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LIGHT CONE FORMULATION ON N=2 YANG MILLS

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March 1985

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LIGHT CONE FORMULATION OF N=2 YANG MILLS [1]

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1 Abstract

We write N=2 Yang Mills in Mandelstam's form of superspace. All the one loop counter terms are computed and the result is proportional to the lagrangian. Some subtleties associated with power counting for light cone integrals are noted.

2 Introduction

It has been known for some time [1] that N=4 supersymmetric Yang Mills has a vanishing beta function. This is most directly seen if the theory is written so as to be perturbatively finite, i.e. it requires no ultraviolet counterterms whatsoever. Mandelstam [2] and Brink et.al. [3] accomplished this by first using null plane quantization and then constructing superspace for the light cone field theory. N=2 theories are generally thought [4] to be finite above one loop and proofs based on a covariant superspace have been offered [5].

The finiteness properties of N=4 light cone theories are immediately apparent from their superspace Feynman rules. It is the manifest supersymmetry and not the (rather complicated) Lorentz properties of the Feynman rules that are responsible for the graph by graph finiteness in these theories.

Since N=2 theories in general have infinities at one loop a light cone superspace formulation cannot exhibit finiteness through power counting. However N=2 light cone superspace is worth investigating to see which, if any, finiteness properties of the N=4 theory remain. Tollsten [6] has already investigated this problem in a superspace formulation different from the one given here.

In section 1 we discuss the N=2 component lagrangian and it's expression in terms of light cone fields. Next, we discuss Mandelstam's [2] form of

superspace.

Section 3 conntains the superspace lagrangian and the corresponding Feynman rules. We also point out a subtlety associated with power counting in the light cone formalism. In section 4 we calculate all the one loop counterterms and find their contribution is proportional to the Lagranngian. An appendix contains notation and some more details on loop integrals.

3 Component Lagrangian

The starting point is the N=2 Yang Mills lagrangian of Fayet [7] and Brink et.al. [8] . It contains a gluon field v_μ, two Weyl spinors λ and ψ, and a complex scalar A. All the fields are massless and in the adjoint representation of the gauge group.The lagrangian is

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}^2 - i\,\overline{\lambda}^{\dot{a}}\,\sigma^{\mu}_{\dot{a}a}D_\mu\lambda^a - i\,\overline{\psi}^{\dot{a}}\,\sigma^{\mu}_{\dot{a}a}D_\mu\psi^a - D_\mu A^* D^\mu A \\ &+ ig\sqrt{2}\lambda^a\left(\psi_a \times A\right) + ig\sqrt{2}\,\overline{\psi}^{\dot{a}}\left(\overline{\lambda}_{\dot{a}} \times A^*\right) + \frac{g^2}{2}\left(A^* \times A\right)^2\end{aligned} \tag{3.1}$$

$$F_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - g\left(v_\mu \times v_\nu\right) \tag{3.2}$$

$$D_\mu\phi = \partial_\mu\phi - g\left(v_\mu \times \phi\right) \tag{3.3}$$

$$\left(A \times B\right)^\alpha = f^{\alpha\beta\gamma}A^\beta B^\gamma \tag{3.4}$$

$f^{\alpha\beta\gamma}$ are the group structure constants while the spinor indices $a, \dot{a}$ run over 1,2. This lagrangian is just an N=1 Yang Mills theory interacting with N=1 matter in the adjoint representation. $\mathcal{L}$ has a global $SU(2)$ invariance with λ, ψ transforming as a doublet. This is the origin of the two supercurrents in the model.

Next we impose the $v_+ = 0$ gauge condition through a delta function in the functional integral. With $v_+ = 0$ the $v_- = v_0 - v_3$ integral is gaussian and the fermion determinants for $\overline{\lambda}^2$, λ^2, $\overline{\psi}^2$, ψ^2 are field independent. So all

these fields can be integrated out and we are left with an expression in which we only integrate over the fields $v = v_1 + iv_2$, , $v^* = v_1 - iv_2$ $\overline{\lambda}^1$, λ^1, $\overline{\psi}^1$, ψ^1, A, and A^*. These are the light cone fields and through them we can linearly realize supersymmetry. We will not display the rather lengthy expression for the light cone lagrangian since it's superspace form is so compact.

4 Light Cone Superspace

By integrating out some of the fields we have not lost supersymmetry because it is really the constraint equations which allow this, and they are themselves supersymmetric. Half of the supersymmetry algebra closes on the light cone fields [9] (e.g. the remaining fields in the functional integral). If we look at the N=2 supercharge algebra,

$$\{Q_a^L, Q_b^M\} = \{\bar{Q}_{L\dot{a}}, \bar{Q}_{M\dot{b}}\} = 0 \tag{4.1}$$

$$\{Q_a^L, \bar{Q}_{M\dot{b}}\} = -2^{3/2}\delta_M^L p_{a\dot{b}} \tag{4.2}$$

$$p_{a\dot{a}} = \begin{pmatrix} -p_- & p \\ p^* & -p_+ \end{pmatrix} \tag{4.3}$$

it is the $a = \dot{a} = 2$ transformations we can realize with light cone fields alone. The supersymmetry transformations are effected by $\sum_{a\dot{a}} \alpha^a Q_a^1 + \beta^a Q_a^2 + \overline{\alpha}^{\dot{a}} \bar{Q}_{1\dot{a}} + \overline{\beta}^{\dot{a}} \bar{Q}_{2\dot{a}}$ where the α's and β's are Grassmann parameters.

Here L,M are internal indices running over 1,2. In particular, the known N=2 transformations specialize for $\alpha^1 = \overline{\alpha}^1 = \beta^1 = \overline{\beta}^1 = 0$ to

$$\delta A = -2^{3/4}\alpha^2\psi^1 - 2^{3/4}\beta^2\lambda^1 \tag{4.4}$$

$$\delta\psi^1 = 2^{3/4}\overline{\alpha}^2 p_+ A - i2^{1/4}\beta^2 p_+ v \tag{4.5}$$

$$\delta\lambda^1 = 2^{3/4}\overline{\beta}^2 p_+ A - i2^{1/4}\alpha^2 p_+ v \tag{4.6}$$

$$\delta v = 2^{5/4} i\overline{\alpha}^2\lambda^1 + 2^{5/4} i\overline{\beta}^2\psi^1 \tag{4.7}$$

It can be checked that the commutator of two supersymmetry transformations (3.3) is consistent with the algebra (3.1) at $a = \dot{a} = 2$. In fact only the light cone fields close part of the supersymmetry algebra without auxillary fields.

At this point we would like to represent the charge algebra

$$\{Q_2^L, Q_2^M\} = \{\overline{Q}_{L2}, \overline{Q}_{M2}\} = 0 \tag{4.8}$$

$$\{Q_2^L, \overline{Q}_{M2}\} = 2^{3/2} p_+ \delta_M^L \tag{4.9}$$

in terms of linear differential operators. We do this by formulating the lagrangian in a superspace whose coordinates are space-time coordinates and two Grassmann parameters θ_1 and θ_2. This is Mandelstam's form [2] of superspace. The absence of θ's makes it analogous to the description of N=1 superspace in terms of the variables $y^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta}$, θ^j. So with the correspondences

$$Q_2^L \to D_L = \frac{i\partial}{\partial\theta_L} - i2^{1/2} p_+ \theta_L \tag{4.10}$$

$$-\overline{Q}_2^L \to \overline{D}_L = \frac{\partial}{\partial\theta_L} + 2^{1/2} p_+ \theta_L \tag{4.11}$$

, $(L = 1, 2)$, we can construct superfields Φ, Φ^* which respond to supersymmetry transformations as

$$\delta\Phi = \sum_{L=1}^{2} \left(\alpha_L D_L + \beta_L \overline{D}_L\right) \Phi \tag{4.12}$$

$$\delta\Phi^* = \sum_{L=1}^{2} \left(-\alpha_L D_L + \beta_L \overline{D}_L\right) \Phi^* \tag{4.13}$$

The explicit superfields can be constructed as

$$\Phi = \frac{1}{2}v + \frac{\theta_1}{2^{1/4}}\lambda^1 + \frac{\theta_2}{2^{1/4}}\psi^1 + \theta_1\theta_2 p_+ A^* \tag{4.14}$$

$$\Phi^* = \frac{1}{2}v^* + \frac{\theta_1}{2^{1/4}}\overline{\lambda}^1 + \frac{\theta_2}{2^{1/4}}\overline{\psi}^1 + \theta_1\theta_2 p_+ A \tag{4.15}$$

and with a little guesswork the lagrangian written in terms of the superfields is

$$\begin{aligned}\mathcal{L} = {} & \Phi^* \bullet \frac{p_\mu^2}{p_+^2} D\Phi + 2ig\frac{D}{p_+}\Phi \bullet \left(\frac{p}{p_+}\Phi^* \times \Phi^*\right) \\ & + 2ig\frac{D}{p_+}\Phi^* \bullet \left(\frac{p^*}{p_+}\Phi \times \Phi\right) + 2g^2\left(\frac{D}{p_+}\Phi \times \Phi^*\right) \bullet \frac{D}{p_+^2}\left(\frac{D}{p_+}\Phi^* \times \Phi\right)\end{aligned} \tag{4.16}$$

. D is defined by $D = D_1 D_2$.

The action is $S = \int d\theta_2 d\theta_1 \mathcal{L}$. A better form for Feynman rules is got by replacing the present fields with

$$\phi^* = \frac{1}{p_+}\Phi^*, \qquad \phi = \frac{D}{p_+^2}\Phi \qquad or \qquad \Phi = \frac{-D}{2}\phi.$$

Then we have

$$\begin{aligned} i\mathcal{L} = {} & -i\phi^* \bullet p_\mu^2 p_+ \phi - 2gp_+\phi \bullet (p\phi^* \times p_+\phi^*) \\ & - \frac{g}{2}D\phi^* \bullet \left(\frac{p^*}{p_+}D\phi \times D\phi\right) - ig^2\left(p_+\phi \times p_+\phi^*\right) \bullet \frac{D}{p_+^2}\left(D\phi^* \times D\phi\right)\end{aligned} \tag{4.17}$$

In this form $\mathcal{L}$ depends on θ only through ϕ and ϕ^*. This is seen by writing out (3.9) using the definition of the D's. There results

$$\begin{aligned} i\mathcal{L} = {} & \phi_a^*\left(-ip_\mu^2 p_+ \delta^{ab}\right)\phi_b - 2gf^{abc}p_+\phi_a p\phi_b^* p_+ \phi_c \\ & + f^{abc}\phi_a^*\Bigg[-\left(p_+ p^*\phi_b\right)\left(\frac{\partial}{\partial\theta_1}\frac{\partial}{\partial\theta_2}\phi_c\right) - \left(p^*\frac{\partial}{\partial\theta_2}\phi_b\right)\left(p_+\frac{\partial}{\partial\theta_1}\phi_c\right) \\ & + \left(p^*\frac{\partial}{\partial\theta_1}\phi_b\right)\left(p_+\frac{\partial}{\partial\theta_2}\phi_c\right) - \left(\frac{\partial}{\partial\theta_1}\frac{\partial}{\partial\theta_2}\frac{p^*}{p_+}\phi_b\right)\left(p_+^2\phi_c\right)\Bigg] \\ & - 2ig^2 f^{aca}f^{dba}\left[\frac{1}{p_+^2}\left(p_+\phi_a p_+\phi_c^*\right)\right]\Bigg[\left(p_+^2\phi_d^*\right)\left(\frac{\partial}{\partial\theta_1}\frac{\partial}{\partial\theta_2}\phi_b\right) \\ & + \left(p_+^2\phi_b\right)\left(\frac{\partial}{\partial\theta_1}\frac{\partial}{\partial\theta_2}\phi_d^*\right) - \left(p_+\frac{\partial}{\partial\theta_1}\phi_d^*\right)\left(p_+\frac{\partial}{\partial\theta_2}\phi_b\right) + \left(p_+\frac{\partial}{\partial\theta_2}\phi_d^*\right)\left(p_+\frac{\partial}{\partial\theta_1}\phi_b\right)\Bigg]\end{aligned} \tag{4.18}$$

From the form (4.18) , which depends only on derivatives in superspace, we can easily write the Feynman rules in momentum space for the x's and the θ's. Functional derivatives of superfields are just delta functions so deriving the Feynman rules is simple. These Feynman rules are given in figure 1, with notation summarized in the appendix. The four point vertex is conveniently expressed as a sum of asymmetric four point vertices. With the

Feynman rules and some light cone integrals (appendix) we can compute the one loop counterterms of the theory. Since we are using the $p_+ \to p_+ + i\epsilon p_-$ or Mandelstam [2] prescription to treat the $1/p_+$ singularities, conventional power counting is valid and we can eliminate many graphs from consideration on this basis. However some refinement of the usual power counting procedure is needed for light cone integrals. In theories with manifest Lorentz covariance (Euclidean covariance in the Wick rotated integrals) all 4 components of momentum in a given loop integration are on the same footing, e.g. the integrand transforms as some tensor representation of $O(4)$. For light cone integrals the transverse p_1, p_2 and longitudinal p_0, p_3 components appear on different footings and Euclidean light cone integrals transform under $O(2) \times O(2)$[2] not $O(4)$. To check the convergence of any multiple integral Weinberg's [10] theorem states that every integration and subintegration must converge, and to ascertain the convergence/divergence of any integration we can use ordinary power counting. Because of the $O(4)$ symmetry in covariant integrals we can use $\int d^4p$ as the lowest subintegration in the nested hierarchy of integrations making up a multiloop integral. As is well known this gives rise to the usual prescription for determining which subtractions are required in a given graph. Due to the $O(2) \times O(2)$ symmetry, in light cone integrals we can no longer use $\int d^4p$ as the lowest subintegration, in it's

[2] In the plus-minus direction the $O(2)$ transformation properties arise from the combination $p_+ \to ip_0 + p_3$ which changes by a phase under rotations.

place we use $\int d^2p_T$ and $\int d^2p_L$. So instead of one 4 dimensional momentum flow in each loop, we must consider two 2 dimensional flows in the application of Weinberg's theorem.

The effect of this modification of power counting properties is mild. For a graph with $n_{3/4}$ three/four point vertices power counting in the transverse dimensions yields a degree of divergence $D_T = 2 - 2n_4 - n_3$. So the only graphs in the theory which diverge in the transverse dimensions are A,C,D,E in figure 2. Of these only A is actually divergent. Had we used the power counting customary to covariant integrals we would have found $D = -2$ for graph A and concluded (wrongly) it's convergence.

Application of this revised form of power counting to N=4 Yang Mills [2] does not change the conclusion that the theory is finite. This is so since all of the potentially divergent graphs in $\int d^2p_T$ are topologically the same as A,C,D,E in figure 2. But since the finiteness of the 2 point function follows from the finiteness of 3 point graphs via the Ward identity these divergences in the propogator must cancel.

5 One Loop Counterterms

First we compute the propagator counterterms. After doing the Grassmann momentum integrals we are left with ordinary four dimensional integrals. In figure 2 the one loop propagator graphs and their corresponding momentum integrals are given. The momentum integral for graph 2A is:

$$I_A = p\,p^*\,J_1 - pp_+J_2 - p^*\,p_+J_3 + p_+^2J_4 \tag{5.1}$$

with

$$J_{1,2,3,4} = \int \frac{d^4q}{(p_\mu - q_\mu)^2\,q_\mu^2\,(p_+ - q_+)^2}\left(1, \frac{q^*}{q_+}, \frac{q}{q_+}, \frac{q\,q^*}{q_+^2}\right) \tag{5.2}$$

. By power counting only J_4 is divergent and only in the transverse dimensions. Calculation of J_4 is in the appendix and the infinite part of I_A is:

$$I_A = \frac{-4i\pi^2}{\epsilon}\frac{p_-}{p_+} \tag{5.3}.$$

For graph B the coresponding loop integral is:

$$p_+pp^*I_1 - p_+^2pI_2 - p_+^2p^*I_3 + p_+^3I_4 \tag{5.4}$$

with

$$I_{1,2,3,4} = \int \frac{d^4q}{q_\mu^2\,(p_\mu + q_\mu)^2}\left(1, \frac{q^*}{q_+}, \frac{q}{q_+}, \frac{qq^*}{q_+^2}\right) \tag{5.5}.$$

Of these integrals only I_1 is actually infinite (see appendix) although they are all logarithmically divergent by power counting. So the infinite part of I_B is,

$$I_B^{infinite} = \frac{2i\pi^2}{\epsilon}p_+p\,p^* \tag{5.6}$$

.

When we take account of the symmetry factors of graphs A and B their combined contribution is proportional to $p_+p_\mu^2$.

For the three and four point functions, only one graph contributes to the corresponding counterterms at one loop. The absence of infinite graphs at one loop with more than 2 propagators follows from a general power counting theorem [11]. Infinite parts for these graphs are proportional to the invariant integral I_1 in (5.5). Figure 3 lists the infinite two, three and four point graphs and their associated counterterms. Notice that taken together the one loop counterterms are proportional to the lagrangian.

6 Conclusion

We have written down the N=2 Yang Mills light cone lagrangian and calculated the complete set of one loop counterterms using the Euclidean or Mandelstam prescription to treat $1/p_+$ singularities. Use of Leibbrandt's $1/p_+$ prescription [12] would produce identical integrals at one loop. As expected from the Lorentz Ward identities the one loop,counterterms taken together are Lorentz invariant and proportional to the original lagrangian. This is a nontrivial test of the $1/p_+$ prescription. We must emphasize the the difference in power counting procedures for light cone and covariant theories. One cannot count all 4 components of momentum together to ascertain covergence, we must seperately consider the transverse and longitudinal integrations. Without these considerations we would conclude the two point counterterm was noncovariant. It would be interesting to apply this prescription to ordinary light cone Yang Mills since we could be assured of it's consistency. The really interesting question is, how do we prove the finiteness properties of N=2 Yang Mills above one loop? Only that power counting and Lorentz Ward identities will be important in the proof.

7 Acknowledgements

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8 Appendix

The notation used in the Feynman rules is

$$p = p_1 + ip_2 \qquad p_+ = p_0 + p_3$$

$$p^* = p_1 - ip_2 \qquad p_- = p_0 - p_3$$

$$(p,q) = pq_+ - qp_+ \qquad (p,q)^* = p^*q_+ - q^*p_+$$

$$[\![p,q]\!] = (p_+\bar{q}_1 - q_+\bar{p}_1)(p_+\bar{q}_2 - q_+\bar{p}_2)$$

$\bar{p}_1, \bar{p}_2, \bar{q}_1, \bar{q}_2$ are the Grassmann momenta.

For the purpose of computing the one loop counterterms we encounter five superficially divergent integrals. They are I_1, I_2, I_3, I_4, J_4 and are given in eq.(4.2) and (4.4). I_1 is trivial to calculate, I_2 can be gotten from I_3, so we only need I_3 , I_4 and J_4. Because of q_+'s in the denominators of our integrals, when we Wick rotate the longitudinal and transverse components appear on a different footing. So calculating a one loop integral is mechanically similar to computing a two loop integral in 2 dimensions.

$$I_3 = \int \frac{d^{2-\epsilon}q dq_0 dq_3}{q_\mu^2 (q_\mu + p_\mu)^2} \frac{q}{q_+} \qquad (a1)$$

We use dimensional regularization in the transverse dimensions. Rotating to euclidean space, $q_+ \to iq_0 + q_3$ so

$$I_3 = i \int \frac{d^{2-\epsilon}q dq_0 dq_3\, q\,(-iq_0 + q_3)}{q_\mu^2 (q_\mu + p_\mu)^2 (q_0^2 + q_3^2)}. \qquad (a2)$$

After combining denominators using Feynman parameters; do the q_0 and q_3 integrals, do the transverse q integral then do the parametric integrals. The result is finite as $\epsilon \to 0$ and is

$$I_3 = -i\pi^2 \frac{p_-}{p^*} \log\left(\frac{-p_\mu^2}{p_+p_-}\right). \qquad (a3)$$

The procedure for calculating I_4 is word for word the same as for I_3. At the end we cannot do the parametric integral in closed form and we must express the answer in terms of the Spence function

$$F(x) = \int_0^x \frac{ds}{s} \log(1+s).$$

$$I_4 = i\pi^2 \frac{p_-}{p_+} \left(\log\left(\frac{-p_+p_-}{p_\mu^2}\right) - \frac{p_\mu^2}{p_+p_-}\left(\frac{\pi^2}{6} + F\left(\frac{-pp^*}{p_\mu^2}\right)\right)\right) \qquad (a4)$$

We will not exhibit J_1, J_2 or J_3 since they are finite by power counting but the author has computed them and they are convergent. The finite part of J_4 is likewise uninteresting so we proceed to directly extract it's infinite part. After Wick rotating:

$$J_4(p) = i\int \frac{d^4q q_T^2}{(p-q)^2 q^2 q_+^2 (p_+ - q_+)^2} \qquad (a5)$$

Calculating the transverse integral first we get:

$$J_4 = i\int \frac{d^2q_L}{q_+^2 (p_+ - q_+)^2}\left(\frac{2\pi}{\epsilon} + finite\right) \qquad (a6)$$

$$= \frac{-4i\pi^2}{\epsilon}\frac{p_-}{p_+^3}$$

9 References

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10 Figure Captions

Figure 1 Feynman rules, momentum flows in arrow's direction.

Figure 2 One loop graphs contributing to the two point function.

Figure 3 The infinite one loop graphs and their corresponding counterterms.

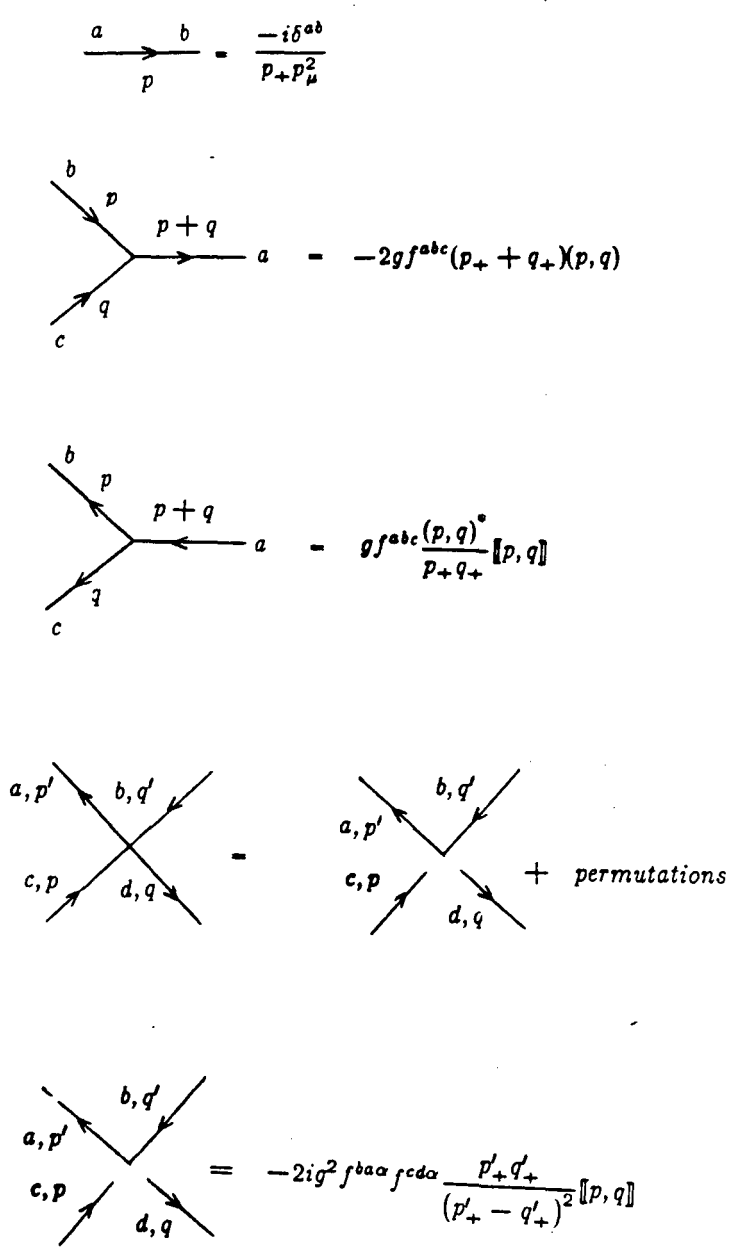

a
b
p
$= \frac{-i\delta^{ab}}{p_+ p_\mu^2}$
b
p
p + q
a
q
c
$= -2gf^{abc}(p_+ + q_+)(p, q)$
b
p
p + q
a
q
c
$= gf^{abc}\frac{(p, q)^*}{p_+ q_+}[\![p, q]\!]$
a, p'
b, q'
c, p
d, q
=
b, q'
a, p'
c, p
d, q
+ permutations
b, q'
a, p'
c, p
d, q
$= -2ig^2 f^{baa} f^{cda} \frac{p'_+ q'_+}{(p'_+ - q'_+)^2}[\![p, q]\!]$

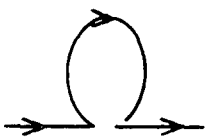
Figure 1

A

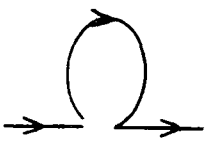
$$p_+^3 \int \frac{d^4q\,(p,q)^*(p,q)}{(p_+ - q_+)^2 q_+^2 (p_\mu - q_\mu)^2 q_\mu^2} = infinite$$

B

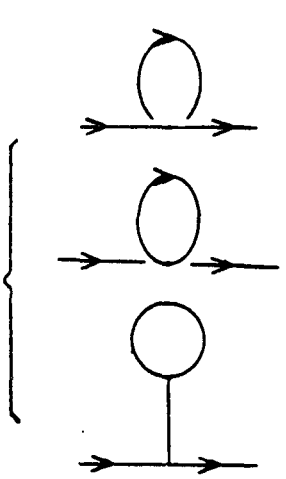
$$p_+ \int \frac{d^4q\,(p,q)^*(p,q)}{q_+^2 q_\mu^2 (q_\mu + p_\mu)^2} = infinite$$

C 

$$p_+^3 \int \frac{d^4q}{(p_+ - q_+)^2 q_\mu^2} = finite$$

D 

$$p_+^3 \int \frac{d^4q}{(p_+ - q_+)^2 q_\mu^2} = finite$$

E 

$$= 0$$

from θ integration or structure constants

figure 2

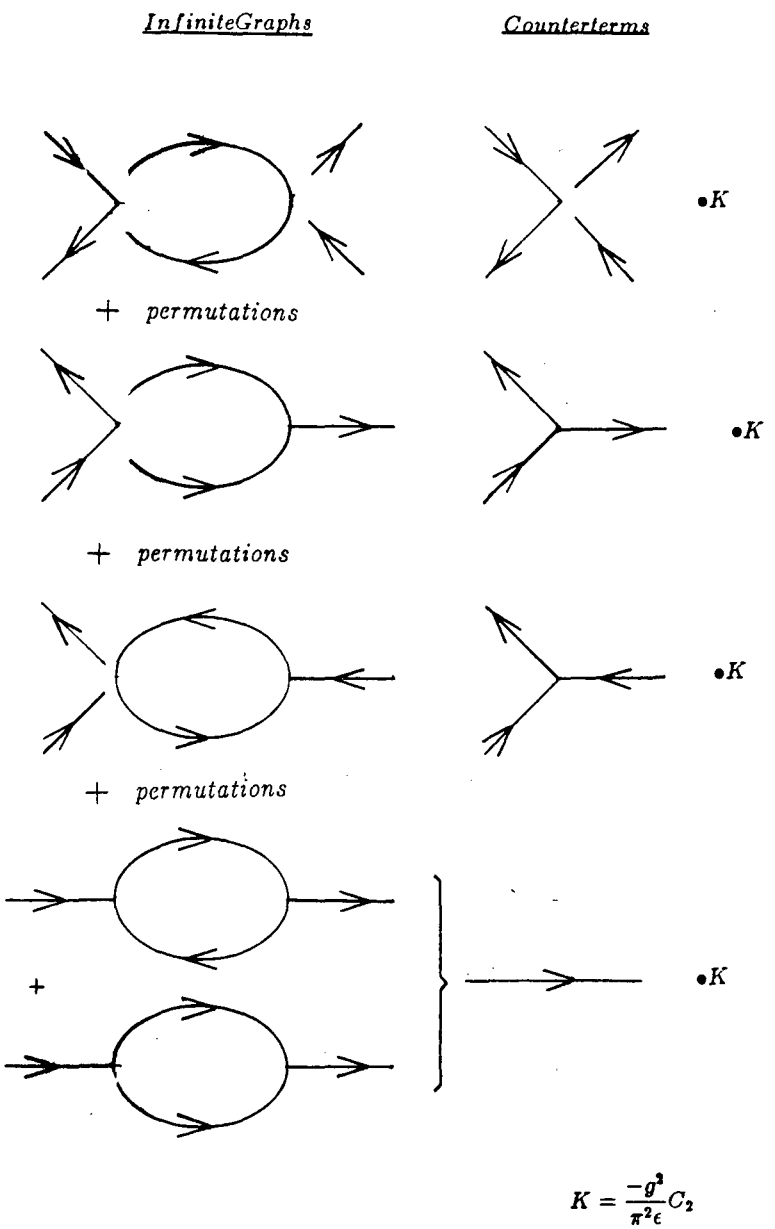

Infinite Graphs
Counterterms
•K
+ permutations
•K
+ permutations
•K
+ permutations
+
•K
$K = \frac{-g^2}{\pi^2 \epsilon} C_2$

Figure 3

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