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FROM PION PRODUCTION TO THE NUCLEAR MATTER
EQUATION OF STATE

J.W. Harris and R. Stock

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"From Pion Production to the Nuclear Matter Equation of State"

J. W. Harris and R. Stock

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"From Pion Production to the Nuclear Matter Equation of State"

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Abstract:

Equilibrium rate calculations are used to show that thermal and chemical equilibrium are approached during the high density stage in central nucleus-nucleus collisions. The total pion multiplicity is established as a probe of the high density stage. The observed pion multiplicities are compared to predictions of a hadrochemical model with Rankine-Hugoniot compression. Assuming a partition of the internal energy per nucleon into thermal and compressional energy fractions, a nuclear matter equation of state is found which is very similar to that derived using an independent intranuclear cascade approach.

The primary objective for studying relativistic nucleus-nucleus collisions is to determine the response of nuclear matter under extreme conditions of high temperature and compression to densities several times that of normal nuclear matter. This information in the form of the nuclear matter equation of state is important for testing field theoretical approaches[1] to nuclear matter and to the understanding of the formation and collapse of supernovae[2] and the stability and structure of neutron stars.[3] The equation of state at high densities can be derived from central relativistic nucleus-nucleus collisions only if observables exist which transcend the final expansion stage of the reaction without considerable change, thus retaining information on the intermediate high density stage. During expansion the system relaxes and the compressional energy developed in the initial compression stage returns to kinetic energy consisting of flow and thermal motion. The total pion multiplicity, strange particle yields and distributions, cluster production yields, and global variables of the nucleon momentum flux have been proposed as observables that are sensitive to the high density stage of the collision.

In the present study we will show that thermal and chemical equilibrium are closely approached during the high density stage and that the pion plus delta-particle abundance freezes-out at the end of this intermediate stage of the reaction to become the final, observed total pion multiplicity. This chemical freeze-out must be distinguished from the thermal freeze-out which occurs at latter times during the expansion near ground state nuclear matter density ρ_0. It is at these later

times that the pion and nucleon temperatures and the pion emission source sizes are determined. Furthermore, the times at which the yields and distributions of strange particles and nucleon cluster abundance are determined will be addressed. The consequences of the time scales on the ability of various observables to probe the high density stage will be discussed. In a previous publication[4] an approach to the equation of state was taken by assuming a partitioning of the internal energy per nucleon into "cold" compressional and thermal fractions[5] as is the case in the hydrodynamical model.[6] The thermal energy during the high density stage was then determined by fitting the intranuclear cascade model[7] to the $\pi + \Delta$ abundance observed experimentally as the total pion multiplicity in central collisions. The cascade model exhibits a constant $\pi + \Delta$ abundance throughout the expansion stage resulting in the proposal to use the final pion yield, which is identical to the $\pi + \Delta$ abundance at high density, as a thermometer for the thermal energy content of that phase. In the present study we will eliminate the use of the cascade model, replacing its predictions for the $\pi + \Delta$ content per baryon and for the compression during the high density phase by use of a hadrochemical model for the thermal part of the equation of state, and the Rankine-Hugoniot relation for the compression. Using an iterative procedure, a relationship between the pion multiplicity, the thermal and compressional energies, and the density will be established. This will again result in a determination of the compressional energy of nuclear matter as a function of density.

A mixture of nucleons, deltas, and pions will approach equilibrium through successive collisions on the microscopic level during the

interpenetration of two colliding nuclei. The rate of approach toward thermal equilibrium is governed by the total cross sections for πN and NN collisions which are above 25 mb at the energies of interest. On the other hand, chemical equilibrium is determined from the inelastic cross sections for $NN \rightarrow N\Delta \rightarrow NN\pi$. These vary rapidly with the c.m. energy and are thus dependent upon the temperature of the system which falls off during expansion. For a mixture of N, Δ, and π gases the build up of the Δ chemical abundance[8] is described by

$$\rho_\Delta(t) = \rho_\Delta^{equ}(1 - e^{-\lambda t}) \tag{1}$$

with

$$\lambda \simeq \frac{\rho_N^2}{2\rho_\Delta^{equ}} \langle \sigma_{NN \leftrightarrow N\Delta} v \rangle \tag{2}$$

where $\rho_\Delta(t)$ and ρ_Δ^{equ} are the Δ densities at time t and in equilibrium, respectively. λ is the rate constant which is proportional to the thermal energy average $\langle \sigma_{NN \leftrightarrow N\Delta} v \rangle$ of the inelastic cross section $\sigma_{NN \leftrightarrow N\Delta}$ times the relative pair velocity v. For a first estimate of the extent of chemical equilibrium in the collision of an equal mass system at 1 GeV/n, we take representative values for the high density stage from Ref. 8: at T = 100 MeV, $\rho_\Delta^{equ} \simeq 0.2\ \rho_N \simeq 0.6 \times \rho_0$ with $\rho_N \simeq 3\rho_0$, and $\langle \sigma v \rangle \simeq 1.5 \times 10^{23} fm^3 s^{-1}$. The reaction time constant is $\tau = \lambda^{-1}$. Thus the time it takes the system to reach a value of $\rho_\Delta(t) = 0.63\rho_\Delta^{equ}$ is $\tau \simeq 6 \times 10^{-24}$s. A similar estimate can be made for thermal equilibrium using the even larger total cross sections which results in a shorter τ. Hence, chemical and thermal equilibrium

will be reached at the end of the high density stage if τ is shorter than the duration $t_{\rho max}$ of the high density stage of the reaction. This is in fact the case for central collisions of Ar + KCl where $t_{\rho max} \simeq 2\tau$ and $^{139}La + ^{139}La$ where $t_{\rho max} \simeq 3.5\tau$ as will be shown below.

Once equilibrium of N, Δ, and π is established near the end of the high density stage, the effects of expansion must be considered. Expansion can in principle occur along any path between isoergic and isentropic. During isoergic expansion, which is assumed in the fireball model,[9] the $\pi + \Delta$ abundance will increase and the temperature will slowly decrease if equilibrium were maintained. On the other hand, for isentropic expansion the system will shrink in momentum space commensurate to its growth in position space and the temperature will drop rapidly with chaotic kinetic energy being converted into radial flow. It has been shown that the cascade model leads to near isentropic expansion,[10] which is assumed in hydrodynamical models. An isentropic expansion will be assumed in order to estimate whether chemical equilibrium among N, Δ, and π can be maintained during the expansion stage. For 1 GeV/n incident energy the system will have a temperature of $T = 60$ MeV at a time corresponding to $\rho = 0.5\ \rho_0$ just before interactions cease. From Ref. 8 the equilibrium delta density will be described by $\rho_\Delta^{equ} \simeq 0.08\ \rho_N = 0.04\ \rho_0$, $\langle\sigma v\rangle \simeq 10^{22} fm^3 s^{-1}$, and the equilibration time will be $\tau \simeq 2 \times 10^{-22} s$. This time is too long for chemical equilibrium to be maintained as the expansion of the system from ρ to $\rho/2$ occurs in $\sim 10^{-23} s$ for Ar + KCl and $^{139}La + ^{139}La$ at 1 GeV/n. Therefore, the system will not readjust its $\pi + \Delta$ abundance at such late

expansion times. Contrary to the assumption made in the fireball and hydrodynamical model studies on pion production,[9,11] the pion yield will not freeze-out at $\rho \sim 0.5 - 1.0\ \rho_0$ but rather at higher densities near the onset of expansion. After this occurs and expansion begins, the system becomes undercooled with respect to the $\pi + \Delta$ abundance. Thermal equilibrium, on the other hand, is maintained until much later times due to the large $\pi N \leftrightarrow \Delta$ cross section resulting in $\langle \sigma_{\pi N \leftrightarrow \Delta} v \rangle \simeq 10^{24} fm^3 s^{-1}$ which is approximately 100 times larger than $\langle \sigma_{NN \leftrightarrow N\Delta \leftrightarrow NN} v \rangle$ at expansion temperatures of $T \simeq 60$ MeV. Therefore, the pion spectral temperature[12] and the pion source size[13] seen in 2π-interferometry refer to late times, but the total π yield which is unaltered by the pion number conserving interchange $\pi N \leftrightarrow \Delta$ is established by the $\pi + \Delta$ abundance at chemical freeze-out and will remain only slightly altered during expansion.

The above observation that thermal and chemical equilibrium of certain reaction products do not necessarily coincide in time is important also for other observables which have been suggested as "primordial probes." Deuteron formation, for example, can only occur during the brief expansion time interval from $\rho \simeq \frac{2}{3}\rho_0$ to $\rho \simeq \frac{1}{3}\rho_0$ where all collisions cease. At densities above about $\frac{2}{3}\rho_0$, no localized clusters exist in nuclear matter. The appropriate cluster chemical equilibration time,[8] $\tau \simeq 10^{-23}$s, is comparable to the time required for expansion from $\frac{2}{3}\rho_0$ to $\frac{1}{3}\rho_0$, $t \simeq \frac{0.3\ A^{1/3}}{\langle v \rangle_R}$ where A is the fireball baryon number and $\langle v \rangle_R$ its radial expansion velocity. With light nuclei in noncentral collisions and for energies above ~ 1 GeV/A the cluster abundance should not equilibrate. The strange particle yield, as another

example, will never reach equilibrium at Bevalac energies as the appropriate equilibration time constant, $\tau \gtrsim 10^{-22}$s, exceeds the duration of the high density stage even for the heaviest nuclei. The strange particle yield will therefore be established primarily in direct NN reaction processes during the early compression stage where conditions near equilibrium are not yet attained.

A dynamical model is necessary to investigate whether the effect of a finite system of colliding nuclear matter leads to drastic departures from the rate considerations given above. The intranuclear cascade code[7] will be used for this purpose. The microscopic processes governing the N, Δ, and π abundances are the reactions $NN \leftrightarrow N\Delta$ and $\Delta \leftrightarrow N\pi$. $NN \leftrightarrow N\Delta$ mediates the production and absorption rate of the $\pi + \Delta$ abundance. The $N\Delta \leftrightarrow NN$ absorption is related to the production cross section by detailed balance. The other important process $\Delta \leftrightarrow N\pi$ is treated according to the isobar model[14] and the finite lifetime of the Δ. The time dependence of various quantities is shown in Fig. 1 for the 1 GeV/n ^{139}La + ^{139}La reaction for impact parameters $b \leqq 3.4$ fm. Shown in Fig. 1a is the density of baryons observed in a sphere of 3 fm diameter about the origin of the c.m. rest frame. A broad plateau is observed from a time of $t = 10$ fm/c, corresponding to the classical interpenetration time for two La nuclei at this relative c.m. velocity, to approximately $t = 16$ fm/c. Thus, the density is relatively stationary for approximately 2×10^{-23}s. Thereafter, the density decreases near-exponentially. The number of baryon-baryon collisions per fm/c is displayed in Fig. 1b. Approximately 200 of the total 278 nucleons of the system are involved by the time $t = 14$ fm/c. In the high density stage there are approximately 90 collisions per fm/c and each participant interacts once

every 5×10^{-24}s. At the mean relative energy of this stage, one inelastic collision per participant occurs every 10^{-23}s. The geometry and surface effects, density inhomogeneities, and other effects of a finite nuclear system thus tend to somewhat lengthen the chemical equilibration time. However, within the high density stage, about two inelastic and three elastic collisions per participant occur and the system should closely approach equilibrium. The time dependence of the $\pi + \Delta$ abundance is shown in Fig. 1c. The final $\pi + \Delta$ yield is established near the end of the high density stage and remains constant throughout expansion. Also plotted as a bar in Fig. 1c is the N, Δ, π chemical equilibrium estimate[8] of the $\pi + \Delta$ abundance. The values agree remarkably well. Therefore, the relatively stationary conditions during the high density stage observed in the dynamics of the cascade model and the predicted rapid chemical equilibration time at these high densities strongly suggest this stage as the only suitable time interval for N, Δ, and π chemical equilibrium. The rapid fall-off in the collision rate with the onset of expansion, as seen in Fig. 1b, is the principal reason for the survival of the $\pi + \Delta$ to participant ratio to late expansion times.

The degree of thermal equilibrium in the system can be investigated in the cascade model and is expressed by the ratio of transverse to longitudinal temperature or, equivalently, the ratio of the mean transverse to mean longitudinal momentum. This is represented by the ratio $R = (2/\pi)\langle|p_\perp|\rangle / \langle|p_\parallel|\rangle$ where $R = 1$ represents a necessary condition, up to first moments of the distributions, for thermal equilibrium. For the baryons in the 3 fm diameter test sphere fixed in the c.m. system origin for 1 GeV/n La + La at $b \lesssim 3.4$ fm, $R \simeq 0.9$ at the end of the high density stage indicating only a

small preponderance of longitudinal motion. Thus, thermal equilibrium is incomplete to within 10% for the finite nuclear system.

We have thus demonstrated that central collisions of medium- to heavy-mass nuclei lead to a near equilibrium $\pi + \Delta$ chemical abundance at the end of the high density stage and that the chemical freeze-out of this abundance occurs with the onset of isentropic expansion. Furthermore, the finite nuclear system was shown to approach thermal equilibrium to within 10% at Bevalac energies. The measured pion multiplicity which is the $\pi + \Delta$ abundance after $\Delta \rightarrow \pi N$ decay can now be used to determine the thermal energy content of the system at the end of the high density stage. To accomplish this, the idealization of complete equilibrium at the end of the high density stage will be assumed, keeping in mind the possible uncertainty estimated to be ±15% due to the nonequilibrium aspects of the finite nuclear system. The thermodynamical model will be used to relate the thermal energy content of the system at high density to the observed pion multiplicity. In this model the total excitation energy per baryon in a system at temperature T and density ρ is given by[5,8,9]

$$\frac{E - m_o^N c^2}{B} = \frac{3}{2} T + \frac{\langle n_\pi \rangle}{B}(\rho,T) \times E^\pi(T) + \frac{\langle n_\Delta \rangle}{B}(\rho,T) \times (m_o^\Delta - m_o^N)c^2 + \frac{W(\rho,T=0)}{B} + E^{flow}(\rho) \qquad (3)$$

where the left-hand side is simply the beam kinetic energy per baryon in the c.m. frame. The first three terms on the right-hand side describe the thermal excitation energy (per baryon) contained in baryons, pions, and in Δ mass excess. The fourth term is the ground state $(T = 0)$ compressional energy per baryon and the last term is the flow kinetic

energy, both at density ρ. These last two terms represent the nonthermal response of the nuclear medium to compression. At the time of maximum density when the final pion multiplicity is established, the compressional energy is a maximum and there is not yet kinetic flow energy which will appear as the system expands. The first three terms constitute the specific heat of the system. Thus, at the time of maximum density Eq. 3 can be written in a simplified form

$$\frac{\varepsilon}{B} = \frac{E_T}{B}(\rho, \frac{n_\pi}{B}, \frac{n_\Delta}{B}) + \frac{E_C}{B}(\rho, T = 0) \tag{4}$$

where ε is the incident c.m. beam energy. E_T represents the thermal excitation while E_C represents the compressional degrees of freedom. Here medium effects on the thermal baryon spectrum such as the softening of the pionic excitation mode in dense nuclear matter are ignored. The medium is assumed to exert its influence through the compressional energy only. The entire thermodynamical ansatz of Eq. 3 is also used in present hydrodynamical models.

In the chemical and cascade models only the thermal excitation energy E_T is considered while the nonthermal fraction E_C is ignored. The total $\pi + \Delta$ multiplicity directly reflects the available thermal energy E_T. The presence of compression simply offsets the thermal energy scale with respect to the beam energy ε. The total pion multiplicity determined from experiment at an energy ε will specify E_T and thereby determine $E_C(\varepsilon) = \varepsilon - E_T$. The resulting compressional energy is $E_C(\varepsilon) = \varepsilon - \varepsilon'$ where $\varepsilon' = E_T$ is the energy, input into the chemical or cascade model, which is required to fit the experimental pion

multiplicity. Since the cascade is a dynamical model it also determines the density ρ which is required to plot $E_c(\varepsilon)$ as a function of ρ. The chemical model will need further dynamics to determine ρ as will be shown below.

Displayed as solid circles in Fig. 2a is the experimental total pion multiplicity per participant nucleon $\langle n_{\pi+\Delta}\rangle/A$ extrapolated to zero impact parameter collisions of Ar + KCl as a function of the beam c.m. energy. The negative pion multiplicity was measured in the streamer chamber as a function of participant proton number in the minimum bias trigger mode at incident lab energies of 566, 772, 977, 1180, 1385, 1609, and 1808 MeV/n at the Bevalac. The techniques employed in the analysis have been described previously.[15] The prediction of the intranuclear cascade model of Cugnon is displayed as a dashed line in Fig. 2a. Also shown as triangles for comparison are two points from the Yariv-Fraenkel cascade[16] after implementation of the most recent experimental cross sections used by Cugnon. These cascade predictions are indeed offset in energy as expected from Eq. 4. These offset values, represented by the length of the arrows in Fig. 2a, are simply $E_c(\varepsilon) = \varepsilon_i - \varepsilon_i'$ where the cascade model at energy ε' gives the same $\langle n_{\pi+\Delta}\rangle/A$ abundance as the experiment at energy ε as already described. $E_c(\varepsilon)$ is interpreted as the compressional energy present in the collision at beam energy ε and is shown in Fig. 2b. The final step necessary to derive the equation of state function $W(\rho)$ is to use the relationship between ε and ρ as given by the model. The values of $W(\rho_i) = \varepsilon_i - \varepsilon_i' - 10$ MeV as a function of the density ρ_i, which is taken as the value at the maximum density plateau in the cascade calculation at energy

ε', are shown as triangles in Fig. 3. The 10 MeV is subtracted to allow for the ground state binding energy of mass 40 nuclei. These results are very similar to those obtained previously in a slightly different analysis.[4]

A second, independent model approach can be undertaken to identify the thermal energy fraction associated with the total pion multiplicity and thus the functional relationship $W(\rho)$. A chemical model[9] which assumes an equilibrium mixture of N, Δ, and π is used to predict the total pion multiplicity. In order to calculate the density which must be input into the chemical model, in addition to the energy per baryon, the Rankine-Hugoniot relation[17]

$$\rho/\rho_0 = \frac{\gamma_{cm}}{1 - \frac{\rho_0 E^{lab}}{2P}} \tag{5}$$

is used where ρ/ρ_0 describes the shock compression in a medium which is at rest in the center of mass, and γ_{cm} is the beam Lorentz factor with respect to the c.m. frame for an incident lab kinetic energy E^{lab}/A. The pressure of the medium,

$$P = \rho^2 \frac{\partial W(\rho)}{\partial \rho} + \rho T \tag{6}$$

can be calculated once $W(\rho)$ and the temperature T are known. The two terms are further constrained by Eq. (4) because $\varepsilon = E_c(\varepsilon) + E_T$, through the dependence of T on E_T. The technique which will be used to calculate the total pion multiplicity in the chemical model will be one of iteration:

1) Assume a "trial" density dependence $\rho^{(o)}(\varepsilon)$ which will be input into the chemical model. The trial function will not matter as long as the iteration technique converges. In fact, using the trial value $\rho/\rho_0 = 2\gamma + 1$ the technique did not converge due to the high estimate of ρ at small ε, $\rho/\rho_0 \simeq 3$ independent of ε. A second steeper ε-dependent "trial" density $\rho/\rho_0 = 2\gamma^2$ was used which led to convergence.

2) Using the density as an input into the chemical model, the total $\pi + \Delta$ multiplicity per participant nucleon can then be calculated as a function of energy. The result is then plotted in a plot similar to Fig. 2a and compared to the data.

3) Analogous to the previous cascade approach the compressional energy $E_c(\varepsilon) = \varepsilon - \varepsilon'$ can be obtained from the new Fig. 2a as a function of energy ε.

4) Using the "trial" relation for $\rho(\varepsilon)$ the $W(\rho)$ dependence is found.

5) This trial function $W(\rho)$ is input into Eq. (6) along with the temperature T, which is obtained from the chemical model for a given thermal energy fraction $E_T = \varepsilon - E_c(\varepsilon) = \varepsilon'$ and density ρ. The pressure P is determined and a new value of the density $\rho^{(1)}$ is calculated.

6) As a test of convergence, $|\Delta\rho| = |\rho^{(i)} - \rho^{(i+1)}|$ can be used. Return to step 2 using $\rho^{(i+1)}$ until $\Delta\rho$ converges to zero. If $|\Delta\rho|$ diverges, return to step 1 where a new trial value of $\rho^{(o)}(E)$ must be implemented. This technique converges after 3 iterations for the trial value $\rho^{(o)} = 2\gamma^2$.

The final results using the above iteration technique for the chemical model are shown in Figs. 2 and 3. In Fig. 2a the chemical model predicts a very similar $\langle n_{\pi+\Delta} \rangle / A$ vs. E dependence to the cascade except at the highest energies. This remaining difference results in a somewhat lower compressional energy $E_c(\varepsilon)$ for the chemical model than in the cascade at most energies as seen in Fig. 2b. The final compressional energies, as a function of density are displayed as circles in Fig. 3.

In summary, in the collision of two relativistic nuclei the reaction processes proceed through three distinct stages: namely, a rapid compression stage, a stationary but short high density stage, and finally a somewhat longer expansion stage. For a mixture of N, Δ, and π it was shown that thermal and chemical equilibrium conditions are approached at the end of the high density stage. The $\pi + \Delta$ chemical abundance freezes-out and is established at this time and is essentially preserved through expansion. However, thermal equilibrium is maintained through the expansion stage with thermal freeze-out occurring much later. The final pion spectra, angular distributions, and source sizes are determined at the time of thermal freeze-out. In order to check the effects of the finite size of nuclear systems on the equilibrium of the N, Δ, and π mixture a dynamical model, the internuclear cascade, was enlisted. These finite size effects were determined to be small of the order $\pm 15\%$ for central collisions of medium to heavy mass nuclei. By comparison, the strange particle abundances will never reach equilibrium due to the low production cross sections at Bevalac energies and will mostly be produced during the compressional stage under pre-equilibrium conditions. At the other extreme, the cluster particle equilibrium was shown to have a

delicate window for occurrence, with the calculated equilibration time being of the order of the expansion volume-doubling time, thereby making equilibrium tenuous. Thus, the observed d-like/p ratios may be lower than those calculated from equilibrium assumptions, particularly for lighter systems and noncentral collisions. This would result in an overestimate of the entropy when determined from the experimental d/p ratios, as has been observed.[18]

From the previous arguments about the $\pi + \Delta$ abundance two different models, the intranuclear cascade and the chemical model assuming the Rankine-Hugoniot pressure-density relation, were used to independently determine the thermal energy fraction of the total energy which goes into pion production. Neither of these models includes compressional degrees of freedom and both overestimate the total pion multiplicity. Using the simple relationship that the thermal plus compressional energies will be the total conserved c.m. energy, the compressional energy was determined from the thermal energy necessary to reproduce the observed total pion multiplicity in each model. The dynamical cascade model also predicts the density in the high density stage immediately determining an equation of state $W(\rho)$. In order to calculate an independent density for the chemical model the Rankine-Hugoniot relationship for shock compression was assumed and the resulting $W(\rho)$ is almost identical to that found in the cascade-data comparison. The present approach to the equation of state is closely related to the suggested use of entropy,[18] as determined from the d/p ratios, to derive information on the high density stage of the collision. Both the $\pi + \Delta$ abundance and the entropy remain constant after the onset of expansion, thereby preserving information about the high density stage.

It should be reiterated that several assumptions have been made in the two approaches. First, all higher order medium connections[19] are neglected and if present are included in the derived compressional energies. Such effects would have a tendency to soften the derived equation of state. Secondly, the cascade model assumes a mean free path which is not small with respect to the size of the system whereas the Rankine-Hugoniot relations, used in the chemical model approach, assume zero mean free path for particles in the system. A further assumption of the Rankine-Hugoniot relations used in the chemical model is that we are dealing with bulk nuclear matter rather than finite nuclear systems. These two approaches are at exteme ends of the spectrum but yield drastically similar results for the equation of state. Perhaps, this simply reflects the fact that equilibrium is reached in both approaches during the high density stage and that the actual chemistry which produces the total pion yield is similar in both cases and insensitive to the dynamics of the approach to and recession from the equilibrium state.

In conclusion, we have to note that the application of equilibrium concepts to calculate reaction rates in order to better understand the dynamics of relativistic nucleus-nucleus collisions has been developed by others[8] over the past ten years. Several authors[17,20] have also suggested the sensitivity of the pion yield to the equation of state of nuclear matter using various theoretical approaches. The present study, in an attempt to make a connection between theoretical and experimental results, introduces a few new arguments about chemical equilibration and freeze-out and derives information on the equation of state of nuclear matter under certain assumptions by restating in context many older concepts.

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Figure Captions

1. Predictions of the cascade model for 1.0 GeV/n $^{139}La + ^{139}La$ collisions for impact parameters $b \leq 3.4$ fm. The time dependence in units of fm/c is shown for (a) the baryon density in units of normal nuclear density ρ_0, (b) the number of baryon-baryon collisions per unit time and (c) the instantaneous $\pi + \Delta$ abundance (scale is on the right). Also displayed as a bar in (c) is the prediction of the hadrochemical model for the equilibrium abundance at the predicted maximum cascade density.

2. (a) The $\pi + \Delta$ abundance per participant nucleon is plotted as a function of c.m. energy (lower axis) and laboratory energy (upper axis) for Ar + KCl. The solid circles are the experimental results extrapolated to zero impact parameter. The dashed and dotted curves correspond to the Cugnon cascade and the chemical model predictions, respectively, and the triangles are predictions of the Yariv-Fraenkel cascade. The horizontal arrows represent the values of the compressional energy per nucleon determined at each experimental point assuming a partition of the equation of state into thermal and compressional energy fractions. (b) The resulting values of the compressional energy per nucleon derived independently from the data-cascade (triangles) and data-chemical model (open circles) comparisons.

3. Values of the compressional energy, minus 10 MeV binding energy, plotted as a function of the density in units of normal nuclear density ρ_0 derived using the data + cascade and data + chemical model (+ Rankine-Hugoniot) techniques described in the text.

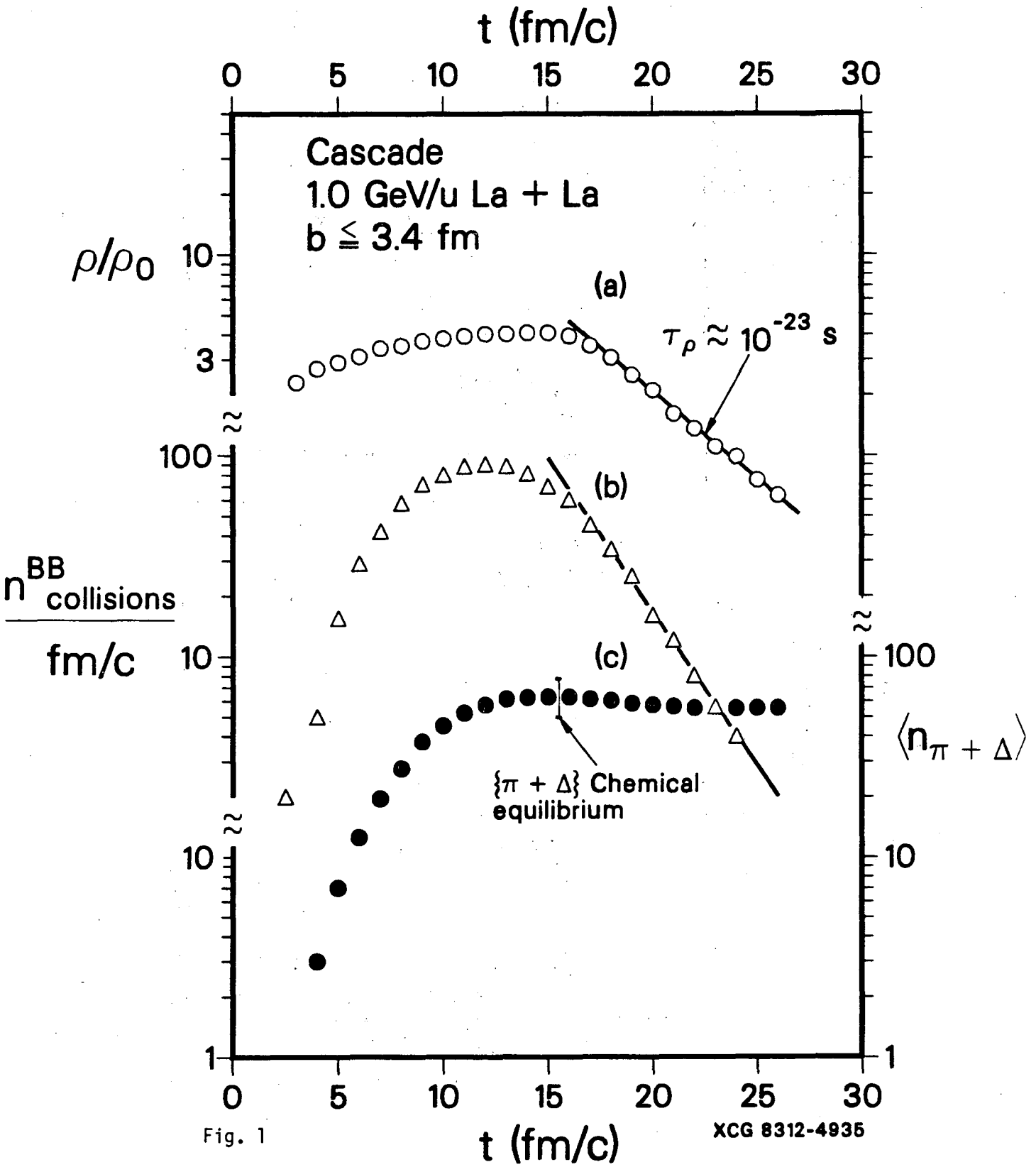
t (fm/c)
0
5
10
15
20
25
30
Cascade
1.0 GeV/u La + La
b ≦ 3.4 fm
ρ/ρ0
10
3
(a)
τρ ≈ 10^-23 s
100
(b)
n^BB collisions / fm/c
10
(c)
100
⟨nπ + Δ⟩
{π + Δ} Chemical equilibrium
10
1
1
0
5
10
15
20
25
30
t (fm/c)
XCG 8312-4935

Fig. 1

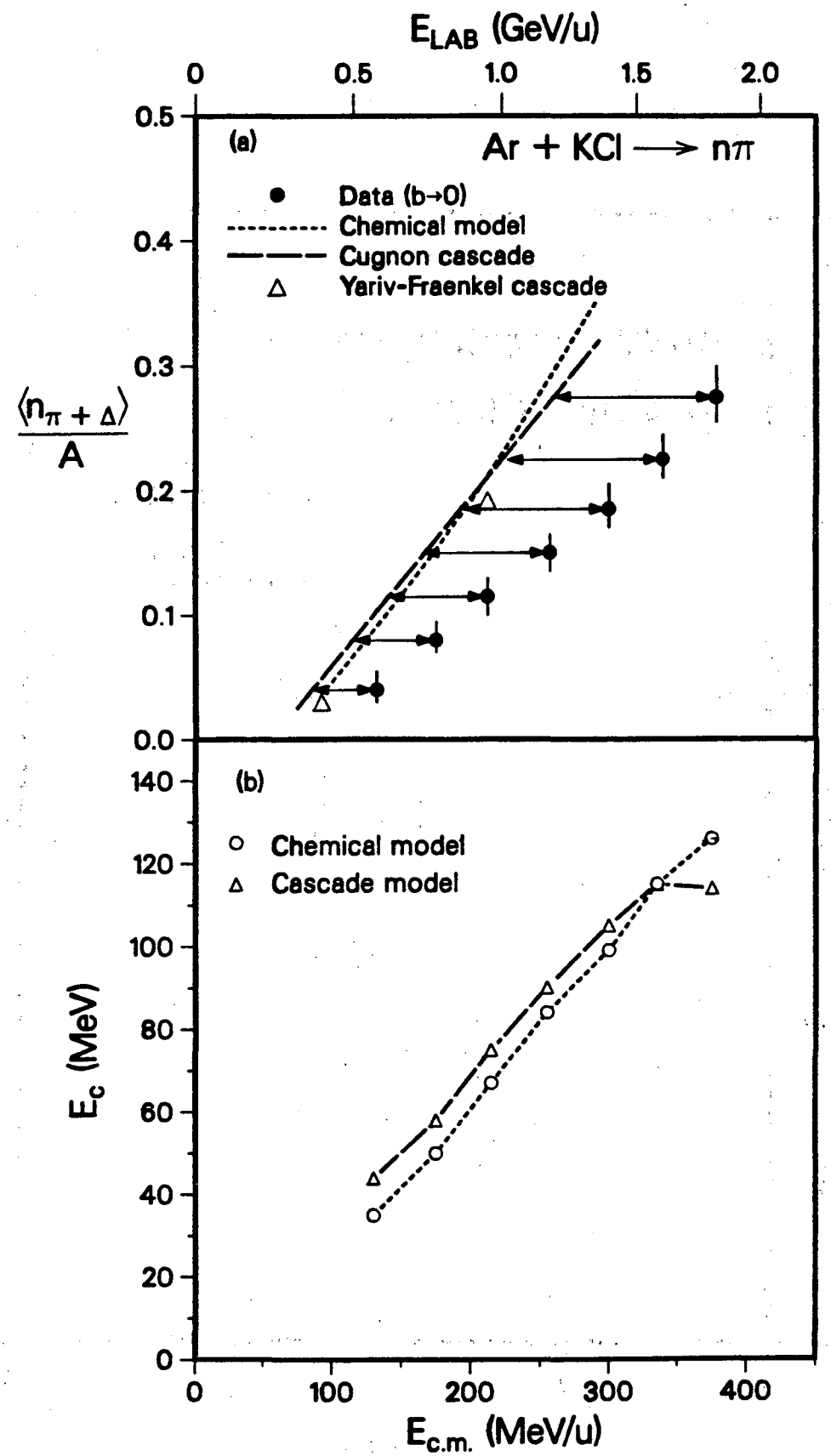

Fig. 2

XCG 8312-4934

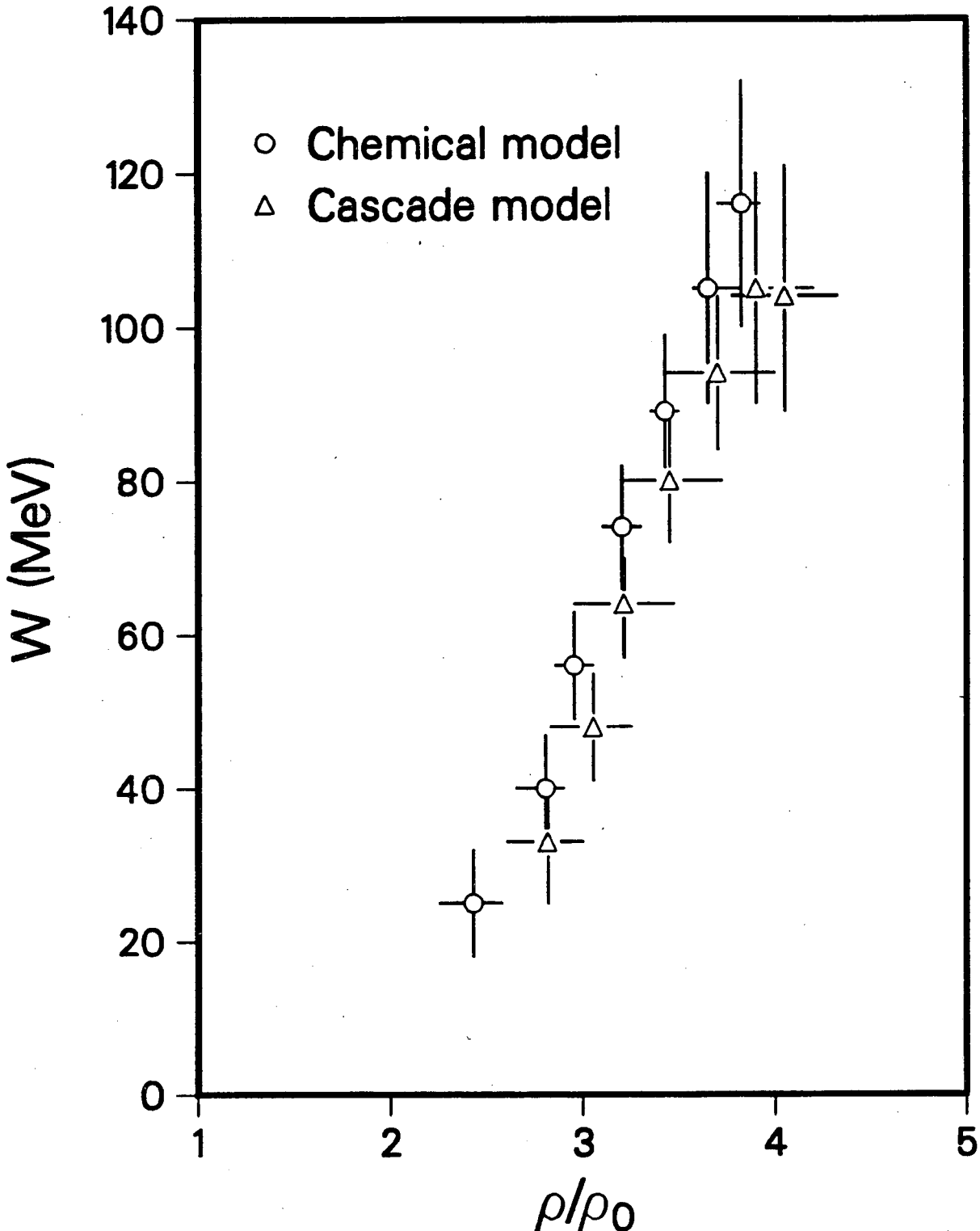

Fig. 3

XCG 8312-4936

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