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A. Smith

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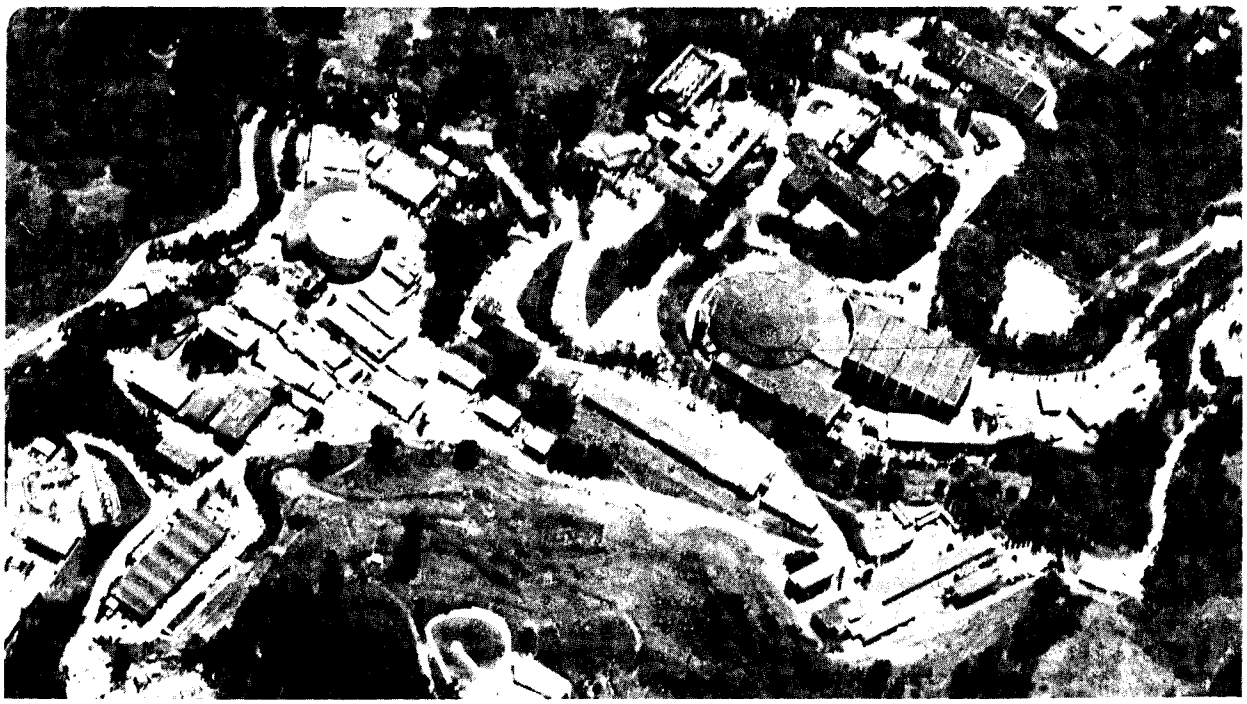

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LIGHT CONE FORMULATION OF N=2 YANG MILLS*

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Abstract

We write N=2 Yang Mills in Mandelstam's form of superspace. All of the one loop counterterms are computed and the result is noncovariant. This result is independent of the regulator used, but does depend on the $1/p_+$ prescription used. The consequences for finiteness proofs are discussed.

*This work was supported by the Director, Office of Energy Research, Office of High Energy Physics and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

Introduction

It has been known for some time [1] that N=4 supersymmetric Yang Mills has a vanishing beta function. This is most directly seen if the theory is written so as to be perturbatively finite, i.e. it requires no ultraviolet counterterms whatsoever. Mandelstam [2] and Brink et.al. [3] accomplished this by first using null plane quantization and then constructing superspace for the light cone field theory. N=2 theories are generally thought [4] to be finite above one loop and proofs based on a covariant superspace have been offered [5].

The finiteness properties of N=4 light cone theories are immediately apparent from their superspace Feynman rules. It is the manifest supersymmetry and not the (rather complicated) Lorentz properties of the Feynman rules that are responsible for the graph by graph finiteness in these theories.

Since N=2 theories in general have infinities at one loop a light cone superspace formulation cannot exhibit finiteness through power counting. However N=2 light cone superspace is worth investigating to see which, if any, finiteness properties of the N=4 theory remain. Tollsten [6] has already investigated this problem in a superspace formulation different from the one given here.

In section 1 we discuss the N=2 component lagrangian and it's expression in terms of light cone fields. Next, we discuss Mandelstam's [2] form of superspace. Section 3 contains the superspace lagrangian and the corresponding Feynman rules. Finally ,in section 4, we conclude with the observation that the one loop counterterms in this theory are noncovariant. Therefore any discussion of finiteness in N=2 theories must be in the context of one loop finite theories. An appendix contains notation and some details concerning the evaluation of light cone integrals.

Component Lagrangian

The starting point is the N=2 Yang Mills lagrangian of Fayet [7] and Brink et.al. [8] . It contains a gluon field v_μ, two Weyl spinors λ and ψ, and a complex scalar A. All the fields are massless and in the adjoint representation of the gauge group.The lagrangian is

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F^2_{\mu\nu} - i\,\overline{\lambda}^{\dot a}\,\sigma^\mu_{\dot a a}D_\mu\lambda^a - i\,\overline{\psi}^{\dot a}\,\sigma^\mu_{\dot a a}D_\mu\psi^a - D_\mu A^* D^\mu A \\ &+ ig\sqrt{2}\lambda^a(\psi_a \times A) + ig\sqrt{2}\,\overline{\psi}^{\dot a}\left(\overline{\lambda}_{\dot a} \times A^*\right) + \frac{g^2}{2}\left(A^* \times A\right)^2\end{aligned} \tag{2.1}$$

$$F_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - g(v_\mu \times v_\nu) \tag{2.2}$$

$$D_\mu\phi = \partial_\mu\phi - g(v_\mu \times \phi) \tag{2.3}$$

$$(A \times B)^\alpha = f^{\alpha\beta\gamma}A^\beta B^\gamma \tag{2.4}$$

$f^{\alpha\beta\gamma}$ are the group structure constants while the spinor indices $a, \dot a$ run over 1,2. This lagrangian is just an N=1 Yang Mills theory interacting with N=1 matter in the adjoint representation. $\mathcal{L}$ has a global $SU(2)$ invariance with λ, ψ transforming as a doublet. This is the origin of the two supercurrents in the model.

Next we impose the $v_+ = 0$ gauge condition through a delta function in the functional integral. With $v_+ = 0$ the $v_- = v_0 - v_3$ integral is gaussian and the fermion determinants for $\overline{\lambda}^2$, λ^2, $\overline{\psi}^2$, ψ^2 are field independent. So all these fields can be integrated out and we are left with an expression in which we only integrate over the fields $v = v_1 + iv_2$, $v^* = v_1 - iv_2$ $\overline{\lambda}^1$, λ^1, $\overline{\psi}^1$, ψ^1, A, and A^*. These are the light cone fields and through them we can linearly realize supersymmetry. We will not display the rather lengthy expression for the light cone lagrangian since it's superspace form is so compact.

Light Cone Superspace

It may appear that by integrating out some of the fields we have lost all vestiges of supersymmetry. That this is not so is well known [9]. In fact half of the supersymmetry algebra closes on the light cone fields (e.g. the remaining fields in the functional integral). If we look at the N=2 supercharge algebra,

$$\{Q_a^L, Q_b^M\} = \{\overline{Q}_{L\dot{a}}, \overline{Q}_{M\dot{b}}\} = 0 \tag{3.1a}$$

$$\{Q_a^L, \overline{Q}_{M\dot{b}}\} = -2^{3/2}\delta_M^L p_{a\dot{b}} \tag{3.1b}$$

$$p_{a\dot{a}} = \begin{pmatrix} -p_- & p \\ p^* & -p_+ \end{pmatrix}$$

it is the $a = \dot{a} = 2$ transformations we can realize with light cone fields alone. The supersymmetry transformations are effected by $\sum_{a\dot{a}} \alpha^a Q_a^1 + \beta^a Q_a^2 + \overline{\alpha}^{\dot{a}}\, \overline{Q}_{1\dot{a}} + \overline{\beta}^{\dot{a}}\, \overline{Q}_{2\dot{a}}$ where the α's and β's are Grassmann parameters.

Here L,m are internal indices running over 1,2. In particular, the known N=2 transformations specialize for $\alpha^1 = \overline{\alpha}^1 = \beta^1 = \overline{\beta}^1 = 0$ to

$$\begin{aligned} \delta A &= -2^{3/4}\alpha^2\psi^1 - 2^{3/4}\beta^2\lambda^1 \\ \delta\psi^1 &= 2^{3/4}\overline{\alpha}^2 p_+ A - i2^{1/4}\beta^2 p_+ v \\ \delta\lambda^1 &= 2^{3/4}\overline{\beta}^2 p_+ A - i2^{1/4}\alpha^2 p_+ v \\ \delta v &= 2^{5/4} i\overline{\alpha}^2\lambda^1 + 2^{5/4} i\overline{\beta}^2\psi^1 \end{aligned} \tag{3.3}$$

It can be checked that the commutator of two supersymmetry transformations (3.3) is consistent with the algebra (3.1) at $a = \dot{a} = 2$. In fact only the light cone fields close part of the supersymmetry algebra without auxillary fields.

At this point we would like to represent the charge algebra

$$\{Q_2^L, Q_2^M\} = \{\overline{Q}_{L2}, \overline{Q}_{M2}\} = 0 \tag{3.4a}$$

$$\{Q_2^L, \overline{Q}_{M2}\} = 2^{3/2} p_+ \delta_M^L \tag{3.4b}$$

in terms of linear differential operators. We do this by formulating the lagrangian in a superspace whose coordinates are space-time coordinates and two Grassmann parameters θ_1 and θ_2. This is Mandelstam's form [2] of superspace. The absence of θ's makes it analogous to the description of N=1 superspace in terms of the variables $y^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta}$, θ^j. So with the correspondences

$$Q_2^L \to D_L = \frac{i\partial}{\partial\theta_L} - i2^{1/2} p_+ \theta_L \tag{3.5a}$$

$$-\overline{Q}_2^L \to \overline{D}_L = \frac{\partial}{\partial\theta_L} + 2^{1/2}p_+\theta_L \tag{3.5b}$$

, $(L = 1,2)$, we can construct superfields Φ, Φ^* which respond to supersymmetry transformations as

$$\delta\Phi = \sum_{L=1}^{2}\left(\alpha_L D_L + \beta_L \overline{D}_L\right)\Phi \tag{3.6a}$$

$$\delta\Phi^* = \sum_{L=1}^{2}\left(-\alpha_L D_L + \beta_L \overline{D}_L\right)\Phi^* \tag{3.6b}$$

The explicit superfields can be constructed as

$$\Phi = \frac{1}{2}v + \frac{\theta_1}{2^{1/4}}\lambda^1 + \frac{\theta_2}{2^{1/4}}\psi^1 + \theta_1\theta_2 p_+ A^* \tag{3.7a}$$

$$\Phi^* = \frac{1}{2}v^* + \frac{\theta_1}{2^{1/4}}\overline{\lambda}^1 + \frac{\theta_2}{2^{1/4}}\overline{\psi}^1 + \theta_1\theta_2 p_+ A \tag{3.7b}$$

and with a little guesswork the lagrangian written in terms of the superfields is

$$\begin{aligned}\mathcal{L} = {} & \Phi^* \bullet \frac{p_\mu^2}{p_+} D\Phi + 2ig\frac{D}{p_+}\Phi \bullet \left(\frac{p}{p_+}\Phi^* \times \Phi^*\right) \\ & + 2ig\frac{D}{p_+}\Phi^* \bullet \left(\frac{p^*}{p_+}\Phi \times \Phi\right) + 2g^2\left(\frac{D}{p_+}\Phi \times \Phi^*\right) \bullet \frac{D}{p_+^2}\left(\frac{D}{p_+}\Phi^* \times \Phi\right)\end{aligned} \tag{3.9}$$

. D is defined by $D = D_1 D_2$.

The action is $S = \int d\theta_2 d\theta_1 \mathcal{L}$. A better form for Feynman rules is got by replacing the present fields with

$$\phi^* = \Phi^*, \qquad \phi = \frac{D}{p_+}\Phi \qquad or \qquad \Phi = \frac{-D}{2p_+}\phi.$$

Then we have

$$\begin{aligned}i\mathcal{L} = {} & -i\phi^* \bullet p_\mu^2 p_+ \phi - 2gp_+\phi \bullet \left(p\phi^* \times p_+\phi^*\right) \\ & - \frac{g}{2}D\phi^* \bullet \left(\frac{p^*}{p_+}D\phi \times D\phi\right) - ig^2\left(p_+\phi \times p_+\phi^*\right) \bullet \frac{D}{p_+^2}\left(D\phi^* \times D\phi\right)\end{aligned} \tag{3.9}$$

In this form $\mathcal{L}$ depends on θ only through ϕ and ϕ^*. This is seen by writing out (3.9) using the definition of the D's. There results

$$
\begin{aligned}
i\mathcal{L} = {} & \phi_a^*(-ip_\mu^2 p_+ \delta^{ab})\,\phi_b - 2g f^{abc} p_+ \phi_a p \phi_b^* p_+ \phi_c \\
& + f^{abc}\phi_a^*\Big[-\Big(p_+ p^* \phi_b\Big)\Big(\frac{\partial}{\partial\theta_1}\frac{\partial}{\partial\theta_2}\phi_c\Big) - \Big(p^* \frac{\partial}{\partial\theta_2}\phi_b\Big)\Big(p_+ \frac{\partial}{\partial\theta_1}\phi_c\Big) \\
& + \Big(p^* \frac{\partial}{\partial\theta_1}\phi_b\Big)\Big(p_+ \frac{\partial}{\partial\theta_2}\phi_c\Big) - \Big(\frac{\partial}{\partial\theta_1}\frac{\partial}{\partial\theta_2}\frac{p^*}{p_+}\phi_b\Big)(p_+^2\,\phi_c)\Big] \\
& - 2ig^2 f^{ac\alpha} f^{db\alpha}\Big[\frac{1}{p_+^2}\Big(p_+\phi_a p_+ \phi_c^*\Big)\Big]\Big[\Big(p_+^2\,\phi_d^*\Big)\Big(\frac{\partial}{\partial\theta_1}\frac{\partial}{\partial\theta_2}\phi_b\Big) \\
& + (p_+^2\,\phi_b)\Big(\frac{\partial}{\partial\theta_1}\frac{\partial}{\partial\theta_2}\phi_d^*\Big) - \Big(p_+ \frac{\partial}{\partial\theta_1}\phi_d^*\Big)\Big(p_+ \frac{\partial}{\partial\theta_2}\phi_b\Big) + \Big(p_+ \frac{\partial}{\partial\theta_2}\phi_d^*\Big)\Big(p_+ \frac{\partial}{\partial\theta_1}\phi_b\Big)\Big]
\end{aligned}
\tag{3.10}
$$

From the form (3.10) , which depends only on derivatives in superspace, we can easily write the Feynman rules in momentum space for the x's and the θ's. Functional derivatives of superfields are just delta functions so deriving the Feynman rules is simple. These Feynman rules are given in figure 1, with notation summarized in the appendix. The four point vertex is conveniently expressed as a sum of asymmetric four point vertices. With the Feynman rules and some light cone integrals (appendix) we can compute the one loop counterterms of the theory. Since we are using the $p_+ \to p_+ + i\epsilon\, p_-$ or Mandelstam [2] prescription to treat the $1/p_+$ singularities, conventional power counting is valid and we can eliminate many graphs from consideration on this basis.

One Loop Counterterms

First we compute the propagator counterterms. After doing the Grassmann momentum integrals we are left with ordinary four dimensional integrals. In figure 2 integrals corresponding to the possible one loop graphs are given. Notice that only one graph is infinite. The corresponding loop integral for this graph is

$$p_+ p p^* I_1 - p_+^2 p I_2 - p_+^2 p^* I_3 + p_+^3 I_4 \tag{4.1}$$

$$I_{1,2,3,4} = \int \frac{d^4 q}{q_\mu^2 (p_\mu + q_\mu)^2}\Big(1, \frac{q^*}{q_+}, \frac{q}{q_+}, \frac{q q^*}{q_+^2}\Big). \tag{4.2}$$

Of these integrals, only I_1 is infinite although they are all logarithmically divergent by power counting. Therefore the one loop counterterm to the propagator is proportional to $p_+ p p^*$ and not $p_+ p_\mu^2$. A two point counterterm not proportional to the quadratic term in the lagrangian leads one to suspect that the one loop counterterms are not Lorentz invariant. This is indeed true if we examine the complete set of one loop counterterms in figure 3. The quantity K is just the infinite part of

$$K = -8ig^2 C_2 \left(\int \frac{d^4p}{p_\mu^2 (p_\mu + X_\mu)^2} \right) \tag{4.3}$$

where X_μ is an external momenta.

What we should glean from figure 3 is that taken together the one loop counterterms are not proportional to the lagrangian. That is if we write the bare lagrangian as

$$\mathcal{L} = \mathcal{L}^{quadratic} + \mathcal{L}^{cubic} + \mathcal{L}^{'cubic} + \mathcal{L}^{quartic}$$

we know the nonlinear Lorentz symmetry links

$$\mathcal{L}^{quadratic} \quad to \quad \mathcal{L}^{cubic} \quad and \quad \mathcal{L}^{'cubic}$$

and also links

$$\mathcal{L}^{cubic} \quad and \quad \mathcal{L}^{'cubic} \quad to \quad \mathcal{L}^{quartic}$$

Demanding Lorentz invariance of any counterterm

$$\mathcal{L}^{counterterm} = a\mathcal{L}^{quadratic} + b\mathcal{L}^{cubic} + c\mathcal{L}^{'cubic} + d\mathcal{L}^{quartic}$$

implies $a = b = c = d$. For the case at hand the two point counterterm does not have the same form as the $\phi^* \phi$ term in the lagrangian. The desired Lorentz invariance cannot therefore follow.

The above result is independent of the method for regulating the ultraviolet divergences but does depend on our $1/p_+$ prescription. This independence from the regulator derives from the fact that all the one loop counterterms are proportional to the same (up to finite parts) integral. People [10] have encountered similar difficulties with ordinary light cone Yang Mills, e.g. noncovariant two point counterterms. However they used a $1/p_+$ prescription different from Mandelstam's.

Conclusion

In conclusion, we point out that any Lorentz Ward identity derived from the bare lagrangian cannot be used to restrict the form of the counterterms. We have explicitly seen this in the case of the two point function. This is thought to be a general characteristic of light cone theories [11] (not necessarily supersymmetric). It's appearance here complicates matters since Lorentz Ward identities will probably play an important role in proving finiteness above one loop for N=2 light cone theories. So in order to discuss finiteness of N=2 theories we shall have to add N=2 matter fields to cancel the one loop infinities. Then we can proceed with an analysis based on the naive Ward identities. Even if the one loop counterterms had been covariant we would still be faced with the difficulty that at three loops and beyond the beta function is subtraction scheme dependent.

Acknowledgements

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Appendix

The notation used in the Feynman rules is

$$p = p_1 + ip_2 \qquad p_+ = p_0 + p_3$$

$$p^* = p_1 - ip_2 \qquad p_- = p_0 - p_3$$

$$(p,q) = pq_+ - qp_+ \qquad (p,q)^* = p^* q_+ - q^* p_+$$

$$[\![p,q]\!] = (p_+\overline{q}_1 - q_+\overline{p}_1)(p_+\overline{q}_2 - q_+\overline{p}_2)$$

$\overline{p}_1, \overline{p}_2, \overline{q}_1, \overline{q}_2$ are the Grassmann momenta.

For the purpose of computing the one loop counterterms we encounter four superficially divergent integrals. They are I_1, I_2, I_3, I_4 and are given in eq.(4.2). I_1 is trivial to calculate, I_2 can be gotten from I_3, so we only need I_3 and I_4.

$$I_3 = \int \frac{d^{2-\epsilon}q\, dq_0\, dq_3}{q_\mu^2 (q_\mu + p_\mu)^2} \frac{q}{q_+} \tag{a1}$$

We use dimensional regularization in the transverse dimensions. Rotating to euclidean space, $q_+ \to iq_0 + q_3$ so

$$I_3 = i \int \frac{d^{2-\epsilon}q\, dq_0\, dq_3\, q(-iq_0 + q_3)}{q_\mu^2 (q_\mu + p_\mu)^2 (q_0^2 + q_3^2)}. \tag{a2}$$

After combining denominators using Feynman parameters; do the q_0 and q_3 integrals, do the transverse q integral then do the parametric integrals. The result is finite as $\epsilon \to 0$ and is

$$I_3 = -i\pi^2 \frac{p_-}{p^*} \log\left(\frac{-p_\mu^2}{p_+\, p_-}\right). \tag{a3}$$

The procedure for calculating I_4 is word for word the same as for I_3. At the end we cannot do the parametric integral in closed form and we must express the answer in terms of the Spence function

$$F(x) = \int_0^x \frac{ds}{s} \log(1+s).$$

$$I_4 = i\pi^2 \frac{p_-}{p_+}\left(\log\left(\frac{-p_+\, p_-}{p_\mu^2}\right) - \frac{p_\mu^2}{p_+\, p_-}\left(\frac{\pi^2}{6} + F\left(\frac{-pp^*}{p_\mu^2}\right)\right)\right) \tag{a4}$$

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Figure Captions

Figure 1 Feynman rules, momentum flows in the arrow's direction.

Figure 2 One loop graphs contributing to the two point function.

Figure 3 The infinite one loop graphs and their corresponding counterterms.

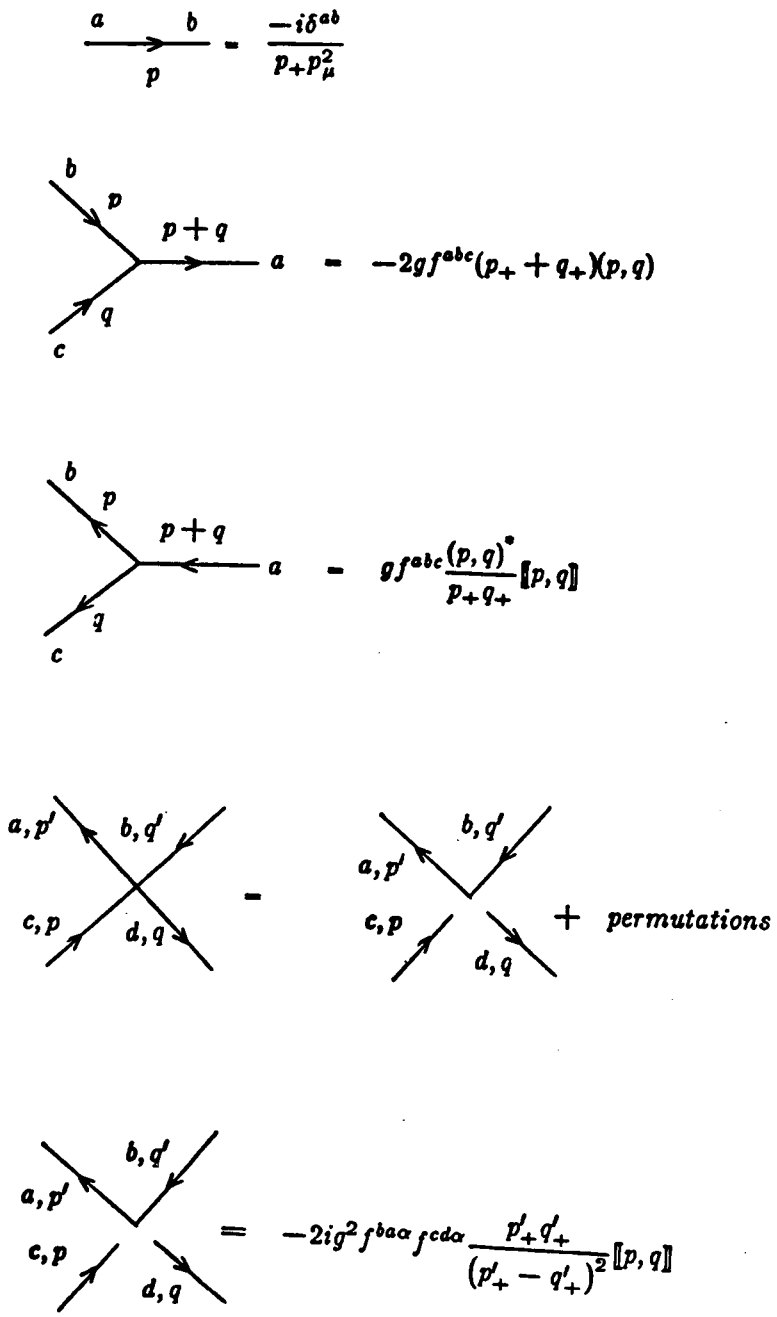
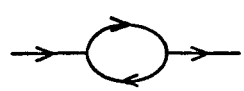
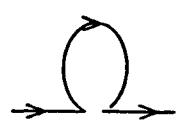

a
b
p
$= \frac{-i\delta^{ab}}{p_+ p_\mu^2}$
b
p
p + q
a
q
c
$= -2gf^{abc}(p_+ + q_+)(p,q)$
b
p
p + q
a
q
c
$= gf^{abc}\frac{(p,q)^*}{p_+ q_+}[\![p,q]\!]$
a, p'
b, q'
c, p
d, q
=
b, q'
a, p'
c, p
d, q
+ permutations
b, q'
a, p'
c, p
d, q
$= -2ig^2 f^{bac} f^{cda} \frac{p'_+ q'_+}{(p'_+ - q'_+)^2}[\![p,q]\!]$

figure 1

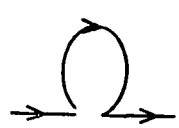
$$p_+^3 \int \frac{d^4q\,(p,q)^*(p,q)}{(p_+ - q_+)^2 q_+^2 (p_\mu - q_\mu)^2 q_\mu^2} = finite$$



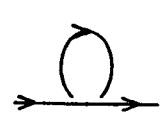
$$p_+ \int \frac{d^4q\,(p,q)^*(p,q)}{q_+^2 q_\mu^2 (q_\mu + p_\mu)^2} = infinite$$

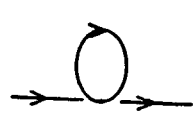


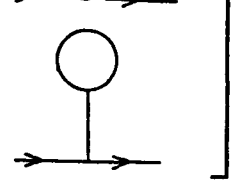
$$p_+^3 \int \frac{d^4q}{(p_+ - q_+)^2 q_\mu^2} = finite$$



$$p_+^3 \int \frac{d^4q}{(p_+ - q_+)^2 q_\mu^2} = finite$$







$$= 0$$

from θ integration or structure constants

figure 2

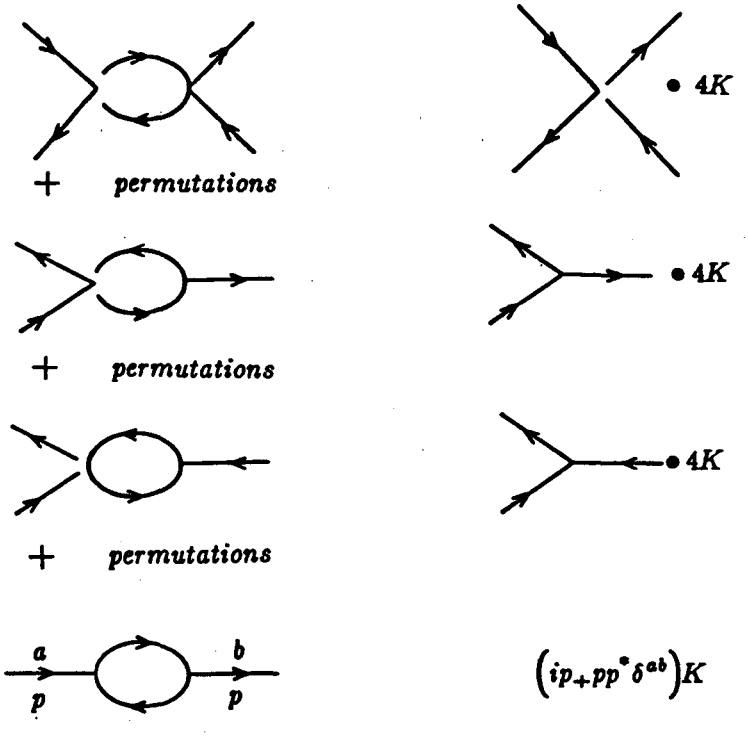

+ permutations
• 4K
+ permutations
• 4K
+ permutations
• 4K
a
p
b
p
$\left(ip_+pp^*\delta^{ab}\right)K$

figure 3

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