LBL-18593
UC-28

# Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

RECEIVED
LAWRENCE
BERKELEY LABORATORY
FEB 11 1985
LIBRARY AND
DOCUMENTS SECTION

## Materials & Molecular Research Division

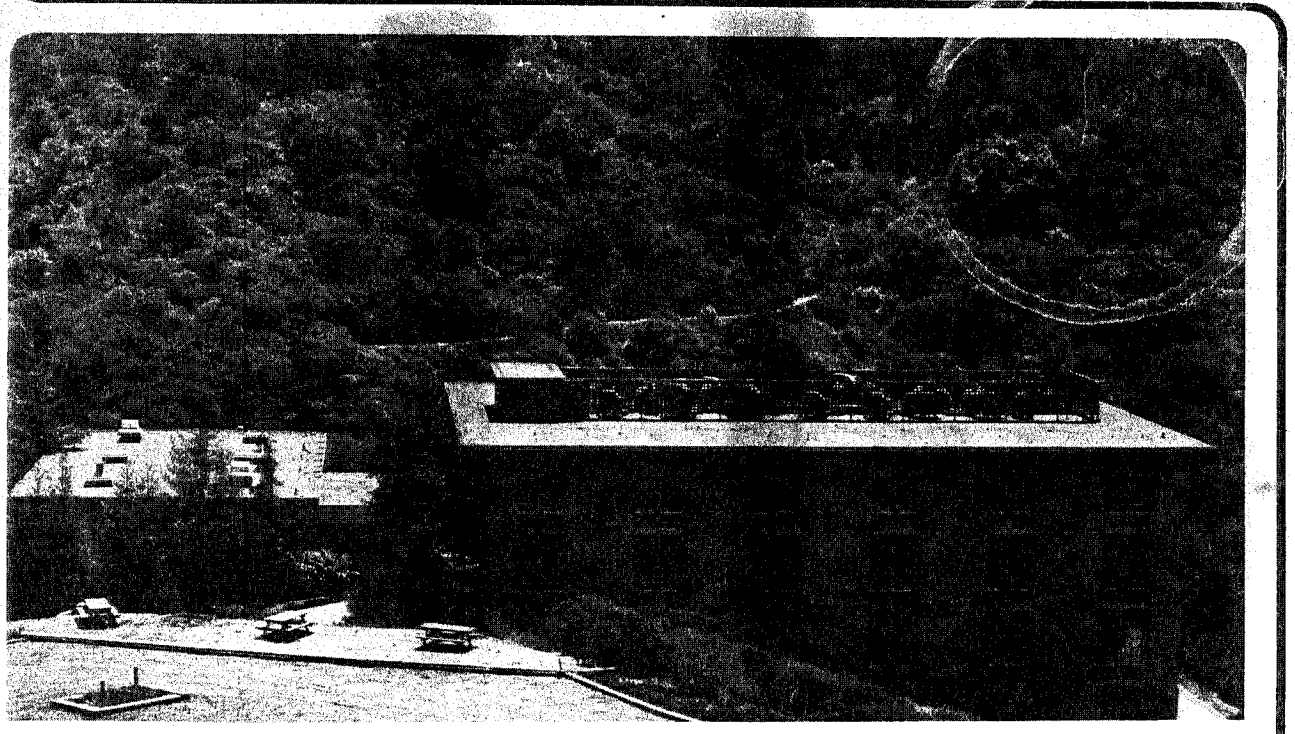
ATOMIC PHYSICS ASPECTS OF A RELATIVISTIC NUCLEAR COLLIDER

H. Gould

*TWO-WEEK LOAN COPY*

*This is a Library Circulating Copy which may be borrowed for two weeks.*

November 1984



Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

# Atomic Physics Aspects of a Relativistic Nuclear Collider

Harvey Gould

*Materials and Molecular Research Division,*
*Lawrence Berkeley Laboratory,*
*University of California,*
*Berkeley California, 94720.*

## I. PREFACE

The purpose of this note is to call attention to some important atomic physics effects and some interesting experiments involving a Relativistic Nuclear Collider. The ideas contained here come from many people (see acknowledgements).

## II. SUMMARY

Atomic collision cross sections involving bare uranium nuclei are large at relativistic energies and will affect the design and operation of a relativistic nuclear collider (RNC). The most significant may be production of electron - positron pairs and muon pairs ($\approx 10^8$ per sec. and 2000 per sec. respectively for a 100 GeV/nucleon collider with a luminosity of $10^{27}$ cm$^2 s^{-1}$). Although the pair production is a direct measure of the luminosity it is also a large source of background and capture of an electron from the pair by one of the nuclei will result in the loss of the ion. Another important loss mechanism is Coulomb excitation of the giant nuclear dipole and giant nuclear quadrupole resonances.

Storing and colliding bare and highly-stripped uranium opens up new possibilities for novel atomic physics experiments and an alternate approach for present experiments. As examples, the use of a collider for experiments to study spontaneous decay of the super-critical state (both positron production and x-ray production) of quasi-atoms of atomic number $Z > 172$, and a storage-ring measurement of the ground state hyperfine structure of hydrogenlike thallium as a test of quantum electrodynamics (QED) are discussed.

## III. ATOMIC PHYSICS RELATED TO THE DESIGN AND OPERATION OF AN RNC

### (a) Introduction

Electron capture from the background gas is the atomic collision process which has been considered most carefully in the design of a relativistic nuclear collider (RNC) and is discussed in section

III. (d) However electromagnetic nuclear excitation of the giant dipole resonance, section III. (e), and electron capture from pair production, section III. (c), may be more important limitations because they scale with beam luminosity. On the positive side, pair production, section III. (b), is a non-destructive real-time measure of the beam luminosity.

We consider four collision processes in the following sections:

b) pair production in atomic collisions between the two uranium nuclei;
c) electron capture from pair production ;
d) electron capture by bare uranium from the residual background gas;
e) electromagnetic nuclear excitation of giant dipole and giant quadrupole resonances.

**(b) Pair Production**

The cross section for producing electron - positron pairs from the Coulomb field of two (bare) nuclei has been considered by a number of authors[1-10]. The mechanism for producing these pairs may be thought of as the virtual photons from a motional Coulomb field scattering from a static Coulomb field (Weitsäcker - Williams method Ref. 1,2). The calculations in Ref. 1 - 10 are approximate and none are considered valid unless $\gamma >> 1$ (where $\gamma = (1-\beta^2)^{-1/2}$ with $\beta = v/c$). A simple formula[1-4] gives an estimate for $\gamma$ not too small.

$$\sigma_{pair} = (28/27\pi)\, \alpha^2\, Z_1^2\, Z_2^2\, r_0^2\, \log^3\gamma \qquad (1)$$

where $\alpha = 1/137$, $Z_1$ and $Z_2$ are the nuclear charges, and $r_0 = e^2/mc^2 = 2.8 \times 10^{-13}$ is the classical electron radius,

Table I shows the cross sections for producing electron - positron pairs in uranium - uranium collisions at collider energies as computed from Eq. 1.

Table I. Pair production cross sections for $U^{92+}$ on $U^{92+}$

| energy/beam GeV/amu | equivalent fixed target energy GeV/amu | cross section kilo-barns |
|---|---|---|
| 3 | 30 | 4 |
| 4 | 48 | 6 |
| 5 | 70 | 8 |
| 10 | 240 | 17 |
| 20 | 880 | 31 |
| 40 | 3400 | 54 |
| 100 | 20400 | 98 |

For a 100 GeV/nucleon RNC operating with a luminosity of $10^{27}\ cm^{-2}\ s^{-1}$ about $10^8$ electron - positron pairs per second are produced in the collision region. As number of muon pairs scales roughly as $(m_e/m_\mu)^2$ about 2000 muon pairs are also produced. Experiments which look for lepton production from nuclear collisions will need to consider this background. Finally, heavier pairs will also be produced in small quantities and the upper limit to the mass of the pairs which can be observed needs to be explored.

The number of pairs produced is linear in the beam luminosity and since these are easy-to-detect particles, a real-time non-destructive measure of the beam luminosity can be established. With a more sophisticated calculation of the cross section, or from measurements of the cross section in fixed targets, an absolute measurement of collider luminosity should be possible.

**(c) Electron Capture from Pair Production**

It is possible for the electron, produced in pair production, to be captured into the K-shell of one of the uranium atoms which produced the pair. Classically, capture occurs if the electron is found within the potential well of the Coulomb field and has a kinetic energy less than the K-shell binding energy. In uranium this means within $\approx$ 580 fm and a kinetic energy of less than 130 keV. (If the two uranium are closer than 580 fm, then the binding energy of the combined system will be larger than 130 keV. By comparison, in collisions at energies close to the Coulomb barrier, uranium nuclei which approach to within 35 fm have a combined binding energy in excess of 1 MeV.)

We can estimate the fraction of pairs produced with electron kinetic energy of 130 keV or less (pair energy between 1.22 MeV and 1.48 MeV), from a differential cross section [3] which depends upon the total pair energy.

$$dQ = (56/9\pi)\,\alpha^2\, Z_1^2\, Z_2^2\, r_0^2\, \log(kE/mc^2)\, \log(k'\gamma mc^2/E)\, dE/E \qquad (2)$$

where k and k' are constants $\approx$ 1 and E is the total energy of the electron-positron pair. Fig. 1 shows

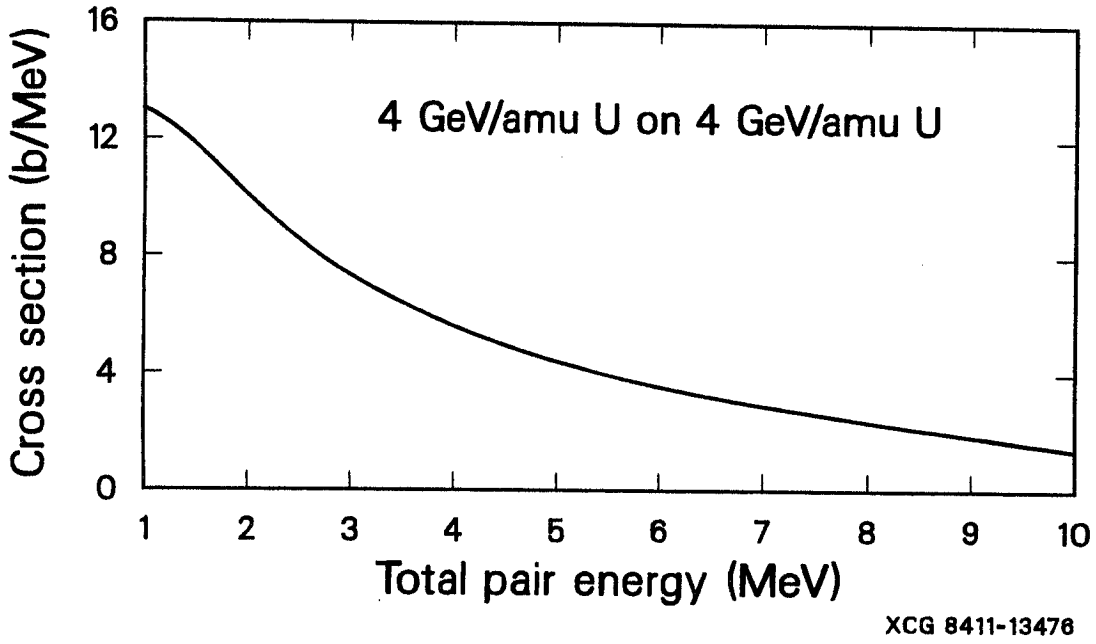



Fig. 1 - Estimate of pair production for uranium on uranium at 4 GeV/amu each beam in units of barns/MeV as a function of the energy of the electron positron pair.

the differential cross sections calculated from Eq. 2 for uranium on uranium at 4 GeV/amu. Although Eq. 2 contains approximations which break down unless E >> 1 MeV, it should still be valid to conclude that a substantial fraction of the pairs are produced at the lower energies.

The cross sections for pair production are highest for pairs produced at lower kinetic energies and for pairs emitted along the beam direction. If these crude estimates are close to reality then the cross sections for producing and capturing an electron, from the collision of two bare uranium nuclei could be several hundred barns. Cross sections significantly greater than 100 barns would set limits on beam survival or luminosity. For example, if the cross section were 1000 barns, then a RNC with luminosity of $10^{25}\ cm^2\ s^{-1}$ two interaction regions would loose nearly $10^8$ ions per hour.

These numbers should be considered a guess and not a calculation, and should not be used to set upper or lower limits. A detailed calculation will be required for more realistic numbers.

**(d) Electron Capture From the Residual Gas**

Bare uranium ions can charge change by capturing an electron from the residual background gas. Radiative electron capture (REC) is the most probable capture mechanism for several GeV/nucleon bare uranium in light gasses. The mean time for REC of an electron by a 4 GeV/nucleon $U^{92+}$ ion in $10^{-9}$ Torr $N_2$ is calculated to be 1/2 day. At higher energies the survival time against REC is expected to be much longer. The calculation is likely to be reliable to the accuracies needed for accelerator design.

Experiments[11,12] to measure charge states and charge changing cross sections of uranium at 0.4 GeV/nucleon and 0.96 GeV/nucleon are consistent with REC being the dominant mechanism for recombination of $U^{92+}$ in low atomic number (Z) targets at these energies. (REC is the inverse of the photoelectric effect. In REC by bare uranium at relativistic energies, an electron is captured, usually into the K-shell, from the continuum, or from a weakly bound system, with the simultaneous emission of a photon.) To first approximation, the REC cross section scales as the number of electrons in the target atom. The energy of the emitted photon, in the rest frame of the uranium, is $(\gamma - 1)\ mc^2 + B_K$ where $B_K$ is the K-shell binding energy.)

Table II. Radiative electron capture cross sections for $U^{92+}$

| Energy GeV/amu | $\sigma_v$ barns | $\sigma_{REC/elect.}$ barns |
|---|---|---|
| 1 | 25. | 11. |
| 2 | 7. | 4. |
| 3 | 4. | 2.6 |
| 4 | 2.5 | 1.8 |
| 10 | 0.8 | 0.6 |
| 20 | 0.3 | 0.3 |
| 40 | 0.13 | 0.14 |
| 100 | 0.05 | 0.05 |

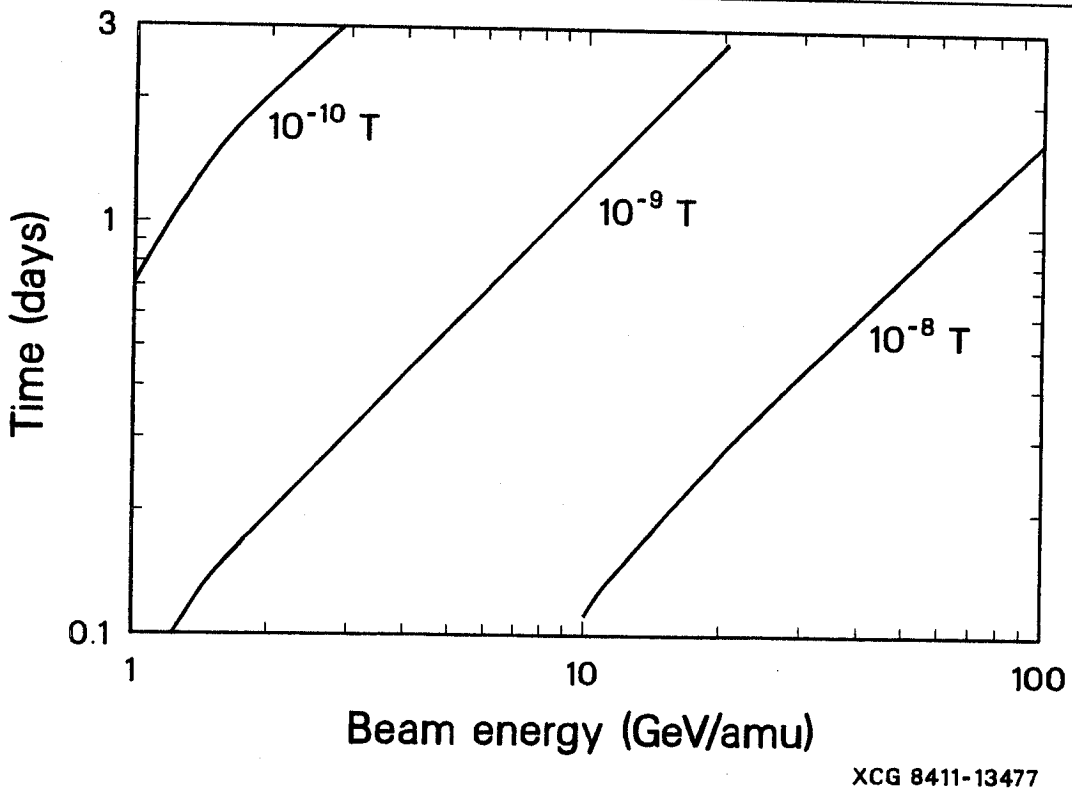



Fig. 2- Mean time for radiative electron capture for $U^{92+}$ in nitrogen at 300 K$^o$

The cross section per target electron for REC by a bare nuclei is given by:

$$\sigma_{REC/elect.} = \frac{[\gamma - 1 + B_K/mc^2]^2 \sigma_\nu}{\gamma^2 - 1} \quad (3)$$

where $\sigma_{REC/elect.}$ is the REC cross section per target electron, and $\sigma_\nu$ is the cross section for photoionizing an electron by a photon of energy $(\gamma - 1)\, mc^2 + B_K$. Table II shows the calculated REC cross sections per target electron for $U^{92+}$ and Fig. 2 shows the mean time for radiative electron capture of an electron by $U^{92+}$ in $N_2$ at $10^{-8}$ Torr, $10^{-9}$ Torr and $10^{-10}$ Torr at 300 K°.

The pressures needed to minimize loss due to radiative electron capture from the background gas are modest and do not preclude long storage times for bare ions.

**(e) Electromagnetic Nuclear Excitation**

The virtual photon field of the moving Coulomb field of the nuclei will electromagnetically excite the nuclear giant dipole and giant quadrupole resonances leading to a break-up of the nuclei. Cross sections for these processes are shown if Fig. 3. The calculations, performed by J.R. Beene and C. Bemis Jr (O.R.N.L.)[13] based upon the work of Winther and Alder[14] show the cross sections for these processes at collider energies to be about 100 barns.

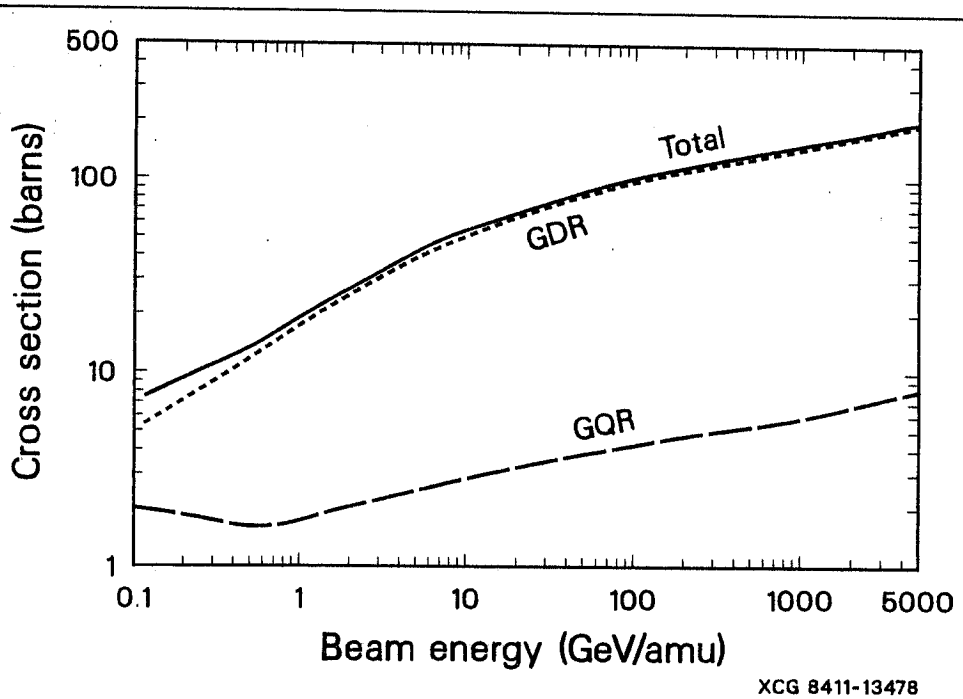



Fig. 3 - Cross sections (From Ref. 13,14). for excitation of the giant dipole resonance (GDR) and giant quadrupole resonance (GQR) as a function of the *equivalent fixed target energy* 48 GeV/nucleon corresponds to a 4 GeV/nucleon collider. See Table I for other equivalent energies values.

These calculated cross sections are high enough to be of concern in designing a high luminosity collider with small beam currents. The count rate for this effect is linear in beam luminosity and might be useful as a monitor of the luminosity. Finally, the break-up of $\approx$ 100 nuclei per second in the interaction region of even a small collider (with a luminosity of $10^{24}$ $cm^{-2}s^{-1}$) will produce a background to the experiment. For uranium, the break-up is in the form of fission, for lead it may be neutron emission.

## IV. EXPERIMENTS

### (a) Decay of the Supercritical Vacuum

*Introduction* -- Experimentals at GSI have recently observed peaks in the energy spectrum of positrons emitted in collisions of U+U, Th+Cm, and U+Cm at energies below the Coulomb barrier[15-17]. The peaks in the positron spectra appear to be consistent with a "spontaneous decay of a super-critical state" of the combined nuclear system, with the system held together for times much longer than the time involved for Rutherford scattering[18-21].

When two nuclei of $Z_1 + Z_2 > 172$ approach to within about 35 fm the binding energy of the the K-shell of their combined Coulomb field exceeds twice the electron rest mass. The lowest energy state of this system is reached when the K-shell is *filled.* Therefore, if a vacancy exists in the K shell, of this system (supercritical vacuum) it can fill spontaneously by decaying to an electron in the K shell and a positron which is emitted. The probability of forming a K vacancy when the two nuclei approach to about 35 fm is of the order of a few percent[22-25]. The time for decay of the K vacancy (super-critical state) is of the order of $10^{-19}$ seconds[26,27]. If the nuclei simply Rutherford scatter, the interaction time is only of the order of $10^{-21}$ seconds and few of the super-critical states would decay. Those which decay in $10^{-21}$ seconds produce a broad distribution of positron energies due to the uncertainty principle.

There are other mechanisms which also yield a *continuous* positron spectra. One is an induced or dynamic process in which the motional Coulomb field of the projectile induces the decay of a K vacancy into a bound electron and an emitted positron. This process adds energy to the system so that the decay can occur even if the binding energy is less than $2mc^2$ (1.1 MeV) (Ref. 28-30). The induced process is similar to pair production discussed earlier, but is more of an adiabatic process, being driven mostly by the large binding energy of the static combined Coulomb field. The induced decay is coherent with spontaneous decay and in the absence of the nuclei remaining together for long times (to promote spontaneous decay) the induced process accounts for almost all of the observed positrons[29,30]. At low collision energies, the probability for induced decay increases rapidly with projectile energy[26]. In looking for the spontaneous decay it is therefore important to do the experiment at the lowest collision energy for which the two nuclei can be brought within the critical distance.

*Collider geometries* -- Greiner and Schmelzer (see fig 2.27 of Ref. 32) have suggested producing low-energy collisions between uranium ions using storage rings with two beams of uranium nuclei rotating in the same direction with a small crossing angle. By adjusting the relative beam energy and crossing angle, the collision energy may be continuously changed. To achieve the highest possible luminosity (which would be roughly $\Delta\beta/\beta \approx 0.1$ L, where L is the luminosity of counter rotating beams) C.

Leeman[33] has suggested a zero-degree crossing angle and a small energy difference between two beams rotating in the same direction in separate storage rings bent into an interaction region. A collider which had a luminosity of $10^{24}\ cm^{-2}\ s^{-1}$ in a counter rotating mode would have a luminosity of roughly $10^{23}\ cm^{-2}\ s^{-1}$ for co-rotating beams and would produce roughly 3 interactions per second. Beam energies of the order of 1 GeV/nucleon would be needed to keep the bare nuclei from capturing electrons in the background gas of the collider.

*Nuclear collider experiments - bare nuclei* -- A relativistic nuclear collider offers some real advantages over the present fixed target experiments, but at the price of making some of the measurements more difficult. The two obvious advantages on a collider are 200% K-shell vacancies and no energy loss in the target. The collider may also allow experiments to look for signatures which can not be observed in unstripped-beam fixed-target experiments.

In the present experiments, only a small fraction of the uranium ions have a K-shell vacancy when they are within the critical distance of each other. Because there can be no decay of the supercritical state without a K vacancy, increasing the percentages of vacancies to 200% is expected to increase the cross section by about a factor of 40 (Ref. 26,34,35).

The dynamic process also depends upon a K vacancy and Greenberg and Vincent[36] have pointed out that the ratio of induced to spontaneous decays would increase if bare nuclei are used. (This is because unlike the spontaneous decay, which can occur only when the two nuclei are close enough to produce a binding energy of $> 2mc^2$, the dynamic process can occur when the nuclei are much further apart. Having K-shell vacancies at larger distances therefore increases the dynamic process relative to the spontaneous decay.) Recently, U. Müller (Ref. 34) has performed calculations which show that using bare nuclei changes this ratio only by about 15%. Consequently, using bare nuclei remains an advantage in increasing the signal and in discriminating against positron production which does not depend upon the number of K-shell vacancies.

Pair production from decay of excited nuclear states, is one such process expected to be insensitive to the number of K-shell vacancies. Consequently the use of bare nuclei decreases the contribution of this mechanism by a factor of 40. (One small caveat: With a vacant K shell, an excited nuclear state above 1 MeV [$2mc^2$] can decay by E0 transition producing an electron - positron pair with the electron captured into the K shell, and emitting a monoenergetic positron. The energy of the peak in the positron spectrum and its width should distinguish this process from positron production by spontaneous decays of the supercritical state[18,19,31]. The E0 decay also exists in the present GSI experiments but has a very low probability because the K vacancy lifetime is only about $10^{-17}$ s and the K shell fills before the excited nuclear state, with a lifetime of $\approx 10^{-13}$ s, can decay[31]. This suggests a possible experiment to measure the cross section for this process at higher energies: Bare nuclei pass through a fixed target and a charge changing collision in coincidence with a mono-energetic positron is observed.)

*Nuclear collider experiments -- elimination of fixed targets* -- In present experiments at GSI[15-17], targets of $\approx 0.5$ mg/cm$^2$ are used and peaks in the positron spectra are observed for bombarding energies between 6.0 MeV/nucleon and 6.2 MeV/nucleon. At these energies, about 0.1 MeV/nucleon is lost in the target[37]. If the energy spread at which nuclei will stick together is much smaller than 0.1 MeV/amu then most of the target thickness contributes only background. Given the intrinsic energy spread of the beam from the linac and non-uniformity in targets, there is a limit to how thin a target

can be used to advantage in fixed target experiments. In a collider there is essentially no energy loss in the target and the possibility of stepping the relative collision energy through small increments appears particularly attractive. Finally, it is in principle possible to do coincidence experiments between detected positrons and nuclei which have undergone a charge changing collision.

The technical problems associated with holding and adjusting the relative beam energy have not yet been evaluated. The energy resolution in a collider is limited by such factors as the ability to cool the beam and the trade off between beam quality and intensity.

*Other signatures* - If one of the beams of bare nuclei is replaced by a hydrogenlike atom then nearly half of the combined systems will form with the electron in the $2\,^2S_{1/2}$ state. The decay rate for the electric dipole transition $2\,^2S_{1/2} \rightarrow 1\,^2S_{1/2}$ for $Z > 173$ is calculated to be $10^{+18}$ seconds.[38,39], or $\approx$ 10% of the spontaneous positron production rate. Because the electric dipole decay rate from the $2\,^2S_{1/2}$ state is much slower than the decay of the super-critical state, if the nuclei stay together longer than $10^{-19}$ seconds then the peak in the photon spectra would be much narrower than the peak in the positron spectra. The angular distribution of the photons might also yield information about the dynamics of the collision.

*Experimental difficulties in colinear colliding beams* -- In addition to needing a small energy spread and precise control over the relative energy of the two beams, there are experimental disadvantages presented by the kinematics of relativistic beams. In present GSI experiments one of the principal tools in the detection of the peaks in the positron spectra is correlation of the positron spectroscopy with the angle through which the projectile is scattered[15]. The positron peaks are only observed in collisions in which the uranium scatters, in the center of mass, through 100 degrees to 130 degrees (back scattering). The kinematics of these collisions in a collider, with the center of momentum moving at roughly 1 GeV/c or higher is such that all of the scattering is highly peaked in the forward direction making the measurement of scattering angles much more difficult. Lower beam energies would make this less of a problem and could be used if the vacuum were better than $10^{-10}$ Torr nitrogen ($10^{-9}$ Torr hydrogen) or if there was frequent replenishment of the beam. To duplicate the count rate in present fixed target experiments it is estimated[40] that a luminosity of $10^{25}\ cm^{-2}\ s^{-1}$ is needed. Depending upon what fraction of the fixed target contributed to the signal in the GSI experiments, and the energy resolution of the collider, a lower luminosity might be used.

**(b) Hyperfine Splitting and Magnetic Moments**

*Hyperfine structure as a test of QED* -- A measurement of the hyperfine splitting of high-Z hydrogenlike atoms provides a test of the QED corrections to the hyperfine splitting in a strong Coulomb field. The largest QED correction to the hyperfine splitting is the correction to the electron magnetic moment due to emission and reabsorption of a virtual photon. The leading order term is the Schwinger correction which is $\alpha/2\pi$ times the Fermi splitting. Terms of order $\alpha(Z\alpha)$ and $\alpha(Z\alpha)^2$ are also present and the calculation of these and higher order terms presents a challenge because it is necessary to consider the energy of the electron bound by the combined Coulomb and magnetic fields of the nucleus. QED corrections to order $\alpha(Z\alpha)$ and $\alpha(Z\alpha)^2$ have so far been calculated by Sapirstein[41] and by Brodsky and Erickson[42] for hydrogen (and muonium) by perturbative techniques. Perturbative QED can not be used for very high-Z atoms where terms of higher order may be larger than the leading

terms. The next term in the expansion, $\alpha(Z\alpha)^3$, can not be neglected at high-Z.

In addition to being a test of QED in the limit of a strong Coulomb and magnetic field, the measurement may be an important test of the leading order terms as well. This is because the relative size of the QED correction increases with Z. In hydrogen the $\alpha(Z\alpha)$ and $\alpha(Z\alpha)^2$ terms contribute less than 10 parts-per-million (ppm) to the hyperfine structure but in thallium (Z = 81) they contribute more than 1%. Tests of the theory in hydrogen are limited at about 1 ppm due to the uncertainty in the proton polarizability. To make a test of equivalent sensitivity in thallium, the uncertainty in the contribution from nuclear polarizability could still be as large as 0.1% of the hyperfine splitting.

*Hyperfine structure anomalies* -- Hyperfine structure (HFS) anomalies arise when comparing the magnetic dipole splitting constants for an isotropic pair from different techniques i.e., NMR (external field) vs optical (internal field). The HFS anomalies have been attributed to the differences in the distribution of nuclear magnetism between members of the pair for which the internal field methods are sensitive. Optical methods however suffer from the inability to calculate the multielectron atom hyperfine structure to sufficient accuracy to yield definitive information on the distribution of nuclear magnetism[43]. Measurement of the hyperfine anomaly in the $1\ ^2S_{1/2}$ ground state of hydrogenlike atoms such as $^{203}$Tl and $^{205}$Tl is a definitive test of the origins of the hyperfine anomalies and of the distribution of nuclear magnetism. Although muonic atoms also involve a one electron wave function they are not likely to be a reliable probe of the distribution of nuclear magnetism because the mass of the muon $\approx 210\ m_e$ is large enough to perturb the distribution inside the nucleus[44].)

*Application of a RNC* Measurements on hydrogenlike thallium using storage rings have been considered by Bemis[45], and fixed target experiments have been formally proposed by Bemis and Gould[46]. The atomic ground state of thallium (I = 1/2) F=1 - F=0 transition energy is calculated to be 4800 $A^o$ without QED corrections. The magnetic dipole decay (M1) rate for F=1 $\rightarrow$ F=0 is $\approx 10^2\ s^{-1}$. Confinement of hydrogenlike thallium in a storage ring would then produce a spectra from the M1 decay of the $1\ ^2S_{1/2}$, F=1 state and optical spectroscopy would be used to determine the ground state hyperfine interval[45]. The determination of this interval for both $^{203}$Tl and $^{205}$Tl (both I =1/2) would address the question of hyperfine structure anomalies because the nuclear magnetic moment ratio is known by external field methods to an accuracy of $< 2$ in $10^7$.

Hydrogenlike thallium atoms traveling together in the same beam would undergo low energy collisions in intra-beam scattering which would repopulate the upper hyperfine level. This would produce a continuous glow from the M1 decay and might be a useful diagnostic of beam density or temperature. The cross sections have not yet been worked out.

The technique used in measuring the ground state hyperfine structure may also allow measurement of the magnetic moment and hyperfine structure of metastable nuclear excited states. In addition, transient magnetic fields of up to $10^{10}$ Tesla produced from motional Coulomb fields may have application for measuring the g factors of high spin states of nuclei.

## V. ACKNOWLEDGMENTS

I am happy to acknowledge the help given by, and to thank, among others: Dr. Jose Alonso, Dr.

Robert Anholt, Dr. James Beene, Dr. Curtis Bemis Jr., Dr. Klaus Berkner, Dr. Barry Berman, Dr. Christopher Bottcher, Professor Jack Greenberg, Dr. Harry Heckman, Dr. Christoph Leeman, Professor Eckert Matthias, Professor Walter Meyerhof, Professor Peter Mohr, Professor Berndt Müller, Mr. Charles Munger, Professor Jonathan Sapirstein, Dr. Walter Schimerling, Professor Gerhardt Soff, and Professor Paul Vincent. Without their assistance I could not have assembled this document. This work was supported by the Director, Office of Energy Research; Office of Basic Energy Sciences, Chemical Sciences Division of the U.S. Department of Energy under Contract No. DE-AC-03-76SF00098. and in part by the Office of High Energy and Nuclear Physics, Nuclear Science Division of the U.S. Department of Energy under Contract No. DE-AC-03-76SF00098.

## VI. REFERENCES

[1] E.J. Williams, Kgl. Dansk. Vid. Selsk. 13, No. 4 (1935).
[2] C.F. v. Weizsäcker, A. Phys. 88, 612 (1934).
[3] H.J. Bhabha, Proc. Roy. Soc. A152, 559 (1935); Proc. Camb. Phil. Soc. 31, 394 (1935).
[4] L. Landau and L. Lifshitz, Phys. Zs. Sov. U. 6, 244 (1934).
[5] Y. Nishina, S. Tomonaga and M. Kobayasi, Sci. Pap. Ins. Phys. Chem. Research, Japan 27, 137 (1935).
[6] G. Racah, Nuvo Cim. 14, 93 (1937).
[7] T. Murota, A Ueda and H. Tanaka, Prog. Theor. Phys. 16, 482 (1956).
[8] J.R. Oppenheimer, Phys. Rev. 47, 146 (1935).
[9] E.C.G. Stueckelberg, Helvetica Physica 8, 324 (1935).
[10] L. Nordheim, J. Phys. et Radium 6, 135 (1935).
[11] H. Gould, D. Greiner, P. Lindstron, T.J.M. Symons, and H. Crawford, Phys. Rev. Lett. 52, 180 (1984); 52, 1654 (1984).
[12] R. Anholt, S.A. Andriamonje, E. Morenzoni, Ch. Stoller, J.D. Molitoris, W.E. Meyerhof, H. Bowman, J.-S Xu, Z.-Z. Xu, J.O. Rasmussen, and D.H.H. Hoffmann, Phys. Rev. Lett. 53, 234, 1984.
[13] J.R. Beene and C. Bemis Jr. Private communication.
[14] A. Winther and K. Alder, Nucl. Phys. A312, 518 (1979).
[15] J. Schweppe, A. Gruppe, K. Bethge, H. Bokemeyer, T. Cowan, H. Folger, J.S. Greenberg, H. Grein, S. Ito, R. Schule, D. Schwalm, K.E. Steibing, N. Trautmann, P. Vincent, and M. Waldschmidt, Phys. Rev. Lett., 51, 2261 (1983).
[16] E. Bedermann, F. Bosch, M. Clemente, F. Güttner, P. Kienle, W. Ko¨nig, Ch. Kozhuharov, B. Martin, B. Povh, H. Tsertos, W. Wagner, and Th. Walcher, GSI Scientific Report 1980, Darmstadt, March. 1981, p. 128 and Verh. Dtsch. Ges. (VI) 16, 806 (1981).
[17] H. Bokemeyer, H. Folger, H. Grein, S. Ito, D. Schwalm, P. Vincent, K. Bethge, A. Gruppe, R. Schulé, M. Waldschnidt, J.S. Greenberg, J. Schweppe, and N. Trautmann, GSI Scientific Report 1980, Darmstadt, March 1981, p. 127, and Verh. Dtsch. Ges. (VI) 16, 795 (1981).
[18] U. Müller, G. Soff, T. de Reus, J. Reinhardt, B. Müller, and W. Greiner, Z. Physik. A313, 263 (1983).
[19] P. Schlüter, T. de Reus, J. Reinhardt, B. Müller, and G. Soff, Z. Physik. A314, 297 (1983).
[20] U. Heinz, J. Reinhardt, B. Müller, W. Greiner, and U. Mu:ller, Z. Physik. A314, 125 (1983).
[21] J. Reinhardt, U. Müller, B Müller, and W. Greiner, Z. Physik. A303, 172 (1981).
[22] T. de Reus, J. Reinhardt, B. Müller, W. Greiner, G. Soff, and U. Müller, J. Phys. B17, 615 (1984).
[23] B. Müller, J. Reinhardt, W. Greiner, and G. Soff, Z. Physik. A311, 151 (1983).

[24] J. Reinhardt, U. Müller, W. Greiner, and B Müller, Phys. Rev. A28, 2558 (1983).
[25] G. Soff, J. Reinhardt, B. Müller, and W. Greiner, Z. Physik. A294, 137 (1980).
[26] J. Reinhardt, B. Müller, and W. Greiner, Phys. Rev. A24, 103 (1981).
[27] B. Müller, Ann. Rev. Nucl. Sci. 26, 351 (1976).
[28] T. Tomada and H.A. Weidenmüller, Phys. Rev. A26, 162 (1982).
[29] H. Backe, L. Handschug, F. Hessberger, E. Kankeleit, L. Richter, F. Weik, R. Willwater, H. Bokemeyer, P. Vincent, Y. Nakayama, and J.S. Greenberg, Phys. Rev. Lett. 40, 1443 (1978).
[30] C. Kozhuharov, P. Kienle, E. Bedermann, H. Bokemeyer, J.S. Greenberg, Y. Nakayama, P. Vincent, H. Backe, L. Handschug, and E. Kankeleit, Phys. Rev. Lett. 42, 376 (1979)
[31] G. Soff, P. Schlüter, and W. Greiner, Z. Physik. A303, 189 (1981).
[32] See for example, Fig. 2.27 in S.J. Brodsky and P.J. Mohr, in *Topics in Current Physics: Quantum Electrodynamics in Strong and Supercritical Fields* edited by I.A. Sellin (Springer, Berlin, 1978), Vol. 5, p.3 This is an excellent review article on QED in high Z atoms.
[33] C. Leeman, Private communication.
[34] U. Müller, *Atomare Anregungen in Schwerionenstössen mt Nuklearem Kontakt* Doctoral Thesis, Johann-Wolfgang-Goethe University, Frankfurt 1984.
[35] G. Soff, Private Communication
[36] J.S. Greenberg and P. Vincent, Private communication.
[37] L.C. Northcliffe and R.F. Schilling, Nucl. Data. Tables 7, 324 (1970).
[38] G. Soff and B. Müller, Z. Physik. A280, 243 (1977).
[39] R. Anholt and J.O. Rasmussen, Phys. Rev. A9, 585 (1974).
[40] B. Müller, Private communication.
[41] J.R. Sapirstein, Phys. Rev. Lett. 51, 985 (1983).
[42] S.J. Brodsky, and G.W. Erickson, Phys. Rev. 148, 26 (1966).
[43] See for example, E. Matthias, H. Rinneberg, R. Beigang, A. Timmermann, J. Neukammer, and D. Schmidt, *To What Extent Can Isotope Shifts and Hyperfine Structure of Rydberg States Provide Nuclear Information?* In Nuclear Science Research Conference Series: Vol 3, *Lasers in Nuclear Physics,* edited C.E. Bemis Jr. and H.K. Carter, (Harwood, N.Y., 1982) p. 107.
[44] P.J. Mohr, Private communication.
[45] C.E. Bemis Jr. , Private communication.
[46] H. Gould and C.E. Bemis Jr., Bevalac Proposal 719H *Polarized Hydrogenlike Uranium,* unpublished.