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Quark Exchange Effects in the Three and Four Nucleon Systems[1]

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ABSTRACT

The quark exchange forces in the three and four nucleon systems are evaluated in the context of a QCD-like potential model. As in the two nucleon system the two body exchange color hyperfine interaction is found to be strongly repulsive. Additional effects due to the delocalization of quarks over three or four nucleons are investigated and found to produce small but non-negligible effects on binding energies, effects whose magnitudes, however, are shown to be sensitive to poorly known details of the short distance features of the nuclear wavefunctions.

1. Introduction

In recent years studies of phenomenological models of hadron structure incorporating what are believed to be the important features of QCD have lead to considerable progress in our understanding of the origins of the NN force[1-14]. In this context the repulsive core of the NN potential is seen to arise, via quark exchange effects, from the color magnetic (hyperfine) piece of one gluon exchange between quarks, while the intermediate range attraction usually ascribed to 2π exchange appears to be due largely to the mutual color polarization of the two nucleons[15]. The role of meson exchange remains at least quantitatively unclear. It has been argued that contributions from mesons other than the pion should be negligible unless the quark substructure of hadrons is anomalously small (i.e. small relative to the inverse QCD deconfinement temperature scale ~ physical hadron sizes)[15]. The matter of producing a phenomenological framework in which such exchange effects can be incorporated in a reliable quantitative manner is, however, more problematic, even if the question of the relative magnitudes of the chiral symmetry breaking and confinement scales in QCD is settled, since the two likely scenarios correspond to rather similar pictures of the averaged suppression of the pion field inside the region

of the nucleon occupied by quarks[a]. The basic physics of the NN system, however, seems well established.

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[a] There are two likely scenarios for the relation of the chiral symmetry breaking (χ) and confinement (C) transition temperatures: $T_\chi \sim (3\text{-}5)T_C$, as suggested by Shuryak[16] based on estimates of the vacuum gluon condensate from QCD sum rules, and $T_\chi \sim T_C$, as suggested by some Monte Carlo results[17]. There are some numerological arguments in favor of the former, eg. the pseudoscalar mixing pattern, which is very natural if $T_\chi \gg T_C \sim m_s$ but requires special dynamics if $T_\chi \sim T_C$. In fact such special dynamics appear possible in the bag model. In addition the accuracy of the gluon condensate obtained from QCD sum rules has been the subject of some recent controversy[18], although the effect seems to be to under- rather than over-estimate, so the situation is far from settled. Although the two scenarios correspond to different phenomenological pictures of the pion coupling, the fact that T_χ is at most $\sim 5T_C$ leads to very similar pictures when one averages over the nucleon volume. In the case $T_\chi > T_C$, the presence of a light quark inside the nucleon produces a region of chirally symmetric vacuum of radius $\sim 1/T_\chi$ and since the extent of the current quark substructure of the pion will also be $\sim 1/T_\chi$, the natural scale for the suppression of the pion field in the vicinity of a light quark is a few times $1/T_\chi$. Implementing such a suppression smoothly with $T_\chi \sim 1$ GeV and averaging over the nucleon volume one finds an average field strength suppression rather similar to what one would expect from a surface-coupled cloudy bag picture, softened to take acount of surface fluctuations, which is the natural qualitative realization of the case $T_\chi \sim T_C$.

Given the state of our knowledge of the NN interaction, a study of the three nucleon system is thus of considerable interest, especially viz a vis nuclear structure. For example, while the results of the behavior of the NN system can be cast in the form of an effective potential, the fact that such a potential represents the effects of both exchange interactions and the mixing of additional states means that the sum of two body forces in multi-nucleon systems cannot necessarily be obtained by using the effective potential extracted from the NN system. This is, of course, related to the question of the presence of exotic degrees of freedom in nuclei. Such configurations are present in the deuteron only at rather small levels owing to the repulsive nature of the exchange hyperfine interaction. This repulsion leads to a diffuse distribution for NN component of the deuteron, thus suppressing significant overlap with the hidden color excitations of the system, which, due to confinement, must be well localized in space. The result is a deuteron which is dominantly a standard nuclear system, at least in terms of composition, although the weakly excited internal degrees of freedom play a crucial role in determining the effective nuclear interaction. Other nuclei, however, are considerably more compact than the deuteron and, even assuming one is able to explain their structure within the present phen-

omenological framework, it is of interest to know whether, with typical separations in such nuclei only slightly greater than the confinement scale, the hyperfine repulsion continues to determine the basic physics of the system and whether or not there are significant admixtures of exotic configurations into the nuclear ground state. ^{3}He and ^{4}He, being among the most compact nuclei, are ideal systems in which to study this question. In addition, such tightly bound nuclei may exhibit interesting delocalization effects. Recall that the internal kinetic energy of a nucleon is large on the scale of nuclear binding. One therefore expects the alleviation of this kinetic energy via delocalization to represent potentially non-trivial binding contributions relative to a description of the system in terms of point-like nucleons. Although such an effect is present, in principle, in the deuteron, the smallness of the spatial exchange integrals means that permutationally distinct configurations of the system do not appreciably interfere and the resulting effect is, in consequence, negligible. One final property of the three nucleon system of interest with regard to nuclear structure is the possible existence of three (and even four) body nuclear forces induced by quark-quark interactions. It has often been suggested that three body forces might be required to explain the binding of ^{3}H, ^{3}He, and estimates of such effects have been obtained in a framework involving nucleons, isobars and meson exchange.[19] Such contributions are, of course, sensitive to the cutoffs used for

meson-baryon vertices and, in addition, further contributions are expected, in dense nuclear matter, from quark exchange effects.

In this paper we investigate quark exchange and delocalization contributions to the energy of three and four nucleon systems. We show that the exchange hyperfine interaction remains strongly repulsive at short distances and that delocalization effects in the kinetic energy as well as three and four body forces arising from quark exchange, produce small but non-negligible corrections to nuclear binding. In what follows we will, for simplicity, consider only the symmetric s-state portion of the nuclear wavefunction. The presence of small admixtures of other components, which will also lead to three and four body forces, we do not attempt to estimate at present. In addition, effects due to pion exchange and admixtures of non-nucleonic configurations into the nuclear ground states, which are presumably responsible for the bulk of the binding of the system, are not considered here. These additional effects rely, for a reasonable estimate of their magnitude, on clarifying the role of the pion and the size of pion vertex cutoffs, and will be the subject of future work.

2. The Model

Calculations have been performed in a QCD-inspired potential model which has previously been applied extensively, and with considerable success, to baryon spectroscopy[20] and decays[21] as well as to the NN problem[15]. In the context of the NN interaction the model is

discussed more fully in Ref. 15). The Hamiltonian for the three (four) nucleon system is

$$H = \sum_{i=1}^{N} \left(m_i + p_i^2/2m_i \right) + \sum_{i<j=1}^{N} \left(H_{conf}^{ij} + H_{hyp}^{ij} \right) \qquad (1)$$

where N = 9(12). Setting $\underset{\sim}{r}_{ij} = \underset{\sim}{r}_i - \underset{\sim}{r}_j$ and

$$S_{ij} = 3\underset{\sim}{S}_i \cdot \underset{\sim}{r}_{ij} \underset{\sim}{S}_j \cdot \underset{\sim}{r}_{ij} / r_{ij}^{\,2} - \underset{\sim}{S}_i \cdot \underset{\sim}{S}_j$$

the two body confining potential H_{conf}^{ij} is given by

$$H_{conf}^{ij} = -\left(e_0 + \tfrac{1}{2} k r_{ij}^2 + U(r_{ij}) \right) \left(\tfrac{1}{2} \vec{\lambda}_i \right) \cdot \left(\tfrac{1}{2} \vec{\lambda}_j \right) \qquad (2)$$

and the two body color hyperfine interaction, H_{hyp}^{ij}, expected from one gluon exchange, by

$$H_{hyp}^{ij} = -\left(\frac{\alpha_s}{m_i m_j} \right) \left(\frac{8\pi}{3} \underset{\sim}{S}_i \cdot \underset{\sim}{S}_j \, \delta^3(\underset{\sim}{r}_{ij}) + S_{ij} r_{ij}^{-3} \right) \left(\tfrac{1}{2} \vec{\lambda}_i \right) \cdot \left(\tfrac{1}{2} \vec{\lambda}_j \right) \qquad (3)$$

The anharmonicity, $U(r_{ij})$, in (2) is meant to represent departures from the harmonic limit, including the attractive short range Coulomb interaction of QCD[a]. The parameters of the model are all determined from

[a] As in Ref. 15), $U(r_{ij})$ is taken to be a δ function for ease of calculation. Smearing over clusters reduces the sensitivity to the actual functional form chosen. We will see later that the three body effects, however, remain sensitive to this choice. While a Coulomb form for U would be more realistic, one is constrained to use fixed cluster sizes owing to the singular nature of the hyperfine interaction, and in this limit it is not possible to satisfy the constraints of the baryon spectrum[20] using the Coulomb form for U in (1).

the work on baryon spectroscopy and are as given in Ref. 15).

3. The Three Nucleon State

Let us denote by (123;456;789) the normalized but not yet fully antisymmetrized state of three nucleons, having total isospin ($I=\frac{1}{2}$, I_z) and total spin ($S=\frac{1}{2}$, $S_z=\frac{1}{2}$) in which nucleon 1 contains quarks 1,2 and 3, nucleon 2 quarks 4,5 and 6, and nucleon 3 quarks 7,8 and 9. In order to simplify matters we deal with only those terms which survive complete antisymmetrization by choosing the state (123;456;789) to be antisymmetric with respect to the interchange of any pair of its constituent nucleons. Denote by A_{ijk} the usual color singlet combination of three quarks

$$A_{ijk} = \frac{1}{\sqrt{6}} \varepsilon_{\alpha\beta\gamma} | q_i^\alpha q_j^\beta q_k^\gamma \rangle \tag{4}$$

and by $\Psi_{ijk;\ell mn;rst}$ the spatial wavefunction of three nucleons (ijk), (ℓmn), (rst) in a relative wavefunction ψ. One has

$$\Psi_{ijk;\ell mn;rst} = \phi(ijk)\phi(\ell mn)\phi(rst)\psi(\underset{\sim}{R}_{ijk;\ell mn;rst}, \underset{\sim}{L}_{ijk;\ell mn;rst}) \tag{5}$$

where $\phi(ijk)$ is the internal spatial wavefunction of the nucleon (ijk) [20]

$$\phi(ijk) = \frac{\alpha^3}{\pi^{3/2}} \exp\left(-\alpha^2(\rho_{ijk}^2 + \lambda_{ijk}^2)/2\right) \tag{6}$$

with

$$\begin{aligned} \rho_{ijk} &= (\underset{\sim}{r}_i - \underset{\sim}{r}_j)/\sqrt{2} \\ \lambda_{ijk} &= (2\underset{\sim}{r}_k - \underset{\sim}{r}_i - \underset{\sim}{r}_j)/\sqrt{6} \end{aligned} \tag{7}$$

and the relative wavefunction ψ is expressed in terms of the usual Jacobi coordinates

$$\begin{aligned} \underset{\sim}{R}_{ijk;\ell mn;rst} &= ((\underset{\sim}{r}_i + \underset{\sim}{r}_j + \underset{\sim}{r}_k)/3 - (\underset{\sim}{r}_\ell + \underset{\sim}{r}_m + \underset{\sim}{r}_n)/3)/\sqrt{2} \\ \underset{\sim}{L}_{ijk;\ell mn;rst} &= (2(\underset{\sim}{r}_r + \underset{\sim}{r}_s + \underset{\sim}{r}_t)/3 - (\underset{\sim}{r}_i + \underset{\sim}{r}_j + \underset{\sim}{r}_k)/3 \\ &\qquad - (\underset{\sim}{r}_\ell + \underset{\sim}{r}_m + \underset{\sim}{r}_n)/3)/\sqrt{6}. \end{aligned} \tag{8}$$

Then, taking $I_z = +1/2$,

$$\begin{aligned} (123;456;789) = \frac{1}{\sqrt{6}} \Psi_{123;456;789} A_{123} A_{456} A_{789} (&p\uparrow p\downarrow n\uparrow - \\ &p\downarrow p\uparrow n\uparrow + p\downarrow n\uparrow p\uparrow - p\uparrow n\uparrow p\downarrow + n\uparrow p\uparrow p\downarrow - \\ &n\uparrow p\downarrow p\uparrow)_{123;456;789} \end{aligned} \tag{9}$$

where

$$(p\uparrow p\downarrow n\uparrow)_{123;456;789} = (p\uparrow)_{123}(p\downarrow)_{456}(n\uparrow)_{789} \tag{10}$$

and

$$(p\uparrow)_{123} = (p^\rho \chi^\rho\uparrow + p^\lambda \chi^\lambda\uparrow)_{123}/\sqrt{2} \quad \text{(etc.)} \tag{11}$$

is the spin-isospin wavefunction for a spin up proton (etc.). The labels ρ, λ refer to transformation properties under the permutation group S_3 [a]. The states p^ρ, p^λ, $\chi^\rho\uparrow$ and $\chi^\lambda\uparrow$ are given by

---

a) Denoting the 2-cycles (ij) of S_3 by π_{ij}, the states ρ, λ, which form a basis for the mixed representation of S_3, transform according to

$$\pi_{12}\rho = -\rho \qquad \pi_{12}\lambda = \lambda$$
$$\pi_{13}\rho = \tfrac{1}{2}\rho + \tfrac{\sqrt{3}}{2}\lambda \qquad \pi_{13}\lambda = \tfrac{\sqrt{3}}{2}\rho - \tfrac{1}{2}\lambda$$

$$
\begin{aligned}
p^{\rho}_{123} &= (u_1d_2-d_1u_2)u_3/\sqrt{2} \\
p^{\lambda}_{123} &= (u_1d_2u_3+d_1u_2u_3-2u_1u_2d_3)/\sqrt{6} \\
\chi^{\rho}_{123\uparrow} &= (\uparrow_1\downarrow_2-\downarrow_1\uparrow_2)\uparrow_3/\sqrt{2} \\
\chi^{\lambda}_{123\uparrow} &= (\uparrow_1\downarrow_2\uparrow_3+\downarrow_1\uparrow_2\uparrow_3-2\uparrow_1\uparrow_2\downarrow_3)/\sqrt{6}
\end{aligned}
\tag{12}
$$

Those involving n and/or $\downarrow$ may be obtained from (12) by spin or isospin lowering. The spatial wavefunctions ϕ, ψ are normalized with respect to the measures $d^3\rho\, d^3\lambda$ and $d^3R\, d^3L$ respectively.

The fully antisymmetrized state of three nucleons may now be obtained by applying the nine-quark antisymmetrizer to (123;456;789) and normalizing. The resulting expression consists of 280 terms, representing the 280 distinct ways of arranging 9 quarks into three identical nucleons. Using the available partial antisymmetries of (ijk;ℓmn;rst) one can write the expectation value of H in the resulting state as

$$
\begin{aligned}
&\langle(123;456;789)|\, H\{|(123;456;789)\rangle - 27\,|(126;453;789)\rangle + \\
&\qquad 54\,|(129;453;786)\rangle + 162\,|(169;452;783)\rangle - \\
&\qquad 36\,|(483;159;726)\rangle\}/N^2 \\
&\equiv \langle(123;456;789)|\, H \sum_i \alpha_i |P_i(123;456;789)\rangle /N^2
\end{aligned}
\tag{13}
$$

where

$$
N^2 = C^{(1)} - 27C^{(2)} + 54C^{(3)} + 162C^{(4)} - 36C^{(5)} \tag{14}
$$

The 9 quark antisymmetrizer has been written down previously by Suzuki and Hecht[22]. In (14) the quantities $C^{(i)}$ are given by

$$
\begin{aligned}
C^{(1)} &= \langle(123;456;789)|(123;456;789)\rangle \\
&\equiv \langle(123;456;789)|P_1|(123;456;789)\rangle \equiv 1 \\
C^{(2)} &= \langle(123;456;789)|(126;453;789)\rangle \\
&\equiv \langle(123;456;789)|P_2|(123;456;789)\rangle \\
C^{(3)} &= \langle(123;456;789)|(129;453;786)\rangle \\
&\equiv \langle(123;456;789)|P_3|(123;456;789)\rangle \\
C^{(4)} &= \langle(123;456;789)|(169;452;783)\rangle \\
&\equiv \langle(123;456;789)|P_4|(123;456;789)\rangle \\
C^{(5)} &= \langle(123;456;789)|(483;159;726)\rangle \\
&\equiv \langle(123;456;789)|P_5|(123;456;789)\rangle
\end{aligned}
\tag{15}
$$

In what follows we will refer to the permutations occurring in the kets of (15) as P_i $(i=1,\ldots,5)$ rather than $P_i(123;456;789)$.

4. The Four Nucleon State

In analogy to section 3 we define the normalized but not yet fully antisymmetrized state of four nucleons, antisymmetric with respect to quark interchange within any given nucleon and also with respect to nucleon interchange, and with total spin and isospin zero, to be

$$
\begin{aligned}
(ijk;\ell mn;rst;uvw) &= \frac{1}{\sqrt{24}}\,\Psi_{ijk;\ell mn;rst;uvw}\, A_{ijk} A_{mn} A_{rst} \\
&\quad A_{uvw} \Big[\sum_P (-1)^P (p\uparrow p\downarrow n\uparrow n\downarrow)\Big]_{ijk;\ell mn;rst;uvw}
\end{aligned}
\tag{16}
$$

where the sum runs over all 4! permutations, P, of $p\uparrow$, $p\downarrow$, $n\uparrow$, $n\downarrow$ and

$$\Psi_{ijk;\ell mn;rst;uvw} = \phi(ijk)\phi(\ell mn)\phi(rst)\phi(uvw) \qquad (17)$$
$$\psi(\underset{\sim}{R}_{ijk;\ell mn;rst;uvw}, \underset{\sim}{L}_{ijk;\ell mn;rst;uvw}, \underset{\sim}{E}_{ijk;\ell mn;rst;uvw})$$

The vector

$$\underset{\sim}{E}_{ijk;\ell mn;rst;uvw} = \left[3(\underset{\sim}{r}_u+\underset{\sim}{r}_v+\underset{\sim}{r}_w)/3 - (\underset{\sim}{r}_i+\underset{\sim}{r}_j+\underset{\sim}{r}_k)/3 - (\underset{\sim}{r}_\ell+\underset{\sim}{r}_m+\underset{\sim}{r}_n)/3 - (\underset{\sim}{r}_r+\underset{\sim}{r}_s+\underset{\sim}{r}_t)/3\right]\sqrt{12} \qquad (18)$$

is the third Jacobi coordinate for the four nucleon system, and ψ is a (symmetric) relative wavefunction of the four nucleons. Applying the twelve quark antisymmetrizer to (16) and using the existing partial antisymmetries one obtains, for the expectation value of H in the fully antisymmetrized state

$$\begin{aligned}
&\langle(123;456;789;10,11,12)|H\Big\{|(123;456;789;10,11,12)\rangle \\
&\quad -54|(126;453;789;10,11,12)\rangle + 216|(129;453;786;10,11,12)\rangle \\
&\quad +648|(169;452;783;10,11,12)\rangle - 144|(483;159;726;10,11,12)\rangle \\
&\quad -486|(1,2,12;453;786;10,11,9)\rangle + 243|(126;453;7,8,12;10,11,9)\rangle \\
&\quad -3888|(169;452;7,8,12;10,11,3)\rangle - 3888|(159;4,2,12;783;10,11,6)\rangle \\
&\quad -3888|(1,5,12;483;726;10,11,9)\rangle - 648|(1,2,12;4,11,6;10,8,9;753)\rangle \\
&\quad -1296|(159;4,8,12;7,11,3;10,2,6)\rangle\Big\}/N^2 \\
&\equiv \langle(123;456;789;10,11,12)|H \sum_i \alpha_i|P_i(123;456;789;10,11,12)\rangle/N^2
\end{aligned} \qquad (19)$$

where

$$\begin{aligned}
N^2 &= \sum_{i=1}^{12}\alpha_i\langle(123;456;789;10,11,12)|P_i(123;456;789;10,11,12)\rangle \\
&= \sum_{i=1}^{12}\alpha_i c^{(i)}
\end{aligned} \qquad (20)$$

with $_i$ the coefficients appearing in the braces in (19) and $P_i(123;456;789;10,11,12)$ the corresponding permutations of (123;456;789;10,11,12) which we will refer to, for brevity, in what follows, as P_i.

5. Evaluating the Energies of the Three and Four Nucleon States

Before evaluating (13) it is instructive to examine the origin of the terms contained therein. In what follows let i,j be particle labels. Then, owing to the color dependence of the two body operators, H^{ij}, in (1), the expectation $\langle(123;456;789)|H^{ij}|(123;456;789)\rangle$ vanishes unless i,j belong simultaneously to either $\{1,2,3\}$, $\{4,5,6\}$ or $\{7,8,9\}$. Similarly $\langle(123;456;789)|H^{ij}|(126;453;789)\rangle$ is zero unless i,j are both elements of either $\{1,2,3,4,5,6\}$ or $\{7,8,9\}$. As a result the two body part of the first two terms of (13) can be recast as

$$\begin{aligned}
&\langle(123;456;789)|\,3\Big(\sum_{i<j=1}^{3}H^{ij}\Big\{|(123;456;789)\rangle - 9|(123;459;786)\rangle\Big\} \\
&\quad -9\sum_{i<j=1}^{6}H^{ij}|(126;453;789)\rangle\Big)/N^2
\end{aligned} \qquad (21)$$

where we have taken advantage of the symmetries under relabelling to obtain this expression. We immediately recognize the last term in (21) as the sum of the two nucleon exchange interactions.

The interpretation of the first two terms is less clear. From (15) we see that they can be combined in the form

$$\langle(123;456;789)|3\sum_{i<j=1}^{3} H^{ij}(1-9C^{(2)})|(123;456;789)\rangle/N^2 \qquad (22)$$

Then, since, as we will see below, $C^{(2)}<0$, $C^{(5)}=0$ and $|54C^{(3)}+162C^{(4)}|\ll 27C^{(2)}$, and since the net contribution of the quark-quark forces to the energy of the nucleon is negative, the effect of these terms is to destabilize the bound three nucleon system relative to the isolated three nucleon state. Note that one need not, in general, have $C^{(2)}<0$ so this destabilization is a characteristic property of the three nucleon system. Note also that the second term in braces in (21), which has been evaluated in terms of direct matrix elements in order to arrive at (22), is a genuine three-body effect. However, we see from (22) that its role is, in fact, similar to that of the normalization constant N^2, i.e. it is a delocalization effect rather than an exchange <u>interaction</u> in the usual sense of the word. For this reason, in what follows, we will distinguish it from the remaining terms in (21), which produce genuine three body exchange forces. A similar analysis holds for (19). The terms involving permutations 2,3,4,5 contain pieces corresponding to internal nucleon interactions modified by delocalization amongst the remaining nucleons, while, owing to the color structure of the two body operators, the P_7 term consists entirely of two body exchange interactions similarly modified. Note that it is common practice to refer to the contributions of P_2, $(P_3+P_4+P_5)$ in (13), (19) and $(P_6+P_7+P_8+P_9+P_{10}+P_{11}+P_{12})$

in (19) as representing, respectively, non-local two, three and four body nuclear forces. Such non-local forces will, therefore, contain the effects of kinetic energy as well as the pseudo-two, three and four body contributions discussed above. In order to make contact with the results of Suzuki and Hecht[22] we will quote our results in this form. In light of the discussion above, however, and the plausibility of attributing modifications in the total kinetic energy to delocalization, we will also present the results in the following form: 1) the effect of delocalization on the single nucleon internal two body forces and also on the reduction of the total kinetic energy relative to the value for point-like nucleons with the same internal kinetic energy as an isolated nucleon and in the same nuclear wavefunction, 2) the remaining non-local two, three and four body nuclear forces, now arising solely from quark interactions among the participating nucleons.

In order to evaluate (13), (19) we require the matrix elements, in the relevant sectors, of the spin, space and color operators appearing in (1), (2), and (3). The calculation of the spin and color matrix elements is straightforward. The results are presented in the Appendix. In order to complete the evaluation, however, we must choose a form for the three and four nucleon relative wavefunctions, Ψ. Since we are interested primarily in the three and four body exchange effects it is the short distance behavior of Ψ which most concerns us. Unfortunately, one must consider this behavior to be rather ill-determined. Not only are hidden color configurations expected to be present in ^{3}He,

^{4}He at the level of a few percent or so[a], but one does not yet know how well the short range parts of NN potentials, which are presumed to represent effects of the exchange hyperfine interaction, actually reproduce the short range quark exchange forces at typical nuclear densities[b]. In addition, since the scale of chiral symmetry breaking in QCD is not yet clarified, and, as a result, one does not have clear guidance in choosing cutoffs for pion coupling vertices, one does not know whether (and if so, to what extent) pionic effects are damped in systems such as ^{3}He, ^{4}He. Such effects, undoubtedly, have a bearing on attempts to rectify the well known discrepancies of the binding energies and charge form factors of the three nucleon system[19], but, while they represent interesting avenues of investigation in their own right, we do not attempt to pursue them in this paper. In order to procede, therefore, consider wavefunctions of the form originally proposed by Fearing[24] and Khanna[25] for the symmetric s-state portion of ψ:

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[a] Sauer and Wiese[23] estimate the percent of hidden color configurations in ^{3}He to be .5%. If one includes also the spatially excited hidden color configurations claimed to produce much of the intermediate range NN attraction[15], this percentage will be increased even further. In ^{4}He, combinatorial factors double the two body exchange amplitudes over those in ^{3}He so that hidden color configurations induced by quark-quark forces could well be present at the level of several %.

[b] As a crude illustration, if one extracts bound state equivalent potentials[15] from the exchange forces alone in the two NN channels and then evaluates the 3N expectations using these potentials for a range of pure Gaussian nuclear wavefunctions, one finds that the resulting expectations reproduce the results of the actual exchange interactions to an accuracy of only about 10% at typical nuclear densities.

$$\psi_{3N} = \frac{1}{N}\exp\left(-\beta_o^2(R^2+L^2)/2\right)\prod_{i<j}\left(1-C\exp\left(-\beta_1^2 R_{ij}^2/2\right)\right)$$
$$\psi_{4N} = \frac{1}{N}\exp\left(-\beta_o^2(R^2+L^2+E^2)/2\right)\prod_{i<j}\left(1-C\exp\left(-\beta_1^2 R_{ij}^2/2\right)\right) \qquad (23)$$

where R, L, E are the nuclear Jacobi coordinates, as in (8), (17), N is the normalization constant relative to these coordinates, and $\underset{\sim}{R}_{ij} = \underset{\sim}{R}_i - \underset{\sim}{R}_j$ is the separation between nucleons i,j. The constants β_o, β_1, C are chosen so as to reproduce, as well as possible, the charge form factors of the systems in question. For ^{3}He we will use the values cited as models I and II in Aerts and Dover[26]:

$$\begin{aligned} &\text{Model I: } \beta_o = 117 \text{ MeV}, \beta_1 = 626 \text{ MeV}, C = 1.0 \\ &\text{Model II: } \beta_o = 154 \text{ MeV}, \beta_1 = 328 \text{ MeV}, C = .925 \end{aligned} \qquad (24)$$

The former yields the correct rms charge radius[27] and the location of the first minimum of the form factor but is generally too small at large q^2. The latter produces too small a charge radius and, correspondingly, too large a form factor at small q^2, but reproduces the position of the first minimum and the correct magnitude in the region of the subsequent maximum. The features of these model wavefunctions should not be taken too literally. In evaluating the form factors, for example, the exchange contributions due to quark antisymmetrization have been neglected. If one includes them one finds an outward shift of the first minimum (.3 fm^{-2} for model II and 1 fm^{-2} for model I) and a 10% reduction in the magnitude of $|F(q^2)|$ in the region of the subsequent maximum. In addition, contributions from the quark pair and meson exchange currents[28], pion induced three body forces and isobars[29] will modify the predictions at larger q^2, as will the presence of hidden color configurations which, being highly localized due to confinement, will pro-

duce effects much like those of 6q, 9q bag-like substructures[30,31]. The use of these model wavefunctions, therefore, represents a first estimate of the effects in question, a comparison of the corresponding results serving as a test of the sensitivity of the calculation to the range of plausible nuclear wavefunctions. For ^{4}He, subject to the same caveats, we employ the following models:

$$\begin{aligned} &\text{Model I: } \beta_0 = 160 \text{ MeV}, \ \beta_1 = 375 \text{ MeV}, \ C = 1.0 \\ &\text{Model II: } \beta_0 = 154 \text{ MeV}, \ \beta_1 = 328 \text{ MeV}, \ C = .925 \end{aligned} \tag{25}$$

The former has been adjusted to fit the charge form factor out into the first minimum, while the latter, which simply employs the parameters of model II in (24), has the first minimum occuring at too small a value of q^2 but does somewhat better in the region of the subsequent maximum. Since the parameters of the two models are reasonably close one obtains a good idea of the sensitivity of the results. The charge form factors for the two models (without exchange effects) are shown in Figure 1. The experimental points are from McCarthy et al.[32], and Frosch et al.[33].

6. Results

The results are summarized in Tables 1-4. In Table 1 we present the direct (D), two body (2B), and three body (3B) contributions of the kinetic energy (KE), hyperfine (HF) and confinement (C) pieces of the Hamiltonian, (1), to the energy of ^{3}He, taking as a two body effect the pseudo-two body contributions discussed in section 5. For reference, the values of the corresponding direct contributions for three nucleons

in the same nuclear wavefunction but with no exchange effects are included in brackets. α is the nuclear cluster parameter of (6). The value employed in baryon spectroscopy[20] is 320 MeV. As an aid to understanding the sensitivity of the resutls to nucleon cluster size we have also performed the calculation with α=240 MeV and α=400 MeV. The former value corresponds to a nucleon whose entire charge radius is produced by its quark substructure. In Table 2 we recast the results of Table 1 in terms of the "true" two and three body exchange forces and delocalization (DELOC) effects on 1) the kinetic energy, and 2) the internal two body forces within individual nucleons. The rationale for doing this has been discussed in section 5. The delocalization energies quoted represent 1) the difference between the total KE and that expected for three nucleons in the same nuclear wavefunction but without exchange effects[a], and 2) the difference between the direct plus pseudo-two body contributions and the expectation value in three isolated nucleons of the two body potential operators, respectively. Positive (negative) entries, therefore, correspond to effective anti-binding (binding) contributions. Tables 3 and 4 present analogous results for ^{4}He. Since the calculations, in this case, are rather lengthy only the value α=320 MeV has been employed. The 2B contributions of

[a] As usual there is an ambiguity regarding the mass parameter for the relative motion of the three quark clusters. The naturally occuring parameter, which follows from (1), is $3m_q$, or greater by about 5% than the nucleon mass. In order to make the comparison of delocalization effects consistent we have evaluated the effectively-point-like-nucleons relative kinetic energy using $3m_q$ rather than M_N.

Table 4 include the pseudo-four body (pseudo-4B) contribution of the P_7 term in (19).

We see, from Tables 2 and 4, clear evidence for the softening of the momentum distributions of quarks in ^{3}He, ^{4}He, an effect solely attributable to delocalization. Interpreting this as an increase in the effective confinement volume of quarks in nuclei we see that the effect is essentially that discussed by Close et al.[34] as a possible explanation of the EMC effect. In the present context the delocalization results in binding contributions, relative to what would be expected for effectively point-like nucleons, of 5-10 MeV in ^{3}He and 15-20 MeV in ^{4}He. The specific values obtained, of course, are subject to uncertainties associated with the use of the non-relativistic form of the quark kinetic energy. In contrast, the effect of delocalization on the internal two body interactions of individual nucleons is effectively repulsive and sufficiently large that the combined delocalization effects are uniformly anti-binding. Since the pseudo-two and three body contributions are negative for both the confinement and hyperfine potentials (the three body contributions, however, being quite small and not affecting the qualitative features of the discussion) this conclusion is strengthened if one chooses to combine pseudo-two and three body with other two and three body effects, as in Table 1. Comparing Table 1 to 2 (or 3 to 4) one sees that the pseudo-two body effects tend to obscure the basically repulsive character of the two body exchange forces. This is especially true in ^{4}He where the pseudo-two body to "true" two body ratio is enhanced by a combinatorial factor of 2 relative to its value in ^{3}He. Considering, therefore, Tables 2 and 4, so as to remove this effect, one sees that the exchange hyperfine interaction remains strongly repulsive and dominates the exchange confinement force, just as in the two nucleon system. The repulsive core nature of this effect is even more evident if one considers wavefunctions of the form (23), but with C=0. In such cases one finds, as β_0 is increased, an increase in the binding contributions due to the exchange confinement interaction, but an even more rapid increase in the repulsive contribution of the exchange hyperfine force. In the model wavefunctions (24), (25) this repulsion is partially alleviated but still produces substantial residual anti-binding. While we have yet to investigate the structure of mixing and mesonic effects, these observations suggest that, while there may be non-trivial admixtures of unusual configurations into the nuclear ground states, the resulting states, not only here, but in heavier nuclei as well, will not be radically exotic in composition.

Let us now turn to the three and four body effects. Note first that Model II_b for ^{3}He differs from that employed by Suzuki and Hecht[22] only in the structure of the short-range piece of the confinement potential. The present result of +.24 MeV for the total three body contribution is, therefore, to be compared with the result -.22MeV obtained in Ref. 22. We are forced to conclude that three body effects are sensitive to the short distance structure of the confinement potential. This contrasts markedly with the case of the deuteron where two body confinement effects were not only small but also considerably smaller

in magnitude than two body hyperfine effects[15]. While one can probably be confident that the order of magnitude of non-local quark induced three body binding contributions in three nucleon systems is a few hundred keV, and thus of importance in understanding the features of the nuclear ground state, any calculated values must be treated with considerable scepticism in the absence of calculations employing a more sophisticated potential in which the hyperfine spatial dependence is regulated, thus allowing polarization of the clusters by the hyperfine force and, thereby, presumably, making the Coulomb form of the short-range confinement potential compatible with the entirety of the baryon spectrum. One also sees, from the Tables, considerable sensitivity of the various three body contributions to the choice of the nuclear wavefunction as well as a strong dependence of the three body hyperfine contribution on nucleon cluster size. Note that, in ^{4}He, we predict sizable (1-2 MeV) three body effects, again sensitive to the short distance structure of the nuclear wavefunction. This is not unexpected since the strength of three body forces in ^{4}He is enhanced by a combinatorial factor of 4 over that in ^{3}He. In light of the model dependence of the ^{3}He results, however, the ^{4}He calculation should be treated as only an order of magnitude estimate. A further understanding of the sensitivity of the results to the short distance part of the nuclear wavefunction is obtained by noting that, for wavefunctions of the form (23) with C=0, it is the three body hyperfine force that dominates the three body confinement force in ^{4}He, and not vice versa. The situation changes, in the presence of pairwise anti-

correlations because the size of three body effects is a result of significant cancellations between the P_3 and P_4 parts of (19) and this cancellation is considerably more sensitive in the case of the spin dependent hyperfine interaction. The same remarks apply to the three nucleon system. Note finally that there are small but non-negligible four body effects (Table 3). If one considers only the four body interactions (Table 4), however, the effects are safely small, so that the bulk of the four body effects arise only as a result of delocalization.

In conclusion, quark exchange effects in light nuclei may play a non-trivial role in determining ground state properties. Reliable estimates of the size and sign of such effects, however, require considerably more sophisticated models of nucleon structure and quark-quark interactions than are presently available. Nonetheless, one must expect important and basically non-nuclear effects to be present unless one is prepared to accept a quark core size considerably smaller than the .6 fm usually advocated in potential models of the nucleon.

TABLE 1: Contributions to the Energy of ^3He[a)]

contribution \ model	I_a	I_b	I_c	II_a	II_b	II_c
KE D	802.3 (809.8)	1414.8 (1420.7)	2201.3 (2206.2)	817.9 (828.6)	1431.1 (1439.6)	2219.0 (2224.9)
HF D	-372.6 (-390.0)	-382.4 (-390.0)	-386.5 (-390.0)	-367.7 (-390.0)	-381.6 (-390.0)	-386.7 (-390.0)
C D	-875.6 (-916.5)	-898.7 (-916.5)	-908.2 (-916.5)	-864.1 (-916.5)	-896.7 (-916.5)	-908.7 (-916.5)
KE 2B	28.5	21.4	15.0	35.2	22.0	12.9
HF 2B	51.0	18.5	7.4	54.7	15.7	5.1
C 2B	-36.3	-15.1	-6.8	-43.5	-15.6	-6.0
TOTAL 2B	43.2	24.8	15.6	43.4	22.1	12.0
KE 3B	.18	.23	.13	1.56	.67	.18
HF 3B	2.00	.29	.04	2.07	.08	-.01
C 3B	-.21	-.19	-.07	-1.75	-.51	-.10
TOTAL 3B	1.97	.33	.10	1.98	.24	.07

[a)] All entries in MeV. The subscripts a,b,c on the model numbers refer to the choices α=240,320,400 MeV, respectively. All other notation as in text.

TABLE 2: Resummed Contributions to the Energy of ^3He[a)]

contribution \ model	I_a	I_b	I_c	II_a	II_b	II_c
DELOC KE	-7.5	-5.9	-4.9	-8.7	-8.5	-5.9
DELOC (HF+C)	38.9	17.0	7.9	50.2	19.1	7.5
HF 2B	56.8	21.0	8.6	62.0	18.4	6.2
C 2B	-22.7	-9.2	-4.1	-26.5	-9.2	-3.5
TOTAL 2B	32.1	11.8	4.5	35.5	9.2	2.7
HF 3B	2.00	.29	.04	2.07	.08	-.01
C 3B	-.21	-.19	-.07	-1.75	-.51	-.10
TOTAL 3B	1.79	.10	-.03	.32	-.43	-.11

[a)] Notation as in Table 1 and the text. All entries in MeV. 2B and 3B results here are solely the result of quark-quark interactions.

TABLE 3: Contributions to the Energy of ^4He[a)]

contribution \ model	I	II
KE D	1840.3 (1939.2)	1849.4 (1929.5)
HF D	-493.5 (-520.0)	-498.4 (-520.0)
C D	-1157.8 (-1222.0)	-1169.4 (-1222.0)
KE 2B	75.3	61.0
HF 2B	31.5	24.4
C 2B	-50.0	-41.0
TOTAL 2B	56.8	44.4
KE 3B	5.19	3.55
HF 3B	.30	.04
C 3B	-3.49	-2.45
TOTAL 3B	2.00	1.14
KE 4B	.40	.24
HF 4B	.45	.27
C 4B	-.23	-.18
TOTAL 4B	.62	.33

a) Notation as in Table 1 and the text. α = 320 MeV. All entries in MeV.

TABLE 4: Resummed Contributions to the Energy of ^4He[a)]

contribution \ model	I	II
DELOC KE	-18.0	-15.4
DELOC (HF+C)	42.9	38.3
HF 2B	43.9	34.6
C 2B	-20.9	-17.1
TOTAL 2B	23.0	17.5
HF 3B	.69	.32
C 3B	-2.57	-1.78
TOTAL 3B	-1.88	-1.46
HF 4B	.10	.05
C 4B	-.06	-.07
TOTAL 4B	.04	-.02

a) Notation as in Table 3 and the text. α = 320 MeV. All entries in MeV. 2B, 3B, and 4B entries are solely the result of quark-quark interactions.

APPENDIX

For compactness let us write the matrix elements of H^{ij} between $\langle(123;456;789)|$ and $|P_k(123;456;789)\rangle$ as $(ij)_k$. Where there is no danger of confusion we will, in addition, drop the subscripts. Then, using the permutational symmetries of $|(ijk;\ mn;rst)\rangle$, one can easily obtain the following relations between the different (ij). Any matrix elements not shown vanish from color considerations.

P_1: (12)=(13)=(23)=(45)=(46)=(56)=(78)=(79)=(89) (A1)

P_2: (13)=(23)=(16)=(26)=(34)=(35)=(46)=(56)
(14)=(15)=(24)=(25)
(78)=(79)=(89) (A2)
(12)=(45)
(36)

P_3: (14)=(15)=(24)=(25)=(17)=(18)=(27)=(28)=(47)=(48)
=(57)=(58)
(13)=(23)=(19)=(29)=(34)=(35)=(46)=(56)=(67)=(68)
=(79)=(89) (A3)
(16)=(26)=(37)=(38)=(49)=(59)
(12)=(45)=(78)
(36)=(39)=(69)

P_4: (24)=(25)=(37)=(38)=(46)=(56)=(79)=(89)
(27)=(28)=(34)=(35)=(49)=(59)=(67)=(68)
(12)=(13)=(16)=(19)
(14)=(15)=(17)=(18)
(47)=(48)=(57)=(58) (A4)
(23)=(69)
(26)=(39)
(29)=(36)
(45)=(78)

P_5: (14)=(16)=(17)=(18)=(24)=(25)=(28)=(29)=(35)=(36)
=(37)=(39)=(47)=(49)=(57)=(58)=(68)=(69)
(12)=(13)=(23)=(45)=(46)=(56)=(78)=(79)=(89) (A5)
(15)=(19)=(26)=(27)=(34)=(38)=(48)=(59)=(67)

Similarly, for $\langle(123;456;789;10,11,12)|H^{ij}|P_k(123;456;789;10,11,12)\rangle$ we obtain the following results where, for $P_1,\ldots,P_5$ the relations (A1),...,(A5), which are still valid, are not explicitly written.

P_1: (10,11)=(10,12)=(11,12)=(12) (A6)

P_2: (10,11)=(10,12)=(11,12)=(78) (A7)

P_3, P_4, P_5: (10,11)=(10,12)=(11,12) (A8)

P_6: (13)=(23)=(1,12)=(2,12)=(34)=(35)=(46)=(56)=(67)=(68)
=(79)=(89)=(9,10)=(9,11)=(10,12)=(11,12)
(14)=(24)=(15)=(25)=(1,10)=(1,11)=(2,10)=(2,11)=(47)=(48)
=(57)=(58)=(7,10)=(7,11)=(8,10)=(8,11)
(16)=(26)=(19)=(29)=(37)=(38)=(3,10)=(3,11)=(49)=(59)
=(4,12)=(5,12)=(6,10)=(6,11)=(7,12)=(8,12) (A9)
(17)=(18)=(27)=(28)=(4,10)=(4,11)=(5,10)=(5,11)
(12)=(45)=(78)=(10,11)
(36)=(69)=(9,12)=(3,12)
(39)=(6,12)

P_7: (13)=(23)=(16)=(26)=(34)=(35)=(46)=(56)=(79)=(89)
=(7,12)=(8,12)=(9,10)=(9,11)=(10.12)=(11,12)
(14)=(15)=(24)=(25)=(7,10)=(8,10)=(7,11)=(8,11) (A10)
(12)=(45)=(78)=(10,11)
(36)=(9,12)

P_8: (47)=(48)=(57)=(58)=(4,10)=(4,11)=(5,10)=(5,11)
(17)=(18)=(1,10)=(1,11)
(24)=(25)=(46)=(56)
(27)=(28)=(6,10)=(6,11)
(2,10)=(2,11)=(67)=(68)
(34)=(35)=(49)=(59)
(37)=(38)=(9,10)=(9,11) (A11)
(3,10)=(3,11)=(79)=(89)

(7,10)=(7,11)=(8,10)=(8,11)
(7,12)=(8,12)=(10,12)=(11,12)
(12)=(16)
(13)=(19)
(14)=(15)
(23)=(69)
(29)=(36)
(2,12)=(6,12)
(3,12)=(9,12)
(4,12)=(5,12)
(78)=(10,11)
(1,12)
(26)
(39)
(45)

P_9: (27)=(28)=(57)=(58)=(2,10)=(2,11)=(5,10)=(5,11)
(37)=(38)=(79)=(89)=(6,10)=(6,11)=(10,12)=(11,12)
(3,10)=(3,11)=(9,10)=(9,11)=(67)=(68)=(7,12)=(8,12)
(12)=(45)=(15)=(24)
(13)=(46)=(19)=(4,12) (A12)
(16)=(34)=(1,12)=(49)
(17)=(18)=(4,10)=4,11)
(1,10)=(1,11)=(47)=(48)
(23)=(2,12)=(56)=(59)

(26)=(35)=(5,12)=(29)

(7,10)=(7,11)=(8,10)=(8,11)

(36)=(9,12)

(39)=(6,12)

(3,12)=(69)

(78)=(10,11)

(14)

(25)

P_{10}: (12)=(13)=(45)=(56)=(27)=(67)=(38)=(48)

(14)=(16)=(25)=(35)=(37)=(47)=(28)=(68)

(2,10)=(2,11)=(3,10)=(3,11)=(4,10)=(4,11)=(6,10)=(6,11)

(29)=(39)=(2,12)=(3,12)=(49)=(69)=(4,12)=(6,12)

(1,10)=(1,11)=(5,10)=(5,11)=(7,10)=(7,11)=(8,10)=(8,11)

(17)=(18)=(57)=(58)

(19)=(59)=(7,12)=(8,12)

(1,12)=(5,12)=(79)=(89) (A13)

(23)=(26)=(34)=(46)

(9,10)=(9,11)=(10,12)=(11,12)

(15)=(78)

(24)=(36)

(9,12)

(10,11)

P_{11}: (15)=(25)=(17)=(27)=(1,10)=(2,10)=(1,11)=(2,11)=(34)=(36)
=(38)=(39)=(47)=(67)=(4,10)=(6,10)=(4,12)=(6,12)
=(58)=(59)=(8,11)=(9,11)=(8,12)=(9,12)

(13)=(23)=(1,12)=(2,12)=(45)=(56)=(4,11)=(6,11)=(78)=(79)
=(8,10)=(9,10)

(14)=(24)=(16)=(26)=(48)=(68)=(49)=(69)=(18)=(19) (A14)
=(28)=(29)

(35)=(37)=(57)=(10,11)=(10,12)=(11,12)

(3,10)=(3,11)=(5,10)=(5,12)=(7,11)=(7,12)

(12)=(46)=(89)

(3,12)=(5,11)=(7,10)

P_{12}: (12)=(13)=(15)=(19)=(23)=(26)=(2,10)=(37)=(3,11)=(45)
=(46)=(48)=(4,12)=(56)=(59)=(6,10)=(78)=(79)
=(7,11)=(89)=(8,12)=(10,11)=(10,12)=(11,12)

(14)=(18)=(1,10)=(1,11)=(24)=(27)=(29)=(2,12)=(35)=(36) (A15)
=(38)=(3,12)=(47)=(4,11)=(57)=(5,10)=(5,12)
=(68)=(69)=(6,11)=(7,10)=(8,10)=(9,11)=(9,12)

(16)=(17)=(25)=(2,11)=(39)=(3,10)=(49)=(4,10)=(58)=(6,12)
=(7,12)=(8,11)

One can also use the antisymmetry of the nuclear spin-isospin wavefunctions and the symmetry of the nucleon spin-isospin wavefunction to show that the following matrix elements from the list above vanish: $(12)_5$, $(15)_5$, $(1,9)_{10}$, $(1,10)_{10}$, $(1,12)_{10}$, $(23)_{10}$, $(9,10)_{10}$, $(15)_{10}$,

$(9,12)_{10}$ and $(10,11)_{10}$. All matrix elements occuring in (13), (19) can be written as a product of matrix elements in the color, space, and joint spin-isospin sectors. Given (4), (9), (11), (12) those involving color or spin-isospin are straightforward to compute. The results are given in Tables A1, A2.

In order to evaluate the spatial matrix elements for ^{3}He it is convenient to choose the natural internal coordinates of the configuration (123;456;789), namely $\underset{\sim}{\ell}_{123}$, $\underset{\sim}{\lambda}_{123}$, $\underset{\sim}{\ell}_{456}$, $\underset{\sim}{\lambda}_{456}$, $\underset{\sim}{\ell}_{789}$, $\underset{\sim}{\lambda}_{789}$, $\underset{\sim}{R}_{123;456;789}$, $\underset{\sim}{L}_{123;456;789}$. In terms of these coordinates, dropping the subscripts on $\underset{\sim}{R}$, $\underset{\sim}{L}$ for concision, one has, in the CM system

$$
\begin{aligned}
\underset{\sim}{r}_1 &= -\underset{\sim}{L}/\sqrt{6} + \underset{\sim}{R}/\sqrt{2} - \underset{\sim}{\lambda}_{123}/\sqrt{6} + \underset{\sim}{\ell}_{123}/\sqrt{2} \\
\underset{\sim}{r}_2 &= -\underset{\sim}{L}/\sqrt{6} + \underset{\sim}{R}/\sqrt{2} - \underset{\sim}{\lambda}_{123}/\sqrt{6} - \underset{\sim}{\ell}_{123}/\sqrt{2} \\
\underset{\sim}{r}_3 &= -\underset{\sim}{L}/\sqrt{6} + \underset{\sim}{R}/\sqrt{2} + 2\underset{\sim}{\lambda}_{123}/\sqrt{6} \\
\underset{\sim}{r}_4 &= -\underset{\sim}{L}/\sqrt{6} - \underset{\sim}{R}/\sqrt{2} - \underset{\sim}{\lambda}_{456}/\sqrt{6} + \underset{\sim}{\ell}_{456}/\sqrt{2} \\
\underset{\sim}{r}_5 &= -\underset{\sim}{L}/\sqrt{6} - \underset{\sim}{R}/\sqrt{2} - \underset{\sim}{\lambda}_{456}/\sqrt{6} - \underset{\sim}{\ell}_{456}/\sqrt{2} \\
\underset{\sim}{r}_6 &= -\underset{\sim}{L}/\sqrt{6} - \underset{\sim}{R}/\sqrt{2} + 2\underset{\sim}{\lambda}_{456}/\sqrt{6} \\
\underset{\sim}{r}_7 &= 2\underset{\sim}{L}/\sqrt{6} - \underset{\sim}{\lambda}_{789}/\sqrt{6} + \underset{\sim}{\ell}_{789}/\sqrt{2} \\
\underset{\sim}{r}_8 &= 2\underset{\sim}{L}/\sqrt{6} - \underset{\sim}{\lambda}_{789}/\sqrt{6} - \underset{\sim}{\ell}_{789}/\sqrt{2} \\
\underset{\sim}{r}_9 &= 2\underset{\sim}{L}/\sqrt{6} + 2\underset{\sim}{\lambda}_{789}/\sqrt{6}.
\end{aligned}
\tag{A16}
$$

from which one obtains, also

$$
\sum_i \left(-\nabla_i^2/2m\right) = -\left(\nabla_R^2+\nabla_L^2\right)/2(3m) - \left(\nabla^2_{\rho_{123}} + \nabla^2_{\lambda_{123}} + \nabla^2_{\rho_{456}} + \nabla^2_{\lambda_{456}} + \nabla^2_{\rho_{789}} + \nabla^2_{\lambda_{789}}\right)/2m \,. \tag{A17}
$$

For ^{4}He the expressions (A16) for $r_1, \ldots, r_9$ are modified by the addition of a factor $-\underset{\sim}{E}/\sqrt{12}$. Since

$$
\begin{aligned}
\underset{\sim}{r}_{10} &= 3\underset{\sim}{E}/\sqrt{12} - \underset{\sim}{\lambda}_{10,11,12}/\sqrt{6} + \underset{\sim}{\ell}_{10,11,12}/\sqrt{2} \\
\underset{\sim}{r}_{11} &= 3\underset{\sim}{E}/\sqrt{12} - \underset{\sim}{\lambda}_{10,11,12}/\sqrt{6} - \underset{\sim}{\ell}_{10,11,12}/\sqrt{2} \\
\underset{\sim}{r}_{12} &= 3\underset{\sim}{E}/\sqrt{12} + 2\underset{\sim}{\lambda}_{10,11,12}/\sqrt{6}
\end{aligned}
\tag{A18}
$$

(A17) is modified by the addition of

$$
-\nabla_E^2/(2(3m)) - (\nabla^2_{\rho_{10,11,12}} + \nabla^2_{\lambda_{10,11,12}})/2m \tag{A19}
$$

(A16), (A18) allow us to construct all quark-quark separations, $\underset{\sim}{r}_{ij}$, as well as the $\underset{\sim}{\ell}_{ijk}$, etc. for any configuration P_i in terms of the natural coordinates of (123;456;789) or (123;456;789;10,11,12). All the spatial integrals then follow straightforwardly and analytically by Gaussian integration.

Table A1 : Color and Spin-Isospin Matrix Elements for ^3He[a)]

k	i,j	$(ij)_k^{col}$	$(ij)_k^{IS}$
1	1,2	-2/3	-1/4
	0	1	1
2	1,2	-2/9	17/108
	1,3	-2/9	-7/108
	1,4	1/9	1/108
	3,6	4/9	25/108
	7,8	-2/9	1/108
	0	1/3	-4/108
3	1,2	-2/27	-43/972
	1,3	-2/27	5/972
	1,4	1/27	3/972
	1,6	-2/27	-11/972
	3,6	4/27	-11/972
	0	1/9	44/972

Table A1 (continued)

4	1,2	-2/27	13/972
	1,4	1/27	-9/972
	2,3	-2/27	-11/972
	3,4	1/27	5/972
	3,6	-2/27	5/972
	3,9	4/27	-7/972
	4,5	-2/27	13/972
	4,6	-2/27	1/972
	4,7	-1/54	-1/972
	0	1/9	-20/972
5	1,4	1/18	-1/162
	0	0	0

a) The labels, k, are as defined in the text. The matrix elements $(ij)^{col}$ and $(ij)^{IS}$ are the color matrix elements of the operator $(\underset{\sim}{\lambda}_i/2).(\underset{\sim}{\lambda}_j/2)$, and the spin-isospin matrix elements of the operator $\underset{\sim}{S}_i \cdot \underset{\sim}{S}_j$, respectively. By i,j=0 (for overlap) we mean that the operators in question are replaced by the identity operation of the appropriate sector.

Table A2: Color and Spin-Isospin Matrix Elements for ^4He[a)]

k	i,j	$(ij)_k^{col}$	$(ij)_k^{IS}$
6	1,2	-2/81	137/8748
	1,3	-2/81	-31/8748
	1,4	1/81	3/8748
	1,6	-2/81	1/8748
	1,7	1/81	5/8748
	3,6	4/81	97/8748
	3,9	4/81	1/8748
	0	1/27	-100/8748
7	1,2	-2/27	-17/2916
	1,3	-2/27	-1/2916
	1,4	1/27	7/2916
	3,6	4/27	-25/2916
	0	1/9	4/2916
8	1,2	-2/81	-47/8748
	1,3	-2/81	25/8748
	1,4	1/81	21/8748
	1,7	1/81	-7/8748
	1,12	-2/81	9/8748
	2,3	-2/81	1/8748

	2,4	-2/81	-23/8748
	2,6	4/81	89/8748
	2,7	1/81	-1/8748
	2,10	1/81	7/8748
	2,12	-2/81	1/8748
	3,4	1/81	-7/8748
	3,6	-2/81	-31/8748
	3,7	-2/81	5/8748
	3,9	4/81	5/8748
	3,10	-2/81	13/8748
	3,12	4/81	-43/8748
	4,5	-2/81	25/8748
	4,7	-1/162	-2/8748
	4,12	1/81	1/8748
	7,8	-2/81	-47/8748
	7,10	1/81	0
	7,12	-2/81	13/8748
	0	1/27	28/8748
9	1,2	-2/81	-19/8748
	1,3	-2/81	17/8748
	1,4	1/81	21/8748
	1,6	1/81	5/8748
	1,7	1/81	-6/8748

	1,10	-1/162	4/8748
	2,3	-2/81	5/8748
	2,5	4/81	1/8748
	2,6	-2/81	13/8748
	2,7	1/81	1/8748
	3,6	1/81	1/8748
	3,7	-2/81	11/8748
	3,9	4/81	-41/8748
	3,10	-1/162	-1/8748
	3,12	1/81	-7/8748
	7,8	-2/81	-19/8748
	7,10	1/324	1/8748
	0	1/27	-4/8748
10	1,2	0	----
	1,4	1/54	-5/8748
	1,7	1/54	-3/8748
	2,10	1/108	3/8748
	3,6	-1/54	4/8748
	3,9	-1/54	2/8748
	0	0	0

11	1,2	-2/81	9/8748
	1,3	-2/81	9/8748
	1,4	-1/162	5/8748
	1,7	1/81	-5/8748
	3,5	-2/81	9/8748
	3,10	-2/81	25/8748
	3,12	4/81	-13/8748
	0	1/27	-36/8748
12	1,2	-1/81	1/5832
	1,4	-1/324	-1/5832
	1,6	5/324	1/5832
	3,4	1/162	1/5832
	0	1/54	-4/5832

[a] All notation as in Table A1. The entries for k=1,...,5, which are identical to those of Table A1, have been omitted.

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FIGURE CAPTIONS

Figure 1: Model Charge Form Factors for ^{4}He

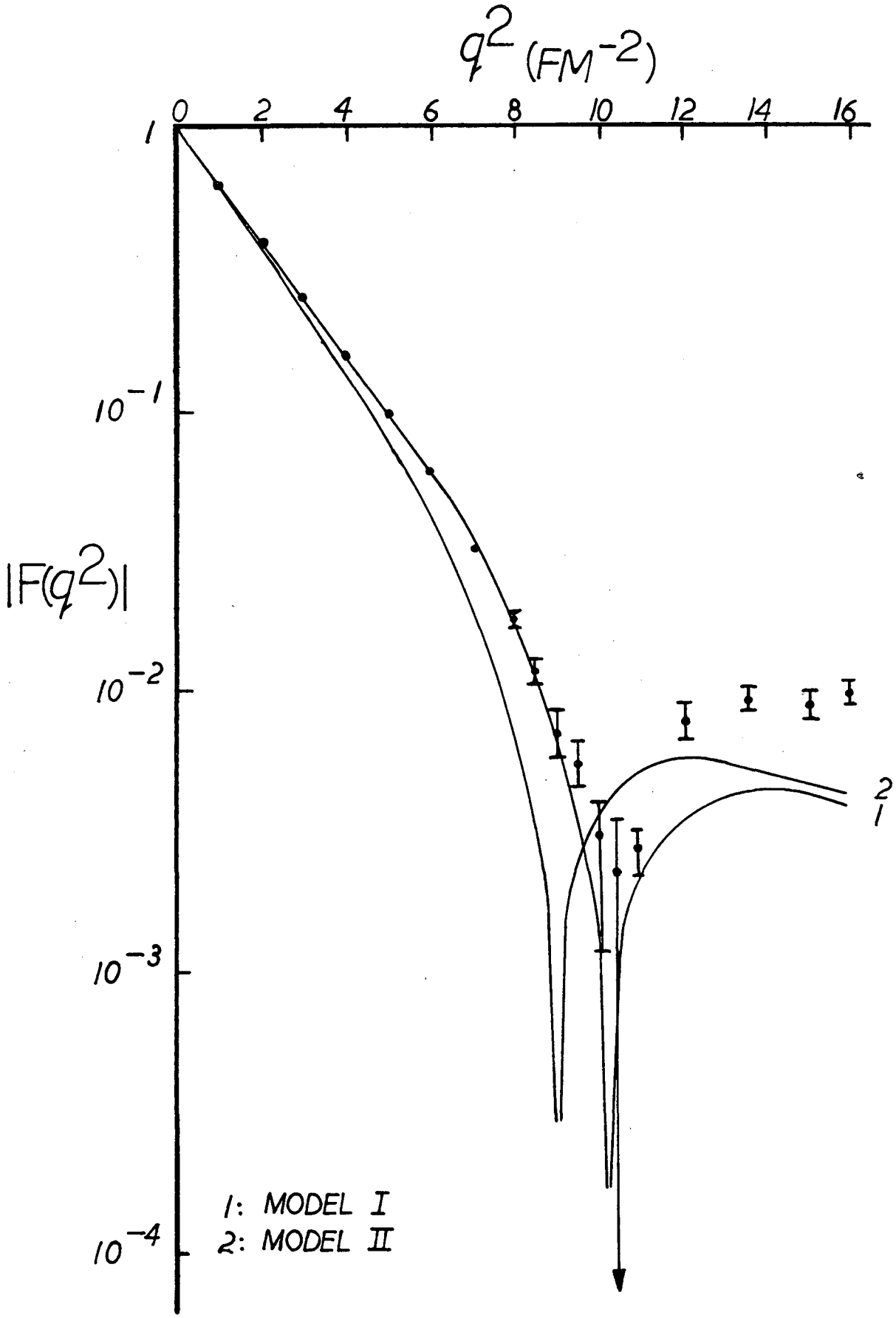
q^2 (FM^{-2})
0
2
4
6
8
10
12
14
16
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$|F(q^2)|$
2
1
1: MODEL I
2: MODEL II

FIG. 1

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