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NOTE ON THE PARTITIONING OF NUCLEI

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Note on the Partitioning of Nuclei

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Abstract

We show that the information theory method of Aichelin and Huefner for calculating the fragment distribution resulting from high energy proton bombardments is an approximation to the Euler number partition. We find that a consistent application of the theory produces fragment distributions substantially steeper than the calculation of Aichelin and Huefner. The quality of fit to the data and its implications are discussed.

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The inclusive charge and mass distributions of high energy proton and heavy ion bombardments are frequently interpreted in terms of a liquid-gas phase transition.[1,2] However, Aichelin and Huefner[3] have recently shown that the charge distributions for high energy p + Kr, Ag, Xe and U can be reproduced by a calculation which is portrayed to produce the least biased distribution. We show that the approach of Aichelin and Huefner is an approximation to the Euler number partition and that such an approximation breaks down for large fragment size. Furthermore, we show that the most reasonable application of these types of theories differs substantially from the previously published work.[3] Although this theory is not inconsistent with the data its applicability to the nuclear case is made uncertain by the lack of appropriate geometrical constraints.

We wish to calculate the part size distribution when an integer is decomposed. As with Aichelin and Huefner, our assumption will be that all decompositions are equally likely.

Let us calculate the number of times a part of size a appears when we fracture an integer N in all of the possible ways. We start by calculating the number of unrestricted partitions P(N) of the integer N by using Euler's recurrence relation[4]

$$P(N) + (-1)^K[P(N - K_1) + P(N - K_2)] + \ldots =$$

$$P(N) - P(N - 1) - P(N - 2) + P(N - 5) + P(N - 7) \ldots = 0 \qquad (1)$$

where $K_1 = (3K^2 - K)/2$, $K_2 = (3K^2 + K)/2$

and $K = 1, 2, 3, 4, 5, \ldots$

For example: $P(1) = 1$, $P(2) = 2$, $P(3) = 3$, $P(4) = 5$, $P(5) = 7$, $P(6) = 11$ etc.

The number of times a part of size a occurs when the integer N is fractured in each of the P(N) ways is

$$P_a(N) = P(N - a) + P(N - 2a) + P(N - 3a) + \ldots P(0) \tag{2}$$

There is no overcounting in equation 2 due to the removal of the part of size a in each successive term. If all fractures are equally likely then $P_a(N)$ is proportional to the probability of producing a part of size a. Aichelin and Huefner calculate the equivalent probability $P_a(N)$ as follows. They define the probability P(m,a) of producing a fragment of size a with a multiplicity m as,

$$\sum_{m=0} P(m,a) = 1 \quad \text{for each } \underline{a}$$

$$\text{and} \quad \sum_{m=0} \sum_{a}^{N} ma\, P(m,a) = N \tag{3}$$

where N is the size of the object being fragmented.

The information contained in P is

$$I = \sum_{m=0} \sum_{a}^{N} P(m,a) \ln P(m,a) \tag{4}$$

Introducing the constraints 3, 4 one obtains the maximum

$$\frac{\delta}{\delta P(m,a)} \sum_{m} \sum_{a}^{N} [P(m,a) \ln P(m,a) - C(a)P(m,a) - DmaP(m,a)] = 0 \tag{5}$$

which gives

$$P(m,a) = C(a)\exp(-Dma) \tag{6}$$

From eq. 3 one obtains

$$C(a) = 1 - \exp(-Da) \tag{7}$$

and from eq. 5 one obtains

$$N = \sum_{m}^{\infty} \sum_{a}^{N} ma[1 - \exp(- Da)]\exp(-Dma)$$

$$= \sum_{a}^{N} a/(\exp(Da) - 1) \simeq D^{-2}\Gamma(2)\zeta(2) \tag{8}$$

where $\zeta(x)$ is Riemann's zeta function.

From which

$$P_a(N) \simeq [\exp 1.28a/\sqrt{N} - 1]^{-1} \tag{9}$$

Conditions 3 and 4 make this problem a saddle point method approximation to the Euler number partition. Therefore the part size distribution from eq. 2 should be identical to that obtained with the method reproduced above. Figure 1 shows the part size distributions for two values of N for both eqs. 2 and 9. We can immediately notice that the two calculations are nearly identical for small fragments. However, the calculations diverge as the part size increases. This divergence can be traced to the upper limit in the

summation over m used in obtaining eq. 7, and in eq. 8, which is taken to be N while it should be int(N/a).

The comparison between the calculation and data presented in ref 3 is obtained by identifying N with Z_0, the total charge minus a certain number of charges presumably directly knocked out by the projectile. However since these theories do not involve any potentials and all "fractures" are equally likely, the only consistent application of these theories is to identify N with the total number of nucleons. Unfortunately eq. 9 shows that the partition theory does not scale trivially from Z to A. In other words the calculation is not invariant with respect to the transformation $A = Zf(Z)$ where $f(Z)$ is a slowly varying function of Z. Therefore the calculation yields dramatically different results if one uses Z rather than A. The point is illustrated in Fig. 2. The solid line is an "inappropriate" calculation of the charge distribution obtained by calculating the number of partitions of $Z_0 = 43$ charges. If we calculate the number of partitions of $A_0 = 99$ and transform the mass distribution into a charge distribution by assuming an average charge per part of $Z = a \times Z_0/A_0$ we obtain the dot-dashed curve. This curve, which, we believe, is the only approach consistent with the spirit of the theory, has a steeper slope than a calculation where the charges are partitioned and the neutral particles ignored (solid line).

The experimental charge distributions[5] and mass distributions[1] are compared to calculations in Figs. 2 and 3 respectively. The data are more or less reproduced by number partitions in either of the versions, but cannot distinguish between them. This approximate agreement is somewhat surprising since both the isospin and geometrical aspects of the problem are ignored.

Some insight into the geometrical aspect of the problem can be gained from the studies on the degradation of a linear polymer.[6] The problem of determining the part size distribution of degraded polymers is not related to partitions (where the order of summands is irrelevant) but rather to permuted partitions. The extension of such a calculation to 2 or 3 dimensional polymer is much more complicated and probably can be dealt with within the framework of percolation theories. Although the bond breaking in a polymer may not be the best illustration of nuclear fragmentation, it indicates the required complications that arise when the dimensionality of the problem is considered. As a consequence we find the connection between number partition and fragmentation somewhat tenuous and we wonder whether the fair comparison between data and theory may not be fortuitous.

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References

1. J. E. Finn et al., Phys. Rev. Lett. 49 (1982) 1321

2. A. D. Panagiotou et al., Phys. Rev. Lett. 52 (1984) 496 and references therein

3. J. Aichelin and J. Huefner, Phys. Lett. 136B (1984) 15

4. J. Riordan, An Introduction to Combinatorial Analysis, John Wiley and Sons, Inc., N.Y. (1967), pg. 117

5. G. D. Westfall et al., C17 (1978) 1368

6. V. S. Nanda and R. K. Pathria, Jour. Chem. Phys. 30 (1959) 27

Figure Captions

Fig. 1

Part size distributions. The solid lines are the calculation using partition theory (eq. 2) while the dashed line uses the expression of Aichelin and Huefner (eq. 9). This second calculation (eq. 9) is normalized to the number theory calculation at a = 1. The thin curve is a distribution given by $a^{-2.64}$ which fits both p + Kr and p + Xe data of ref. 1.

Fig. 2

The charge distributions calculated in various ways, see text.

Fig. 3

Mass distribution calculated from number partitions. The dots are the data of Finn et al.[1]

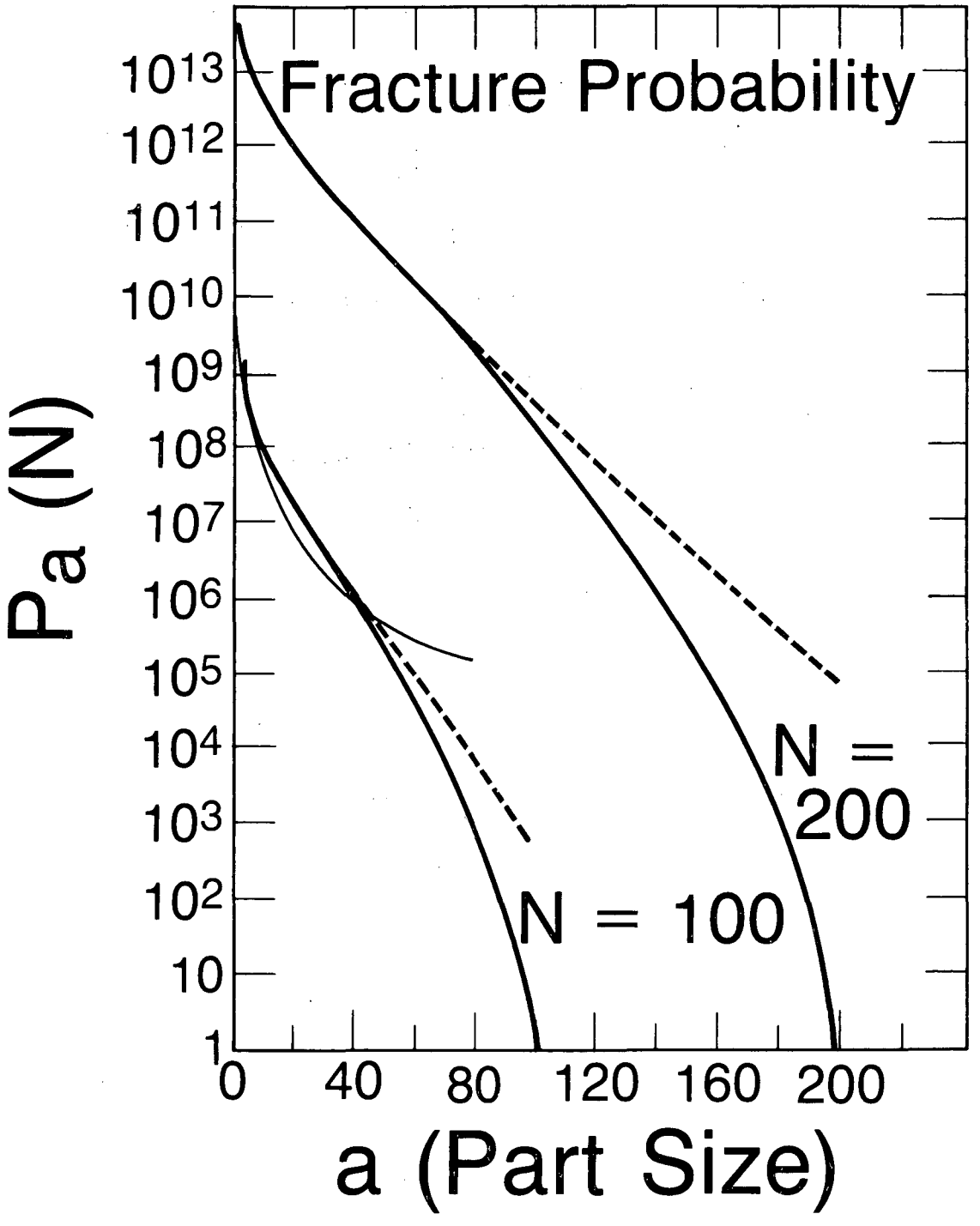

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Fig. 1

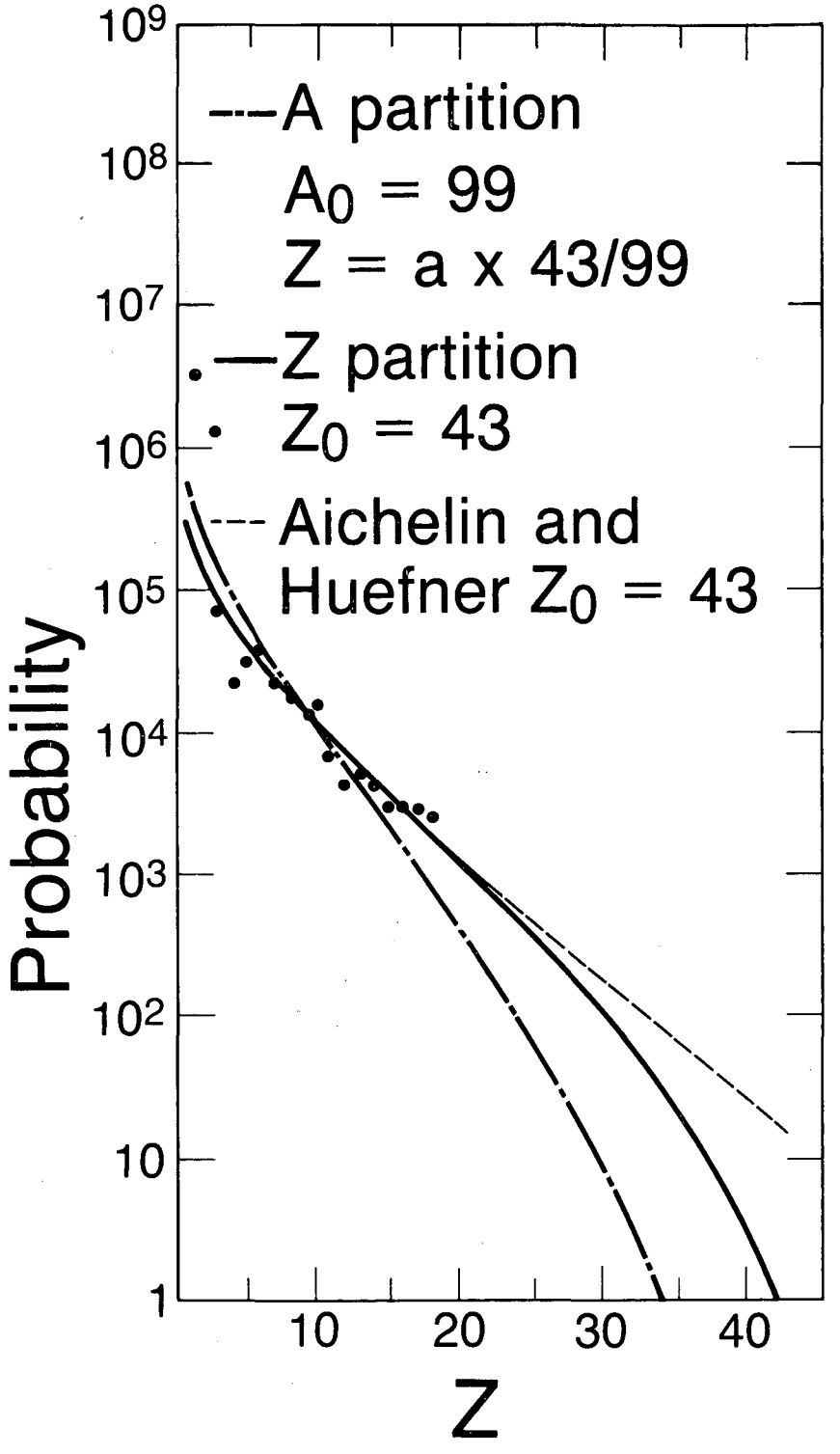

Fig. 2

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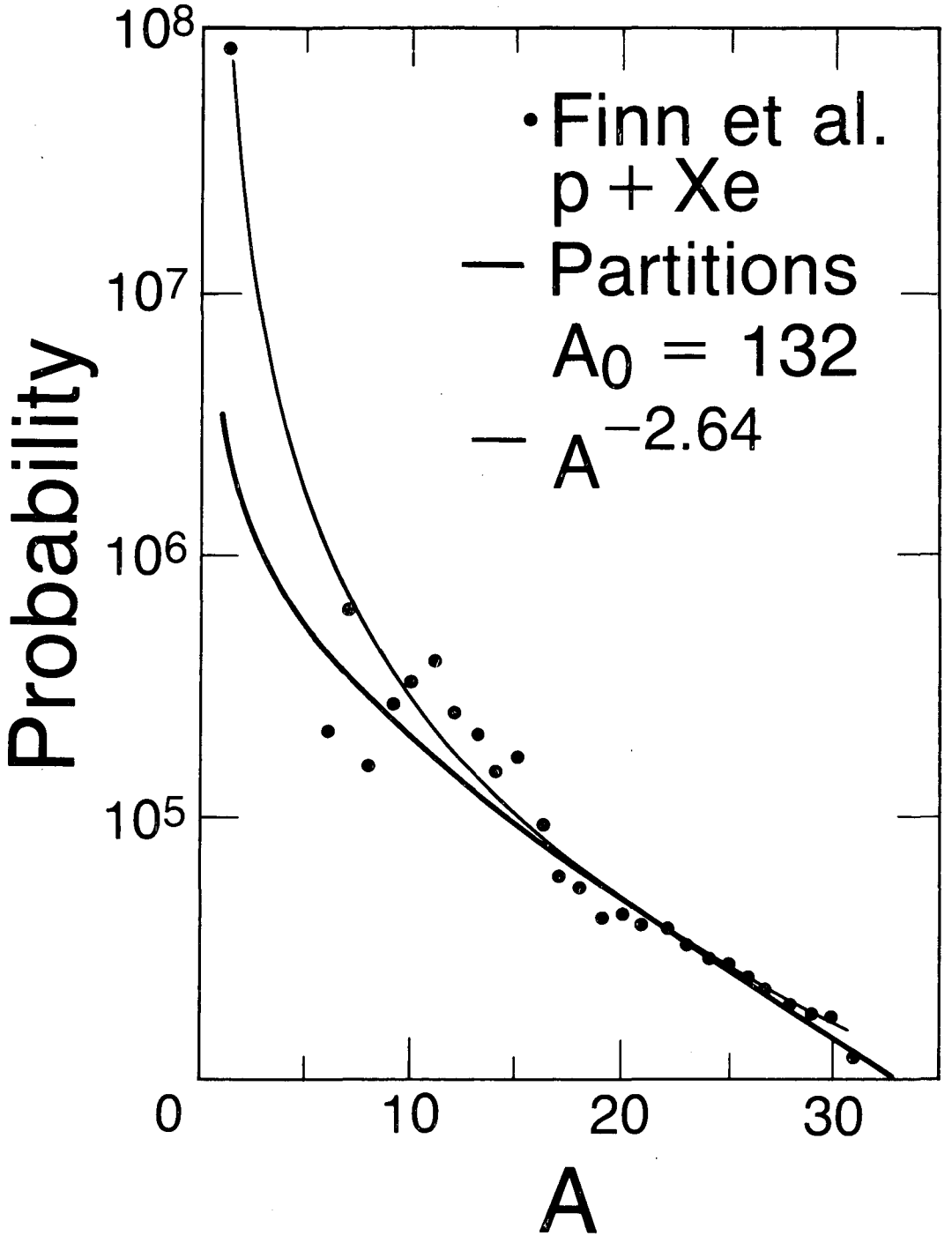

Fig. 3

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