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June 1984

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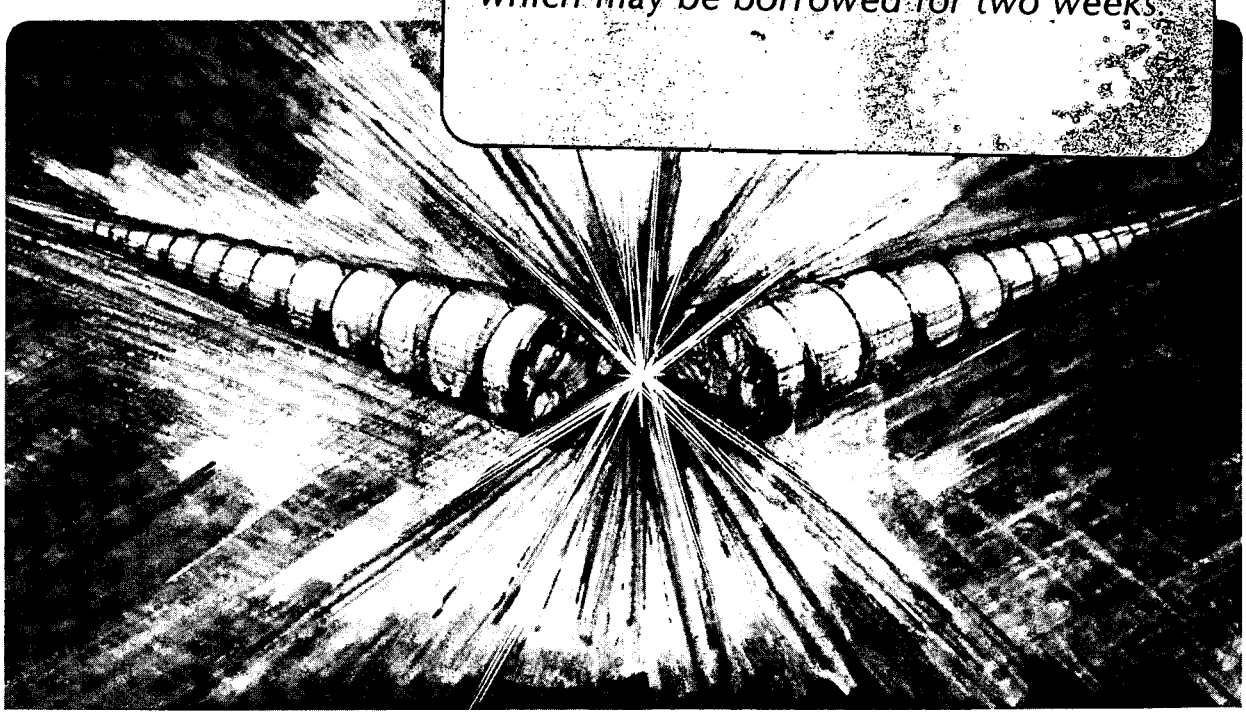

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Ponderomotive Hamiltonian and Lyapunov Stability for Magnetically Confined Plasma in the presence of r.f. Field*

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Abstract

The self-consistent interaction of high-frequency antenna-generated field with low-frequency plasma motion is derived for multifluid dynamics and magnetohydrodynamics. Field Hamiltonians and Poisson brackets are obtained, for the low-frequency evolution of plasma, low-frequency field, and high-frequency amplitude. Casimir functionals are combined with the Hamiltonians to form Lyapunov functionals, yielding stability criteria. Application to ponderomotive stabilization of unstable equilibria is discussed.

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Recent experiments on r.f. stabilization of the flute instability of plasmas confined in mirror machines [1] have stimulated new theoretical research to describe the nonlinear interaction of high frequency fields with low frequency modes of the plasma [2-4]. Such a stabilization process, if successful, would allow the use of axisymmetric mirrors for plasma confinement, implying simpler design and better transport properties than for minimum-B mirrors. The chief aim of the theoretical work is to determine stability criteria and the threshold intensity of r.f. field necessary to achieve stabilization.

The conventional description of the stabilizing effect of high frequency field is based on consideration of ion drifts [5-7]. It is argued that stabilization is achieved when the ion ponderomotive drift balances curvature and magnetic gradient drifts, so that the charge separation produced by unfavorable average curvature is reversed. This approach, however, ignores the mutual interaction between the particles and fields. The necessary <u>self-consistent</u> treatment is given in Ref. 8, for the system composed of plasma, low frequency field, and high frequency field. This system is represented by a field Hamiltonian with ponderomotive term, and its associated Poisson bracket.

We shall use a Lagrangian action principle, averaged over the fast time scale (the period of the applied antenna current), to derive the ponderomotive Hamiltonian and Poisson bracket for each of two models: the multifluid plasma and ideal magnetohydrodynamics. The Hamiltonian formulation in the Lagrangian description is converted to the Eulerian description. The resulting evolution equations, for oscillation center densities, momentum densities and low frequency electromagnetic field, involve ponderomotive forces and magnetization current created by the r.f. field. The Hamiltonian formalism in

the Eulerian description is degenerate, i.e., there exist Casimir functionals, whose Poisson bracket vanishes with any other functional of the Eulerian dynamical variables. It allows the use of recently developed techniques [9] that establish sufficient conditions for Lyapunov stability. The Hamiltonian is combined with the Casimir functionals to form a Lyapunov functional, whose local extrema in functional space are stable equilibria. By this method we shall derive sufficient criteria for stability of plasma equilibria in the presence of the r.f. field, and discuss the application of these stability criteria to axisymmetric mirrors.

The system we consider has four components: first, the plasma, considered as a continuous medium, whose state at time t is represented by the position field $\vec{r}(z^\circ,t)$ (z° is the Lagrangian coordinate in some reference state, with $n^\circ(z^\circ)dz^\circ$ particles in volume element dz°); second, the electromagnetic field, represented by the vector potential $\vec{A}(\vec{x},t)$, with $\vec{x}$ an Eulerian spatial coordinate (we choose the radiation gauge); third, the antenna, modeled by an applied high-frequency current density $\vec{j}_a(\vec{x})\exp(-i\omega t)$; finally, a gravitational field $-\nabla\psi(\vec{x})$ to mimic the unfavorable average curvature of the magnetic field.

A complete description of the dynamics is provided by the total Lagrangian action, including the action of the plasma, of the electromagnetic field, and of the antenna. Variation of the action with respect to the position field $\vec{r}(z^\circ,t)$ yields the Newton-Lorentz equations, and with respect to the potential $\vec{A}(x,t)$ yields the Maxwell equations with plasma and antenna current sources.

Since the frequency ω of the antenna field is typically of order of the ion gyrofrequency, much larger than the rate γ at which the flute instability develops, a separation of time scales is appropriate. We represent the total motion $\vec{r}_{tot}(z^\circ,t)$ of a particle as the sum of the low frequency motion

$\vec{r}_c(z^\circ,t)$ of its oscillation center, and of the high frequency oscillation, with amplitude $\vec{r}(z^\circ,t)$ modulated at low frequency: $\vec{r}_{tot}(t) = \vec{r}_c(t) + \mathrm{Re}\,[\vec{r}(t)\exp(-i\omega t)]$. We use a similar representation for the field: the vector potential $\vec{A}_{tot}(\vec{x},t)$ is the sum of a slow (low frequency) component $\vec{A}_s$ and of a high frequency component of amplitude $\vec{A}$: $\vec{A}_{tot}(\vec{x},t) = \vec{A}_s(\vec{x},t) + \mathrm{Re}[\vec{A}(\vec{x},t)\exp(-i\omega t)]$.

The action, expressed in terms of these variables, is expanded to second order in the amplitudes and averaged over the fast time scale, assuming that no resonance takes place. Terms of order γ/ω are neglected. The new form of the action S is the sum of three contributions:

$$S = S_c + S_s - \int dt\, V. \tag{1}$$

The oscillation-center action is

$$S_c = \int dt \int n^\circ dz^\circ [\tfrac{1}{2} m\dot{\vec{r}}_c^{\,2} + (q/c)\dot{\vec{r}}_c\cdot\vec{A}_s(\vec{r}_c,t) - \Psi(\vec{r}_c)]; \tag{2}$$

a sum over species is implicit. The slow-field action is

$$S_s = \int dt \int d^3x\, [(\partial\vec{A}_s/\partial t)^2/(8\pi c^2) - (\mathrm{curl}\,\vec{A}_s)^2/8\pi]\,. \tag{3}$$

The ponderomotive energy V collects terms in the high frequency amplitude:

$$V = \int d^3x\, [-(\vec{A}^*\cdot\overleftrightarrow{\varepsilon}\cdot\vec{A})(\omega^2/16\pi c^2) + |\mathrm{curl}\,\vec{A}|^2/16\pi - \mathrm{Re}(\vec{j}_a^{\,*}\cdot\vec{A})/2c]. \tag{4}$$

Here $\overleftrightarrow{\varepsilon}$ is the hermitian (since we have excluded resonances) dielectric tensor at frequency ω. The equation satisfied by the high frequency field amplitude $\vec{A}$, obtained by setting to zero the variation of V with respect to $\vec{A}$, is the driven wave equation

$$(\omega^2/c^2)\overleftrightarrow{\varepsilon}\cdot\vec{A} - \mathrm{curl}\,\mathrm{curl}\,\vec{A} = -(4\pi/c)\vec{j}_a. \tag{5}$$

It is important to note that the dielectric tensor $\overleftrightarrow{\varepsilon}$, the wave solution $\vec{A}$, the ponderomotive energy V, and the actions S_{oc} and S_s are functionals of the fields $\vec{r}_c(z^\circ,t)$ and $\vec{A}_s(\vec{x},t)$. Thus, in the cold plasma approximation adopted in this paper, the dielectric tensor is a local function [10] of the oscillation center densities $n_\sigma(\vec{x},t)$ and of the slow magnetic field $\vec{B}(\vec{x},t)$, which will be precisely defined later. The variation of the new action (1) with respect to its variables $\vec{r}_c$ and $\vec{A}_s$ provides the complete set of Lagrangian equations for the system. The ponderomotive effects appear as a ponderomotive force $\vec{F}_p$ on an oscillation center, $\vec{F}_p(z^\circ,t) = -\nabla(\delta V/\delta n_\sigma)$ at $\vec{x} = \vec{r}_c(z^\circ,t)$, and as a magnetization current density $\vec{j}_m(\vec{x},t) = \mathrm{curl}(-\delta V/\delta\vec{B})$ in Ampere's law. The ponderomotive potential and the magnetization are [11]

$$\delta V/\delta n_\sigma(\vec{x}) = -[\vec{A}^* \cdot (\partial\overleftrightarrow{\varepsilon}/\partial n_\sigma)\cdot\vec{A}]\ (\omega^2/16\pi c^2), \tag{6}$$

$$-\ \delta V/\delta\vec{B}\ (\vec{x}) = [\vec{A}^* \cdot (\partial\overleftrightarrow{\varepsilon}/\partial\vec{B}) \cdot \vec{A}]\ (\omega^2/16\pi c^2). \tag{7}$$

respectively. This Lagrangian description leads to a number of useful results [8] and is appropriate for comparison with particle simulations, but for many applications an Eulerian description is more convenient. We show that the latter is provided by the appropriate Hamiltonian structure of the system. Following standard procedure [12], we derive from the expression (1) of the action the fields $\vec{p}_c(z^\circ,t)$ and $\vec{\pi}(\vec{x},t)$, canonically conjugate to $\vec{r}_c(z^\circ,t)$ and $\vec{A}_s(\vec{x},t)$. The Poisson bracket in terms of these variables is of course the canonical one. The corresponding Hamiltonian is obtained as the Legendre tranform of the Lagrangian, and is a functional of $\vec{r}_c$, $\vec{p}_c$, $\vec{A}_s$, and $\vec{\pi}$. Thus one finds the conjugate variables, $\vec{p}_c(z^\circ,t) = \delta S/\delta\dot{\vec{r}}_c = n^\circ[m\dot{\vec{r}}_c + (q/c)\vec{A}_s(\vec{r}_c,t)]$ and $\vec{\pi}(x,t) = \delta S/\delta(\partial\vec{A}_s/\partial t) = (\partial\vec{A}_s/\partial t)/4\pi c^2$, the canonical Poisson brackets $\{\vec{r}_c(z^\circ), \vec{p}_c(z^{\circ\prime})\} = \overleftrightarrow{I}\ \delta(z^\circ - z^{\circ\prime})$ and $\{\vec{A}_s(\vec{x}),\vec{\pi}(\vec{x}')\} =$

$\vec{I}\,\delta(\vec{x}-\vec{x}')$, and the Hamiltonian [13] $H = V + \int n^\circ dz^\circ[\frac{1}{2}m\dot{\vec{r}}_c^2 + \psi(\vec{r}_c)] + \int d^3x\,[(\partial\vec{A}_s/\partial t)^2/8\pi c^2 + (\mathrm{curl}\,\vec{A}_s)^2/8\pi + \Sigma_\sigma U_\sigma(n_\sigma)]$. It so happens that H can be expressed entirely in terms of the Eulerian fields and that the Poisson bracket of any two of these fields is expressible as a function of themselves [14,15]. The Eulerian dynamical fields for a cold plasma are the oscillation center densities $n_\sigma(x,t) = \int n^\circ dz^\circ\,\delta[\vec{x}-\vec{r}_c(z^\circ,t)]$ and flux densities $\vec{g}_\sigma(\vec{x},t) = \int n^\circ dz^\circ\,\dot{\vec{r}}_c\,\delta[\vec{x}-\vec{r}_c(z^\circ,t)]$ or velocities $\vec{u}_s=\vec{g}_s/n_s$, and the slow fields $\vec{B} = \mathrm{curl}\,\vec{A}_s$ and $\vec{E} = -c^{-1}\partial\vec{A}_s/\partial t$.

In these Eulerian variables, the Poisson bracket becomes the Spencer-Kaufman bracket [14,15] and the Hamiltonian is the total energy: $H = V + \int d^3x\,\Sigma_\sigma[\frac{1}{2}n_\sigma m_\sigma u_\sigma^2 + n_\sigma\psi_\sigma + U_\sigma(n_\sigma)] + \int d^3x\,(E^2 + B^2)/8\pi$, where V is now to be considered as a functional of the fields $n_\sigma(x)$ and $\vec{B}(x)$. This Hamiltonian structure yields the evolution of any functional F of the Eulerian fields according to $dF/dt = \{F,H\}$. Thus we find the complete system of equations (species label suppressed):

$m(\partial\vec{u}/\partial t + \vec{u}\cdot\nabla\vec{u}) = q(\vec{E} + \vec{u}\times\vec{B}/c) - \nabla(\psi + dU/dn + \delta V/\delta n)$,

$\partial\vec{B}/\partial t = -c\,\mathrm{curl}\,\vec{E}$, $\qquad \partial n/\partial t + \nabla\cdot(n\vec{u}) = 0$,

$\mathrm{curl}(\vec{B} + 4\pi\delta V/\delta\vec{B}) = (4\pi/c)\Sigma_\sigma qn\vec{u} + c^{-1}\partial\vec{E}/\partial t$, in which the ponderomotive forces and magnetization current appear, as defined in Eqs. (6), (7).

One important consequence of this Hamiltonian formulation is that it allows the use of Arnold's stability method [9,16]. This method produces simple criteria sufficient for stability of certain nonstatic equilibria, and is generalizable to nonlinear stability analysis [9,17]. It exploits the existence of Casimir functionals for degenerate Poisson brackets, in order to construct Lyapunov functionals. We first discuss the two-dimensional multifluid case, with $\vec{B}$ in the $\hat{z}$ direction. It is known [17] that a first family of Casimir functionals is given by $C_1 = \int dxdy\,\Sigma_\sigma\,n_\sigma\,\Phi_\sigma(Z_\sigma)$,

where $Z_\sigma = (w_\sigma + \Omega_\sigma)/n_\sigma$, $w_\sigma = \hat{z}\cdot\text{curl}\,\vec{u}_\sigma$ is the vorticity, Ω_σ is the signed gyrofrequency, and Φ_σ is an arbitrary function of its argument. A second family of Casimirs is $C_2 = \int dxdy\,(\Sigma_\sigma\, q_\sigma n_\sigma - \nabla\cdot\vec{E}/4\pi)\phi(\vec{x})$, where ϕ is an arbitary function of $\vec{x}$.

Our Lyapunov functional is the sum of the Hamiltonian and the Casimirs: $H_c = H + C_1 + C_2$. The functional H_c can in fact be considered as a Hamiltonian, equivalent to H since they generate the same evolution: for any functional G, one has $\{G,H\}=\{G,H_c\}$.

The critical points of the Lyapunov functional H_c are <u>equilibrium solutions</u> of the system. The first variation δH_c vanishes for all variations of the Eulerian fields, which provides the set of equilibrium equations: $\vec{E} = -\nabla\phi$; $\quad B^2/4\pi + B\delta V/\delta B + \Sigma_\sigma\,\Omega\, d\Phi/dZ = 0$;
$mn\vec{u} = \hat{z}\times\nabla Z d^2\Phi/dZ^2$; $\quad \frac{1}{2}mu^2 + \Psi + \delta V/\delta n + dU/dn + q\phi + \Phi - Zd\Phi/dZ = 0$,
for each species. The functional derivatives of V are given by Eqs. (6) and (7), and the high frequency field $\vec{A}$ is a solution of Eq. (5). It is easy to check that the solution of this system is stationary: $\partial n/\partial t=\{n,H\}=\{n,H_c\}=0$, etc.

The equilibrium is linearly stable if the Lyapunov functional H_c is locally a minimum at the stationary point, i.e. if its second variation is a positive definite quantity. The second variation, $\delta^2 H_c$, is the conserved Hamiltonian for the motion linearized around the equilibrium solution where δH_c vanishes, see. see Ref. 9. Consequently, when the quadratic form $\delta^2 H_c$ is positive definite, it provides a norm which is preserved by the linearized equations; hence Lyapunov stability in terms of this norm is implied for the linearized motion. Thus, for the two dimensional multifluid plasma case, the <u>sufficient stability condition</u> is

$$0 < 2\,\delta^2 H_c = \int d^3x\, \Sigma_\sigma\, mn|\delta\vec{u} + \vec{u}\delta n/n|^2 + \int d^3x\, [(\delta\vec{E})^2 + (\delta\vec{B})^2]/4\pi$$
$$+ \int d^3x\, \Sigma_\sigma (\delta n)^2 (d^2U/dn^2 - mu^2/n) + \int d^3x\, \Sigma_\sigma (\delta Z)^2 d^2\Phi/dZ^2$$
$$+ \int d^3x \int d^3x' [\delta B(\vec{x})\, \delta B(\vec{x}')\, \delta^2 V/\delta B \delta B' + 2\, \Sigma_\sigma\, \delta n(\vec{x})\, \delta B(\vec{x}')\, \delta^2 V/\delta n \delta B'$$
$$+ \Sigma_{\sigma\sigma'}\, \delta n_\sigma(\vec{x})\, \delta n_{\sigma'}(\vec{x}')\, \delta^2 V/\delta n_\sigma(\vec{x}) \delta n_{\sigma'}(\vec{x}')],$$

where $\delta Z = (\hat{z}\cdot \mathrm{curl}\, \delta\vec{u} + \Omega\delta B/B - Z\, \delta n)/n$.

In particular, the second variation $\delta^2 H_c$ is positive definite if the equilibrium flows are subsonic: $mu^2 < nd^2U/dn^2$, if the equilibrium is such that $d^2\Phi/dZ^2 > 0$, and if the second variation of the ponderomotive energy V is positive definite. The self-consistent modification of the high frequency field is the physical process described by the second variation of V, which is given explicitly by

$$-(16\pi c^2/\omega^2)\delta^2 V = \int d^3x\, \vec{A}^*\cdot \overleftrightarrow{\varepsilon}^{(2)}\cdot \vec{A} - \iint d^3x\, d^3x'\, \vec{K}^*(\vec{x})\cdot \overleftrightarrow{G}(\vec{x},\vec{x}')\cdot \vec{K}(\vec{x}'),$$

where $\overleftrightarrow{G}$ is the Green's function of the operator $\overleftrightarrow{\varepsilon} - c^2 \mathrm{curl}^2/\omega^2$, evaluated at equilibrium; $\vec{K} = \overleftrightarrow{\varepsilon}^{(1)}\cdot \vec{A}$; $\overleftrightarrow{\varepsilon}^{(1)} = \delta B\, \partial\overleftrightarrow{\varepsilon}/\partial B + \Sigma_\sigma\, \delta n_\sigma\, \partial\overleftrightarrow{\varepsilon}/\partial n_\sigma$, and $\overleftrightarrow{\varepsilon}^{(2)} = \frac{1}{2}(\delta B)^2\, \partial^2\overleftrightarrow{\varepsilon}/\partial B^2 + \Sigma_\sigma\, \delta B\, \delta n_\sigma\, \partial^2\overleftrightarrow{\varepsilon}/\partial B \partial n_\sigma$.

We have previously [8] shown that V is equal to the antenna inductive energy. This interpretation allows one to draw the important conclusion that <u>the self-consistent modification of the fields is stabilizing if the antenna inductance, in the presence of plasma, is minimum for the equilibrium configuration</u>. Note that the ponderomotive effects influence the stability, not only be contributing directly to the perturbed energy, but also by modifying the equilibrium, and therefore the functions Φ_σ.

A similar analysis can be carried out for the two-dimensional magnetohydrodynamic model, as we outline briefly. Now the Hamiltonian is a functional of the fluid density $n(x,y;t)$, the magnetic field $B(x,y;t)$, and the velocity field $\vec{u}(x,y;t)$ contained in the x-y plane:
$H = V + \int dxdy\, (\frac{1}{2} mnu^2 + n\Psi + U + B^2/8\pi)$. The associated Poisson

bracket is that of Morrison and Greene [16]. The resulting evolution equations are: $\partial n/\partial t + \nabla\cdot(n\vec{u}) = 0$, $\partial B/\partial t + \nabla\cdot(B\vec{u}) = 0$, $mn(\partial\vec{u}/\partial t + \vec{u}\cdot\nabla\vec{u}) = -\nabla(B^2/8\pi) - B\nabla(\delta V/\delta B) - n\nabla(\psi + dU/dn + \delta V/\delta n)$, and the Casimir functional associated with the Poisson bracket is [9] $C = \int dxdy\ n\,\Phi(B/n)$, where Φ is an arbitrary function of its argument $Y=B/n$. The appropriate Lyapunov functional is, as before, $H_c = H + C$. From $\delta H_c = 0$, we obtain the stationary equilibrium equations: $u=0$; $B/4\pi + \delta V/\delta B + d\Phi/dY = 0$; $\psi + \delta V/\delta n + dU/dn + \Phi - Y d\Phi/dY = 0$.

The linear stability condition obtained from $\delta^2 H_c > 0$ is

$$0 < 2\,\delta^2 H_c = \int d^3x\,[mn(\delta u)^2 + (\delta B)^2/4\pi + (\delta n)^2 d^2U/dn^2 + (\delta Y)^2(B^2/n)d^2\Phi/dY^2]$$
$$+ \int d^3x \int d^3x'[\delta n(\vec{x})\delta n(\vec{x}')\delta^2V/\delta n\delta n' + \delta B(\vec{x})\delta B(\vec{x}')\delta^2V/\delta B\delta B' +$$
$$2\delta n(\vec{x})\delta B(\vec{x}')\delta^2V/\delta n\delta B'],$$

where $\delta Y/Y = \delta B/B - \delta n/n$. The equilibrium is certainly MHD stable if the second variation of V is positive definite, and if the equilibrium is such that $d^2\Phi/dY^2 > 0$.

It is interesting to note the connection of the second variation of the Lyapunov functional H_c with the ΔW variational principle derived in Ref. 8. If one expresses the variations δn (etc.) in terms of the plasma displacement $\vec{\xi}(x,t)$, i.e., $\delta n = -\nabla\cdot(n\vec{\xi})$, $\delta B = -\nabla\cdot(B\vec{\xi})$, and $\delta u = \delta\vec{\xi}/\delta t$, then the condition $\delta^2 H_c > 0$ is <u>equivalent</u> to $\Delta W > 0$.

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