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QUARK EXCHANGE EFFECTS IN THE THREE NUCLEON SYSTEM

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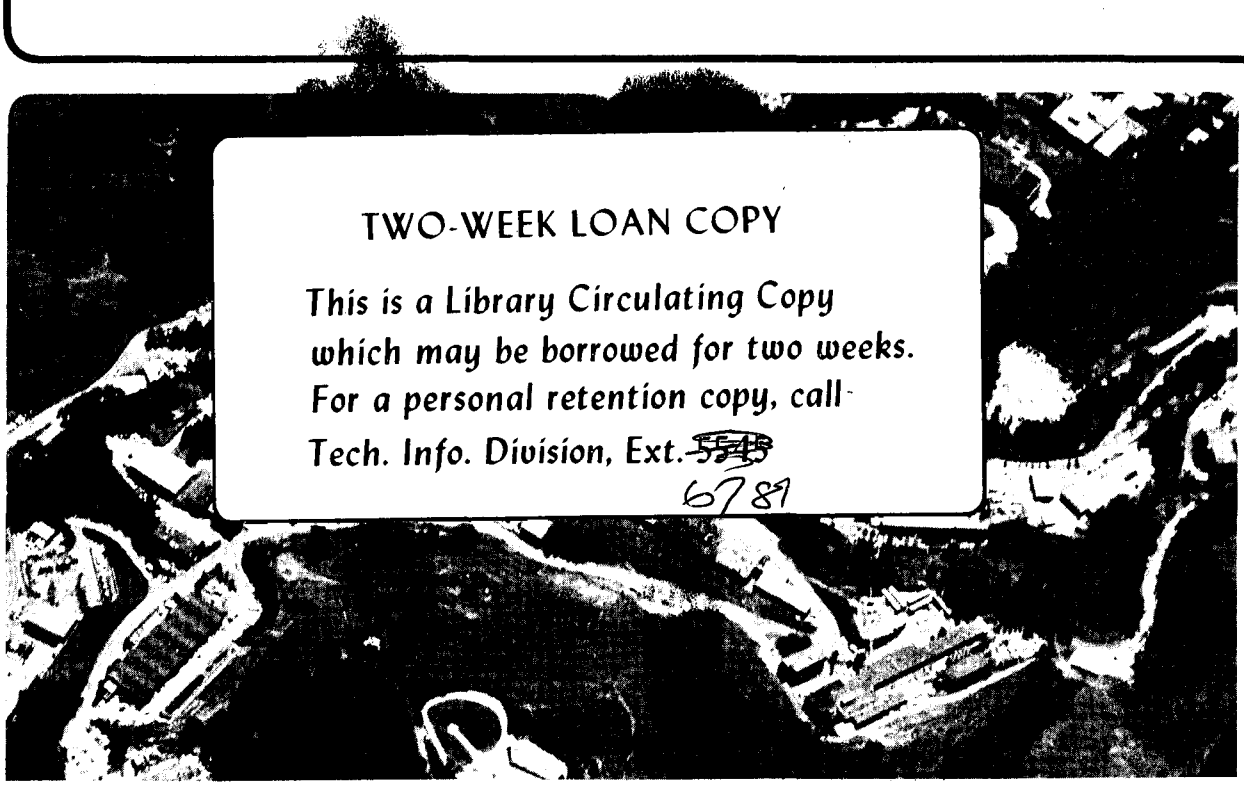

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Quark Exchange Effects in the Three Nucleon System[1]

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ABSTRACT

The quark exchange forces in the three nucleon system are evaluated in the context of a QCD-like potential model. As in the two nucleon system the two-body exchange color hyperfine interaction is found to be strongly repulsive. Additional effects due to the delocalization of quarks over all three nucleons are investigated and found to be considerably larger than the usual nuclear three-body effects generated by two pion exchange.

1. Introduction

In recent years studies of phenomenological models of hadron structure incorporating what are believed to be the important features of QCD have lead to considerable progress in our understanding of the origins of the NN force[1-1]. In this context the repulsive core of the NN potential is seen to arise, via quark exchange effects, from the color magnetic (hyperfine) piece of one gluon exchange between quarks, while the intermediate range attraction usually ascribed to 2π exchange appears to be due largely to the mutual color polarization of the two nucleons[15]. The role of meson exchange remains at least quantitatively unclear. It has been argued that contributions from mesons other than the pion should be negligible unless the quark substructure of hadrons is anomalously small (i.e. small relative to the inverse QCD deconfinement temperature scale $\sim$ physical hadron sizes)[15]. The matter of producing a phenomenological framework in which such exchange effects can be incorporated in a reliable quantitative manner is, however, more problematic, even if the question of the relative magnitudes of the chiral symmetry breaking and confinement scales in QCD is settled, since the two likely scenarios correspond to rather similar pictures of the averaged suppression of the pion field inside the region

of the nucleon occupied by quarks[a]. The basic physics of the NN system, however, seems well established.

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[a] There are two likely scenarios for the relation of the chiral symmetry breaking (χ) and confinement (C) transition temperatures: $T_\chi \sim (3\text{-}5)T_C$, as suggested by Shuryak[16] based on estimates of the vacuum gluon condensate from QCD sum rules, and $T_\chi \sim T_C$, as suggested by some Monte Carlo results[17]. There are some numerological arguments in favor of the former, eg. the pseudoscalar mixing pattern, which is very natural if $T_\chi \gg T_C \sim m_s$ but requires special dynamics if $T_\chi \sim T_C$. In fact such special dynamics appear possible in the bag model. In addition the accuracy of the gluon condensate obtained from QCD sum rules has been the subject of some recent controversy[18], although the effect seems to be to under- rather than over-estimate , so the situation is far from settled. Although the two scenarios correspond to different phenomenological pictures of the pion coupling, the fact that T_χ is at most $\sim 5T_C$ leads to very similar pictures when one averages over the nucleon volume. In the case $T_\chi > T_C$, the presence of a light quark inside the nucleon produces a region of chirally symmetric vacuum of radius $\sim 1/T_\chi$ and since the extent of the current quark substructure of the pion will also be $\sim 1/T_\chi$, the natural scale for the suppression of the pion field in the vicinity of a light quark is a few times $1/T_\chi$. Implementing such a suppression smoothly with $T_\chi \sim 1$ GeV and averaging over the nucleon volume one finds an average field strength suppression rather similar to what one would expect from a surface-coupled cloudy bag picture, softened to take acount of surface fluctuations, which is the natural qualitative realization of the case $T_\chi \sim T_C$.

Given the state of our knowledge of the NN interaction, a study of the three nucleon system is thus of considerable interest, especially viz a vis nuclear structure. For example, while the results of the behavior of the NN system can be cast in the form of an effective potential, the fact that such a potential represents the effects of both exchange interactions and the mixing of additional states means that the sum of two body forces in multi-nucleon systems cannot necessarily be obtained by using the effective potential extracted from the NN system. This is, of course, related to the question of the presence of exotic degrees of freedom in nuclei. Such configurations are present in the deuteron only at rather small levels owing to the repulsive nature of the exchange hyperfine interaction. This repulsion leads to a diffuse distribution for NN component of the deuteron, thus suppressing significant overlap with the hidden color excitations of the system, which, due to confinement, must be well localized in space. The result is a deuteron which is dominantly a standard nuclear system, at least in terms of composition, although the weakly excited internal degrees of freedom play a crucial role in determining the effective nuclear interaction. Other nuclei, however, are considerably more compact than the deuteron and, even assuming one is able to explain their structure within the present phen-

omenological framework, it is of interest to know whether, with typical separations in such nuclei only slightly greater than the confinement scale, the hyperfine repulsion continues to determine the basic physics of the system and whether or not there are significant admixtures of exotic configurations into the nuclear ground state. ^{3}H and ^{3}He, being among the most compact nuclei, are ideal systems in which to study this question. In addition, such tightly bound nuclei may exhibit interesting delocalization effects. Recall that the internal kinetic energy of a nucleon is large on the scale of nuclear binding. One therefore expects the alleviation of this kinetic energy via delocalization to represent potentially non-trivial binding contributions relative to a description of the system in terms of point-like nucleons. Although such an effect is present, in principle, in the deuteron, the smallness of the spatial exchange integrals means that permutationally distinct configurations of the system do not appreciably interfere and the resulting effect is, in consequence, negligible. One final property of the three nucleon system of interest with regard to nuclear structure is the possible existence of three body nuclear forces induced by quark-quark interactions. It has often been suggested that three body forces might be required to explain the binding of ^{3}H, ^{3}He [19], although the dynamical nature of the NN force has not been taken into account in arriving at these conclusions. Nonetheless, there are sources

for such an interaction and, in dense nuclear matter, contributions arising from quark exchange may well be important.

In this paper we investigate quark exchange and delocalization contributions to the energy of the three nucleon system. We show that the exchange hyperfine interaction remains strongly repulsive at short distances, and that both delocalization effects in the kinetic energy and three body forces arising from quark exchange are significant on the scale of nuclear binding. Effects due to pion exchange and admixtures of non-nucleonic configurations into the three nucleon ground state, which are presumably responsible for the binding of the system, are not considered here, but will be the subject of future work.

2. The Model

Calculations have been performed in a QCD-inspired potential model which has previously been applied extensively, and with considerable success, to baryon spectroscopy[20] and decays[21] as well as to the NN problem[15]. In the context of the NN interaction the model is discussed more fully in Ref. 15). The Hamiltonian for the three nucleon system is

$$H = \sum_{i=1}^{9} \left(m_i + p_i^2/2m_i \right) + \sum_{i<j=1}^{9} \left(H_{conf}^{ij} + H_{hyp}^{ij} \right) \qquad (1)$$

where, with $\underset{\sim}{r}_{ij} = \underset{\sim}{r}_i - \underset{\sim}{r}_j$ and

$$\underset{\sim}{S}_{ij} = 3\underset{\sim}{S}_i \cdot \underset{\sim}{r}_{ij} \underset{\sim}{S}_j \cdot \underset{\sim}{r}_{ij} / r_{ij}^{\ 2} - \underset{\sim}{S}_i \cdot \underset{\sim}{S}_j$$

the two body confining potential H^{ij}_{conf} is given by

$$H^{ij}_{conf} = -\left(e_0 + \tfrac{1}{2} k r_{ij}^2 + U(\underset{\sim}{r}_{ij})\right)\left(\tfrac{1}{2}\underset{\sim}{\lambda}_i\right)\cdot\left(\tfrac{1}{2}\underset{\sim}{\lambda}_j\right) \qquad (2)$$

and the two body color hyperfine interaction, H^{ij}_{hyp}, expected from one gluon exchange, by

$$H^{ij}_{hyp} = -\left(\frac{\alpha_s}{m_i m_j}\right)\left(\frac{8\pi}{3}\underset{\sim}{S}_i \cdot \underset{\sim}{S}_j\, \delta^3(\underset{\sim}{r}_{ij}) + \underset{\sim}{S}_{ij}\, r_{ij}^{-3}\right)\left(\tfrac{1}{2}\underset{\sim}{\lambda}_i\right)\cdot\left(\tfrac{1}{2}\underset{\sim}{\lambda}_j\right) \qquad (3)$$

The anharmonicity, $U(r_{ij})$, in (2) is meant to represent departures from the harmonic limit, including the attractive short range Coulomb interaction of QCD[a]. The parameters of the model are all determined from the work on baryon spectroscopy and are as given in Ref. 15).

3. The Three Nucleon State

Let us denote by (123;456,789) the normalized but not yet fully antisymmetrized state of three nucleons, having total

---

[a] As if Ref. 15), $U(r_{ij})$ is taken to be a δ function for ease of calculation. Smearing over clusters reduces the sensitivity to the actual functional form chosen.

isospin ($I=1/2, I_z$) and total spin ($S=1/2, S_z=+1/2$) in which nucleon 1 contains quarks 1,2 and 3, nucleon 2 quarks 4,5 and 6 and nucleon 3 quarks 7,8 and 9. In order to simplify matters we deal with only those terms which survive complete antisymmetrization by choosing the state (123;456;789) to be antisymmetric with respect to the interchange of any pair of its constituent nucleons. Denote by A_{ijk} the usual color singlet combination of three quarks

$$A_{ijk} = \frac{1}{\sqrt{6}} \varepsilon_{\alpha\beta\gamma} | q_i^\alpha q_j^\beta q_k^\gamma \rangle \tag{4}$$

and by $\Psi_{ijk;\ell mn;rst}$ the spatial wavefunction of three nucleons (ijk), (ℓmn), (rst) in a relative wavefunction ψ. One has

$$\Psi_{ijk;\ell mn;rst} = \phi(ijk)\phi(\ell mn)\phi(rst)\psi(\underset{\sim}{R}_{ijk;\ell mn;rst}, \underset{\sim}{L}_{ijk;\ell mn;rst}) \tag{5}$$

where $\phi(ijk)$ is the internal spatial wavefunction of the nucleon (ijk) [20]

$$\phi(ijk) = \frac{\alpha^3}{\pi^{3/2}} \exp\left(-\alpha^2(\rho_{ijk}{}^2 + \lambda_{ijk}{}^2)/2\right) \tag{6}$$

with

$$\begin{aligned} \underset{\sim}{\rho}_{ijk} &= (\underset{\sim}{r}_i - \underset{\sim}{r}_j)/\sqrt{2} \\ \underset{\sim}{\lambda}_{ijk} &= (2\underset{\sim}{r}_k - \underset{\sim}{r}_i - \underset{\sim}{r}_j)/\sqrt{6} \end{aligned} \tag{7}$$

and the relative wavefunction ψ is expressed in terms of the usual Jacobi coordinates

$$\underset{\sim}{R}_{ijk;\ell mn;rst} = ((\underset{\sim}{r}_i+\underset{\sim}{r}_j+\underset{\sim}{r}_k)/3 - (\underset{\sim}{r}_\ell+\underset{\sim}{r}_m+\underset{\sim}{r}_n)/3)/\sqrt{2}$$

$$\underset{\sim}{L}_{ijk;\ell mn;rst} = (2(\underset{\sim}{r}_r+\underset{\sim}{r}_s+\underset{\sim}{r}_t)/3 - (\underset{\sim}{r}_i+\underset{\sim}{r}_j+\underset{\sim}{r}_k)/3 - (\underset{\sim}{r}_\ell+\underset{\sim}{r}_m+\underset{\sim}{r}_n)/3)/\sqrt{6}. \tag{8}$$

Then, taking $I_z = +1/2$,

$$(123;456;789) = \frac{1}{\sqrt{6}}\Psi_{123;456;789}A_{123}A_{456}A_{789}(p\uparrow p\downarrow n\uparrow - p\downarrow p\uparrow n\uparrow + p\downarrow n\uparrow p\uparrow - p\uparrow n\uparrow p\downarrow + n\uparrow p\uparrow p\downarrow - n\uparrow p\downarrow p\uparrow)_{123;456;789} \tag{9}$$

where

$$(p\uparrow p\downarrow n\uparrow)_{123;456;789} = (p\uparrow)_{123}(p\downarrow)_{456}(n\uparrow)_{789} \tag{10}$$

and

$$(p\uparrow)_{123} = (p^\rho\chi^\rho\uparrow + p^\lambda\chi^\lambda\uparrow)_{123}/\sqrt{2} \quad \text{(etc.)} \tag{11}$$

is the spin-isospin wavefunction for a spin up proton (etc.). The labels ρ,λ refer to transformation properties under the permutation group S_3[a]. The states p^ρ, p^λ, $\chi^\rho\uparrow$ and $\chi^\lambda\uparrow$ are given by

---

a) Denoting the 2-cycles (ij) of S_3 by π_{ij}, the states ρ, λ, which form a basis for the mixed representation of S_3, transform according to

$$\pi_{12}\rho = -\rho \qquad \pi_{12}\lambda = \lambda$$

$$\pi_{13}\rho = \tfrac{1}{2}\rho + \tfrac{\sqrt{3}}{2}\lambda \qquad \pi_{13}\lambda = \tfrac{\sqrt{3}}{2}\rho - \tfrac{1}{2}\lambda$$

$$
\begin{aligned}
p^{\rho}_{123} &= (u_1d_2-d_1u_2)u_3/\sqrt{2} \\
p^{\lambda}_{123} &= (u_1d_2u_3+d_1u_2u_3-2u_1u_2d_3)/\sqrt{6} \\
\chi^{\rho}_{1\uparrow 23} &= (\uparrow_1\downarrow_2-\downarrow_1\uparrow_2)\uparrow_3/\sqrt{2} \\
\chi^{\lambda}_{1\uparrow 23} &= (\uparrow_1\downarrow_2\uparrow_3+\downarrow_1\uparrow_2\uparrow_3-2\uparrow_1\uparrow_2\downarrow_3)/\sqrt{6}
\end{aligned}
\tag{12}
$$

Those involving n and/or $\downarrow$ may be obtained from (12) by spin or isospin lowering. The spatial wavefunctions ϕ, ψ are normalized with respect to the measures $d^3\rho\, d^3\lambda$ and $d^3R d^3L$ respectively.

The fully antisymmetrized state of three nucleons may now be obtained by applying the nine-quark antisymmetrizer to (123;456;789) and normalizing. The resulting expression consists of 280 terms, representing the 280 distinct ways of arranging 9 quarks into three identical nucleons. Using the available partial antisymmetries of (ijk;ℓmn;rst) one can write the expectation value of H in the resulting state as

$$
\begin{aligned}
\langle(123;456;789)|\, H\{&|(123;456;789)\rangle - 27\,|(126;453;789)\rangle + \\
&54\,|(129;453;786)\rangle + 162\,|(169;452;783)\rangle - \\
&36\,|(483;159;726)\rangle\}/N^2
\end{aligned}
\tag{13}
$$

where

$$
N^2 = 1 - 27C^{(1)} + 54C_c^{(2)} + 162C_y^{(2)} - 36C^{(3)} \tag{14}
$$

with

$$
\begin{aligned}
c^{(1)} &= \langle(123;456;789)|(126;453;789)\rangle \\
c_c^{(2)} &= \langle(123;456;789)|(129;453;786)\rangle \\
c_y^{(2)} &= \langle(123;456;789)|(169;452;783)\rangle \\
c^{(3)} &= \langle(123;456;789)|(483;159;726)\rangle.
\end{aligned}
\tag{15}
$$

4. Evaluating the Energy of the Three Nucleon State

Before evaluating (13) it is instructive to examine the origin of the terms contained therein. In what follows let i,j be particle labels. Then, owing to the color dependence of the two body operators, H^{ij}, in (1), the expectation $\langle(123;456;789)|H^{ij}|(123;456;789)\rangle$ vanishes unless i,j belong simultaneously to either $\{1,2,3\}$, $\{4,5,6\}$ or $\{7,8,9\}$. Similarly $\langle(123;456;789)|H^{ij}|(126;453;789)\rangle$ is zero unless i,j are both elements of either $\{1,2,3,4,5,6\}$ or $\{7,8,9\}$. As a result the two body part of the first two terms of (13) can be recast as

$$
\begin{aligned}
\langle(123;456;789)|\, 3\Big(\sum_{i<j=1}^{3} H^{ij}\Big\{ &|(123;456;789)\rangle - 9|(123;459;786)\rangle\Big\} \\
&-9\sum_{i<j=1}^{6} H^{ij}|(126;453;789)\rangle\Big)/N^2
\end{aligned}
\tag{16}
$$

where we have taken advantage of the symmetries under relabelling to obtain this expression. We immediately recognize the last term in (16) as the sum of the two nucleon exchange interactions.

The interpretation of the first two terms is less clear. From (15) we see that they can be combined in the form

$$\langle(123;456;789)|3\sum_{i<j=1}^{3} H^{ij}(1-9C^{(1)})|(123;456;789)\rangle/N^2 \qquad (17)$$

Then, since, as we will see below, $C^{(1)}<0$, $C^{(3)}=0$ and $|54C_c^{(2)}+162C_y^{(2)}|\ll 27C^{(1)}$, and since the net contribution of the quark-quark forces to the energy of the nucleon is negative, the effect of these terms is to destabilize the bound three nucleon system relative to the isolated three nucleon state. Note that one need not, in general, have $C^{(1)}<0$ so this destabilization is a characteristic property of the three nucleon system. Note also that the second term in braces in (16), which has been evaluated in terms of direct matrix elements in order to arrive at (17), is a genuine three-body effect. However, since it does not represent what is usually referred to as an exchange interaction we will distinguish if from the remaining terms in (13), which produce genuine three body exchange forces, in what follows.

In order to evaluate (13) we require the matrix elements, in the relevant sectors, of the spin, space and color operators appearing in (1), (2) and (3). The calculation of the spin and color matrix elements is straightforward. The results are presented in the Appendix. In order to complete the evaluation,

however, we must choose a form for the three nucleon relative wavefunction, Ψ, in (5). Lacking a full dynamical calculation one is unable to solve for Ψ variationally and so we will follow common practice and choose

$$\Psi(\underset{\sim}{R},\underset{\sim}{L}) = \frac{\beta^3}{\pi^{3/2}} \exp\left(-\beta^2(R^2+L^2)/2\right) \tag{18}$$

As we will see, the two body exchange hyperfine interaction is repulsive and of relatively short range, which suggests that it might, in fact, be appropriate to build pairwise anti-correlations into (18). Such anti-correlations, while favored by the residual hyperfine forces, will be disfavored on the grounds of kinetic energy, especially in tightly bound systems, but lacking an understanding of the binding mechanisms at work we are, at present, unable to say anything about the balance between these effects. Nonetheless, in an average sense, (18) is a reasonable ansatz and allows us, by varying β, to investigate the relative importance of various contributions to the three nucleon energy, and how these contributions vary with the size of the system. In understanding the implications of our results for the A=3 nuclei we will be guided by the charge radius of our nine quark system. As in the case of the nucleon this should be somewhat smaller that the physical charge radius owing to the existence of a peripheral pion cloud. The charge radius of the quark

distribution is readily obtained by substituting $\sum_i Q_i r_i^2 / \sum_i Q_i$ for H in (13), where Q_i is the charge operator for the ith quark. Details are given in the Appendix. The physical charge radii of ^3H, ^3He (1.69 fm[22] and 1.93 fm[23], respectively) correspond to $\beta \sim 128$ MeV, 108 MeV respectively. Values somewhat larger than this, probably by of order 10-20 MeV, will be appropriate for the quark distributions of these systems. The information necessary to calculate the spatial matrix elements required for an evaluation of either the charge radius or energy expectation in a state with relative wavefunction ψ as in (18) are given in the Appendix.

5. Results

The results are presented in Figures 1-4. Figure 1 shows the effect of quark delocalization on the total kinetic energy. Plotted is the delocalization energy as a function of β, where, by delocalization energy we mean the amount by which the kinetic energy is lowered relative to its value in a state with the same spatial inter-nucleon wavefunction but no anti-symmetrization of the quarks in different nucleons[a]. The in-

---

[a] As usual there is an ambiguity regarding the mass parameter for the relative motion of the three quark clusters. The naturally occuring parameter, which follows from (1), is $3m_q$, or greater by about 5% than the nucleon mass. In order to make the comparison of delocalization effects consistent we have evaluated the effectively-point-like-nucleons relative kinetic energy using $3m_q$ rather than M_N.

ternal nucleon kinetic energy in this case is taken to be that of an isolated nucleon. We see clear evidence for the softening of the momentum distribution of the quarks in the three nucleon system relative to that of isolated nucleons. Similar effects, expected in heavy nuclei, have, of course, been suggested as one of the sources of the EMC effect[24]. In ^{3}H and ^{3}He we see that this effect corresponds to a binding contribution of order 5–8 MeV relative to what one would expect for a nuclear system consisting of effectively point-like nucleons. In Figure 2 we plot the contributions of the non-exchange forces produced by the two body pieces of H to the energy of the system. As discussed previously the pseudo-three-body contribution is attractive, but, when, combined with the decrease in magnitude of the direct contributions due to normalization, the net effect is anti-binding. One might also interpret this as a result of delocalization, especially in light of the fact that the bulk of the contribution comes from the short ranged hyperfine and U pieces of H. However, certain $\Delta\Delta$ channels have two body normalization coeffieients $C^{(1)} > 0$, so such an interpretation should be viewed with some caution. Nonetheless, we see, over the range of values of β appropriate to the ^{3}H, ^{3}He systems, a net anti-binding effect of order 10-20 MeV, again significant on a scale of the corresponding nuclear binding energies.

Figure 3 displays the effects of the two body exchange forces. The repulsive nature of the exchange hyperfine interaction is clearly in evidence, confirming the persistence of this feature of NN interactions in going from the deuteron to multi-nucleon systems. Although we have yet to investigate the structure of mixing and mesonic effects which presumably produce the net binding of the system, this result gives us reasonable hope that, while there may be non-trivial admixtures of unusual configurations into the ground state of the system, the resulting state, not only here, but in heavier nuclei as well, will not be radically exotic in composition. Finally, in Figure 4, we present the contributions of the three body exchange forces to the energy of the system. Note that both the hyperfine and confinement interactions produce repulsive effects, the net anti-binding contribution being of order 1-3 MeV for the ^3H, ^3He systems. Note also that the effect is a short range one, dying away rapidly to zero as β is decreased. Combining the results we see that the net effect of non-two-body-exchange forces in the three nucleon system is anti-binding and of magnitude 5-10 MeV. Such effects, arising from the quark substructure of nucleons and not traditionally included in nuclear physics, are therefore likely to be of importance in understanding nuclear structure unless one is prepared to accept a value of considerably less that the .6 fm represented by the potential model value of α in (6) for the size of the quark substructure of the nucleon.

APPENDIX

For compactness let us write the matrix elements of H^{ij} between $\langle(123;456;789)|$ and $|(123;456;789)\rangle$, $|(126;453;789)\rangle$, $|(129;453;786)\rangle$, $|(169;452;783)\rangle$ and $|(483;159;726)\rangle$ by $(ij)_D$, $(ij)_S$, $(ij)_C$, $(ij)_Y$ and $(ij)_T$, respectively. Where there is no danger of confusion we will, in addition, drop the subscripts. Then, using the permutational symmetries of $(ijk;\ell mn;rst)$, one can easily obtain the following relations between the different (ij). Any matrix elements not shown vanish from color considerations.

$$D:\quad (12)=(13)=(23)=(45)=(46)=(56)=(78)=(79)=(89) \tag{A1}$$

$$\begin{aligned} S:\quad & (13)=(23)=(16)=(26)=(34)=(35)=(46)=(56) \\ & (14)=(15)=(24)=(25) \\ & (78)=(79)=(89) \\ & (12)=(45) \\ & (36) \end{aligned} \tag{A2}$$

$$\begin{aligned} C:\quad & (14)=(15)=(24)=(25)=(17)=(18)=(27)=(28)=(47)=(48) \\ & \qquad =(57)=(58) \\ & (13)=(23)=(46)=(56)=(79)=(89) \\ & (19)=(29)=(34)=(35)=(67)=(68) \\ & (16)=(26)=(37)=(38)=(49)=(59) \\ & (12)=(45)=(78) \end{aligned} \tag{A3}$$

$$
\begin{aligned}
Y:\quad & (24)=(25)=(37)=(38)=(46)=(56)=(79)=(89)\\
& (27)=(28)=(34)=(35)=(49)=(59)=(67)=(68)\\
& (12)=(13)=(16)=(19)\\
& (14)=(15)=(17)=(18)\\
& (47)=(48)=(57)=(58)\\
& (23)=(69)\\
& (26)=(39)\\
& (29)=(36)\\
& (45)=(78)
\end{aligned}
\tag{A4}
$$

$$
\begin{aligned}
T:\quad & (14)=(16)=(17)=(18)=(24)=(25)=(28)=(29)=(35)=(36)\\
& \quad =(37)=(39)=(47)=(49)=(57)=(58)=(68)=(69)\\
& (12)=(13)=(23)=(45)=(46)=(56)=(78)=(79)=(89)\\
& (15)=(19)=(26)=(27)=(34)=(38)=(48)=(59)=(67)
\end{aligned}
\tag{A5}
$$

One can use the total symmetry of the nucleon spin-isospin wavefunction to obtain the additional relations

$$
\begin{aligned}
(12)_T &= 0\\
(15)_T &= 0
\end{aligned}
\tag{A6}
$$

All matrix elements occuring in (13) can be written as a product of matrix elements in the color, space and joint spin-isospin sectors. Given (4), (9), (11), (12) those involving color or spin-isospin are straightforward to compute. The results are given in Table A1. We will also require the expectations of the charge operators Q_i in order to calculate the charge raduis of the three nucleon state. Denoting the expectation values of one

body operators by $(i)_D$ etc. and noting that the T expectation of Q_i vanishes due to zero color overlap the following relations are useful.

$$
\begin{aligned}
&D: \quad (1)=(2)=(3)=(4)=(5)=(6)=(7)=(8)=(9) \\
&S: \quad (1)=(2)=(4)=(5) \\
&\qquad (7)=(8)=(9) \\
&\qquad (3)=(6) \\
&C: \quad (1)=(2)=(4)=(5)=(7)=(8) \\
&\qquad (3)=(6)=(9) \\
&Y: \quad (2)=(3)=(6)=(9) \\
&\qquad (4)=(5)=(7)=(8) \\
&\qquad (1)
\end{aligned}
\tag{A7}
$$

They again follow from permutational symmetries. The spin-isospin expectations of the Q_i are given in Table A2.

In order to evaluate spatial matrix elements it is convenient to choose the natural internal coordinates of the configuration (123;456;789), namely $\boldsymbol{\rho}_{123}$, $\boldsymbol{\lambda}_{123}$, $\boldsymbol{\rho}_{456}$, $\boldsymbol{\lambda}_{456}$, $\boldsymbol{\rho}_{789}$, $\boldsymbol{\lambda}_{789}$, $\mathbf{R}_{123;456;789}$, $\mathbf{L}_{123;456;789}$. In terms of these coordinates, dropping the subscripts on R, L for concision, one has, in the CM system

$$
\begin{aligned}
\underset{\sim}{r}_1 &= -\underset{\sim}{L}/\sqrt{6} + \underset{\sim}{R}/\sqrt{2} - \underset{\sim}{\lambda}_{123}/\sqrt{6} + \underset{\sim}{\rho}_{123}/\sqrt{2} \\
\underset{\sim}{r}_2 &= -\underset{\sim}{L}/\sqrt{6} + \underset{\sim}{R}/\sqrt{2} - \underset{\sim}{\lambda}_{123}/\sqrt{6} - \underset{\sim}{\rho}_{123}/\sqrt{2} \\
\underset{\sim}{r}_3 &= -\underset{\sim}{L}/\sqrt{6} + \underset{\sim}{R}/\sqrt{2} + 2\underset{\sim}{\lambda}_{123}/\sqrt{6} \\
\underset{\sim}{r}_4 &= -\underset{\sim}{L}/\sqrt{6} - \underset{\sim}{R}/\sqrt{2} - \underset{\sim}{\lambda}_{456}/\sqrt{6} + \underset{\sim}{\rho}_{456}/\sqrt{2} \\
\underset{\sim}{r}_5 &= -\underset{\sim}{L}/\sqrt{6} - \underset{\sim}{R}/\sqrt{2} - \underset{\sim}{\lambda}_{456}/\sqrt{6} - \underset{\sim}{\rho}_{456}/\sqrt{2} \\
\underset{\sim}{r}_6 &= -\underset{\sim}{L}/\sqrt{6} - \underset{\sim}{R}/\sqrt{2} + 2\underset{\sim}{\lambda}_{456}/\sqrt{6} \\
\underset{\sim}{r}_7 &= 2\underset{\sim}{L}/\sqrt{6} - \underset{\sim}{\lambda}_{789}/\sqrt{6} + \underset{\sim}{\rho}_{789}/\sqrt{2} \\
\underset{\sim}{r}_8 &= 2\underset{\sim}{L}/\sqrt{6} - \underset{\sim}{\lambda}_{789}/\sqrt{6} - \underset{\sim}{\rho}_{789}/\sqrt{2} \\
\underset{\sim}{r}_9 &= 2\underset{\sim}{L}/\sqrt{6} + 2\underset{\sim}{\lambda}_{789}/\sqrt{6}.
\end{aligned}
\tag{A8}
$$

(A8) allows us to construct the $\underset{\sim}{\rho}_{ijk}$ etc. for any configuration (ijk;ℓmn;rst) in terms of those for (123;456;789). From (A8) we also obtain

$$
\sum_i \left(-\nabla_i^2/2m\right) = -\left(\nabla_R^2+\nabla_L^2\right)/2(3m) - \left(\nabla^2_{\rho_{123}} + \nabla^2_{\lambda_{123}} + \nabla^2_{\rho_{456}} + \nabla^2_{\lambda_{456}} + \nabla^2_{\rho_{789}} + \nabla^2_{\lambda_{789}}\right)/2m \ . \tag{A9}
$$

Acting on $\Psi_{123;456;789}$ with (A9) produces a factor which is quadratic in the 123;456;789 coordinates. The expressions for the spatial expectations of r_i^2, r_{ij}^2 and the kinetic energy are then readily obtained by writing r_i^2, r_{ij}^2 and the result of acting with (A9) in terms of coordinates which diagonalize the exponents of $\Psi^*_{123;456;789}\Psi_{ijk;\ell mn;rst}$, where ijk;$\ell$mn;rst run over

D, S, C, Y, T. An appropriate set of coordinates is presented below for each case. The notation is $\underset{\sim}{R}=\underset{\sim}{R}_{123;456;789}$ and $\underset{\sim}{L}=\underset{\sim}{L}_{123;456;789}$ and the coordinates are chosen so as to have unit Jacobean with respect the the natural coordinates of the (123;456;789) configuration.

D: $\underset{\sim}{\ell}_{123}, \underset{\sim}{\lambda}_{123}, \underset{\sim}{\ell}_{456}, \underset{\sim}{\lambda}_{456}, \underset{\sim}{\ell}_{789}, \underset{\sim}{\lambda}_{789}, \underset{\sim}{R}, \underset{\sim}{L}$

S: $\underset{\sim}{\ell}_{123}, \underset{\sim}{\ell}_{456}, \underset{\sim}{\ell}_{789}, \underset{\sim}{\lambda}_{789}, \underset{\sim}{L}, \underset{\sim}{\lambda}_a, \underset{\sim}{\lambda}_b, \underset{\sim}{R}$

where

$$\underset{\sim}{\lambda}_b = (\underset{\sim}{\lambda}_{123}+\underset{\sim}{\lambda}_{456})/2$$
$$\underset{\sim}{\lambda}_a = \underset{\sim}{\lambda}_{123}-\underset{\sim}{\lambda}_{456}+ 2\sqrt{3}(3\alpha^2-\beta^2)\underset{\sim}{R}/(15\alpha^2+4\beta^2)$$

C: $\underset{\sim}{\ell}_{123}, \underset{\sim}{\ell}_{456}, \underset{\sim}{\ell}_{789}, \underset{\sim}{R}, \underset{\sim}{L}, \underset{\sim}{\lambda}_a, \underset{\sim}{\lambda}_b, \underset{\sim}{\lambda}_c$

where

$$\underset{\sim}{\lambda}_a = (\underset{\sim}{\lambda}_{123}+\underset{\sim}{\lambda}_{456}+\underset{\sim}{\lambda}_{789})/3 \tag{A10}$$
$$\underset{\sim}{\lambda}_b = \underset{\sim}{\lambda}_{123}-\underset{\sim}{\lambda}_{789}+\sqrt{3}(3\alpha^2-\beta^2)\underset{\sim}{R}/(6\alpha^2+\beta^2)$$
$$\underset{\sim}{\lambda}_c = \underset{\sim}{\lambda}_{456}-\underset{\sim}{\lambda}_{123}/2-\underset{\sim}{\lambda}_{789}/2-3(3\alpha^2-\beta^2)\underset{\sim}{L}/2(6\alpha^2+\beta^2)$$

Y: $\underset{\sim}{\ell}_{231}, \underset{\sim}{\ell}_{456}, \underset{\sim}{\ell}_{789}, \underset{\sim}{L}, \underset{\sim}{\lambda}_a, \underset{\sim}{\lambda}_b, \underset{\sim}{\lambda}_c, \underset{\sim}{R}'$

where

$$\underset{\sim}{\lambda}_a = (\underset{\sim}{\lambda}_{231}+\underset{\sim}{\lambda}_{456}+\underset{\sim}{\lambda}_{789})/3$$
$$\underset{\sim}{\lambda}_b = \underset{\sim}{\lambda}_{456}-\underset{\sim}{\lambda}_{789}+\sqrt{3}(9\alpha^4-\beta^4)\underset{\sim}{\ell}_{231}/(36\alpha^4+39\alpha^2\beta^2+\beta^4)$$

$$\underset{\sim}{\lambda}_c = \underset{\sim}{\lambda}_{231} - \underset{\sim}{\lambda}_{456}/2 - \underset{\sim}{\lambda}_{789}/2$$

$$\underset{\sim}{R}' = \underset{\sim}{R} + 2(3\alpha^2 - \beta^2)\underset{\sim}{\lambda}_b/\sqrt{3}\,(15\alpha^2 + 13\beta^2)$$
$$-6\alpha^2(3\alpha^2 - \beta^2)\underset{\sim}{\rho}_{123}/(36\alpha^4 + 78\alpha^2\beta^2 + 5\beta^4)$$

A common formulation of this type is not possible for the matrix elements $\langle \delta^3(r_{ij}) \rangle_{D,S,C,Y,T}$. However, since Gaussian integrals can be expressed in terms of the determinants of the quadratic forms occurring in their exponents, these quantities are readily obtained by, for example, expressing the arguments of the δ-functions and the D, S, C, Y, and T exponents in terms of the (123;456;789) coordinate basis. Since the resulting expressions are cumbersome and not particularly enlightening we do not record them here.

Table A1 : Color and Spin-Isospin Matrix Elements[a)]

σ	i,j	$(ij)_\sigma^{col}$	$(ij)_\sigma^{IS}$
D	1,2	-2/3	-1/4
	0	1	1
S	1,2	-2/9	17/108
	1,3	-2/9	-7/108
	1,4	1/9	1/108
	3,6	4/9	25/108
	7,8	-2/9	1/108
	0	1/3	-4/108
C	1,2	-2/27	-43/972
	1,3	-2/27	5/972
	1,4	1/27	3/972
	1,6	-2/27	-11/972
	1,9	-2/27	5/972
	3,6	4/27	-11/972
	0	1/9	44/972

Table A1 (continued)

Y	1,2	-2/27	13/972
	1,4	1/27	-9/972
	2,3	-2/27	-11/972
	3,4	1/27	5/972
	3,6	-2/27	5/972
	3,9	4/27	-7/972
	4,5	-2/27	13/972
	4,6	-2/27	1/972
	4,7	-1/54	-1/972
	0	1/9	-20/972
T	1,4	1/18	-1/162
	0	0	0

a) The labels, σ, are as defined in the text. The matrix elements $(ij)^{col}$ and $(ij)^{IS}$ are the color matrix elements of the operator $(\underset{\sim}{\lambda}_i/2).(\underset{\sim}{\lambda}_j/2)$, and the spin-isospin matrix elements of the operator $\underset{\sim}{S}_i.\underset{\sim}{S}_j$, respectively. By i,j=0 (for overlap) we mean that the operators in question are replaced by the identity operation of the appropriate sector.

Table A2: Spin-Isospin Expectations of Q_i [a)]

σ	i	$(i)^{IS}_{\sigma}$
D	1	2/9
S	1	0
	3	-6/243
	7	-2/243
C	1	6/729
	3	10/729
Y	1	-2/729
	2	-4/729
	4	-3/729

a) All notation as in the text and Table A1.

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FIGURE CAPTIONS:

Figure 1: Kinetic Energy Delocalization Effects

Figure 2: Non-Exchange Forces in the Three Nucleon System
1=direct antibinding contributions, 2=pseudo-three-body binding contribution, 3=net anti-binding contribution

Figure 3: Two-Body Exchange Forces in the Three Nucleon System
1=hyperfine anti-binding contribution, 2=confinement binding contribution, 3=net anti-binding contribution

Figure 4: Three-Body Exchange Forces in the Three Nucleon System
1=hyperfine anti-binding contribution, 2=confinement anti-binding contribution, 3=net antibinding contribution

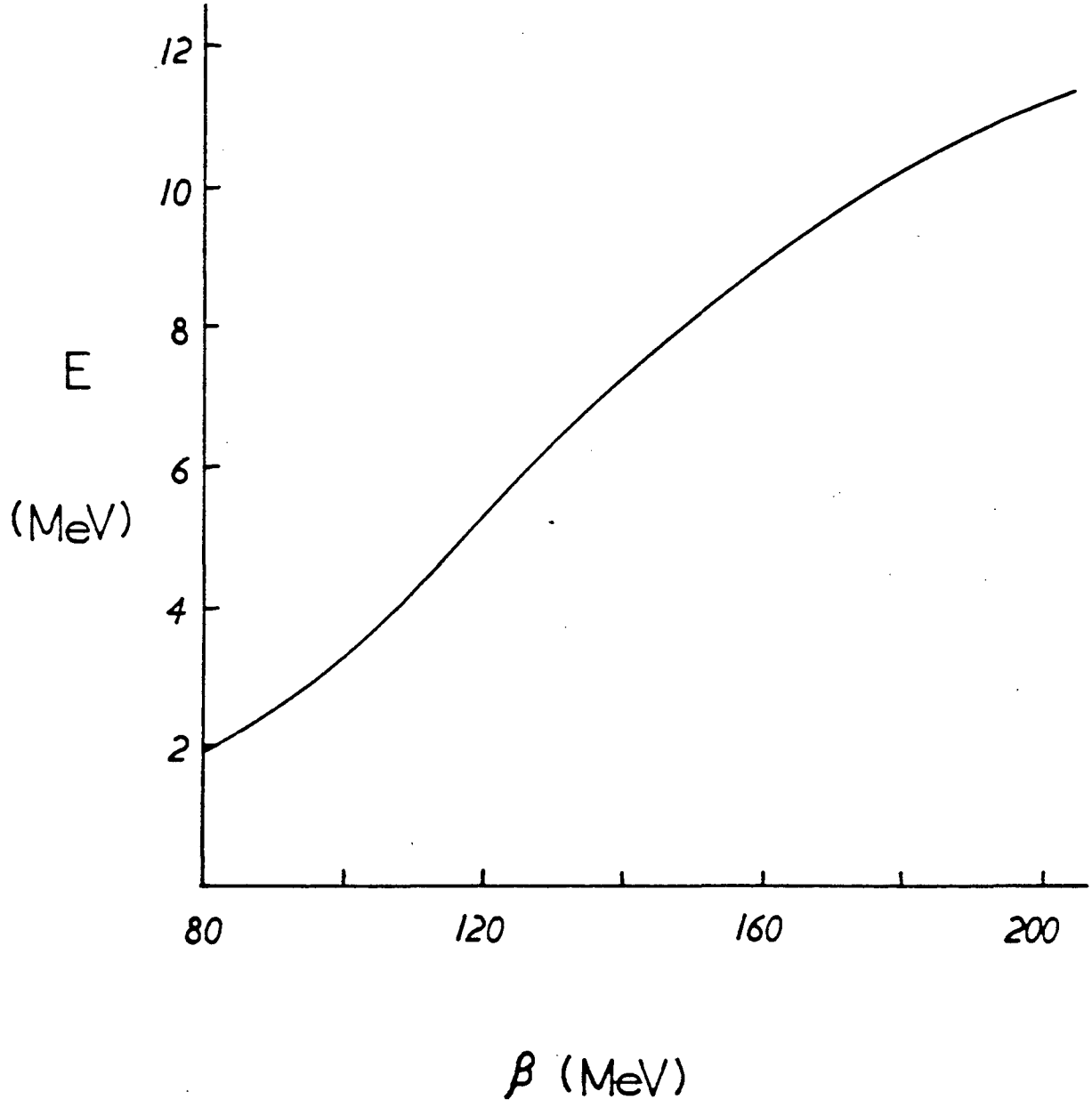

FIGURE 1

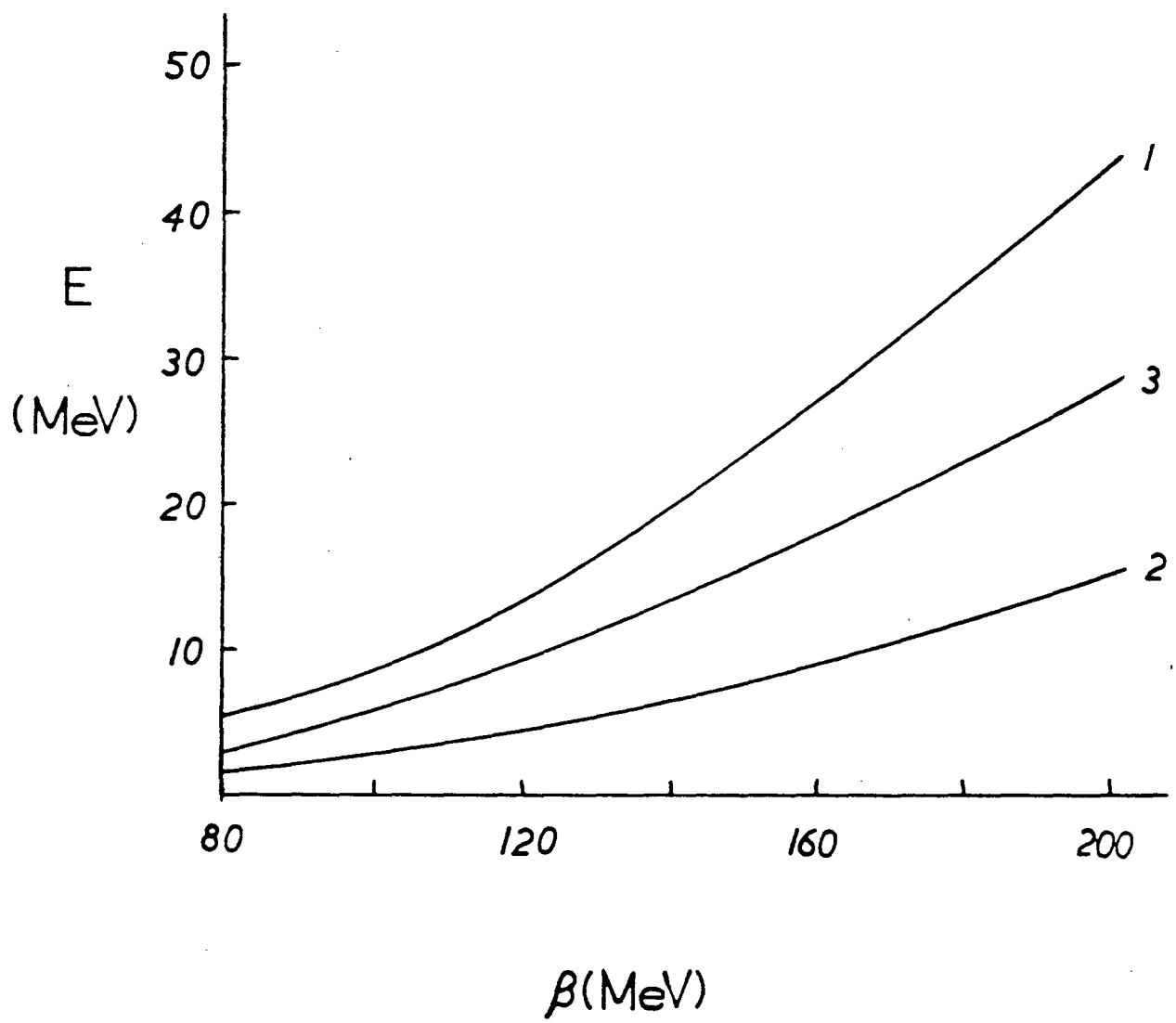

FIGURE 2

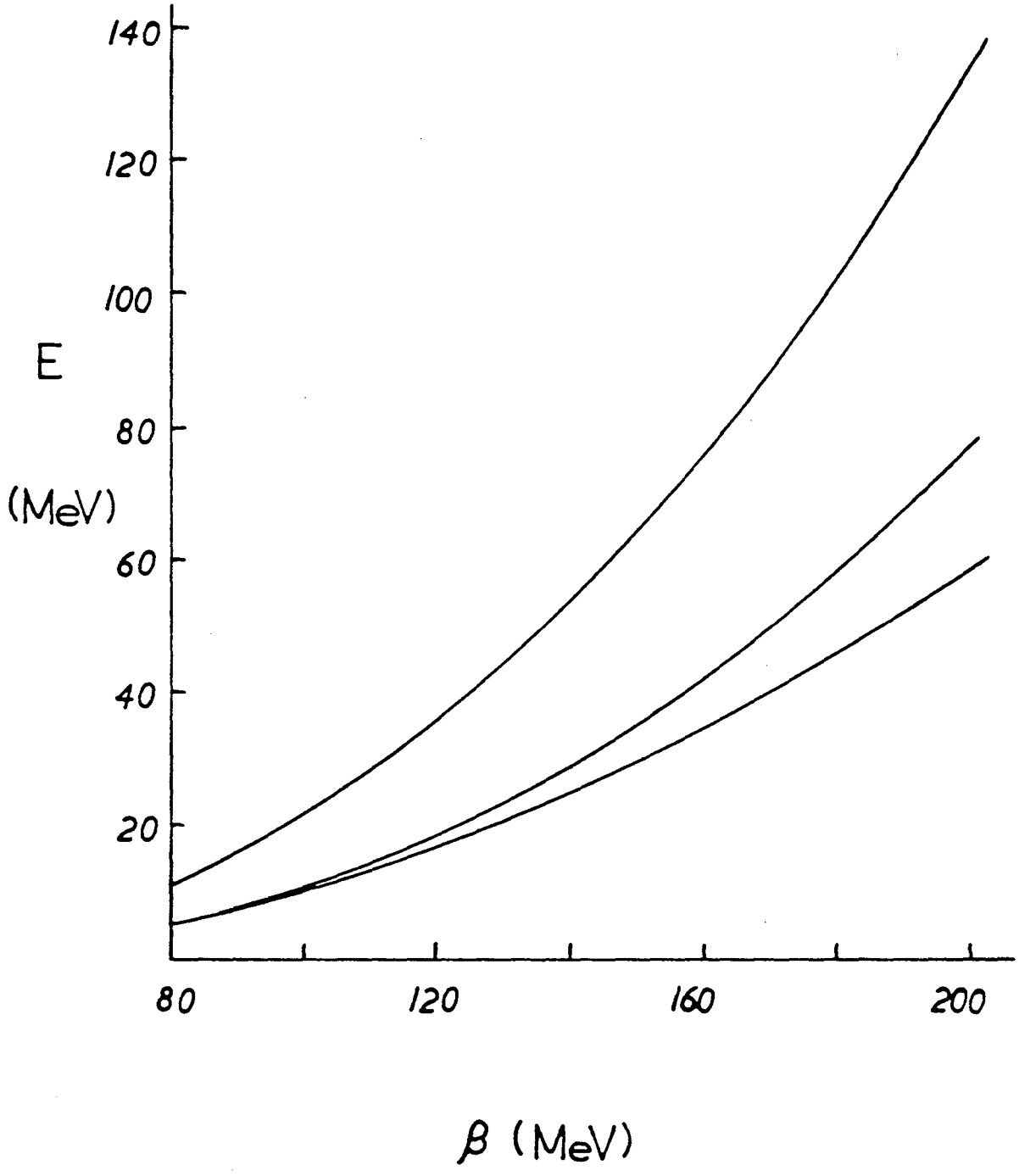

FIGURE 3

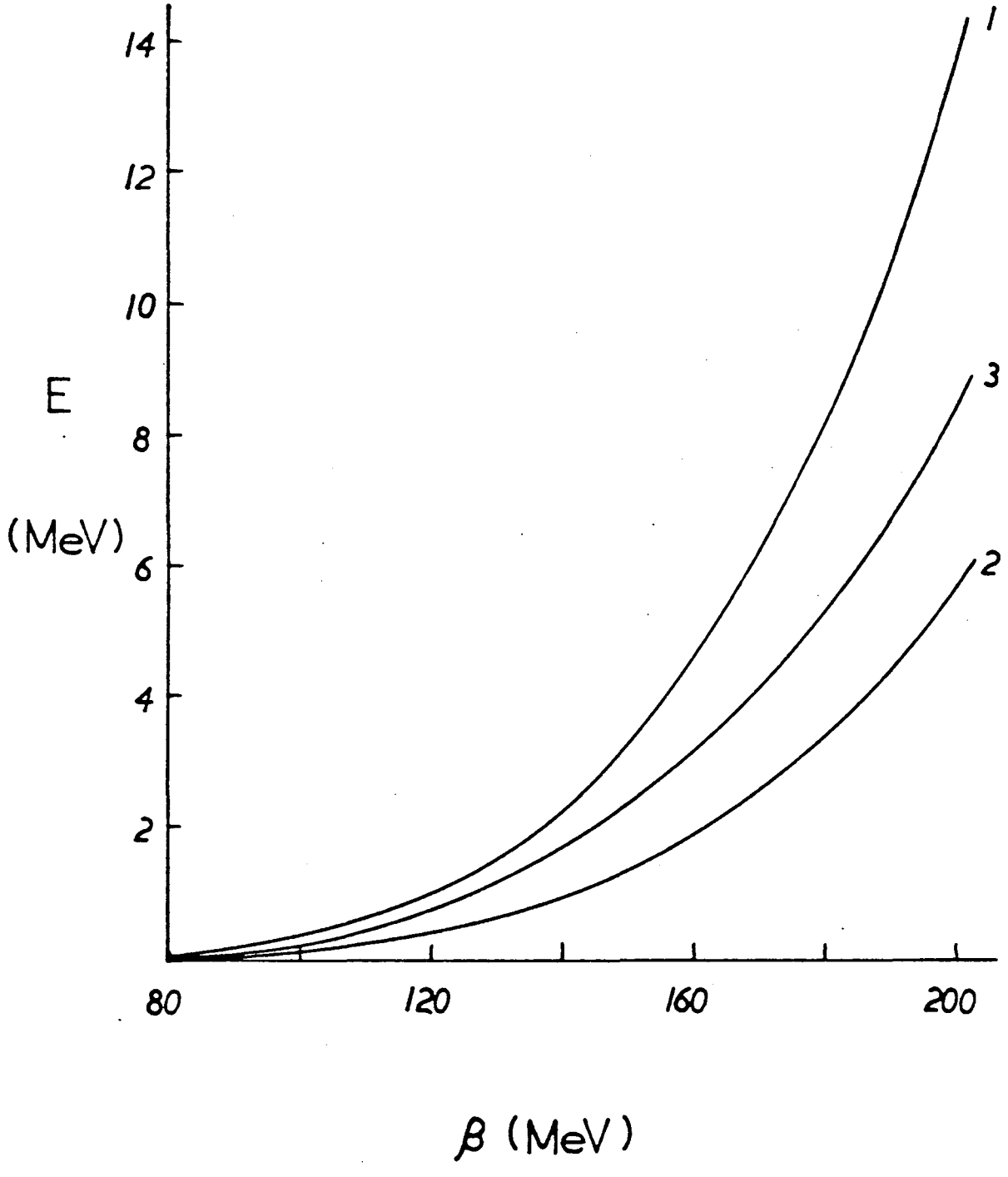

FIGURE 4

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