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Bell's Theorem and the Foundations of Quantum Physics

Henry P. Stapp

The idea that causal influences propagate only forward in time, from earlier cause to later effect, and no faster than light, can be shown to be mathematically incompatible with experimentally verified predictions of quantum theory.

Einstein and Bohr were the principal protagonists in an epic debate of the issue: Can quantum-mechanical description of physical reality be considered complete? Bohr, one of the two main founders of quantum theory, maintained that the quantum-mechanical description of physical reality is complete, even though the formalism must be regarded merely as a tool for making predictions about our observations of atomic processes, rather than as a description of the external situation itself. Einstein defended the more traditional view that a complete description of physical reality should describe also the external situation as it supposedly exists independently of observation, not merely our observations of it.[1]

Bohr's ideas about the interpretation of the mathematical formalism of quantum theory are the foundation of orthodox (Copenhagen) interpretation of quantum theory. In his own words "...the formalism does not allow pictorial representation along accustomed lines, but aims directly at establishing relations between observations obtained under well defined conditions."[2] "Strictly speaking, the

mathematical formalism of quantum theory ... merely offers rules of calculation for the deduction of expectations about observations obtained under well-defined experimental conditions specified by classical physical concepts."[3] "...the appropriate physical interpretation of the symbolic quantum-mechanical formalism amounts only to predictions, of determinant or statistical character, pertaining to individual phenomena appearing under conditions defined by classical physical concepts."[4] Bohr acknowledged that the notion that a description of this kind could be a complete description of physical reality entailed "a radical revision of our attitude towards physical reality."[5]

The critical issue in the Einstein-Bohr debate was the meaning of "physical reality." To get a logical grip on this concept Einstein, together with two younger colleagues, Podolsky and Rosen, introduced the following <u>criterion of physical reality</u>: "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."[6]

To use this criterion it is necessary to satisfy the condition "without in any way disturbing a system." This requires some assumption about causal influences. Here Einstein, Podolsky, and Rosen (EPR) relied upon an idea drawn from the theory of relativity: causal influences propagate always forward in time, from earlier cause to later effect, and never faster than light. From this locality condition, together with the criterion of physical

reality and the quantum-mechanical formalism itself, EPR were apparently able to prove, by logical analysis, the incompleteness of the quantum-mechanical description of physical reality.

The locality condition needed for the EPR argument should, according to Einstein, be held "absolutely fast." Bohr likewise appeared to regard it as beyond question. However, this condition is now known to be mathematically incompatible with certain predictions of quantum theory. These predictions have been accurately confirmed[7] under experimental conditions essentially equivalent to those needed for the EPR argument. Hence it may now be safely concluded that this locality condition is incorrect.

The failure of this firmly held belief about the spacetime structure of causal influences invalidates the EPR argument. But, more importantly, it alters basic ideas about the nature of physical reality. For it revises understandings of the way in which quantum theory accomodates the theory of relativity. This revision has a profound impact on the question of the proper interpretation of quantum theory. For it opens the way to a thoroughly objective interpretation of quantum theory that meets Einstein's demand for description of the external situation, and is moreover in close accord with the intuitive ideas of Heisenberg, and probably those of most contemporary quantum physicists as well. The previously perceived need to regard the quantum-mechanical formalism as merely a tool for making predictions about observations appears in the light of the

failure of EPR locality to be simply an artifact of the tacit acceptance by the founders of quantum theory of a belief about the spacetime structure of causal influences that is mathematically incompatible with quantum theory itself.

This possibility for a thoroughly objective interpretation of quantum theory, opened up by recognition of the failure of earlier beliefs about locality, appears to have no immediate impact on atomic physics per se. But, it may be of crucial importance for the unification of the sciences. For to bring together into a single coherent framework of thought our knowledge from the fields of, for example, cosmology, biology, and psychology, on the basis of their common atomic underpinnings, one must deal with experimental situations that do not conform to the format demanded by the orthodox (Copenhagen) interpretation of Bohr. That format is admirably suited to experiments that arise in the realm of atomic and subatomic physics, but is ill-suited to experimental circumstances that can arise in these other domains of science.

In this article, I shall first describe in more detail the locality condition upon which the EPR argument rests, review the opinions about it expressed by Einstein and Bohr, and then demonstrate the mathematical incompatibility of this locality condition with the predictions of quantum theory itself. This result is a generalization of a theorem by J. S. Bell.[8] Next, I shall describe the revisions of ideas about the nature of physical

reality and about the interpretation of quantum theory demanded by the failure of this earlier idea about the spacetime structure of causal influences. Finally, I shall describe the impact of these revised ideas upon the program of enlarging the scope of quantum theory to provide an adequate foundation for the unification of science.

The locality condition

The locality condition needed for the EPR argument expresses certain ideas drawn from the theory of relativity. This condition is formulated within the general framework of quantum theory. In accordance with the ideas of Bohr the quantum formalism is regarded merely as a tool for making predictions about phenomena appearing under specified experimental conditions. And these experimental conditions are considered to be freely chosen by experimenters.

The relevant experimental situation involves two regions R_1 and R_2 that are limited both in space and in time, and situated so that nothing can travel from any point in either region to any point in the other region without moving either faster than light or backward in time. Thus, the two regions should be far apart in space, compared to their time durations, and approximately simultaneous. (See Fig. 1) Regions that enjoy this property are said to be spatially (or spacelike) separated. The experiment is set up so that within each region three things will happen: a choice will be made between two alternative possible

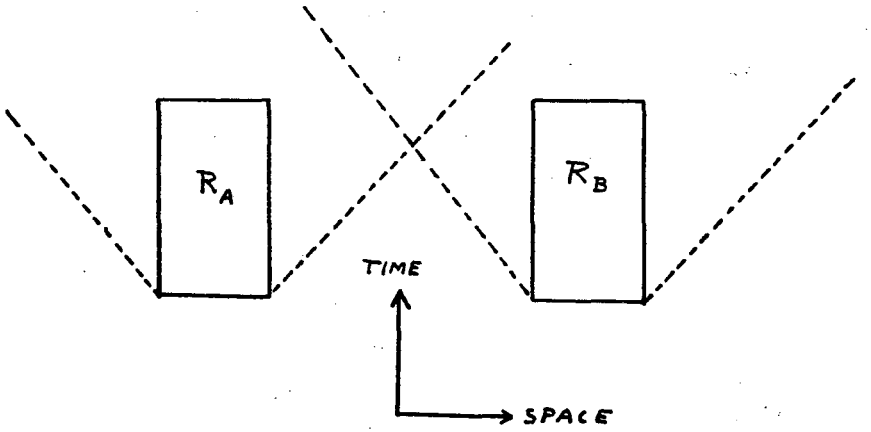

Fig. 1. Two regions are situated so that no information or influence can travel from any point in either region to any point in the other region without traveling either faster than light or backward in time. The dotted lines indicate the boundaries of the regions limited by this speed-of-light condition. The theory of relativity suggests that a choice made freely in one region cannot affect or influence observable phenomena appearing in the other.

measurements, the chosen measurement will be performed, and the result of that measurement will be observed.

According to the theory of relativity causal influences propagate no faster than light and only forward in time. Thus EPR assumed that a system S_2 in R_2 cannot in any way be disturbed by what is done with a system S_1 in R_1. Einstein later asserted: "But on one assumption we must, in my opinion, hold absolutely fast: the real factual situation of the system S_2 is independent of what is done with S_1, which is spatially separated from the former."[9]

In this later statement Einstein refers to "the real factual situation of the system S_2." It is crucial to the EPR argument itself that the concept of physical reality be introduced only through the criterion of physical reality. According to this criterion the EPR candidates for physical reality are certain values that can be predicted with certainty. These values are values of observables that can be measured in R_2: they are defined in terms of results of experiments that can be performed in R_2. In the context of the EPR argument the "system" referred to by the EPR criterion of physical reality consists essentially of these values, and the essential locality assumption is this: phenomena appearing in R_2 cannot depend on which measurement is freely chosen and performed in R_1. The word "cannot" signifies that the restriction is on the possibilities themselves: the possibilities for the phenomena appearing in R_2 under any conditions chosen in R_2 are limited to those that do not depend on which experiment is freely

chosen and performed in R_1. This statement of the locality condition is the weakest general formulation that allows the EPR argument to be carried through. It does not introduce any superfluous ontological elements that would undermine the EPR argument. In particular, it is not based on the prejudicial idea that the results of the measurements in R_2 are predetermined, or that anything actually exists in R_2 beyond the phenomena that actually appear there. It merely limits the possibilities for what can appear there.

Accepting this locality condition the EPR argument proceeds as follows: one can, by using quantum theory and the result of an experiment freely chosen and performed in R_1, predict with certainty the value of either one of two incompatible observables pertaining to system S_2 in R_2. Neither of these values, as defined by phenomena appearing in R_2 if the associated observable is measured, can be disturbed in any way by what is done in R_1. Consequently, on the basis of the EPR criterion of physical reality, both values are elements of physical reality. But the quantum-mechanical description does not allow both of these values to be simultaneously well defined. Thus the quantum-mechanical description is not a complete description of physical reality.

The EPR conclusion was challenged by Bohr.[10] Most of Bohr's reply was not directed at the EPR argument itself. The major part of his reply was devoted to showing that his usual argument for the practical completeness of quantum theory -- i.e., completeness with respect to

predictions about observations -- holds also in the physical situation considered by EPR. But Bohr also challenged EPR argument itself, on the basis of the meaning of "physical reality." He argued that the concept of physical reality appropriate to quantum theory is different from that of classical physics. The essence of quantum theory consists of the predictions that can be made by using it. But making predictions requires knowledge. According to quantum theory there are limitations on this knowledge associated with the uncertainty principle. And the form of these limitations depends upon which experiments one decides to do. Bohr showed explicity how the limitations in available knowledge arising from the choice of measurement on S_1 influence what kinds of predictions can be made about S_2. Thus to the extent that "physical reality" is that which our physical theories allow us to predict about a system one can say that what is done with S_1 does disturb the physical reality S_2.

Bohr summarizes his argument as follows:

> "From our point of view we now see that the wording of the above-mentioned criterion of physical reality proposed by Einstein, Podolsky and Rosen contains an ambiguity as regards the meaning of the expression "without in any way disturbing a system." Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of <u>an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system</u>. Since these conditions constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete."

Bohr's reply to the EPR argument does not directly challenge the EPR locality condition. Bohr fully endorses the idea that a choice between alternative possible experiments is to be considered a "free choice." He grants that the process of measurement in R_1 "does not directly interfere with the particle concerned," i.e., with the particle in R_2. He affirms that "Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation [i.e., S_2] during the last critical stages of the measuring procedure [on S_1]."

Although Bohr does not challenge the EPR locality assumption directly, he does maintain that the physical reality S_2 is disturbed by what is done with S_1. But this disturbance is epistemological: it is an influence on predictions via change in available knowledge, rather than a direct mechanical disturbance.

I turn now to the proof of the incompatibility of EPR locality with the statistical predictions of quantum theory. First the experimental set-up must be described in more detail.

The Experimental Set-Up

The experiment has two distinct phases. In the first phase two beams of low energy spin-1/2 particles are caused to collide near a point P lying at the center of several concentric spherical arrays of particle detectors. Each array covers a complete sphere, except for two escape holes, located at polar extremities. The escape holes in the various spherical arrays are lined up so that two particles, one from each beam, can scatter near P and then escape from the set of arrays through these holes. The two particles of this pair will then be traveling in opposite directions, along approximately known trajectories. Fast electronics is set up to keep track of all the incoming and scattered particles, to identify pairs of escaping particles, and to label each such pair by a different index i from the set (1, 2, ..., n). Information about the calculated time t_i of the collision of the two particles of each escaping pair i is sent ahead to the two parts of the second phase of the experiment.

The second phase involves two separate measuring devices D_A and D_B. The geometry is such that one particle from each pair i will enter D_A, and the other will enter D_B. The conditions on D_A and D_B are similar, and the symbol D_j stands for either one of them. (See Fig. 2)

The device D_j contains a Stern-Gerlach apparatus. This apparatus has a preferred direction, which is oriented approximately perpendicularly to the trajectories of the particles passing through it. The azimuthal orientation of

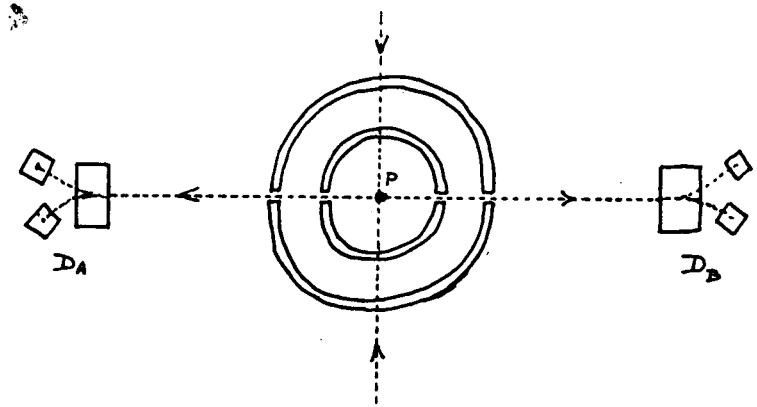

Fig. 2. The Experimental Set-Up. A pair of incident particles scatter near a point P, then escape through the polar escape holes in the concentric rings of particle counters. Each particle of the pair will enter one of the two devices D_A or D_B, where it will be deflected into either a certain "preferred" direction, or in the opposite direction.

this preferred direction is specified by an angle θ_j. (See Fig. 3) Each particle entering the apparatus will be deflected either along this preferred direction or in the opposite direction. Two particle detectors are placed behind the apparatus so that one or the other is activated according to whether the deflection is along the preferred direction or in the opposite direction.

The fast electronics is rigged to anticipate the activation by a particle from each pair i of one or the other of the two detectors in device D_j, and to assign to a variable r_{ji} the value +1 or -1 according to whether the deflection is along the preferred direction or in the opposite direction. The set of n values r_{ji} for all i in the set (1, 2, ..., n) is called the result r_j. It is arranged that this result will be recorded and visually displayed.

The device D_j is under the care of an experimenter E_j. Two possible values θ_j' and θ_j'' of the angle θ_j are fixed in advance, and it is arranged that a signal will be given to the experimenter E_j shortly before the arrival at D_j of the bunched set of particles from the n pairs i. Upon receipt of this signal the experimenter E_j will freely choose one or the other of the two angles, θ_j' or θ_j'', then place the preferred direction of the device D_j at this setting, and finally observe the visually displayed result r_j.

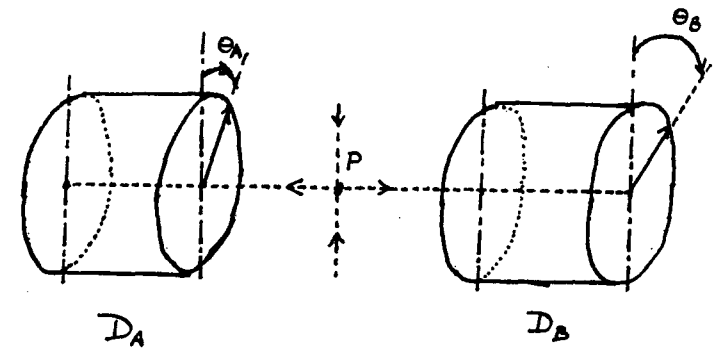

Fig. 3. Each device D_j has a preferred direction d_j, which is placed in the position specified by the azimuthal angle θ_j.

The time interval from the beginning of the processes of choosing the setting θ_j to the observation of the final result r_j is called T_j. The spatial region occupied by D_j and E_j is called $\bar{R}_j$. The set of points $(t,\vec{r})$ such that t is in T_j and $\vec{r}$ is in $\bar{R}_j$ is the spacetime region R_j.

The critical requirement on the experimental set up is that the two regions R_A and R_B be "spacelike separated." This means that no signal originating at any point in either region can reach any point in the other region without traveling either faster than light or backward in time.

In the experiment of Aspect, Dalibard, and Roger[2] the "experimenters" E_A and E_B were mechanical devices: the processes of "choosing" the setting and "observing" the results were done mechanically, rather than by human beings.

The Predictions of Quantum Theory

Quantum theory provides statistical predictions about phenomena appearing under appropriate experimental conditions. For each appropriate set of experimental specifications s there is a corresponding set Λ_s of <u>conceivable possibilities</u> λ_s for the phenomena appearing under conditions s. Quantum theory assigns a probability $P(\lambda_s)$ to each conceivable possibility λ_s.

In the situation described above there are four alternative possible experimental specifications s. They correspond to the four alternative possible pairs (θ'_A,θ'_B), (θ'_A,θ''_B), (θ''_A,θ'_B), and (θ''_A,θ''_B). The angles θ'_A, θ''_A, θ'_B, and θ''_B are specified to be 0°, 45°, 45°, and -45°, respectively. Thus we have the four alternative possibilities

$$a = (\theta'_A,\theta'_B) = (0°,\ 45°) = (\theta^a_A,\theta^a_B), \tag{1a}$$

$$b = (\theta'_A,\theta''_B) = (0°,\ -45°) = (\theta^b_A,\theta^b_B), \tag{1b}$$

$$c = (\theta''_A,\theta'_B) = (45°,\ 45°) = (\theta^c_A,\theta^c_B), \tag{1c}$$

and

$$d = (\theta''_A,\theta''_B) = (45°,\ -45°) = (\theta^d_A,\theta^d_B). \tag{1d}$$

Let s be any one of these four quantities. There are 4^n conceivable possibilities λ_s. These 4^n conceivable possibilities are labelled by the 4^n symbols

$$1_s,\ 2_s,\ 3_s,\ \ldots,\ \text{and}\ m_s,$$

where $m = 4^n$. Each conceivable possibility λ_s corresponds to a set of 2n numbers

$$\{r_{Ai}(\lambda_s),\ r_{Bi}(\lambda_s)\},$$

where i runs from 1 to n, and each number $r_{ji}(\lambda_s)$ is either +1 or -1 according to whether the deflection in device D_j of the particle from pair i is along the preferred direction of this device or in the opposite direction.

Quantum theory assigns a definite value to each probability $P(\lambda_s)$. From these values the following property follows: for any fixed s in the set (a, b, c, d), and any $\epsilon > 0$, the set of conceivable possibilities λ_s such that the equation

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\lambda_s)\ r_{Bi}(\lambda_s) + \cos(\theta^s_A-\theta^s_B) = 0 \tag{2}$$

fails by more than $\pm\epsilon$ have a total probability that tends to zero as n tends to infinity. Thus, for any $\epsilon > 0$, however small, one can, by taking n sufficiently large, exclude as prohibitively unlikely those conceivable possibilities for which (2) fails by more than $\pm\epsilon$.

The four special cases of (2) are

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\lambda_a)\, r_{Bi}(\lambda_a) = -1/\sqrt{2}, \tag{3a}$$

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\lambda_b)\, r_{Bi}(\lambda_b) = -1/\sqrt{2}, \tag{3b}$$

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\lambda_c)\, r_{Bi}(\lambda_c) = -1, \tag{3c}$$

and

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\lambda_d)\, r_{Bi}(\lambda_d) = 0. \tag{3d}$$

Locality Conditions

In quantum theory the choice of the experiment is considered to be a free choice. No restrictions are placed on the manner in which the choice is made, and the predictions do not depend on how this choice is made. Thus in the present context it is unreasonable to ascribe any special significance to the logical possibility of a correlation arising via influences from their common past between the choice of experiment made in one region and

results obtained in the other. There is, in principle, nothing to prevent our making these choices in some way that would render negligible any such correlations. Accordingly, we shall consider the choices of the experiments to be free, in the sense that the aforementioned possible correlations are assumed not to exist.

The information about which conditions are freely chosen in the region R_1, which can be either R_A or R_B, cannot get to the other region R_2 without traveling either faster than light or backward in time. In this situation, ideas from the theory of relativity suggest that the following locality condition should hold: the possibilities for the phenomena appearing in R_2 under any conditions chosen in R_2 are limited to those that do not depend on which experiment is freely chosen and performed in R_1.

Let us recall the definitions $a = (\theta'_A, \theta'_B)$, $b = (\theta'_A, \theta''_B)$, $c = (\theta''_A, \theta'_B)$, and $d = (\theta''_A, \theta''_B)$. Let $\hat{\lambda}_a$ be any one of the 4^n possible λ_a. The locality condition imposes the following four conditions on the conceivable possibilities:

1) If under freely chosen conditions a the phenomena appearing would be $\hat{\lambda}_a$ then under freely chosen conditions b the phenomena appearing would be a $\hat{\lambda}_b$ satisfying

$$r_{Ai}(\hat{\lambda}_b) = r_{Ai}(\hat{\lambda}_a). \tag{4a}$$

2) If under freely chosen conditions a the phenomena appearing would be $\hat{\lambda}_a$ then under freely chosen condi-

tions c the phenomena appearing would be a $\hat{\lambda}_c$ satisfying

$$r_{Bi}(\hat{\lambda}_c) = r_{Bi}(\hat{\lambda}_a). \tag{4b}$$

3) If under freely chosen conditions b the phenomena appearing would be $\hat{\lambda}_b$ then under freely chosen conditions d the phenomena appearing would be a $\hat{\lambda}_d$ satisfying

$$r_{Bi}(\hat{\lambda}_d) = r_{Bi}(\hat{\lambda}_b). \tag{4c}$$

4) If under freely chosen conditions c the phenomena appearing would be $\hat{\lambda}_c$ then under freely chosen conditions d the phenomena appearing would be a $\hat{\lambda}_d$ satisfying

$$r_{Ai}(\hat{\lambda}_d) = r_{Ai}(\hat{\lambda}_c). \tag{4d}$$

The condition 1, for example, limits the possibilities for phenomena appearing in R_A under the condition $\theta_A = \theta'_A$, chosen in R_A, to results that do not depend on the choice between θ'_B and θ''_B made in R_B. The other three conditions impose analogous limitations on the possibilities.

These four conditions, taken together, entail that if the phenomena appearing under freely choosen conditions a would be $\hat{\lambda}_a$ then the phenomena appearing under the three alternative possible conditions b, c, and d would be three conceivable possible results $\hat{\lambda}_b$, $\hat{\lambda}_c$, and $\hat{\lambda}_d$, respectively, that are restricted by the four conditions

(4). Thus, the original $\hat{\lambda}_a$ appears as part of a quartet $(\hat{\lambda}_a, \hat{\lambda}_b, \hat{\lambda}_c, \hat{\lambda}_d)$ that satisfies all four equations (4). The same four equations are obtained no matter which of the four values of s is taken as the starting point. Therefore all of the conceivable possibilities allowed by the locality condition are members of quartets of conceivable possibilities $(\hat{\lambda}_a, \hat{\lambda}_b, \hat{\lambda}_c, \hat{\lambda}_d)$ that satisfy (4).

One tacit assumption has been made here. In the contemplation of the conceivable possibilities it is assumed that under any one of the four alternative possible conditions s the phenomena appearing would be some <u>single</u> conceivable possible result λ_s. It might seem that this is no assumption at all. For whichever choice is made we observe only one result. Thus the tacit assumption seems to be nothing more than a recognition of what one must mean by the conceivable possibilities.

But there is a subtle point, which is in fact exploited by the so-called many-worlds interpretation of quantum theory. This is the fact that one part of the conceivable result λ_s appears in one region R_1 and the other part appears in the other region R_2. These two parts cannot appear in the consciousness of any single observer until some later time, when the information about the results in the two regions has been brought together into a single region.

In the many-worlds interpretation[11-14] the observer in each region splits into four different real persons, each observing a different choice of experiment in his region, or

a different experimental result there. These four observers occupy the same spacetime region, but are oblivious of each other. The four observers from region R_1 can travel and meet the four observers from R_2, and they can then compare notes. At this point the two independent sets of four observers are transformed into one set of sixteen pairs of observers, with each of the sixteen pairs containing one of the four observers from R_1 and one from R_2.

This strange-sounding picture of reality arises from interpreting the probability function of quantum theory as a representation of reality itself. The probability function behaves as a probability function should: it splits into effectively independent branches representing the different distinguishable possibilities, with each branch carrying an appropriate "weight" or probability. But in the many-worlds interpretation each branch, which is ordinarily thought to represent only a possibility, represents a part of reality itself.

The tacit assumption mentioned above fails in the many-worlds picture of nature, and no conflict between locality and quantum theory arises. This fact could be used to support the many-worlds interpretation, if failure of locality were regarded as more objectionable than the unbounded proliferation of unobservable real worlds. But we shall accept the normal idea that under any one of the four alternative possible conditions s precisely one of the 4^n conceivable possible results λ_s will appear.

Proof of Contradiction

Equations (3) represent restrictions on the conceivable possibilities arising from the statistical prediction of quantum theory. Equations (4) represent restrictions on the conceivable possibilities arising from the locality condition. These two conditions are, however, mathematically incompatible: it is not possible to satisfy both of them. To prove this consider any quartet $(\hat{\lambda}_a, \hat{\lambda}_b, \hat{\lambda}_c, \hat{\lambda}_d)$ satisfying (4). Let the arbitrarily small $\epsilon > 0$ associated with (3) be first set to zero. Then (3c) gives

$$r_{Ai}(\hat{\lambda}_c) = -r_{Bi}(\hat{\lambda}_c). \tag{5}$$

Then (3), (4), and (5) give

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\hat{\lambda}_a)\, r_{Bi}(\hat{\lambda}_a) = -1/\sqrt{2}, \tag{6a}$$

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\hat{\lambda}_a)\, r_{Bi}(\hat{\lambda}_b) = -1/\sqrt{2}, \tag{6b}$$

and

$$\frac{1}{n}\sum_{i=1}^{n} r_{Bi}(\hat{\lambda}_a)\, r_{Bi}(\hat{\lambda}_b) = 0. \tag{6c}$$

Equations (6), and the fact that each r_{Ai} and r_{Bi} is either +1 or -1, entail that

$$\frac{1}{n}\sum_{i=1}^{n} (\sqrt{2}\, r_{Ai}(\hat{\lambda}_a) + r_{Bi}(\hat{\lambda}_a) + r_{Bi}(\hat{\lambda}_b))^2$$

$$= 2 + 1 + 1 - 2 - 2 = 0. \tag{7}$$

The fact that each $r_{Ai}(\hat{\lambda}_a)$, $r_{Bi}(\hat{\lambda}_a)$, and $r_{Bi}(\hat{\lambda}_b)$ is either +1 or -1 entails that

$$\frac{1}{n}\sum_{i=1}^{n} (\sqrt{2}\, r_{Ai}(\hat{\lambda}_a) + r_{Bi}(\hat{\lambda}_a) + r_{Bi}(\hat{\lambda}_b))^2$$

$$> (\sqrt{2} - 2)^2. \tag{8}$$

Equations (7) and (8) entail

$$0 > (\sqrt{2} - 2)^2. \tag{9}$$

This is a mathematical contradiction. Small changes associated with an arbitrarily small but nonzero value of ϵ cannot undo this large discrepancy. Hence, the conditions (3) and (4), which express the requirements of quantum theory and EPR locality, respectively, are mathematically incompatible.

Impact on the Theory of Relativity

The failure of EPR locality has no practical impact on the theory of relativity, provided quantum theory is, from a practical standpoint, complete. Failure of EPR locality means that phenomena appearing in R_2 cannot in general be independent of the choice made in R_1. However, one cannot determine, either by experiment or by theory, what "would have happened" if the other choice had been made. So the "nonlocal influence" entailed by the failure of EPR locality cannot be directly observed. It was therefore an unwarranted and unnecessary requirement on the

part of Einstein (and all others who were involved on both sides of the EPR controversy) to demand the absence of nonlocal influences of the kind ruled out by EPR locality.

It is a trivial consequence of the quantum formalism that the <u>predictions</u> about phenomena arising in R_2 are independent of the choice made in R_1. In the quantum formalism the choice between the two alternative possible experiments on the system in region R_1 is represented by a choice between two different decompositions of the unit matrix I_1 for that system into a sum of projection operators. If one were to specify the particular result that occurs in region R_1 then there would be a corresponding prediction for result in R_2. This prediction would depend on which result is specified in R_1, and hence also on which of the two possible experiments is performed in R_1. But if one has no information about which particular result occurs in R_1 then to obtain the prediction for R_2 one must sum over the various possible results in R_1. This sum gives, for each of the two alternative possible choices of measurements in R_1, the same unit matrix I_1. Consequently the predictions for R_2 are independent of which experiment is chosen in R_1.

Because the <u>statistical predictions</u> for R_2 are independent of which experiment is chosen in R_1 one might naturally think that the <u>individual results</u> could at least be imagined to have this same independence property, particularly if the regions R_1 and R_2 are spacelike separated. Bell's theorem, in its generalized form, asserts

that one cannot even imagine the individual results to have this independence property, without contradicting, in certain cases, the statistical predictions of quantum theory.

Irrelevance of Determinism

The incompatibility of locality with the predictions of quantum theory was first noted by Bell.[8] The first part of his analysis went as follows: using the fact, which follows from equation (2), that $r_{Ai} = -r_{Bi}$ for $\theta_A = \theta_B$, and assuming locality, he argued that "Since we can predict in advance the result [in R_2 for any setting θ_2] by previously measuring the [result in R_1 with $\theta_1 = \theta_2$], it follows that the result of any such measurement must actually be predetermined." The two brackets appearing in this quotation contain translations into the present terminology of Bell's phrases [of measuring in any chosen component of $\vec{\sigma}_2$] and [same component of $\vec{\sigma}_1$].

For a result in R_2 to be predetermined it must be determined at earlier (or previous) times relative to R_2. The "previously" occurring in Bell's argument refers to the time of a spacelike separated event. According to relativistic ideas <u>neither</u> of two spacelike separated events is earlier than the other, in any meaningful physical sense. Yet Bell's argument requires that each be earlier than the other.

Bell used this preliminary argument to justify restricting his attention to deterministic theories. He

himself apparently did not regard the preliminary argument as conclusive, for he later studied nondeterministic theories. In any case, determinism is usually regarded not as a part of the conclusion of Bell's work but rather as part of the assumption. Another assumption is the idea of hidden variables. Thus Bell's main result is generally stated in the form: no deterministic hidden-variable theory that reproduces the statistical predictions of quantum theory can be local.

This limited form of the nonlocality result is important to investigators who want to go back to classical ideas, and interpret quantum theory as a normal statistical theory of an underlying deterministic reality: Bell's result shows that a broad class of theories representing such possibilities must be nonlocal. However, for mainstream physicists, who had already rejected both determinism and hidden variables, Bell's result carried no suggestion of any failure of locality. Rather, it buttressed convictions about determinism and hidden variables.

It was soon pointed out that Bell's assumptions of determinism and hidden variables can be circumvented.[15] The main part of Bell's argument starts from the assumption that there is a set of variables $(\theta_A, \theta_B, \lambda)$ such that the results $r_A = +1$ and $r_B = +1$ can be expressed as functions of these variables, with r_A independent of θ_B, and r_B independent of θ_A. In accordance with the normal ideas of classical statistical mechanics one can then write the

expectation value of the product $r_A r_B$ under conditions (θ_A, θ_B) as

$$\langle r_A r_B ; \theta_A, \theta_B \rangle = \int \rho(\lambda) r_A(\theta_A, \lambda)\ r_B(\theta_B, \lambda)\ d\lambda\ , \qquad (10)$$

where the characters of both the set of hidden variables and of the weight function $\rho(\lambda)$ are left unspecified. Regardless of the characters of λ and $\rho(\lambda)$ Bell shows that the form (10) cannot reproduce certain predictions of quantum theory.

The argument in this original form rests on the assumed existence of the set of deterministic hidden variables λ. But if one identifies the possible values of with the possible values of the set of indices i that label the n pairs of particles, and, furthermore, and concentrates weight 1/n on each of these integers, the integral becomes

$$\frac{1}{n}\sum_{i=1}^{n} r_A(\theta_A, i)\ r_B(\theta_B, i).$$

The <u>hidden</u> variables λ are thereby eliminated: they are replaced by the <u>observed</u> electronic signals that identify the various pairs i. The quantities $r_B(\theta_B, i) = r_{Bi}(\theta_B)$ and $r_A(\theta_A, i) = r_{Ai}(\theta_A)$ can then be interpreted in terms of the conceivable possibilities for phenomena that can appear under conditions (θ_A, θ_B), as was previously discussed. Then the only assumption outside quantum theory itself is the locality condition, which is formulated within the logical framework provided by quantum theory. In this way Bell's

theorem is converted to the statement that quantum theory is nonlocal, in the sense that it is incompatible with EPR locality.

Irrelevance of Realism

Some authors have linked nonlocality to the question of realism. Clausen and Shimony[16] start their review article with a definition: "Realism is a philosophical view, according to which external reality is assumed to exist and have definite properties whether or not they are observed by someone." Those authors go on to say: "Consequently, it can now be asserted with reasonable confidence that either the thesis of realism or that of locality must be abandoned. Either choice will drastically change our concepts of reality and of space-time."

d'Espagnat[17] asserts: "Even if quantum theory is considered to be no more than a set of rules it is still in conflict with a view of the world many people would consider obvious and natural. This view is based on three assumptions, or premises that must be accepted without proof. One is realism, the doctrine that regularities in observed phenomena are caused by some physical reality whose existence is independent of human observers."

Both sets of authors suggest that one way to retain locality would be to abandon realism. Indeed, the proofs of the failure of locality given by these authors do assume realism. But the proof given here does not. If this proof had been based on the assumed existence of functions

$r_{Ai}(\theta_A,\theta_B)$ and $r_{Bi}(\theta_A,\theta_B)$ that represent the results that would appear if the freely chosen conditions are (θ_A,θ_B) then it might conceivably be claimed that there was a hidden assumption akin to realism. But the proof given above makes no such assumption, and involves no presumption that anything actually exists beyond the phenomena that actually appear. Hence rejection of realism cannot save locality: the conflict is with quantum theory itself. In fact, as will be discussed next, it is precisely by accepting rather than rejecting a thoroughly objective realistic viewpoint that one can most naturally accomodate, and in fact quantitatively explain, the failure of EPR locality.

Interpretations of Quantum Theory

The central tenet of the orthodox (Copenhagen) interpretation of quantum theory is that "...the appropriate physical interpretation of the symbolic quantum-mechanical formalism amounts only to predictions, of determinant or statistical character, pertaining to individual phenomena appearing under conditions defined by classic physical concepts."[18,19]

Bohr argued convincingly for the practical completeness of the theory, considered as a theory of predictions about observations of atomic phenomena.

Heisenberg's version[20] of the Copenhagen interpretation provides a partial understanding of the nature of the underlying reality. According to Heisenberg the probability function that occurs in quantum theory

"represents a mixture of two things, partly a fact and partly our knowledge of a fact." The knowledge aspect is analogous to what occurs in classical statistical mechanics. The other aspect is an objective "potentia" or "tendency for events". These events are actual events that can arise from the confluence of the measuring device with the atomic system represented by the probability function. "The probability function combines objective and subjective elements. It contains statements about possibiliites or better tendencies ("potentia" in Aristotelian philosphy) and these statements are completely objective, they do not depend on any observer; and it contains statements about our knowledge of the system, which of course are subjective insofar as they may be different for different observers. In ideal cases the subjective element in the probability function may be practically negligible as compared to the objective one. The physicists then speak of a 'pure case'".

Heisenberg's conception of nature in terms of the ideas of potentia and actual events is similar to one developed by the logician and philosopher A. N. Whitehead.[21] To maintain logical coherence and avoid contradictions it is probably necessary to adopt Whitehead's idea that nature is a cumulative process: each actual event adds to the totality of what is fixed and definite. Each such event is conditioned by, but not determined by, the events occurring prior to it in the cumulative process. Each event may be considered to logically include all prior

events, and a well-ordered sequence of events is then defined by logical inclusion.[22]

The experiment under consideration involves three spacetime regions: R_A, R_B, and R_0. The region R_0 is where the initial collisions of the pairs of particles occur. These three regions and the strips joining them form the V-shaped region shown in Fig. 4.

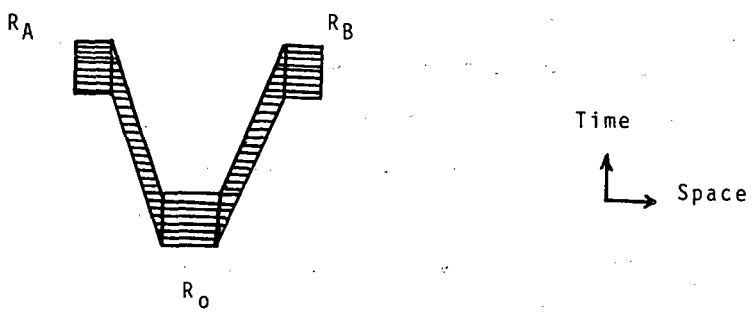

Fig. 4. A spacetime diagram showing the two experimental regions R_A and R_B, the region R_0 where the initial collision occurs, and the spacetime region outside of which the probability functions of the particles of the n pairs i are effectively zero.

The regions outside the V-shaped region in Fig. 4 can be filled with sliding walls of lead without affecting the statistical predictions of quantum theory for this experiment. But even weak magnetic fields within the V-shaped region will affect these predictions, and hence also any influences associated with a failure of locality. Hence it is reasonable to suppose that any "nonlocal influences"

associated with the failure of EPR locality are associated with the V-shaped region.

Analysis of the experiment from a "potentia" point of view is straightforward. The actual events associated with the preparation in R_0 leave the atomic system in a certain state. This state is represented by a wave function that is defined over all spacetime. It represents certain "potentia" for the events in R_A and R_B.

The spin part of this wave function is time independent. It is the singlet state Ψ with components $\Psi_{\alpha,\beta}$. The two indices α and β are the spin indices associated with the two particles. They can take the values 1/2 and -1/2, and the components of Ψ are

$$\Psi_{1/2,1/2} = 0$$
$$\Psi_{-1/2,-1/2} = 0$$
$$\Psi_{1/2,-1/2} = 1$$
$$\Psi_{-1/2,1/2} = -1.$$

There are many events in the fundamental well-ordered sequence. The next one of interest to us is either the one involving the confluence of the atomic system and measuring devices in region R_A, or the analogous event in R_B. Suppose the event in R_B comes first in the well-ordered sequence. Then, with equal probabilities, one of two states will be actualized. The new state corresponds to a new wave function defined over all spacetime. In particular, the wave function throughout the V-shaped spacetime region suddenly changes. If the spin state actualized in R_B is the state ψ_B with components $\psi_{B,\beta}$ then the spin state in R_A

suddenly becomes, according to the standard rules of quantum theory,

$$\Psi_\alpha = \Psi_{\alpha,1/2}\,\psi^*_{B,1/2} + \Psi_{\alpha,\,-1/2}\,\psi^*_{B,-1/2}.$$

This defines the new "potentia" for events in R_A. This new potentia depends on ψ_B, which depends on how the device was set in R_B. The probability for the actualization in R_A of a state ψ_A with components $\psi_{A,\alpha}$ is given by the rules of quantum theory as

$$\left|\Psi_{1/2}\,\psi^*_{A,1/2} + \Psi_{-1/2}\,\psi^*_{A,-1/2}\right|^2.$$

This calculation is standard. But the description of what is being calculated is expressed in terms of the idea of the "potentia" for actual events, rather than in terms of knowledge of observers. Expressed in this way there is no mystery about the nonlocal influence: the potentia in R_A for the actual event occurring there depends on the choice of experiment made in R_B. Hence the phenomena appearing in R_A can depend on the choice made in R_B.

This way of understanding the failure of EPR locality is based on interpreting quantum theory objectively, in terms of objective potentia. Prior to Bell's theorem such an interpretation seemed untenable. For it entails, as just shown, a dependence of phenomena appearing in one region upon the choice made by an experimenter in a spacelike separated region. Such a

dependence was deemed contrary to the theory of relativity, and Heisenberg did not consistently uphold the objective point of view. At the critical point where, from an objective point of view, the events in region R_B disturb the objective potentia in R_A, Heisenberg shifts to a subjective interpretation of the wave function: "The observation itself changes the probability function discontinuously; it selects of all possible events the actual one that has taken place. Since through the observation our knowledge of the system has changed discontinuously, its mathematical representation has also undergone the discontinuous change and we speak of a quantum jump. When the old adage 'Natura non facit saltus' is used as a basis for criticism of quantum theory, we can reply that certainly our knowledge can change suddenly and this fact justifies the use of the term 'quantum jump.'

"Therefore, the transition from the 'possible' to the 'actual' takes place during the act of observation. If we want to describe what happens in an atomic event, we have to realize that the word 'happens' can apply only to the observation, not to the state of affairs between two observations. It applies to the physical, not the psychical act of observation, and we may say that the transition from the 'possible' to the actual 'actual' takes place as soon as the interaction of the object with the measuring device, and thereby with the rest of the world has come into play; it is not connected with the act of registration of the result in the mind of the observer. The discontinuous change in the

probability function, however, takes place with the act of registration, because it is the discontinuous change in our knowledge in the instant of registration that has its image in the discontinuous change in the probability function."

The sudden change in the probability function, considered as a representation of the physicist's knowledge, is completely normal: it occurs also in classical statistical mechanics. But what about the sudden change in the objective potentia associated with the physical, not psychical, act of observation? Heisenberg shifts attention from this objective change by shifting, in effect, at the critical stage, to an essentially subjective interpretation of the wave function.

But why did Heisenberg shift over to the subjective interpretation? The reasons are evidently two. First, the full development of a thoroughly objective interpretation would mean specifying precise details of the process of actualization that appear to have no bearing at all upon the predictions that one can make about experiments on atomic systems. This reason is a purely practical one. Second, the admission of a sudden objective change would have created a conflict with the prevailing relativistic ideas about the spacetime structure of causal influences. In the subjective interpretation the sudden change in the wave function in a distant region represents nothing more than a change in our knowledge. Hence it can exert no nonlocal causal influence.

Proof of the failure of EPR locality completely reverses the situation: the objective interpretation explains this failure automatically and quantitatively, whereas the subjective interpretation accords with the idea that locality should hold, rather than fail. Moreover, the need perceived by Bohr for "a radical revision of our attitude towards physical reality" appears in the light of the failure of EPR locality to be an artifact of Bohr's attempt to compensate, through epistemological analysis, and his idea of an indirect nonlocal influence via change in available knowledge, for his tacit acceptance of a prejudice about the spacetime structure of causal influences that is mathematically incompatible with quantum theory itself.

Bohr stressed that in the description of a quantum-mechanical process the entire experimental arrangement must be considered as a whole. The failure of EPR locality provides an understanding of the reason for this: if two parts of a system have interacted previously then the response of one part to its own environment cannot, in general, be considered independent of the environmental conditions <u>encountered</u> by the other part. Quantum-mechanical effects consequently imbue physical processes with a feature of macroscopic wholeness or nonseparability that is alien to classical physics. Bohr's focus on epistemological considerations, rather than upon mathematical and thoroughly objective ones, tends more to obscure than lay bare the origins of this feature of wholeness that characterizes quantum phenomena.

Consequences for Science in General

Given the practical completeness of quantum theory apparently proved by Bohr the question arises: what practical difference does it make how we think about these things?

The answer lies in the fact that the format for quantum theory used in the Copenhagen interpretation covers only a limited class of situations, namely those that occur in atomic and subatomic physics. This format involves some classically described devices that prepare an atomic system, and some classically described devices that respond to the atomic system. The preparing and responding devices must be very large, for in this case they act in an important way on their general environment, and hence cannot be adequately described quantum theoretically unless the entire universe is included in the description, which is, of course, not feasible within the framework of a practical description. The interaction with the environment therefore introduces disturbances of phase relationships that make classical description of these devices both possible and necessary. On the other hand, the atomic system must be very small, so that disturbances due to interaction with the enviroment can be either ignored, or adequately represented by a prescribed classical background field. Thus in cases to which the Copenhagen format can be applied the interface between the part of the universe described classically and the part

described quantum-mechanically is a boundary of very special type: it separates a huge system from a tiny system.

The question is: how can we go beyond this special case? Every physical system is composed in some sense of atomic or subatomic systems, and its detailed properties and behavior therefore depend on quantum-mechanical effects. How, in general, can we describe these effects? How, for example, can we accurately describe the quantum-mechanical underpinnings of the growth and behavior of a biological system?

Bohr said "it is obviously a quite open question whether the information we have acquired of the laws describing atomic phenomena provides us with a sufficient basis for tackling the problem of living organisms."[23] Einstein said that the orthodox forumulation of quantum theory "constitutes an optimum formulation of [certain] connections" but "offers no useful point of departure for future developments."[24]

To get an idea of the problem involved consider a living virus. It is to some extent an isolated system. Yet it interacts continually with its environment. If it were to be treated as a quantum-mechanical system, weakly interacting with its environment, then it would be both the quantum-mechanical system described by the wave function, and, by virtue of its size, also the preparing and responding device. The behavior of the virus fixes the "settings of this device". Hence the virus acts as the experimenter who fixes the experimental conditions under

which the quantum phenomena appear. The virus also records within its structure the results of the quantum-mechanical acts. Thus the virus combines into one entity the quantum-mechanical system, the preparing and responding devices, the experimenter who fixes the experimental conditions, and the "observer" that registers the results. In the format of the Copenhagen interpretation these systems are separate, and are described in different ways. Consequently, the Copenhagen format does not provide an adequate basis for a precise description of quantum-mechanical effects in the growth and behavior of biological systems.

The essential point is that Bohr's proof of the practical completeness of quantum theory pertains to a particular way of exploring quantum effects in atomic phenomena. There is no quarantee that, and Bohr never claimed that, every manifestation of a quantum-mechanical effect in every area of science must arise only in experiments of the particular kind he considered.

Science is today highly fragmented: there are many disciplines, each with its own language and basic concepts. As scientific knowledge grows, each discipline tends to move into boundary regions where it interfaces with others. Coherent reconciliation of the diverse disciplines at their interfaces demands an over-arching theoretical framework.

An element of commonality of these diverse disciplines is the atomic or subatomic constitution of all processes. At the atomic and subatomic levels quantum-

mechanical effects become important. Thus quantum mechanics, in some form, must be woven into the fabric of any unified scientific conception of nature. The form provided by the observer-oriented Copenhagen interpretation is ill-suited for this, for the reasons just described. And to place the human observer at the center of our understanding of the birth and evolution of the universe, or of life itself, or of consciousness itself, by forcing scientific theory to follow the preparation-response format appropriate to atomic physics would impose an unwarranted egocentric restriction on the form that useful theories might take.

The challenge of enlarging the scope of guantum theory to provide a foundation for the unification of the sciences has not yet been seriously taken up by scientists. There have, however, been many proposals for interpreting quantum theory in objective ways. These proposals provide different ways for tackling phenomena not adequately covered by the Copenhagen format. Three prominent candidates are: (1) the many-worlds interpretation[11-14], in which the wave function of quantum theory is interpretated as a representation of reality itself, not merely of the probabilities or tendencies for actual events; (2) the classical-world interpretation[25], in which there is a single real world of essentially classical type, which is buffeted about by a real quantum potential,[26-29] or by some stochastic disturbance;[30-33] and (3) the "potentia" interpretation already described. All

three models yield predictions that accord with those of the Copenhagen interpretation in the domain of atomic physics proper, but none has been developed to the point of providing unambiguous predictions outside the domain covered by the Copenhagen format. Whether such predictions can be deduced, and confronted with empirical data from other domains of science, remains to be seen. But it would seem unnatural that an aspect of nature so profound as the element of wholeness of physical processes induced by quantum nonseparability and nonlocality would have no ramifications in the development of life, consciousness, and the cosmos.

Within this broad context the impact of Bell's theorem, and its generalization, is to liberate scientific thought by freeing it from both a false prejudice about the spacetime structure of causal influences and from a restrictive observer-based interpretation of quantum theory. As regards this latter point it is interesting that the impact of these theorems is precisely opposite to what it first appeared to be. Bell's theorem was originally formulated within the context of the EPR argument. That argument was designed to show the incompleteness of the quantum-mechanical description of physical reality. Bell's result invalidated the argument by demonstrating the incompatibility of its two main premises, locality and the validity of the statistical predictions of quantum theory. Hence Bell's work appeared to support Bohr's position on the interpretation of quantum theory, and repudiate Einstein's

idea that physical theory should be about the external situation itself, not merely our observations of it.

Einstein also believed that the underlying physical theory should be local and deterministic. Bell's theorem, in its original form, did eliminate that possibility. But Einstein's deeper conviction that the theory should deal with the external situation itself, not merely our observations of it, is supported by the generalized form of Bell's theorem. For the observer-based Copenhagen interpretation gives no understanding of the failure of EPR locality, and in fact allowed Bohr to accept the validity of EPR locality, whereas the objective interpretation based on Heisenberg's idea of objective potentia gives a direct and quantitative understanding of the required nonlocal influences.

Discussion of the Proof of Nonlocality

The proof of nonlocality given above is austere: the mathematical manipulations provide very little intuitive understanding of what is going on. With the overall picture now in place, it is useful to go back and discuss the proof in more detail.

A basic feature of the proof is that it deals simultaneously with the four members of a quartet (λ_a, λ_b, λ_c, λ_d) of conceivable possible results of four alternative possible experiments, only one of which can actually be performed. Hence a critical question naturally arises: Can one really prove anything definite about nature

from an argument based on the simultaneous consideration of quantities that cannot simultaneously exist? This question, which would be important even in the context of classical physics, is given far greater importance in the present circumstance by the fact that quantum theory categorically rejects, as a matter of principle, the idea that the results of mutually incompatible experiments can be considered simultaneously well defined. The four alternative experiments a, b, c, and d, are excellent examples of mutually incompatible experiments: at most one of them can be actually performed in any single given experimental instance. Yet the proof rests crucially on the simultaneous consideration of all four members of the quartet (λ_a, λ_b, λ_c, λ_d).

An important distinction must be made here. It is the logical distinction between a conjunction of the kind: "X is true" and "X is false", and a conjunction of the kind: "If Y is true then X is true" and "If Y is false then X is false". The first conjunction is a logical contradiction, whereas the second conjunction is not: it is equivalent to the statement that X is equivalent to Y.

In quantum theory there are "incompatible observables" such as the coördinate variable x and its conjugate momentum variable p. The quantum formalism provides no way of representing a state in which both x and p have well-defined values. On the other hand, the theory also asserts that the physical measurements of x and p are mutually incompatible: the two measurements cannot be

performed simultaneously. But then there is no need for the theory to represent a state in which both x and p have well-defined values.

It is a basic principle of orthodox quantum thinking that if a measurement of x is performed then one must not even imagine that p has a well defined value. The mathematical formalism does not support this idea, and paradoxes are avoided by banning the very thought. Thus orthodox quantum thinking categorically rejects conjunctions such as: "the value of p is $\hat{p}$" and "the value of x is $\hat{x}$". Such a statement is, from the point of view of quantum theory, a contradiction.

This conjunction is logically different from the conjunction: "If x is measured then x will be found to have a value $\hat{x}$" and "If p is measured then p will be found to have a value $\hat{p}$". Since x and p cannot be simultaneously measured there is no possibility of deducing from the above conjunction the "contradictory" conjunction: "x has value $\hat{x}$" and "p has value $\hat{p}$".

On the other hand, even the single statement "If x is measured then x will be found to have a value $\hat{x}$" is itself rejected in orthodox quantum thinking. For it presupposes, or suggests, that "the value that x will be found to have if x is measured" is already fixed, or well defined. That is, it presupposes determinism, which is itself rejected by orthodox quantum thinking, or in any case not presupposed. The statement that "if x were to be measured then x would be found to have the value $\hat{x}$" is

likewise rejected.

On the other hand, quantum thinking allows statements such as "if x were to be measured then some value for x would appear". Orthodox quantum thinking also allows conjunctions such as: "if x were to be measured then some value of x would appear" and "if p were to be measured then some value of p would appear". Since x and p cannot be simultaneously measured one cannot deduce from this allowed conjunction the disallowed conjuction: "a value of x would appear" and "a value of p would appear".

One of our key assumptions is this: "For every s in the set (a, b, c, d), if the experiment s is actually performed then some single result λ_s will actually appear." This assumption is a conjunction of statements about mutually incompatible measurements. But it leads to no contradiction with either the quantum formalism, or the idea of nondeterminism. For a value λ_s is asserted to appear only under the condition that the associated experiment is actually performed. And there is no presumption that the value that will appear under those conditions is determined prior to the creation of those conditions. Since the four conditions are mutually exclusive, one cannot deduce the simultaneous existence, or appearance, or well-definedness of any two λ_s. Thus our set of conjunctions does not contradict quantum thinking. In fact, it is a normal part of quantum thinking.

Let the assumption just stated be called "single result". And let us temporarily accept an "orthodox"

position consisting of the following assumptions:

1. Free Choice. The choices of the experimenters can be regarded as free.
2. Single Result. For each s in the set (a, b, c, d), if s is actually performed then <u>some</u> single result λ_s will actually appear.
3. Validity of Q.T. No matter which s is actually performed, the predictions of quantum theory can be expected to hold for the result λ_s that actually appears.
4. Only Actual Results. Unperformed experiments have no results, and one must not even imagine that they do.
5. Non-Determinism. The result of an actually performed experiment becomes fixed and well defined within the spacetime region where the process actually occurs, not before.
6. Strong Locality. Information and causal influences propagate only into the "future light cone", i.e., only forward in time, and no faster than light.

These six assumptions produce an immediate contradiction far simpler than the one discovered by Bell. For if the results r_{Ai} and r_{Bi} become fixed and well defined only in R_A and R_B, respectively, and information and causal influences propagate only into the future light core, as indicated in Fig. 1, then there is no way to obtain, for

$\theta_A = \theta_B$, and almost all values of i, the connection $r_{Ai} = -r_{Bi}$ demanded by 3.

This difficulty is the basis of the EPR argument. If one accepts assumptions 1 through 3, which are basic to orthodox quantum thinking, and accepts locality, then 5, hence also 4, must apparently be abandoned. This fact was turned by EPR into an argument for the incompleteness of quantum theory.

On the other hand, if one stays with quantum thinking, and accepts 1 through 5, then one must apparently abandon the locality assumption 6. This locality assumption comes from the theory of relativity, but not in any direct or compelling way. For it is entailed by the theory-of-relativity requirement not that "influences" propagate no faster than light, but only that signals propagate no faster than light. (A "signal" is a controlled, interpretable message.) Thus, if one adheres to the normal quantum precepts then there is already, even without Bell's contribution, an apparent need to abandon the strong idea of locality represented by 6.

This argument is not a complete proof of nonlocality. For there is the logical possibility of abandoning the "orthodox ideas" 4 and 5 without abandoning the more fundamental ideas 1, 2, and 3. Indeed, if one insists on 1, 2, 3, and 6, then it appears that one is forced to accept the idea that the results for all the alternative possible settings are "predetermined:" the "particles" must evidently contain the information about how they will respond to any possible setting. For under

conditions 1, 2, 3, and 6, there appears to be no other way to understand the result that $r_{Ai} = -r_{Bi}$ for $\theta_A = \theta_B = \theta$, for any value of θ. This is evidently the point that Bell was trying to make in his preliminary argument.

This way of escaping nonlocality, by abandoning the quantum ideas 4 and 5, was blocked by Bell. For Bell showed, in effect,[15] that there is no set of values $r_{Ai}(\theta'_A)$, $r_{Ai}(\theta''_A)$, $r_{Bi}(\theta'_B)$, and $r_{Bi}(\theta''_B)$, for i = 1 to n, that satisfy the predictions of quantum theory in all four alternative possible cases. A simple proof of this result is given in Fig. 5 and its caption. With this way of escape blocked one seems forced to the conclusion that locality fails: property 6 is incompatible with properties 1, 2, and 3 regardless of whether one accepts or rejects properties 4 and 5.

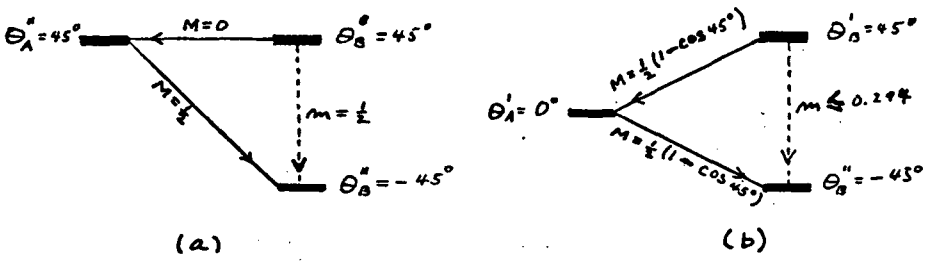

Fig. 5. A simple proof of Bell's theorem can be obtained as follows.[34] Let each pair i for which $r_{Ai}(\theta_A) = -r_{Bi}(\theta_B)$ be called a θ_A to θ_B "match". Let each pair i for which $r_{Ai}(\theta_A) = r_{Bi}(\theta_B)$ be called a θ_A to θ_B "mismatch." Quantum theory predicts that the fractions of θ_A to θ_B "matches" and "mismatches" approach $\frac{1}{2}(1 + \cos(\theta_A - \theta_B))$ and $\frac{1}{2}(1 - \cos(\theta_A - \theta_B))$ respectively, as the number n of pairs i approaches infinity. The expected fractions M of θ_A to θ_B mismatches in the various cases are shown in Fig. 5. Consider first the setting in $\theta_A = \theta''_A = 45°$ shown in Fig. 5a. Taking in tandem the fraction of mismatches M=0 and $M = \frac{1}{2}$ from θ'_B to θ''_A and then from θ''_A to θ''_B one concludes that the expected fraction of θ'_B to θ''_B mismatches $r_{Bi}(\theta'_B) = -r_{Bi}(\theta''_B)$ is $m = \frac{1}{2}$. Consider next the setting $\theta'_A = 0°$ shown in figure 5b. The same argument now shows that m is no greater than the sum of the two fractions $M = \frac{1}{2}(1-\cos 45°)$ and $M = \frac{1}{2}(1-\cos 45°)$. That is, $M \leq (1-\cos 45°) \approx 0.293$. This contradicts the earlier result $m = \frac{1}{2}$. Consequently, there is no quartet of n vectors $[(r_{Ai}(\theta'_A), \ldots, r_{An}(\theta'_A)), (r_{Ai}(\theta''_A), \ldots, r_{An}(\theta''_A)),$

$(r_{Bi}(\theta'_B), \ldots, r_{Bn}(\theta'_B)), (r_{Bi}(\theta''_B), \ldots, r_{Bn}(\theta''_B)]$, that satisfies all four predictions of quantum theory. Hence one must reject either the locality requirement that the results in each region are independent of which experiment is chosen in the other region, or the assumption that the results of the unperformed experiments can be considered definite, albeit unknown. This last assumption is called "contrafactual definiteness", and sometimes, euphemistically and inaccurately, "realism". In local deterministic hidden-variable theories of the kind considered by Bell the results of the unperformed experiments are well-defined, though unknown. Orthodox quantum thinking rejects both deterministic hidden-variable theories and contrafactual definiteness, and thus avoids drawing from Bell's theorem, in its original form, any suggestion of a failure of locality.

What remains to be achieved by the argument given earlier concerns fine points. First, there is the task of putting together, with no cracks, the two parts of the informal argument given earlier: one needs a rigorous proof of nonlocality. Second, there is the task of weakening the locality assumption 6. The "strong locality" assumption 6 refers to the transfer of information or causal influence without reference to any controllable cause. But the idea of a "causal influence" becomes clearest only when one has a controllable cause, and Einstein formulated his locality requirement only under this condition.

The logical question at issue can be made clear by formulating it in terms of "black boxes". Suppose there are two black boxes, connected together by a communication cable. Each black box has two buttons and a display register, but its internal constitution is completely unknown. Seated before each box is an observer. The display register on each box suddenly flashes the message "press a button", and the observer then whimsically chooses one of the two buttons and presses it. Shortly thereafter a number from 1 to 2^n, for some large n, appears briefly on the register of the first box, and a number in the same range appears on the register of the second box. Each observer records the button he pressed and the number that appeared on his box, and the process is then repeated until an unmistakable correlation is observed between the numbers appearing on the two boxes for each of the four alternative possible combinations of the two buttons.

It can be presumed that the number appearing on each box may depend very sensitively on the time in microseconds at which the button on that box is pressed, and may also depend on various times of decays of radioactive nuclei in the box. So the numbers appearing can be regarded as completely unpredictable and nonpredetermined: a pair of numbers appears in each actual trial, but which numbers actually appear may be completely undetermined before the buttons are actually pressed. And "what would have happened if the observe had pressed the other buttom" may likewise be totally ill-defined.

The connection between the two boxes is of one of the two types, I or II. A connection of type I is always open, and the information about which button is pressed on either box can be transmitted to the other box before the number appears there. But a connection of type II is closed, or broken, before the "press a button" message appears. In this case no information about which button is pressed on either box can pass to the other box before the number appears there. Thus for a type II connection the possibilities for what can appear at either box, under either condition chosen there, are limited to those that do not depend upon which button is pressed on the other box.

Suppose now that the numbers appearing on the two boxes in each trial are identified with the appropriate two sequences from the set of four sequences $[(r_{Ai}(\theta'_A), \ldots, r_{An}(\theta'_A)); (r_{Ai}(\theta''_A), \ldots, r_{An}(\theta''_A)); (r_{Bi}(\theta'_B), \ldots, r_{Bn}(\theta'_B))$; and $(r_{Bi}(\theta''_B), \ldots, r_{Bn}(\theta''_B))]$, and that the

numbers appearing are then found to accord with the quantum-theoretical expectations. And suppose that the number of trials is so great that the possibility of a statistical fluke can be disregarded: the correlations that appear can be assumed not to represent some highly unlikely statistical fluctuation. Then we may ask: Can we logically deduce from assumptions 1, 2, and 3 that the connection between the two boxes is not of type II?

The answer is yes. The easiest way to see this is to imagine first that the boxes contain computers that are plotting a strategy for displaying numbers that will accord with the conditions 1, 2, and 3. The very best they can do is to specify exactly which numbers will be displayed in any given situation. Property 1 means that either button might be pressed on each box, but the information about which button will be pressed is not available to the computer until the button is pressed. Thus the strategy must allow for either possibility at each box, and the quantum predictions must, by virtue of property 3, hold no matter which combination of buttons is pressed.

The fact that a type II connection is broken before the buttons are pressed means that even under the most acute planning the possibilities for what can be displayed by each box are limited to those that do not depend on which button is pressed at the other box. But then the possibilities for the responses in the four alternative possible cases are correlated. In particular, the possibilities for what can appear under the alternative possible conditions a, or b, or

c, or d, are logically limited to quartets (λ_a, λ_b, λ_c, λ_d) of alternative possible results that satisfy the four conditions (4). But it was proved that if the results that can possibly appear in the alternative possible experimental situations are limited to possible results correlated in the way demanded by (4) then it is not possible to satisfy the demands 1 and 3 that the predictions of quantum theory be satisfied no matter which of the four alternatives is freely chosen: if the connection of type II then there is no possibility that the results displayed can consistently conform to the predictions of quantum theory, if the choices between the four alternatives are freely made. Indeed, if (as required by 1) the choices of the observer, are independent of what is happening in the box, before the button is pressed, and are random, and if the connection between boxes is of type II, then the displayed results must, with a very high probability, "disagree" with the quantum predictions in about one-quarter or more of a long series of trials. And this "disagreement" can be made arbitrarily large (in terms of standard deviations) by taking n sufficiently large.

The introduction of the vagaries associated with the elements of chance or indeterminism can only make even more difficult the already impossible task of trying to satisfy 1, 2, and 3. The locality limitation (4) on the conceivable possibilities arises directly from the break in communication alone, without regard to any other condition about how the results that actually appear are generated.

The essential point is this: the question of whether or not a system that consistently produces numbers in accordance with the requirements 1, 2, and 3 must have a nonlocal link is a logical question that can be answered by examining the logical restrictions on the conceivable possibilities. The theoretical use in this logical analysis of a quartet of conceivable possibilities does not entail the actual simultaneous physical existence of all four quantities of the quartet, or of any one of them. What can be logically deduced is that if a system, be it deterministic or stochastic or whatever, conforms to the requirements 1, 2, and 3, then what appears in one region must, in some cases, depend on which choice made in a second region spatially separated from the former.

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