LBL-16846
c2

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

RECEIVED
LAWRENCE
BERKELEY LABORATORY
MAR 14 1984
LIBRARY AND
DOCUMENTS SECTION

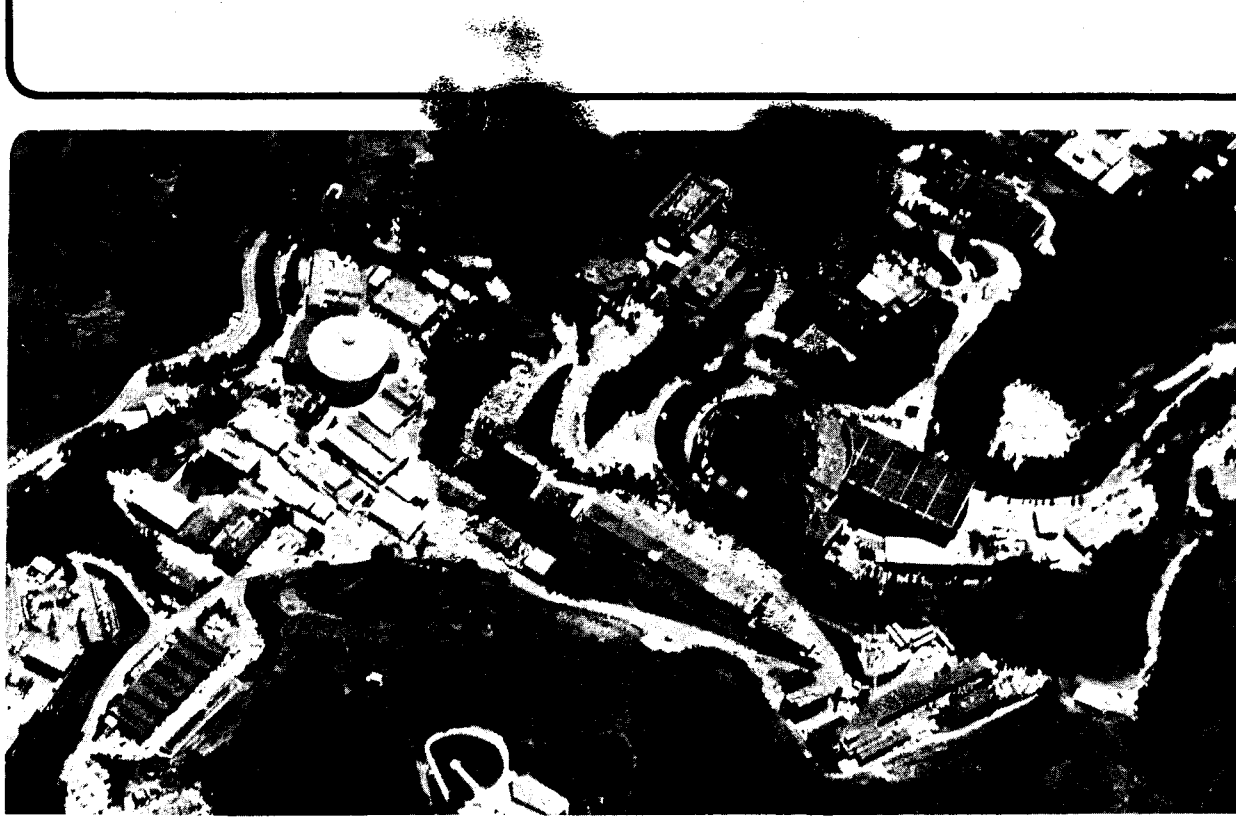
Physics Division

Presented at the Monopole '83 Conference,
Ann Arbor, MI, October 6-9, 1983

MONOPOLE CATALYSIS: AN OVERVIEW

S. Dawson

November 1983



Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

LBL-16846
c.2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

November 1983 LBL-16846

Monopole Catalysis: An Overview*

Talk presented at the Monopole '83 Conference held in Ann Arbor, MI from October 6-9, 1983.

Sally Dawson

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Abstract

A summary of the talks presented in the topological workshop on monopole catalysis at this conference is given. We place special emphasis on the conservation laws which determine the allowed monopole-fermion interactions and on catalysis as a probe of the structure of a grand unified theory.

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

I. Introduction

At this conference, there were nine talks presented dealing with the general topic of monopole catalysis of baryon number violating processes. We attempt — with no pretense at completeness — to summarize these talks here. The talks fall into two general categories. The first is a discussion of the allowed fermion-monopole scattering processes. In the presence of a finite size monopole, it is not *a priori* clear that charge is conserved in all interactions. Charge conservation in the fermion-monopole system and its relationship to gauge invariance is the focus of Sec. II of this paper. The second topic discussed in this workshop was the connection between catalysis and the underlying structure of a grand unified theory. In Sec. III, we discuss the monopole catalysis resulting from monopoles with charges larger than a Dirac charge and from monopoles with a different topology than those of the $SU(5)$ grand unified theory.

One of the most interesting developments in the study of monopole catalysis was not discussed in this workshop. This is the hope, as given by Callan at this conference, of performing a reliable calculation of the total cross section and branching ratios for monopole catalysis of baryon number violating processes. The interested reader is referred to Callan's contribution to these proceedings.

II. Conservation Laws and Charge Conservation

The use of conservation laws to study monopole-fermion scattering has been discussed by Sen [1]. We summarize his results here. For simplicity, consider the Georgi-Glashow $SU(2)$ model. In this model, monopoles are produced when the gauge symmetry is broken by the vacuum expectation value of a triplet of Higgs, ϕ, to a residual $U(1)$ symmetry. The model contains two Dirac doublets of fermions,

$$\psi_i = \begin{pmatrix} a_i^+ \\ b_i^- \end{pmatrix}_L \qquad i = 1, 2 \tag{1a}$$

with charges,

$$Q_{em} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \tag{1b}$$

and is described by the Lagrangian,

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{i=1}^{2} \overline{\psi}_i \not{D} \psi_i + \frac{1}{2} |D_\mu \phi|^2 + V(\phi). \tag{2}$$

(In the $SU(5)$ grand unified model, we make the identifications, $a_1 = e^-$, $b_1 = \overline{d}_3$, $a_2 = \overline{u}_2$, $b_2 = \overline{u}_1$).

The theory can be described by eight charges,

$$\begin{aligned} Q_{a_i(b_i)} &= \int d^3x \, \overline{\psi}_{a_i(b_i)} \, \gamma^0 \, \psi_{a_i(b_i)}, \\ Q^5_{a_i(b_i)} &= \int d^3x \, \overline{\psi}_{a_i(b_i)} \, \gamma^0 \, \gamma_5 \, \psi_{a_i(b_i)}. \end{aligned} \tag{3}$$

It is possible to construct linear combinations of these charges which correspond to global symmetries of the Lagrangian of Eq. (2) and hence are conserved in all scattering processes. For example, $\mathcal{L}$ is invariant under the transformation $\psi_1 \to e^{i\alpha_1}\psi_1$ which corresponds to the conserved charge,

$$S_1 = Q_{a_1} + Q_{b_2}. \tag{4}$$

The other conserved charges are

$$\begin{aligned} S_2 &= Q_{a_2} + Q_{b_2}, \\ S_3 &= (Q^5_{a_1} + Q^5_{b_1}) - (Q^5_{a_2} + Q^5_{b_2}), \\ S_4 &= (Q_{a_1} - Q_{b_1}) + (Q_{a_2} - Q_{b_2}) = -2Q_{em}. \end{aligned} \tag{5}$$

Thus far we have assumed that the monopole is pointlike. In this case, the mass gap between a monopole and a dyon is infinite and so electric charge, $(-\frac{1}{2}S_4)$, must be conserved within the fermion system alone.

Monopole fermion scattering is further constrained because only the $\vec{J} = 0$ partial wave can penetrate to the monopole core. The conserved angular momentum is $\vec{J} = \vec{L} + \vec{S} + \vec{T}$ and so in the $\vec{J} = 0$ partial wave,

$$\hat{r} \cdot (\vec{S} + \vec{T}) = 0. \tag{6}$$

For an outgoing particle, $\hat{r} \cdot \vec{S}$ is the helicity and so there is a relationship between the helicity of a particle and its position in the doublets of Eq. (1):

Allowed incoming states: $a_R^-,\ b_L^+,\ a_L^+,\ b_R^-$

Allowed outgoing states: $a_L^-,\ b_R^+,\ a_R^+,\ b_L^-$ (7)

The helicity constraints and the conserved charges completely specify the allowed scattering process. Examples of allowed interactions are,

$$\begin{aligned} a_{1_L}^+ + a_{2_R}^- &\to b_{1_L}^- + b_{2_R}^+, \\ a_{1_L}^+ + b_{1_L}^+ &\to a_{2_R}^+ + b_{2_R}^+. \end{aligned} \tag{8}$$

Note that both helicity violating and helicity conserving processes are allowed.

The role of the boundary conditions at the monopole core deserves special mention here. We must impose boundary conditions which conserve S_1, S_2, S_3, and S_4 since these correspond to symmetries of the full theory. However, the most general set of boundary conditions which conserves $S_1, \ldots, S_4$ violates baryon number. It is therefore not possible to remove baryon number violating processes from the theory by changing the boundary conditions.

If the monopole has a finite size R_0, the picture changes [2,3]. The mass difference between the monopole and the dyon is no longer infinite, but is of order $1/R_0$. Since the boundary conditions on the fermion fields at the monopole core,

$$\psi_{a_i}(r = R_0) = \gamma^0 \psi_{b_i}(r = R_0), \tag{9}$$

violate charge, the conservation of charge in the monopole fermion system is not obvious at first glance.

Sen [1] has noted that charge conservation for a finite size monopole can be verified as follows. The charge deposited on the monopole core, $\tilde{Q}$, can be computed from the radial electric field, E_r,

$$\tilde{Q} = (r^2 E_r)_{r=R_0}. \tag{10}$$

The total charge of Q_T of the monopole fermion system is $\tilde{Q}$ plus the fermion charge. Q_T commutes with the Hamiltonian and so is conserved in all interactions. An example of a process where charge is deposited on the monopole core is:

$$a_{1_L}^+ + \text{Monopole}(Q_{em} = 0) \to a_{1_L}^- + \text{Dyon}(Q_{em} = -1). \tag{11}$$

Both Sen [1] and Yan [3] have noted that the amplitude for such processes is suppressed by powers of R_0,

$$\langle \overline{\psi}(r)\, \overline{\gamma}_5\, \psi(r) \rangle \sim \left[\frac{R_0 r}{(r + R_0)^2} \right]^{2/N}, \tag{12}$$

where N is the number of Dirac fermion doublets. In a realistic grand unified theory, R_0 is of order $1/M_x$, so the amplitude for creating a dyon is suppressed by powers of μ/M_x, where μ is an appropriate low energy scale and M_x is the unification scale.

In his talk at the conference, Yan made the interesting observation that it is possible to obtain a Lagrangian for fermion-monopole

interactions which manifestly conserves charge. The usual decomposition of the monopole potential is not gauge invariant. When the potential is written in a gauge invariant fashion, charge conservation becomes an exact symmetry of the Lagrangian.

III. Catalysis as a Probe of the GUT Structure

It is an important phenomenological question to determine if different unified theories yield different selection rules for monopole catalysis. Unfortunately, however, this does not appear to be the case.

Most GUTs contain monopoles which catalyze $p \to e^+\pi^0$. Any GUT in which the $SU(3) \times SU(2) \times U(1)$ groups are unified in an $SU(N)$ group in which the fermions are embedded in the fundamental representation such that they decompose under $SU(5)$ as a [5] plus $N-5$ singlets will have the same monopole catalysis of proton decay as the $SU(5)$ model, (i.e. $p \to e^+\pi^0$ will be the dominant decay mode of the proton).

Even introducing supersymmetry does not radically change the predictions for monopole catalysis. Since it is the gauge degree of freedom which is important for monopole catalysis, the supersymmetric $SU(5)$ GUT will catalyze the same proton decay events as the ordinary $SU(5)$ model. This model has the interesting feature that "ordinary" proton decay proceeds predominantly through Higgs exchange which yields $p \to \mu^+K^0$, while proton decay by monopole catalysis gives $p \to e^+\pi^0$.

We turn now to the catalysis induced by monopoles with a different topology from those of the $SU(5)$ model. At this meeting, London [4] spoke about Z_N monopoles — monopoles whose charges are additive modulo N. For example, a Z_2 monopole is its own anti-monopole. Such monopoles arise in a GUT theory where $SO(10)$ is broken to $SU(4) \times SU(2) \times SU(2)$ which is then broken to $SU(3) \times SU(2) \times U(1)$. At the first stage of symmetry breaking, Z_2 monopoles are produced with $eg = 1/2$. These monopoles catalyze the same proton decay as $SU(5)$ monopoles. At the second stage of the symmetry breaking "ordinary", Z, monopoles are produced with $eg = 1$. These monopoles do not catalyze proton decay.

We next consider monopoles with charges larger than the

minimal Dirac charge. The interactions of fermions with these monopoles was discussed by Schellekens [5] at this meeting. He has solved the Dirac equation for fermions interacting with a monopole of arbitrary strength to find the allowed fermion monopole scattering processes. The trick is to find a basis in which the Dirac equation reduces to N non-interacting doublets, where N is an effective number of fermion doublets. The problem is then equivalent to that solved by Rubakov. It is no longer the $J = 0$ partial wave which interacts with the monopole to produce baryon number violating interactions, but rather it is the $J = T - 1/2$ partial wave ($|eg| = T$).

The rules for constructing the effective doublets are easily found. The Dirac potential for the monopole can be written as

$$\vec{A}_D = Q_M(1 - \cos\theta)\frac{\hat{\phi}}{r\sin\theta}, \tag{13}$$

where Q_M is in a representation of the unified gauge group. For a spherically symmetric monopole,

$$Q_M = I_3 - T_3, \tag{14}$$

where $\vec{I}$ is an $SU(2)$ generator which commutes with Q_M and $\vec{T}$ is the generator of the $SU(2)$ group which defines the monopole-fermion interactions. The Dirac quantization condition, classical stability for a non-Abelian monopole, and the charge-triality relationship suffice to determine Q_M uniquely for a given eg.

The Dirac quantization condition requires that,

$$\exp(4\pi i Q_M) = 1, \tag{15}$$

while non-Abelian stability for $SU(3)$ requires that if

$$Q_M = \alpha Q_{em} + Q_c, \tag{16}$$

where Q_{em} and Q_c are generators of $U(1)_{em}$ and $SU(3)_{\text{color}}$ and α is a constant, then

$$Q_c = (q_1, \ldots, q_n), \quad \text{with } q_i - q_j = 0, \pm\frac{1}{2}, \tag{17}$$

for every i, j. Finally, if the only colored fields are color triplets,

$$Q_c = -\frac{1}{3} + m, \tag{18}$$

where m is an integer. The monopole charge eg is then determined in terms of Q_{em} and $Y_c = (\frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$,

$$\begin{array}{ll} eg = \frac{1}{2}, & Q_M = \frac{1}{2}(Q_{em} + Y_c) \\ eg = 1, & Q_M = Q_{em} + Y_c \\ eg = \frac{3}{2} & Q_M = \frac{3}{2}Q_{em} \end{array} \tag{19}$$

The rules for finding the effective doublets which interact with a monopole of arbitrary strength eg, are:

1. Find a basis in which T_3 and Q_M are diagonal,
2. The doublets are formed with fermions which have equal and opposite T_3 and different values of Q_M, and
3. The degeneracy of each doublet is $2|T_3|$.

For example the $eg = \frac{3}{2}$ monopole in $SU(5)$ interacts with fermions in the following representations, [6]

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ e^+ \end{pmatrix}_L \quad \begin{pmatrix} e^- \\ \bar{d}_3 \\ \bar{d}_2 \\ \bar{d}_1 \end{pmatrix}_L \quad \begin{pmatrix} \bar{u}_3 \\ \bar{u}_2 \\ \frac{1}{\sqrt{2}}(u_1 + \bar{u}_1) \\ u_2 \\ u_3 \end{pmatrix}_L . \tag{20}$$

Using the rules given above, there are twelve effective doublets,

$$3 \times \begin{pmatrix} d_1 \\ e^+ \end{pmatrix}_L , \quad 3 \times \begin{pmatrix} e^- \\ \bar{d}_1 \end{pmatrix}_L , \quad 4 \times \begin{pmatrix} \bar{u}_3 \\ u_3 \end{pmatrix}_L , \quad 2 \times \begin{pmatrix} \bar{u}_2 \\ u_2 \end{pmatrix}_L . \tag{21}$$

The $eg = \frac{3}{2}$ monopole is particulary interesting because its interactions are purely electromagnetic ($Q_M = \frac{3}{2} Q_{em}$).

The allowed interactions can then be found by constructing the conserved charges. The simplest allowed process is

$$d_{3_L} + u_{1R} + \text{Monopole} \rightarrow e_L^+ + \bar{u}_{2R} + \text{Monopole}. \tag{22}$$

The next simplest allowed process involves nine particles in the incoming state.

Finally, we turn to a discussion of the connection between the zero energy states of the theory and catalysis. The presence of a zero energy bound state, (*i.e.* one for which $\partial_0 \psi = 0$), depends critically upon the structure of the mass terms in the theory. The $SU(5)$ GUT with a [24] and a [5]-plet of Higgs does not possess such zero energy states, while the Georgi-Glashow $SU(2)$ model with a triplet of Higgs bosons does. Hence if catalysis depends on the existence of these zero energy states, (as claimed by Walsh [7] at this meeting), the presence or absence of catalysis would be a sensitive probe of the Higgs structure of a grand unified theory. Walsh's argument is as follows: Rubakov's calculation [8] of the baryon number violating condensates which are formed in the presence of a monopole relies on the use of the cluster property to find the expectation value of two operators at infinite time separation. The claim by Walsh is that since the calculation is performed at an infinite time separation, it is sensitive to the zero energy states of the theory and catalysis will not occur without such states.

We disagree with Walsh's argument for a number of reasons. The first is that his argument is concerned with the use of the cluster property. It is possible, however, to calculate the expectation value of some condensates, (baryon number violating, but chirality conserving), without the use of the cluster property. Such condensates certainly lead to cross sections of strong interaction magnitude. The second reason for disagreement is more subtle. The physics of monopole catalysis is occuring through the boundary conditions and the anomaly at short distances near the monopole core. The effects of the fermion mass terms are important at distances of the order of $1/M_{\text{fermion}}$ and should not affect the physics near the monopole core. In sum, we do not believe that one particle zero energy bound states are a necessary prerequisite for catalysis.

IV. Conclusion

Finally, we will mention briefly several other interesting talks which were given in this workshop. Craigie [9] has examined the effects of including the non-Abelian generators in the calculation of the baryon number violating condensates. His conclusion is that catalysis proceeds at strong interaction rates even when these non-Abelian interactions are included. There were also talks at this meeting by Grossman [10] and Fiorentini [11]. We have not discussed these talks since their results are well covered in the literature.

The conclusion to be drawn is that catalysis is in good shape — the selection rules and the conservation of charge in the monopole-fermion system are well understood. The baryon number violating processes which are catalyzed by monopoles have been examined in a variety of unified theories and the catalysis of $p \to e^+\pi^0$ seems to be a general effect. It remains only to calculate the magnitude of the cross section and the branching ratios!

Acknowledgment

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

References

[1] A. Sen, talk given at this conference and Phys. Rev. **D28**, 876 (1983).

[2] Y. Kazama and A. Sen, 'On the Conservation of Electric Charge Around a Monopole of Finite Size", Fermilab-Pub-83/58-THY, KUNS695 HE 83/12 (1983).

[3] T.M. Yan, talk given at this conference and "Breaking of Conservation Laws Induced by Magnetic Monopole", CLNS 83/563 (1983).

[4] D. London, talk given at this conference and D. London, J. Rosner, E. Weinbert, "Magnetic Monopoles with Z_N Charges", EFI 83-39-Chicago (1983).

[5] A.N. Schellekens, talk given at this conference and preprint in preparation.

[6] S. Dawson and A.N. Schellekens, Phys. Rev. **D27**, 2119 (1983).

[7] T. Walsh, talk given at this conference and T. Walsh, P. Weisz, and T.T. Wu, DESY 83-022 (1983).

[8] V. Rubakov, Nucl. Phys. **B203**, 311 (1982).

[9] N. Craigie, talk given at this conference and "Status of Rubakov-Callan Effect", IC/83/131 (1983).

[10] G. Grossman, Phys. Rev. Lett. **50**, 464 (1983).

[11] G. Fiorentini, Phys. Lett. **124B**, 29 (1983).

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720