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DISSIPATIVE RESISTANCE AGAINST CHANGES IN THE MASS ASYMMETRY DEGREE OF FREEDOM IN MACROSCOPIC NUCLEAR DYNAMICS: THE COMPLETED WALL-AND-WINDOW FORMULA

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Dissipative Resistance Against Changes in the Mass Asymmetry Degree of Freedom in Macroscopic Nuclear Dynamics: The Completed Wall-and-Window Formula

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Abstract

The wall-and-window formula, describing the macroscopic energy dissipation for two nuclei in relative motion, is generalized to include the dissipation associated with a time rate of change of the mass asymmetry degree of freedom. The additional term is crucial for the possibility of understanding the existence of deep-inelastic nuclear reactions.

1. Introduction

When an atomic nucleus is idealized as a system of weakly interacting nucleons in a time-dependent mean-field potential well, the nature of the macroscopic dynamics describing the time evolution of the nuclear shape is believed to be intimately related to the nature of the nucleonic motions inside the nucleus: whether ordered or chaotic. A particularly simple limiting form of the macroscopic dynamics -- the Chaotic Regime Dynamics -- results from the assumption of chaotic nucleonic motions (refs. 1-5). This dynamics is characterized by the presence of strong "one-body" dissipative forces associated with an irreversible flow of energy from the collective to the particle degrees of freedom. Simple formulae for the rate of this energy dissipation (considered as a function of the rate of change of the nuclear configuration) can then be derived either when the nuclear shape is everywhere convex (the mononuclear regime) or, in the opposite limit, when there is a pronounced constriction in the shape, dividing the configuration into weakly communicating pieces (e.g. the dinuclear regime). In the former case the rate of collective energy dissipation, $-dE/dt$, is described by the "wall formula", in the latter case by the "wall-and-window formula". (See, for example, ref. 6). In the present paper we describe a generalization of the conventional wall-and-window formula that takes into account the mass transfer between the two parts of a dinuclear system.

2. The wall and the wall-and-window formulae

The wall formula is the following integral extended over the surface specifying the nuclear shape:

$$-\frac{dE}{dt} = \rho v \oint d\sigma(\dot{n} - D)^2 \quad . \tag{1}$$

Here ρ is the mass density of the nucleus, v is the mean speed of the nucleons in the nucleus and $\dot{n}$ is the normal velocity of an element $d\sigma$ of the nuclear surface. The quantity D (a function of position $\vec{r}$ on the surface and a functional of the velocity field $\dot{n}$) is the normal component of a drift velocity impressed on the nucleons about to strike $d\sigma$ by the presence of collective translational and rotational components in the velocity field $\dot{n}$. [In refs. 1, 2 the drift $D[\dot{n},\vec{r}]$ is estimated by assuming a physically plausible two-parameter functional form for its space dependence, and determining the two (vector) parameters by the requirements of conservation of linear and angular momenta.]

The wall-and-window formula for collective energy dissipation in a system consisting of two pieces, 1 and 2, which are in relative motion and which communicate through a small window of area a, may be written as

$$-\frac{dE}{dt} = \rho v \oint_1 d\sigma(\dot{n} - D_1)^2 + \rho v \oint_2 d\sigma(\dot{n} - D_2)^2 + \frac{1}{4}\rho v(2\dot{z}^2 + \dot{x}^2)a \quad . \tag{2}$$

The first two terms (analogous to eq. 1) represent the wall-formula dissipation for the two pieces, each with its own impressed drift D_1 or

D_2. These drifts are determined from the requirement that, apart from the forces acting through the window, the linear and angular momenta of each piece should be conserved (i.e., the requirement that the self-force induced on each piece by the wall-formula dissipation should not contribute to changes of linear and angular momenta). The last term in eq. 2 is the window formula for the energy dissipation associated with the relative motion of the pieces, the motion having velocity components $\dot{z}$ and $\dot{x}$ along and at right angles to the normal through the window a.

In the past several years equations 1 and 2 have been used in a number of applications in the context of nuclear fission and nucleus-nucleus collisions (refs. 1-10), but it is clear on reflection that eq. 2 is incomplete in an important respect: it does not address itself to the degree of freedom describing the relative sizes of the two pieces, i.e., to the mass or volume asymmetry degree of freedom. As we shall show, this incompleteness is readily remedied, and the "completed wall-and-window formula" then reads as follows:

$$-\frac{dE}{dt} = \text{eq. } 2 + \frac{16}{9}\frac{\rho v}{a}\dot{V}_1^2 \quad , \tag{3}$$

where $\dot{V}_1$ is the rate of change of the volume of piece 1, say.

3. The mobility and dissipation coefficients

In order to derive the dissipation coefficient $16\rho v/9a$ in eq.3 we proceed analogously to the derivation of eq. 2. We idealize the nucleus as a gas of nearly independent particles in a container consisting of two pieces with

volumes V_1 and V_2 containing A_1 and A_2 particles, respectively, and communicating through a small window of area a. The total volume is assumed to be approximately constant (in accordance with the approximate incompressibility of nuclei) but the part V_1 is imagined to be changing at a rate $\dot{V}_1$ so that there is a net current of particles, say $\dot{A}_1$, through the window. ($\dot{A}_1$ is equal to $\dot{V}_1$ times the nearly constant particle-number density ν_1.) We assume that, associated with a given configuration and state of motion of the system, there is a macroscopic energy E consisting of potential and kinetic parts. If the kinetic part can be considered as negligible, i.e. if the motion is overdamped (the validity of this assumption will be discussed in section 5) the energy E is just the potential energy (consisting, for example, of a constant volume energy and shape-dependent surface and electrostatic energies). The (negative) rate of change of E will then be equal to the rate of dissipation of energy from collective to particle degrees of freedom. If the expression for this rate of change can be brought to a form where $\dot{V}_1^2$ or $\dot{A}_1^2$ is a factor (see eq.3) then the remaining cofactor will define the desired dissipation coefficient. With this in mind we consider the following identity:

$$-\frac{dE}{dt} = \left(-\frac{dE}{dA_1}\right)\frac{dA_1}{dt} \quad , \tag{4}$$

which displays the quantity $(-dE/dA_1)$, the (generalized) force associated with the mass asymmetry degree of freedom. Now, under fairly general conditions, which would include ordinary viscous hydrodynamics as well as the flow of electric current through a resistor, the current of particles between two systems induced by some driving force is expected to be proportional to

this driving force. As a result we anticipate a proportionality relation of the form

$$\frac{dA_1}{dt} = c\left(-\frac{dE}{dA_1}\right) \quad , \tag{5}$$

where c is a constant. (In the analogy with Ohm's law, $\frac{dA_1}{dt}$ would be the current, $-\frac{dE}{dA_1}$ the potential and c the conductivity*). Substituting in eq. 4 we have

$$-\frac{dE}{dt} = \frac{1}{c}\left(\frac{dA_1}{dt}\right)^2 \quad , \tag{6}$$

so that the sought-for dissipation coefficient (with respect to mass asymmetry) is the inverse of the proportionality coefficient between particle current and driving force -- the mobility coefficient of ref. 11.

4. Estimates of the dissipation coefficient

An estimate of the mobility coefficient has already been given in ref. 11 in connection with a transport treatment of dinuclear reactions. We shall go over this derivation in the present context, provide alternative estimates and discuss some consequences.

*This analogy was pointed out to us by S. Bjørnholm and was also noted by H. Feldmeier in ref. 14.

4.1 The simple Fermi-gas model.

Let the nuclei on the two sides of the window be idealized as two potential wells filled with Fermi gases up to energy levels $-S_1$ and $-S_2$ as measured with respect to the potential at infinity (which is taken as the zero of the energy scale). Thus S_1 is the separation energy for removing to infinity a particle from system 1 (in the presence of system 2). Similarly for the separation energy S_2. The difference between the separation energies, S_1-S_2, is also the difference between the chemical potentials on the two sides of the window and is thus the driving force for the transfer of particles from system 2 to system 1:

$$-\frac{dE}{dA_1} = S_1 - S_2 \quad . \tag{7}$$

Let the particle densities, mean particle speeds (equal to 3/4 of the Fermi velocities), Fermi kinetic energies and potential depths for the Fermi gases adjacent to the two sides of the window be denoted by ν_1, v_1, T_1, U_1 and ν_2, v_2, T_2, U_2, respectively. Then the number of particles entering the window per unit time in the direction from 2 to 1 is $\frac{1}{4}\,\nu_2 v_2 a$ and in the direction from 1 to 2 is $\frac{1}{4}\,\nu_1 v_1 a$ (ref. 11). If there is no discontinuity in the potential at the window (i.e., if $U_1 = U_2$), so that all particles entering the window also get through, the net current of particles into system 1 is

$$\frac{dA_1}{dt} = \frac{1}{4}\,a(\nu_2 v_2 - \nu_1 v_1) \quad . \tag{8}$$

Using eqs. 5, 7 it follows that the mobility coefficient c may be written as

$$c = \frac{\frac{1}{4}\, a(\nu_2 v_2 - \nu_1 v_1)}{S_1 - S_2} \quad . \tag{9}$$

If the difference in the properties of the Fermi gases on the two sides of the window can be regarded as small, we may write, approximately

$$c = -\frac{1}{4}\, a\, \frac{d(\nu v)}{dS} \tag{10}$$

$$\approx \frac{1}{4}\, a\, \frac{d(\nu v)}{dT} \quad . \tag{11}$$

(Eq. 11 follows from eq. 10 if the potential depths are assumed to be velocity-independent, so that $T_2 - T_1 = S_1 - S_2$.)

The derivative of νv with respect to T is readily evaluated by recalling that for a Fermi gas the mean particle speed is proportional to $\nu^{1/3}$ and the Fermi kinetic energy to $\nu^{2/3}$. Hence

$$\frac{d(\nu v)}{dT} = \frac{\nu v}{T}\, \frac{d(\ln \nu v)}{d(\ln T)} = \frac{\nu v}{T}\, \frac{d(\ln \nu^{4/3})}{d(\ln \nu^{2/3})} = \frac{2\nu v}{T} \quad . \tag{12}$$

In the above, T, ν, v may be considered as referring to the mean values of the properties of the Fermi gases on the two sides of the window. It follows that

$$c = \frac{a\nu v}{2T} \tag{13}$$

and eq. 6 becomes

$$-\frac{dE}{dt} = \frac{2T}{a\nu v}\dot{A}_1^{\,2} \quad . \tag{14}$$

Changing the variable A_1 to the volume variable V_1, viz.

$$A_1 = V_1\nu = V_1\rho/m \quad ,$$

where m is the nucleon mass, and using

$$T = \frac{1}{2}\, m\ (\text{Fermi velocity})^2 = \frac{1}{2}\, m(\frac{4}{3}\, v)^2$$

we find

$$-\frac{dE}{dt} = \frac{16}{9}\,\frac{\rho v}{a}\,\dot{V}_1^2 \quad . \tag{15}$$

We should stress that the above dissipation term, eq. 15, is "new" in the context of dynamical descriptions of nuclear reactions employing the one-body wall-and-window formula. The physical content of this term is, however, present in ref. 11, which deals with the mass transport in nuclear collisions. It was shown in that reference that the overdamped motion of the mass asymmetry may be written as

$$\dot{A}_1 = F_A N' \quad ,$$

where F_A, equal to $S_1 - S_2$, is the appropriate driving force and N' is the mobility coefficient. In the Fermi-gas model, underlying both ref. 11 and

the present estimates, N' is the derivative, with respect to the Fermi energy, of the one-sided particle current through the window ($N = \frac{1}{4} a\nu v$). Thus

$$N' = \frac{\partial}{\partial T} N = \frac{2}{T} N = \frac{a\nu v}{2T} \quad .$$

Since the friction coefficient is the inverse of the mobility coefficient, the above expression for N', based on ref. 11, reproduces exactly the present eq. 14. It follows that in all calculations based on ref. 11 the strong hindrance in the mass flow associated with eq. 15 is present.

4.2 Fermi gas in a velocity-dependent potential.

As stressed by Weisskopf in ref. 12, nuclear saturation requires that the potential felt by a particle in a Fermi-gas model of a nucleus should be a (decreasing) function of the particle's kinetic energy. It follows that for the two Fermi gases on the two sides of the window the potential depths U_1 and U_2 felt by the particles at the top of the Fermi sea must be somewhat different, and that $S_1 - S_2 \neq T_2 - T_1$. In this case the net current of particles into nucleus 1 is

$$\frac{dA_1}{dt} = \frac{1}{4} a \frac{d(\nu v)}{dT} (T_2 - T_1)$$

$$= \frac{1}{4} a \frac{d(\nu v)}{dT} \frac{dT}{dS} (S_2 - S_1) \quad , \qquad (16)$$

where the factor dT/dS is related to the effective mass of the nucleons at the

Fermi surface and is not, in general, equal to unity. Thus, if the dependence of the potential depth on kinetic energy t is written as $U(t) = U_0 - kt$, then a particle with the Fermi energy experiences a potential $U_0 - kT$, and the separation energy is given by

$$S = (U_0 - kT) - T \quad , \tag{17}$$

from which it follows that $(-dS/dT) = 1 + k$. On the other hand, the total energy (kinetic plus potential) of a nucleon with momentum p and mass m is

$$t - U(t) = -U_0 + (1 + k)t = -U_0 + (1 + k)\frac{p^2}{2m}$$

$$= -U_0 + \frac{p^2}{2m^*} \quad , \tag{18}$$

where m^* is the effective mass, given by

$$\frac{m^*}{m} = \frac{1}{1 + k} = -\frac{dT}{dS} \quad . \tag{19}$$

It follows that in place of eq. 15 we would have

$$-\frac{dE}{dt} = \frac{16}{9}\frac{\rho v}{a}\frac{m}{m^*}\dot{V}_1^2 \quad . \tag{20}$$

4.3 Compressible Fluid Model

It is instructive to go through an explicit calculation of the mobility coefficient c in eq. 11 using the model of a slightly compressible fluid, according to which the energy of a fictitious uncharged nucleus with A particles and radius R is assumed to be given by a sum of a volume energy (including a compressibility correction) and a surface energy, i.e.

$$E = -c_1 A + \frac{1}{2} AK\left(\frac{R - R_o}{R_o}\right)^2 + 4\pi r^2 \gamma \ . \tag{21}$$

Here c_1 is the volume energy coefficient, K is the compressibility coefficient (of the order of 200 MeV), γ is the specific surface energy and R_o, equal to $r_o A^{1/3}$, is the radius that the nucleus would have at standard nuclear matter density. The actual equilibrium radius of the compressible nucleus is obtained by setting dE/dR equal to zero, which leads to

$$\frac{R}{R_o} = 1 - \frac{2c_2}{K} A^{-1/3} \quad , \tag{22}$$

where c_2 stands for $4\pi r_o \gamma$. Substituting in eq. 21, the optimized binding energy is

$$E(A) = -c_1 A + c_2 A^{2/3} - \frac{2c_2^2}{K} A^{1/3} + \ldots \quad , \tag{23}$$

so that the separation energy is

$$S = -\frac{dE}{dA} = c_1 - \frac{2}{3} c_2 A^{-1/3} + \ldots \quad . \tag{24}$$

The flux factor νv, if assumed to scale, as before, according to the Fermi-gas model (with the inverse fourth power of the radius) would be proportional to

$$\left(1 - \frac{2c_2}{K} A^{-1/3}\right)^{-4} = 1 + \frac{8c_2}{K} A^{-1/3} + \ldots \quad . \tag{25}$$

Consider now two such slightly compressible nuclei, with mass numbers A_1 and A_2, brought into contact. The difference in their separation energies (chemical potentials) is

$$S_1 - S_2 = -\frac{2}{3} c_2 \left(A_1^{-1/3} - A_2^{-1/3}\right) \tag{26}$$

and the difference in the flux factors is

$$\nu_1 v_1 - \nu_2 v_2 = \nu v \frac{8c_2}{K} \left(A_1^{-1/3} - A_2^{-1/2}\right) \quad , \tag{27}$$

where νv is some average between $\nu_1 n_1$ and $\nu_2 v_2$. Substituting in eq. 9 we find

$$c = \frac{a\nu v}{K/3} \quad . \tag{28}$$

This is similar to eq. 13, but with the Fermi energy T replaced by K/6. Eq. 15 would then be replaced by

$$-\frac{dE}{dT} = \frac{16}{9}\frac{\rho v}{a}\left(\frac{K/6}{T}\right)\dot{V}_1^2 \quad . \tag{29}$$

Eq. 15 is recovered if the compressibility of a Fermi gas is assigned the value 6T. It may be verified that this is indeed the value which describes the response of the energy of a Fermi gas of fixed volume to small deviations from a uniform density.

In practice, the factor m/m* in eq. 20, as well as the factor K/6T in eq. 29, are not very different from unity, so that the unrefined eq. 15 may continue to be used for rough estimates.

5. Is the Mass Asymmetry Mode Overdamped?

In this section we shall argue that one is indeed justified in disregarding the kinetic energy associated with the rate of change of the mass asymmetry because any kinetic energy originally present would be dissipated away in a very short time.

Let us estimate the kinetic energy associated with the mass asymmetry degree of freedom by considering the flow of matter through a window of radius r. Let us write the kinetic energy associated with the flow as

$$KE = \frac{1}{2}\,(\text{mass of moving matter})\ u^2$$

$$= \frac{1}{2}\,(\rho\pi r^2 d)u^2 \quad , \tag{30}$$

where u is the average flow velocity across the window and d an effective length specifying the amount of fluid affected by the flow. For example, in the case of an irrotational, incompressible flow through a circular aperture, Rayleigh found d = r (ref. 13). Using this result for the purpose of an order of magnitude estimate, and making the substitution $\dot{V}_1 = ua$, we find

$$KE \approx \frac{1}{2} \frac{\rho r}{a} \dot{V}_1^2 \quad .$$

Combining this with eq. 15 we find for the e-folding time for the dissipation of the kinetic energy the value

$$\frac{KE}{-d(KE)/dt} = \frac{9}{64} \left(\frac{2r}{\nu}\right) \quad . \tag{31}$$

Thus, in the presence of a dissipation in the form of eq. 15, the kinetic energy would become small after a time of the order of about one seventh of the time taken by an average nucleon to traverse the diameter of the window. For small windows, to which the present discussion is directed, this is, indeed, a very short time.

We should stress that the above macroscopic estimate most certainly does not do justice to the intricate processes taking place in the small and rapidly opening window region, whose size is comparable both to the spacing between the nucleons and to their quantal wavelengths. The estimate is meant to be relevant to idealizations that are committed from the beginning to a macroscopic treatment and in which the problem arises as to the relative importance of dissipative and inertial forces. The question to what extent

the dominance by dissipation of the mass transfer process survives in a microscopic, quantal treatment is not answered by these estimates.

6. Summary

We have completed the wall-and-window macroscopic energy dissipation formula by including the dissipation associated with the relative volume changes of the two interacting nuclei in a dinuclear configuration. The relevant dissipation coefficient is made up of the same physical quantities as the rest of the Wall-and-Window formula (i.e., the flux factor ρv and the window area a). The fact that the new dissipation term is inversely proportional to a means that it dominates for small window areas and may be expected to effectively freeze out the asymmetry degree of freedom in dinuclear reactions, a condition essential for the ability of a model of nucleus-nucleus collisions to account for the existence of strongly damped (deep-inelastic) scattering. It has been verified (ref. 15) that without the new term this type of scattering is altogether absent in dynamical model calculations of the type of refs. 7-10, the mass flow between the nuclei being far too rapid. Inclusion of the new term reduces the mass drift drastically and we are in the process of ascertaining the degree of agreement with experiments that may be obtained when the completed wall-and-window formula is used.

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