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Nuclear Compression Effects on Pion Production in Nuclear Collisions

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Abstract:

We show that the method of analyzing the pion excitation function proposed by Stock *et al* may determine only a part of the nuclear matter equation of state. With the addition of missing kinetic energy terms the implied high density nuclear equation of state would be much stiffer than expected from conventional theory. A stiff equation of state would also follow if shock dynamics with early chemical freeze out were valid.

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One of the prime objectives of high energy heavy ion research is to probe the properties of nuclear matter at high temperatures and densities. Recent data on collective flow phenomena [1] and the systematics of pion production [2] are particularly interesting and may ultimately provide useful constraints on the nuclear matter equation of state. In this note we concentrate on the implications of the pion data. First we show that the difference between the observed and calculated pion multiplicities[3] provides a constraint on only a part of the nuclear matter energy functional, $W(\rho, T)$ = energy per baryon at density ρ and temperature T. We calculate the kinetic energy terms which must be added to compare with theoretical models for W. Second, we calculate the pion multiplicity assuming that the pion abundance is frozen out at the point when the nuclear density reaches the Rankine-Hugoniot shock condition, as suggested in Ref.[4]. We find with both approches that the equation of state would have to be extremely stiff at high densities if the assumptions were valid.

The observed [2] negative pion multiplicity, $N_{\pi^-}^{obs}(E)$, produced in central nuclear collisions increases approximately linearly with incident kinetic energy E. While cascade calculations that neglect mean field effects [3] - [6] also lead to a linear dependence of $N_{\pi^-}^{cas}(E)$ on E, the calculated values systematically over estimate the observed multiplicity. In fact most models [7] - [9] except hydrodynamics [10] over estimate the pion yield. Stock *et al* [3] suggested that this discrepancy could be exploited to extract information about the nuclear matter equation of state at high densities and temperatures. They noted that if thermal and chemical equilibrium is reached in such collisions, then the number of pions should depend only on the available thermal kinetic energy per nucleon, $K(E_{cm})$. For a given center of mass kinetic energy per nucleon, E_{cm}, $K(E_{cm}) = E_{cm} - U(E_{cm})$, where $U(E_{cm})$ is potential energy per nucleon at the point where the pion abundance freezes out. That potential energy, on the other hand, clearly depends on the baryon density, $\rho(E_{cm})$, at the freeze out point. Since one expects ρ to be an increasing function of energy, the energy dependence of the pion multiplicity should then be sensitive to the form of $U(\rho)$. With momentum dependent forces U could depend on temperature as well as density, but we assume as in Refs.[3][4] that it depends mainly on density for temperatures of interest.

To extract $U(\rho)$ from the data Stock *et al*[3] noted that the cascade calculation explicitly neglects potential energy. Therefore, N_π^{cas} is the pion multiplicity under the condition that $K = E_{cm}$. Hence, they proposed to extract the compression energy by solving

$$N_\pi^{cas}(E_{cm} - \Delta E(E_{cm})) = N_\pi^{obs}(E_{cm}) \ , \tag{1}$$

for $\Delta E(E_{cm})$ and estimating the compression $\rho(E_{cm})$ from the cascade calculation. They then found a remarkable similarity between the curve $W_S(\rho) \equiv \Delta E(\rho) - 10\text{MeV}$ and the energy functional, $W(\rho, T = 0)$, for compressibility modulus K^∞=250 MeV. ($W(\rho, T = 0)$ is also referred to as the zero temperature equation of state of symmetric nuclear matter.) They were therefore tempted to identify $W_S(\rho)$ with $W(\rho, 0)$. However, W_S does not include the Fermi degeneracy kinetic energy $K_N(\rho, T = 0)$, and thus cannot correspond to $W(\rho, 0)$ directly. Furthermore, the cascade models employed treat Fermi motion in way that simply adds on the average the ground state kinetic energy per nucleon, $K_N(\rho_0, 0) \approx 21$

MeV, to the center of mass kinetic energy per nucleon, E_{cm}. Therefore, ΔE is neither the potential energy per nucleon, U, nor the equation of state, $W(\rho, 0)$. The correct connection is

$$W(\rho, T) = K(\rho, T) + U(\rho) \ , \tag{2}$$

where

$$U(\rho) = \Delta E(\rho) - 21\text{MeV} \ , \tag{3}$$

and

$$K(\rho, T) = \sum_{\alpha} K_{\alpha}(\rho, T) \tag{4}$$

where K_α is the kinetic energy per baryon for $\alpha = N, \Delta_{33}, \pi, \ldots$. In the limit $T = 0$, K reduces to

$$K(\rho, 0) = K_N(\rho, 0) \approx (21\text{MeV})(\rho/\rho_0)^{\frac{2}{3}} \tag{5}$$

where $\rho_0 \approx 0.145\text{fm}^{-3}$ is the saturation density. Note that the conventional [10] compression energy functional, $W_c(\rho)$, is given by $W_c(\rho) = K_N(\rho, 0) + U(\rho)$. The popular quadratic or linear forms of the equation of state refer to W_c rather than to U or W_S. Therefore, in order to compare the empirical[3][4] energy function $W_S(\rho) = \Delta E(\rho) - 10\text{MeV}$ to theoretical nuclear models for $W(\rho, 0)$ we must use

$$\begin{aligned} W(\rho, 0) &= W_S(\rho) + (K_N(\rho, 0) - K_N(\rho_0, 0) + 10\,\text{MeV}) \\ &= \Delta E(\rho) + 21\,\text{MeV}((\rho/\rho_0)^{2/3} - 1) \ . \end{aligned} \tag{6}$$

From Refs.[3][4] the empirical W_S can be parametrized for $2.5\rho_0 \lesssim \rho \lesssim 4\rho_0$ as

$$W_S(\rho) = 80(\rho/\rho_0) - 200 \pm 20\,\text{MeV} \ . \tag{7}$$

In Fig.1 the empirical equation of state, given Eq.(6,7), is shown by the shaded region. For comparison we show a modern "state of the art" nuclear equation state labeled FP from Ref.[11]. Note how much smaller are the compression energies of the theoretical curve. Of course, much stiffer equations of state can be parameterized using nonlinear sigma models[12]. Curve B shows an example from Ref.[12] of one set of parameters that leads to a very stiff equation of state at high densities while reproducing the conventional results at densities below $\sim 1.5\rho_0$. Known nuclear properties are therefore not inconsistent with the empirical equation of state. Nevertheless, the empirical equation of state is much stiffer at high densities than expected from orthodox nuclear theory.

The reliability of that empirical equation of state depends on the assumption made about the freeze out density, $\rho(E_{cm})$, at which the pion to baryon ratio ceases to change. However, as clearly shown in Ref.[13] the pion fraction not only depends on that freeze out density but also on the dynamical path leading toward that density. In thermal models [7] [8] the evolution proceeds isoergically. The pion multiplicity therefore grows as the volume increases until freeze out. On

the other hand, in hydrodynamical models [10] expansion proceeds isentropically and thermal energy is converted into collective flow kinetic energy. Therefore the pion multiplicity decreases until freeze out is reached. In the past [7] [13] it was argued that freeze out should occur below normal nuclear density. This explains why thermal models tend to over predict and hydrodynamical models tend to under predict the pion multiplicity. However, a surprizing result obtained from the cascade study of Ref.[4] was that the pion abundance froze out very early in the collision. In fact, within 20% accuracy the pion plus delta multiplicity was fixed at the time when the compression reached its highest value $\rho_{max}(E_{cm})$. The value of ρ_{max} in turn depends on dynamical details of the compression phase unless the Rankine-Hugoniot shock conditions are attained. We show below that the cascade calculations support shock formation, but only if finite range effects are taken into account. If the following conditions are satisfied

1. shock densities are attained
2. pion chemical equilibrium is reached in the shock zone
3. the pion abundance freezes out before the shocked matter expands appreciably

then the pion to baryon ratio is in fact fixed by kinematics and the sought after equation of state.

In Ref.[4] the above assumptions were used to derive the equation of state independent of the cascade model used with Eq(1). They found that the empirical equation of state and the above assumptions also led to a satisfactory fit of the pion data. Because of the importance of this independent determination of the equation of state we recalculated the Rankine- Hugoniot conditions using Eq.(2) and the following ansatz for $U(\rho)$:

$$U(\rho) = -B - K_N(\rho, 0) + \frac{K^\infty}{18\rho_0^2}(\rho - \rho_0)^2 \ , \tag{8}$$

where $B \approx 8MeV$ is the binding energy and K^∞ is the compressibility modulus. Note that the zero temperature energy $K_N(\rho, 0)$ must be removed from the quadratic term in order for nuclear matter to saturate at ρ_0. With eqs.(2,8) the pressure is given by

$$p(\rho, T) = \rho^2 \frac{\partial U(\rho)}{\partial \rho} + \sum_a p_a(\rho, T) \tag{9}$$

where p_a is the usual thermal pressure which we evaluate numerically using relativistic kinematics and Bose or Fermi distributions. We note that in Ref.[4] $\partial U/\partial \rho$ in Eq.(9) was incorrectly replaced by $\partial W_c/\partial \rho$. This explains why they could fit the pion data with an apparently softer equation of state than we find below.

The Rankine-Hugoniot equation [14] relate the shock compression to the energy density, $\epsilon = \rho W(\rho, T)$, and pressure, p, in the shock as

$$\left(\frac{\rho}{\rho_0}\right)^2 = \frac{(\epsilon + p_0)(\epsilon + p)}{(\epsilon_0 + p)(\epsilon_0 + p_0)} \ , \tag{10}$$

where for a given $E_{cm} = (\gamma_{cm} - 1)m_N$ the energy density and pressure are further constrained by

$$\gamma_{cm}^2 = \frac{(\epsilon + p_0)(\epsilon_0 + p)}{(\epsilon + p)(\epsilon_0 + p_0)} \ . \tag{11}$$

Given the conditions, ϵ_0, p_0, of the incoming nuclear matter and its fluid velocity, eqs.(10,11) determine the temperature, T, and chemical potential, μ, in the shock. The equation of state enters through the dependence of ϵ and p on T and μ.

Once T and μ are known the pion to baryon ratio in chemical equilibrium is given by

$$\frac{N_\pi}{A} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3}\left\{\frac{3}{e^{\omega_\pi/T} - 1} + \frac{16}{e^{(\omega_\Delta - \mu)/T} + 1}\right\} \ . \tag{12}$$

In (12) $\omega_\alpha = (p^2 + m_\alpha^2)^{\frac{1}{2}}$ for $\alpha = \pi, \Delta$ is the single particle spectrum of pions and Δ_{33} resonances respectively.

The dependence of ρ, T, and N_π/A on cm kinetic energy are shown in Figs.(2,3). First consider the curves labeled 1 and 2 corresponding to the case with $U = 0$. Curve 1 corresponds to the pure ideal gas case for a mixture of nucleons, pions and Δ_{33} resonances with $W(\rho, T) = K(\rho, T)$. Indicated by the + symbols are the results of cascade calculations [4] [5] for Ar+Ca. We note the large difference between the shock densities in the ideal gas case and the cascade results. This difference is due to finite range of hadronic forces as implemented by the cascade prescription of scattering two particles at the distance of closest approach. Indeed as past studies [15] have shown the density reached is sensitive to the details of the scattering prescription. Physically the finite range induces an effective excluded volume effect that enhances the pressure in the system while leaving the energy functional invariant. For a hard sphere gas the enhancement factor to lowest order is $(1 + b\rho)$, where $b = 2\pi d^3/3$ in terms of the hard sphere diameter d. In curve 2 we have implemented this Enskog correction [16] by enhancing the thermal part of the pressure (9) as

$$p(\rho, T) = p_c(\rho) + (p_T(\rho, T) - p_N(\rho, 0))(1 + b\rho) \ , \tag{13}$$

where $p_T = \sum_\alpha p_\alpha(\rho, T)$. Note that the degeneracy contribution $p_N(\rho, 0)$ included in p_c is not enhanced in (13). We chose $d = 1$fm for illustration for curve 2. We observe that the Enskog corrected nucleon, pion, Δ_{33} gas reproduces the cascade results much better than curve 1. Therefore, the cascade results can be interpreted as supporting the contention [4] that shock densities are reached in finite nuclear systems only when the nonideal nature of the cascade equation of state is taken into account. However, we note that the correction to the ideal gas pressure in (13) may only be a crude approximation to the cascade situation. It would be useful to carry out cascade studies that map out the particular equation of state corresponding to the scattering prescriptions adopted.

While the compression in Fig.(2a) is sensitive to both the Enskog correction and the nuclear potential, the temperature in the shock is sensitive only to the potential. In general, the temperature is smaller with increasing value of K^∞ in eq.(8). However, T is sensitive to the functional form of $U(\rho)$ as well. For curve 5

we modified the term proportional to K^∞ by replacing $(\rho-\rho_0)^2/\rho_0^2$ by a "linearized" form $(\rho-\rho_0)^2/(\rho\rho_0)$. Note that for higher energies the temperatures in curves 3 and 5 are nearly the same although K^∞=250 MeV for curve 3 and K^∞=800 MeV for curve 5. This is because the two equations of state have similar compression energies at high densities. Furthermore, since the pion to baryon ratio is mostly sensitive to T and not ρ , we find in Fig.(3) that curves 3 and 5 are also similar there. These results show that the pion to baryon ratio is not sensitive to the compressibility modulus, K^∞, at saturation density. Only the compression energy at densities $\gtrsim 2\rho_0$ matter in this connection. Neither equation of state 3 or 5 have large enough compression energy to fit the data. In Fig.(1) curve 3 indicates the range of densities and compression energies that follow from the Rankine- Hugoniot equation for center of mass kinetic energies between 100 and 400 MeV per nucleon for the quadratic equation of state with $K^\infty = 250$ MeV.

On the other hand, a good fit to the pion data is obtained using a pathological K^∞=800 MeV quadratic equation of state labeled 4 in Fig.(1). This equation of state has 10 times more compression energy at $3\rho_0$ than the conventional one and is even stiffer than the empirical one. We therefore see that the equation of state deduced assuming shock dynamics is not necessarily consistent with the one deduced assuming Eq.(1). On the other hand, the empirical equation of state can also reproduce the pion data as shown by curve S. For curve S we have fit the empirical potential with K^∞=250 MeV and have set $K_N = 0$ in Eq.(8) as in Ref[4]. This equation of state lies 40 to 50 MeV above curve 3 in Fig.1. Obviously, the pion excitation function only determines at best a lower bound on the compression energy at high densities. With this approach, however, the pion data cannot be fit with an equation of state lying below the shaded region in Fig. 1.

Given the unusual form of the deduced equation of state it is necessary to analyze critically the assumptions made. Perhaps the most questionable assumptions is that the pions drop out of chemical equilibrium before the shocked matter has a chance to expand. That shock densities are reached is plausible from our analysis. On the other hand, the cascade calculations[4] indicate freeze out at shock densities only to 20% accuracy. Note that the discrepancy between curve 2 and the cascade points in Fig.(3) is of that same order. If freeze out at shock densities were exact, then the cascade points would lie on top of curve 2. Our results suggest that idealized shock calculations may systematically overpredict the pion abundance. Allowing for some isentropic expansion would obviously lead to a softer equation of state. In fact, curve 3 could be consistent with the data within the uncertainties of the freeze out process. More detailed cascade studies on the A and E dependence of the time evolution of the pion abundance are necessary to clarify that process.

At the same time it is also important to continue to look for possible systematic errors in the present cascade codes that could contribute to the apparent stiffness of the equation of state via Eq.(1). In Ref.[6] for example a number of technical details associated with binding prescriptions, Pauli blocking, and Fermi motion were shown to lead variations of the pion abundance at the level of 20%. Further insights may come from cascade calculations that include mean field effects. Recently, there has been progress [9] [18] toward constructing such models. In Ref.[18] the pion yield was reproduced assuming only moderately stiff equations of state. However, the

complex interplay between dynamical details and the equation of state have not been sorted out. Much work remains before convincing constraints on that elusive equation of state can be extracted from pion and collective flow data.

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Figure captions:

1. The energy per baryon at zero temperature as a function of density. Curve FP corresponds to a modern variational calculation[11]. Curve B is typical

of non-linear mean field models and is taken from Ref.[12] with $K^{\infty} = 240$ MeV and $m^*/m = .65$. The shaded region is the empirical equation of state[3][4] with corrections as given by Eq.(6). Curves 3 and 4 correspond to equations of state used in shock calculations for cm energies between 0.1 and 0.4 GeV per nucleon.

2. The baryon density (A) and the temperature (B) as a function of center of mass kinetic energy per nucleon. Curves labeled 1 and 2 corrspond to the case $W(\rho, T) = K(\rho, T)$ including pions, nucleons, and Δ_{33} with Enskog diameter $d = 0, 1$ fm resp., *cf.* (13). Curves 3 and 4 include the potential (8) with K^{∞} =250,800 MeV resp.. Curve 5 corresponds to the "linearized" potential with K^{∞}=800 MeV. The + symbols correspond to maximum densities achieved at zero impact parameter in cascade calculations[4] for Ar+Ca.

3. The pion to baryon ratio as a function of E_{cm}. Curves labeled as in Figs.(1,2). Solid dots are data points extrapolated to zero impact parameter[4] for Ar+Ca. Curve S corresponds to a fit to the empirical[4] equation of state where in Eq.(8) K_N set to zero and K^{∞}=250 MeV. In Fig.(1) this equation of state lies 40 to 50 MeV above curve 3.

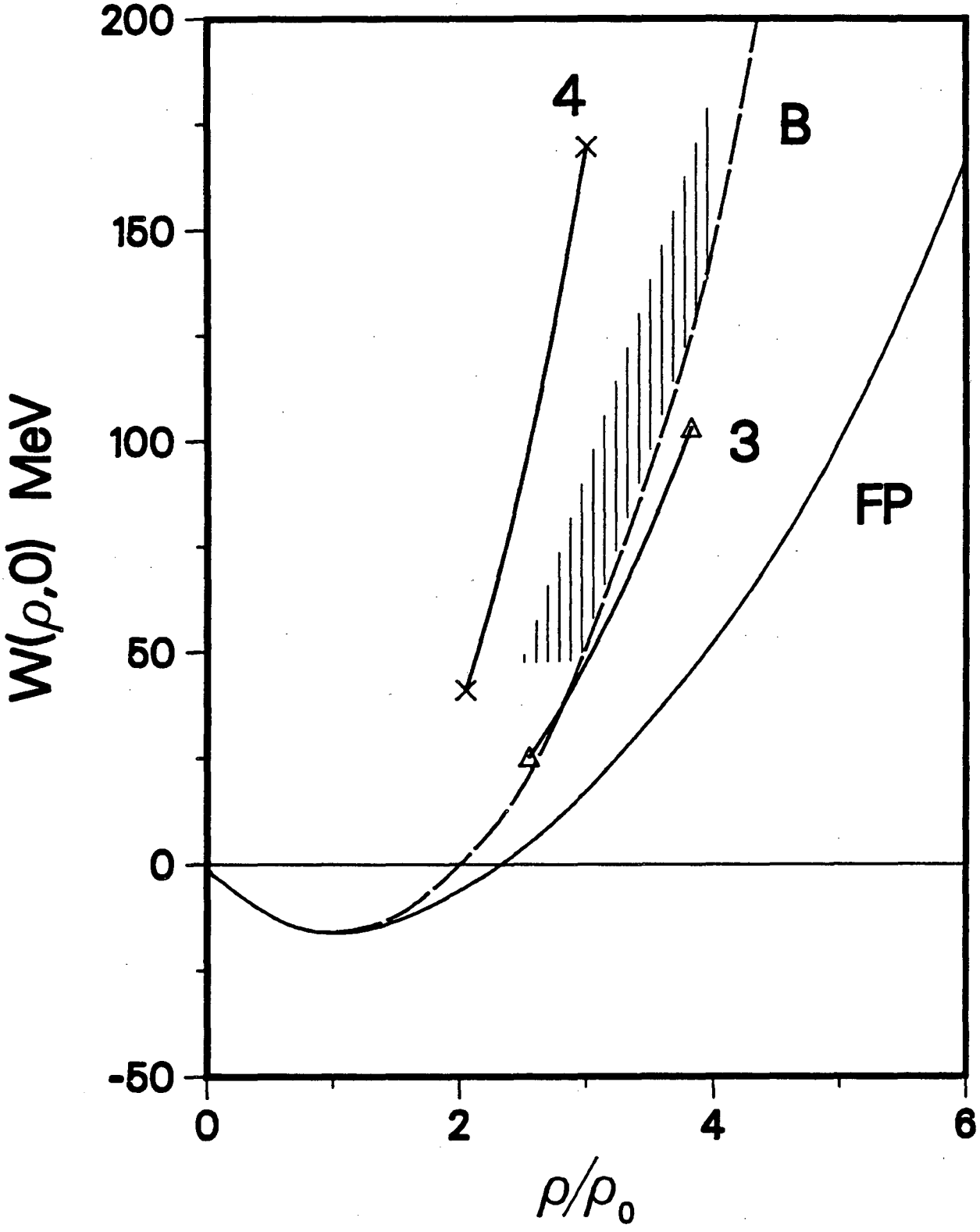

Fig. 1

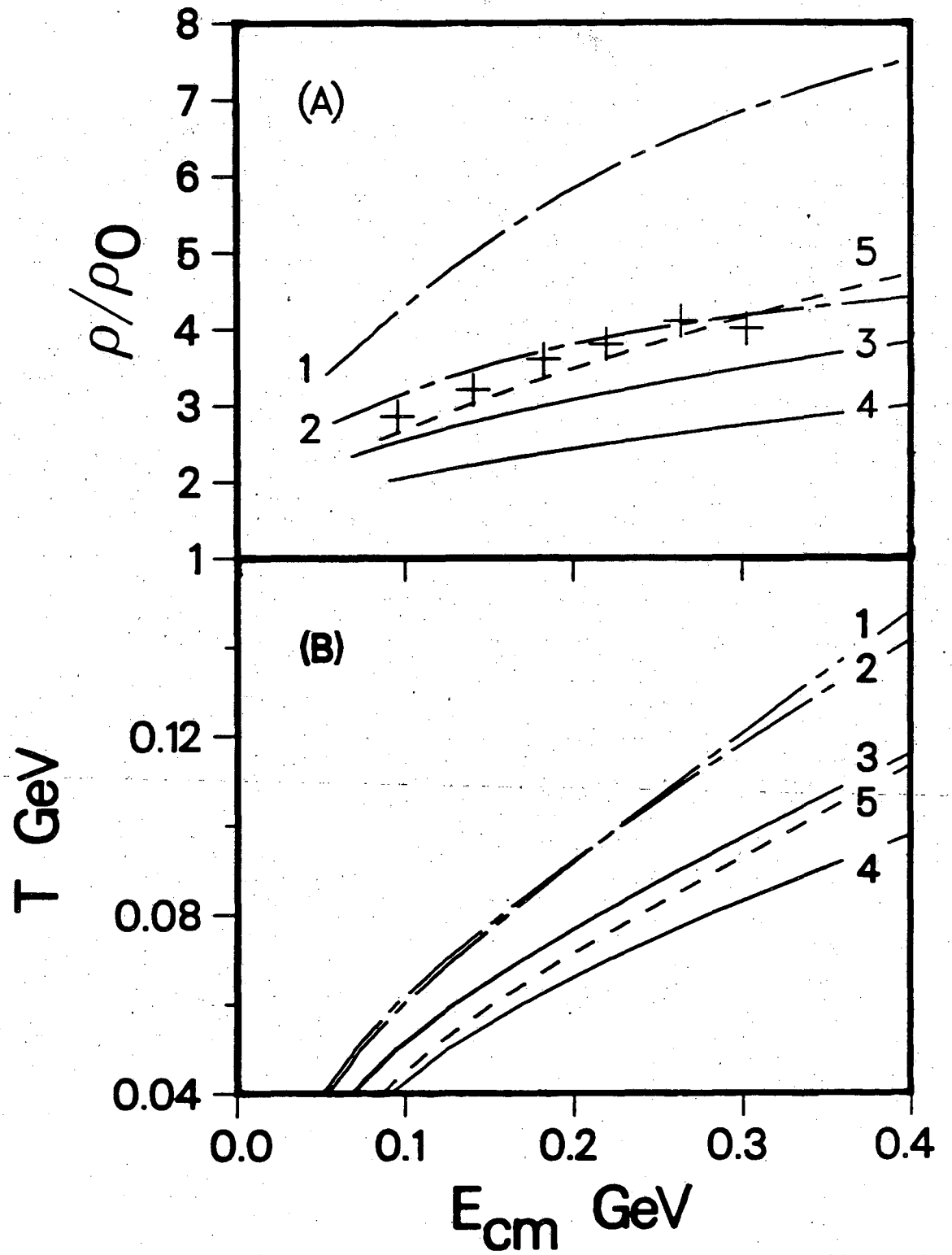

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Fig. 2

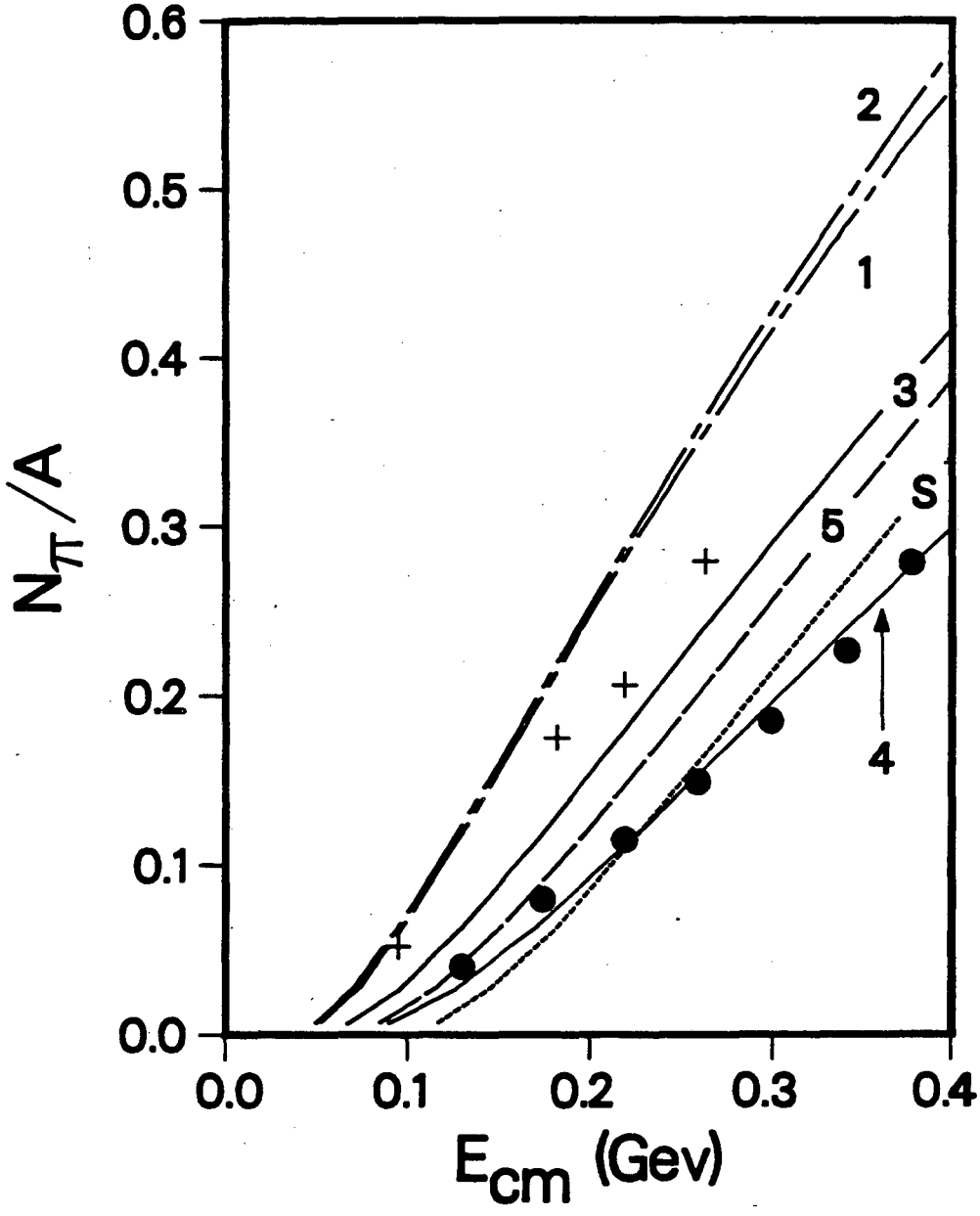

Fig. 3

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