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ELASTODYNAMIC STRESS INTENSITY FACTORS OF AN INTERFACE FINITE-WIDTH CRACK

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July 1984

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Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

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LBL-18051
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San Diego, CA, July 9-13, 1984

Elastodynamic Stress Intensity Factors of an Interface Finite-Width Crack

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ABSTRACT

Many applications in industry utilize a layered elastic structure in which a relatively thin layer of one material is bonded to a much thicker substrate. Often the fabrication process is imperfect and cracks occur at the interface.

This paper is concerned with the plane strain, time-harmonic problem of a single elastic layer of one material on a half space of a different material with a single crack at the interface. The derivation of the required Green's functions for dilational and rotational point sources in the uncracked layered half space are outlined here. These functions are used with the appropriate form of Green's integral theorem to derive the scattered field potentials for arbitrary incident fields in the cracked layered half space. These potentials are then used in turn to reduce the problem to a system of singular integral equations for determining the gradients of the crack opening displacements in the scattered field. The integral equations are analyzed to determine the crack tip singularity, which is found, in general, to be oscillatory, as it is in the corresponding static problem of an interface crack. For many material combinations of interest, however, the crack tip singularity in the stress field is one-half power, as in the case of a homogeneous material. In the numerical work of this work attention is restricted to this class of composites and the integral equations are solved numerically to determine the Mode I and Mode II stress intensity factors as a function of a dimensionless wave number for various ratios of crack length to layer depth. The results are presented in graphical form and are compared with previously published analyses for the special cases where such results are available.

INTRODUCTION

Many structures, both man-made and of natural origin, are composed of different elastic materials formed in layers. Often the layers are bonded together along common faces, but it can happen that the bonding is not perfect and flaws occur as cracks or regions of poor bonding in the interface. It is of importance to be able to detect these interface cracks, and one of the most practical methods for accomplishing this, in the cases of engineering interest, utilizes the scattering of elastic waves and the subsequent detection of these scattered waves by appropriate transducers. The goal of this work is to contribute to the theoretical basis for detecting the interface flaw by these means.

The previously published works that bear directly on this problem are those of Neerhoff [1], which considered the corresponding problem for the case of anti-plane strain, and the paper by Keer et al. [2], which was concerned with the plane strain problem of a crack parallel to the boundary in a homogeneous half space. Also the paper by Farnell and Adler [3], that studied the various types of free surface waves that can exist in a half space with a single layer on top, as well as the paper by Bogy and Gracewski [4] on the same topic, are of major significance to this work. This paper is based on the Ph.D. dissertation of Yang [10].

The method utilized here to solve the plane strain interface flaw problem is similar to that used by Neerhoff [1] for the antiplane problem. First we formulate the problem, where the elastic field is separated into the incident and scattered fields. The incident field is left arbitrary in the analysis until a particular choice is made later on for the purpose of obtaining numerical results. In Yang [10] the Green's functions were derived for dilatational and rotational sources in the layer as well as the substrate. These Green's functions are used in conjunction with the appropriate form of Green's integral theorem, to reduce the problem to a system of singular integral equations. These integral equations are analyzed to determine the order of singularity in the stress field at the crack tip, and it is found that this singularity is of the oscillatory type, except for a certain class of composites. The methods of Muskhelishvili [5] and Erdogan and Gupta [6] are then utilized to reduce these singular integral equations to a system of algebraic equations suitable for numerical solution.

These equations are solved numerically for several sets of composites, and the results are presented in the form of Mode I and Mode II stress intensity factors plotted as functions of dimensionless wave number for various ratios of layer depth to crack length.

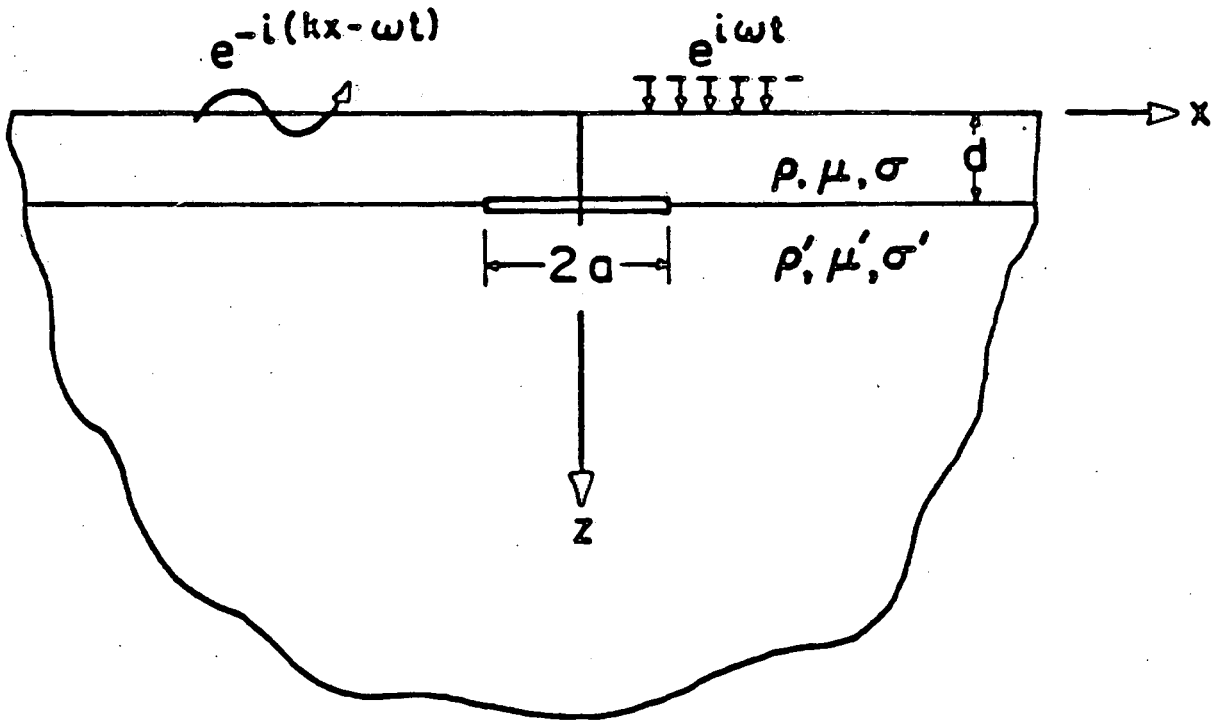

Fig. 1. Layered half space with interface crack and incident waves.

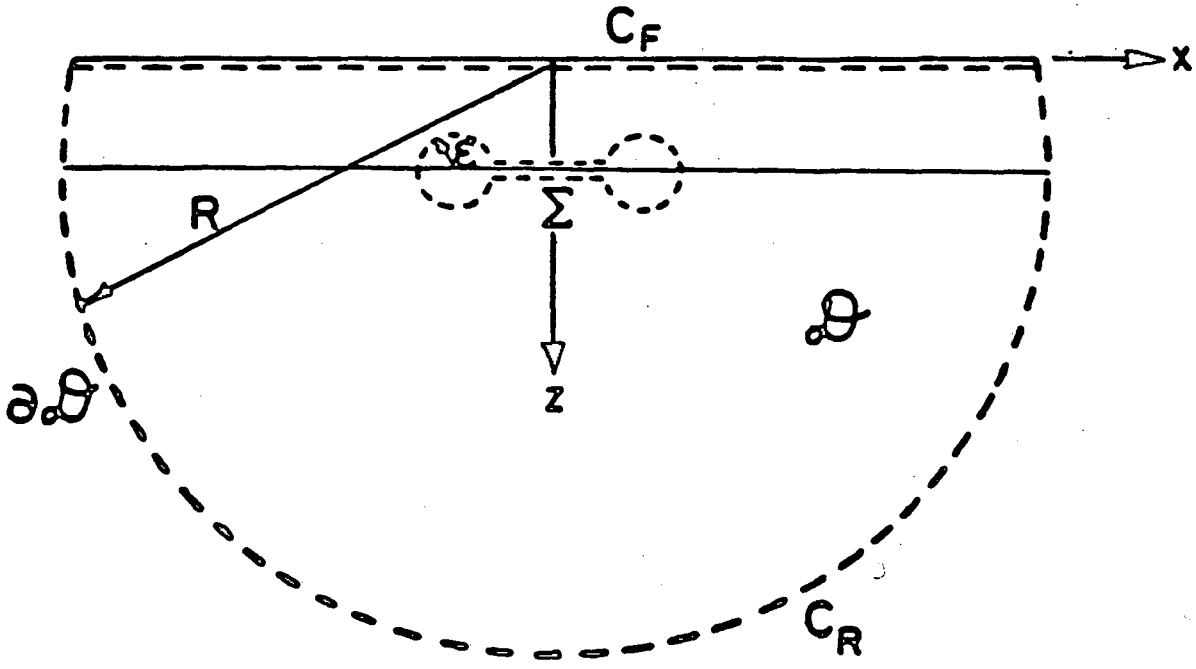

Fig. 2. Domain to which divergence theorem is applied.

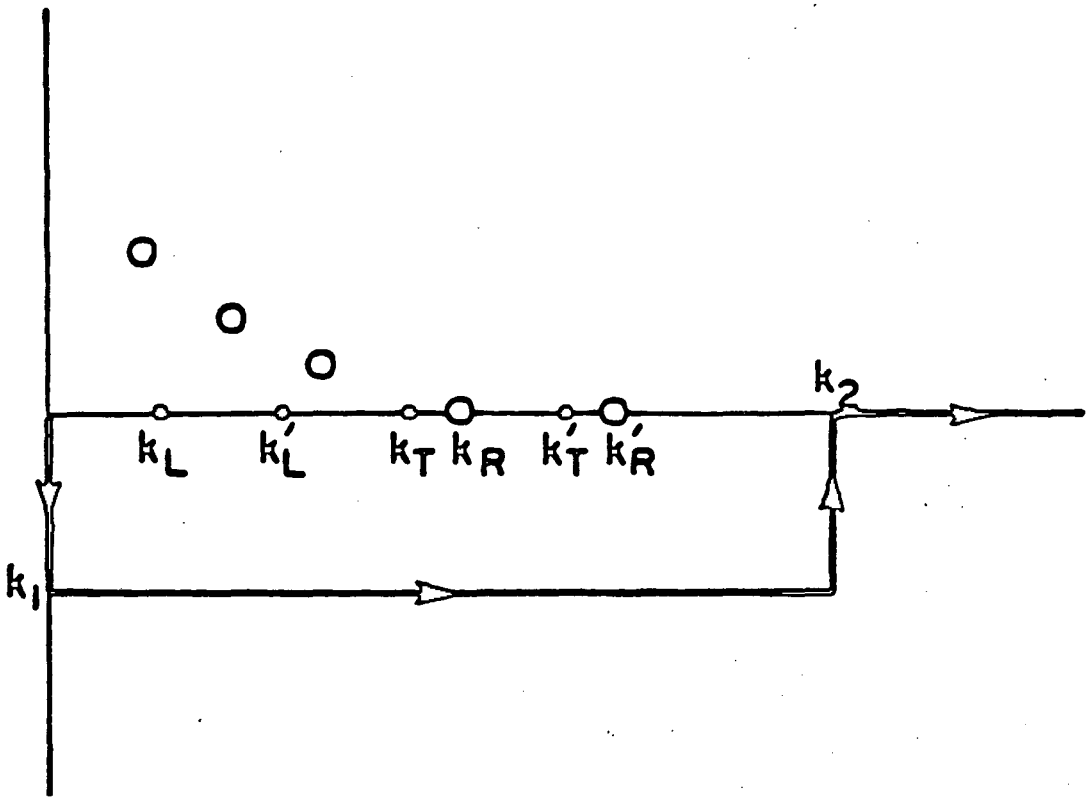

Fig. 3. Path of integration of evaluating semi-infinite integrals.

THEORY

We consider the steady time-harmonic plane-strain problem for the cracked layered elastic half space shown in Fig. 1.

The time-reduced form of the displacement equations of motion for an isotropic media is

$$(\lambda+\mu)\nabla\nabla\underline{u} + \mu\nabla^2\underline{u} + \rho\omega^2\underline{u} = -\rho\underline{f} \ ,$$

The displacement and stress can be decomposed into the sum of incident and scattered parts according to

$$\underline{u} = \underline{u}^{(i)} + \underline{u}^{(s)}, \ \underline{\tau} = \underline{\tau}^{(i)} + \underline{\tau}^{(s)}.$$

The solution of the scattered field problem formulated here is obtained in the following sections by a method of integral equations in a manner similar to that used for the corresponding anti-plane problem in Ref. [1]. This method utilizes Green's functions corresponding to point sources in the uncracked layered half space.

Use the Green's functions derived in Ref. [10] for the uncracked layered half space to reduce the problem formulated for the cracked layered half space to a system of integral equations. For this purpose we make use of Green's (divergence) theorem in the following form (for 3-D elasticity solution).

$$\iint_D (u_i^A \tau_{ij,j}^A) dA = \oint_{\partial D} (u_i^A \tau_{ij}^B - u_i^B \tau_{ij}^A) n_j ds \ ,$$

where ∂D denotes the boundary of the region D and n is the outward unit normal to ∂D as shown in Fig. 2. Letting $\underline{u}^A = \underline{u}^{(s)}$, $\underline{u}^B = \underline{u}_L$ and $\underline{u}^A = \underline{u}^{(s)}$, $\underline{u}^B = \underline{u}_T$, respectively, where $u^{(s)}$ is the displacement of the scattered field and $\underline{u}_L$, $\underline{u}_T$ denote the dilatational and rotational wave fields associated with the Green's functions, we can derive the potentials $\phi^{(s)}(\underline{x}_p)$, $\psi^{(s)}(\underline{x}_p)$ for the scattered field.

Then we can obtain the result:

$$\phi^{(s)}(\underline{x}_p) = -\frac{1}{\mu k_f^2} \int_{-a}^{a} [u_x^{(s)}]\tau_{zx}^L + [u_z^{(s)}]\tau_{zz}^L dx, \ \ 0 < z_p < d$$

$$-\frac{1}{\mu' k_T'^2} \int_{-a}^{a} [u_x^{(s)}]\tau_{zx}^L + [u_z^{(s)}]\tau_{zz}^L dx, \ \ d < z_p < \infty$$

$$\phi^{(s)}(\underline{x}_p) = -\frac{1}{\mu k_f^2} \int_{-a}^{a} [u_x^{(s)}]\tau_{zx}^T + [u_z^{(s)}]\tau_{zz}^T dx, \ \ 0 < z_p < d$$

$$-\frac{1}{\mu' k_T'^2} \int_{-a}^{a} [u_x^{(s)}]\tau_{zx}^T + [u_z^{(s)}]\tau_{zz}^T dx, \ \ d < z_p < \infty$$

in which

$$[u_x^{(s)}] = \lim_{z\to d^+} u_x^{(s)} - \lim_{z\to d^-} u_x'^{(s)}$$

$$[u_z^{(s)}] = \lim_{z\to d^+} u_z^{(s)} - \lim_{z\to d^-} u_z'^{(s)}$$

and the integrals are taken along the crack and therefore the quantities τ_{zx}^{L}, etc. are evaluated along z = d.

Next we compute the interface tractions corresponding to the scattered wave potentials and impose the traction free conditions on the crack face to obtain integral equations for determining the unknown crack opening displacements and obtain the following pair of integral equations

$$\int_{-a}^{a} [\int_{0}^{\infty} \frac{p_1(k)}{\Delta(k)k} \sin[k(x-x_p)]dk] f_x(x)$$

$$-[\int_{0}^{\infty} \frac{ip_2(k)}{\Delta(k)k} \cos[k(x-x_p)]dk] f_z(x)\, dx = \frac{\pi}{\mu\mu'} \tau_{zx}^{(i)}(x_p)$$

$$\int_{-a}^{a} [\int_{0}^{\infty} \frac{ip_2(k)}{\Delta(k)k} \cos[k(x-x_p)]dk] f_x(x)$$

$$+[\int_{0}^{\infty} \frac{p_3(k)}{\Delta(k)k} \sin[k(x-x_p)]dk] f_z(x)\, dx = \frac{\pi}{\mu\mu'} \tau_{zz}^{(i)}(x_p), \quad -a < x_p < a.$$

where

$$f_x(x) = \frac{d}{dx}[u_x^{(s)}(x)], \quad f_z(x) = \frac{d}{dx}[u_z^{(s)}(x)]$$

and

$$\int_{-a}^{a} f_x(x)\, dx = 0, \quad \int_{-a}^{a} f_z(x)\, dx = 0.$$

The problem has therefore been reduced to the system of integral equations, which need to be solved for f_x and f_z for prescribed $\tau_{zx}^{(i)}$, $\tau_{zz}^{(i)}$, and subject to the resultant conditions above.

The integral equations we obtained appear to be of the first kind but the kernel functions contain infinite integrals which may have singularities. We know that the function $\Delta(k)$ has zeros corresponding to the propagating surface modes of the type studied in Ref. [3]. These roots may occur on or off the real k-axis and they determine poles in the complex k-plane of the integrands in the kernels. We must determine their locations for given geometry, material parameters, and frequency and take them into account in our numerical solution.

In addition to the zeros of $\Delta(k)$, we must investigate the behavior of the kernel integrands in the limit $k \to \infty$ and $k \to 0$. Also, the multi-valued functions γ_L, γ_T, γ_L' and γ_T', defined as $\gamma_L^2 = k_L^2 - k^2$, etc., have branch points at $\pm k_L$, $\pm k_T$, $\pm k_L'$ and $\pm k_T'$, respectively. We choose the branchs such that $Re(\gamma_L) \geqslant 0$, $Re(\gamma_T) \geqslant 0$ on the path of integration.

Making use of the asymptotic expressions to remove the singularities from the kernels, we obtain the following integral equations

$$-\frac{\pi\delta_2}{\delta_0} f_z(x_p) + \frac{\delta_1}{\delta_0} \int_{-a}^{a} \frac{f_x(x)}{x-x_p}\, dx + \int_{-a}^{a} L(x_p,x) f_x(x) dx$$

$$-i \int_{-a}^{a} M(x_p,x)f_z(x)dx = \frac{\pi}{\mu\mu'} \tau_{zx}^{(i)}(x_p),$$

$$\frac{\pi\delta_2}{\delta_0} f_x(x_p) + \frac{\delta_1}{\delta_0} \int_{-a}^{a} \frac{f_x(x)dx}{x-x_p} + i \int_{-a}^{a} M(x_p,x)f_x(x)dx$$

$$-i \int_{-a}^{a} L^*(x_p,x)f_z(x)dx = \frac{\pi}{\mu\mu'} \tau_{zz}^{(i)}(x_p).$$

The kernels $L(X_p,x)$, $M(x_p x)$ are regular, which means that the system has only Cauchy singularities.

We next analyze the singular integral equations to determine the order of singularity in the solution functions $f_x(x)$ and $f_z(x)$. Here we follow the technique in Ref. [5] and define $F_x(x)$, $F_z(x)$ through

$$f_x(x) = F_x(x)/(a^2-x^2)^\alpha,\ f_z(x) = F_z(x)/(a^2-x^2)^\alpha,\ |x| < a$$

where $Re(\alpha) < 1$ and F_x, F_z are Holder continuous. Using the result

$$\phi(x_p) = \frac{1}{\pi} \int_{-a}^{a} \frac{f_x(x)}{x-x_p} dx = \frac{F_x(-a)\cot(\pi\alpha)}{(2a)^\alpha(a+x_p)^\alpha} - \frac{F_x(a)\cot(\pi\alpha)}{(2a)^\alpha(a-x_p)^\alpha}$$

together with a similar result for $f_z(x)$, and Eq. (4.11), we obtain, by analyzing the dominant part and taking the limit $x_p \rightarrow a$ (or $x_p \rightarrow -a$),

$$\delta_1 F_x(a)\cot(\pi\alpha) + \delta_2 F_z(a) = 0$$

$$\delta_2 F_x(a) - \delta_1 F_z(a)\cot(\pi\alpha) = 0$$

This system requires, for a nontrivial solution,

$$(\frac{\delta_2}{\delta_1})^2 \sin^2(\pi\alpha) + \cos^2(\pi\alpha) = 0,$$

which is the same as the equation that determines the stress singularity at the tip of an interface crack in static problems (see Ref. [7]). In fact, it can be shown that (for plane strain)

$$\frac{\delta_2}{\delta_1} = \beta = \frac{\mu'(1-2\sigma)-\mu(1-2\sigma')}{2\mu'(1-\sigma)+2\mu(1-\sigma')}$$

which is one of the composite parameters introduced in Ref. [8].

The solution for α is

$$\alpha = \frac{1}{2} \pm \frac{i}{\pi} \tanh^{-1}(\beta) = \frac{1}{2} + \frac{i}{2\pi} \ln (\frac{1+\beta}{1-\beta}).$$

Therefore, the well-known oscillating singularity occurs in the solution of the integral equations, unless $\beta = 0$. If $\beta = 0$, the crack tip singularity is 1/2, as in the case of a homogeneous material. We will restrict our numerical solution of the integral equations to this case.

We now approximate the system of integral equations by a corresponding set of algebraic equations by first introducing dimensionless variables then decomposing the problem into its physically symmetric and anti-symmetric parts with respect to x. Next we recall the approximation formulas from Erdogan and

Gupta [6], when the functions $f_x(x)$, $f_z(x)$ appear in integrals with Cauchy kernels or with regular kernels. For functions appearing outside of integrals we use Lagrange interpolation polynominals together with the symmetries. Then we replace the integral equations to their algebraic approximation, which provides two complex two complex ($n \times n$) linear algebraic systems for determining the 2n complex unknown $F_x^S(\eta_i)$, $F_x^A(\eta_i)$, $F_z^S(\eta_i)$, $F_z^A(\eta_i)$, $i = 1,2,...,n/2$. We may write the complex system symbolically in matrix form as:

$$E^L F^L, \quad L = S,A$$

in which

$$E^L = E_R^L + iE_I^L, \quad F^L = F_R^L + iF_I^L, \quad T^L = T_R^L + iT_I^L.$$

Then we obtain two real linear ($2n \times 2n$) systems

$$\begin{bmatrix} E_R^L & -E_I^L \\ E_I^L & -E_R^L \end{bmatrix} \begin{bmatrix} F_R^L \\ F_I^L \end{bmatrix} = \begin{bmatrix} T_R^L \\ T_I^L \end{bmatrix}, \quad L = S,A$$

We are now ready to solve numerically the system except for the singularity in the integrands of the integrals. These singularities are poles that occur in the complex k-plane at the zeros of $\Delta(k)$. It was shown by Farnell and Adler [3] that in the "loading" case, for which the shear wave velocity of the layer is less than that of the substrate, only one Rayleigh-type pole occurs, and it is on the real k-axis. However, for the "stiffening" case, for which the substrate shear wave velocity is greater, more than one such pole may occur, depending on the frequency. In both cases, it is found (see Bogy and Gracewski [4]), that the poles never occur in the fourth quadrant ($Re(k) > 0$, $Im(k) < 0$) of the k-plane.

Two different ways have been used by others for dealing with these poles. Kundu and Mal [9] use a technique of removing the singularities from the integrands. The other technique, used by Neerhoff [1] and Keer et al. [2], merely deforms the contour of integration below the real k-axis, as shown in Fig. 3, so that no poles occur on the path of integration. We chose the later method in this work.

NUMERICAL RESULTS

Various numerical results can be obtained from our analysis. We shall restrict the presentation here to the dynamic stress intensity factors K_I and K_{II}, at the crack tips, due to an incident wave resulting from a harmonic uniform normal traction applied at the boundary $z = 0$.

Numerical calculations were carried out for three different material pairs: nickel/iron, aluminum/zinc, and nickel/gold. Both materials were considered as layer and substrate in each case. The material parameters were slightly adjusted in each case in order to satisfy the condition $\beta = 0$. The actual values for the material parameters used are given in Ref. [10].

Figure 4a and 4b, respectively, show K_I, and K_{II} versus $k_R'd$ for various ratios of layer thickness d to crack half length for three sets of material combinations. One set is for a nickel layer with iron substrate, one set is for iron layer with iron substrate, and the third set is for an iron layer on a nickel substrate. The material parameters for these two materials are not very different and the computations for this case caused less difficulties than for other cases in which the materials are radically different.

Figures 5a,b show K_I and K_{II}, respectively, as functions of $k_R'd$ for various values of d/a for the case of an aluminum layer on a zinc substrate. Figures 6a,b show the corresponding results for a zinc layer on an aluminum substrate.

Finally, Figs. 7a,b show K_I and K_{II}, respectively, as functions of $k_R'd$ for various values of d/a for the case of a nickel layer on a gold substrate. Figures 8a,b show the corresponding results for a gold layer on a nickel substrate. This pair of materials had the greatest mismatch of material parameters of the three pairs considered.

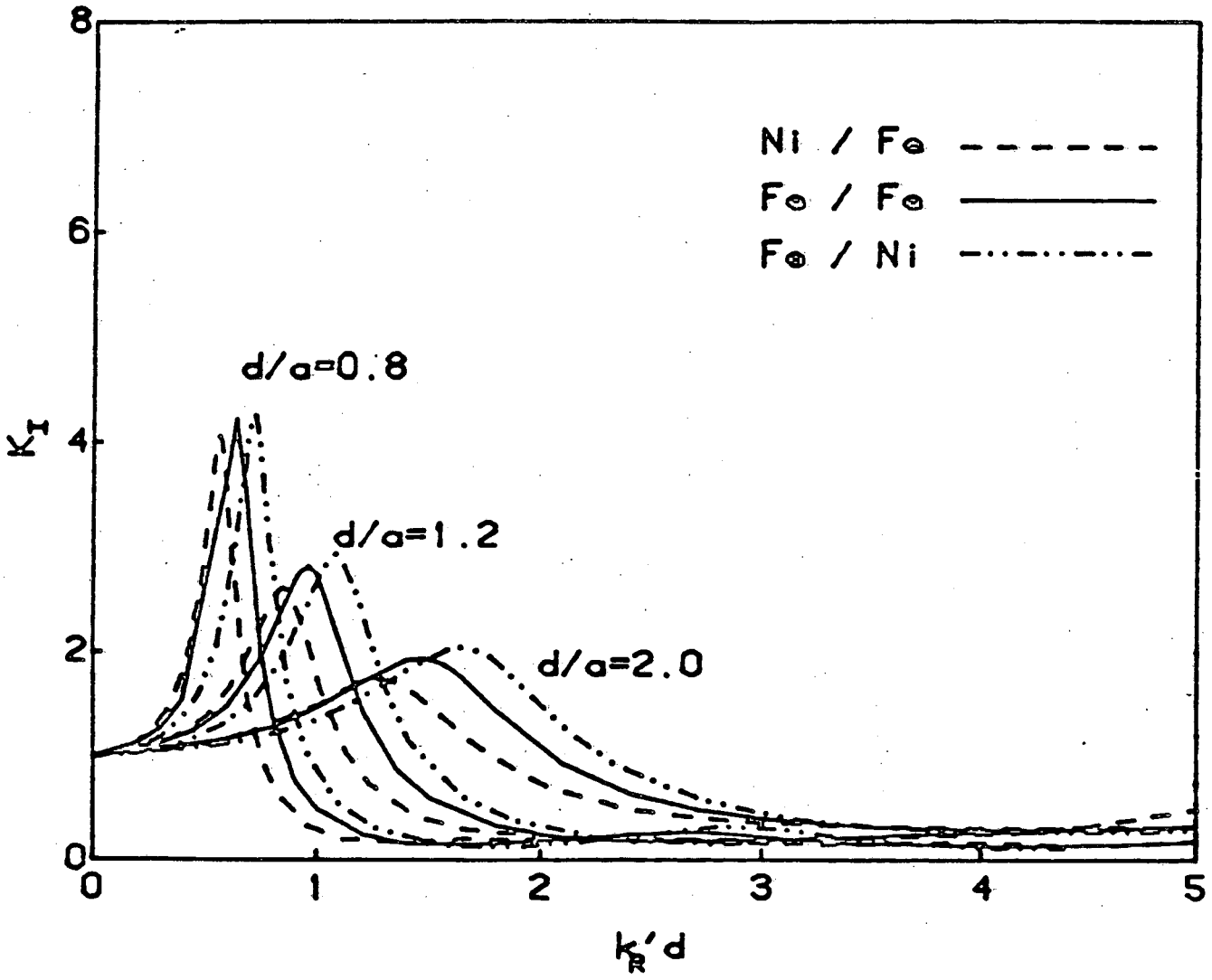

Fig. 4a Mode I stress intensity factor for different combinations of materials – layer/substrate.

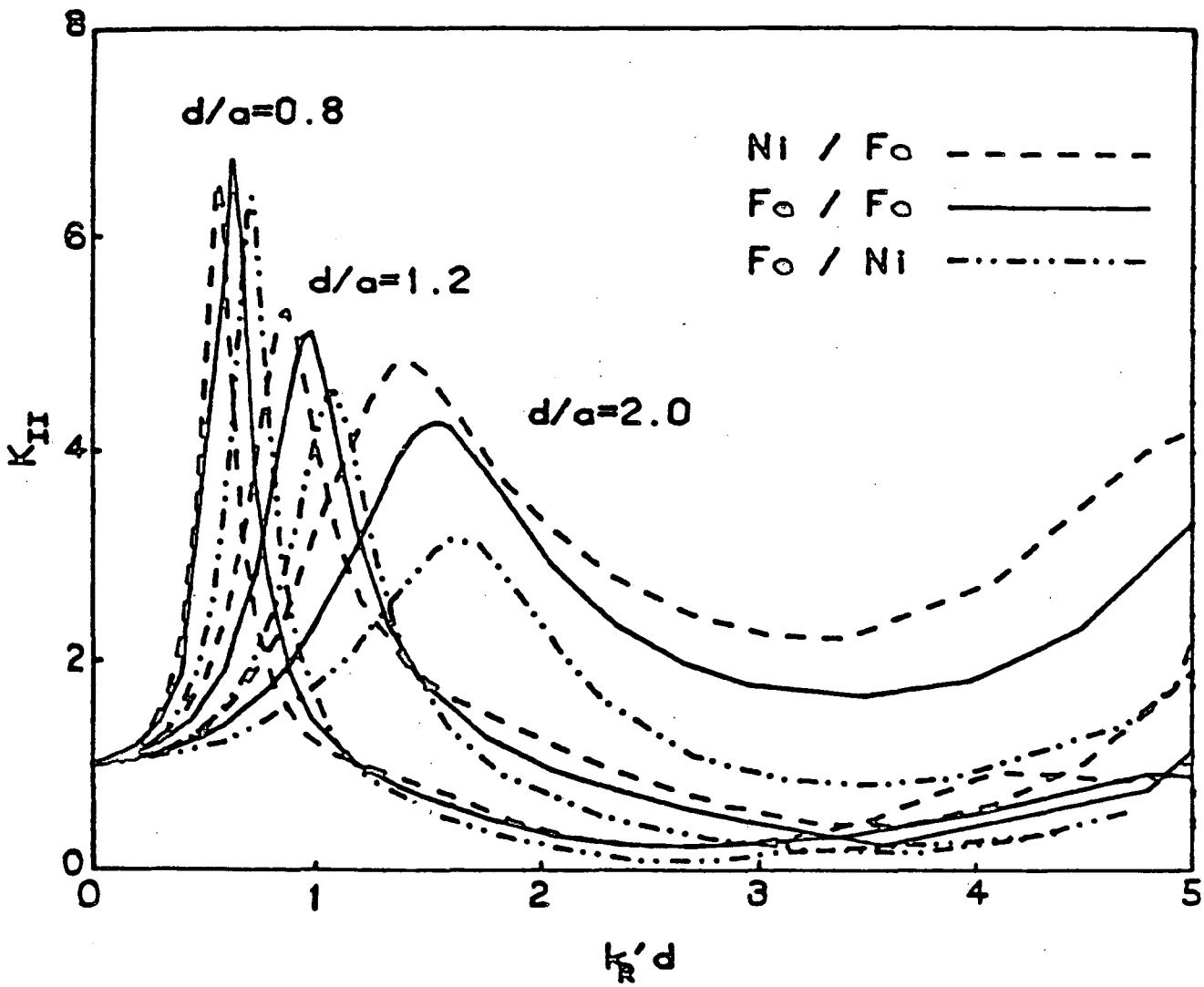

Fig. 4b. Mode II stress intensity factor in correspondence with Fig. 4a.

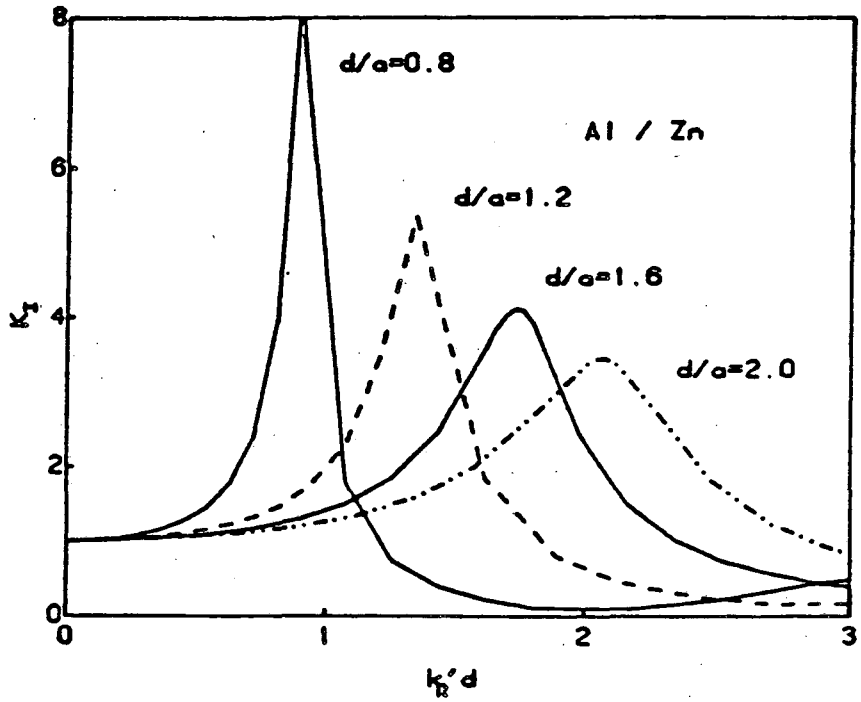

Fig. 5a. Mode I stress intensity factor for Al/Zn.

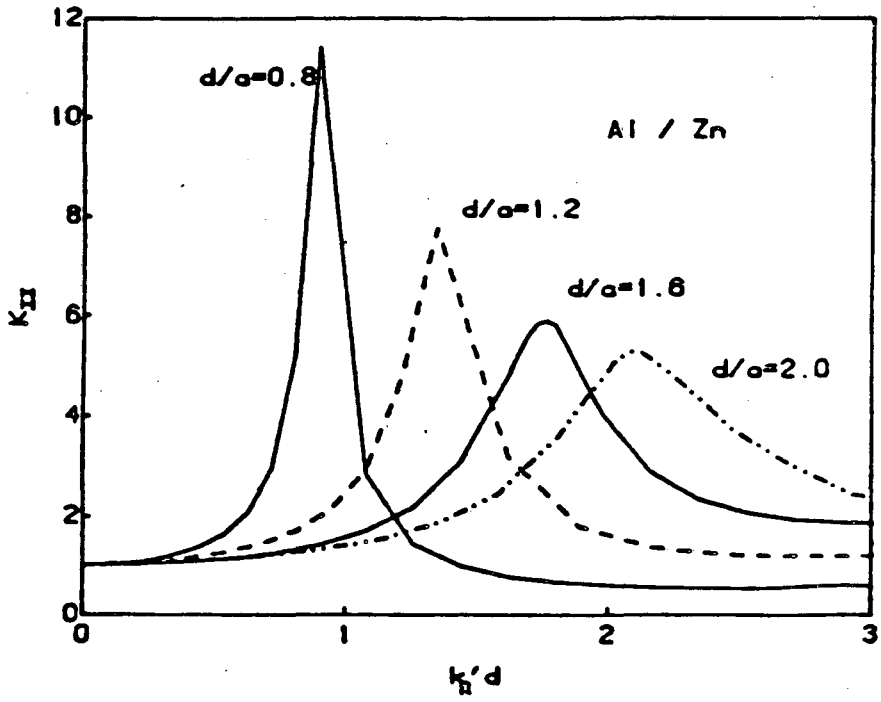

Fig. 5b. Mode II stress intensity factor for Al/Zn.

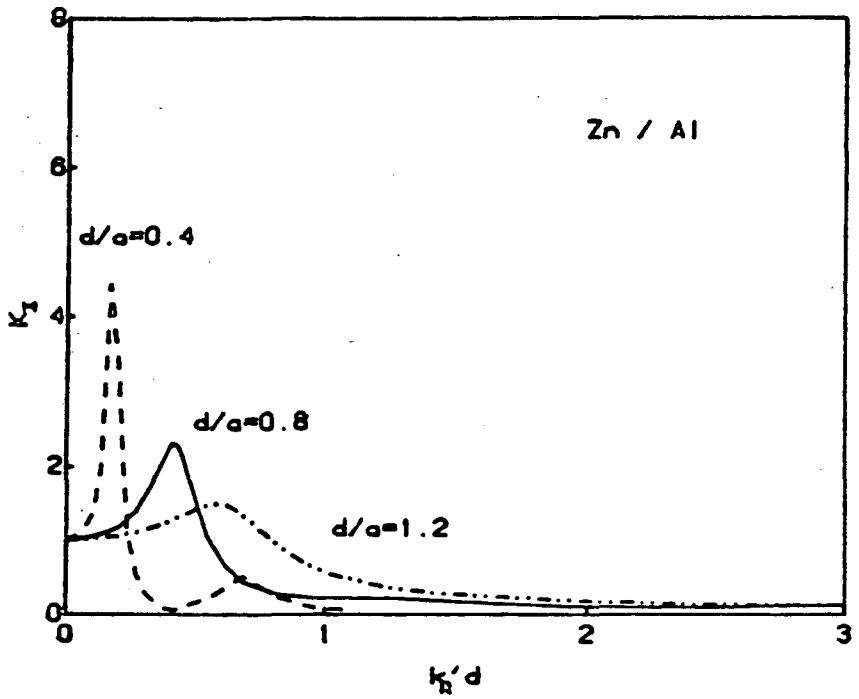

Fig. 6a. Mode I stress intensity factor for Zn/Al.

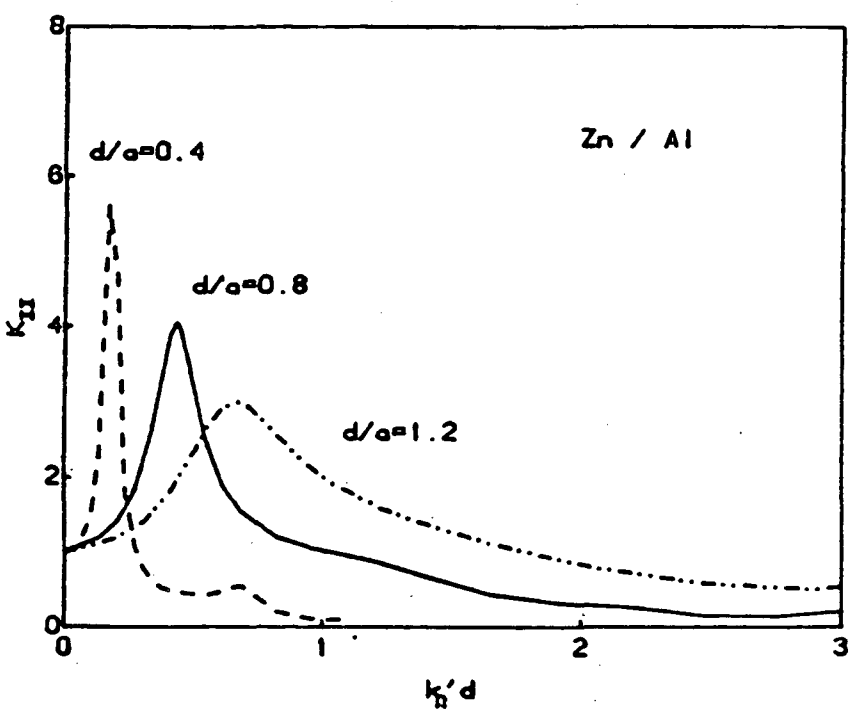

Fig. 6b. Mode II stress intensity factor for Zn/Al.

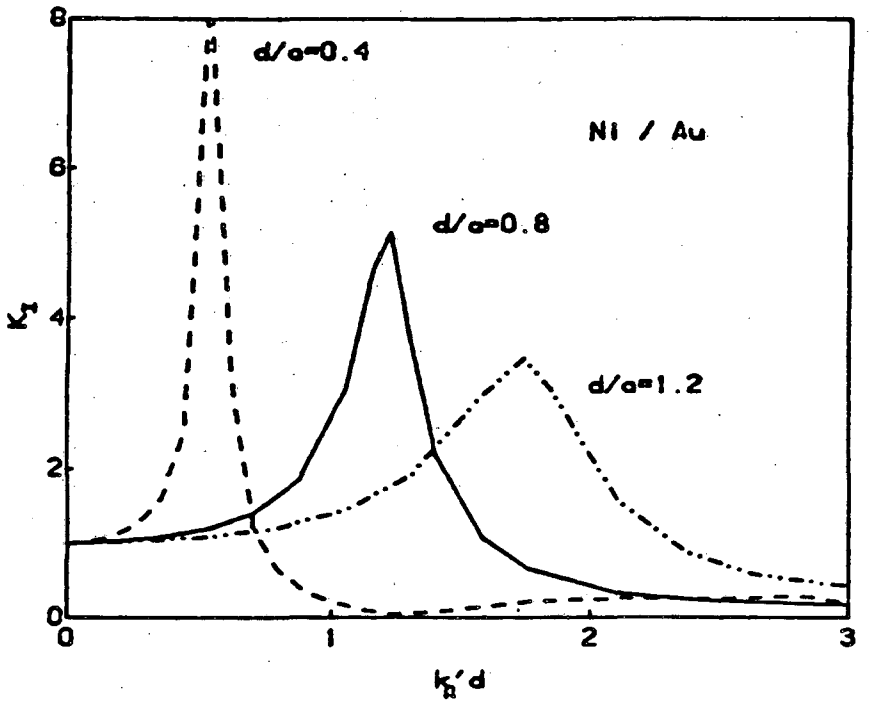

Fig. 7a. Mode I stress intensity factor for Au/Ni.

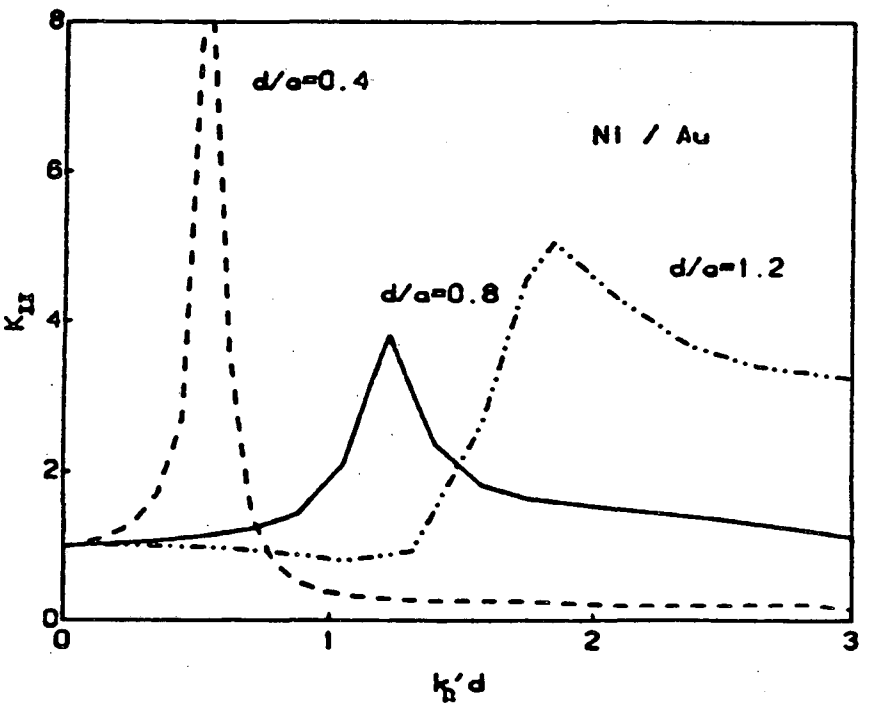

Fig. 7b. Mode II stress intensity factor for Au/Ni.

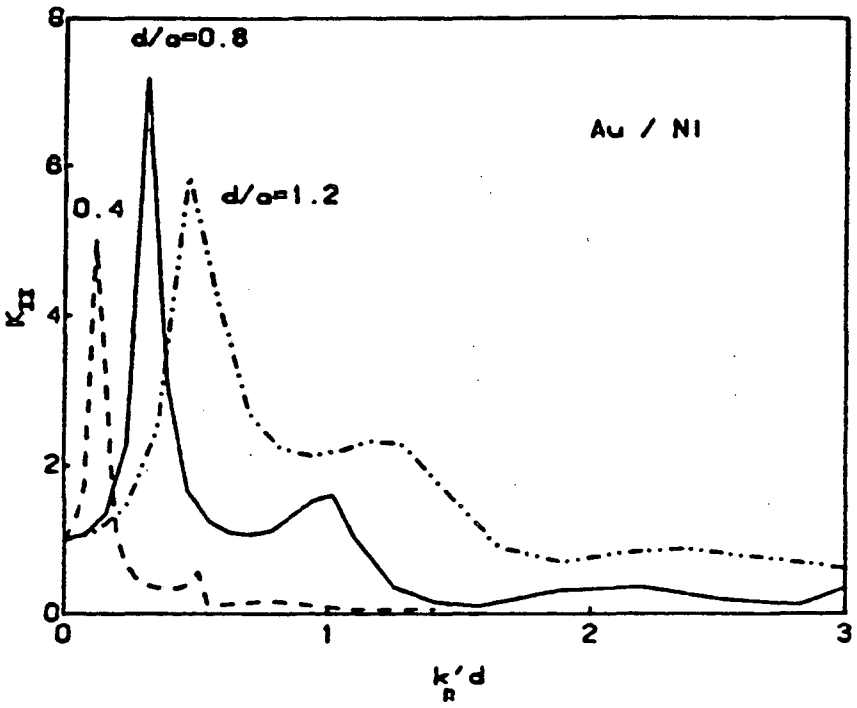

Fig. 8a. Mode I stress intensity factor for Ni/Au.

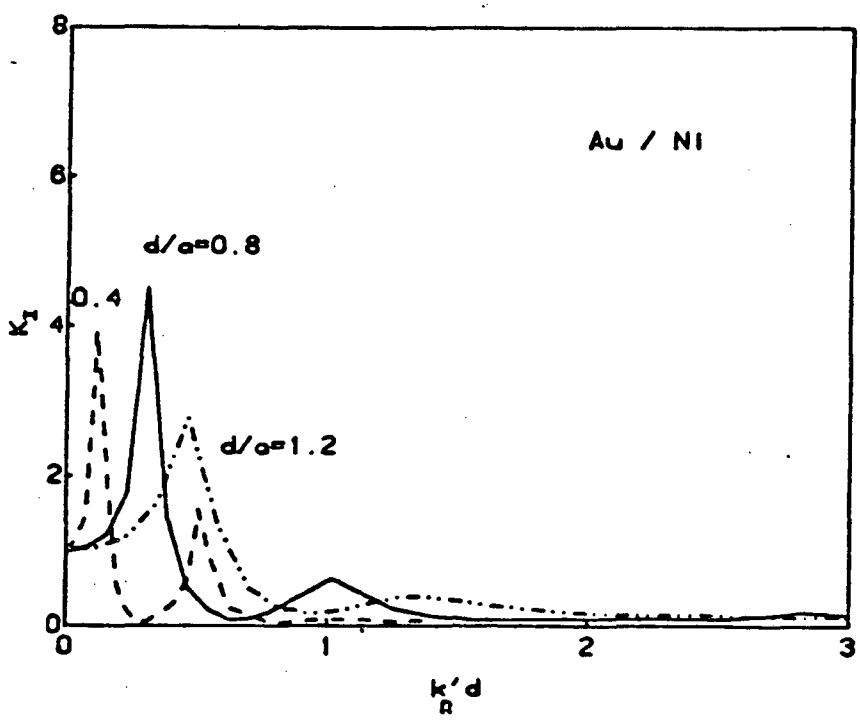

Fig. 8b. Mode II stress intensity factor for Ni/Au.

DISCUSSION

In order to optimize accuracy and computation time, judicious choices of the two parameters k_1 and k_2, that determine the deformed path, are required. k_2 must be chosen beyond the Rayleigh poles corresponding to the two materials, and hence it depends on frequency. The value of k_1 needs to be large enough to offset the path sufficiently from the singular points but not too large because the magnitudes of the hyperbolic functions increase rapidly with the imaginary part of k. In our calculations we found that the choices of k_1 and k_2 are highly dependent of ω, d and c_T/c_T' in order to get accurate and consistent results.

Some difficulties were encountered in the numerical computations, especially when the material parameters for the layer and substrate were quite different and for the larger values of $k_R'd$. Further work is planned to investigate these problems in more detail.

CONCLUSIONS

We have presented the plane strain solution for the problem of elastic wave scattering from an interface crack in a layered half space for the case of a single layer. The stress field has an oscillatory singularity, in general, at the crack tips, but it is a non-oscillatory square root singularity for a certain class of composites. The numerical solution of the integral equations was restricted to this class of composites and to the case when the incident wave is generated by harmonic uniform normal tractions on the boundary.

The stress intensity factors K_I and K_{II} show resonances at specific values of $k_R'd$ and the locations as well as peak values of these resonances depend on the material combinations. As shown in Figs. 6 and 7 we observe that the $k_R'd$ value for resonance is smaller, for a pair of materials, in the loading case ($c_T < c_T'$) than it is in the stiffening case ($c_T > c_T'$).

ACKNOWLEDGEMENT

This paper was prepared under U.S. Department of Energy Contract No. DE-AC03-76SF00098 with Lawrence Berkeley Laboratory.

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