LBL-18362
c.2

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

RECEIVED
LAWRENCE
BERKELEY LABORATORY

OCT 9 1984

LIBRARY AND
DOCUMENTS SECTION

Materials & Molecular Research Division

Presented at the 1984 Applied Superconductivity Conference, San Diego, CA, September 10-13, 1984; and submitted to the IEEE Transactions on Magnetics

DC SQUID AS A TUNED RADIOFREQUENCY AMPLIFIER

C. Hilbert and J. Clarke

September 1984

TWO-WEEK LOAN COPY
This is a Library Circulating Copy
which may be borrowed for two weeks

LBL-18362
c.2

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

DC SQUID AS A TUNED RADIOFREQUENCY AMPLIFIER

Claude Hilbert and John Clarke

Department of Physics
University of California
Berkeley, California 94720
and

Materials and Molecular Research Division
Lawrence Berkeley Laboratory
Berkeley, California 94720

Abstract

The design criteria for a tuned radiofrequency amplifier based on a dc SQUID are briefly discussed. A practical amplifier involving a planar dc SQUID with a 4-1/4-turn spiral input coil and operated at 4.2K had a gain of 18.6 ± 0.5 dB and a noise temperature of 1.7 ± 0.5K at 93 MHz. These results are in satisfactory agreement with predicted values.

Introduction

In an earlier paper,[1] we described a radiofrequency amplifier involving an untuned input circuit coupled to a dc Superconducting QUantum Interference Device (SQUID). However, for certain applications, notably nuclear magnetic resonance (NMR) and nuclear quadrupole resonance (NQR), it is advantageous to tune the input circuit. In this paper, we describe briefly the design and operation of a tuned radiofrequency amplifier based on a dc SQUID.

Theory

Figure 1 shows a dc SQUID in a tuned amplifier configuration. The SQUID has an inductance L, and is biased with a current I. Each Josephson junction has a critical current I_o, a self-capacitance C, and a shunt resistance R. There is a voltage V(t) across the SQUID and a current J(t) circulating around the SQUID loop. The magnetic flux threading the SQUID is Φ. The SQUID is coupled to an input coil with inductance L_i via a mutual inductance $M_i = \alpha(LL_i)^{1/2}$. In series with the input coil is a resistor, R_i, a capacitor, C_i, and a pick-up coil with an inductance L_p. A voltage V_i may be applied to the input circuit; alternatively, a signal may be generated by means of a changing magnetic flux in the pick-up coil, as would be the case, for example, for NMR or NQR. Not shown in Fig. 1 is the parasitic capacitance between the SQUID and the input coil which, in some cases, may influence the behavior of the SQUID quite significantly.[2]

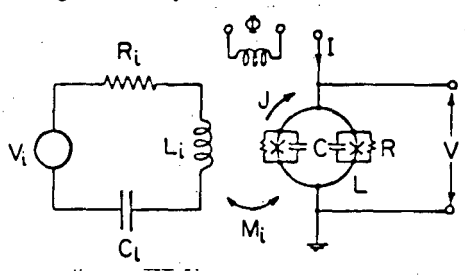

Figure 1. Configuration of tuned radiofrequency amplifier.

In general, the impedance of the input circuit will be modified by the presence of the SQUID, while the parameters of the SQUID will be modified by the presence of the input circuit.[3-5] We first discuss the effect of the dynamic impedance of the SQUID on the input circuit. The flux-to-current transfer function of the SQUID, $J_\Phi \equiv (\partial J/\partial\Phi)_I$, is related to the dynamic inductance, $\mathscr{L}$, and dynamic resistance, $\mathscr{R}$, by the relation[3]

$$J_\Phi = -\frac{1}{\mathscr{L}} - \frac{j\omega}{\mathscr{R}}, \tag{1}$$

where $\omega/2\pi$ is the signal frequency. The transfer function is a property of the SQUID at the Josephson frequency, $\omega_J/2\pi$, and one observes a time-averaged value at the signal frequency, which is assumed to be small compared with $\omega_J/2\pi$. In general, J_Φ will be modified by the presence of the input circuit.[3,4] The total inductance of the input is easily shown to be[5]

$$L_p + L_i' = L_p + L_i + \Delta L_i = L_p + L_i(1 - \alpha^2 L/\mathscr{L}). \tag{2}$$

We note that $\mathscr{L}$ is a strong function of the current and flux biases, and can be of either polarity. In the same way, the dynamic resistance of the SQUID induces a resistance in the input circuit. However, for the SQUID that we use in our experiments, there is a parasitic capacitance between the SQUID and the input coil, and the change in the input resistance turns out to be dominated by capacitive feedback from the SQUID output to the input circuit. If we approximate the distributed capacitance between the SQUID and the input coil by a single capacitor, C_p, a simple model predicts[5]

$$R_i' = R_i + \Delta R_i = R_i - \omega_o^2 M_i V_\Phi C_p L_i, \tag{3}$$

where $f_o = \omega_o/2\pi$ is the resonant frequency of the input circuit. For certain designs of amplifier, these corrections to the input impedance can be substantial. Since both ΔL_i and ΔR_i depend on the flux in the SQUID, they vary with the signal amplitude, thereby introducing nonlinearities in the gain and bandwidth of the input circuit, and increasing the noise temperature above the optimum value. Thus, it is important to design the tuned amplifier so as to minimize these effects.

The presence of the input circuit implies that the transfer functions V_Φ and J_Φ and the spectral densities of the low frequency noise $S_V(f) = 2\gamma_V k_B TR$, $S_J(f) = 2\gamma_J k_B T/R$ and $S_{VJ}(f) = 2\gamma_{VJ} k_B T$ may differ from their values for the bare SQUID;[3,4] here $S_V(f)$ and $S_J(f)$ are the spectral densities of V(t) and J(t), and $S_{VJ}(f)$ is the cross-spectral density of V(t) and J(t). It can be shown that the noise voltage across the SQUID coupled to the input circuit of Fig. 1 takes the form[6]

$$V_N^C(\omega) = V_N^R(\omega) + \frac{V_\Phi^R \alpha_{eff}^2 L J_N^R(\omega)(R_i + 1/j\omega C_i)}{R_i + j\omega(L_i + L_p) + 1/j\omega C_i - J_\Phi^R\, \alpha_{eff}^2 L(R_i + 1/j\omega C_i)}. \tag{4}$$

Here, $V_N^R(\omega)$ and $J_N^R(\omega)$ refer to the voltage and current noises of a SQUID with inductance reduced to $L(1 - \alpha_{eff}^2)$, and V_Φ^R and J_Φ^R are the corresponding transfer functions;[6] we define $\alpha_{eff}^2 = \alpha^2/(1 + L_p/L_i)$. The nu-

merator of the second term on the right hand side was obtained previously by Tesche[6] for the case $L_p = 0$.

In the limit of a high Q of present interest, at the resonant frequency $\omega_o = [C_i(L_i' + L_p)]^{-1/2}$ Eq. (3) simplifies to

$$V_N^C(\omega) = V_N^R(\omega) - \frac{j\omega_o M_i^2 V_\phi^R J_N^R(\omega)}{R_i'}, \tag{5}$$

where, for a SQUID in which capacitive feedback effects dominate, the reflected resistance R_i' is given by Eq. (3). As we will demonstrate in due course, the optimal operation of the amplifier requires $\alpha_{eff}^2 Q = 1$. Hence, in the high Q limit, $\alpha_{eff}^2 \ll 1$ and the reduced SQUID parameters resume their bare values.

Thus, in the high Q limit, Eq. (3) reduces to the result of Clarke, Tesche, and Giffard[7] except for the modification of the input resistance. [In ref. 7, the minus sign in Eq. (5) was erroneously replaced by a plus sign; this did not affect any of the results for the tuned amplifier.] We can write the noise temperature of the amplifier at the resonant frequency in the form[7]

$$T_N(f_o) = \frac{\omega_o^2 L_i RT}{2\alpha^2 L R_i V_\phi^2}\left[\gamma_V\left(\frac{R_i'}{\omega L_i}\right)^2 + \gamma_J\left(\frac{\alpha^2 L V_\phi}{R}\right)^2\right]. \tag{6}$$

At resonance, $T_N(f_o)$ is independent of γ_{VJ}. Minimizing Eq. (6) with respect to R_i to obtain the optimal source resistance, we find

$$R_i^{opt} = [(R_{io}^{opt})^2 + (\Delta R_i)^2]^{1/2}, \tag{7}$$

where

$$R_{io}^{opt} = (\gamma_J/\gamma_V)^{1/2}\omega M_i^2 V_\phi/R \tag{8}$$

is the optimal source impedance when the effects on the SQUID on the input resistance can be neglected. The optimized noise temperature is

$$T_N^{opt} = (\gamma_V\gamma_J)^{1/2}\omega T/V_\phi. \tag{9}$$

Equation (8) has an important consequence. To a reasonable approximation,[6] $(\gamma_J/\gamma_V)^{1/2} = 1$ and $V_\phi = R/L$, so that

$$R_{io}^{opt} = \alpha^2\omega L_i. \tag{10}$$

As a result, the quality factor of the tuned input circuit neglecting contributions from the SQUID, $Q = \omega_o(L_i + L_p)/R_i$, takes the optimal value

$$Q = (1 + L_p/L_i)/\alpha^2 \tag{11}$$

or

$$Q\alpha_{eff}^2 = 1, \tag{12}$$

where $\alpha_{eff}^2 = \alpha^2(1 + L_p/L_i)$. Thus, it becomes evident that a reasonably high value of Q (say 100) implies a small value of α_{eff}^2 (say 0.01). If α^2 is close to unity, as is the case for the present SQUID, one must introduce a large series inductance, L_p, to satisfy the optimization condition, Eq. (11).

We now examine the influence of ΔL_i and ΔR_i on the input circuit in the light of Eq. (11). The fractional change in the resonant frequency due to ΔL_i is

$$\left|\frac{\Delta f_o}{f_o}\right| = \frac{1}{2}\left|\frac{\Delta L_i}{L_i + L_p}\right| = \frac{1}{2\alpha^2 Q}\left|\frac{\Delta L_i}{L_i}\right| = \frac{1}{2Q}\left|\frac{L}{\mathscr{L}}\right|. \tag{13}$$

This fractional change is independent of frequency, and inversely proportional to Q. The fractional change in Q is

$$\left|\frac{\Delta Q}{Q}\right| = \left|\frac{\Delta R_i}{R_i^{opt}}\right| = \frac{\omega_o M_i |V_\phi| C_p}{\alpha^2} \tag{14}$$

and scales as ω_o. In the next section, we will give numerical estimates of $|\Delta f_o/f_o|$ and $|\Delta Q/Q_o|$.

Experiment

We used a planar, thin-film SQUID with a 4-1/4-turn spiral input coil.[1,9] The parameters had the approximate values $L = 400$ pH, $I_o = 10$ μA, $\beta = 2LI_o/\Phi_o = 4$, $R = 8\ \Omega$, $C = 0.5$ pF, $\beta_c = 0.8$, $L_i = 5.6$ nH, $M_i = 1$ nH and $\alpha^2 = 0.6$. The measured value of V_ϕ was about 3×10^{10} s^{-1} ($= 1.5\ R/L$). The self-resonance of the input coil was at about 450 MHz with a Q higher than 100, so that one could use this amplifier at frequencies up to about 400 MHz.

We first measured the changes in the impedance of the input circuit due to its coupling to the SQUID at a frequency of 27 MHz. At this relatively low frequency we were able to connect a spectrum analyzer with a high input impedance across the high-Q input circuit without loading it substantially.[5] The maxima of $|L/\mathscr{L}|$ occurred at $\Phi = n\Phi_o/2$ and their value never exceeded 0.6 at any bias parameters, so that the maximum fractional change in input inductance, $|\Delta L_i/L_i| = \alpha^2|L/\mathscr{L}|$, was never greater than 40%. In practice, one operates an amplifier with a flux bias near $(n \pm 1/4)\Phi_o$. Over the working range of the amplifier, say within $\pm\Phi_o/10$ of the operating point, $|L/\mathscr{L}|$ was always less than 0.2 and the fractional change in input inductance was correspondingly smaller, roughly 15%.

The change in input resistance, ΔR_i, scales with V_ϕ and thus is a maximum near the operating point of the amplifier. At 27 MHz, we found the maximum value of $|\Delta R_i|$ to be about 30 mΩ. Hence at 100 MHz, we expect $|\Delta R_i|$ not to exceed 0.4 Ω, a correction of about 20% to the optimal input resistance ($\alpha^2\omega L_i$) of about 2 Ω. Within $\pm\Phi_o/10$ of the operating point, however, ΔR_i should not change by more than about 7%.

These preliminary measurements together with Eqs. (10), (13) and (14) allow us to make the following predictions: $|\Delta f_o/f_o| \leq 1/3Q$ and $|\Delta Q/Q| \leq 0.2$ for any bias condition, and $T_N^{opt} = 1.1$K at 100 MHz and at 4.2K, using the values $\gamma_V = 25$ and $\gamma_J = 6$ (obtained from an analog simulator with $\beta = 4$). The predicted power gain, $M_i^2 V_\phi^2/R_i R_D$, is about 17 dB for a SQUID dynamic output resistance, R_D, of 8 Ω and an input resistance, R_i, of 2 Ω.

We tested the performance of our dc SQUID as a tuned radiofrequency amplifier at frequencies near 100 MHz. The input resistance, R_i, was 2 Ω and the input capacitance, C_i, was 20 pF. A cold, 50-Ω signal source was connected across the input resistance. The input circuit had a resonant frequency of 113 MHz and an average Q of about 36. The resonant frequency corresponds to a stray series inductance in the input circuit, L_p, of about 90 nH, so that $\alpha_{eff}^2 = 0.036$. Hence the value of $Q\alpha_{eff}^2$ was about 1.3, close to the optimal value. The maximum shift in the resonant frequency, f_o, as the flux bias of the SQUID was varied over a flux quantum was ± 0.8 MHz or less than 1% of f_o, as expected. The fractional change in Q between $\Phi = (n + 1/4)\Phi_o$ and $\Phi = (n - 1/4)\Phi_o$ was ± 25%, agreeing approximately with our prediction and furthermore verifying the fact that ΔR_i scales with ω_o^2. The associated change in power gain was ± 2.7 dB.

We then measured the gain and the noise temperature

of the tuned SQUID amplifier with the same values of R_i and C_i. For the measurements, we enclosed the input resistor in a vacuum can so that the temperature of the resistor could be raised above the bath temperature by means of a heater. By measuring the total noise out of the amplifier as a function of the temperature of R_i, we were able to determine the noise temperature of the amplifier quite accurately.[1] However, in this configuration the stray inductance was increased somewhat, and the resonant frequency was reduced to 93 MHz. At a bath temperature of 4.2K, the measured gain was 18.6 ± 0.5 dB, while the noise temperature was 1.7 ± 0.5K. Considering the uncertainty in the values of the SQUID parameters and, in particular, our neglect of the effects of the parasitic capacitance on the SQUID noise characteristics and transfer functions, we feel that the agreement between predicted and measured performance is quite satisfactory.

Concluding Remarks

We have seen that the optimum noise temperature for a tuned amplifier is obtained when the conditions $R_i = \alpha^2\omega L_i$ and $Q\alpha^2_{eff} = 1$ are satisfied. When Q is reasonably large and α^2_{eff} correspondingly small, the mutual influences of the SQUID and input circuit on each other become relatively unimportant. The requirement of low α^2_{eff} would probably be met most advantageously by increasing the spacing between the SQUID and the input coil, thereby reducing the parasitic capacitance substantially, possibly to an insignificant level. In the present configuration, however, the coupling coefficient α^2 was relatively high (≈ 0.6), and a low effectively coupling coefficient $\alpha^2_{eff} = \alpha^2/(1 + L_p/L_i)$ was obtained by introducing a large series inductance L_p. The performance obtained with this amplifier at 93 MHz and 4.2K, namely a gain of 18.2 ± 0.5 dB and a noise temperature of 1.7 ± 0.5K, was in satisfactory agreement with predictions.

Acknowledgements

We are indebted to John Martinis for helpful discussions on the theory of noise in SQUID amplifiers. We acknowledge the use of the Microelectronics Facility in the Electronics Research Laboratory of the Electrical Engineering and Computer Science Department at U.C. Berkeley. This work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division of the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

References

1. C. Hilbert and J. Clarke, "Radio-frequency Amplifier Based on a dc Superconducting Quantum Interference Device," Appl. Phys. Lett., Vol. 43, pp. 694-696, 1983.

2. C. D. Tesche, "Analysis of a Double-Loop DC SQUID," J. Low Temp. Phys., Vol. 47, pp. 365-410, 1982.

3. R. H. Koch, Ph.D. Thesis, University of California, Berkeley, 1982 (unpublished).

4. C. D. Tesche, "Optimization of dc SQUID Linear Amplifiers and the Quantum Noise Limit," Appl. Phys. Lett., Vol. 41, pp. 490-492, 1982; C. D. Tesche, "Analysis of Strong Inductance Coupling on dc SQUID Systems," IEEE Trans. Magn., Vol. MAG-19, pp. 458-460, 1983; C. D. Tesche, "Quantum Limit Constraints on dc SQUID Amplifiers," in Noise in Physical Systems and 1/f Noise, M. Savelli, G. Lecoy and J-P. Nougier, eds., Elsevier Science Publishers B.V., pp. 137-140, 1983.

5. C. Hilbert and J. Clarke, "Input Impedance of an Amplifier Based on a dc Superconducting Quantum Interference Device," Appl. Phys. Lett., Oct. 1984 (in press).

6. J. Martinis and J. Clarke (unpublished).

7. J. Clarke, C. D. Tesche, and R. P. Giffard, "Optimization of dc SQUID Voltmeter and Magnetometer Circuits," J. Low Temp. Phys., Vol. 37, pp. 405-420, 1979.

8. C. D. Tesche and J. Clarke, "DC SQUID: Noise and Optimization," J. Low Temp. Phys., Vol. 29, pp. 301-331, 1977; C. D. Tesche and J. Clarke, "DC SQUID Current Noise," J. Low Temp. Phys., Vol. 37, pp. 397-403, 1979.

9. M. B. Ketchen and J. M. Jaycox, "Ultra-low-noise Tunnel Junction DC SQUID with a Tightly Coupled Planar Input Coil," Appl. Phys. Lett., Vol. 40, pp. 736-738, 1982.

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720