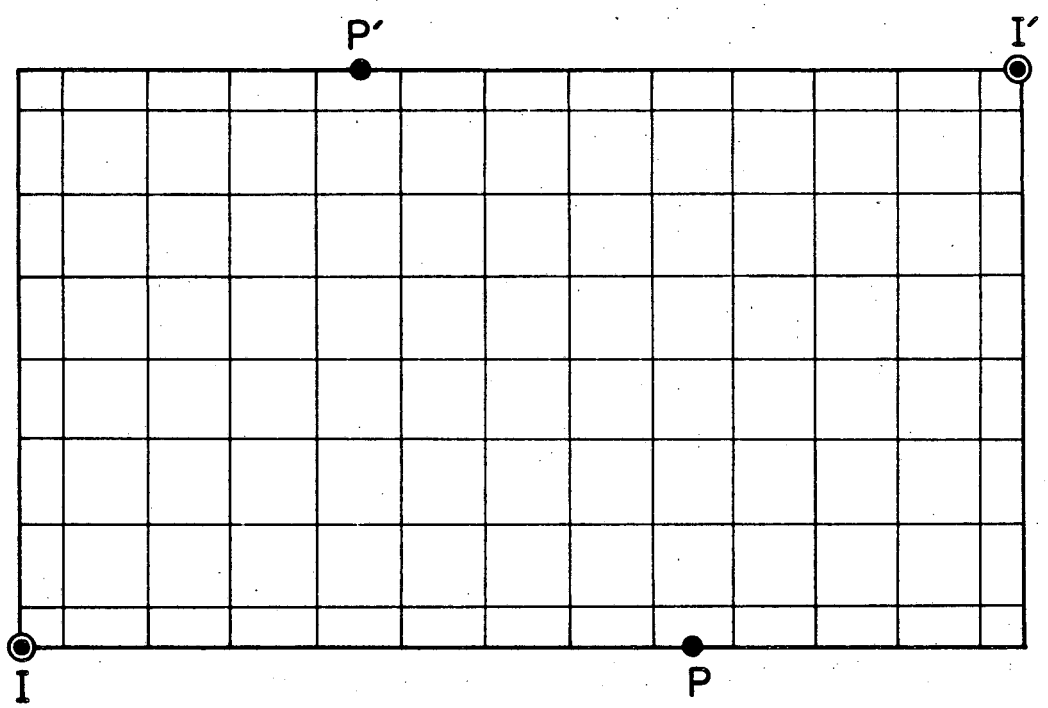


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Figure 6: Grid with approximately square blocks for 1/2 of a seven-spot.

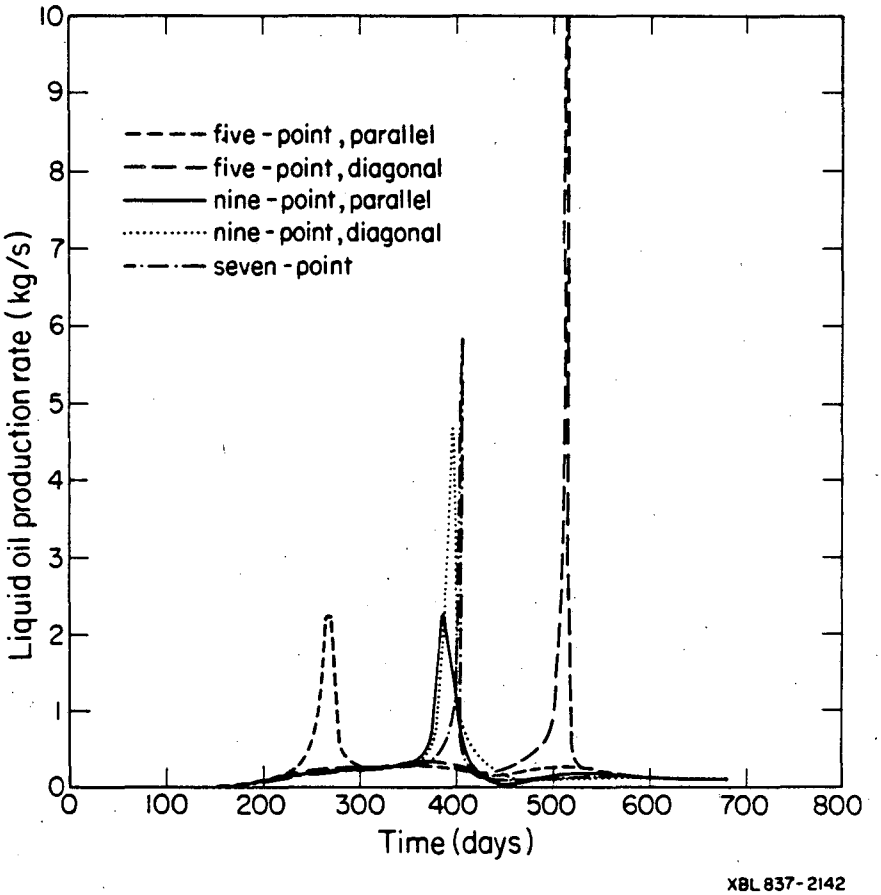

Figure 7: Liquid phase oil production rate for five-spot steam flood.

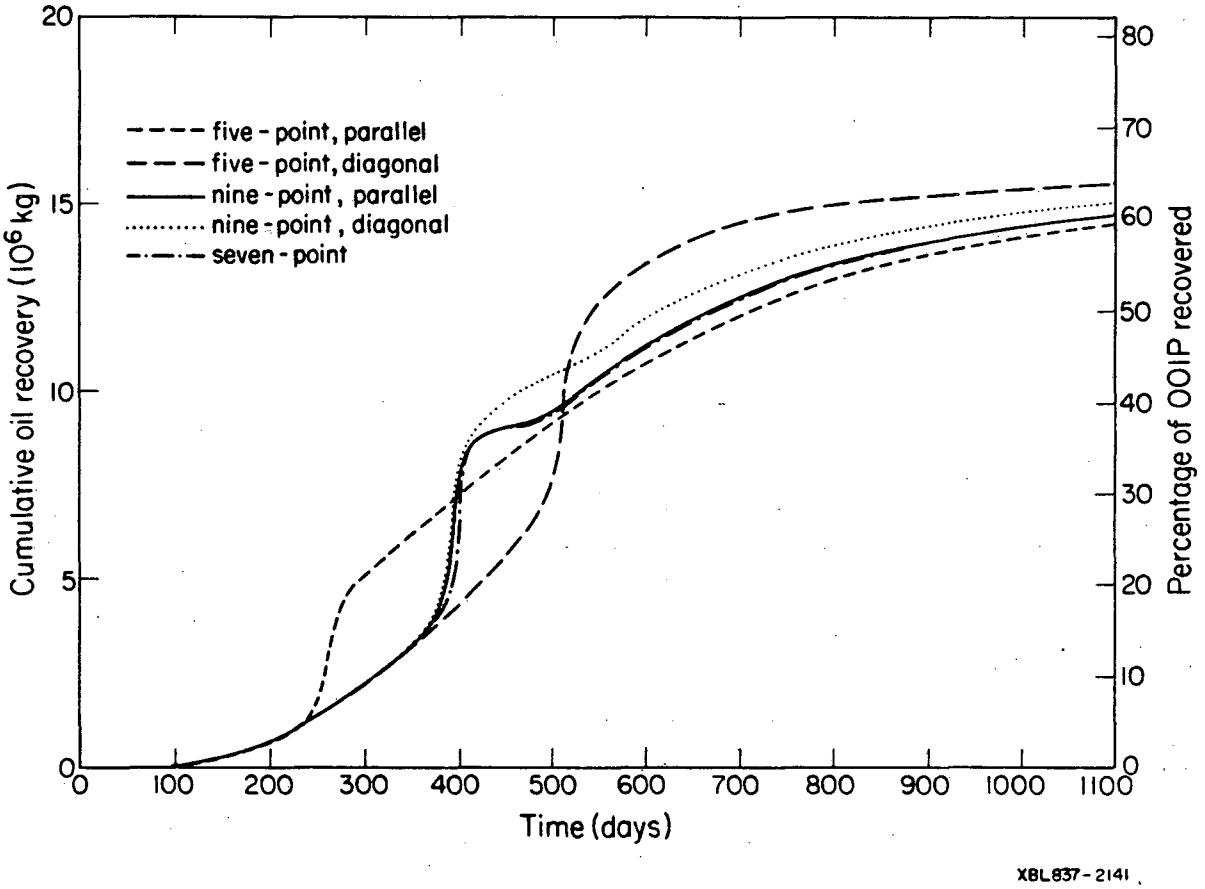

Figure 8: Cumulative oil recovery for five-spot.

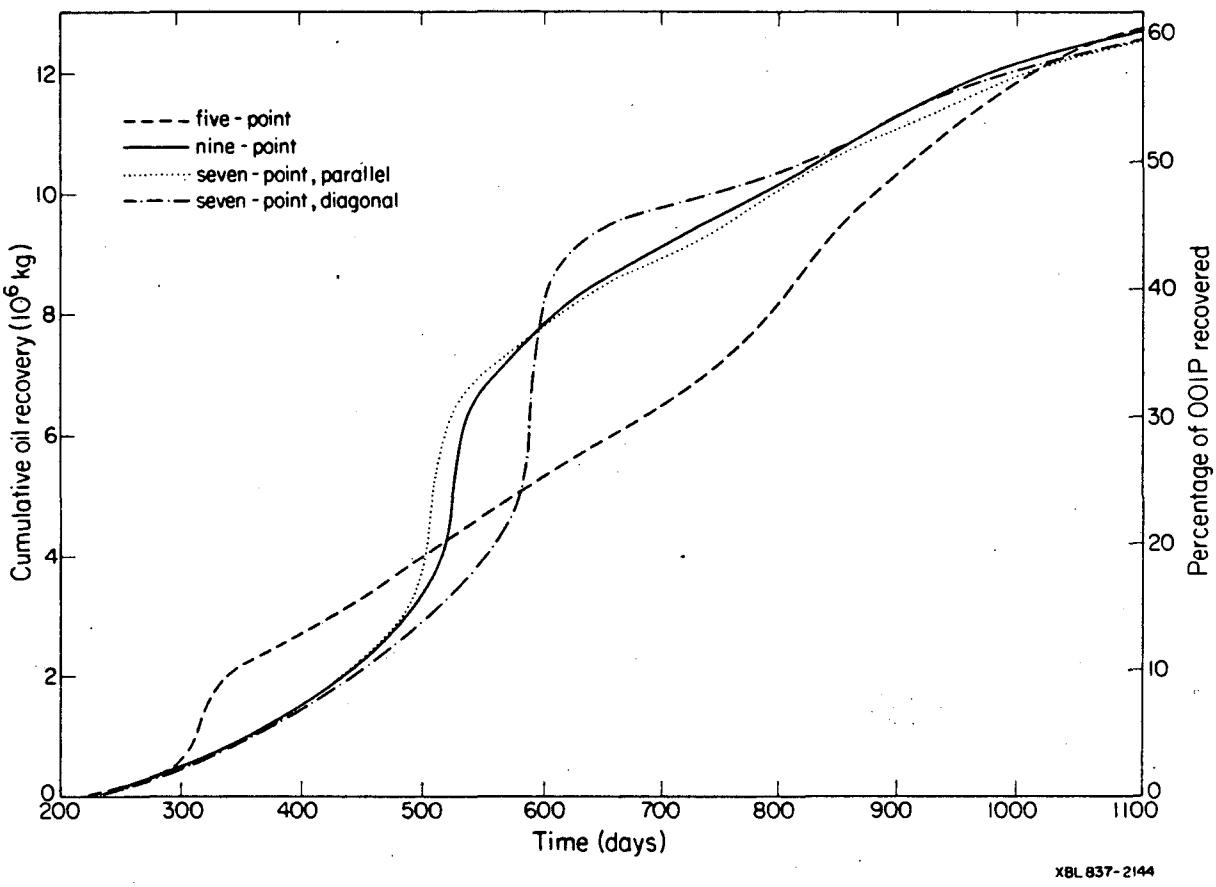

Figure 9: Cumulative oil recovery for seven-spot.

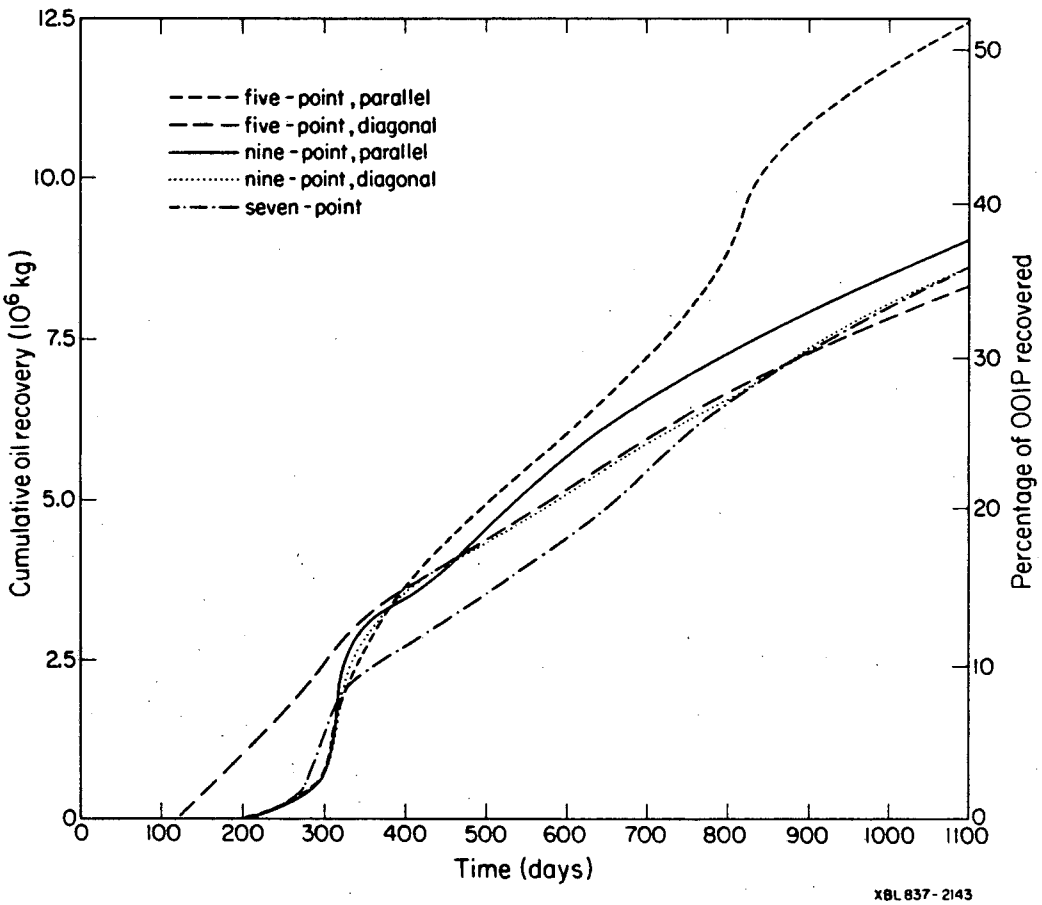

Figure 10: Cumulative oil recovery for nine-spot.

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