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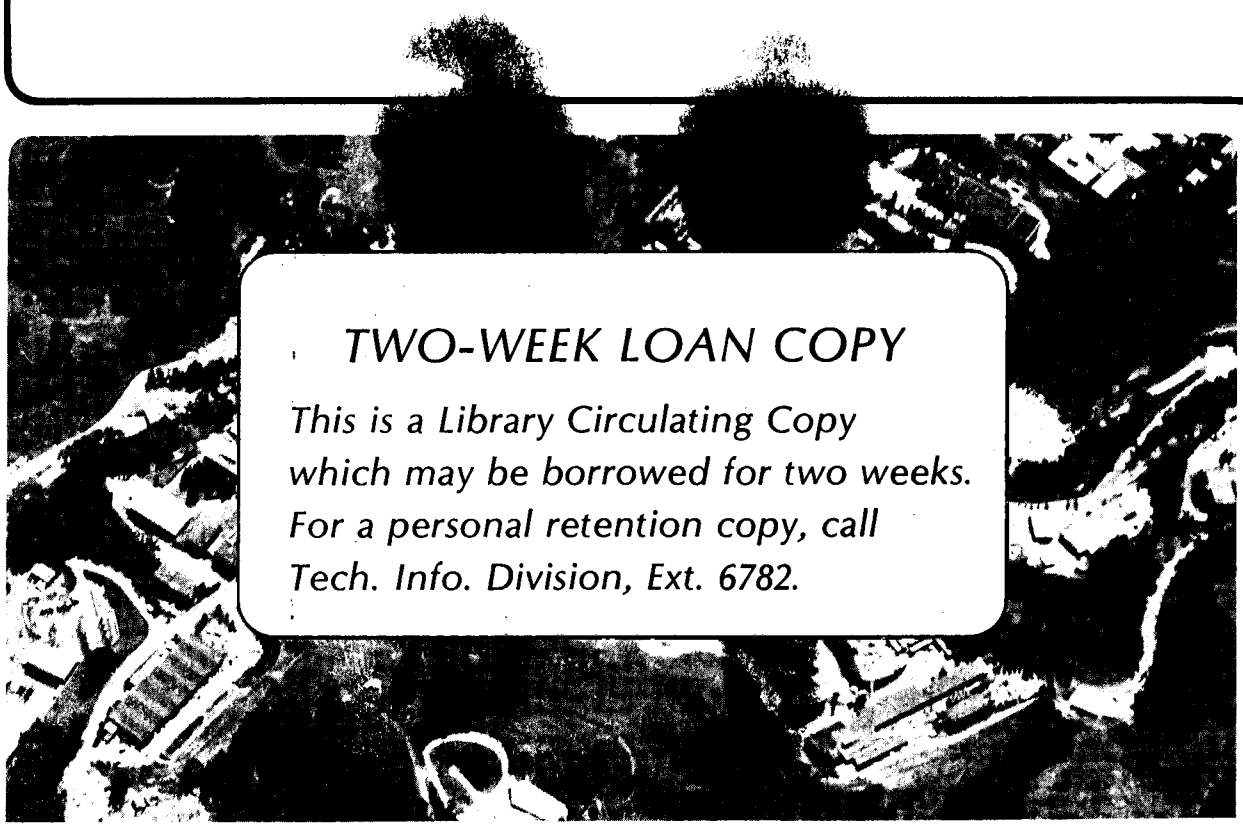
BELL'S THEOREM AND THE FOUNDATIONS OF QUANTUM PHYSICS

H.P. Stapp

April 1984



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BELL'S THEOREM AND THE FOUNDATIONS OF QUANTUM PHYSICS

(to be published in the American Journal of Physics)

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Abstract

A locality property expressing the idea that causal influences can propagate only forward in time, from earlier cause to later effect, and no faster than light is shown to be mathematically incompatible with certain predictions of quantum theory. The contradiction is proved without invoking any additional assumptions such as realism or hidden variables that contravene conventional quantum thinking. The failure of this locality property refines ideas about the theory of relativity, and opens the way to an objective interpretation of quantum theory that may expand its domain of applicability outside the one circumscribed by the orthodox Copenhagen interpretation.

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Einstein and Bohr were the principal protagonists in an epic debate of the issue: Can quantum-mechanical description of physical reality be considered complete? Bohr, one of the two main founders of quantum theory, maintained that the quantum-mechanical description of physical reality is complete, even though the formalism must be regarded merely as a tool for making predictions about our observations of atomic processes, rather than as a description of the external situation itself. Einstein defended the more traditional view that a complete description of physical reality should describe also the external situation as it supposedly exists independently of observation, not merely our observations of it.[1]

Bohr's ideas about the interpretation of the mathematical formalism of quantum theory are the foundation of orthodox (Copenhagen) interpretation of quantum theory. In his own words "...the formalism does not allow pictorial representation along accustomed lines, but aims directly at establishing relations between observations obtained under well defined conditions."[2] "Strictly speaking, the

mathematical formalism of quantum theory ... merely offers rules of calculation for the deduction of expectations about observations obtained under well-defined experimental conditions specified by classical physical concepts."[3] "...the appropriate physical interpretation of the symbolic quantum-mechanical formalism amounts only to predictions, of determinant or statistical character, pertaining to individual phenomena appearing under conditions defined by classical physical concepts."[4] Bohr acknowledged that the notion that a description of this kind could be a complete description of physical reality entailed "a radical revision of our attitude towards physical reality."[5]

The critical issue in the Einstein-Bohr debate was the meaning of "physical reality." To get a logical grip on this concept Einstein, together with two younger colleagues, Podolsky and Rosen, introduced the following criterion of physical reality: "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."[6]

To use this criterion it is necessary to satisfy the condition "without in any way disturbing a system." This requires some assumption about causal influences. Here Einstein, Podolsky, and Rosen (EPR) relied upon an idea drawn from the theory of relativity: causal influences propagate always forward in time, from earlier cause to later effect, and never faster than light. From this locality condition, together with the criterion of physical

reality and the quantum-mechanical formalism itself, EPR were apparently able to prove, by logical analysis, the incompleteness of the quantum-mechanical description of physical reality.

The locality condition needed for the EPR argument should, according to Einstein, be held "absolutely fast". Bohr, as we shall see, likewise appeared to regard it as beyond question. However, this condition can be shown to be mathematically incompatible with certain predictions of quantum theory. The predictions of quantum have been accuractely confirmed under very diverse experimental conditions including, recently,[7] conditions similar to those needed for the EPR argument. Hence it now appears safe to conclude that this locality condition is incorrect.

The failure of this firmly held belief about the spacetime structure of causal influences invalidates the EPR argument. But, more importantly, it alters basic ideas about the nature of physical reality. For it revises understandings of the way in which quantum theory accomodates the theory of relativity. This revision has a profound impact on the question of the proper interpretation of quantum theory. For it opens the way to a thoroughly objective interpretation of quantum theory that meets Einstein's demand for description of the external situation, and is moreover in close accord with the intuitive ideas of Heisenberg, and probably those of most contemporary quantum physicists as well. The previously perceived need to regard the quantum-mechanical formalism as merely a tool for making predictions about observations appears in the light of the

failure of EPR locality to be simply an artifact of the tacit acceptance by the founders of quantum theory of a belief about the spacetime structure of causal influences that is mathematically incompatible with quantum theory itself.

This possibility for a thoroughly objective interpretation of quantum theory, opened up by recognition of the failure of earlier beliefs about locality, appears to have no immediate impact on atomic physics per se. But, it may be of crucial importance for the unification of the sciences. For to bring together into a single coherent framework of thought our knowledge from the fields of, for example, cosmology, biology, and psychology, on the basis of their common atomic underpinnings, one must deal with experimental situations that do not conform to the format demanded by the orthodox (Copenhagen) interpretation of Bohr. That format is well suited to experiments that arise in the realm of atomic and subatomic physics, but is ill-suited to experimental circumstances that can arise in these other domains of science.

In this article, I shall first describe in more detail the locality condition upon which the EPR argument rests, review the opinions about it expressed by Einstein and Bohr, and then demonstrate the mathematical incompatibility of this locality condition with the predictions of quantum theory itself. This result is a generalization of a theorem by J. S. Bell.[8] Next, I shall describe the revisions of ideas about the nature of physical

reality and about the interpretation of quantum theory demanded by the failure of this earlier idea about the spacetime structure of causal influences. Finally, I shall describe the impact of these revised ideas upon the program of enlarging the scope of quantum theory to provide an adequate foundation for the unification of science.

The locality condition

The locality condition needed for the EPR argument expresses certain ideas drawn from the theory of relativity. This condition is formulated within the general framework of quantum theory. In accordance with the ideas of Bohr the quantum formalism is regarded merely as a tool for making predictions about phenomena appearing under specified experimental conditions. And these experimental conditions are considered to be freely chosen by experimenters.

The relevant experimental situation involves two regions R_1 and R_2 that are limited both in space and in time, and situated so that nothing can travel from any point in either region to any point in the other region without moving either faster than light or backward in time. Thus, the two regions should be far apart in space, compared to their time durations, and approximately simultaneous. (See Fig. 1) Regions that enjoy this property are said to be spatially (or spacelike) separated. The experiment is set up so that within each region three things will happen: a choice will be made between two alternative possible

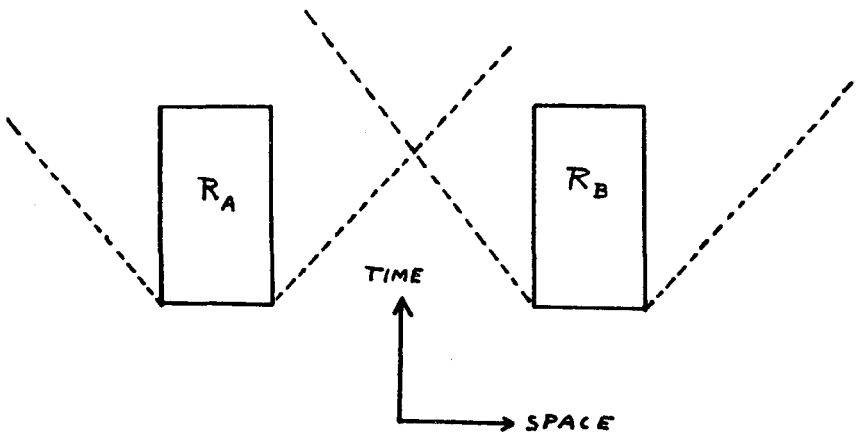

Fig. 1. Two regions are situated so that no information or influence can travel from any point in either region to any point in the other region without traveling either faster than light or backward in time. The dotted lines indicate the boundaries of the regions limited by this speed-of-light condition. The theory of relativity suggests that a choice made freely in one region cannot affect or influence observable phenomena appearing in the other.

measurements, the chosen measurement will be performed, and the result of that measurement will be observed.

According to the theory of relativity causal influences propagate no faster than light and only forward in time. Thus EPR assumed that a system S_2 in R_2 cannot in any way be disturbed by what is done with a system S_1 in R_1. Einstein later asserted: "But on one assumption we must, in my opinion, hold absolutely fast: the real factual situation of the system S_2 is independent of what is done with S_1, which is spatially separated from the former."[9]

In this later statement Einstein refers to "the real factual situation of the system S_2." It is crucial to the EPR argument itself that the concept of physical reality be introduced only through the criterion of physical reality. According to this criterion the EPR candidates for physical reality are certain values that can be predicted with certainty. These values are values of observables that can be measured in R_2: they are defined in terms of results of experiments that can be performed in R_2. In the context of the EPR argument the "system" referred to by the EPR criterion of physical reality consists essentially of these values, and the essential locality assumption is this: phenomena appearing in R_2 cannot depend on which measurement is freely chosen and performed in R_1. The word "cannot" signifies that the restriction is on the possibilities themselves: the possibilities for the phenomena appearing in R_2 under any conditions chosen in R_2 are limited to those that do not depend on which experiment is freely

chosen and performed in R_1. This statement of the locality condition is the weakest general formulation that allows the EPR argument to be carried through. It does not introduce any superfluous ontological elements that would undermine the EPR argument. In particular, it is not based on the prejudicial idea that the results of the measurements in R_2 are predetermined, or that anything actually exists in R_2 beyond the phenomena that actually appear there. It merely limits the possibilities for what can appear there.

Accepting this locality condition the EPR argument proceeds as follows: one can, by using quantum theory and the result of an experiment freely chosen and performed in R_1, predict with certainty the value of either one of two incompatible observables pertaining to system S_2 in R_2. Neither of these values, as defined by phenomena appearing in R_2 if the associated observable is measured, can be disturbed in any way by what is done in R_1. Consequently, on the basis of the EPR criterion of physical reality, both values are elements of physical reality. But the quantum-mechanical description does not allow both of these values to be simultaneously well defined. Thus the quantum-mechanical description is not a complete description of physical reality.

The EPR conclusion was challenged by Bohr.[10] Most of Bohr's reply was not directed at the EPR argument itself. The major part of his reply was devoted to showing that his usual argument for the practical completeness of quantum theory -- i.e., completeness with respect to

predictions about observations -- holds also in the physical situation considered by EPR. But Bohr also challenged EPR argument itself, on the basis of the meaning of "physical reality." He argued that the concept of physical reality appropriate to quantum theory is different from that of classical physics. The essence of quantum theory consists of the predictions that can be made by using it. But making predictions requires knowledge. According to quantum theory there are limitations on this knowledge associated with the uncertainty principle. And the form of these limitations depends upon which experiments one decides to do. Bohr showed explicity how the limitations in available knowledge arising from the choice of measurement on S_1 influence what kinds of predictions can be made about S_2. Thus to the extent that "physical reality" is that which our physical theories allow us to predict about a system one can say that what is done with S_1 does disturb the physical reality S_2.

Bohr summarizes his argument as follows:

> "From our point of view we now see that the wording of the above-mentioned criterion of physical reality proposed by Einstein, Podolsky and Rosen contains an ambiguity as regards the meaning of the expression "without in any way disturbing a system." Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of <u>an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system</u>. Since these conditions constitute an inherent element of the description of any phenomenon to which

> the term "physical reality" can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete."

Bohr's reply to the EPR argument does not directly challenge the EPR locality condition. Bohr fully endorses the idea that a choice between alternative possible experiments is to be considered a "free choice." He grants that the process of measurement in R_1 "does not directly interfere with the particle concerned," i.e., with the particle in R_2. He affirms that "Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation [i.e., S_2] during the last critical stages of the measuring procedure [on S_1]."

Although Bohr does not challenge the EPR locality assumption directly, he does maintain that the physical reality S_2 is disturbed by what is done with S_1. But this disturbance is epistemological: it is an influence on predictions via change in available knowledge, rather than a direct mechanical disturbance.

I turn now to the proof of the incompatibility of EPR locality with the statistical predictions of quantum theory. First the experimental set-up must be described in more detail.

The Experimental Set-Up

The experiment has two distinct phases. In the first phase two beams of low energy spin-1/2 identical particles are caused to collide near a point P lying at the center of several concentric spherical arrays of particle detectors. Each array covers a complete sphere, except for two escape holes, located at polar extremities. The escape holes in the various spherical arrays are lined up so that two particles, one from each beam, can scatter near P and then escape from the set of arrays through these holes. The two particles of this pair will then be traveling in opposite directions, along approximately known trajectories. Fast electronics is set up to keep track of all the incoming and scattered particles, to identify pairs of escaping particles, and to label each such pair by a different index i from the set (1, 2, ..., n). Information about the calculated time t_i of the collision of the two particles of each escaping pair i is sent ahead to the two parts of the second phase of the experiment.

The second phase involves two separate measuring devices D_A and D_B. The geometry is such that one particle from each pair i will enter D_A, and the other will enter D_B. The conditions on D_A and D_B are similar, and the symbol D_j stands for either one of them. (See Fig. 2)

The device D_j contains a Stern-Gerlach apparatus. This apparatus has a preferred direction, which is oriented approximately perpendicularly to the trajectories of the particles passing through it. The azimuthal orientation of

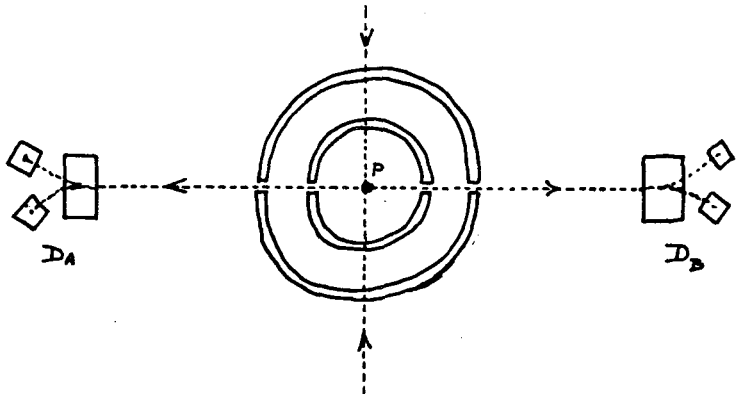

Fig. 2. The Experimental Set-Up. A pair of incident particles scatter near a point P, then escape through the polar escape holes in the concentric rings of particle counters. Each particle of the pair will enter one of the two devices D_A or D_B, where it will be deflected into either a certain "preferred" direction, or in the opposite direction.

this preferred direction is specified by an angle θ_j. (See Fig. 3) Each particle entering the apparatus will be deflected either along this preferred direction or in the opposite direction. Two particle detectors are placed behind the apparatus so that one or the other is activated according to whether the deflection is along the preferred direction or in the opposite direction.

The fast electronics is rigged to anticipate the activation by a particle from each pair i of one or the other of the two detectors in device D_j, and to assign to a variable r_{ji} the value +1 or -1 according to whether the deflection is along the preferred direction or in the opposite direction. The set of n values r_{ji} for all i in the set (1, 2, ..., n) is called the result r_j. It is arranged that this result will be recorded and visually displayed.

The device D_j is under the care of an experimenter E_j. Two possible values θ'_j and θ''_j of the angle θ_j are fixed in advance, and it is arranged that a signal will be given to the experimenter E_j shortly before the arrival at D_j of the bunched set of particles from the n pairs i. Upon receipt of this signal the experimenter E_j will freely choose one or the other of the two angles, θ'_j or θ''_j, then place the preferred direction of the device D_j at this setting, and finally observe the visually displayed result r_j.

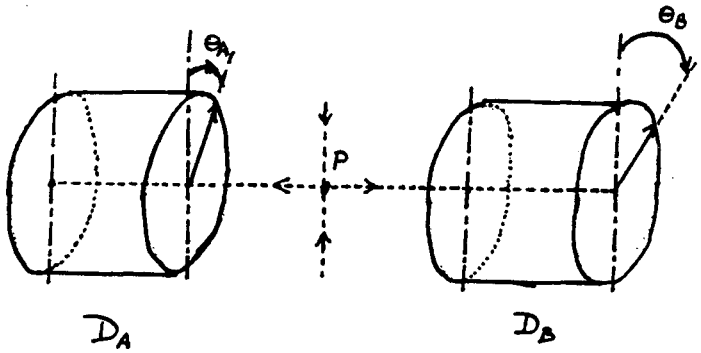

Fig. 3. Each device D_j has a preferred direction d_j, which is placed in the position specified by the azimuthal angle θ_j.

The time interval from the beginning of the processes of choosing the setting θ_j to the observation of the final result r_j is called T_j. The spatial region occupied by D_j and E_j is called $\bar{R}_j$. The set of points $(t,\vec{r})$ such that t is in T_j and $\vec{r}$ is in $\bar{R}_j$ is the spacetime region R_j.

The critical requirement on the experimental set up is that the two regions R_A and R_B be "spacelike separated." This means that no signal originating at any point in either region can reach any point in the other region without traveling either faster than light or backward in time.

In the experiment of Aspect, Dalibard, and Roger[2] the "experimenters" E_A and E_B were mechanical devices: the processes of "choosing" the setting and "observing" the results were done mechanically, rather than by human beings.

The Predictions of Quantum Theory

Quantum theory provides statistical predictions about phenomena appearing under appropriate experimental conditions. For each appropriate set of experimental specifications s there is a corresponding set Λ_s of <u>conceivable possibilities</u> λ_s for the phenomena appearing under conditions s. Quantum theory assigns a probability $P(\lambda_s)$ to each conceivable possibility λ_s.

In the situation described above there are four alternative possible experimental specifications s. They correspond to the four alternative possible pairs (θ'_A, θ'_B),

(θ'_A, θ''_B), (θ''_A, θ'_B), and (θ''_A, θ''_B). The angles θ'_A, θ''_A, θ'_B, and θ''_B are specified to be 0°, 45°, 45°, and -45°, respectively. Thus we have the four alternative possibilities

$$a = (\theta'_A, \theta'_B) = (0°, 45°) = (\theta^a_A, \theta^a_B), \tag{1a}$$

$$b = (\theta'_A, \theta''_B) = (0°, -45°) = (\theta^b_A, \theta^b_B), \tag{1b}$$

$$c = (\theta''_A, \theta'_B) = (45°, 45°) = (\theta^c_A, \theta^c_B), \tag{1c}$$

and

$$d = (\theta''_A, \theta''_B) = (45°, -45°) = (\theta^d_A, \theta^d_B). \tag{1d}$$

Let s be any one of these four quantities. There are 4^n conceivable possibilities λ_s. These 4^n conceivable possibilities are labelled by the 4^n symbols

$$1_s, 2_s, 3_s, \ldots, \text{ and } m_s,$$

where $m = 4^n$. Each conceivable possibility λ_s corresponds to a set of 2n numbers

$$\{r_{Ai}(\lambda_s), r_{Bi}(\lambda_s)\},$$

where i runs from 1 to n, and each number $r_{ji}(\lambda_s)$ is either +1 or -1 according to whether the deflection in device D_j of the particle from pair i is along the preferred direction of this device or in the opposite direction.

Quantum theory assigns a definite value to each probability $P(\lambda_s)$. From these values the following property follows: for any fixed s in the set (a, b, c, d), and any $\epsilon > 0$, the set of conceivable possibilities λ_s such that the equation

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\lambda_s)\, r_{Bi}(\lambda_s) + \cos(\theta^s_A - \theta^s_B) = 0 \tag{2}$$

fails by more than $\pm\epsilon$ have a total probability that tends to zero as n tends to infinity. Thus, for any $\epsilon > 0$, however small, one can, by taking n sufficiently large, exclude as prohibitively unlikely those conceivable possibilities for which (2) fails by more than $\pm\epsilon$.

The four special cases of (2) are

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\lambda_a)\, r_{Bi}(\lambda_a) = -1/\sqrt{2}, \tag{3a}$$

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\lambda_b)\, r_{Bi}(\lambda_b) = -1/\sqrt{2}, \tag{3b}$$

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\lambda_c)\, r_{Bi}(\lambda_c) = -1, \tag{3c}$$

and

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\lambda_d)\, r_{Bi}(\lambda_d) = 0. \tag{3d}$$

Locality Conditions

In quantum theory the choice of the experiment is considered to be a free choice. No restrictions are placed on the manner in which the choice is made, and the predictions do not depend on how this choice is made. Thus in the present context it is unreasonable to ascribe any special significance to the logical possibility of a correlation arising via influences from their common past between the choice of experiment made in one region and

results obtained in the other. There is, in principle, nothing to prevent our making these choices in some way that would render negligible any such correlations. Accordingly, we shall consider the choices of the experiments to be free, in the sense that the aforementioned possible correlations are assumed not to exist.

The information about which conditions are freely chosen in the region R_1, which can be either R_A or R_B, cannot get to the other region R_2 without traveling either faster than light or backward in time. In this situation, ideas from the theory of relativity suggest that the following locality condition should hold: the possibilities for the phenomena appearing in R_2 under any conditions chosen in R_2 are limited to those that do not depend on which experiment is freely chosen and performed in R_1.

Let us recall the definitions $a = (\theta'_A, \theta'_B)$, $b = (\theta'_A, \theta''_B)$, $c = (\theta''_A, \theta'_B)$, and $d = (\theta''_A, \theta''_B)$. Let $\hat{\lambda}_a$ be any one of the 4^n possible λ_a. The locality condition imposes the following four conditions on the conceivable possibilities:

1) If under freely chosen conditions a the phenomena appearing would be $\hat{\lambda}_a$ then under freely chosen conditions b the phenomena appearing would be a $\hat{\lambda}_b$ satisfying

$$r_{Ai}(\hat{\lambda}_b) = r_{Ai}(\hat{\lambda}_a). \tag{4a}$$

2) If under freely chosen conditions a the phenomena appearing would be $\hat{\lambda}_a$ then under freely chosen conditions c the phenomena appearing would be a $\hat{\lambda}_c$ satisfying

$$r_{Bi}(\hat{\lambda}_c) = r_{Bi}(\hat{\lambda}_a). \tag{4b}$$

3) If under freely chosen conditions b the phenomena appearing would be $\hat{\lambda}_b$ then under freely chosen conditions d the phenomena appearing would be a $\hat{\lambda}_d$ satisfying

$$r_{Bi}(\hat{\lambda}_d) = r_{Bi}(\hat{\lambda}_b). \tag{4c}$$

4) If under freely chosen conditions c the phenomena appearing would be $\hat{\lambda}_c$ then under freely chosen conditions d the phenomena appearing would be a $\hat{\lambda}_d$ satisfying

$$r_{Ai}(\hat{\lambda}_d) = r_{Ai}(\hat{\lambda}_c). \tag{4d}$$

The condition 1, for example, limits the possibilities for phenomena appearing in R_A under the condition $\theta_A = \theta'_A$, chosen in R_A, to results that do not depend on the choice between θ'_B and θ''_B made in R_B. The other three conditions impose analogous limitations on the possibilities.

These four conditions, taken together, entail that if the phenomena appearing under freely choosen conditions a would be $\hat{\lambda}_a$ then the phenomena appearing under the three alternative possible conditions b, c, and d would be three conceivable possible results $\hat{\lambda}_b$, $\hat{\lambda}_c$, and $\hat{\lambda}_d$, respectively, that are restricted by the four conditions

(4). Thus, the original $\hat{\lambda}_a$ appears as part of a quartet $(\hat{\lambda}_a, \hat{\lambda}_b, \hat{\lambda}_c, \hat{\lambda}_d)$ that satisfies all four equations (4). The same four equations are obtained no matter which of the four values of s is taken as the starting point. Therefore all of the conceivable possibilities allowed by the locality condition are members of quartets of conceivable possibilities $(\hat{\lambda}_a, \hat{\lambda}_b, \hat{\lambda}_c, \hat{\lambda}_d)$ that satisfy (4).

One tacit assumption has been made here. In the contemplation of the conceivable possibilities it is assumed that under any one of the four alternative possible conditions s the phenomena appearing would be some <u>single</u> conceivable possible result λ_s. It might seem that this is no assumption at all. For whichever choice is made we observe only one result. Thus the tacit assumption seems to be nothing more than a recognition of what one must mean by the conceivable possibilities.

But there is a subtle point, which is in fact exploited by the so-called many-worlds interpretation of quantum theory. This is the fact that one part of the conceivable result λ_s appears in one region R_1 and the other part appears in the other region R_2. These two parts cannot appear in the consciousness of any single observer until some later time, when the information about the results in the two regions has been brought together into a single region.

In the many-worlds interpretation[11-14] the observer in each region splits into four different real persons, each observing a different choice of experiment in his region, or

a different experimental result there. These four observers occupy the same spacetime region, but are oblivious of each other. The four observers from region R_1 can travel and meet the four observers from R_2, and they can then compare notes. At this point the two independent sets of four observers are transformed into one set of sixteen pairs of observers, with each of the sixteen pairs containing one of the four observers from R_1 and one from R_2.

This strange-sounding picture of reality arises from interpreting the probability function of quantum theory as a representation of reality itself. The probability function behaves as a probability function should: it splits into effectively independent branches representing the different distinguishable possibilities, with each branch carrying an appropriate "weight" or probability. But in the many-worlds interpretation each branch, which is ordinarily thought to represent only a possibility, represents a part of reality itself.

The tacit assumption mentioned above fails in the many-worlds picture of nature, and no conflict between locality and quantum theory arises. This fact could be used to support the many-worlds interpretation, if failure of locality were regarded as more objectionable than the unbounded proliferation of unobservable real worlds. But we shall accept the normal idea that under any one of the four alternative possible conditions s precisely one of the 4^n conceivable possible results λ_s will appear.

Proof of Contradiction

Equations (3) represent restrictions on the conceivable possibilities arising from the statistical prediction of quantum theory. Equations (4) represent restrictions on the conceivable possibilities arising from the locality condition. These two conditions are, however, mathematically incompatible: it is not possible to satisfy both of them. To prove this consider any quartet $(\hat{\lambda}_a, \hat{\lambda}_b, \hat{\lambda}_c, \hat{\lambda}_d)$ satisfying (4). Let the arbitrarily small $\epsilon > 0$ associated with (3) be first set to zero. Then (3c) gives

$$r_{Ai}(\hat{\lambda}_c) = -r_{Bi}(\hat{\lambda}_c). \tag{5}$$

Then use of (3a); (3b) plus (4a); and (3d) plus (4c), (4d), (5), and (4b) give, respectively,

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\hat{\lambda}_a)\, r_{Bi}(\hat{\lambda}_a) = -1/\sqrt{2}, \tag{6a}$$

$$\frac{1}{n}\sum_{i=1}^{n} r_{Ai}(\hat{\lambda}_a)\, r_{Bi}(\hat{\lambda}_b) = -1/\sqrt{2}, \tag{6b}$$

and

$$\frac{1}{n}\sum_{i=1}^{n} r_{Bi}(\hat{\lambda}_a)\, r_{Bi}(\hat{\lambda}_b) = 0. \tag{6c}$$

Equations (6), and the fact that each r_{Ai} and r_{Bi} is either +1 or -1, entail that

$$\frac{1}{n}\sum_{i=1}^{n} (\sqrt{2}\, r_{Ai}(\hat{\lambda}_a) + r_{Bi}(\hat{\lambda}_a) + r_{Bi}(\hat{\lambda}_b))^2$$

$$= 2 + 1 + 1 - 2 - 2 = 0. \tag{7}$$

The fact that each $r_{Ai}(\hat{\lambda}_a)$, $r_{Bi}(\hat{\lambda}_a)$, and $r_{Bi}(\hat{\lambda}_b)$ is either +1 or -1 entails that

$$\frac{1}{n}\sum_{i=1}^{n} (\sqrt{2}\, r_{Ai}(\hat{\lambda}_a) + r_{Bi}(\hat{\lambda}_a) + r_{Bi}(\hat{\lambda}_b))^2$$

$$> (\sqrt{2} - 2)^2. \tag{8}$$

Equations (7) and (8) entail

$$0 > (\sqrt{2} - 2)^2. \tag{9}$$

This is a mathematical contradiction. Small changes associated with an arbitrarily small but nonzero value of ϵ cannot undo this large discrepancy. Hence, the conditions (3) and (4), which express the requirements of quantum theory and EPR locality, respectively, are mathematically incompatible.

Impact on the Theory of Relativity

The failure of EPR locality has no practical impact on the theory of relativity, provided quantum theory is, from a practical standpoint, complete. Failure of EPR locality means that one cannot restrict what "could appear" in each region to possibilities that are independent of the choice made by the experimenter in the other region. However, one cannot determine, either by experiment or by quantum theory, what "would have appeared" if the other choice had been made. So the "nonlocal influence" entailed by the failure of EPR locality cannot be directly observed. It was therefore an unwarranted and unnecessary requirement on the

part of Einstein (and all others who were involved on both sides of the EPR controversy) to demand the absence of nonlocal influences of the kind ruled out by EPR locality.

It is a trivial consequence of the quantum formalism that the predictions about phenomena arising in R_2 are independent of the choice made in R_1. In the quantum formalism the choice between the two alternative possible experiments on the system in region R_1 is represented by a choice between two different decompositions of the unit matrix I_1 for that system into a sum of projection operators. If one were to specify the particular result that occurs in region R_1 then there would be a corresponding prediction for result in R_2. This prediction would depend on which result is specified in R_1, and hence also on which of the two possible experiments is performed in R_1. But if one has no information about which particular result occurs in R_1 then to obtain the prediction for R_2 one must sum over the various possible results in R_1. This sum gives, for each of the two alternative possible choices of measurements in R_1, the same unit matrix I_1. Consequently the predictions for R_2 are independent of which experiment is chosen in R_1.

Because the statistical predictions for R_2 are independent of which experiment is chosen in R_1 one might naturally think that the individual results could at least be imagined to have this same independence property, particularly if the regions R_1 and R_2 are spacelike separated. Bell's theorem, in its generalized form, asserts

that one cannot even imagine the individual results to have this independence property, without contradicting, in certain cases, the statistical predictions of quantum theory.

Irrelevance of Determinism

The original version of Bell's theorem asserted that "No deterministic hidden-variable theory that reproduces the statistical predictions of quantum theory can be local." This theorem would be useful to those who would like to go back to classical ideas, and try to understand the predictions of quantum theory in terms of deterministic hidden variables. The theorem shows that any such model must be nonlocal. However, for mainstrem physicists,[15] who had already rejected both determinism and hidden variables, Bell's result carried no implication of any nonlocality in nature. It merely buttressed convictions about determinism and hidden variables.

Actually, however, in an argument preliminary to his main proof, Bell "derived" the existence of deterministic hidden variables from a certain idea about locality. This argument was rather cryptic, and other workers who have tried to use Bell's reasoning to circumvent the assumption of determinism have found it necessary to introduce an assumption of "realism" or "objectivity" to

justify their use of hidden variables. On the other hand, the present proof uses neither hidden variables nor any assumption about determinism. It therefore achieves Bell's original intention of starting from locality alone.

Irrelevance of Realism

Some authors have linked nonlocality to the question of realism. Clausen and Shimony[16] start their review article with a definition: "Realism is a philosophical view, according to which external reality is assumed to exist and have definite properties whether or not they are observed by someone." Those authors go on to say: "Consequently, it can now be asserted with reasonable confidence that either the thesis of realism or that of locality must be abandoned. Either choice will drastically change our concepts of reality and of space-time."

d'Espagnat[17] asserts: "Even if quantum theory is considered to be no more than a set of rules it is still in conflict with a view of the world many people would consider obvious and natural. This view is based on three assumptions, or premises that must be accepted without proof. One is realism, the doctrine that regularities in observed phenomena are caused by some physical reality whose existence is independent of human observers."

Both sets of authors suggest that one way to retain locality would be to abandon realism. Indeed, the proofs of the failure of locality given by these authors do assume realism. But the proof given here does not. If this proof had been based on the assumed existence of either hidden variables or functions

$r_{Ai}(\theta_A,\theta_B)$ and $r_{Bi}(\theta_A,\theta_B)$ that represent the results that would appear if the freely chosen conditions are (θ_A,θ_B) then it might be claimed that there was a hidden assumption akin to realism. But the proof given above makes no such assumption, and involves no presumption that anything actually exists beyond the phenomena that actually appear. Hence rejection of realism, as defined above, cannot save EPR locality. On the other hand, as is discussed next, acceptance of a thoroughly objective realistic viewpoint that entails neither hidden variables nor any other ideas alien to quantum theory allows one to naturally accomodate, and in fact quantitatively explain, the failure of EPR locality.

Interpretations of Quantum Theory

The central tenet of the orthodox (Copenhagen) interpretation of quantum theory is that "...the appropriate physical interpretation of the symbolic quantum-mechanical formalism amounts only to predictions, of determinant or statistical character, pertaining to individual phenomena appearing under conditions defined by classic physical concepts."[18,19]

Bohr argued convincingly for the practical completeness of the theory, considered as a theory of predictions about observations of atomic phenomena.

Heisenberg's version[20] of the Copenhagen interpretation provides a partial understanding of the nature of the underlying reality. According to Heisenberg the probability function that occurs in quantum theory

"represents a mixture of two things, partly a fact and partly our knowledge of a fact." The knowledge aspect is analogous to what occurs in classical statistical mechanics. The other aspect is an objective "potentia" or "tendency for events". These events are actual events that can arise from the confluence of the measuring device with the atomic system represented by the probability function. "The probability function combines objective and subjective elements. It contains statements about possibiliites or better tendencies ("potentia" in Aristotelian philosphy) and these statements are completely objective, they do not depend on any observer; and it contains statements about our knowledge of the system, which of course are subjective insofar as they may be different for different observers. In ideal cases the subjective element in the probability function may be practically negligible as compared to the objective one. The physicists then speak of a 'pure case'".

Heisenberg's conception of nature in terms of the ideas of potentia and actual events is similar to one developed by the logician and philosopher Alfred North Whitehead.[21] To maintain logical coherence and avoid contradictions it is probably necessary to adopt Whitehead's idea that nature is a cumulative process: each actual event adds to the totality of what is fixed and definite. Each such event is conditioned by, but not determined by, the events occurring prior to it in the cumulative process. Each event may be considered to logically include all prior

events, and a well-ordered sequence of events is then defined by logical inclusion.[22]

The experiment under consideration involves three spacetime regions: R_A, R_B, and R_o. The region R_o is where the initial collisions of the pairs of particles occur. These three regions and the strips joining them form the V-shaped region shown in Fig. 4.

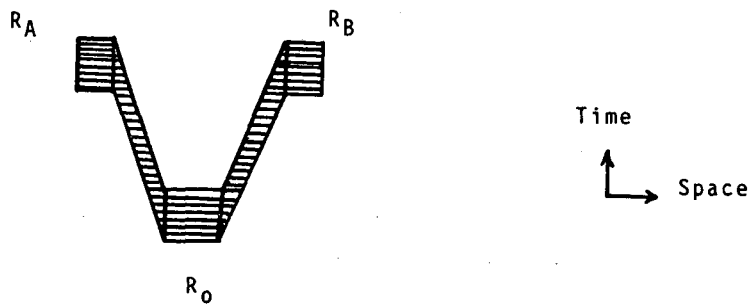

Fig. 4. A spacetime diagram showing the two experimental regions R_A and R_B, the region R_o where the initial collision occurs, and the spacetime region outside of which the probability functions of the particles of the n pairs i are effectively zero.

The regions outside the V-shaped region in Fig. 4 can be filled with sliding walls of lead without affecting the statistical predictions of quantum theory for this experiment. But even weak magnetic fields within the V-shaped region will affect these predictions, and hence also any influences associated with a failure of locality. Hence it is reasonable to suppose that any "nonlocal influences"

associated with the failure of EPR locality are associated with the V-shaped region.

Analysis of the experiment from a "potentia" point of view is straightforward. The actual events associated with the preparation in R_o leave the atomic system in a certain state. This state is represented by a wave function that is defined over all spacetime. It represents certain "potentia" for the events in R_A and R_B.

The spin part of this wave function is time independent. It is the singlet state Ψ with components $\Psi_{\alpha,\beta}$. The two indices α and β are the spin indices associated with the two particles. They can take the values 1/2 and -1/2, and the components of Ψ are

$$\Psi_{1/2,1/2} = 0$$
$$\Psi_{-1/2,-1/2} = 0$$
$$\Psi_{1/2,-1/2} = 1$$
$$\Psi_{-1/2,1/2} = -1.$$

There are many events in the fundamental well-ordered sequence. The next one of interest to us is either the one involving the confluence of the atomic system and measuring devices in region R_A, or the analogous event in R_B. Suppose the event in R_B comes first in the well-ordered sequence. Then, with equal probabilities, one of two states will be actualized. The new state corresponds to a new wave function defined over all spacetime. In particular, the wave function throughout the V-shaped spacetime region suddenly changes. If the spin state actualized in R_B is the state ψ_B with components $a_{B,\beta}$ then the spin state in R_A

suddenly becomes, according to the standard rules of quantum theory, a state with components

$$a_\alpha = a^*_{B,1/2}\,\Psi_{\alpha,1/2} + a^*_{B,-1/2}\,\Psi_{\alpha,-1/2}$$

This defines the new "potentia" for events in R_A. This new potentia depends on Ψ_B, which depends on how the device was set in R_B. The probability for the actualization in R_A of a state Ψ_A with components $a_{A,\alpha}$ is given by the rules of quantum theory as

$$\left| a^*_{A,1/2}\, a_{1/2} + a^*_{A,-1/2}\, a_{-1/2} \right|^2$$

This calculation is standard. But the description of what is being calculated is expressed in terms of the idea of the "potentia" for actual events, rather than in terms of knowledge of observers. Expressed in this way there is no mystery about the nonlocal influence: the potentia in R_A for the actual event occurring there depends on the choice of experiment made in R_B. Hence the phenomena appearing in R_A can depend on the choice made in R_B.

This way of understanding the failure of EPR locality is based on interpreting quantum theory objectively, in terms of objective potentia. Prior to Bell's theorem such an interpretation seemed untenable. For it entails, as just shown, a dependence of phenomena appearing in one region upon the choice made by an experimenter in a spacelike separated region. Such a

dependence was deemed contrary to the theory of relativity, and Heisenberg did not consistently uphold the objective point of view. At the critical point where, from an objective point of view, the events in region R_B disturb the objective potentia in R_A, Heisenberg shifts to a subjective interpretation of the wave function: "The observation itself changes the probability function discontinuously; it selects of all possible events the actual one that has taken place. Since through the observation our knowledge of the system has changed discontinuously, its mathematical representation has also undergone the discontinuous change and we speak of a quantum jump. When the old adage 'Natura non facit saltus' is used as a basis for criticism of quantum theory, we can reply that certainly our knowledge can change suddenly and this fact justifies the use of the term 'quantum jump.'

"Therefore, the transition from the 'possible' to the 'actual' takes place during the act of observation. If we want to describe what happens in an atomic event, we have to realize that the word 'happens' can apply only to the observation, not to the state of affairs between two observations. It applies to the physical, not the psychical act of observation, and we may say that the transition from the 'possible' to the actual 'actual' takes place as soon as the interaction of the object with the measuring device, and thereby with the rest of the world has come into play; it is not connected with the act of registration of the result in the mind of the observer. The discontinuous change in the

probability function, however, takes place with the act of registration, because it is the discontinuous change in our knowledge in the instant of registration that has its image in the discontinuous change in the probability function."

The sudden change in the probability function, considered as a representation of the physicist's knowledge, is completely normal: it occurs also in classical statistical mechanics. But what about the sudden change in the objective potentia associated with the physical, not psychical, act of observation? Heisenberg shifts attention from this objective change by shifting, in effect, at the critical stage, to an essentially subjective interpretation of the wave function.

But why did Heisenberg shift over to the subjective interpretation? The reasons are evidently two. First, the full development of a thoroughly objective interpretation would mean specifying precise details of the process of actualization that appear to have no bearing at all upon the predictions that one can make about experiments on atomic systems. This reason is a purely practical one. Second, the admission of a sudden objective change would have created a conflict with the prevailing relativistic ideas about the spacetime structure of causal influences. In the subjective interpretation the sudden change in the wave function in a distant region represents nothing more than a change in our knowledge. Hence it can exert no nonlocal causal influence.

Proof of the failure of EPR locality completely reverses the situation: the objective interpretation explains this failure automatically and quantitatively, whereas the subjective interpretation accords with the idea that locality should hold, rather than fail. Moreover, the need perceived by Bohr for "a radical revision of our attitude towards physical reality" appears in the light of the failure of EPR locality to be an artifact of Bohr's attempt to compensate, through epistemological analysis, and his idea of an indirect nonlocal influence via change in available knowledge, for his tacit acceptance of a prejudice about the spacetime structure of causal influences that is mathematically incompatible with quantum theory itself.

Bohr stressed that in the description of a quantum-mechanical process the entire experimental arrangement must be considered as a whole. The failure of EPR locality provides an understanding of the reason for this: if two parts of a system have interacted previously then the response of one part to its own environment cannot, in general, be considered independent of the environmental conditions <u>encountered</u> by the other part. Quantum-mechanical effects consequently imbue physical processes with a feature of macroscopic wholeness or nonseparability that is alien to classical physics. Bohr's focus on epistemological considerations, rather than upon mathematical and thoroughly objective ones, tends more to obscure than lay bare the origins of this feature of wholeness that characterizes quantum phenomena.

[Added note. Heisenberg describes the "potentia" notion as part of the Copenhagen interpretation. I have, accordingly, identified it with his name, since Bohr does not use it. As far as I know this idea was first set down in the quantum-theoretic literature in David Bohm's text book Quantum Theory.]

Consequences for Science in General

Given the practical completeness of quantum theory apparently proved by Bohr the question arises: what practical difference does it make how we think about these things?

The answer lies in the fact that the format for quantum theory used in the Copenhagen interpretation covers only a limited class of situations, namely those that occur in atomic and subatomic physics. This format involves some classically described devices that prepare an atomic system, and some classically described devices that respond to the atomic system. The preparing and responding devices must be very large, for in this case they act in an important way on their general environment, and hence cannot be adequately described quantum theoretically unless the entire universe is included in the description, which is, of course, not feasible within the framework of a practical description. The interaction with the environment therefore introduces disturbances of phase relationships that make classical description of these devices both possible and necessary. On the other hand, the atomic system must be very small, so that disturbances due to interaction with the enviroment can be either ignored, or adequately represented by a prescribed classical background field. Thus in cases to which the Copenhagen format can be applied the interface between the part of the universe described classically and the part

described quantum-mechanically is a boundary of very special type: it separates a huge system from a tiny system.

The question is: how can we go beyond this special case? Every physical system is composed in some sense of atomic or subatomic systems, and its detailed properties and behavior therefore depend on quantum-mechanical effects. How, in general, can we describe these effects? How, for example, can we accurately describe the quantum-mechanical underpinnings of the growth and behavior of a biological system?

Bohr said "it is obviously a quite open question whether the information we have acquired of the laws describing atomic phenomena provides us with a sufficient basis for tackling the problem of living organisms."[23] Einstein said that the orthodox forumulation of quantum theory "constitutes an optimum formulation of [certain] connections" but "offers no useful point of departure for future developments."[24]

To get an idea of the problem involved consider a living virus. It is to some extent an isolated system. Yet it interacts continually with its environment. If it were to be treated as a quantum-mechanical system, weakly interacting with its environment, then it would be both the quantum-mechanical system described by the wave function, and, by virtue of its size, also the preparing and responding device. The behavior of the virus fixes the "settings of this device". Hence the virus acts as the experimenter who fixes the experimental conditions under

which the quantum phenomena appear. The virus also records within its structure the results of the quantum-mechanical acts. Thus the virus combines into one entity the quantum-mechanical system, the preparing and responding devices, the experimenter who fixes the experimental conditions, and the "observer" that registers the results. In the format of the Copenhagen interpretation these systems are separate, and are described in different ways. Consequently, the Copenhagen format does not provide an adequate basis for a precise description of quantum-mechanical effects in the growth and behavior of biological systems.

The essential point is that Bohr's proof of the practical completeness of quantum theory pertains to a particular way of exploring quantum effects in atomic phenomena. There is no quarantee that, and Bohr never claimed that, every manifestation of a quantum-mechanical effect in every area of science must arise only in experiments of the particular kind he considered.

Science is today highly fragmented: there are many disciplines, each with its own language and basic concepts. As scientific knowledge grows, each discipline tends to move into boundary regions where it interfaces with others. Coherent reconciliation of the diverse disciplines at their interfaces demands an over-arching theoretical framework.

An element of commonality of these diverse disciplines is the atomic or subatomic constitution of all processes. At the atomic and subatomic levels quantum-

mechanical effects become important. Thus quantum mechanics, in some form, must be woven into the fabric of any unified scientific conception of nature. The form provided by the observer-oriented Copenhagen interpretation is ill-suited for this, for the reasons just described. And to place the human observer at the center of our understanding of the birth and evolution of the universe, or of life itself, or of consciousness itself, by forcing scientific theory to follow the preparation-response format appropriate to atomic physics would impose an unwarranted egocentric restriction on the form that useful theories might take.

Within this broad context the impact of Bell's theorem, and its generalization, is to liberate scientific thought by freeing it from both a false prejudice about the spacetime structure of causal influences and from a restrictive observer-based interpretation of quantum theory. As regards this latter point it is interesting that the impact of these theorems is precisely opposite to what it first appeared to be. Bell's theorem was originally formulated within the context of the EPR argument. That argument was designed to show the incompleteness of the quantum-mechanical description of physical reality. Bell's result invalidated the argument by demonstrating the incompatibility of its two main premises, locality and the validity of the statistical predictions of quantum theory. Hence Bell's work appeared to support Bohr's position on the interpretation of quantum theory, and repudiate Einstein's

idea that physical theory should be about the external situation itself, not merely our observations of it.

Einstein also believed that the underlying physical theory should be local and deterministic. Bell's theorem, in its original form, did eliminate that possibility. But Einstein's deeper conviction that the theory should deal with the external situation itself, not merely our observations of it, is supported by the generalized form of Bell's theorem. For the observer-based Copenhagen interpretation gives no understanding of the failure of EPR locality, and in fact allowed Bohr to accept the validity of EPR locality, whereas the objective interpretation based on Heisenberg's idea of objective potentia gives a direct and quantitative understanding of the required nonlocal influences.

Discussion of the Proof of Nonlocality

The proof of nonlocality given above is austere: the mathematical manipulations provide very little intuitive understanding of what is going on. With the overall picture now in place, it is useful to go back and discuss the proof in more detail.

A basic feature of the proof is that it deals simultaneously with the four members of a quartet (λ_a, λ_b, λ_c, λ_d) of conceivable possible results of four alternative possible experiments, only one of which can actually be performed. Hence a critical question naturally arises: Can one really prove anything definite about nature

from an argument based on the simultaneous consideration of quantities that cannot simultaneously exist? This question, which would be important even in the context of classical physics, is given far greater importance in the present circumstance by the fact that quantum theory categorically rejects, as a matter of principle, the idea that the results of mutually incompatible experiments can be considered simultaneously well defined. The four alternative experiments a, b, c, and d, are excellent examples of mutually incompatible experiments: at most one of them can be actually performed in any single given experimental instance. Yet the proof rests crucially on the simultaneous consideration of all four members of the quartet (λ_a, λ_b, λ_c, λ_d).

An important distinction must be made here. It is the logical distinction between a conjunction of the kind: "X is true" and "X is false", and a conjunction of the kind: "If Y is true then X is true" and "If Y is false then X is false". The first conjunction is a logical contradiction, whereas the second conjunction is not: it is equivalent to the statement that X is equivalent to Y.

In quantum theory there are "incompatible observables" such as the coordinate variable x and its conjugate momentum variable p. The quantum formalism provides no way of representing a state in which both x and p have well-defined values. On the other hand, the theory also asserts that the physical measurements of x and p are mutually incompatible: the two measurements cannot be

performed simultaneously. But then there is no need for the theory to represent a state in which both x and p have well-defined values.

It is a basic principle of orthodox quantum thinking that if a measurement of x is performed then one must not even imagine that p has a well defined value. The mathematical formalism does not support this idea, and paradoxes are avoided by banning the very thought. Thus orthodox quantum thinking categorically rejects conjunctions such as: "the value of p is $\hat{p}$" and "the value of x is $\hat{x}$". Such a statement is, from the point of view of quantum theory, a contradiction.

This conjunction is logically different from the conjunction: "If x is measured then x will be found to have a value $\hat{x}$" and "If p is measured then p will be found to have a value $\hat{p}$". Since x and p cannot be simultaneously measured there is no possibility of deducing from the above conjunction the "contradictory" conjunction: "x has value $\hat{x}$" and "p has value $\hat{p}$".

On the other hand, even the single statement "If x is measured then x will be found to have a value $\hat{x}$" is itself rejected in orthodox quantum thinking. For it presupposes, or suggests, that "the value that x will be found to have if x is measured" is already fixed, or well defined. That is, it presupposes determinism, which is itself rejected by orthodox quantum thinking, or in any case not presupposed. The statement that "if x were to be measured then x would be found to have the value $\hat{x}$" is

likewise rejected.

On the other hand, quantum thinking allows statements such as "if x were to be measured then some value for x would appear". Orthodox quantum thinking also allows conjunctions such as: "if x were to be measured then some value of x would appear" and "if p were to be measured then some value of p would appear". Since x and p cannot be simultaneously measured one cannot deduce from this allowed conjunction the disallowed conjuction: "a value of x would appear" and "a value of p would appear".

One of our key assumptions is this: "For every s in the set (a, b, c, d), if the experiment s is actually performed then some single result λ_s will actually appear." This assumption is a conjunction of statements about mutually incompatible measurements. But it leads to no contradiction with either the quantum formalism, or the idea of nondeterminism. For a value λ_s is asserted to appear only under the condition that the associated experiment is actually performed. And there is no presumption that the value that will appear under those conditions is determined prior to the creation of those conditions. Since the four conditions are mutually exclusive, one cannot deduce the simultaneous existence, or appearance, or well-definedness of any two λ_s. Thus our set of conjunctions does not contradict quantum thinking. In fact, it is a normal part of quantum thinking.

Let the assumption just stated be called "single result". And let us temporarily accept an "orthodox"

position consisting of the following assumptions:

1. Free Choice. The choices of the experimenters can be regarded as free.
2. Single Result. For each s in the set (a, b, c, d), if s is actually performed then <u>some</u> single result λ_s will actually appear.
3. Validity of Q.T. No matter which s is actually performed, the predictions of quantum theory can be expected to hold for the result λ_s that actually appears.
4. Only Actual Results. Unperformed experiments have no results, and one must not even imagine that they do.
5. Non-Determinism. The result of an actually performed experiment becomes fixed and well defined within the spacetime region where the process actually occurs, not before.
6. Strong Locality. Information and causal influences propagate only into the "future light core", i.e., only forward in time, and no faster than light.

These six assumptions produce an immediate contradiction far simpler than the one discovered by Bell. For if the results r_{Ai} and r_{Bi} become fixed and well defined only in R_A and R_B, respectively, and information and causal influences propagate only into the future light core, as indicated in Fig. 1, then there is no way to obtain, for

$\theta_A = \theta_B$, and almost all values of i, the connection $r_{Ai} = -r_{Bi}$ demanded by 3.

This difficulty is the basis of the EPR argument. If one accepts assumptions 1 through 3, which are basic to orthodox quantum thinking, and accepts locality, then 5, hence also 4, must apparently be abandoned. This fact was turned by EPR into an argument for the incompleteness of quantum theory.

On the other hand, if one stays with quantum thinking, and accepts 1 through 5, then one must apparently abandon the locality assumption 6. This locality assumption comes from the theory of relativity, but not in any direct or compelling way. For it is not entailed by the theory-of-relativity requirement that <u>signals</u> propagate no faster than light. (A "signal" is a controlled, interpretable message.) Thus, if one adheres to the normal quantum precepts then there is already, even without Bell's contribution, an apparent need to abandon the strong idea of locality represented by 6.

This argument is not a complete proof of non-locality. For there is the logical possibility of abandoning the "orthodox ideas" 4 and 5 without abandoning the more fundamental ideas 1, 2, and 3. Indeed, if one insists on 1, 2, 3, and 6, then it appears that one is forced to accept the idea that the results for all the alternative possible settings are "predetermined:" the "particles" must evidently contain the information about how they will respond to any possible setting. For under

conditions 1, 2, 3, and 6, there appears to be no other way to understand the result that $r_{Ai} = -r_{Bi}$ for $\theta_A = \theta_B = \theta$, for any value of θ. This is the point that Bell made in his preliminary argument.

This way of escaping nonlocality, by abandoning the quantum ideas 4 and 5, was blocked by Bell. For Bell showed, in effect, that there is <u>no</u> set of values $r_{Ai}(\theta'_A)$, $r_{Ai}(\theta''_A)$, $r_{Bi}(\theta'_B)$, and $r_{Bi}(\theta''_B)$, for i = 1 to n, that satisfy the predictions of quantum theory in all four alternative possible cases. A simple proof of this result is given in Fig. 5 and its caption. With this way of escape blocked one seems forced to the conclusion that locality fails: property 6 is incompatible with properties 1, 2, and 3 regardless of whether one accepts or rejects properties 4 and 5.

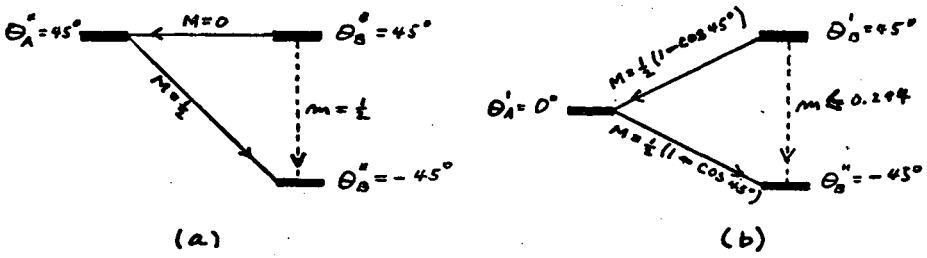

Fig. 5. A simple proof of Bell's theorem can be obtained as follows.[25] Let each pair i for which $r_{Ai}(\theta_A) = -r_{Bi}(\theta_B)$ be called a θ_A to θ_B "match". Let each pair i for which $r_{Ai}(\theta_A) = r_{Bi}(\theta_B)$ be called a θ_A to θ_B "mismatch." Quantum theory predicts that the fractions of θ_A to θ_B "matches" and "mismatches" approach $\frac{1}{2}(1 + \cos(\theta_A - \theta_B))$ and $\frac{1}{2}(1 - \cos(\theta_A - \theta_B))$ respectively, as the number n of pairs i approaches infinity. The expected fractions M of θ_A to θ_B mismatches in the various cases are shown in Fig. 5. Consider first the setting in $\theta_A = \theta''_A = 45°$ shown in Fig. 5a. Taking in tandem the fraction of mismatches M=0 and $M = \frac{1}{2}$ from θ'_B to θ''_A and then from θ''_A to θ''_B one concludes that the expected fraction of θ'_B to θ''_B mismatches $r_{Bi}(\theta'_B) = -r_{Bi}(\theta''_B)$ is $m = \frac{1}{2}$. Consider next the setting $\theta'_A = 0°$ shown in figure 5b. The same argument now shows that m is no greater than the sum of the two fractions $M = \frac{1}{2}(1-\cos 45°)$ and $M = \frac{1}{2}(1-\cos 45°)$. That is, $M \leq (1-\cos 45°) \approx 0.293$. This contradicts the earlier result $m = \frac{1}{2}$. Consequently, there is no quartet of n vectors $[(r_{Ai}(\theta'_A), \ldots, r_{An}(\theta'_A)), (r_{Ai}(\theta''_A), \ldots, r_{An}(\theta''_A)),$

$(r_{Bi}(\theta'_B), \ldots, r_{Bn}(\theta'_B)), (r_{Bi}(\theta''_B), \ldots, r_{Bn}(\theta''_B))]$, that satisfies all four predictions of quantum theory. Hence one must reject either the locality requirement that the results in each region are independent of which experiment is chosen in the other region, or the assumption that the results of the unperformed experiments can be considered definite, albeit unknown.

What remains to be achieved by the argument given earlier concerns fine points. First, there is the task of putting together, with no cracks, the two parts of the informal argument given above. Second, there is the task of weakening the locality assumption 6. The "strong locality" assumption 6 refers to the transfer of information or causal influence without reference to any controllable cause. But the idea of a "causal influence" becomes clearest only when one has a controllable cause, and Einstein formulated his locality requirement only under this condition.

The logical question at issue can be made clear by formulating it in terms of "black boxes". Suppose there are two black boxes, connected together by a communication cable. Each black box has two buttons and a display register, but its internal constitution is completely unknown. Seated before each box is an observer. The display register on each box suddenly flashes the message "press a button", and the observer then whimsically chooses one of the two buttons and presses it. Shortly thereafter a number from 1 to 2^n, for some large n, appears briefly on the register of the first box, and a number in the same range appears on the register of the second box. Each observer records the button he pressed and the number that appeared on his box, and the process is then repeated until an unmistakable correlation is observed between the numbers appearing on the two boxes for each of the four alternative possible combinations of the two buttons.

It can be presumed that the number appearing on each box may depend very sensitively on the time in microseconds at which the button on that box is pressed, and may also depend on various times of decays of radioactive nuclei in the box. So the numbers appearing can be regarded as completely unpredictable and nonpredetermined: a pair of numbers appears in each actual trial, but which numbers actually appear may be completely undetermined before the buttons are actually pressed. And "what would have happened if the observe had pressed the other buttom" may likewise be totally ill-defined.

The connection between the two boxes is of one of the two types, I or II. A connection of type I is always open, and the information about which button is pressed on either box can be transmitted to the other box before the number appears there. But a connection of type II is closed, or broken, before the "press a button" message appears. In this case no information about which button is pressed on either box can pass to the other box before the number appears there. Thus for a type II connection the possibilities for what can appear at either box, under either condition chosen there, are limited to those that do not depend upon which button is pressed on the other box.

Suppose now that the numbers appearing on the two boxes in each trial are identified with the appropriate two sequences from the set of four sequences $[(r_{Ai}(\theta'_A), \ldots, r_{An}(\theta'_A)); (r_{Ai}(\theta''_A), \ldots, r_{An}(\theta''_A)); (r_{Bi}(\theta'_B), \ldots, r_{Bn}(\theta'_B))$; and $(r_{Bi}(\theta''_B), \ldots, r_{Bn}(\theta''_B))]$, and that the

correlation functions occurring in (3) are then calculated in the four alternative cases, and compared to the quantum predictions given in (3).

The following questions can now be posed. Suppose that we can assume that the choices between the two buttons made by the experimenters are independent, and random, with equal weights for the two alternative possibilities. And suppose we can also assume that the information about which choices are made becomes available to the black-box-processes that determines what appears on the registers only when the buttons are pressed. Can we then deduce that the connection is of type I, not type II, from the assumption that in an increasing sequence of trials of the kind described above the distribution of observed correlation parameters will, with probability one, move into conformity with the predictions of quantum theory?

The answer is yes. One can see this by examining the restrictions on the conceivable possibilities imposed by our assumptions, in the case of a type II connection. These assumptions mean that the choice of result $R_A = (R_{A1}, \ldots, R_{An})$ must be made without knowledge of, or dependence upon, the choice of experiment (button) in region B, and that the two possibilities for this choice are equally likely. And the choice of result R_B must be made without knowledge of, or dependence upon, the choice of experiment in region A, and that the two possibilities for this choice are equally likely.

Because n can be made arbitrarily large the epsilontic questions associated with probabilities are not an essential problem, and we can use the terminology that a certain result (R_A, R_B) is allowed by quantum theory: e.g., $(R_A, R_B) \in Q(D_A', D_B')$ means that the pair of results (R_A, R_B) conforms to the quantum predictions for the case $D_A = D_A'$ and $D_B = D_B'$.

Although the choice of result R_B must be independent of the choice between D_A' and D_A'' it cannot be generally independent also of the result R_A occurring in region A. For then there could be no nonzero correlation functions, contrary to (3). So the choice of R_B must in general depend on some restriction imposed on the allowed set of R_A, or vice versa. For example, it is conceivable that before the communication link is broken the value of R_A in the case D_A' is fixed at a value R_A' by some random or stochastic process. If this value R_A' of R_A in the case D_A' were thus available in the process of choosing R_B then one could satisfy the requirements of quantum theory in the case that D_A is D_A' by requiring R_B to lie in either $S_B(R_A', D_A', D_B') = \{R_B';\ (R_A', R_B') \in Q(D_A', D_B')\}$ or $S_B(R_A', D_A', D_B'') = \{R_B'';\ (R_A', R_B'') \in Q(D_A', D_B'')\}$, according to whether the choice of D_B is D_B' or D_B''. This single allowed value R_A' could be expanded to a <u>set</u> of allowed values $\bar{R}_A'$ if the intersection of the sets $S_B(\bar{R}_A', D_A', D_B')$ over this set of $\bar{R}_A'$ were nonempty, and if the intersection of the sets $S_B(\bar{R}_A', D_A', D_B'')$ were likewise nonempty. For then R_B could be taken to be any value in the intersection of the sets $S_B(\bar{R}_A', D_A', D_B')$, if the

choice of D_B is D_B', or any value in the intersection of the sets $S_B(\bar{R}_A', D_A', D_B'')$ if the choice of D_B is D_B''. The quantum predictions could then be satisfied if the choice of D_A were D_A' no matter whether the choice of D_B is D_B' or D_B''.

However, there is a 50% chance that the choice of D_A would be D_A'' instead of D_A'. And the choice of R_B cannot depend whether the choice of D_A is D_A' or D_A''. Thus the choice of R_B must be compatible also with the possibility that the choice of D_A is D_A''. This means that there must be some triple (R_B', R_B'', R_A'') such that R_B' lies in the intersection of the sets $S_B(\bar{R}_A', D_A', D_B'')$ and R_B'' lies in the intersection of the sets $S_B(\bar{R}_A', D_A', D_B'')$, as specified above, and such that, moreover, R_A'' satisfies $(R_A'', R_B') \in Q(D_A'', D_B')$, and $(R_A'', R_B'') \in Q(D_A'', D_B'')$. For otherwise there would be at least a 25% probability that the predictions of quantum theory would fail. Using only the weaker conditions that $R_B' \in S_B(R_A', D_A', D_B')$ and $R_B'' \in S_B(R_A', D_A', D_B'')$ one obtains a quartet of conditions on the quartet of values $(R_A', R_A'', R_B', R_B'')$: $(R_A', R_B') \in Q(D_A, D_B')$, $(R_A', R_B'') \in Q(D_A', D_B'')$, $(R_A'', R_B') \in Q(D_A'', D_B')$, and $(R_A'', R_B'') \in Q(D_A'', D_B'')$. But these conditions have no solution: that is precisely the content of our main result that equations (3) and (4) are incompatible. Therefore there is no way that any block-box-process can generate results that conform to the predictions of quantum theory if the communication link is of type II. To obtain results in conformity with the quantum predictions the connection must be of type I, and this corresponds in

the Bell-type experiment to a communication link that involves faster-than-light transfers.

The illustration has been couched in a language of black-boxes for concreteness. But the results under consideration are actually appearances in the consciousness of the community of communicating observers. Thus there is no essential assumption of realism. The important assumption is rather that there be only one such community. If there were more that one such community, as there are in the many-worlds picture of nature, then the appearances would have to be labelled by an index that identifies which community of communicating observers the appearances are appearing to, and our argument would fail.

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