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WATER WAVE MACHINE FOR DEMONSTRATING GROUP VELOCITY

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Nothing beats water waves for demonstrating phase and group velocity. The trouble is, you need a long path, because the waves travel fast. A ripple tank is too small. A pleasing way is to get in a canoe in the middle of a smooth lake and rock the boat a half-dozen times. One easily sees that the individual waves travel faster than the group of waves. Another way that works is to throw stones into a smooth pond and watch the expanding circular wave pocket. Both of these ways have inconveniences and limitations. Neither enables you to easily measure, without taking a moving picture and playing it back in slow motion, the famous factor of 2: phase velocity is about twice the group velocity, for deep water waves of more than a few centimeters wavelength.

Recently I discovered on the Berkeley campus a rectangular outdoor fountain, built for architectural and aesthetic purposes.[1] This pool has enabled me to easily make the best "real time" demonstration of group versus phase velocity that I have seen. It occurs to me that such fountains may exist on other campuses, hence this note.[2]

The rectangular pool is 14 ft wide, 25 ft long, with water about 1 ft deep. Near one end a pipe projects above the water surface and spouts water several feet into the air. To eliminate the distracting waves from the fountain I wade out and jam a styrofoam cup over the pipe. That turns it off.

At one end of the pool is my wave machine. It consists of a 3/4-in. by 12-in. wooden board, 14 ft long, with a "broom handle" fastened to the middle. For the handle I use a "push squeegee" that I stole from a janitor's closet. It is like a push broom except that in place of the broom there is a 12-in. by 6-in. board (with a squeegee on one edge). This board I simply C-clamp to my 14-ft board.

At the other end of the pool I built a "resistive load" to terminate the waves without reflection. The termination consists of a sloping "beach," making an angle of about 30 degrees to the horizontal. The waves run up the beach and dribble back without reflecting. The beach consists of a 14-ft-long 12-in.-wide board. One edge of this board rests on a concrete ledge at the end of the pool. The other edge rests in the water about an inch below the surface. The water supports (floats) that edge. Wave energy is degraded mostly in the waves running up the beach and dribbling back, but perhaps partly also in their lifting and lowering the floating edge of the board. The termination works well. Without it the wave packets reflect, return to the wave generator, and there reflect again. The pool then takes an inconveniently long time to calm down. With the beach installed the reflection is practically eliminated. The beach can be removed in a few seconds, so as to demonstrate its effectiveness as a terminating load. That in itself is an interesting demonstration.

To make a wave packet I hold the handle horizontal so the wave-machine board is vertical, floating with about half of its 12-in. width submerged. I push on the handle perhaps eight times and stop. When the sun is out each wave acts as a cylindrical lens that focuses the sunlight to a bright band on the bottom of the pool, making it easy (and beautiful) to see the waves. Alternatively, the wave profile is easily seen at the lateral ends of the waves (at the sides of the pool). After making a packet I can, if I wish, walk at the group velocity along the side of the pool so as to keep the packet under close observation. It is fascinating to watch individual waves appear at the rear of the packet, travel through the packet, and disappear at the front. It is like looking through a window past which an infinite wave train travels,

the waves coming into view at the rear edge of the window, traveling past the window and leaving at the front.

The frequency at which I push the handle is limited by the inertia of the board and water, and my strength. Typical rates for my machine are 2 to 3 Hz. That gives wave packets with predominant wavelengths of about 1 ft to $\frac{1}{2}$ ft, with group velocities of about 1 ft/sec and phase velocities about 2 ft/sec. If I wish to make packets with frequencies up to about 5 Hz I can do so by letting the board float flat on the water and jiggling the handle so as to "rock the boat."

With my machine I can easily make a rough measurement of the ratio of phase to group velocity with my naked eye as follows: I shake the wave machine a certain counted number of times. Call that number N (waves made). Then I stop generating waves and watch the group. I count the number of waves in the group at a given instant. (I can repeat that count several times before the packet reaches the beach.) Call that number N(waves in group), Amazingly, N(waves in group) is less than N(waves made)! By the time the last wave is made, the first waves made have traveled to the front of the group and disappeared! With a sketch representing the situation at the instant when the last wave was made, with a mark for each wave made (including those that have disappeared out the front of the group) the reader may easily convince herself that the ratio of phase velocity to group velocity is given by

$$\frac{v(\text{phase})}{v(\text{group})} = \frac{N(\text{waves made})}{N(\text{waves in group})} . \qquad (1)$$

I usually generate a packet by pushing the machine 10 times. When I count the waves in the packet I get sometimes 4, sometimes 5, sometimes 6. I conclude that at frequencies of 2 or 3 Hz the phase velocity is about twice the group velocity. That agrees with the textbooks.[3]

References

1. The fountain is located in an outdoor raised courtyard south of Latimer Hall.

2. Now that I have seen how beautifully this pool works, I would build a pool about 25 ft long, 1 or 2 ft wide, and 2 ft deep, aligned north and south, if I didn't already have the fountain.

3. The dispersion relation for deep water gravity waves is $\omega = \sqrt{g\,k}$. That gives $v(\text{phase}) = \omega/k = \sqrt{g/k}$, and $v(\text{group}) = d\omega/dk = \frac{1}{2}\sqrt{g/k}$.

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