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PART I: AN ESTIMATE OF THE POMERANCHUKON-RHO CUT FOR PI-NUCLEON SCATTERING
PART II: FIXED POLES AND FESR FOR K^+p SCATTERING

Mark Dubovoy
(Ph.D. thesis)

June 4, 1973

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PART I: AN ESTIMATE OF THE POMERANCHUKON-RHO CUT FOR PI-NUCLEON SCATTERING*

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ABSTRACT

We show that it is plausible to find Nonsense Wrong Signature Zeroes (NWSZ) in inclusive reactions. We briefly review how they appear in certain models, and then give some model independent arguments that make their existence in certain cases a strong possibility. In particular, by using Finite Missing Mass Sum Rules (FMMSR) we show that the triple-Regge residue $g_{P\rho}{}^{\rho}(t)$ probably has a NWSZ at $t = 0$. We then propose a parametrization of $g_{P\rho}{}^{\rho}(t)$ for small t and proceed to estimate this particular coupling phenomenologically.

Using our estimate of $g_{P\rho}{}^{\rho}(t)$ we estimate the contribution of the ρ-P cut to the total πN cross section and calculate the polarization in $\pi^- p$ charge exchange scattering as a ρ-pole ρ-P cut interference. We find that it is possible to obtain a fairly good fit to the polarization for values of our parameters that are consistent with experiment, and furthermore, we find that our fit to the polarization is very sensitive to the small t behavior of $g_{P\rho}{}^{\rho}(t)$.

I. NONSENSE-WRONG SIGNATURE ZEROES IN INCLUSIVE REACTIONS

Due to the work of Mueller[1] in which inclusive cross sections are related to a certain discontinuity of the three-to-three forward amplitude, much interest arises in the Regge properties of three-to-three scattering amplitudes. In particular, it is interesting to study whether certain properties that we are familiar with from two-to-two scattering amplitudes are still retained in certain kinematical regions of three-to-three amplitudes.

In this section we will specialize in the problem of Nonsense-Wrong Signature Zeroes (NWSZ) in inclusive reactions, where they appear and why, what processes are useful to test these ideas, and finally, how Finite Missing Mass Sum Rules (FMMSR) may help us test the former ideas when certain experimental data are not available.

In what follows, we shall use an extension of the conventional definitions of nonsense and wrong signature.[2] Let us consider the reaction $a + b \to c$ + anything, and define $s = (p_a + p_b)^2$; $t = (p_b - p_c)^2$; $M^2 = (p_a + p_b - p_c)^2$. The diagram for this reaction in the triple Regge limit[3] is shown in Fig. 1. The signature factor in this case is given by[4]

$$\xi(t) = \frac{\tau + \exp\{-i\pi(\alpha_3(0) - \alpha_1(t) - \alpha_2(t))\}}{\sin \pi(\alpha_3(0) - \alpha_1(t) - \alpha_2(t))}$$

where τ is the product of the three signatures of reggeons 1, 2, and 3 respectively. Since in all the diagrams we will deal with, $\tau = 1$, we see that $\xi(t)$ becomes infinite when $\alpha_3(0) - \alpha_1(t) - \alpha_2(t) = -2, -4, -6, \cdots$. We will call this a right-signature point, whereas

$$\alpha_3(0) - \alpha_1(t) - \alpha_2(t) = -1,-3,-5,\cdots \qquad (1)$$

will be referred to as a wrong-signature point. Notice that when Eq. (1) is satisfied, $\xi(t)$ is finite. Also, following the usual definition of nonsense,[2] we see by looking at the triple-Regge vertex that both sets of points mentioned above are nonsense.

A. Appearance of NWSZ in Inclusive Reactions

1. Dual Model

In the dual resonance model one can study single particle distributions, and actually evaluate these distributions explicitly in certain kinematical regions.

Let us consider again the reaction

$$a + b \rightarrow c + \text{anything}$$

and define the invariant variables as usual:

$$s = (p_a + p_b)^2 \qquad t = (p_b - p_c)^2$$

$$M^2 = (p_a + p_b - p_c)^2 \ .$$

The relevant diagram for this reaction in the triple-Regge region[3] (which will be the interesting kinematical region in our case) is shown in Fig. 1. Gordon and Veneziano[5] obtain an explicit formula for the cross section of this process by calculating the discontinuities of elastic six-point functions in the tree approximation. In particular, for a diagram of the form shown in Fig. 2 they use the amplitude

$$B_6^{(1b)} = \int\int_0^1\int dxdydz\, x^{-\alpha_s-1}\, y^{-\alpha_{\bar{s}}-1}\, z^{-\alpha_M-1}\, (1-x)^{-\alpha_t-1}$$

$$\times\ (1-y)^{-\alpha_t-1}\, (1-z)^{-\alpha_0-1}\, (1-xz)^{t+2\alpha_0}$$

$$\times\ (1-yz)^{t+2\alpha_0}\, (1-xyz)^{-2\alpha_0}$$

where we have defined $\bar{s} = (p_a' + p_b')^2$; $\alpha_3(0) = \alpha_0$; $\alpha_1(t) = \alpha_2(t) = \alpha(t) = \alpha_t$, $\alpha(s) = \alpha_s$, $\alpha(\bar{s}) = \alpha_{\bar{s}}$, and $\alpha(M^2) = \alpha_M$.

Then, by taking the limit when $\mathrm{Re}\,\alpha_s$, $\mathrm{Re}\,\alpha_{\bar{s}}$, and $\mathrm{Re}\,\alpha_M \rightarrow -\infty$ with $\mathrm{Re}\,\alpha_s/\mathrm{Re}\,\alpha_M \rightarrow \infty$ and the discontinuity across the α_M cut, one ends up with the explicit expression

$$\frac{d\sigma}{d^3p_c/p_{c0}} \underset{\substack{s\rightarrow\infty \\ 1\ll\alpha_M\ll\alpha_s \\ t\ \text{fixed}}}{\sim} \frac{\alpha_s^{\alpha_0-1}\, \alpha'^2\, \gamma^4\, \Gamma^2(-\alpha_t)}{\pi^2\, \Gamma(1+\alpha_0-2\alpha_t)}$$

$$\times \left(\frac{\alpha_s}{\alpha_M}\right)^{-2\alpha_t-\alpha_0} \cos^2[\pi\alpha(t)/2]$$

where γ is the coupling constant. From the factor $\Gamma(1+\alpha_0-2\alpha_t)$ in the denominator of this expression, it is clear that the dual amplitude exhibits zeros when $\alpha_0 - 2\alpha_t = -1,-2,\cdots$. These zeroes are nonsense zeroes analogous to the ones that appear in two-to-two amplitudes as will be shown later. Since the Gordon-Veneziano model is an exchange degenerate dual resonance model, there are nonsense zeroes at both right- and wrong-signature points.

2. Feynman Graph Model

Chang, Gordon, Low, and Treiman[6] have performed a simple model calculation, based on a single type multiladder Feynman diagram where nonsense zeroes also appear. Their model is based on the diagram of Fig. 3, where the boxes describe off-mass-shell $2 \to 2$ amplitudes for which they adopt the Regge properties corresponding to ladder graphs. The Regge trajectory function associated with momentum transfer t is $\alpha(t)$, and the unsignatured amplitude associated with the corresponding two boxes is taken to be

$$R_t = \frac{-[(k - p_2)^2]^{\alpha(t)}}{\sin \pi\alpha(t)} \beta_t\left(k^2, (k + p_1 - p_2)^2, t\right) .$$

Analogously the trajectory function associated with momentum transfer zero is α_0, and the amplitude associated with the corresponding box is

$$R_0 = \frac{-[(k + p + p_1 + p_2)^2]^{\alpha_0}}{\sin \pi\alpha_0} \beta_0\left((k + p_1 - p_2)^2, (k + p_1 - p_2)^2\right) .$$

Without going into the details of their calculation, we briefly mention the results. Chang et al. obtain an explicit formula for the single-particle inclusive amplitude A in the asymptotic region when $M^2 \to \infty$, $\frac{s}{M^2} \to \infty$,[6] i.e., the asymptotic limit of the triple-Regge region. Their method of calculation consists of taking the above mentioned limit, and then to use a spectral representation based on the identity

$$\frac{-(-s)^\alpha}{\sin \pi\alpha} = \frac{1}{\pi} \int_0^\infty ds' \left(\frac{s'^\alpha}{s' - s - i\epsilon}\right) \qquad -1 < \alpha < 0$$

in order to carry out the k integration in the Feynman diagram. Noticing that this identity does not hold for $\alpha > 0$, one must do an analytic continuation after the k-space integration has been carried out.

The final formula Chang et al. obtain is as follows,

$$A \underset{\substack{s/M^2 \to \infty \\ M^2 \to \infty}}{\sim} \frac{(M^2)^{\alpha_0} \left(\frac{s}{M^2}\right)^{2\alpha_t} \Gamma(\alpha_0 + 1)\, \Gamma(1 - 2\alpha_t)}{16\pi^2 \sin^2 \pi\alpha_t\, \Gamma(1 + \alpha_0 - 2\alpha_t)}$$

$$\times \int d\mu_1^2 d\mu_2^2\, \rho(\mu_1^2, \mu^2, t) \int du\, x_4\, dx_4\, \delta(u + x_4 - 1) u^{-\alpha_0 - 2\alpha_t}$$

$$\times \left[t\, x_4\, u + \mu^2 x_4 + \mu_1^2 u\right]^{2\alpha_t - 1}$$

where the variables u, μ_1, μ_2, and x_4 are integration variables defined in a convenient way, and ρ is a double-spectral function.[6] Again, we can see that in this model the factor $\Gamma(1 + \alpha_0 - 2\alpha_t)$ appears in the denominator, so that in this model one expects the same result as in the dual model: vanishing of the amplitude when

$$\alpha_0 - 2\alpha_t = -1, -2, \cdots \quad . \qquad (2)$$

3. Double O(2,1) Expansion

A much better plausibility argument for the existence of NSWSZ in the amplitude of a single particle inclusive process, in the triple-Regge region (as long as the existence of Nonsense-Wrong-Signature Fixed Poles is neglected), can be found by performing a double O(2,1) expansion. Here the advantage lies in the fact that the argument is essentially model independent.

One can define functions that play a completely analogous role to the one the more familiar e-functions, or functions of the second kind play in the usual reggeology of two-body reactions. In particular, if one defines

$$d^{\ell}_{\mu,-}(\xi) = \frac{\Gamma(\ell + 1 - i\mu)}{\pi\Gamma(\ell + 1)} e^{\mu\pi} e^{i\pi/2(\ell+1)} Q_{\ell}^{i\mu}(i \sinh \xi)$$

$$d^{\ell}_{\mu,+}(-\xi) = d^{\ell}_{\mu,-}(\xi)$$

where the continuation to $z = i \sinh \xi$ is made above the singularity at $z = 1$, it has been shown by Jones et al.[7] (hereby referred to as JLY) that one can make generalized $O(2,1)$ expansions for asymptotic growing amplitudes, and in particular if one defines

$$f_{\lambda}(\alpha,\xi) = \frac{A(\alpha,\xi)}{(\cosh \alpha)^{\lambda_1}(\cosh \xi)^{\lambda_2}}$$

where λ_1 and λ_2 are such that $f_{\lambda}(\alpha,\xi)$ is square integrable in α and $\sinh \xi$ then one can make a Mandelstam-Sommerfeld-Watson transformation, the result being [see Eq. (2.13) in Ref. 7]

$$f_{\lambda}(\xi) = \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} \left(\frac{d\ell}{2i}\right)\left(\frac{2\ell + 1}{\tan \pi\ell}\right)\left\{a_{\mu}^{(1)}(\ell)\left[d^{\ell *}_{\mu,+}(\xi) - \left(\frac{\cosh \mu\pi}{\cos \ell\pi}\right)\right.\right.$$

$$\left. \times\ d^{\ell *}_{\mu,-}(\xi) - \frac{\pi\ \Gamma(\ell + 1)\ d^{-\ell-1 *}_{\mu,-}(\xi)}{\cos \ell\pi\ \Gamma(-\ell)\ \Gamma(-i\mu + \ell + 1)\ \Gamma(i\mu + \ell + 1)}\right]$$

$$+ a_{\mu}^{(2)}(\ell)\left[d^{\ell *}_{\mu,-}(\xi) - \left(\frac{\cosh \mu\pi}{\cos \ell\pi}\right) d^{\ell *}_{\mu,+}(\xi)\right.$$

Equation (3) continued next page

Equation (3) continued

$$\left.\left. - \frac{\pi\ \Gamma(\ell + 1)\ d^{-\ell-1 *}_{\mu,+}(\xi)}{\cos \ell\pi\ \Gamma(-\ell)\ \Gamma(i\mu + \ell + 1)\ \Gamma(-i\mu + \ell + 1)}\right]\right\} \qquad (3)$$

where

$$a_{\mu}^{(1)}(\ell) = \frac{1}{2\pi}\int_0^{\infty} d(\sinh \xi)\ d^{\ell}_{\mu,+}(-\xi)\ f_{\lambda}(-\xi)$$

and

$$a_{\mu}^{(2)}(\ell) = \frac{1}{2\pi}\int_0^{\infty} d(\sinh \xi)\ d^{\ell}_{\mu,+}(-\xi)\ f_{\lambda}(\xi) \ .$$

As in the two-to-two body case, using the symmetry properties of JLY's d-functions around $\mathrm{Re}\ \ell = -\frac{1}{2}$, one can easily see that only the terms containing a $d^{-\ell-1}$ survive. Actually the JLY expansion is similar to the usual $O(3)$ expansion in terms of e-functions, and as a matter of fact[8,9]

$$d^{\ell}_{\mu,-}(z) = \frac{2^{\ell}\ \Gamma(\ell + 1 - i\mu)\ \Gamma(\ell + 1 + i\mu)}{\pi\ \Gamma(2 + 2\ell)} \exp\{i\pi/2(\ell + 1)\}$$

$$\times\ (z + 1)^{\frac{1}{2}i\mu}\ (z - 1)^{-\frac{1}{2}-i\mu-\ell-1}\ F\left(\ell + i\mu + 1,\ \ell + 1,\ 2\ell + 2,\ \frac{2}{1 - z}\right) \qquad (4a)$$

and

$$e^{\ell}_{-i\mu,0}(z) = (-1)^{-i\mu+\ell+1}\, 2^{\ell}\, \frac{\Gamma(\ell+1)}{\Gamma(2\ell+2)}\, (1-z)^{-\frac{i\mu}{2}-\ell-1}\, (z+1)^{i\mu/2}$$

$$\times\ [\Gamma(\ell - i\mu + 1)\, \Gamma(\ell + i\mu + 1)]^{\frac{1}{2}}\, F\left(\ell + i\mu + 1,\ \ell + 1,\ 2\ell + 2, \frac{2}{1-z}\right) \tag{4b}$$

so that comparing (4a) and (4b) we see that the JLY d-functions differ from the usual e-functions by a normalization factor of

$$\frac{[\Gamma(\ell - i\mu + 1)\, \Gamma(\ell + i\mu + 1)]^{\frac{1}{2}}}{\Gamma(\ell+1)}\ \exp\left(i\pi/2(\ell+1)\right)\ (-1)^{\frac{1}{2}i\mu - 2(\ell+1)} \tag{5}$$

(we shall omit the phase factor from now on, since it is irrelevant for our argument). This is not unexpected, since one can go continuously from the group $O(2,1)$ to $O(3)$. Let us therefore extract this factor, and change the normalization of the JLY d-functions to match that of the usual e-functions. This implies that this factor has to be included in our amplitude, but the amplitude now satisfies the usual partial wave unitarity equation, whereas before it did not, due to the spurious factor in Eq. (5). After this is done, we can substitute in the asymptotic form of Eq. (3) where upon substitution of Eq. (4a) [with the factor in Eq. (5) extracted] and repeated use of the identity

$$\Gamma(\alpha) = \pi[\sin \pi\alpha\, \Gamma(1-\alpha)]^{-1}$$

the factor f in front of $a^{(1)}_{\mu}(\ell)$ is such that

$$f \propto [\sin \pi(\ell + i\mu) \sin \pi(\ell - i\mu)]^{-1/2} [\Gamma(i\mu + \ell + 1)\Gamma(-i\mu + \ell + 1)]^{-3/2}\ .$$

Now, in the triple-Regge region we have (see Fig. 1) from the interpretation of the index μ as an analytically continued helicity index,[10]

$$\ell = \alpha_3(0)$$

$$i\mu = -\alpha_2(t) - \alpha_1(t)$$

so that if ℓ is either integer or half integer or if $i\mu$ is either integer or half integer

$$f \propto 1/\{\sin \pi(\ell + i\mu)[\Gamma(i\mu + \ell + 1)\, \Gamma(-i\mu + \ell + 1)]^{3/2}\}$$

$$= \{\sin \pi[\alpha_3(0) - \alpha_2(t) - \alpha_1(t)]\}^{-1} \{\Gamma\left(\alpha_3(0) - \alpha_1(t) - \alpha_2(t) + 1\right)$$

$$\times\ \Gamma\left(\alpha_3(0) + \alpha_1(t) + \alpha_2(t) + 1\right)\}^{-3/2}$$

and therefore as $\alpha_3(0) - \alpha_2(t) - \alpha_1(t) \to -N$ with N a positive integer, f has a square root zero. In this case, one can use exactly the same analyticity argument as in two-body reactions, to establish the existence of the NSWSZ.[11] If neither ℓ nor $i\mu$ are integer or half integer, then we have a linear zero. We see that the usual arguments for the existence of NSWSZ in two-body reactions follow through for single particle inclusive reactions and therefore unless the triple-Regge residue has a fixed pole at the point of interest, we expect to find a linear NSWSZ for inclusive reactions, whenever

$$\alpha_3(0) - \alpha_2(t) - \alpha_1(t) = -1, -3, -5, \cdots$$

if the triple-Regge signature $\tau = +1$.

Two comments are in order at this stage: First, that in the Feynman diagram model we have neglected nonplanar diagrams. If one includes nonplanar diagrams (analogous to third double spectral function effects in two-body processes) singularities may appear which actually cancel the zeroes of the amplitude when

$$\alpha_3(0) - \alpha_1(t) - \alpha_2(t) = -1, -3, \cdots$$

i.e., precisely at the wrong-signature points.[12] Second, that if one neglects nonplanar diagrams, or if one assumes that our argument using the JLY expansion is correct (and one neglects multiplicative fixed poles) then the result for the existence of the NWSZ is consistent with the vanishing of the triple pomeranchukon vertex at $t = 0$ if $\alpha_P(0) = 1$.

B. Processes that are Useful to Test for the Existence of NWSZ

As we have seen in the former chapter, the existence of NWSZ in inclusive reactions implies that the triple pomeranchukon coupling is zero at $t = 0$ if $\alpha_P(0) = 1$. This is consistent with earlier results by other methods,[13] as long as $\alpha_P(0) = 1$. Therefore, if we had extensive data in the triple-Regge region for different values of t, as well as different energies and missing masses for the reaction

$$p + p \rightarrow p + \text{anything}$$

we could hopefully isolate the different Regge terms, and test for the existence of a NWSZ. For example, if one could isolate a $P'PP$ vertex, in which the P' is the trajectory at momentum transfer zero, if we assume $\alpha_{P'}(0) = 0.5$ and $\alpha_P(t) = 1 + 0.3t$ then we expect a

NWSZ to appear at $t_0 \approx -0.8$ since at this value $\alpha_{P'}(0) - 2\alpha_P(t_0) = -1$. Notice that the value of t_0 depends on the pomeranchukon trajectory slope.

Now, let us consider instead diagrams of the form shown in Fig. 4. If $\alpha_3(t) = \alpha_2(t)$, then Eq. (1) is satisfied at $t = 0$ for any trajectory $\alpha_3(t)$, and therefore if we are at a point of wrong (triple Regge) signature we expect a nonsense zero at $t = 0$. Notice that for three boson trajectories the equality $\tau_P \tau_{\alpha_3} \tau_{\alpha_2} = +1$ where τ_α represents the signature of the reggeon α is indeed satisfied. Therefore, let us try to isolate diagrams that satisfy the former conditions, by taking suitable combinations of reactions. We introduce the following notation: $\rho(b; c|a)$ represents $(d^2\sigma/dtdM^2)$ in the triple-Regge region for the reaction $a + b \rightarrow c$ + anything where c is a fragment of b. Then, if we consider the combination

$$\rho(p; p|\pi^-) - \rho(p; p|\pi^+)$$

we obtain the diagram of Fig. 5a, and we see that this diagram satisfies all the conditions we want, and therefore we predict

$$\rho(p; p|\pi^+) \rightarrow \rho(p; p|\pi^-) \quad \text{when} \quad t \rightarrow 0 \tag{6}$$

where we have ignored nonleading triple-Regge terms.

There are a couple of points that need further clarification. In the triple-Regge region[3]

$$s^2 \frac{d^2\sigma}{dtdM^2} \propto \left(\frac{s}{M^2}\right)^{\alpha_1(t)+\alpha_2(t)} (M^2)^{\alpha_3(0)} \tag{6a}$$

(see Fig. 1). What we have referred to as nonleading diagrams are diagrams like the ones shown in Fig. 5b, and it is clear from Eq. (6a) that as long as (s/M^2) is large enough, the nonleading terms will be negligible; however, one might object that at the values of (s/M^2) and M^2 currently available in experiments, such diagrams may be important. In particular in Fig. 5b, the πpp coupling is very large, so if the triple-Regge coupling is not small, this diagram might be non-negligible, even for fairly large values of (s/M^2). Therefore, we must be aware of the fact that Eq. (6) holds only if the nonleading triple-Regge terms are negligible, an assumption that should be further investigated when more detailed fits and values for triple-Regge couplings are available.

Another point that needs to be made clear is that from Eq. (6a) alone we know that if we keep M^2 large and fixed, and let s grow,

$$\rho(p;\, p|\pi^+) \rightarrow \rho(p';\, p|\pi^-) \text{ like } s^{\alpha_P(t)+\alpha_\rho(t)-2} .$$

Therefore, what Eq. (6) really means is that in the neighborhood of $t = 0$, the two distributions should approach each other much faster than $s^{\alpha_P(t)+\alpha_\rho(t)-2}$.

The data for the π^- reaction are already available,[14] unfortunately the data for the π^+ reaction are still unavailable. We expect that in the near future one will be able to test Eq. (6) directly, and find out whether there is a NWSZ at $t = 0$ for this particular example.

If the NWSZ does not appear, we can interpret this in several ways: Either there is a nonsense-wrong-signature fixed pole which is multiplicative and cancels the zero, or the triple-Regge residue is

singular like $\frac{1}{t}$ as $t \rightarrow 0$, or the nonleading terms still have a sizeable contribution to the cross section.

One can find other combinations that will isolate diagrams of the wanted form, for example if one assumes weak exchange degeneracy $(\alpha_{P'} = \alpha_\rho)$ the combination

$$\rho(\pi^-;\, \pi^-|p) - \rho(\pi^+;\, \pi^+|p)$$

isolates a diagram similar to Fig. 5a (simply replace $p \longleftrightarrow \pi$ in this diagram). This combination has the added advantage that because of G-parity conservation it does not allow any of the reggeons to be a π, a fact that greatly reduces the number of nonleading diagrams, and actually gets rid of the most dangerous ones. Therefore, this particular combination will be very useful to us later on.

Instead of isolating the ρ exchanges, one can isolate the ω exchanges (provided the ϕ is neglected with respect to the ω) and then combinations of the form

$$\rho(n;\, n|k^-) - \rho(n;\, n|k^+) + \rho(p;\, p|k^-) - \rho(p;\, p|k^+)$$

or

$$\rho(p;\, p|\bar{p}) + \rho(p;\, p|\bar{n}) - \rho(p;\, p|p) - \rho(p;\, p|n)$$

isolate diagrams similar to Fig. 5a, but with ω exchanges instead of ρ exchanges These last combinations however, are obviously much harder to test.

C. How Finite Missing Mass Sum Rules (FMMSR) may be Useful to Test the Existence of NWSZ

Since we do not have enough data to test relation (6) directly, the question arises whether one can use other data that might be available and still try to test this relation. This can indeed be done, under certain assumptions. Let us define the following quantities:

$$\nu \equiv p_a\cdot(p_b - p_c) = \tfrac{1}{2}(M^2 - t - m_a^{\,2})$$

$$\eta \equiv p_a\cdot(p_b + p_c) = \tfrac{1}{2}(s_{ab} - s_{a\bar{c}} - m_b^{\,2} - m_c^{\,2}) \ .$$

In a completely analogous way to the way in which one derives the usual finite energy sum rules for two-body reactions, one can derive FMMSR in the inclusive case.[4] The resulting right moment (i.e., without fixed pole terms) sum rules at fixed η and t are as follows,

$$\int_0^N d\nu\ \nu^n \Big[E_c \frac{d^3\sigma}{dp_c}(a + b \to c + \text{anything})$$

$$+ (-1)^{n+1} E_b \frac{d^3\sigma}{dp_b}(a + c \to b + \text{anything})$$

$$= \sum_{i,j,k} \left(1 + (-1)^{n+1} \tau_i\, \tau_j\, \tau_k\right) \left(\frac{\eta}{N}\right)^{\alpha_i(t)+\alpha_j(t)-1}$$

$$\times \frac{N^{\alpha_k(0)+n}\ \xi_i(t)\ \xi_j(t)\ \beta_{b\bar{c}}^{\ i}(t)\ \beta_{b\bar{c}}^{\ j*}(t)\ g_{ij}^{\ k}(t)\ \beta_{a\bar{a}}^{\ k}(0)}{\alpha_k(0) + n + 1 - \alpha_i(t) - \alpha_j(t)}$$

where the τ_i are the usual Regge signatures, $\xi_i(t) = [\tau_i + e^{-i\pi\alpha_i(t)}]/(\sin \pi\alpha_i(t))$; $\beta_{b\bar{c}}^{\ i}$, $\beta_{b\bar{c}}^{\ j}$, $\beta_{a\bar{a}}^{\ k}$ are the usual two-body scattering reduced residues, and $g_{ij}^{\ k}(t)$ is the triple reggeon vertex. We can actually lump some factors together, and write the right-hand side as

$$(\text{r.h.s}) = \sum_{i,j,k} \left(1 + (-1)^{n+1} \tau_i\, \tau_j\, \tau_k\right) \left(\frac{\eta}{N}\right)^{\alpha_i(t)+\alpha_j(t)-1}$$

$$\times\ N^{\alpha_k(0)+n} \frac{\gamma_{ij}^{\ k}(t)}{\alpha_k(0) + n + 1 - \alpha_i(t) - \alpha_j(t)}$$

where we have absorbed the ξ_i's and the two-body residues into the γ. We see that these sum rules relate the low missing mass region with the triple-Regge region, just like finite energy sum rules relate the high- and low-energy regions in the 2-body case.

Assuming particles b and c are identical and choosing $n = 1$, we are left with

$$\int_0^N d\nu\ \nu \left[2\ E_c \frac{d^3\sigma}{dp_c}\right] = \sum_{i,j,k} (1 + \tau_i\, \tau_j\, \tau_k)$$

$$\times \left(\frac{\eta}{N}\right)^{\alpha_i(t)+\alpha_j(t)-1} \frac{N^{\alpha_k(0)+1}\ \gamma_{ij}^{\ k}(t)}{\alpha_k(0) - \alpha_i(t) - \alpha_j(t) + 2} \ .$$

Since we want to isolate the diagram of Fig. 5a, we take the appropriate linear combination, and obtain

$$\int_0^N d\nu\ \nu \left[E_{\text{outgoing proton}} \frac{d^3\sigma}{dp_{\text{proton}}} (\pi^- p \to p + \text{anything}) - E_{\text{outgoing proton}} \frac{d^3\sigma}{d^3p_{\text{proton}}} (\pi^+ p \to p + \text{anything}) \right]$$

$$= \left(\frac{\eta}{N} \right)^{\alpha_\rho(t)+\alpha_P(t)-1} \frac{N^{\alpha_\rho(0)+1}\ \gamma_{P\rho}{}^{\rho}(t)}{\alpha_\rho(0) - \alpha_\rho(t) - \alpha_P(t) + 2} \qquad (7)$$

We have again neglected nonleading triple-Regge diagrams in Eq. (7).

If we can determine $\gamma_{P\rho}{}^{\rho}(t)$ from Eq. (7) we just have to see whether $\gamma_{P\rho}{}^{\rho}(t)$ has a zero for $t = 0$ or not. In order to apply this method, what we need are data for the low missing mass region, as opposed to trying to test Eq. (6) directly. As a first approximation, one can try to saturate the left-hand side of Eq. (7) with the quasi two-body low missing mass reactions only. Since we are interested in P, P', and ρ exchanges only, the relevant low-energy diagrams are as shown in Fig. 6. Now, as is well known from the single particle inclusive kinematics,

$$E_c \frac{d^3\sigma}{d^3p_c} = \left(\frac{s}{\pi} \right) \frac{d^2\sigma}{dt dM^2}$$

and in our approximation,

$$\frac{d^2\sigma}{dt dM^2} = \sum_R \left. \frac{d\sigma}{dt} \right|_{a+b \to c+R} \delta(M^2 - M_R{}^2) = \frac{1}{2} \sum_R \left. \frac{d\sigma}{dt} \right|_{a+b \to c+R}$$

$$\times\ \delta(\nu - \nu_R) \ .$$

In this approximation, the ℓ.h.s. of Eq. (7) simply becomes:

$$(\ell.\text{h.s.}) = \left(\frac{s}{2\pi} \right) \left\{ \nu_\pi \left[\frac{d\sigma}{dt} (\pi^- p \to p\pi^-) - \frac{d\sigma}{dt} (\pi^+ p \to p\pi^+) \right] + \nu_{A_1} \left[\frac{d\sigma}{dt} (\pi^- p \to pA_1{}^-) - \frac{d\sigma}{dt} (\pi^+ p \to pA_1{}^+) \right] + \nu_{A_2} \left[\frac{d\sigma}{dt} (\pi^- p \to pA_2{}^-) - \frac{d\sigma}{dt} (\pi^+ p \to pA_2{}^+) \right] \right\}$$

so that with accurate elastic scattering and A-production data in πp reactions one can calculate the (ℓ.h.s.) of Eq. (7) in a very simple way, and therefore test whether $\gamma_{P\rho}{}^{\rho}(t)$ has a linear zero at $t = 0$.

Since at the present time, there are no data available for the reaction $\pi^+ p \to A^+ p$ (from now on A will represent either A_1 or A_2), we will try to relate this reaction to the reaction $\pi^- p \to A^- p$ by assuming that there is only Pomeranchukon exchange in exotic channels, thus giving rise to certain exchange degeneracy conditions. We will also make a slight digression, and show how this can be combined with SU(3) actually to predict a certain F/D ratio. The relevant s-channel exotic reactions, as well as the exchange degeneracy conditions that we will use repeatedly are shown in Fig. 7.

Let us denote by (Rab) the coupling of reggeon R to particles a and b, and let us also define

$$f_R \equiv (Rpp)(R\pi^- A^+)$$

then we see that

$$\frac{d\sigma}{dt}(\pi^- p \to A^- p) = \left| f_P + f_{P'} s^{\alpha_{P'}(t)-1} + f_\rho s^{\alpha_\rho(t)-1} \right|^2$$

$$= \left| f_P + f_\rho (1 + \kappa) s^{\alpha(t)-1} \right|^2$$

where we have assumed the exchange degeneracy condition $\alpha(t) = \alpha_{P'}(t) = A_\rho(t)$ and we have defined

$$\kappa \equiv \frac{f_{P'}}{f_\rho} = \frac{(P'pp)(P'\pi^- A^+)}{(\rho pp)(\rho\pi^- A^+)} \quad . \tag{8}$$

From the SU(2) relation

$$(\rho^0 \pi^- A^+) = -(\rho^0 \pi^+ A^-)$$

we can write

$$\frac{d\sigma}{dt}(\pi^+ p \to A^+ p) = \left| f_P + f_\rho (-1 + \kappa) s^{\alpha(t)-1} \right|^2$$

so that if we know f_P, f_ρ and κ we can determine the cross section for A^+ production. We will now try to determine the former three quantitites from the A^- production data by using the exchange degeneracy conditions of Fig. 7 as well as exact SU(3) symmetry. For simplicity we will first assume that ω is pure octet and ϕ is pure singlet. Later on we will show that our result holds even if there is ω-ϕ mixing, and is independent of the mixing angle.

From exchange degeneracy we can write:

$$\frac{(P'pp)}{(\rho pp)} = \frac{(\omega pp)}{(\rho pp)} \frac{(P'pp)}{(\omega pp)} = \frac{(\omega pp)}{(\rho pp)} \frac{(\omega k^+ k^-)}{(P'k^+k^-)}$$

and using exchange degeneracy again,

$$\kappa \equiv \frac{(P'pp)(P'\pi^- A_2^+)}{[(\rho pp)(\rho\pi^- A_2^+)]} = - \frac{(\omega pp)}{(\rho pp)} \frac{(\omega k^+k^-)}{(P'k^+k^-)} \frac{(\rho\pi^-\pi^+)}{(P'\pi^-\pi^+)}$$

$$= - \frac{(\omega pp)}{(\rho pp)} \frac{(\omega k^+k^-)}{(P'k^+k^-)} \frac{(P'k^+k^-)}{(\rho k^+k^-)}$$

so that

$$\kappa = - \frac{(\omega pp)}{(\rho pp)} \frac{(\omega k^+k^-)}{(\rho k^+k^-)}$$

and by using SU(3) Clebsch-Gordan coefficients,[15]

$$\frac{(\omega k^+k^-)}{(\rho k^+k^-)} = \sqrt{3} \quad .$$

Therefore, the only thing we are left to calculate is $(\omega pp)/(\rho pp)$. We will assume only (electric) vector coupling, since we are interested in the forward direction and then the $\sigma_{\mu\nu}$ term can be neglected. The expression for electric VBB coupling is well known,[16] and we have

$$\frac{(\omega pp)}{(\rho pp)} = \frac{(3 - 4\alpha)}{\sqrt{3}}$$

where the coupling is α times the D coupling plus $(1 - \alpha)$ times the F coupling. Therefore, our final result for exact SU(3) symmetry, without ω-ϕ mixing is

$$\kappa = 4\alpha - 3 \quad . \tag{9}$$

Equation (9) implies that if we know α we know κ.

Instead of calculating κ in this manner however, we will do the converse: we can calculate κ from exchange degeneracy alone, and predict α. Indeed,

$$\kappa = \frac{(P'pp)}{(\rho pp)} \frac{(P'\pi^- A_2^+)}{(\rho\pi^- A_2^+)} = -\frac{(P'pp)}{(\rho pp)} \frac{(\rho\pi^-\pi^+)}{(P'\pi^-\pi^+)}$$

$$= -\frac{(P'pp)}{(\rho pp)} \frac{(P'\pi^-\pi^+)}{(\rho\pi^-\pi^+)} \frac{(\rho\pi^-\pi^+)^2}{(P'\pi^-\pi^+)^2}$$

or

$$\kappa = -\frac{(P'pp)(P'\pi\pi)}{(\rho pp)(\rho\pi\pi)} \tag{10}$$

which can be calculated directly from experimental data.[17,18] From the Michael et al. compilation one obtains

$$\kappa = -5.54 \quad .$$

While from the Rarita et al. data, the result is

$$\kappa = -5.57 \quad .$$

The prediction for α is

$$\alpha_{\text{predicted}} \approx -0.62 \quad .$$

This value is in reasonable agreement with other estimates.[19]

We wish to show now that this result holds even if ω-ϕ mixing is included. In this case we write:

$$P' = f_8 \sin\theta + f_1 \cos\theta$$

$$\omega = \omega_8 \sin\theta + \omega_1 \cos\theta$$

$$\phi = \omega_8 \cos\theta - \omega_1 \sin\theta \quad .$$

Because of c-invariance, if $\mathcal{P}$ is a pseudoscalar meson,

$$(\omega_1 \mathcal{P}\mathcal{P}) = 0 \quad . \tag{11}$$

Also, since the $(pp\phi)$ coupling is known to be very small, we have

$$(\omega_8 pp) \cos\theta - (\omega_1 pp) \sin\theta = 0$$

or,

$$(\omega_1 pp) = (\omega_8 pp) \frac{\cos\theta}{\sin\theta}$$

which tells us that

$$(\omega pp) = (\omega_8 pp) \sin\theta + (\omega_8 pp) \frac{\cos^2\theta}{\sin\theta}$$

$$= \frac{(\omega_8 pp)}{\sin\theta} (\sin^2\theta + \cos^2\theta) = \frac{\omega_8 pp}{\sin\theta} \quad .$$

On the other hand, from Eq. (11),

$$(\omega kk) = (\omega_8 kk) \sin\theta$$

and therefore

$$(\omega pp)(\omega kk) = (\omega_8 pp)(\omega_8 kk)$$

so that our result for α holds with ω-ϕ mixing included, regardless of the value of the mixing angle.

Now that we have our numerical value for κ, we have to determine f_P and f_ρ from the experimental data for A^- production. We would like to mention at this point that although it is not obvious that the value of κ is the same for the A_1 and A_2 cross sections, if the A_1 is a member of the 1^+ SU(3) meson nonet, the value of κ is the same in both cases. This can be verified by changing A_2 to A_1 in Fig. 7, and going through the same procedure leading to Eq. (10). Although the value of κ in Eq. (10) is the same for the A_1 and the A_2 reactions, it is important to notice that f_P and f_ρ are in principle different for the A_1 and A_2 reactions.

The formula

$$\frac{d\sigma}{dt} = A\, t\, e^{-Bt}$$

with $B = (8 \pm 1)(\mathrm{GeV}/c)^2$ seems to fit the A_2 production experimental cross section.[14] In other words, the helicity flip amplitude seems to dominate, and for $t = 0$ at $p_{lab} = 40$ GeV/c the A_2 production cross section is extremely small.

Furthermore, at $t = 0$, $\nu_\pi = 0$, so that the only term that contributes to the FMMSR is the A_1 term. Then, from Eq. (7), we see that if we saturate the ℓ.h.s. of the FMMSR with quasi-two body low missing mass reactions as we have done, we expect $\gamma_{P\rho}{}^{\rho}(0)$ to vanish only if the difference

$$\frac{d\sigma}{dt}(\pi^- p \to pA_1{}^-) - \frac{d\sigma}{dt}(\pi^+ p \to pA_1{}^+) = 0 \qquad (12)$$

at $t = 0$. The experimental data are by no means conclusive at the present time, however, we mention that from pure reggeology one

expects $\gamma_{P\rho}{}^{\rho}(0)$ to vanish only when the ℓ.h.s. of Eq. (12) goes to zero faster than $s^{-\frac{1}{2}}$ as s increases; on the other hand, if the intercept of the ρ-trajectory is $1/2$ one expects this difference to go to zero exactly like $s^{-\frac{1}{2}}$ (for $t = 0$). Therefore, $\gamma_{P\rho}{}^{\rho}(0)$ vanishes as long as the ρ-trajectory decouples from $A_1\pi$ in the forward direction. This last statement is consistent with experiment[20] so that we believe that there is a very good chance that a NSWSZ is indeed present in $\gamma_{P\rho}{}^{\rho}(0)$.

Later on we will show still another way of making the existence of the NSWSZ plausible, so we rest this subject for the time being, by reminding the reader that our approximation of saturating the ℓ.h.s. of the sum rule with resonances might prove to be a bad one. Perhaps there are other 3π states we are unaware of, or maybe even the saturation scheme might prove not to be a good approximation, however, with the tools available to us, this is the best we can do at the present time.

II. AN ESTIMATE OF $g_{P\rho}{}^{\rho}(t)$

We have seen in Chapter I that it is very plausible that $g_{P\rho}{}^{\rho}(t)$ has a NWSZ at $t = 0$. In view of the above, as well as some other reasons outlined below, we propose the following parametrization of this triple-Regge residue, for small t:

$$g_{P\rho}{}^{\rho}(t) = a' \, t \, e^{\gamma t} \quad . \tag{13}$$

We know of course, from our former discussion, that the difference

$$\rho(\pi^-; \pi^-|p) - \rho(\pi^+; \pi^+|p)$$

if measured accurately for many values of t would give us the t-dependence of $g_{P\rho}{}^{\rho}(t)$ whether Eq. (13) is valid or not, however, since these data are not available yet, we will assume that Eq. (13) is valid and we will try to estimate a'. Unfortunately we do not know the value of γ, and therefore from now on γ will be considered as a free parameter. In Chapter III it will become obvious why we have introduced this extra parameter into the picture.

Let us therefore give another motivation for our parametrization in Eq. (13), and then show how we can estimate a'.

Let us assume that a usual two-body Regge residue $\beta_i(t)$ can be parametrized as $\beta_i(t) = \beta_i(0)\, e^{c_i t}$ where t is in GeV^2. We know that the contribution of a particular triple-Regge diagram to the cross section can be written as[3]

$$\frac{d^2\sigma}{dt dM^2} = \frac{1}{16\pi s^2} \beta_1(t)\, \beta_2(t)\, \beta_3(0)\, g_{12}{}^3(t) \left(\frac{s}{M^2}\right)^{\alpha_1(t)+\alpha_2(t)} (M^2)^{\alpha_3(0)} \tag{14}$$

where 1, 2, and 3 refer to the three reggeons, and we have omitted particle labels for the time being. We see from Eq. (14) that under our assumptions the t dependence is given by

$$\frac{d^2\sigma}{dt dM^2} \propto e^{(\alpha_1'+\alpha_2')t \log(s/M^2)}\, e^{(c_1+c_2)t}\, g_{12}{}^3(t) \tag{15}$$

where $\alpha_i(t) = \alpha_i(0) + \alpha_i' \, t$. Equation (15) clearly shows a strong exponential cutoff in t, so that if one integrates inclusive distributions over all values of t, the small t region dominates. We consider, in particular, $\alpha_1 = \alpha_P$, $\alpha_2 = \alpha_3 = \alpha_\rho$. Let us assume now that $\alpha_P(t) = 1$ and $\alpha_\rho(t) = 0.5 + t$ and integrate Eq. (14) over the triple-Regge region in t and M^2 to get the contribution to the total cross section coming from this particular diagram, and denoted by σ,

$$\sigma \equiv \int_{M_1^2}^{M_2^2} \int_{|t_{min}|}^{\infty} \frac{d^2\sigma}{dt dM^2}\, d|t| dM^2$$

$$= \frac{1}{16\pi s^{\frac{1}{2}}} \int_{M_1^2}^{M_2^2} \frac{dM^2}{M^2} \int_{|t_{min}|}^{\infty} d|t| \; \beta_\rho(t)\, \beta_P(t)\, \beta_\rho(0)\, g_{P\rho}{}^{\rho}(t) \left(\frac{s}{M^2}\right)^t \tag{16}$$

where we have denoted the limits of integration in the missing mass as M_1 and M_2, and t_{min} can be obtained from the inclusive kinematics. The general expression for t_{min} (see Fig. 8) is[21]

$$t_{min} = -\mu^2 \left(\frac{M^2 - m^2}{s}\right)^2 \Big/ \left(1 - \frac{M^2 - m^2}{s}\right) \quad . \tag{16a}$$

It is important to note that M_2^2 is a finite fraction of s, whereas M_1^2 is just the mass of the minimal cluster allowed by the kinematics such that a triple-Regge expansion is still valid. Therefore

$$\left.\begin{array}{l} \dfrac{M_2^2}{s} \to \text{const.} \\[2ex] \dfrac{M_1^2}{s} \to 0 \end{array}\right\} \text{ as } s \to \infty \quad .$$

With these properties in mind, if we now write

$$\beta_\rho(t)\,\beta_P(t) = \beta_\rho(0)\,\beta_P(0)\,e^{ct}$$

Equation (16) becomes

$$\sigma = \left(\frac{1}{16\pi s^{\frac{1}{2}}}\right)\int_{M_1^2}^{M_2^2}\left(\frac{dM^2}{M^2}\right)\int_{|t_{min}|}^{\infty}|dt|\;\beta_\rho(0)\,\beta_P(0)\,\beta_\rho(0)\,g_{P\rho}{}^{\rho}(t)$$

$$\times\; e^{ct}\left(\frac{s}{M^2}\right)^t$$

$$= \left(\frac{\beta_\rho(0)\beta_P(0)\beta_\rho(0)}{16\pi s^{\frac{1}{2}}}\right)\int_{M_1^2}^{M_2^2}\left(\frac{dM^2}{M^2}\right)\int_{|t_{min}|}^{\infty}|dt|\;g_{P\rho}{}^{\rho}(t)$$

$$\times\; e^{\left(c+\log(s/M^2)\right)t} \quad .$$

Let us first assume that $g_{P\rho}{}^{\rho}(t) = \text{constant} = g$ and define

$$a = c + \log\left(\frac{s}{M_2^2}\right)$$

$$b = c + \log\left(\frac{s}{M_1^2}\right) \quad ,$$

then,

$$\sigma = -\left(\beta_\rho(0)\,\beta_P(0)\,\beta_\rho(0)\,g/16\pi s^{\frac{1}{2}}\right)\int_{M_1^2}^{M_2^2}\frac{dM^2}{M^2}$$

$$\times\; \frac{\exp\left\{\left[c + \log\left(\frac{s}{M^2}\right)\right]t_{min}\right\}}{c + \log\left(\frac{s}{M^2}\right)} \quad .$$

Even though in the reactions we will be considering t_{min} is so small that one can approximate the exponential by 1, we will keep the first order term and write

$$\sigma \approx -\left(\beta_\rho(0)\,\beta_P(0)\,\beta_\rho(0)\,g/16\pi s^{\frac{1}{2}}\right)\int_{M_1^2}^{M_2^2}\frac{dM^2}{M^2}$$

$$\times\; \frac{1 + \left[c + \log\left(\frac{s}{M^2}\right)\right]t_{min}}{c + \log\left(\frac{s}{M^2}\right)} \quad .$$

Upon performing the M^2 integration we obtain

$$\sigma \approx \frac{\beta_\rho(0)\,\beta_P(0)\,\beta_\rho(0) g}{16\pi}\, s^{-\frac{1}{2}} \left\{ \log\left(\frac{b}{a}\right) - \mu^2 \left[\lambda - \frac{M_1^{\,2}}{s} + \log\left(\frac{\lambda - 1}{\frac{M_1^{\,2}}{s} - 1} \right) \right] \right\}$$

where we have used Eq. (16a), neglected m^2 with respect to M^2, and we have defined

$$\lambda \equiv \frac{M_2^{\,2}}{s} \quad .$$

We see that as $s \to \infty$ and $t \to 0$ σ does not behave as

$$\sigma = \text{const.} \times s^{\alpha_\rho(0)-1}$$

because we have an extra factor of $\log(\log s)$ i.e., $\log b$ coming from the zeroth order term in t_{min}.

On the other hand, had we assumed that $g_{P\rho}{}^{\rho}(t) \to 0$ as $t \to 0$ we would not have encountered this problem. For example, if we assume $g_{P\rho}{}^{\rho}(t) = a't$ for small t then the zeroth order term in t_{min} (denoted by σ_0) becomes

$$\sigma_0 = \left\{ \frac{1}{a} - \frac{1}{b} \right\} \frac{a' s^{-\frac{1}{2}}}{16\pi}\, \beta_\rho(0)\, \beta_P(0)\, \beta_\rho(0)$$

which does have the right asymptotic behavior (again, the first order term in t_{min} poses no problem, as the reader can easily verify).

Finally, in the next chapter we will see that when we try to calculate the contribution of the ρ-P cut to the total πN cross section, the integrals involved diverge, unless $g_{P\rho}{}^{\rho}(t)$ vanishes at least as t^{ϵ} $(\epsilon > 0)$ when $t \to 0$, and furthermore, analyticity excludes a fractional power behavior as $t \to 0$. In view of all the

reasons mentioned above, we believe that Eq.(13) is very likely to hold for small t.

Let us now estimate the value of a'. It is a matter of straightforward algebra to show that the product of the ρ and P signatures gives

$$\mathrm{Re}\{\xi_\rho(t)\, \xi_P{}^*(t)\} = \frac{\sin[\pi(\alpha_P - \alpha_\rho)/2]}{\left(\cos\left(\pi \frac{\alpha_\rho}{2}\right) \sin\left(\pi \frac{\alpha_P}{2}\right)\right)}$$

which for our trajectories gives

$$\mathrm{Re}\{\xi_\rho(t)\, \xi_P{}^*(t)\} \approx 1 \qquad \text{for small } t.$$

Therefore,

$$\rho(\pi^-;\, \pi^-|p) - \rho(\pi^+;\, \pi^+|p) \cong K'$$

$$\equiv \left(\frac{4}{16\pi s^2}\right) \beta_{\pi\pi P}(t)\, \beta_{\pi\pi\rho}(t)\, \beta_{pp\rho}(0)\, g_{P\rho}{}^{\rho}(t) \left(\frac{s}{M^2}\right)^{\alpha_P(t)+\alpha_\rho(t)}$$

$$\times\ (M^2)^{\alpha_\rho(0)} \quad .$$

On the other hand, from the data on Regge residues[22] we know that $\beta_{\pi\pi P}(t)\, \beta_{\pi\pi\rho}(t) \propto e^{2t}$, where t is in GeV^2,

$$\therefore K' = \left(\frac{1}{4\pi s^2}\right) \beta_{\pi\pi P}(0)\, \beta_{\pi\pi\rho}(0)\, \beta_{pp\rho}(0)\, e^{2t}\, g_{P\rho}{}^{\rho}(t) \left(\frac{s}{M^2}\right)^{1.5+t}$$

$$\times\ (M^2)^{\frac{1}{2}} \quad ,$$

so that if we call

$$K \equiv \int_{|t_{min}|}^{\infty} K' |dt|$$

and use Eq. (13),

$$K = -(1/4\pi s^2)\, \beta_{\pi\pi P}(0)\, \beta_{\pi\pi\rho}(0)\, \beta_{pp\rho}(0)\, s^{3/2}\, (M^2)^{-1}\, a'$$

$$\times \left\{ t \exp[(\log(s/M^2) + 2 + \gamma)t]/(\log(s/M^2) + 2 + \gamma) - \exp[(\log(s/M^2) + 2 + \gamma)t]/(\log(s/M^2) + 2 + \gamma)^2 \right\}_{t=t_{min}}^{-\infty} .$$

Finally, from the inclusive kinematics, for these particular reactions,

$$t_{min} = \left(\frac{M^2}{s}\right)\left(m_\pi^{\,2} - \frac{m_\pi^{\,2}}{\left(1 - \frac{M^2}{s}\right)}\right)$$

so that if s/M^2 is 3 or 4, we see that $t_{min} \approx 0$ and then

$$K = \frac{1}{4\pi s^2}\, \beta_{\pi\pi P}(0)\, \beta_{\pi\pi\rho}(0)\, \beta_{pp\rho}(0)\, s^{3/2} (M^2)^{-1}\, a' / \left(\log \frac{s}{M^2} + 2 + \gamma\right)^2 . \tag{17}$$

Notice that for a certain value of K, $|a'|$ increases as γ increases.

From the above, we see that if we know K we can estimate a'. A quantity usually measured in inclusive experiments is

$$F = \int dp_\perp^{\,2} \left(\frac{E^*}{\pi\, p^*_{max}}\right) \frac{d^2\sigma}{dx\,dp_\perp^{\,2}}$$

where x is the well-known Feynman variable $x = (P^*/P^*_{max})$ and the asterisk represents c.m. quantities. Since from kinematics,

$$\frac{d\sigma}{dM^2} \equiv \int_{\text{all } t} \frac{d^2\sigma}{dt\,dM^2}\, dt = \left(\frac{\pi}{s(x)}\right) F$$

K is obviously just the difference between two F distributions, and it turns out that we do have the data we need[23] and we can estimate a'. The data are presented at $p_{lab} \simeq 8$ GeV/c, and what is given is the F distribution for each reaction as a function of x. Due to the fact that the difference

$$\rho(\pi^-;\, \pi^-|p) - \rho(\pi^+;\, \pi^+|p)$$

is comparable to (and sometimes even smaller than) the experimental error in the distributions themselves, our estimate for a' is very rough. Furthermore, since we would like to keep M^2 large and s/M^2 large, and at the same time $\rho(\pi^-;\, \pi^-|p) - \rho(\pi^+;\, \pi^+|p)$ larger than the experimental errors, the range of data we can use is greatly restricted. For $x \geq 0.8$, the experimental errors are as large or larger than the difference of the distributions; while for $x = 0.6$, s/M^2 is already becoming small ($s/M^2 \approx 2.5$). Therefore, we choose $x = 0.7$ (i.e., $M^2 = 4.54$ GeV^2, $s/M^2 = 3.35$) for our estimate. We mention that estimates of a' at different values of x are consistent with the values of a' obtained at $x = 0.7$.

From Eq. (17) and the experimental data we obtain

$$a'/(\log(s/M^2) + 2 + \gamma)^2 = 0.92 \pm 0.69 \ \text{GeV}^{-4}$$

which gives

$$a' = -9.6 \pm 7.2 \text{ GeV}^{-4} \qquad \text{for } \gamma = 0$$

$$a' = -16.5 \pm 12.4 \text{ GeV}^{-4} \qquad \text{for } \gamma = 1$$

$$a' = -25.4 \pm 19.0 \text{ GeV}^{-4} \qquad \text{for } \gamma = 2$$

$$a' = -36 \pm 27 \text{ GeV}^{-4} \qquad \text{for } \gamma = 3$$

$$\vdots$$

etc.

We conclude this chapter by reminding the reader that with our conventions, $g_{P\rho}{}^{\rho}(t)$ is in GeV^{-2}, and our normalization coincides with that of Rajaraman,[24] and also by mentioning that even though the errors are very large, our estimate will still enable us to study the ρ-P Regge cut in the next chapter.

III. AN ESTIMATE OF THE P-ρ CUT FOR πN SCATTERING

Regge cuts have been discussed in the past by many authors,[25] and more recently, a great deal of interest has arisen in actually trying to obtain the magnitude of the discontinuity across them and apply this result to phenomenological analyses.[26,27]

In the present chapter we will assume that Gribov's formula[28] holds for the P-ρ cut in πN scattering, and using this formula as well as our results of previous chapters we will try to estimate the contribution of the cut to physical observables like the total cross section and the polarization in πN charge exchange scattering. Our calculation is in a way similar to that of Muzinich et al.,[27] so we urge the reader to familiarize himself with this work which is particularly useful to us because it also contains a very detailed derivation of Gribov's formula for the discontinuity across the cut.

A. An Estimate of the P-ρ Cut Contribution to the Total πN Cross Section

Gribov's formula for the discontinuity across a two-reggeon cut is derived by studying the asymptotic behavior of the Feynman diagram shown in Figs. 9a and 9b. There is no proof that this formula holds for strong interactions in general, since it was derived only for a particular Feynman diagram. However, we will assume that Gribov's formula is valid for the ρ-P cut in πN scattering (see Fig. 10). We shall not derive the formula in the present work, but we refer the reader to the paper by Muzinich et al.[27] as well as the review article by Collins.[29]

A few comments are in order at this point. Some authors disagree with Gribov's sign for the discontinuity across a two-reggeon cut. Abarbanel[30] isolates N-reggeon irreducible amplitudes using the

unitarity relation and the multiperipheral model of production and obtains a sign opposite to that of Gribov. Chew[31] argues on physical grounds that Abarbanel's sign is correct. On the other hand, White[32] seems to agree with Gribov's sign by doing an independent calculation. The basic difference between reggeon calculus and field theory computations, and the simple unitarity construction through pole contributions is that reggeon calculus and field theory calculations allow consideration of two-reggeon contributions to the full elastic amplitude, and the total discontinuity of these amplitudes includes terms which do not arise in the unitarity construction.[33]

Let us now return to our estimate of the ρ-P cut. In what follows, we will use the usual A' and B amplitudes for πN scattering as defined by Singh.[34] In terms of A' and B, experimental quantities are given by

$$\sigma_T(s) = \frac{1}{p} \operatorname{Im} A'(s, t = 0) \tag{18}$$

$$\frac{d\sigma}{dt}(s,t) = \frac{1}{\pi s}\left(\frac{m}{4p^*}\right)^2 \left[\left(1 - \frac{t}{4m^2}\right) |A'|^2 - \left(\frac{t}{4m^2}\right) \frac{(4m^2p^2 + st)}{(4m^2 - t)} |B|^2\right] \tag{19}$$

$$P(s,t) = -\left(\frac{\sin\theta}{16\pi s^{\frac{1}{2}}}\right) \frac{\operatorname{Im}(A'B^*)}{\frac{d\sigma}{dt}} \tag{20}$$

where s is the invariant square of the total energy, p is the pion lab. momentum, p^* is the c.m. momentum, θ is the c.m. angle

m is the nucleon mass and $P(s,t)$ is the polarization parameter defined relative to the normal $\vec{p}_i \times \vec{p}_f$ where p_i and p_f are the initial and final pion momenta. The contribution of the forward cut to Im A' is given by[27-29]

$$\operatorname{Im} A'_{cut}(s,0) = \left(\frac{1}{32\pi s}\right)\int_{-\infty}^{0} dt\, \phi_{P\rho}(t)\, N_{P\pi\to\rho\pi}(t)\, N_{PN\to\rho N}(t) \times \left(\frac{s}{s_0}\right)^{\alpha_P(t)+\alpha_\rho(t)} \tag{21}$$

where[27,35]

$$N_{P\pi\to\rho\pi}(t) = \int_0^{\bar{M}_0^2} A'_{P\pi\to\rho\pi}(\bar{M}^2,t)\, d\bar{M}^2 - \frac{\beta_{\rho\pi\pi}(0)\, g_{P\rho}{}^{\rho}(t)}{1 + \alpha_\rho(0) - \alpha_P(t) - \alpha_\rho(t)} \times (\bar{M}_0^2)^{1+\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t)} \tag{22}$$

$$N_{PN\to\rho N}(t) = \int_0^{\bar{M}_0^2} A'_{PN\to\rho N}(\bar{M}^2,t)\, d\bar{M}^2 - \frac{\beta_{\rho NN}(0)\, g_{P\rho}{}^{\rho}(t)}{1 + \alpha_\rho(0) - \alpha_P(t) - \alpha_\rho(t)} \times (\bar{M}_0^2)^{1+\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t)} . \tag{23}$$

Here $\bar{M}^2 = M^2 - t - m_\pi^2$ (M is the missing mass) $s_0 = 1\ \mathrm{GeV}^2$, $\beta_{\rho\pi\pi}$ and $\beta_{\rho N\bar{N}}$ are the usual nonflip two-body residues, and

$$\phi_{P\rho} = 2\operatorname{Re}\{\xi_P \xi_\rho\}$$

where ξ_i is the usual signature factor. The absorptive parts of the $P\pi \to \rho\pi$ and $PN \to \rho N$ reggeon-particle amplitudes are represented by $A'_{P\pi\to\rho\pi}(\bar{M}^2, t)$ and $A'_{PN\to\rho N}(\bar{M}^2, t)$, respectively; N can be recognized as the residue of the nonsense wrong-signature fixed pole in the appropriate reggeon-particle scattering amplitude at $J = \alpha_P(t) + \alpha_\rho(t) - 1$ (for forward cuts both reggeons are at the same mass). Equations (22) and (23) are obtained from the FMMSR.[35] Notice that the integrals involving $g_{P\rho}{}^{\rho}(t)$ diverge unless $g_{P\rho}{}^{\rho}(t)$ vanishes at least as t^{ϵ} $(\epsilon > 0)$ when $t \to 0$, a result we mentioned before.

Since we are only interested in a rough estimate of the cut, we will make the following approximations: We assume $\alpha_\rho = 0.5 + t$, $\alpha_P = 1$.[36] Then,

$$\xi_P \xi_\rho = -i \frac{[\cos \pi(\alpha_P + \alpha_\rho)/2 - i \sin \pi(\alpha_P + \alpha_\rho)/2]}{\sin \frac{\pi\alpha_P}{2} \cos \frac{\pi\alpha_\rho}{2}}$$

so that

$$\mathrm{Re}(\xi_P \xi_\rho) = - \frac{\sin \pi(\alpha_P + \alpha_\rho)/2}{\sin \frac{\pi\alpha_P}{2} \cos \frac{\pi\alpha_\rho}{2}} \approx -1 \quad \text{for small } t;$$

on the other hand, we will assume that $A_{P\pi\to\rho\pi}$ is dominated by the π pole, and $A_{PN\to\rho N}$ is dominated by the N pole. With this in mind, we can write

$$\int_0^{\bar{M}_0^2} A_{P\pi}(\bar{M}^2, t)\, d\bar{M}^2 = \beta_{P\pi\pi}(t)\, \beta_{\rho\pi\pi}(t) \tag{24a}$$

$$\int_0^{\bar{M}_0^2} A_{PN\to\rho N}(\bar{M}^2, t) = \beta_{PNN}(t)\, \beta_{\rho NN}(t) \tag{24b}$$

and choose $\bar{M}_0^2 = 1$ in the rest of the calculation. Equations (24a) and (24b) might seem a little bit surprising, for example one might ask about higher contributions like A_1 and A_2 in Eq. (24a) or, N^* contributions to Eq. (24b). Let us therefore forget about the cut for one moment, and show that Eqs.(24a) and (24b) are indeed a reasonable approximation. First, let us calculate the contributions of the A_1 and A_2 to the integral in Eq. (24a). From the data of Antipov et al.[14] we know that for the reaction $\pi^- p \to A_2^- p$ the cross section can be parametrized as

$$\frac{d\sigma}{dt} = -A\, t\, e^{Bt} \quad \text{with } B = 8\ \mathrm{GeV}^{-2} \text{ and } A = 900\ \mu\mathrm{b/GeV}^4.$$

Analogously, for the reaction $\pi^- p \to A_1^- p$ one has

$$\frac{d\sigma}{dt} = a\, e^{bt} \quad \text{with } b = 6.7\ \mathrm{GeV}^{-2} \text{ and } a = 218\ \mu\mathrm{b/GeV}^2.$$

We immediately see that A_2 production is dominated by the flip amplitude, and for small t the A_2 contribution is expected to be negligible. In order to estimate the A_1 contribution, we shall use a logical (although phenomenologically unfounded) extension of Carlitz-Green and Zee's P-P' universality.[37] Using the same notation as in Chapter I for Regge couplings, we will assume that

$$\frac{(\pi^- A_1^+ P)}{(\pi^- A_1^+ P')} = \frac{(ppP)}{(ppP')} \approx 1 \quad . \tag{25}$$

Furthermore, we saw in Chapter I [see Eq. (10)] that in the exchange degenerate case

$$\kappa \equiv \frac{(ppP')(\pi^- A_1^+ P')}{(pp\rho)(\pi^- A_1^+ \rho)} = -5.5 \quad . \tag{26}$$

Since Antipov et al.'s data are measured at $s = 40\ \mathrm{GeV}^2$, we will assume that P-exchange dominates differential cross sections, so that at $t = 0$

$$\frac{d\sigma}{dt}(\pi^- p \to A_1^- p) = 0.56\ \mathrm{GeV}^{-4} = \left(\frac{1}{16\pi}\right) |(\pi^- A_1^+ P)(ppP)|^2 ,$$

from here and the known value of ppP we can calculate $(\pi^- A_1^+ P)$ (up to a sign) and then using Eqs. (25) and (26) we can estimate $(\rho\pi^- A_1^+)$. In our normalization we find

$$(P\pi\pi)(\rho\pi\pi) \approx 34 \quad \text{while} \quad |(P\pi^- A_1^+)(\rho\pi^- A_1^+)| \approx 0.4 \quad .$$

By an entirely analogous reasoning, the A_2 contribution to Eq. (24a) is also shown to be negligible:

$$|\rho\pi^- A_2^+|^2 = 0.61 \quad .$$

Now, regarding the N^* contribution to Eq. (24b), from the data of Foley et al.[38] for the Roper $N^*(1400)$ production the cross section is given as

$$\frac{d\sigma}{dt}\left(\pi^- p \to \pi^- N^*(1400)\right) = 6.4 \times e^{12.1t}\ \mathrm{GeV}^{-4}$$

$$\frac{d\sigma}{dt}\left(\pi^+ p \to \pi^+ N^*(1400)\right) = 6.2 \times e^{15.6t}\ \mathrm{GeV}^{-4}$$

$s \approx 31\ \mathrm{GeV}^2$. Therefore, at $t = 0$

$$\frac{d\sigma}{dt}(\pi^- p \to \pi^- N^*) = \left(\frac{1}{16\pi}\right)|f_P + f_\rho(1 + k)\, s^{-\frac{1}{2}}|^2$$

$$\frac{d\sigma}{dt}(\pi^+ p \to \pi^+ N^*) = \left(\frac{1}{16\pi}\right)|f_P + f_\rho(-1 + k)\, s^{-\frac{1}{2}}|^2$$

where we have assumed $\alpha_{P'}(0) = \alpha_\rho(0) = 1/2$, we have written $f_R \equiv (\pi\pi R)(NN^* R)$ for any reggeon R, and we have defined

$$k \equiv \frac{f_{P'}}{f_\rho} \quad .$$

If we now use universality again (i.e., $f_P \simeq f_{P'}$) we have

$$16\pi\left[\frac{d\sigma}{dt}(\pi^- p \to \pi^- N^*) - \frac{d\sigma}{dt}(\pi^+ p \to \pi^+ N^*)\right] \approx 4\, f_P[f_\rho(s^{-\frac{1}{2}} + s^{-1}] \quad .$$

Since f_P is known, we can calculate f_ρ and we find

$$(NNP)(NN\rho) \simeq 35 \qquad |(NN^* P)(NN^* \rho)| \simeq 0.36$$

so that the N^* contribution to Eq. (24b) is of the order of 1%. The contributions of other N^* resonances turn out to be negligible too, and actually even smaller than the contribution of the $N^*(1400)$. For example, by using the same reasoning as above we find for $N^*(1600)$,

$$|(NN^* P)(NN^* \rho)| \approx 0.26$$

and so on. Actually, for $N^*(1400)$ one can estimate $(NN^* \rho)$ independently from the N^* production charge exchange data,[39] and the result is identical to the one we just mentioned. Therefore, we feel that Eq. (24) is a reasonable approximation, and we can now get back to the estimate of $\sigma_c \equiv p \times \mathrm{Im}\, A'_{cut}(s,0)$.

Once Eq. (24) is established, we see that knowledge of $g_{P\rho}{}^{\rho}(t)$ is all we need to calculate σ_c, and this is precisely the triple-Regge residue we estimated in Chapter II. Since the experimental errors in the determination of a' are fairly large, and we do not know the value of γ, we present our results as follows: we choose $a' = -9.6\ \mathrm{GeV}^{-4}$ which is the central value for $\gamma = 0$, (see our discussion at the end of Chapter 2), and then we plot the ratio (σ_c/σ_{pole}) as a function of s, for different values of γ.

Here σ_{pole} is the ρ-pole contribution to the $\pi^- p$ total cross section. We should mention that in our calculation we have parametrized the two-body Regge residues as simple exponentials:

$$\beta_{PNN}(t) = \beta_{PNN}(0)\, e^{2t}$$

$$\beta_{\rho\pi\pi}(t) = \beta_{\rho\pi\pi}(0)\, e^{t}$$

$$\beta_{\rho NN}(t) = \beta_{\rho NN}(0)\, e^{t} \quad \text{(nonflip residue)}$$

$$\beta_{P\pi\pi}(t) = \beta_{\rho\pi\pi}(0)\, e^{t} \ .$$

This parametrization is consistent with the Regge fits we are using[22] and the fact that we have simple exponentials will become extremely useful for computational purposes below.

Our results are shown in Fig. 11. We have not said much about the a' dependence of the cut yet. We will talk about it in detail after we calculate the polarization for πN charge exchange scattering. In the meantime, let us see what happens when one wants to calculate the contribution of the cut to the B amplitude, what to do about nonforward cuts, etc.

B. Calculation of Nonforward Cuts

When we wish to calculate other observables, like for example, the differential cross section or the polarization we must know the B amplitude also. Calculating the forward contribution of the ρ-P cut to the B amplitude does not pose a big problem. We can write equations entirely analogous to Eqs. (21), (22), and (23), with the only basic difference that we take the $\beta_{\rho NN}$ to be the flip residue instead of the nonflip residue. For example, if we define $\beta_{\rho NN}^{\Delta h=1} \equiv \beta'_{\rho NN}$ where Δh is the change in helicity at the vertex in question, we have

$$\nu\ \mathrm{Im}\ B_{cut}(s,0) = \frac{1}{32\pi s}\int_{-\infty}^{0} dt\ N'_{P\pi\to\rho\pi}(t)\ N'_{PN\to\rho N}(t) \times \left(\frac{s}{s_0}\right)^{\alpha_P(t)+\alpha_\rho(t)}$$

where $\nu \equiv \frac{s-u}{4m}$ and,

$$N'_{P\pi\to\rho\pi}(t) = N_{P\pi\to\rho\pi}(t) \approx \beta_{P\pi\pi}(t)\,\beta_{\rho\pi\pi}(t) - \frac{\beta_{\rho\pi\pi}(0)\, g_{P\rho}{}^{\rho}(t)}{(1+\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t))}\,(\bar{M}_0{}^2)^{1+\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t)}$$

and,

$$N_{PN\to\rho N}(t) \approx \beta_{PNN}(t)\,\beta'_{\rho NN}(t) - \frac{\beta'_{\rho NN}(0)\, g_{P\rho}{}^{\rho}(t)}{(1+\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t))} \times (\bar{M}_0{}^2)^{1+\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t)} \ .$$

Again, $\beta'_{\rho NN}$ can be approximated by a simple function. It turns out that

$$\beta'_{\rho NN}(t) \approx \text{constant}$$

is a good approximation for small t.[40] This approximation will of course break down for $|t|$ near $0.5\ \text{GeV}^2$ since we have not included the usual nonsense wrong signature zero when α_ρ vanishes, however, the approximation is fairly good for, say $|t| \leq 0.3\ \text{GeV}^2$.

So far, we have been able to calculate forward cuts in terms of fairly simple functions, and integrals that can be done analytically. On the other hand, we know from kinematics that the polarization vanishes at $t = 0$, therefore we would like to calculate the contributions of nonforward cuts to the A' and B amplitudes. This is a slightly more complicated problem, however, we shall soon see that as long as all our Regge residues are expressed in terms of exponential functions, we still get an analytic expression for the cut.

When $t \neq 0$, instead of having a single integral over t, we have a double integral.[27-29,41] Let us forget about factors multiplying the integral, and just say that we have an integral of the form

$$H(t) = \left(\frac{1}{\pi}\right) \int dt_1 \int dt_2\ \theta(-\Delta)\ [\Delta(t_1,t_2,t)]^{-\frac{1}{2}}\ g_1(t_1)\ g_2(t_2) \tag{27}$$

where

$$\Delta(t_1,t_2,t) \equiv t_1^2 + t_2^2 + t^2 - 2t_1t - 2t_2t - 2t_1t_2 \ .$$

The functions g_1 and g_2 are essentially the N functions of Eqs. (22) and (23). Therefore, let us assume that

$$g_i(t_i) = e^{\gamma_i t_i} \qquad i = 1,2 \ . \tag{28}$$

Equation (28) only reflects the fact that we can approximate all our Regge residues by exponentials, so that if we can obtain a closed expression for the integral in Eq. (27) when Eq. (28) holds, we will be able to calculate nonforward cuts analytically.

In order to evaluate the integral in Eq. (27), let us make the following change of variables:

$$\frac{(t_1 + t_2)}{2} - \frac{t}{4} = u; \qquad t_1 = u + \frac{t}{4} + (ut)^{\frac{1}{2}}\ z$$

$$\frac{(t_1 - t_2)}{[2(ut)^{\frac{1}{2}}]} = z\ ; \qquad t_2 = u + \frac{t}{4} - (ut)^{\frac{1}{2}}\ z$$

which implies that

$$(1 - z^2)^{\frac{1}{2}} = \frac{[-\Delta(t_1,t_2,t)]^{\frac{1}{2}}}{[2(ut)^{\frac{1}{2}}]} \ ,$$

remembering that the phase space boundary is given by $\Delta = 0$, i.e., $z^2 - 1 = 0$, we have

$$H(t) = \left(\frac{1}{\pi}\right) \int_{-\infty}^{0} du \int_{-1}^{+1} dz\ g_1(t_1)\ g_2(t_2)(1 - z^2)^{\frac{1}{2}}$$

which upon substitution of Eq. (28) becomes,

$$H(t) = \frac{1}{\pi}\int_{-\infty}^{0} du \int_{-1}^{+1} \frac{dz}{(1-z^2)^{\frac{1}{2}}} \exp\left\{(r_1 + r_2)\left(u + \frac{t}{4}\right) + (ut)^{\frac{1}{2}}(r_1 - r_2)z\right\}$$

$$= \int_{-\infty}^{0} du\, e^{(r_1+r_2)(u+t/4)} \times \frac{1}{\pi}\int_0^{\pi} d\phi\, e^{(ut)^{\frac{1}{2}}(r_1-r_2)\cos\phi}$$

where $z = \cos\phi$. Put now $u = -v$, all the integrals are known[42] and we have

$$H(t) = \int_0^{\infty} dv\, e^{-(r_1+r_2)(v-t/4)}\, I_0[(r_1 - r_2)(-vt)^{\frac{1}{2}}]$$

$$= \frac{e^{(r_1+r_2)t/4}}{r_1 + r_2}\, e^{-\frac{(r_1-r_2)^2 t}{4(r_1+r_2)}}$$

so that finally,

$$H(t) = \frac{e^{\frac{r_1 r_2}{r_1+r_2} t}}{r_1 + r_2} . \qquad (29)$$

Equation (29) is precisely what we wanted. It tells us that as long as we use exponential functions, the only difference between forward and nonforward cuts is the extra "nonforward correction factor" $e^{\frac{r_1 r_2}{r_1+r_2} t}$. We are not quite done, however. In Eqs. (22) and (23) we were able to use the triple-Regge residue of inclusive reactions, because the calculation of a forward cut guarantees that the mass of

one of the three legs in a triple-Regge coupling is zero. When we go to nonforward cuts, however, this is no longer true, and the use of the triple-Regge residue of inclusive reactions is no longer justified. Since we are interested in small values of t, ($|t| \leq 0.3$ GeV^2), and we have to make some assumption about the value of this triple-Regge coupling, we will assume that the coupling is the same even when all three reggeon masses are nonvanishing, in other words, we will keep on using $g_{P\rho}{}^{\rho}$ as the coupling, even though as we said before this is not justified.

C. Estimate of the Polarization for $\pi^- p$ Charge Exchange Scattering

We have seen in the former section that we are able to estimate the contribution of the ρ-P cut for both the A' and the B amplitudes as long as t is small. In this section we will assume that the polarization in $\pi^- p$ charge exchange scattering is produced by the combined effect of the ρ-pole and the ρ-P cut,[43] so that once we know the cut contribution to the amplitudes we can immediately calculate the polarization from Eq. (20). Our results will obviously depend on the value of r we choose, and furthermore, they are extremely sensitive to the value of a', as can be seen by recalling from Eqs. (21), (22), and (23) that the cut contribution to the amplitude has linear as well as quadratic terms in a'. In fact, these terms have opposite signs, so changing a' will greatly affect the cut contribution, and perhaps even the sign of the cut contribution. This effect is further enhanced by the fact that in Eq. (20) either products or squares of amplitudes appear, so that one has terms of order as large as $(a')^4$, and several terms have opposite relative

signs. We also notice that as $|a'|$ becomes larger and larger, the quadratic terms in a' must dominate the amplitudes, and then the amplitudes become so large in magnitude that the cross section as calculated from Eq. (19) becomes much larger than the experimental value. In particular, if the cut contribution to the B amplitude is too large, we see from Eq. (19) that as t goes away from zero, $d\sigma/dt$ will become too large, and this might bring down the value of the polarization considerably, as soon as we move away from $t = 0$. This phenomenon can actually be seen in the graphs that we will present shortly. We also remind the reader that if we decide to calculate the polarization for $-t \approx 0.5\ \mathrm{GeV}^2$, our approximation for $\beta'_{\rho NN}$ breaks down, and again $\frac{d\sigma}{dt}$ from Eq. (19) becomes too large. Therefore, the predicted polarization beyond say $-t = 0.3\ \mathrm{GeV}^2$ cannot be taken seriously.

As for the s-dependence, our model predicts a decrease proportional to $(\log s)^{-1}$ as s increases.

Because of our unknown parameters, as well as all our assumptions, we do not feel that it is useful to present very detailed numerical predictions. Instead, we present the results at $p_{lab} = 5$ GeV/c for several typical values of a', γ, and t in Fig. 12. We see that it is possible to obtain a fairly good fit to the data. We have also compared the cross sections to those of Giacomelli et al.[44] For the highest values of $|a'|$ presented the predicted cross sections are too large. However, when we get a good fit to the polarization data, the cross sections are also reasonable (say, within 30% of the experimental data). Since in terms like

$$\mathrm{Im}\ \xi_\rho(\xi_{cut})^* = -\frac{\sin\left(\pi(\alpha_\rho - \alpha_c)/2\right)}{\left(\cos\frac{\pi\alpha_\rho}{2}\cos\frac{\pi\alpha_c}{2}\right)},$$

the slope of the pomeranchukon trajectory is important,[45] we have taken $\alpha_P = 1 + 0.3t$. Notice that this affects very little our former estimate of the cut amplitudes.[36]

The experimental data shown in Fig 12 are those of Yokosawa,[46] and we see that the s dependence of our model is also consistent with the data (see Fig. 12c).

CONCLUSION

We have studied the possibility of the existence of NWSZ in inclusive reactions in Chapter I, both in specific models as well as in a model independent way. In particular, we showed that it is very likely that $g_{P\rho}{}^{\rho}(t)$ has a NWSZ at $t = 0$. With this basis, we estimated $g_{P\rho}{}^{\rho}(t)$ for small t in terms of an unknown parameter γ and a parameter a' which is determined up to order of magnitude only, due to experimental errors. Then, in Chapter III we showed how to estimate the ρ-P Regge cut contribution to πN scattering in terms of $g_{P\rho}{}^{\rho}(t)$ and under certain approximations we estimated the polarization for $\pi^-\rho$ charge exchange scattering. We found that it is possible to get a fit consistent with experiment for certain values of γ and a' which are also consistent with experiment. Even though our calculation is very rough (and we caution the reader not to take our numbers too seriously), we feel that the results are encouraging, even though much work needs to be done both, theoretically and experimentally to determine whether Gribov's formula is valid in general, and whether the $\pi^- p \to \pi^0 n$ polarization is indeed produced by the type of pole-cut interference we have worked with.

More important than the precise numerical results however, is the fact that we have shown the possibility that triple-Regge residues might be very closely connected with physical observables (like the polarization) in two-body reactions. This we feel is our most interesting result.

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FOOTNOTES AND REFERENCES

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33. R. Blankenbecler, SLAC-TN-72-13 (Sept. 1972, unpublished), see also T. L. Neff, Phys. Letters 43B, 391 (1973).

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35. M. B. Einhorn et al., Ref. 4; A. I. Sanda, Phys. Rev. D6, 280 (1972); J. Kwiecinski, Nuovo Cimento Letters 3, 619 (1972).

36. When we calculate the polarization in πN CEX scattering later on, it will be important to give the Pomeranchukon a slope, however, for the time being we will keep $\alpha_P(t) = 1$. Since the

pomeranchukon slope is about 0.3 we can for now safely neglect terms that depend on this slope for small t as the reader can easily verify.

37. R. Carlitz, M B. Green, and A. Zee, Phys. Rev. D4, 3439 (1971).

38. K. J. Foley et al., Phys. Rev. Letters 19, 397 (1967).

39. R. B. Bell et al., Phys. Rev. Letters 20, 164 (1967).

40. V. Barger and R. J. N. Phillips, Ref. 20; see also A. D. Martin and T. D. Spearman, Elementary Particle Theory (North Holland Publishing Co., Amsterdam, 1970), p. 457.

41. J. Finkelstein and M. Jacob, Nuovo Cimento 56A, 681 (1968).

42. Bateman Manuscript Project Ref. 8.

43. It is well known that a single reggeon R can produce no polarization because $\xi_R \xi_R^*$ is real and then Eq. (18) gives $P = 0$.

44. G. Giacomelli, P. Pini, and S. Stagni, A compilation of πN scattering data, Preprint CERN/HERA 69-1 (unpublished).

45. The cut trajectory is given by (See Collins, Ref. 10)

$$\alpha_c = \alpha_P(0) + \alpha_\rho(0) - 1 + \frac{\alpha_P' \alpha_\rho'}{\alpha_P' + \alpha_\rho'} t \approx 0.5 + 0.23t$$

if $\alpha_P = 1 + 0.3t$. If the pomeranchukon were flat, then $\alpha_c \simeq 0.5$, and therefore we see that $\alpha_\rho - \alpha_c$ is considerably affected even by a small pomeranchukon slope.

46. A. Yokosawa, Proceedings Conference on Phenomenology in Particle Physics, Ref. 20.

FIGURE CAPTIONS

Fig. 1. Diagram for the reaction $a + b \to c + x$ in the triple-Regge region where c is a fragment of b.

Fig. 2. Dual model diagram used by Gordon and Veneziano to obtain an explicit formula for the cross section.

Fig. 3. Feynman diagram that gives an explicit expression for the single particle inclusive amplitude in the triple-Regge region.

Fig. 4. Triple-Regge diagrams where a NWSZ is expected.

Fig. 5. (a) Diagram obtained by subtracting $\rho(p;\, p|\pi^+)$ from $\rho(p;\, p|\pi^-)$.

(b) Two of several nonleading diagrams obtained by subtracting $\rho(p;\, p|\pi^+)$ from $\rho(p;\, p|\pi^-)$.

Fig. 6. Quasi-two body reactions used to saturate the ℓ.h.s. of Eq. (7).

Fig. 7. s-channel exotic reactions and exchange degeneracy conditions they imply.

Fig. 8. A particle "b" of mass μ colliding with a particle "a" of mass m, and fragmenting into a particle "c" of mass μ.

Fig. 9. (a) A diagram which when summed over all possible numbers of rungs gives rise to a Regge cut.

(b) The Feynman diagram of Fig. 8a with the ladders replaced by Regge poles R_1 and R_2.

Fig. 10. ρ-P cut in πN scattering.

Fig. 11. Ratio of ρ pole to ρ-P cut contributions to $\sigma_T(\pi^- p)$ as a function of s and γ.

Fig. 12. $\pi^- p$ charge exchange polarization data, and predictions of our model. (a) for $\gamma = 2$; (b) for $\gamma = 1$; (c) experimental data at 8 GeV/c.

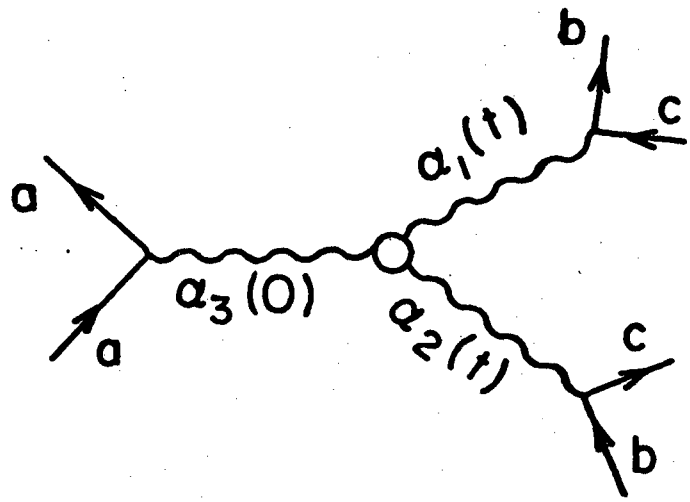

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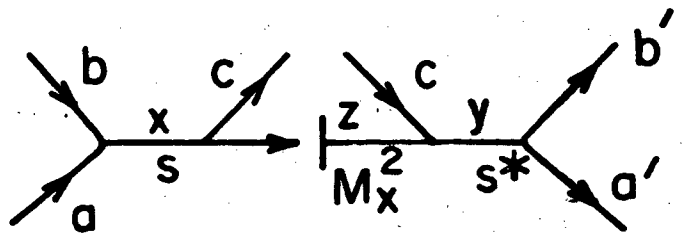
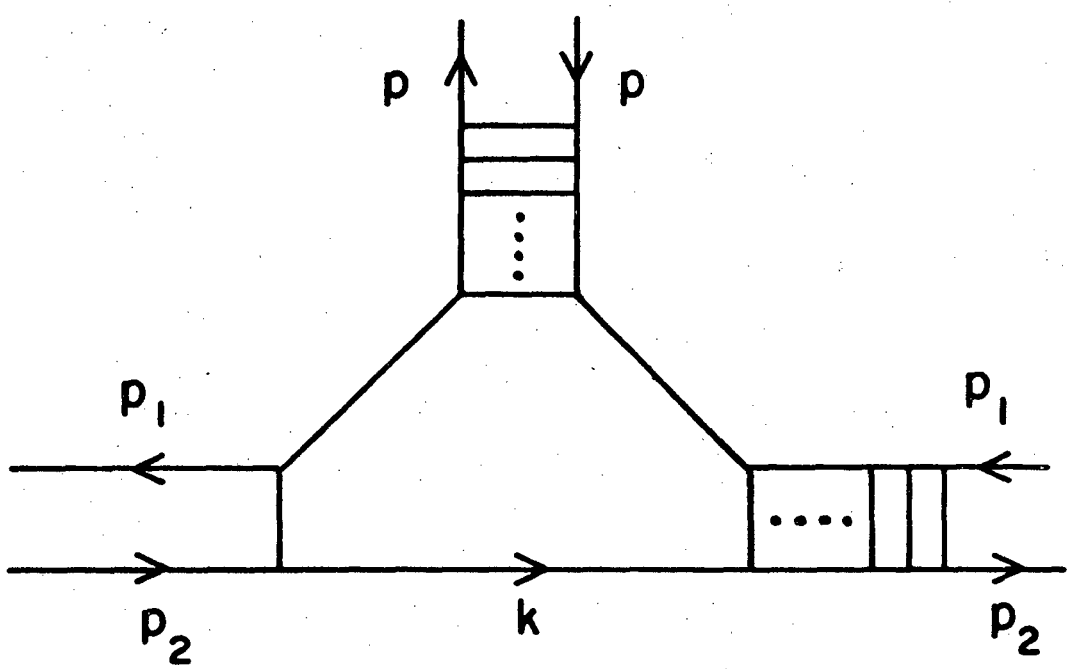
Fig. 1.



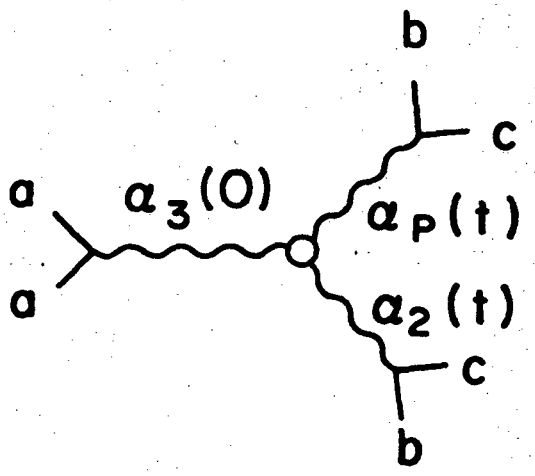
XBL736-3069

Fig. 2.



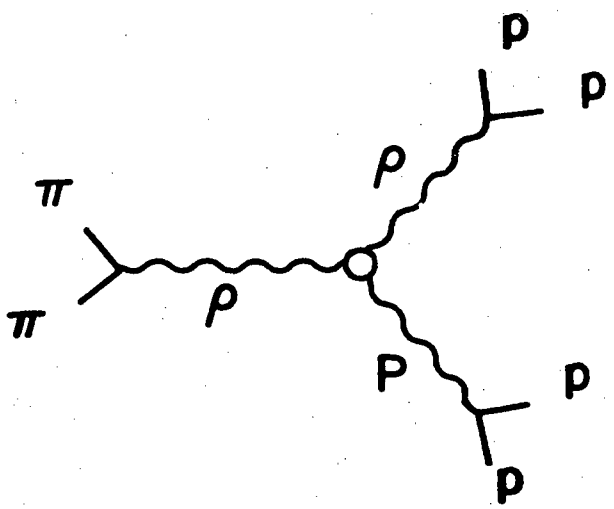
XBL736 - 3068

Fig. 3.



XBL736-3067

Fig. 4.



XBL736-3066

Fig. 5a.

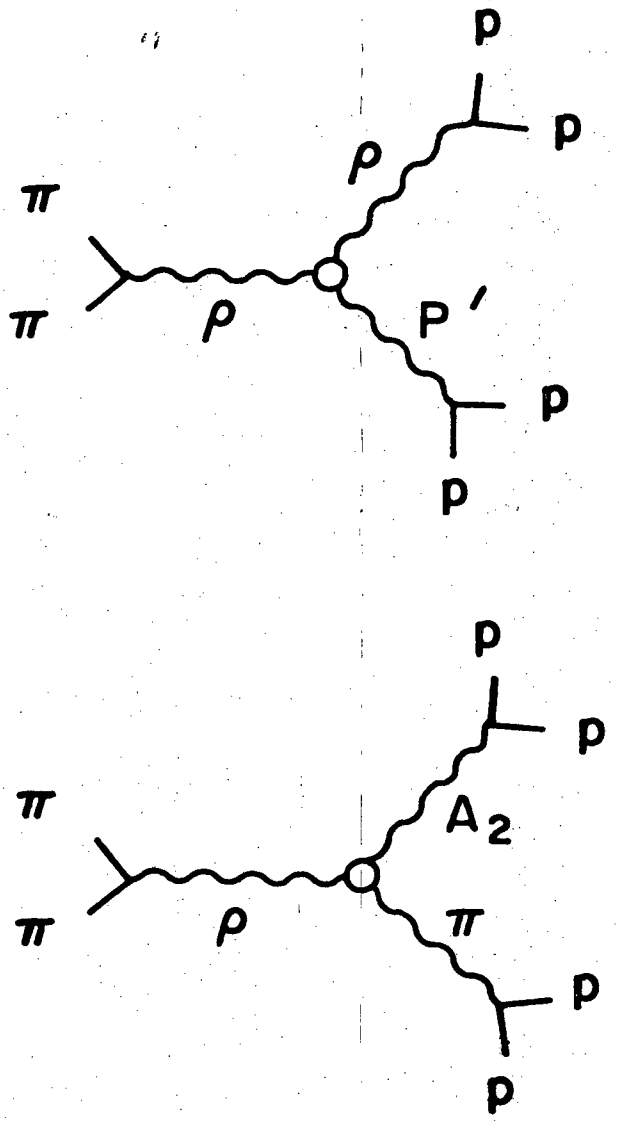


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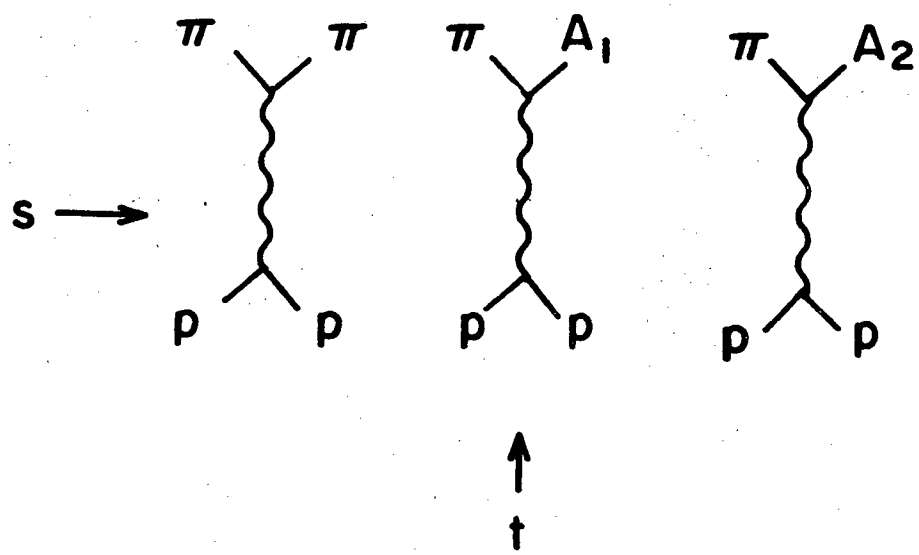
Fig. 5b.



XBL 736 - 3064

Fig. 6.

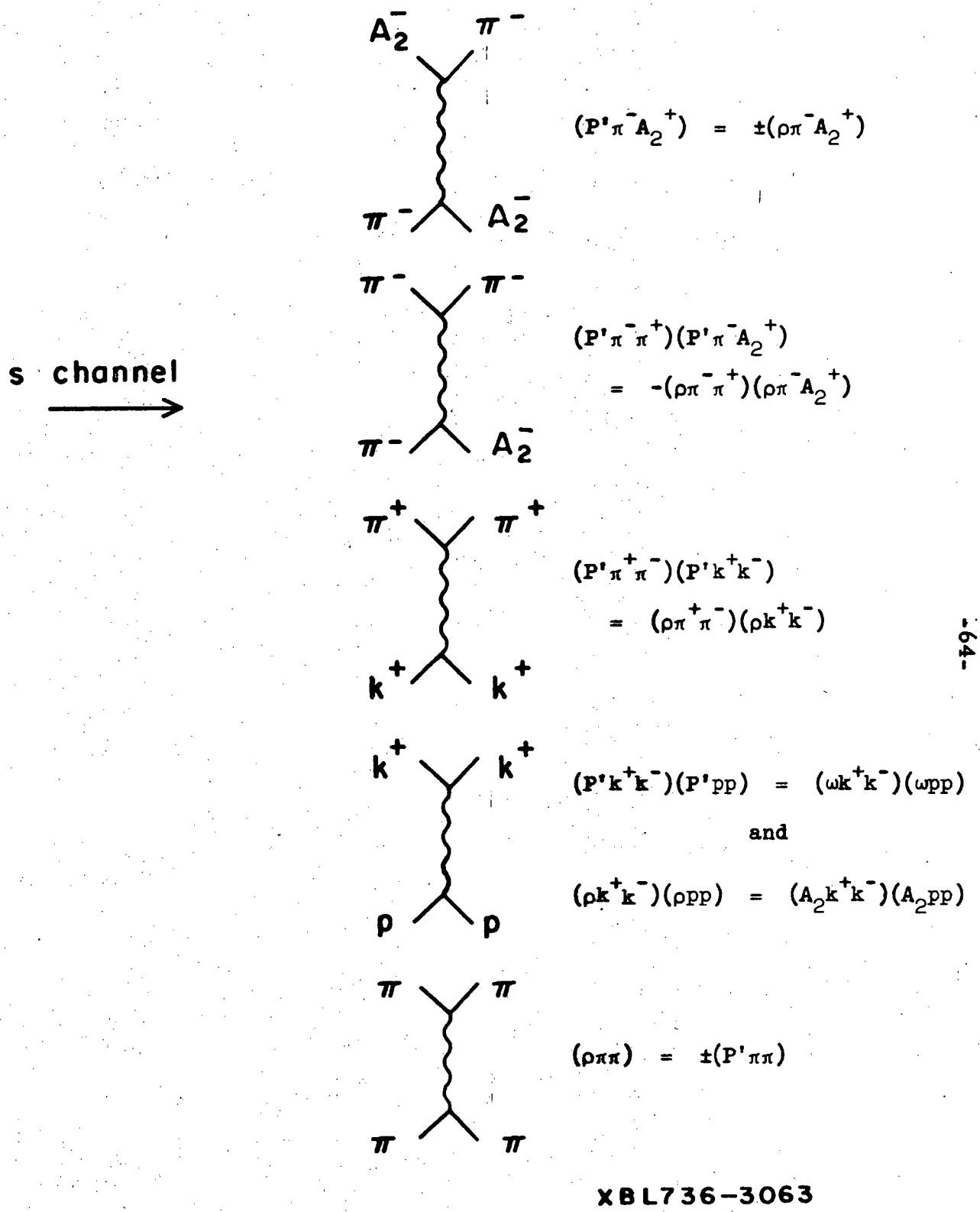
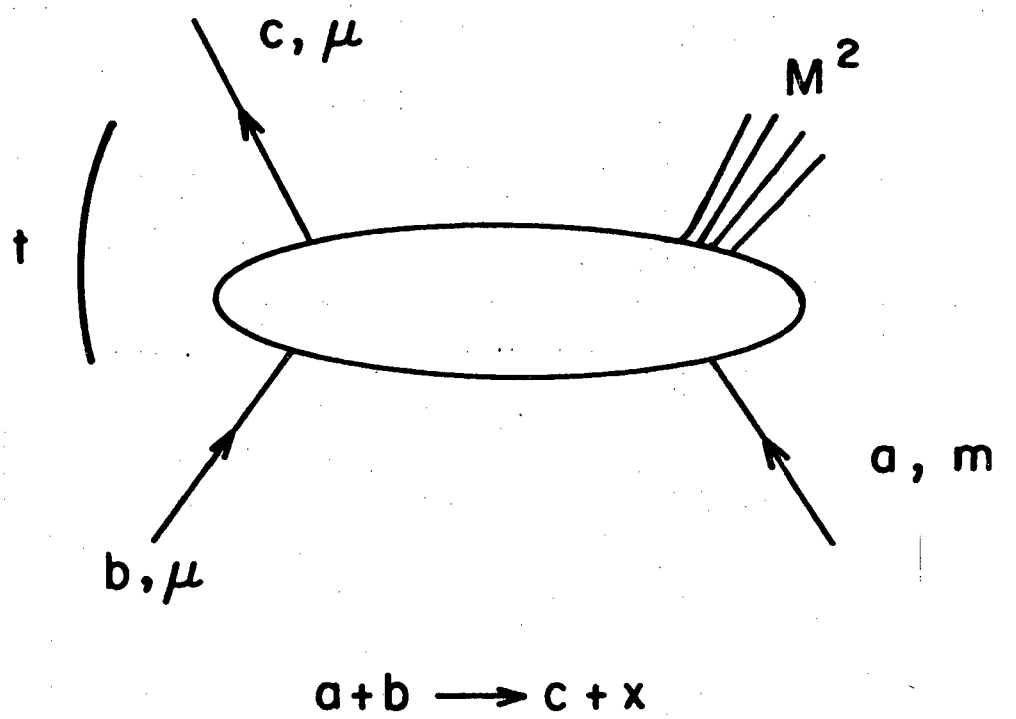

s channel
A_2^-
π^-
π^-
A_2^-
$(P'\pi^-A_2^+) = \pm(\rho\pi^-A_2^+)$
π^-
π^-
π^-
A_2^-
$(P'\pi^-\pi^+)(P'\pi^-A_2^+)$
$= -(\rho\pi^-\pi^+)(\rho\pi^-A_2^+)$
π^+
π^+
k^+
k^+
$(P'\pi^+\pi^-)(P'k^+k^-)$
$= (\rho\pi^+\pi^-)(\rho k^+k^-)$
k^+
k^+
p
p
$(P'k^+k^-)(P'pp) = (\omega k^+k^-)(\omega pp)$
and
$(\rho k^+k^-)(\rho pp) = (A_2k^+k^-)(A_2pp)$
π
π
π
π
$(\rho\pi\pi) = \pm(P'\pi\pi)$
XBL736-3063

Fig. 7.



XBL736-3062

Fig. 8.

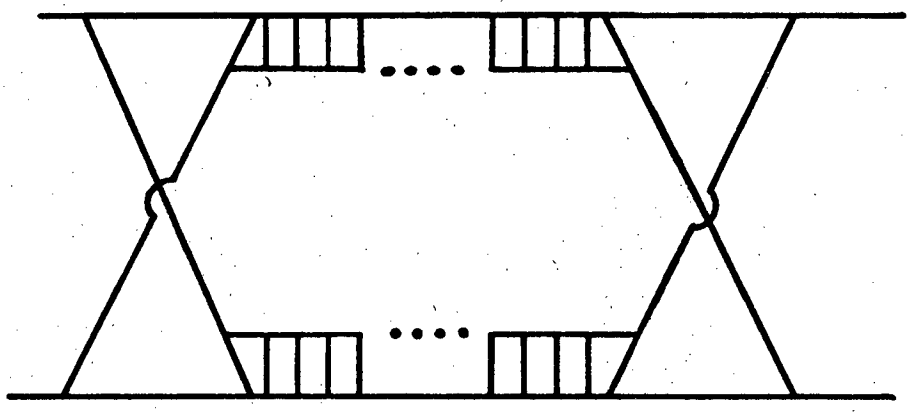
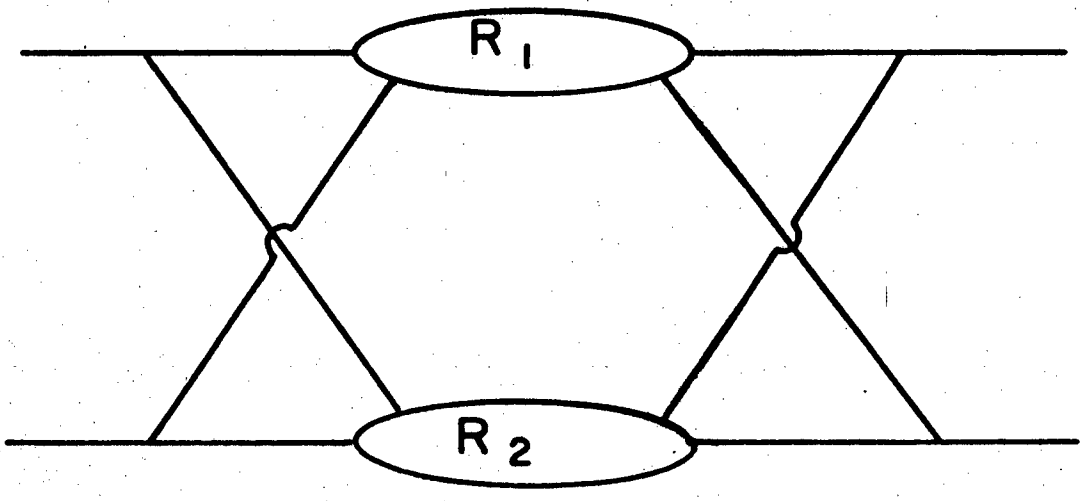
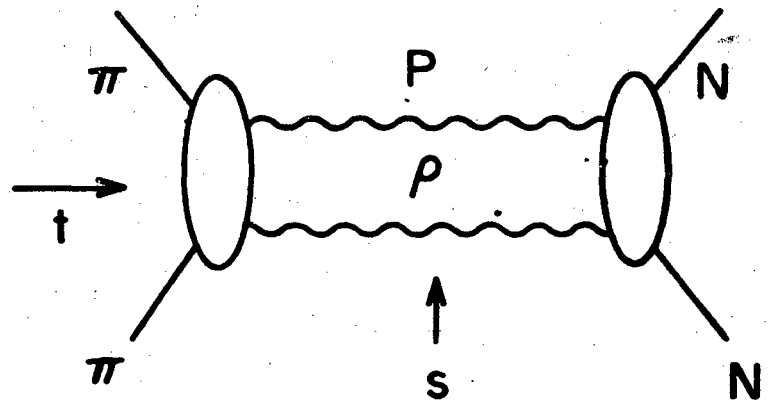


Fig. 9a.



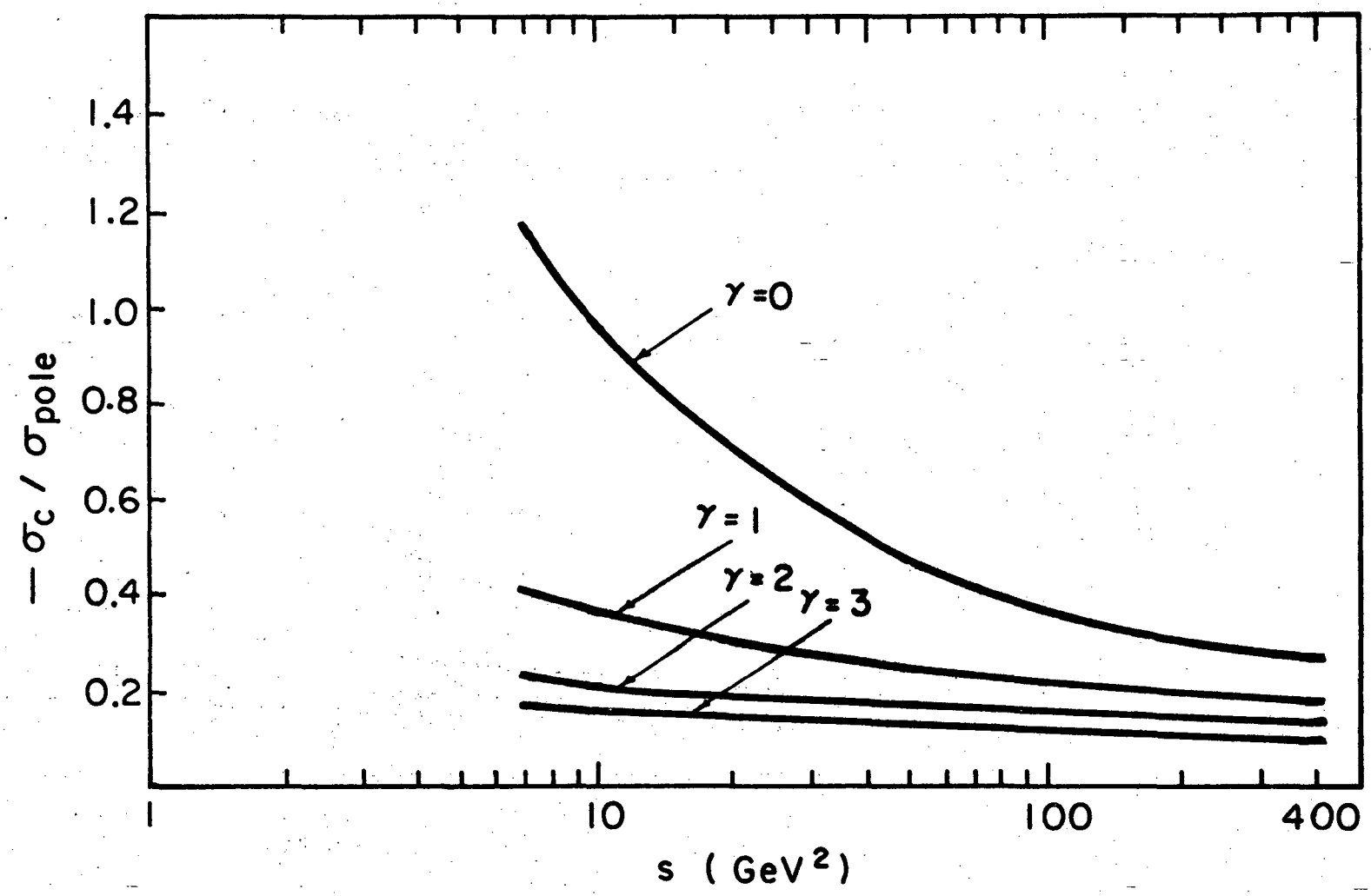
XBL736-3061

Fig. 9b.



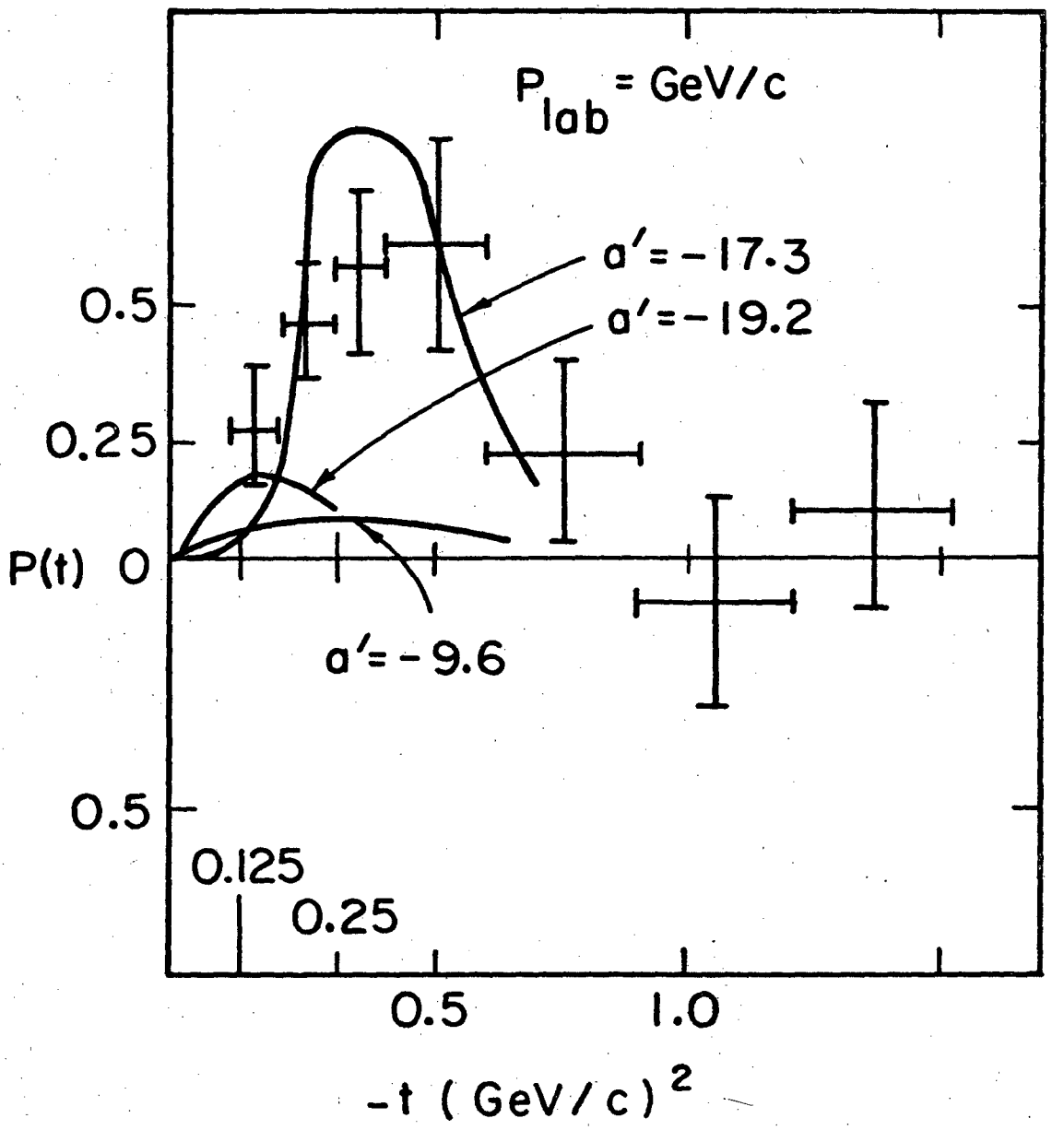
XBL736-3060

Fig. 10.



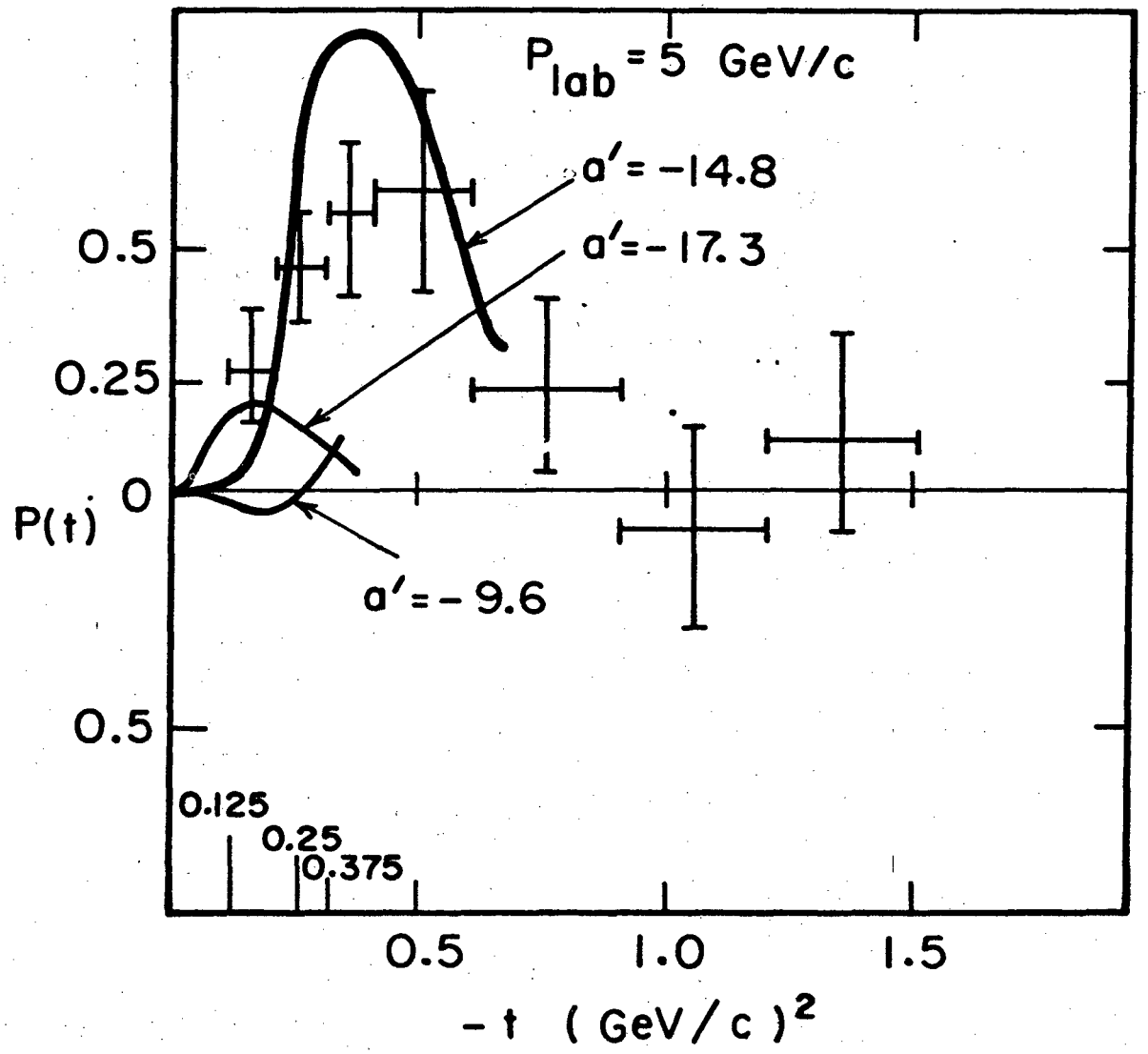
XBL736-3059

Fig. 11.



XBL736-3058

Fig. 12a.



XBL736-3057

Fig. 12b.

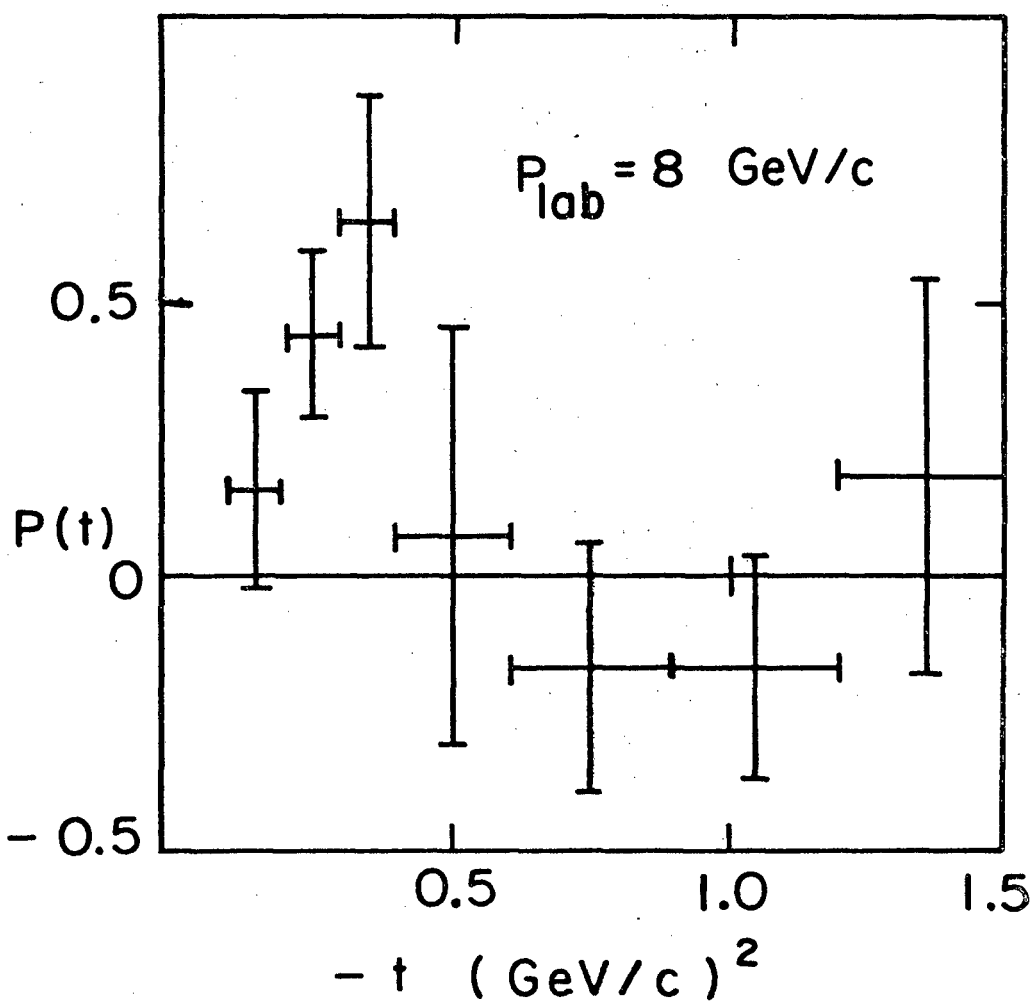


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Fig. 12c.

PART II: FIXED POLES AND FESR FOR K^+p SCATTERING

0 0 0 0 0 9 0 0 0 7 2

Finite-Energy Sum Rules and Fixed Poles for K^+p Elastic Scattering*

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Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 1 October 1971)

Assuming that exchange degeneracy is exact, the residue of the Pomeranchukon trajectory for K^+p elastic scattering is calculated from finite-energy sum rules and the low-energy phase shifts. It is shown that in order to be consistent with some recent results on s-channel helicity conservation for Pomeranchukon-dominated processes, at least one fixed pole must be included in the B amplitude, and in particular, this consistency is achieved by introducing a fixed pole at $J=0$.

I. INTRODUCTION

It has been suggested by Gilman *et al.*[1] that s-channel helicity conservation might be a general property of all Pomeranchukon exchange amplitudes.

Furthermore, some recent work by Langacker[2] shows that s-channel helicity conservation is very likely to hold approximately at high s for elastic processes, as a simple constraint of parity conservation and unitarity. If one assumes that exchange degeneracy is exact, one has only Pomeranchukon exchange in K^+p elastic scattering, and therefore this process seems to be ideally suited for studying s-channel helicity conservation. We will assume throughout that the Pomeranchukon is a simple Regge pole, and that the trajectory is given by

$$\alpha_P(t)=1. \tag{1}$$

The whole analysis has also been carried out using a slope of 0.3 GeV^{-2} for the Pomeranchukon trajectory[3]; however, the final results are practically identical to those obtained by using Eq. (1). This implies that even if the Pomeranchukon cut is included, our conclusion remains unchanged.

II. ANALYSIS AND RESULTS

A. Amplitudes and FESR

We employ integer-moment finite-energy sum rules (FESR) of the standard form.[4] The notation for the kinematics of the reaction $K^+p\to K^+p$ is shown in Fig. 1. The amplitude can be written in terms of the invariant amplitudes A, B as

$$M(s,t,u)=\bar{u}(p')[-A(s,t,u)+\tfrac{1}{2}i\gamma\cdot(q+q')B(s,t,u)]u(p).$$

Here the differential cross section is given by

$$\frac{d\sigma}{dt}=\frac{1}{64\pi s p^2}|M|^2.$$

It is a matter of straightforward calculation to obtain the s-channel helicity nonflip and flip amplitudes

$$\begin{aligned} g_{++}&=-\cos\tfrac{1}{2}\theta_s[2mA+(2Es^{1/2}-2m^2)B],\\ g_{+-}&=-\sin\tfrac{1}{2}\theta_s(2EA+2m\omega B),\end{aligned} \tag{2}$$

as well as the t-channel helicity nonflip and flip amplitudes

$$\begin{aligned} f_{++}&=\frac{A'}{(t-4m^2)^{1/2}},\\ f_{+-}&=\frac{\phi^{1/2}}{(t-4m^2)^{1/2}}B,\end{aligned} \tag{3}$$

where E and ω are the c.m. energies of the nucleon and the kaon, respectively, and $\phi=4tp_t^2q_t^2\sin^2\theta_t$ is the Kibble function. We have introduced the definition[5]

$$A'=(4m^2-t)A+m(s-u)B. \tag{4}$$

It is convenient to use the variables $\nu\equiv s-u/4m$ and t. Amplitudes with even or odd behavior under $\nu\to-\nu$ may be formed in an obvious way,

$$\begin{aligned} A'^{(\pm)}(\nu,t)&=\tfrac{1}{2}[A'(\nu,t)\pm A'(-\nu,t)],\\ B^{(\pm)}(\nu,t)&=\tfrac{1}{2}[B(\nu,t)\mp B(-\nu,t)],\end{aligned}$$

and one can derive the following sum rules for these amplitudes[6]:

$$\frac{1}{\nu_1^{2n+2}}\int_0^{\nu_1}\nu^{2n+1}\,\mathrm{Im}A'^{(+)}(\nu,t)d\nu=\sum_i\frac{a_i^+(t)}{\alpha_i^+(t)+2n+2}\left(\frac{\nu_1}{\nu_0}\right)^{\alpha_i^+(t)}, \tag{5a}$$

where the $\alpha_i^+(t)$ are all the Regge trajectories contributing to the amplitude $A'^{(+)}$, and where ν_0 is just a scale factor. Analogously,

$$\frac{1}{\nu_1^{2n+1}}\int_0^{\nu_1}\nu^{2n}\,\mathrm{Im}A'^{(-)}(\nu,t)d\nu=\sum_i\frac{a_i^-(t)}{\alpha_i^-(t)+2n+1}\left(\frac{\nu_1}{\nu_0}\right)^{\alpha_i^-(t)}, \tag{5b}$$

$$\frac{1}{\nu_1^{2n}}\int_0^{\nu_1}\nu^{2n}\,\mathrm{Im}B^{(+)}(\nu,t)d\nu=\sum_i\frac{b_i^+(t)}{\alpha_i^+(t)+2n}\left(\frac{\nu_1}{\nu_0}\right)^{\alpha_i^+(t)}, \tag{5c}$$

-73-

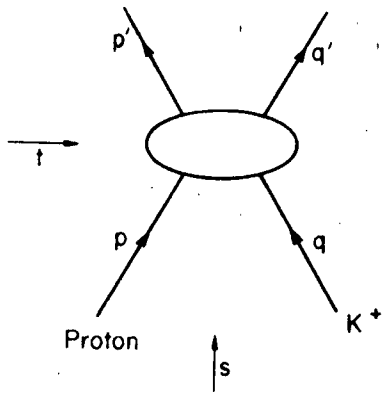

FIG. 1. Kinematics of the reaction. μ = kaon mass; m = proton mass.

and

$$\frac{1}{\nu_1^{2n+1}}\int_0^{\nu_1}\nu^{2n+1}\mathrm{Im}B^{(-)}(\nu,t)d\nu = \sum_i \frac{b_i^-(t)}{\alpha_i^-(t)+2n+1}\left(\frac{\nu_1}{\nu_0}\right)^{\alpha_i^-(t)}. \tag{5d}$$

The amplitude A' is given by $A' = A'^{(+)} + A'^{(-)}$, so that one is tempted to simply add (5a) and (5b) to get a sum rule for the full amplitude A'; however, one immediately notices that whereas (5a) involves an odd-moment sum rule, (5b) involves an even-moment sum rule. This problem can be overcome by taking a wrong-moment sum rule for one of the amplitudes (say, $A'^{(-)}$) and allowing for the possibility of nonsense–wrong-signature fixed poles.[7] We would like to remind the reader that a fixed pole in the partial-wave amplitude at a nonsense value of J with the wrong signature has no effect on the asymptotic behavior of the physical amplitude.

When we write the sum rule for A', we will have contributions from both α_i^+ and α_i^- as well as contributions from the fixed poles, and due to exchange degeneracy the contribution coming from an α_i^+ will be canceled by the contribution of an exchange-degenerate partner α_i^-, except for the Pomeranchukon contribution, which is the only one that survives. The sum rule is therefore simplified and it reads

$$S_n \equiv \frac{1}{\nu_1^{2n+2}}\int_0^{\nu_1}\nu^{2n+1}\mathrm{Im}A'(\nu,t)d\nu = \frac{a_P(t)}{\alpha_P(t)+2n+2}\left(\frac{\nu_1}{\nu_0}\right)^{\alpha_P(t)} + \text{F.P.} \tag{6a}$$

Here F.P. represents the fixed-poles contribution, which we have not written explicitly since we will discuss it later. Similar considerations lead to (wrong-moment sum rule for $B^{(-)}$)

$$S_n' \equiv \frac{1}{\nu_1^{2n}}\int_0^{\nu_1}\nu^{2n}\mathrm{Im}B(\nu,t)d\nu = \frac{b_P(t)}{\alpha_P(t)+2n}\left(\frac{\nu_1}{\nu_0}\right)^{\alpha_P(t)} + \text{F.P.} \tag{6b}$$

It is worth mentioning at this point that the possibility of fixed J-plane poles not associated with the third double-spectral function at right-signature points has been proposed by Finkler.[8] These poles do contribute to the asymptotic behavior of the scattering amplitude, and in particular to the real part of the physical amplitude.[9–13] Unfortunately, from our FESR formulas [for instance (6b)] it is impossible to tell whether F.P. is generated by a wrong-signature pole in $B^{(-)}$ or by a right-signature pole in $B^{(+)}$.

B. Input

We use as input the low-energy phase-shift analysis of Kato *et al.*[14] (hereafter referred to as "Yokosawa's solution"), which seems to be the most recent and accurate at present. In particular, we use their solution I, which seems to be favored with respect to solution II.[15] In this solution the $S_{1/2}$ phase shifts are repulsive.

Since attractive $S_{1/2}$ phase shifts have also been proposed in the past, we also performed our analysis using solution II of Ayed *et al.*[16] (hereafter referred to as "Bareyre's solution"), which is a typical solution of this kind, and has been used by Meyers and Salin in their work on K^+p scattering.[17] This solution, however, does not show the correct threshold behavior, and furthermore is incompatible with forward dispersion relations.[18]

In the case of Yokosawa's solution, a linear interpolation for the phase shifts has been performed between $\delta_l = 0$ at threshold and δ_l at $p_{\text{lab}} = 0.52$ GeV/c, which is the lowest value of the momentum in Yokosawa's analysis. This interpolation is consistent with S-wave dominance and k^{2l+1} behavior of phase shifts at low energies, and as a matter of fact it turns out that the contribution from the low-energy part of the integrals in the FESR's is quite small, and therefore the results are insensitive to this interpolation.

The cutoff values used for the upper limits of the integrals in the FESR's are those values of ν corresponding to (a) $p_{\text{lab}} = 2.53$ GeV/c in the case of Bareyre's solution, and (b) $p_{\text{lab}} = 1.89$ GeV/c in the case of Yokosawa's phase shifts.

C. Results Neglecting the Contribution of Fixed Poles

Throughout this section, we will assume that the fixed-poles contribution is negligible in all our sum rules, and we will do our calculations using S_0 and

S_0' in (6). To leading order in s, the condition for s-channel helicity conservation is that the amplitude A vanish, or, from Eq. (4), that the energy-independent dimensionless ratio

$$R \equiv \frac{m(s-u)B}{A'} = 1. \tag{7}$$

In the case of Bareyre's solution, and for small t, we obtain $R \approx 1$. A plot of R for different values of t is shown in Fig. 2 for Yokosawa's solution. The values of R for Yokosawa's solution are clearly inconsistent with s-channel helicity conservation. As we will show later, however, different moment FESR's are not compatible with each other unless one introduces at least one additional J-plane singularity.

D. Results Including One Fixed Pole

If one assumes that there is only one fixed pole in a certain amplitude, one can in principle calculate its position and residue by using different moment sum rules; however, in practice this method is highly unreliable because one has to calculate certain ratios that are very sensitive to small errors in the sum rules.[19] Therefore, we will not attempt to calculate the position and residue of the fixed pole in this fashion.

On the other hand, we can still estimate very roughly the magnitude of the relative contribution of the fixed pole to the sum rule as follows[20]: Assume that the fixed pole has a trajectory ϵ and a residue γ. Let us also assume that $\epsilon \neq 0$. Then, using the fact that $\alpha_P(t) = 1$, from (6) we have

$$S_0' = b^+(t)\frac{\nu_1}{\nu_0} + \frac{\gamma}{\epsilon}\left(\frac{\nu_1}{\nu_0}\right)^{\epsilon},$$

$$S_1' = b^+(t)\frac{\nu_1}{3\nu_0} + \frac{\gamma}{\epsilon+2}\left(\frac{\nu_1}{\nu_0}\right)^{\epsilon},$$

so that the quantity

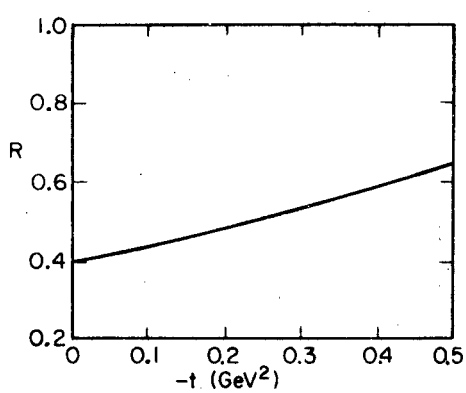

FIG. 2. R as a function of t for Yokosawa's solution I, without including fixed poles.

$$B_{\text{R.C.}} \equiv \frac{3S_1' - S_0'}{3S_1' + S_0'}$$

should give us a good idea of the relative contribution of the fixed pole to the B amplitude. The same argument holds in the case $\epsilon = 0$, for which

$$S_0' = b_P(t)\frac{\nu_1}{\nu_0} + \gamma, \tag{8}$$

$$S_1' = b_P(t)\frac{\nu_1}{3\nu_0}. \tag{9}$$

By the way, from the above formulas we see that if there is a fixed pole at $J = 0$ in B, one should use the sum rule S_1' and not S_0' in computing b_P. It is now obvious that for A' one has a relative contribution given by

$$A'_{\text{R.C.}} = \frac{3S_0 - 5S_1}{5S_1 + 3S_0}.$$

The results are shown in Fig. 3, and it is evident

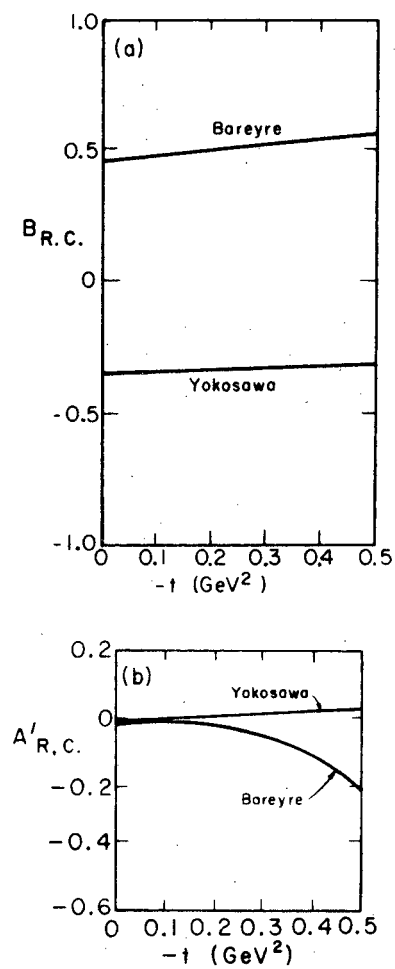

FIG. 3. (a) $B_{\text{R.C.}}$ as a function of t. (b) $A'_{\text{R.C.}}$ as a function of t.

that there is a negligible relative contribution to A', whereas there is a very sizeable contribution to B, for both sets of phase shifts.

One might interpret this contribution as coming from broken exchange degeneracy; however, the very flat K^+p total cross section indicates that exchange degeneracy holds very accurately for this process (at least at $t=0$).

An alternative interpretation (which, by the way, happens to be consistent with Finkler's predictions) is that the additional contribution comes from a fixed pole at $J=0$ in the B-amplitude sum rule; i.e., we use (9) to calculate $b_P(t)$, and combine this with (8) to obtain γ. Once these quantities are known, one can easily calculate R [see Eq. (7)] by using the asymptotic behavior of A' and B in Regge theory. Furthermore, one can calculate the ratio [see Eq. (2)]

$$R' = \frac{g_{+-}/s^{1/2}\sin\frac{1}{2}\theta_s}{g_{++}/\cos\frac{1}{2}\theta_s} \times m$$

for any large value of s. We show our results in Figs. 4–6. All the results are shown at the cutoff value of the FESR integrals, and in Fig. 6 we also include a plot of R' for $s=10$ GeV^2. Bareyre's solution is clearly incompatible with s-channel helicity conservation. For Yokosawa's solution,

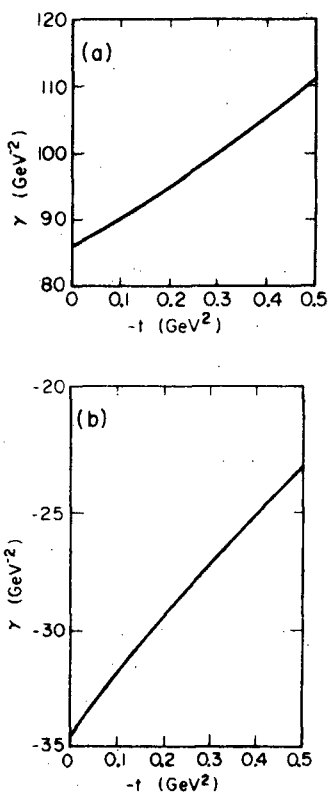

FIG. 4. (a) Residue of fixed pole for Bareyre's solution. (b) Residue of fixed pole for Yokosawa's solution.

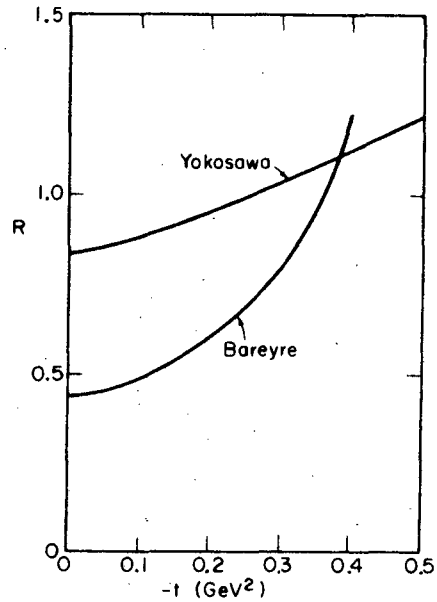

FIG. 5. R as a function of t, including one fixed pole at $J=0$.

R is seen to be very close to 1, and R' is very small and decreasing as s increases, in agreement with some earlier work on KN scattering.[21] Our results are in sharp contradiction with those of Meyers and Salin after we introduce the fixed pole at $J=0$. It is now only fair to ask what happens if instead of using (6), one uses a wrong-moment sum rule for $A'^{(+)}$ and a wrong-moment sum rule for $B^{(+)}$. We find that if one does the calculation in this manner, there is no evidence of a fixed pole at $J=-1$, and furthermore, all our previous results are essentially unchanged.

III. CONCLUSIONS

As we have mentioned before, the repulsive $S_{1/2}$ wave solution seems to be favored at the present time, and therefore we will draw our conclusions

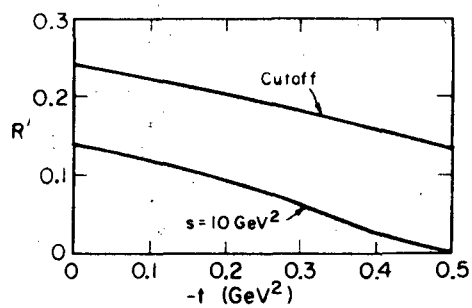

FIG. 6. R' as a function of t and energy in Yokosawa's solution, including one fixed pole.

from the results given by Yokosawa's solution. We would like to point out that Yokosawa's solution is a typical solution of this kind, and very similar solutions have previously been proposed in the literature.[17] Therefore, we expect our qualitative results to hold not only for Yokosawa's phase shifts, but also for the other similar $S_{1/2}$ repulsive phase-shift solutions.

The FESR's indicate that in order to have s-channel helicity conservation for K^+p elastic scattering, at least one fixed pole must be included in the B amplitude, and in particular, s-channel helicity conservation is achieved by introducing a fixed pole at $J=0$. Furthermore, there is no evidence of fixed poles in A'. Unfortunately, we cannot test Finkler's prediction, since we have no way of determining whether our pole has right or wrong signature; and for that matter, we may even have a combination of both types of poles contributing to our FESR's.

A final remark is in order here. Yokosawa's solution seems to indicate some evidence for an exotic baryon resonance with strangeness +1. If this is the case, and if duality holds for this type of resonance, the fixed pole introduced here may be reinterpreted as one or more very low lying ordinary Regge trajectories with fairly large residues.

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[9]The real part of the K^+p amplitude does have contributions from the ordinary Regge trajectories other than the Pomeranchukon, and therefore a right-signature–nonsense fixed pole will be largely masked by the ordinary trajectories unless the scattering angle is very large, i.e., close to 90°. Since a detailed discussion of the properties of right-signature fixed poles is outside the scope of this paper, we refer the reader to Finkler's paper (Ref. 8), as well as to some other discussions of the underlying assumptions and some criticism by Squires (Refs. 10–13).

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