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September 1984

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Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

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A Two-Zone Model for Conduction Heat Transfer from a Particle to a Surrounding Gas at Arbitrary Knudsen Number

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Introduction

The rate of heat transfer from small particles suspended or immersed in a gas to the surrounding gas is an important quantity in many areas of research. It plays a role in determining ice formation in noctilucent clouds and in governing airborne aerosol equilibrium temperatures.[1] The aerosol particles may have a temperature that is significantly different than that of the atmosphere. This heat transfer rate is also important in combustion systems where soot or other particles are present.[2] A process in which knowledge of the particle-gas energy exchange rate is especially important is the use of small particles suspended in a gas to directly capture solar energy.[3] It is this latter application that inspired this paper.

The heat transfer from a particle to a surrounding gas may be broken into three regimes depending on the value of the Knudsen number (Kn), defined as λ/d, where λ is the gas molecule mean free path and d is the characteristic dimension of the body in the gas (d is the diameter if the body is a sphere). For $Kn < 10^{-3}$ the continuum approximation applies and different equations are available to calculate the heat transfer rate depending on whether the losses are purely conductive, or also convective, in nature.[4] For $Kn > 10$ free molecular flow conditions prevail near the particle; again a fairly well recognized expression for the heat transfer based on molecular collisions exists.[1] In the transition region, $10^{-3} < Kn < 10$, analytical modeling of heat transfer is difficult because neither a continuum nor a kinetic theory approach is strictly correct. Currently, the standard approach to calculate heat transfer in this region is almost entirely empirical.[5] In the following, a simple treatment of particle-gas heat transfer is derived for arbitrary Kn; in particular, it applies in the transition region. The resulting expression is shown to approach the correct limits as Kn goes to zero or to infinity.

Analysis

The model used is the following: a spherical particle with radius a is stationary in an infinite gaseous medium with temperature T_∞ *as* $r \to \infty$. (r is the radial coordinate with origin at the center of the sphere as illustrated in Fig. 1). The

This work was supported by the Solar Fuels and Chemicals Program through the San Francisco Operations Office and the Division of Energy Storage Technology of the Office of Energy Systems Research for the Assistant Secretary for Conservation and Renewable Energy under Contract No. DE-AC03-76SF00098 of the U. S. Department of Energy.

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region around the sphere is divided into two zones. Outside a sphere of radius $\lambda+a$ continuum conduction is assumed to hold, where λ is the mean free path of the gas molecules. Within one mean free path of the surface it is assumed that the gas molecules collide only with the particle and not with one another. The effect of convection is neglected. The molecules striking the particle are assumed to have a Maxwellian velocity distribution at temperature T_B, the zone boundary temperature. The particle is maintained at a fixed temperature T_p; energy is supplied or removed by radiation or chemical reaction. Here the particle temperature is assumed to be greater that the gas temperature, but the same analysis applies in the other case.

First, the steady state conduction equation is solved in spherical coordinates for the temperature field outside a radius of $a+\lambda$. For radial symmetry,

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=0$$

with the boundary condition $T = T_\infty$ as $r \to \infty$. The solution is

$$T = \frac{A}{r} + T_\infty \tag{1}$$

with constant A to be determined.

To find the energy being carried to the particle we first calculate its collision rate with the gas molecules. From kinetic theory, the molecular flux density crossing a plane in one direction is[6]

$$\Gamma = n_g\left(\frac{k_B T}{2\pi m_g}\right)^{1/2} \tag{2}$$

if a Maxwellian velocity distribution at temperature T is assumed. Here n_g is the molecule number density, m_g is the mass of a molecule and k_B is Boltzmann's constant. Applying this to the zone boundary, a total inward flow of $4\pi(\lambda+a)^2\Gamma$ results. However, not all these molecules strike the particle. If the molecules are randomly directed as they cross the boundary, then an analysis similar to that for obtaining shape factors in radiation heat transfer will show that only the fraction $a^2/(a+\lambda)^2$ will hit the particle.[7] Thus the particle suffers $4\pi a^2\Gamma$ collisions per unit time.

From kinetic theory, the average energy of a molecule striking the particle surface is $\epsilon_{int} + 2k_B T_B$, where ϵ_{int} is the internal energy if the molecule is not monatomic.[6] We note that $2k_B T_B$, and not $(3/2)k_B T_B$, appears because faster molecules hit the surface with greater frequency than do slower molecules and carry more kinetic energy. The average energy of the colliding molecule multiplied by the collision rate gives the total energy into the particle. To calculate the energy flowing away from the particle, the energy of a molecule as it leaves the particle must be determined. From kinetic theory it can be assumed that

the molecules leave with a Maxwellian velocity distribution at a temperature between T_p and T_B, written as $T_{out} = (T_p - T_B)\alpha + T_B$, where α is termed the accommodation coefficient.[8] If $\alpha = 1$ (perfect accommodation) the molecules leave at T_p; if $\alpha = 0$ no energy is exchanged and they leave at T_B. In principle, α may be different for translational energy exchange than for internal energy exchange; here an average α is employed.

The average energy exchanged in a collision is $(\epsilon_{int(out)} + 2k_BT_{out}) - (\epsilon_{int(B)} + 2k_BT_B)$. If we are in a temperature range where the specific heat is roughly constant with temperature this can be simplified to $\alpha(C_vm_g + k_B/2)(T_p - T_B)$ where C_vm_gT has replaced $\epsilon_{int} + 3/2k_BT$, and C_v is the specific heat of the gas. Combining this with the collision rate gives

$$Q = 4\pi a^2 n_g \alpha \left(\frac{k_B T_B}{2\pi m_g}\right)^{1/2} \left(C_v m_g + \frac{k_B}{2}\right)\left(T_p - T_B\right) \tag{3}$$

where Q is the rate of heat transfer from the particle to the gas; note that T_B is not yet known. In steady state the total outward heat flow Q is independent of r. Then the heat leaving the zone boundary to the bulk of the gas due to conduction is

$$\left[-4\pi\left(\lambda + a\right)^2 k \frac{\partial T}{\partial r}\right]_{r=\lambda+a} = Q = 4\pi\left(\lambda + a\right)^2 \frac{kA}{\left(\lambda + a\right)^2} \tag{4}$$

Using eqns. 3 and 4 we have a second boundary condition whose application yields the following expression for A:

$$A = \frac{a^2}{k} n_g \alpha \left(\frac{k_B T_B}{2\pi m_g}\right)^{1/2} \left(C_v m_g + \frac{k_B}{2}\right)\left(T_p - T_B\right) \tag{5}$$

With A determined, the continuum temperature distribution (1) may be used to solve for T_B. The result is :

$$T_B = \frac{T_\infty + \dfrac{\alpha\Phi}{Kn(2Kn + 1)\pi} T_p}{1 + \dfrac{\alpha\Phi}{Kn(2Kn + 1)\pi}} \tag{6}$$

where $Kn = \lambda/2a$ and $\Phi = 32/75$ for a monatomic gas and 48/95 for a diatomic gas. Equation 6 gives the temperature one mean free path from the particle. In deriving equation 6 the thermal conductivity has been replaced by its kinetic theory value[8]:

$$k = constant \times \pi C_v m_g n_g \lambda \left(\frac{k_B T_B}{2\pi m_g}\right)^{1/2} \tag{7}$$

where the constant is 25/16 for a monatomic gas and 19/16 for a diatomic gas. To get the constant the diatomic gas was assumed to have its rotational modes excited, but not its vibrational modes. Eucken's formula with $\gamma = 7/5$ was used. If the molecule can also store energy in vibration (this depends on the gas and the temperature range), then the constant must be adjusted.

Now that T_B has been evaluated, it can be used in the expression for the heat transfer Q giving:

$$Q = \frac{4\alpha a k \Phi (T_p - T_\infty)}{Kn + \dfrac{\Phi \alpha}{(2Kn + 1)\pi}} \tag{8}$$

As a check on the validity of equation 8, the limits as Kn approaches zero and infinity can be evaluated. As $Kn \to 0$, $Q \to 4\pi a k (T_p - T_\infty)$. If Q is set equal to an equivalent convection loss $4h\pi a^2(T_p - T_\infty)$ then

$$\frac{2ah}{k} = Nu = 2,$$

a familiar result from continuum heat transfer for a motionless fluid.

As $Kn \to \infty$

$$Q = 4\alpha a k \frac{\Phi}{Kn}(T_p - T_\infty)$$

where the fact that $T_B \to T_\infty$ as $Kn \to \infty$ has been used. When k is replaced by equation 7 this becomes the formula used by Fiocco and Grams[1] corresponding to a temperature jump at the particle surface.

Results and Discussion

If the boundary temperature is nondimensionalized as

$$\frac{T_B - T_\infty}{T_P - T_\infty} = \Theta$$

a quantity that varies between 0 and 1 is obtained. This quantity is plotted in Fig. 2. As Kn approaches 0, T_B approaches T_p and the continuum temperature gradient (with no temperature jump) results. For air at STP this corresponds to a particle diameter of $\geq$ 15 μm. As Kn increases toward infinity T_B goes to T_∞ and a temperature jump at the surface appears. This happens for particles of diameter less than .075 μm in air at STP. By calculating Kn for a particle of interest, reference to this plot reveals to what extent there is an effective temperature gradient around it.

The Nusselt number as a function of Kn can also be calculated for the general case by setting Q in (6) equal to $4h\pi a^2(T_p - T_\infty)$ and solving for Nu = 2ah/k. This result is plotted in Fig. 3. Again by finding the appropriate Kn, Nu can easily be determined from the graph. Also shown in Fig. 3 is a value of Nu taken from Clift[5] for the transition region. Clift does not state the origin of this result, only that it matches the data of Takao.[9] One final point to note is that Nu decreases as Kn increases, and the heat flux per area from a particle is $q = (Nuk/2a)(T_p - T_\infty)$. So as the particle is made smaller, q increases as long

as Nu does not decrease faster than a (which it does not). This means that it is increasingly difficult for a small particle to be at a different temperature than the surrounding gas as the particle size decreases. Particles in the upper atmosphere may maintain different relative temperatures only because k is so small (see results of Fiocco and Grams).

It is interesting to note how Θ and Nu are affected by α. As α decreases, so does Θ for a given Kn. This signifies that the molecules at the boundary are closer to the bulk gas temperature than when α is unity. Therefore, the free molecular flow approximation improves for smaller values of α. Nu also decreases with α, as expected, since energy exchange is more difficult. At high Kn, Nu, and hence the rate of heat transfer, is linear with α. As Kn goes to zero, all α effects disappear and the main resistance to heat flow is the bulk gas conductivity, which explains why α is unimportant for macroscopic heat transfer.

Conclusion

In this paper we have developed a simple two zone model for heat transfer from a particle to a surrounding gas at arbitrary Kn. Convection was neglected, and a Maxwellian distribution for the gas was used, but the treatment is applicable to many cases of interest. The expression for the heat transfer rate depends on Kn and was shown to reach the appropriate limits as Kn goes to zero or infinity. The primary value of the equation, however, lies in its ability to predict the energy exchange at values of $Kn \approx 1$ where a useful expression did not previously exist. Other workers, such as Takao or Schmulian[10] have done more involved kinetic theory treatments that have yielded less readily applicable results. The fact that our result is both based on the physics of the problem, and is simple to use, is satisfying.

Nomenclature

a particle radius (m)
A constant in eq. (5) (m-°K)
C_v specific heat (J/g-°K)
Gr Grashof number
h convection heat transfer coefficient (W/°K)
k thermal conductivity of gas (W/m-°K)
k_B Boltzmann constant (J/°K)
Kn Knudsen number
m_g mass of the gas molecule (g)
n_g number density of gas molecules ($1/m^3$)
Nu Nusselt number
q rate of heat transfer per area from particle to gas (W/m^2)
Q rate of heat transfer from particle to gas (W)
r radial coordinate (m)
T_B zone boundary temperature (°K)
T_g gas temperature (°K)
T_p temperature of Particle (°K)
T_∞ temperature far from particle (°K)
α accommodation coefficient
ϵ molecule internal energy (J)
λ mean free path of gas molecules (m)
Γ number flux density ($1/m^2$-sec)
Φ gas-dependent constant in eq. 6
Θ nondimensional temperature

References

1) G. Fiocco and G. Grams, "Equilibrium Temperatures of Small Particles in the Earth's Upper Atmosphere (50-110 km)" *Journal of Atmospheric and Terrestrial Physics*, Oxford, (1975)

2) R. H. Essenhigh, "Combustion and Flame Propagation in Coal Systems, A Review" *Sixteenth Symposium on Combustion* pp. 353-374 (1976)

3) A. J. Hunt, "A New Solar Thermal Receiver Utilizing a Small Particle Heat Exchanger," *14th Intersociety Energy Conversion Engineering Conference*, Boston, MA (1979)

4) R. B. Bird, *Transport Phenomena*, p.409,413, John Wiley, New York (1960)

5) Roland Clift, *Bubbles, Drops, and Particles*, Academic Press, New York (1978)

6) C. L. Tien, *Statistical Thermodynamics*, McGraw-Hill, New York (1979)

7) R. Siegel, *Thermal Radiation Heat Transfer*, p. 830; McGraw-Hill, New York (1981)

8) Earle Kennard, *Kinetic Theory of Gases*, McGraw-Hill, New York (1938)

9) K. Takao, "Heat Transfer from a Sphere in a Rarefied Gas," *Rarefied Gas Dynamics* 3rd Symposium Vol. II, Academic Press, New York (1963)

10) R. Schmulian, "Nearly Free Molecular Heat Transfer from a Sphere," *Rarefied Gas Dynamics* 6th Symposium Vol I., Academic Press, New York (1969)

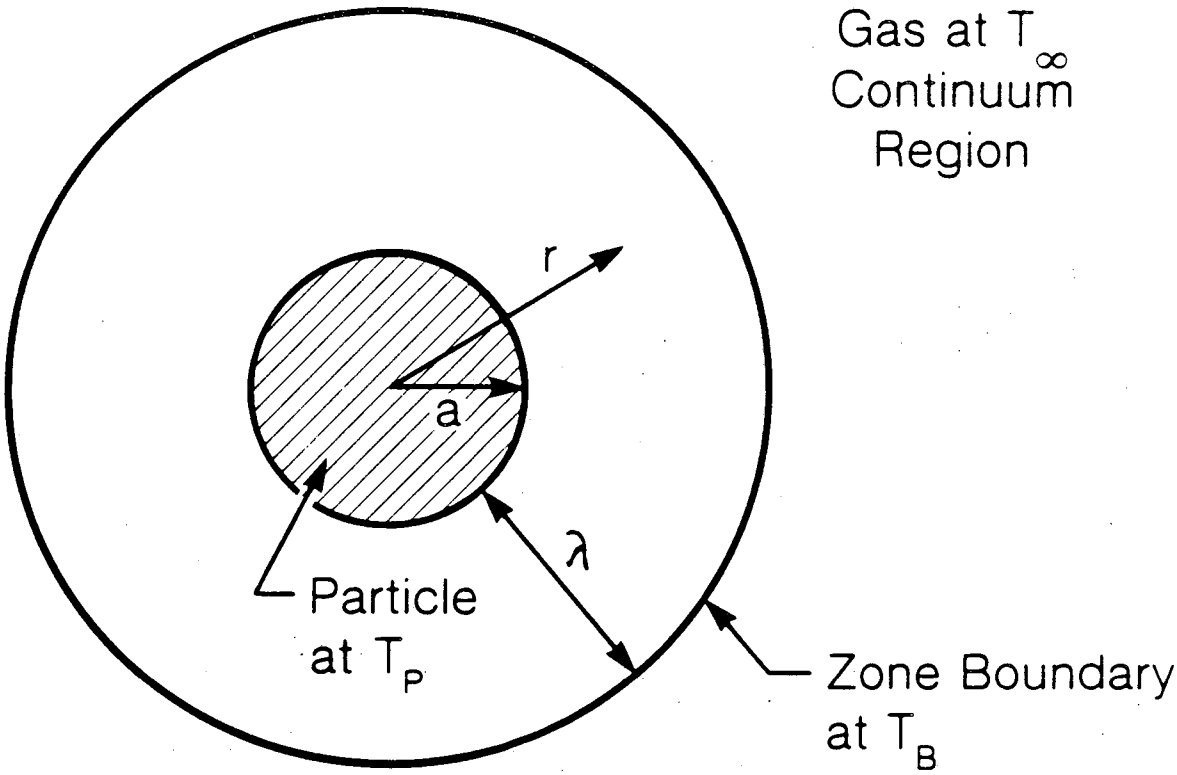

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Fig. 1: Two-Zone Model for Heat Transfer Between Particle and Gas

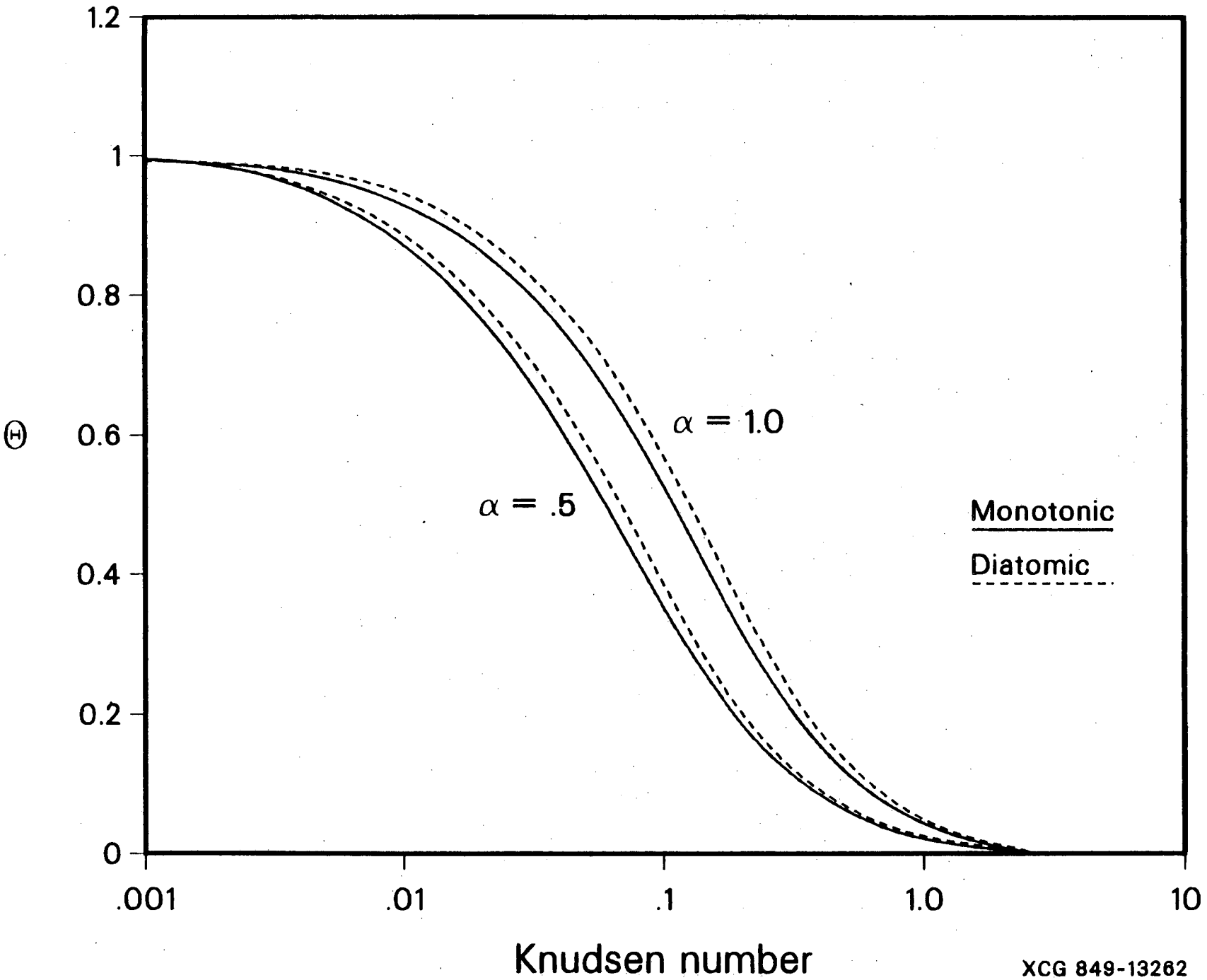

Fig. 2: Nondimensional Temperature vs. Knudsen Number

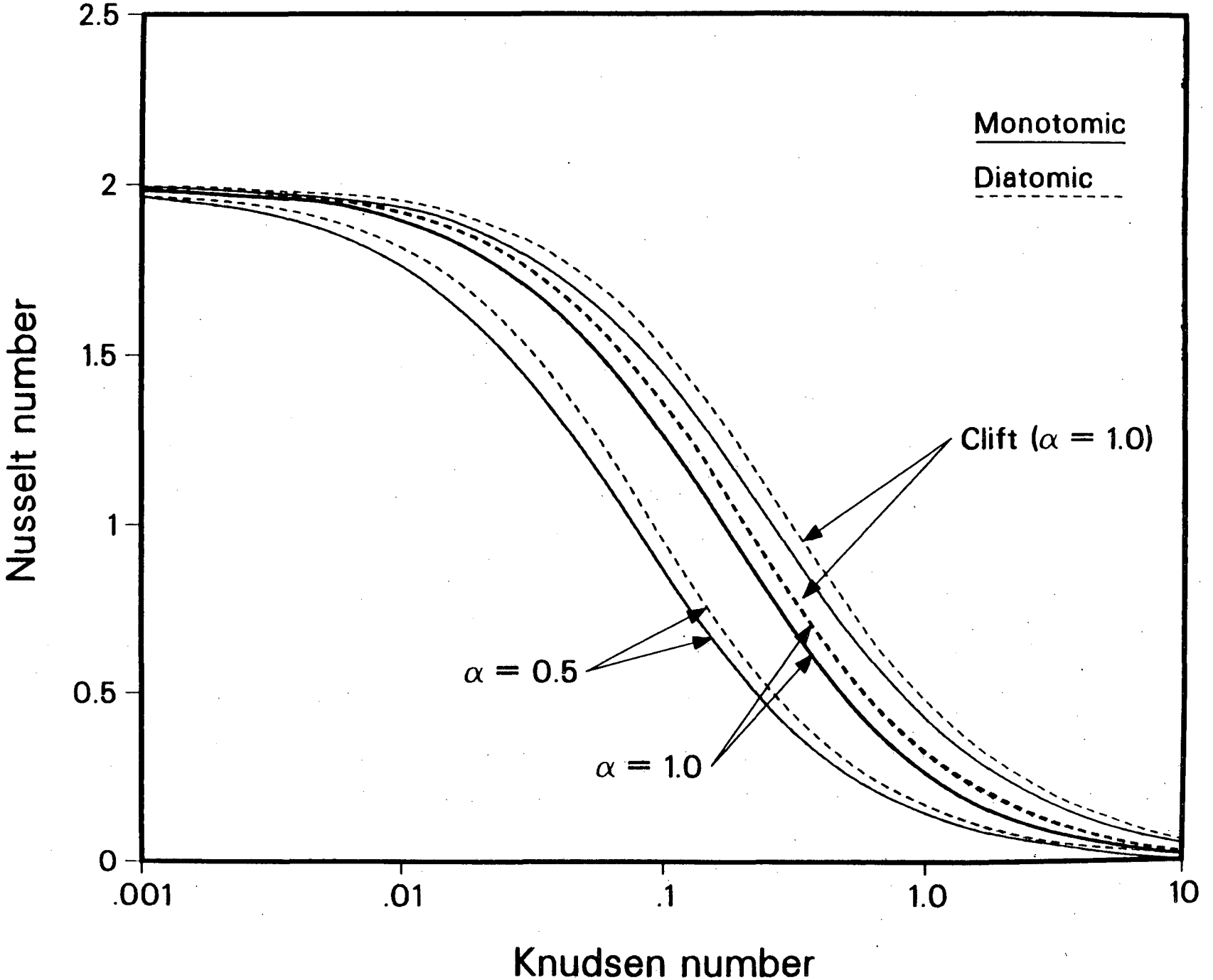

Fig. 3: Nusselt Number vs. Knudsen Number

XCG 849-13263

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

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