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I. QUANTUM NUMBERS AND TOPOLOGICAL
STRUCTURE OF VERTICES

D. Issler

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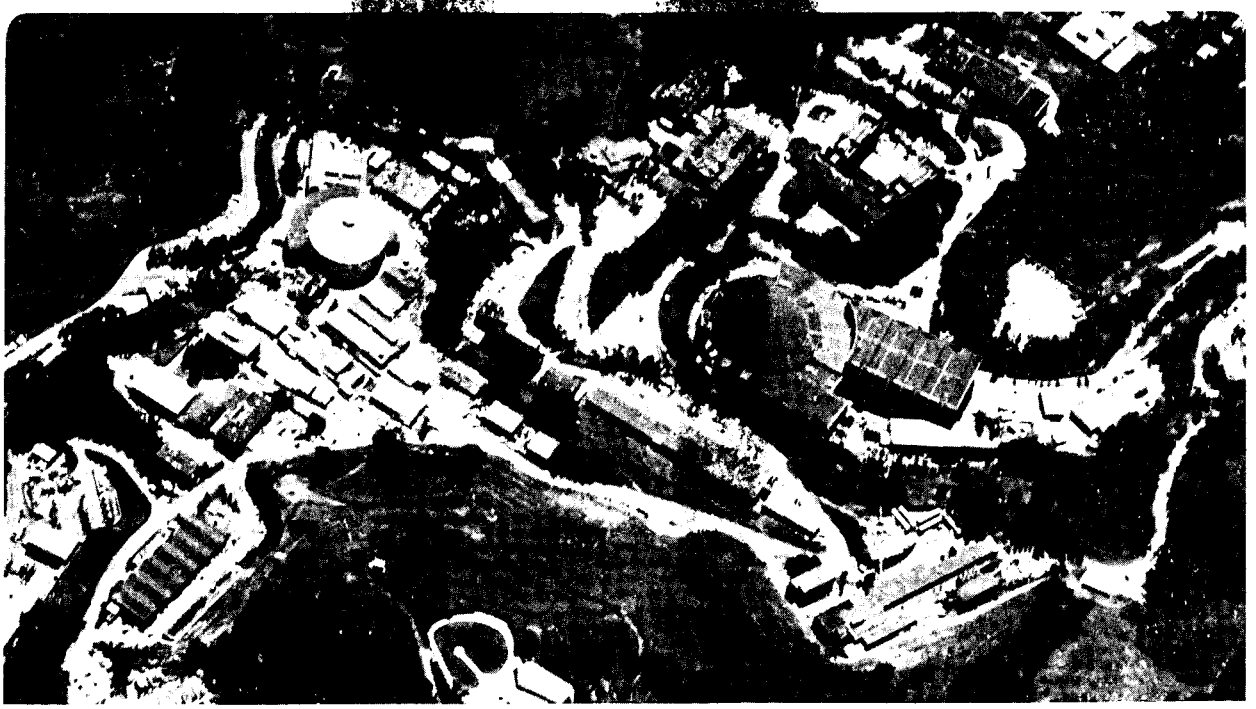

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TOPOLOGICAL THEORY OF LEPTONS AND SCALAR BOSONS

(I) Quantum Numbers and Topological Structure of Vertices

Dieter Issler* †
Lawrence Berkeley Laboratory
and
Department of Physics
University of California
Berkeley, California 94720

ABSTRACT

Consistency with the hadronic sector of topological particle theory (TPT) limits the nonhadron spectrum to eight chiral gauge bosons, four generations of isodoublet Dirac leptons (e, μ, τ, λ plus neutrinos) and eight scalar neutral bosons $H_{e\mu}$, $H_{e\tau}, H_{\mu\lambda}, H_{\tau\lambda}$ plus their conjugates. These bosons $H_{GG'}$ change lepton generation and carry $+1$ (-1) unit of a conserved lepton generation number L_G $(L_{G'})$. In TPT, particles are represented by boundary segments of (abstract) orientable two-dimensional surfaces whose internal structure determines the details of the interactions. The surfaces corresponding to chiral Yang-Mills couplings are constructed and supplemented by vertices involving leptons and H bosons; mutual consistency determines them almost uniquely. Feynman rules are developed and it is seen that lepton – H-boson couplings break the chiral gauge symmetry $U(2)_R \times U(2)_L$, leaving $U(1)_{em}$ intact. Following the patterns of purely nonhadronic interactions, proposals are made for direct couplings between elementary hadrons and leptons or H bosons. They are found to break the symmetry between lepton generations. The dynamical aspects of this theory will be discussed in a supplementary paper.

*Address after January 1, 1986: Institut f. Theor. Physik, Universität Bern, Sidlerstrasse 5, CH-3012 Bern, Switzerland

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1 Introduction: The Bootstrap Approach to Electroweak Interactions

The continuing verification of its low-energy predictions has established the Glashow-Salam-Weinberg (GSW) model of electroweak interactions [1] as the standard by which any new model must be judged. Independently of further development of the experimental situation, however, theory is forced to go beyond the GSW model because it contains too many parameters which must be taken from experiment. In particular, the Higgs mechanism for spontaneous symmetry breaking should rather be viewed as effective description of an unknown underlying dynamics operating presumably at TeV energies. The observed generation patterns of leptons and quarks is another enigma in the framework of the GSW model for which ultimately a dynamical explanation has to be found.

Due to the success of the GSW model, almost all attempts at answering those open questions were based on local quantum field theory (QFT) and gauge symmetries. It is interesting to observe, though, how arduous efforts to incorporate into particle physics a consistent quantum theory of gravitation led to the current interest in superstring theories. Two aspects are particularly relevant: String theories describe extended objects in space-time, which have Regge behaviour built in from the beginning, and the consistency requirements of anomaly cancellation and unitarization (through loop diagrams) almost uniquely determine the internal symmetry group and fix the dimensionality of space-time (before compactification).

Finite size of elementary excitations and self-consistency are also central in topological particle theory (TPT) of hadrons which – like the string theories – grew out of dual models of strong interactions [2] and their graphical representation through Harari-Rosner (HR) quark-line diagrams [3]. The topological complexity of the two-

dimensional surfaces spanned by the HR quark lines (corresponding to the world sheet of the string) furnishes a small parameter in the sense of a $1/N$ expansion [4,5]; a similar *topological expansion* can be defined for string models.

Profound differences exist, however, between the two approaches with respect to their fundamental dynamical assumptions: Strings are extended objects even in the absence of interactions, and one may write string theories as conformally invariant two-dimensional local QFT on the string's world sheet. TPT, on the other hand, does not presuppose continuous space-time and the picture of a moving string does not apply. Instead a contraction principle is postulated which allows closed quark lines to be shrunk away under specified circumstances [6]. In conjunction with the unitarity equation, contractions create an infinite number of coupled nonlinear discontinuity equations which constitute a bootstrap system for elementary (non-polynomial) vertex functions [5,6]. Finite size and Regge behaviour (with curved trajectories) are seen as a consequence of the nonlinear and circular nature of bootstrap dynamics. Although neither the existence nor the uniqueness of a solution to these equations have been shown so far, different approximations [7,8] yielded encouraging and compatible results.

The contraction principle not only controls the analytic structure of TPT vertex functions but also limits embellishments of the HR diagrams with additional lines [6]. TPT exploits this circumstance for a self-consistent determination of discrete hadron quantum numbers by associating the latter with the orientations of lines, patches and patch boundaries [6,9,10] (a short description is given in Sect.2). These considerations are thus completely different from the ones which determine internal symmetry groups in string theories.

Over the past years many consistency requirements were recognized, analyzed and incorporated in TPT, leading to a coherent theory of hadrons. When its phe-

nomenological consequences were investigated, the theory underwent a shift of energy scale similar to that of string models when they were applied to gravity rather than to the strong interactions; in the case of TPT, the fundamental scale was inferred to be in the TeV range [11]. Leading components of the topological expansion create very small, parton-like bound states, some of which have masses in the GeV range. Higher-order components can then be treated as residual interactions among partons if the energies involved remain below the fundamental TeV scale.

In its original form the bootstrap idea arose from experience with strong interactions, resting heavily on analyticity, and had little to say about (discrete) quantum numbers. When incorporation of topological ideas widened the scope of the bootstrap approach, electroweak interactions emerged as a valuable testing ground for the new concept. (Through Stapp's work on the infrared problem associated with the massless photon [12], the difficulties it poses in a framework based on the analytic S matrix also began to appear in a more positive light, see below.) But by which means may one hope to bootstrap the electroweak particles?[1]

As a first step the topological building blocks of hadrons were seen also to accommodate the presently known gauge bosons and leptons as well as a comparable number of not yet observed states [13–16], namely four gauge bosons ($W_R^{\pm}$, $Z^{0\prime}$ and $Z^{0\prime\prime}$), a fourth lepton generation and a set of eight generation-changing scalar bosons. Section 3 adapts those early proposals to the recent changes in the underlying structure of TPT [9].

At the next stage, topological and/or dynamical principles have to be found which are capable of determining the interactions of nonhadrons with themselves (Sects.4 and 5) and with hadrons (Appendix A). The observed strength of electroweak interactions is not compatible with the strong-interaction mechanisms found

[1] TPT does not presently attempt to incorporate a quantum theory of gravity.

in low-complexity diagrams, nor is the masslessness of the photon. Electroweak vertex topologies will thus have to possess a higher degree of complexity and to guarantee absolute conservation of the electromagnetic current. The absence of duality dissolves certain topological consistency requirements operative at the strong-interaction level, but it is likely that new considerations limiting the complexity of electroweak surface structures will be found to eliminate the remaining ambiguities in the present proposals.

Elementary nonhadrons in TPT are massless, and the theory does not contain elementary Higgs bosons to spontaneously break symmetries and give masses. Also, electroweak interactions are presumably too weak to form massless scalar bound states which could dynamically accomplish that. The capacity of elementary baryons and hexons to break electroweak symmetries was recognized in earlier work by Chew, Finkelstein and Poénaru [17,18]. In particular, a preliminary analysis of the gauge-boson spectrum [19] showed that its lower half is almost identical to that of the GSW model while the other half will acquire TeV masses. Appendix A considers the topological aspects of extending those ideas to leptons and electroweak scalar bosons, based on which [II] and [20] discuss the spectrum of leptons and scalar bosons semi-quantitatively. The latter become very massive (on the order of the fundamental scale m_0). The huge mass ratios of leptons are reproduced as a consequence of weak chiral-symmetry breaking and absolute conservation of all lepton-generation numbers; the fourth-generation charged lepton λ is expected somewhere between m_{W_L} and m_0, maybe detectable at the SSC. TPT thus appears capable of reproducing the low-energy successes of the GSW model and also makes predictions which may be tested in the foreseeable future through high-precision measurements at low energies (μ and p decay, e.g.) and at ultra-high energies (λ lepton).

The major conceptual problem which has to be left open here concerns the principle that determines the values of the (unrenormalized) electroweak coupling constants. (All nonhadron masses may be rigourously calculated in terms of electroweak coupling constants as soon as adequate nonperturbative techniques have been developed for the strong interactions.) The order-of-magnitude equality of the fundamental strong-interaction coupling constant[2] g_0 with the electric charge hints towards an intimate connection between the two [21]. (In [II], a model based on assumptions different from the present ones will be briefly discussed to illustrate one possible approach to this fundamental problem.)

The answer to that question is most certainly related to the reason for the existence of nonhadrons. The results of [II] very strongly suggest that a consistent topological theory of nonhadrons is not possible without hadrons, but conversely there is no obvious inconsistency in the hadronic sector that would require the existence of electroweak particles.

Chew has repeatedly called attention to the fact that a fully consistent bootstrap theory has to incorporate a theory of measurement [22]. Thereby meaning must be given to *macroscopic* space-time and classical objects. It was conjectured that exact masslessness and the vector nature of the photon are essential for this purpose, the latter allowing macroscopic objects to be electrically neutral. The former property entails arbitrarily soft interactions of photons with charged particles whose identities are not changed thereby – a hallmark of ideal classical measurement. Another consequence is the so-called infrared catastrophe which has been shown by Stapp [12] to amount to the classical electromagnetic field associated with the quantum process under consideration and so to provide an essential link between micro- and

[2]The weight of leading orders in the topological expansion is very substantially enhanced by the high multiplicities of closed quark and diquark loops.

macro-world. No equally plausible reason has been found why electrons are needed in the theory, but it is conceivable that measurement apparatus depends on the large-scale structures (atoms, molecules, solids) which become possible with light charged particles that do not interact strongly.

2 Basic Notions of TPT

This section summarizes the central notions and rules of TPT for the reader's convenience, as they will be extensively used in this paper. Full accounts of the many consistency considerations that determine these rules are given in Ref.[10] and also in Refs.[6,9,23].

2.1 The Topological Expansion

The fundamental hadronic vertex functions of TPT are represented by single-vertex Feynman graphs F which are then embedded in a two-dimensional bounded surface Σ (classical surface Σ_C in papers before 1985). Σ is globally oriented and at this stage also planar; a cyclic ordering is thereby imposed on the legs leaving the vertex (Fig.1). F divides the boundary $\partial\Sigma$ of Σ into segments which correspond to the quark lines of HR graphs. The global orientation of Σ naturally leads to the identification of mesons with quark-antiquark states.

Multi-vertex surfaces are obtained from simpler surfaces by connected sum $\Sigma_1 \# \Sigma_2 = \Sigma_{1,2}$: Certain Feynman lines of Σ_1 and Σ_2 are linked and corresponding portions of $\partial\Sigma$ adjacent to the new internal Feynman line(s) are identified and erased[3] (Fig.2).The global orientations of the initial surfaces have to match so that the resulting surface is again globally oriented. Before and after the plug each external Feynman line is unambiguously associated with exactly one portion of $\partial\Sigma$ which is characteristic of the particle type and designated by π.

The surfaces associated with tree diagrams are always planar, but more complicated surface topologies can arise with Feynman loops. This complexity may completely be characterized by a set of integer-valued indices g_i in the case of

[3] Chirality switches (see Sect.2.3) provide for an exception.

two-dimensional surfaces. When properly defined, the indices are zero for the disk-like topologies[4] associated with fundamental vertex functions (the so-called "zero-entropy" level) and satisfy the inequalities

$$g_i^{(1,2)} \geq g_i^{(1)} + g_i^{(2)} \tag{1}$$

or at least

$$g_i^{(1,2)} \geq \max\{g_i^{(1)}, g_i^{(2)}\} \tag{2}$$

where $g_i^{(1,2)}$ is the i-th index associated with $\Sigma_1 \# \Sigma_2$. Three such indices are g_1, the genus of the infinitesimally thickened Feynman graph, satisfying the "strong" entropy property (1); g_4, the genus of the full surface Σ minus g_1 (also satisfying (1))[5]; and

$$g_2 := g_1 + b - 1, \tag{3}$$

essentially measuring the number of disconnected boundary components b containing end points e of the Feynman graph (g_2 only satisfies (2)).

The physical significance of an expansion in increasing topological "entropy" rests on the observation that higher-entropy terms have fewer closed quark loops (and hence less numerical weight) than terms with lower entropy [4,5]. The multiplicity of a closed Feynman loop can reach 32·31 in TPT at the fundamental level of the strong interactions; an expansion in "lost quark loops" is expected to converge fairly rapidly while a Feynman-Dyson expansion in the coupling constant would be hopeless.

Only surfaces with $g_1 = g_4 = 0$ can be plugged into other such surfaces without necessarily creating a surface with higher complexity. From this fact results

[4]The surface Σ with the Feynman graph and additional lines – to be discussed below – will often be referred to as a "topology".

[5]$g_4 = 0$ holds for all topologies containing mesons only. Multi-feathered baryon surfaces may have, and electroweak topologies always have, $g_4 > 0$.

the possibility for a bootstrap system of nonlinear dynamical equations which are expected to determine the fundamental single-vertex functions [5,7,8]. The role of the topological expansion in determining the order of summation of electroweak amplitudes and the consequences thereof will be discussed in [II].

2.2 Multi-Feathered Surfaces and Junction Lines

The representation of amplitudes involving baryons requires three-feathered surfaces [24]; the line of contact between the feathers is called a *junction line* (JL). It correlates the global orientations of the feathers as shown in Fig.3. When cut along the JL, this figure decomposes into three independent sheets; such need not be the case in more complicated situations where Σ may be multi-feathered but only one- or two-sheeted – the term "feather" will then be used in connection with the neighbourhood of the JL while the term "sheet" refers to the global structure of Σ.

One of the three feathers is set apart by its carrying the Feynman graph (sheet or feather #1); F is not allowed to cross a JL. Consistent plugging rules require that the remaining two feathers be distinguishable; this is achieved by means of so-called *colour lines* (f_c lines) – introduced by Finkelstein [25] – which are oppositely directed on feathers #2 and #3 (see Fig.3) [9]. f_c lines maintain their orientations but the quark content (flavour, spin, etc.) may be permuted at plugging points [25]. Despite its threeness topological colour is quite different from the colour degree of freedom on which QCD is built; e.g., no $SU(3)_C$ symmetry group is associated with the former, and there are no elementary coloured gauge bosons in TPT.

2.3 Quantum Numbers from Topological Orientations

In order to give a topological meaning to electric charge and isospin, Finkelstein introduced a set of lines which closely parallel the Feynman graph and colour lines in strong-interaction topologies [26]; they are shown in the simple situation of Fig.3. Finkelstein lines associated with JL's are designated by τ_J, all others by τ_φ. Following $\partial\Sigma$ in the sense of the global orientation from a Feynman-line or colour-line end point, the first Finkelstein-line end encountered is given a $-$ label, its other end a $+$ label. If τ is intrinsically oriented from $-$ to $+$ it is said to carry one unit of electric charge ("c"), and no electric charge ("n") when directed from $+$ to $-$. Hadronic τ_J lines are always charged and contribute $-e$ to the total charge of baryons [6].

τ_φ lines induce a further segmentation of $\partial\Sigma$. The following boundary units may be defined (Fig.4):

1. φ units: They touch a Feynman-line end point e or colour-line end point e_c on one side and a τ_φ end point on the other side (including the latter); no JL end point j intervenes. They carry the same $\pm$ label as their τ_φ end point.

2. δ units: Starting at a τ_φ end point without including it, they stretch to the midpoint of a quark line and never contain a j. Their $\pm$ label is opposite to that of the adjacent φ unit.

3. Y units: From a point j three legs spread out and end, respectively, at a Feynman-line end e with accompanying τ_φ end point or at the end points e_c of two colour lines with opposite directions (see also Fig.3). As for φ units the $\pm$ label is inherited from the corresponding τ_φ end point.

Each component of $\partial\Sigma$ can be built from the above defined units whereby $\pm$ labels consistently alternate. It follows from Fig.4 that only the following se-

quences (from left to right in the sense of the global orientation of Σ) may occur: $\varphi^-\varphi^+$, $\varphi^-\delta^+$; $\varphi^+\varphi^-$, φ^+Y^-; $\delta^+\delta^-$; $\delta^-\varphi^+$; Y^+Y^-, $Y^+\varphi^-$. Fig.5 shows the particle portions π of elementary hadrons. (Hexons are baryonium-like states required by duality.)

Feynman, colour and Finkelstein lines also create a patching of Σ. Patches whose patch boundary does not contain φ, Y or δ units are called "white" and are not given any orientations. In all other cases one orients patches and patch boundaries independently. If the orientation agrees with the one induced by the global orientation of Σ, it is called O or U, respectively; otherwise, P or D. Patches with hadronic Y units along their boundaries are invariably oriented U and their boundaries O, for reasons of dynamical consistency at the zero-entropy level.

All strong-interaction surfaces built from zero-entropy topologies by connected sum share a few important properties:

1. Except for "white" patches, all patch boundaries contain exactly one pair ($+$ and $-$) of boundary units; only one kind of unit occurs in a given patch.

2. Only "white" patches and patches with φ units may locate along Feynman lines.

3. There is always a "white" patch or a patch with φ units between two patches with Y or δ units.

Anticipating a future role for Feynman lines in the accurate topological representation of momentum and of Lorentz transformations, patches adjacent to F and generally patches with φ units are called *spinor patches* while all other patches are called *scalar*, their orientations supposed invariant under Lorentz transformations. It is generally true in TPT that the Feynman lines separate spinor patches from

one another whereas all other lines in Σ separate spinor patches from scalar patches [9].

In line with the distinction between spinor and scalar patches is the interpretation of spinor-patch orientations as *spin up (U)* and *spin down (D)*, and of spinor-patch-boundary orientations as *ortho (O)* and *para (P)* chirality. The latter correspond to projection operators $(1 \pm \gamma_5)/2$ acting on Dirac four-spinors. According to Stapp's analysis in terms of M functions [27] the topological orientations U and D can be interpreted as physical spin only in a Lorentz frame in which the two particles sharing the patch in question have collinear momenta. While this circumstance is of little immediate concern to the questions under study in this paper,it will play a central role in all attempts at a topological understanding of the Lorentz invariance of the theory.

In the same paper Stapp showed that the S matrix requirement of Hermitian analyticity requires both smooth, chirality- and spin-preserving plugs *and* chirality-reversing plugs where the spin orientation may or may not change. When chirality reverses, the pair of φ units of $\partial\Sigma$ must not be erased; they form an internal component of $\partial\Sigma$, called a *gauge hole* (Fig.6) because of its similarity to, and connection with, gauge-boson π's[6] (see Sect.3). It can be shown [28] that a chiral switch on a quark line corresponds to an operator $p \cdot \gamma / m_0$ in Dirac space where p_μ is the momentum of the Feynman line along which the switch occurs; m_0 is the common mass of all elementary hadrons.

As mentioned above, the orientations of scalar patches associated with hadronic Y units are frozen at U and O. All four combinations (U, O), (U, P), (D, O) and (D, P) are allowed for scalar patches with δ units. Being invariant under Lorentz

[6]The word *belt* is used to designate $\partial\Sigma$ without internal (gauge hole) components; the belt consists only of particle portions π_i.

transformations, they lead to four internal quantum numbers – quark generations – which are separately conserved in strong interactions.

3 Nonhadronic Boundary Portions

Extension of TPT from strong to electroweak interactions requires nonhadrons to be represented by portions π of $\partial\Sigma$ which must be different from hadronic π's but also consist of φ, δ or Y boundary units; new boundary units would entail arbitrariness. A Feynman-line end point may occur only between φ^+ and φ^-, Y^+ and Y^- or $\varphi^\pm$ and $Y^\mp$. It will be seen below that nonhadronic π's consist of exactly these combinations of boundary units, no addition of $\delta^\pm$ or $(\varphi^\pm\delta^\mp)$ units occurring.

3.1 Gauge Bosons

The vector nature of gauge bosons requires them to consist of a pair of φ units; they can be distinguished from mesons only if no δ units are added. Earlier proposals [26,15,23] did not recognize the difference between φ and δ units and sought to distinguish gauge bosons from mesons by means of an additional closed surface Σ_Q.

Two considerations allow inference that a gauge boson should occupy a closed, disconnected portion of $\partial\Sigma$ (Fig.7): (*i*) The minimal coupling in gauge theories,

$$\gamma \cdot \partial \longrightarrow ie A \cdot \gamma, \tag{4}$$

finds its topological counterpart in a substitution principle replacing gauge holes (Fig.6) by a Feynman line and a gauge-boson π. (*ii*) Open gauge-boson π's prevent satisfactory gauge-boson–hadron couplings (single δ units occur which cannot unambiguously be attributed to any particle). Also it will result from Sect.4 that (chiral) Yang-Mills couplings can be much better represented with closed π's.

It is readily seen that gauge bosons form isoquartets (isotriplet plus isosinglet). It turns out that only the O^-P^+ and P^-O^+ states, corresponding to right-handed and left-handed vectors, are able to interact consistently and hence are admissible.

3.2 Leptons

As mentioned above, leptonic π's have to consist at least of one φ and one Y unit with opposite $\pm$ labels. Assuming that there are no spin-3/2 leptons, options reduce to $Y^\pm\varphi^\mp$ and $Y^\pm(\varphi^\mp\delta^\pm)$. In the latter case the scalar patch associated with the δ unit would yield four lepton generations exactly parallel to the four quark generations. However, it will be seen below that Y units in nonhadrons also carry a four-valued generation degree of freedom. Thus phenomenological considerations as well as the precedent of the gauge bosons, where δ units are also absent, very strongly favour the $Y^\pm\varphi^\mp$ structure shown in Fig.8.

In contrast to hadrons where all Y orientations have to be frozen for the sake of consistency (see Sect.2.3), leptons will never be involved in contractions so that there is no reason to freeze all Y orientations here. An exception is the orientation of τ_J where the absolute conservation of the electric current does not allow "*c*": In order to maintain its masslessness the photon must be able to couple to any charged Finkelstein line wherever there is a gauge hole to which the substitution (4) can be applied. (This rule follows from the graphical proof of the Ward identity in QED by straightforward extension to the present situation.) It will be seen in Sect.5 that gauge bosons are not able to couple consistently to the τ_J in a lepton, and Appendix A will present semihadronic topologies where Finkelstein lines connect a leptonic Y to a hadronic φ. Such topologies will break the Ward identity unless nonhadronic τ_J's are frozen at "*n*".

The orientation of a spinor patch has been associated with spin[7], the corresponding patch-boundary orientation with chirality [9]. These notions are intimately connected with the Poincaré group, and correspondingly spinor patches are

[7] More precisely, this orientation depicts the spin component along the $+z$ direction in a frame where both particles sharing that patch have collinear momenta along z.

located either along a Feynman line or to the right of a colour line. Scalar patches being detached from the "influence" of the Feynman graph, their orientations do not undergo Lorentz transformations (freezing those orientations for hadronic Y's would otherwise break Lorentz invariance!) and cannot be interpreted as spin and chirality [16] but represent internal degrees of freedom, *i.e.*, lepton generations in the present case. In anticipation of the symmetry breaking pattern discussed in the sequel to this paper [II] they are numbered in the following way:

$$G=1:\ (OD)_Y,\ \ G=2:\ (PD)_Y,\ \ G=3:\ (PU)_Y,\ \ G=4:\ (OU)_Y. \qquad (5)$$

Lepton-generation number may be defined as the number of neutral Y^+ minus the number of neutral Y^- with the respective patch and patch-boundary orientations within a particle's π:

$$L_G\ :=\ N(Y^+_{n,G}) - N(Y^-_{n,G}). \qquad (6)$$

It should be remarked at this point that an outgoing electron, say, is represented by a $\varphi^- Y^+$ boundary segment where τ_φ carries one unit of electric charge *away* from the electron. This situation is opposite to the convention for outgoing quarks which are represented by a $\delta^-\varphi^+$ segment, corresponding to a spinor $\overline{u}(p,s)$ in the usual way. Thus, if one wants to adhere to traditional spinor assignments, a charge conjugation transformation must be applied to all nonhadronic φ units (including those on gauge bosons and gauge holes). As discussed in Appendix D of Ref.[9], the C operation turns φ^- into φ^+ and reverses the topological spin orientation without changing chirality. A φ^- unit with O chirality, say, appears in incoming right-handed quarks, outgoing left-handed antiquarks, incoming right-handed antileptons, outgoing left-handed leptons or in both incoming and outgoing left-handed gauge bosons.

In summary, the present proposal entails four generations of isodoublet Dirac

leptons, as did all previous proposals. What has changed in the course of time is the set of orientations that make up lepton generation. The following sections will show that the consistency considerations recognized by now have removed many arbitrary features of the earlier proposals.

3.3 H Bosons

The contraction principle (or planar duality) operative at the low-entropy levels of strong interactions requires the existence [30,6] of hexons as a consequence of the existence of elementary baryons. Subjecting a hexon's π to the same truncation that transforms a baryonic π into a leptonic π results in a Y^-Y^+ belt portion with two JL's and two τ_J lines frozen at n, see Fig.9 (the parallelism between hadronic and nonhadronic belt portions was first stressed in Ref.[15]). No compelling theoretical *raison d'être* for such particles has been found yet, but their important role in symmetry breaking and creation of lepton masses [20 and II] is a strong advocate in their favour.

Fig.9(a) shows the belt of the surface that would correspond to the free propagation of a Y^-Y^+ particle (called H boson for the shape of its π) if one simply used the truncated hexon-π. Boundary, colour lines, τ lines and JL's run parallel; the colour lines appear particularly redundant. Investigation of the detailed structure of the full surfaces for propagation and interactions of H bosons reveals a certain lack of complexity. If, however, the belt portion of a single H is closed on itself, the colour lines acquire a more important role in that they link disconnected boundary components on the same feather (Fig.9(b)). This boundary structure also shows close resemblance to the gauge-boson π of Fig.7 – a similarity that will acquire deeper significance in Sect.4 through the matching complexity (genus) of gauge-boson and

H-boson topologies[8].

Not having φ units in their π, H bosons are scalars (in TPT pseudoscalars can only be built from a $\varphi^-\varphi^+$ pair). The τ_J orientations are frozen at n for the same reason as in leptons, so H bosons are electrically neutral and will, in particular, not couple to the photon whose τ_φ orientations are c.

Lepton-generation degrees of freedom are associated with the Y units of H bosons, characterizing them as $H_{GG'}$ $(G, G' = 1, ..., 4)$. $H_{GG'}$ carries -1 unit of L_G and $+1$ unit of $L_{G'}$, the total lepton number being zero.

The total number of H-boson states requires further discussion. In connection with problems of chiral symmetry breaking in an intermediate proposal [16], J. Finkelstein was the first to suggest that only one half of the combinations GG' are realized as physical H particles. The subsequent development of the phenomenological aspects of the theory [20, II] has given strong support to this idea, but the underlying topological reason or dynamical origin of such a suppression of states has not fully been elucidated yet. There is, however, a striking parallelism between H's and gauge bosons where the scalar/pseudoscalar combinations O^-O^+ and P^-P^+ do not appear. Given the important role of chirality in the dynamics of massless particles, it appears reasonable to suppose that an as yet unrecognized topological principle is at work requiring the two halves of massless bosons to have opposite patch-boundary orientations[9]. Under this assumption only the following H-boson

[8] It is clearly not possible to close leptonic π's on themselves because they contain an odd number of ends (two ends of Y legs, one φ end) in agreement with Fermi–Dirac statistics. There is, however, the option of connecting two Y legs belonging to the *same* lepton, forming a tadpole-like structure as in an early proposal [29]. According to the rules of Ref.[9] this is not admissible because + and − units would no longer alternate along the belt.

[9] This principle appears to manifest itself in a rule that requires tangency points between Feynman lines and Finkelstein or colour lines to be accompanied by a reversal of (O, P) labels on the corresponding patch boundaries if the latter are independent. The same reversal of (O, P) labels is required to occur where exactly two τ or colour lines impinge on $\partial\Sigma$ at the same point, separating independent patch-boundary components. Such a rule immediately excludes O^-O^+ or P^-P^+ scalars (see Fig.7), exhibits the chiral nature of the couplings in Figs.10–12 and reduces the number of physical H-boson states (see Fig.9(b)). No constraints arise, however, for lepton quantum numbers. It appears likely that the number of such tangency points will be relevant to the general definition of a chiral-complexity index g_3.

states are allowed: H_{12} and H_{21}, H_{13} and H_{31}, H_{24} and H_{42}, H_{34} and H_{43}. Fig.9(b) in conjunction with Appendix D of Ref.[9] shows that $H_{GG'}$ and $H_{G'G}$ are related by charge conjugation, *i.e.*, $H_{G'G} = (H_{GG'})^\dagger$ in a notation borrowed from field theory.

In view of the present ignorance in this matter it is reassuring that all the results in [II] would still hold if H bosons were required to have opposite scalar *patch* orientations rather than opposite *patch-boundary* orientations, but it is crucial that they are "off-diagonal" in one of these two variables.

4 Kinetic-Energy Topologies of Nonhadrons

This section concentrates on the detailed surface structures describing the propagation of leptons and H bosons; interactions pose somewhat different questions and will be dealt with separately in the following section. Electroweak topologies differ fundamentally from their strong-interaction counterparts in that they show intrinsic complexity even at the level of elementary vertices; also, there are explicit kinetic-energy vertices. Because Yang-Mills topologies reveal the necessity of those features most clearly, they will be given a preliminary discussion here, the results of which are then applied and adapted to leptons and H bosons.

4.1 Gauge-Boson Topologies

In TPT, spin is always accompanied by the chiral degree of freedom which is eliminated by summation in strong interactions but shows up in the physical spectrum of nonhadrons because of the assumed masslessness of their elementary counterparts. There are thus right-handed (V_R, P^-O^+) and left-handed (V_L, O^-P^+) vector bosons for which self-interactions according to chiral Yang-Mills theory are assumed:

$$\begin{aligned}\mathcal{L} \;=\; -\tfrac{1}{8}\,\mathrm{Tr}\,\Big[&\big(\partial_{[\mu}V^L_{\nu]} + ig_L V^L_{[\mu}V^L_{\nu]}\big)\,\big(\partial^{[\mu}V_L^{\nu]} + ig_L V_L^{[\mu}V_L^{\nu]}\big)\\ &+\big(\partial_{[\mu}V^R_{\nu]} + ig_R V^R_{[\mu}V^R_{\nu]}\big)\,\big(\partial^{[\mu}V_R^{\nu]} + ig_R V_R^{[\mu}V_R^{\nu]}\big)\Big]\;,\end{aligned} \tag{7}$$

where V_L and V_R are 2×2 isospin matrix fields[10]. It has not been possible to find consistent representations for interactions of scalar/pseudoscalar bosons (O^-O^+ and P^-P^+) which will therefore be ignored in the following. The isospin trace may be immediately recognized as a Chan–Paton factor [30], corresponding in the topology to a ring of Finkelstein lines as in HR diagrams and hadron surfaces.

[10] As long as interactions with hadrons are disregarded, the nonhadronic sector of TPT may be treated

According to (7), V_L and V_R do not interact directly with each other; this implies that chiral reversals take place as one passes from the φ^- unit of one particle to its neighbour's φ^+ unit. The same pattern will be found in the lepton kinetic energy and the lepton–gauge-boson vertex and already indicates the intrinsic complexity of elementary electroweak vertices. In Fig.10 the chiral reversals are economically achieved through tangency points between the Feynman graph F and the Finkelstein lines[11] (the precise location of those points does not appear to be important): Each φ belongs to a different patch-boundary component, and one may postulate the rule that chiral orientations always reverse at tangency points of τ and f or f_c lines if the patch-boundary components involved are independent. (See footnote 9 in Sect.3.) Tangency points are not new to TPT: When they occur in plugs of hadronic Y units they are inconsequential because they separate "white" patches. Colour-switching hadron plugs lead to tangency of f and/or f_c lines, no chiral reversal need occur there. Finally, tangency points of τ lines and $\partial\Sigma$ occur when gauge holes are created.

The Lagrangian (7) contains two quartic terms with different correlations of gauge-boson polarizations which have to be represented by two different topologies. Going into a frame of reference in which the particles with matching polarization indices have collinear momenta shows that spinor patches have to be united as indicated in Fig.10 by means of dotted circles with numbers. Such handles create patches with uniform spin orientations, but with two φ units on independent patch-boundary components; the chiralities of the two φ units in the same patch are indeed

like the perturbation expansion of local QFT because vertex functions are free of singularities. The Feynman rules given in Sect.6 imply, however, that the model is nonrenormalizable – the softening of loop effects through hadron-nonhadron interactions is essential in TPT and makes electroweak particles extended objects at the m_0 scale.

[11] A different proposal has been made by Dougherty [31] in which patches are (nonorientable) Möbius strips and the topological representations of chirality and spin are interchanged.

opposite because an odd number of tangency points separates them.

These spin-correlating handles (SCH) are a new feature of the theory; like colour switches in strong interactions they lead to entropy $g_4 > 0$ (the Feynman graphs in Fig.10 are planar, $g_1 = 0$, but Σ has genus $g_4 = 8$). SCH's must be regarded as a consequence of zero mass (requiring chiral switches) and are indicative of the lack of duality in electroweak interactions[12]. It is to be expected that SCH's and tangency points will also be central to lepton and H-boson topologies. Presently it is an open question whether SCH's are truly fundamental entities in TPT or whether they are ultimately a consequence of strong-interaction dynamics.

Applying the substitution principle (4) in the reverse sense produces Fig.11 from Fig.10 (there will also be the cyclic permutations of A, B and C). As a novel feature the gauge hole does not touch an intermediate momentum line but the vertex; by convention it associates with the momentum of the particle which precedes it in the cyclic sequence. That such asymmetry be required by the Yang–Mills amplitudes is puzzling but one may hope for better understanding as soon as momentum finds an adequate topological representation.

4.2 The Need for Kinetic-Energy Topologies

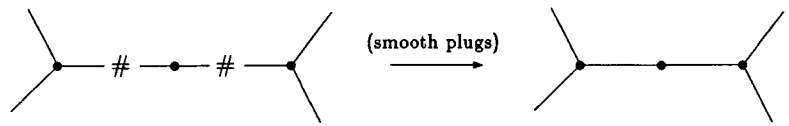
One may apply the reversed substitution principle once again, to the particle opposite to the gauge hole in Fig.11, and obtain Fig.12. (It turns out that gauge holes are always separated by a momentum line in elementary topologies.) It is readily identified with the kinetic-energy terms in (7).

The true novelty for TPT in Fig.12 conceals itself behind the simple operation that led to it: There are no kinetic-energy topologies in strong interactions where propagators are created by plugging together two topologies along one or more matching particle portions. Each intermediate Feynman line corresponds to a factor $(p^2 - m_o^2 - i\epsilon)^{-1}$, and there are numerator factors depending on the way in which the connected sum was done (chiral switches, colour switches) [28]. If the same procedure is applied in a gauge-boson plug, the two π's of the intermediate particle must not be identified because of chiral switching. Instead they touch at the Feynman graph and at a further point where the Finkelstein lines connecting the vertices are pinched, forming a pair of gauge holes. There is thus no correlation of spins across the plug line; as in the hadronic case the newly created gauge holes have to be interpreted as $p\cdot\gamma$ operators. This can be shown to amount to an unusual choice of gauge and is unwanted at this stage.

[12]This conclusion is not in contradiction with the derivation of Yang–Mills amplitudes from the zero-slope limit [32] of dual models by Neveu and Scherk because they did not consider the chiral doubling which is so prominent here.

The only satisfactory alternative found so far consists in elevating the kinetic energy to the status of a (trivial) vertex intervening between nontrivial vertices and projecting onto the modes that are allowed to propagate. Between nontrivial and trivial vertices the nonhadrons propagate freely, no distinction being made between physical and unphysical modes. This concept translates into the topological rule that nontrivial vertex topologies are never plugged directly but only onto kinetic-energy topologies in a perfectly smooth way without any chiral switches, handles, etc. In analogy with the strong-interaction rules [28], those links correspond to factors $(p^2 - i\epsilon)^{-1}$. Denoting the kinetic-energy terms by K_G,K_H and K_l, respectively, one obtains the following analytic expression from the plugged topological

structures:



$$V_1(p,\ldots) \qquad K(p) \qquad V_2(p,\ldots) \longrightarrow V_1\tilde{\Delta}_F(p^2)K(p)\tilde{\Delta}_F(p^2)V_2 \tag{8}$$

In particular, for H bosons and gauge bosons,

$$\begin{aligned}
\tilde{\Delta}_F(p^2)\, K_H(p)\, \tilde{\Delta}_F(p^2) &= (p^2 - i\epsilon)^{-1}\, p^2\, (p^2 - i\epsilon)^{-1} = (p^2 - i\epsilon)^{-1}, \\
\tilde{\Delta}_F(p^2)\, K_G(p)\, \tilde{\Delta}_F(p^2) &= (p^2 - i\epsilon)^{-1}(-g_{\mu\nu}p^2 + p_\mu p_\nu)(p^2 - i\epsilon)^{-1} \\
&= \frac{-g_{\mu\nu} + p_\mu p_\nu/p^2}{p^2 - i\epsilon}.
\end{aligned} \tag{9}$$

When dealing with leptons, spinors appear explicitly. The plugs between kinetic-energy and interaction surfaces do not involve chiral switching and must not project onto a subspace of all lepton states. From

$$\sum_s \frac{1 \pm \gamma_5}{2} u(p,s)\, \overline{u}(p,s) \frac{1 \mp \gamma_5}{2} = \frac{1}{2} \cdot N \cdot \frac{1 \pm \gamma_5}{2},$$

where N is the normalization of the spinors, one sees that one must choose $N = 2\sqrt{p^2}$ for massless particles for dimensional reasons. Then proceeding as before one obtains

$$\begin{aligned}
(\sqrt{p^2}\tilde{\Delta}_F(p^2))K_l(p)(\sqrt{p^2}\tilde{\Delta}_F(p^2)) &= \sqrt{p^2}(p^2 - i\epsilon)^{-1}\, \not{p}\, (p^2 - i\epsilon)^{-1}\sqrt{p^2} \\
&= \frac{\not{p}}{p^2 - i\epsilon}.
\end{aligned} \tag{10}$$

From the point of view of local QFT the propagator rules developed here can hardly be understood, even though the results have the familiar form: In the graphical expansion of Lagrangian QFT the propagator lines correspond to the inverse of the kinetic-energy operator,there are no trivial vertices. In the present S matrix

framework, on the other hand, a propagator line represents a pole of S even where the notions of a Lagrangian and of a kinetic-energy operator do not exist, as is the case for the strong interactions. Where kinetic-energy topologies appear, they are an expression of additional complexity.

4.3 Kinetic-Energy Topologies for Leptons and H Bosons

Due to the circular nature of bootstrap theory all the consistency constraints that will eventually arise in interaction topologies should be anticipated in the construction of kinetic-energy topologies for H bosons and leptons. Therefore guesswork is unavoidable to some extent, but the following properties of Σ are most likely general:

1. Σ is orientable; τ lines always have a $+$ and a $-$ end; f lines separate spinor (or "white" spinor) patches while f_c and τ lines separate spinor from scalar patches.

2. No new topological elements beyond those used in hadron and gauge-boson topologies occur.

3. Surfaces involving nonhadrons are noncontractible and show intrinsic complexity.

Three special properties are postulated for nonhadron topologies:

4. Even in the presence of JL's, Σ is one-sheeted.

5. "White" spinor patches are absent.

6. All elementary nonhadrons have the same degree g_4 of topological complexity.

(4.) and (5.) reflect the fact that electroweak topologies do not allow contractions. Postulate (6) prevents nonhadron Σ's from becoming too simple or too complicated; Yang–Mills vertices were seen to require $g_4 = 8$. It has been noted at an early stage of TPT [6] that each JL in a single-sheeted Σ contributes two units of g_4.[13] Hence, kinetic-energy surfaces will require three SCH's for leptons and only two for H bosons. No statement is made about the chiral-complexity index g_3 because an adequate general definition for it is still lacking.

If successful, requirement (6) may indicate some kind of Fermi–Bose symmetry (neither standard field-theoretical supersymmetry nor topological supersymmetry of zero-entropy M functions [33]) among gauge bosons, leptons and H bosons: Such symmetry is compatible with the topological expansion only if related topologies have equal complexity.

In constructing the H kinetic-energy surface it is convenient to concentrate on feather #1 (carrying the Feynman graph) first. (9) requires two gauge holes which also eliminate the "white" spinor patches associated with hadronic JL's (Fig.13(a)). Two handles unite patches across Feynman lines so that spinor patches have a pair of φ units along their boundaries and Σ will eventually acquire genus $g_4 = 8$ when the remaining feathers are added. The spin correlations correspond to $p_A \cdot p_B$ according to the rules given in Sect.4.1.

The patches α and β in Fig.13(a) each have two independent boundary components which, however, cannot be oriented at will because the gauge holes impose correlations. Such inconsistency can be removed by linking the two components. Tangency points between f and f_c lines – shown in Fig.13(c) – accomplish this task

[13]Globally orientable surfaces with fewer sheets than feathers allow definition of closed paths which cross the JL twice and circumscribe a Möbius strip rather than a disk. In a single-sheeted Σ two independent Möbius strips occur for each JL, each strip corresponding to one unit of genus.

in an almost unique way[14]. Fig.13(b) explains how the "white" spinor patches ϵ are eliminated by inserting feathers #2 and #3 into the "non-white" spinor patches α and β. One may now note that the structure of Fig.13(c) is quite similar to that of the first term in Fig.12, the JL's a and b corresponding to handles **1** and **2**.

As mentioned above, the presence in lepton surfaces of a single JL calls for three handles to match the genus of gauge-boson surfaces. As Fig.14(a) exemplifies one can easily imagine lepton topologies with fewer handles, but as the same example indicates there are some unsatisfactory features: When the substitution principle (4) is applied, right-handed gauge bosons are found to couple to left-handed leptons etc. Furthermore, the entire JL side of a lepton appears inert while there is considerable ambiguity about the location of feathers #2 and #3.

The first problem can be dealt with by introducing chiral switches between lepton φ's and gauge-hole φ's, calling for two SCH's. Fig.14(b) makes it evident that the chiral structure very much resembles that of gauge bosons. Still another SCH needs to be introduced to reach $g_4 = 8$; at the same time a more interesting role needs to be found for the non-principal feathers. The most satisfactory structure recognized so far is shown in Fig.14(c). One SCH connects the φ and Y sides while **1** and **2** are analogous to handles in gauge-boson topologies (the patches α and β have two disconnected boundary components each). It was found necessary to place the gauge hole in the Y side so as to obtain an odd number of SCH's. This step simultaneously upgrades the JL side and assigns a unique location to the colour

[14]Feathers #2 and #3 may be assigned in different ways to α and β. A possibility to remove this ambiguity is to demand that the orientations of the colour lines agree with the orientation of the spinor patch boundary of which they form a part. While no inconsistency has developed from such a rule so far, it has not found application in a different context either, so the question remains open.

lines[15].

[15] As in the case of H bosons, colours #2 and #3 may be interchanged, but again this question is of no concern here.

5 Topologies for Nonhadronic Interactions

5.1 General Considerations

In this section, surfaces representing the self-interactions of leptons and H bosons as well as their interactions with each other and with gauge bosons will be discussed. Non-gauge interactions are not much constrained by symmetry considerations, so topological principles have to be used to select the admissible vertex types. Such criteria, however, are not given *a priori* but must be bootstrapped themselves. It is then not surprising that some topologies presented below are still tentative, but their phenomenological consequences (discussed in [II]) support their main features.

The most fundamental assumption in this context is that all nonhadron topologies share certain characteristics. In the following, four criteria abstracted from Figs.10–14 are assumed to apply generally to all nonhadron topologies:

1. All elementary nonhadron surfaces have $g_1 = 0$ and $g_4 = 8$.

2. A SCH is associated with each ("non-white") spinor patch. Disconnected boundary components of the same patch have opposite orientations (*i.e.*, there is a chiral switch).

3. Gauge bosons couple according to the substitution principle.

4. The number of patches in an elementary surface is limited to 4 or 5; no "white" spinor patches are admitted.

Given the central role of chirality in the dynamics of massless particles, one suspects that there should be a general principle determining the chiral complexity (an entropy index) of elementary nonhadron surfaces. While it is easy to define a

chiral entropy index g_3 for strong interactions, no satisfactory way has been found yet to extend its definition to electroweak interactions. As will be seen, a large share of the remaining uncertainties in vertex topologies is connected to this open question.

5.2 Conservation of Lepton Generation Numbers

Before discussing in detail interaction surfaces, a simple topological fact with striking phenomenological consequences needs to be stressed: There are neither vertices nor tangency points along JL's. Accordingly, a single (three-feathered) patch lies along a JL with the same patch and patch-boundary orientations from one end to the other. The frozen orientations associated with hadronic JL's imply smooth plugging rules for JL patches in strong interactions. It was argued in Sect.4.2 that electroweak surfaces are also plugged smoothly, kinetic-energy topologies being inserted between non-trivial vertices. This rule again ensures the continuity of JL-patch orientations. The immediate consequence thereof is the *absolute and separate conservation of baryon number and all four lepton-generation numbers.* It will be seen in Appendix A and in [II] that the symmetry between lepton generations is broken and that a mass hierarchy develops, but different lepton generations nevertheless do not mix in this theory.

It may be noted in this context that quark generation is associated with δ units where no JL is present to impose an equally rigid conservation law – hadronic JL's guarantee baryon number conservation. One may expect that electroweak interactions of hadrons will at some level induce quark mixing in TPT.

5.3 Couplings Involving Gauge Bosons

Starting from Fig.14(c) it is straightforward to construct the topology for the coupling between gauge bosons and leptons (Fig.15) with the help of the substitution principle. The gauge boson is clearly seen to couple to the isospin of the leptons in the standard fashion, and left-handed gauge bosons couple to left-handed currents only, etc.

The third requirement of Sect.5.1 excludes couplings of a $V^{n\bar{n}}$ gauge boson to the leptonic τ_J line even though a corresponding surface could be constructed. The minimum-coupling prescription is presumably connected to the ultimate reason for the existence of gauge bosons and should thus be considered an overriding principle for the time being.

The substitution procedure may also be applied to the gauge holes appearing in the H-boson kinetic energy (Fig.13(c)), resulting in the cubic and quartic couplings shown in Fig.16. The frozen τ_J orientations of H bosons admit coupling only to gauge bosons of the $V^{n\bar{n}}$ type (a source of isospin-symmetry breaking). The quartic term involves one left-handed and one right-handed gauge boson and thereby provides the first example of mixing between V_L and V_R at the one-loop level. There are two different cubic terms: The one shown in Fig.16 gives an amplitude $\propto p_B \cdot \epsilon_C^{R(L)}$ while the topology with the gauge hole and the gauge boson interchanged corresponds to $p_A \cdot \epsilon_C^{L(R)}$. The substitution principle requires the two terms to be added with relative + sign, as is also the case with the two chiral possibilities. One obtains

$$p_A \cdot \epsilon_C^L + p_B \cdot \epsilon_C^R + p_A \cdot \epsilon_C^R + p_B \cdot \epsilon_C^L = -p_C \cdot \epsilon_C^R - p_C \cdot \epsilon_C^L \tag{11}$$

which vanishes if the gauge bosons are fully transverse.

5.4 Self-Interactions of H Bosons and Leptons

Because of the constraint on the quantum numbers of H bosons (discussed in Sect.3), self-interaction terms for H bosons always contain an even number of particles: One may think of the JL's as being arranged in a circle, linking "neighbouring" H bosons. With each JL goes a definite (O, P) bosonic label that is required to always change from one JL to the next in the sequence, hence the circle can be completed only with an even number of switches. Moreover, the topology of a term $\propto H^{2n}$ contains $2n$ JL's which contribute $4n$ to g_4. The latter being restricted to 8, kinetic-energy topologies have two handles and quartic terms none ($n = 2$) while terms with $n \geq 3$ are excluded. It is then easy to see that the surface representing a quartic term will have eight patches, four of which are "white" spinor patches; in the absence of handles they cannot be united and criterion (4.) cannot be met. The conclusion is that elementary H-boson self-interactions are not allowed.

Multilepton vertices with $2n$ fermions have n JL's and hence $4 - n$ handles to attain genus 8. Before handles are inserted, there are n JL patches, n "white" bosonic patches and also n "white" spinor and n "non-white" spinor patches. Each handle can lower the total patch number at most by two, hence only $n = 2$ (*i.e.*, the four-fermion vertex) needs to be considered. The "raw" surface without handles (Fig.17(a)) shows that the patches α and β and also the (bosonic JL) patches γ and δ must remain separate in order to avoid spinor patches with more than two φ boundary units and to keep the JL patches independent. If the two handles are used to connect ϵ with ς and η with θ, the total patch number is still too high to comply with the proposed rule (4.). There is a way to insert the handles so as to arrive at a surface with just four patches, none of them "white" (Fig.17(b)). However, one notices that particles A and C are not connected to other particles via

Finkelstein lines; instead their τ lines link the Y and φ units of the *same* particle – an unacceptable feature[16]. It is reassuring to see that vertices with dimensionful coupling constants are being excluded by the adopted topological postulates.

5.5 H-Boson–Lepton Couplings

Finally one wants to consider couplings involving both H bosons and leptons. Arguments similar to those used in the previous two cases show that only the cubic $H\bar{l}l$ vertex with two JL's and two handles is able to attain a sufficiently low number of patches. The topological requirements thus have again eliminated all interaction terms with dimensionful coupling constants.

Presently they are, however, not restrictive enough to single out one specific surface structure as the only consistent representation of the $H\bar{l}l$ vertex. One may suspect that the proper extension of the notion of chiral entropy will have considerable impact on the precise form of this chiral-symmetry-breaking interaction. In the meantime a discussion of the most promising candidate topology will persuade the reader that this particular vertex will eventually find a firm place in TPT.

One may try to develop the $H\bar{l}l$ topology either from the H-boson kinetic-energy topology, Fig.13(c), by "opening up" the Feynman line of B and inserting a pair of φ units, or from the lepton kinetic energy, Fig.14(c), by "cutting in two" the JL and readjusting the patch structure appropriately. The latter approach will not be pursued here because it leads to rather complicated structures which hold little promise, the τ_φ line containing two or more tangency points or the two JL's being treated differently.

[16]The only precedent – the so-called vacuon – is briefly discussed in Appendix A.3. The reason requiring τ-line connection between the two Y units in a vacuon does, however, not apply to the present situation.

Fig.18 shows the presently favoured structure, emphasizing its kinship with Fig.13(c). The H boson A occupies its own belt component and the leptons share the other one, as in the lepton–gauge-boson case. The H couples to each of the leptons through one of its JL's and one of its τ_J lines while the τ_φ line links the leptons. The generation labels on H thus determine those on the leptons; the latter are either both charged or both neutral.

In transforming Fig.13(c) into Fig.18, the gauge holes had to be removed whereupon the two SCH's became obsolete in their previous positions. In order to prevent the spinor patches between A and B or A and C from becoming "white", the handles **3** and **4** are rearranged in a symmetrical fashion so as to unite those patches with the "non-white" spinor patch between B and C into one patch α with a single patch-boundary component, as befits a scalar coupling. Tangency points between f and f_c lines are used for this purpose in the same way as in Fig.13(c), with the usual ambiguity arising from the colour assignments.

One of the somewhat unsatisfactory features of Fig.18 is to be seen in the passive role of all τ lines, which do not have tangency points. The low chiral complexity of this vertex type is chiefly responsible for the relative simplicity of Fig.18. On the other hand, the boundary of patch α is unusually complicated, being tangent to itself at four points and consisting of three different τ lines, four colour lines, two φ units and several pieces of Feynman lines. This is to be compared with strong-interaction spinor patches built from just two φ units, one τ_φ line and some Feynman lines, or with spinor patches in elementary gauge-boson topologies which have two components, each composed of Feynman-line pieces, one φ unit and a τ-line segment. The number of boundary segments appears to compensate for the lack of chiral complexity in patch α. Undoubtedly, some kind of topological entropy is associated with the number and type of line segments in patch boundaries. Such

an index would take the value zero in strong-interaction topologies and measure the presence of electroweak interactions. It is to be expected that its precise definition will depend crucially on the properties of semihadronic vertices.

6 Feynman Rules and Symmetries

6.1 Shorthand Graphs

The surface structures discussed in this paper are so complicated that it is essential – for all practical purposes – to develop a graphical shorthand more similar to standard Feynman graphs and also to give the Feynman rules which follow from the elementary kinetic-energy and vertex topologies. It should be obvious from the preceding discussions that the physical content of the theory may adequately be summarized in such rules and shorthand graphs but that only the full set of relations and consistency constraints among the detailed topologies allows inference of those rules.

The corresponding problem for the strong interactions was solved some years ago [28], and the most adequate graphical shorthand was found to be the combination of Feynman (momentum-line) and Harari–Rosner (quark-line) graphs [3] and was called "thickened Landau graph" in Ref.[6]. All particle properties except momentum are put as labels onto the quark lines which carry them from one particle to another. Chiral and colour switching can easily be represented, and all other quantities are conserved in strong interactions. The Feynman graph represents the singularity structure of the amplitude.[17]

The above-discussed nonhadron topologies exhibit a similar flow pattern for isospin and lepton-generation number (the coupling between H bosons and $V^{n\bar{n}}$ vector bosons, Fig.16, is anomalous in this respect). In purely nonhadronic graphs, the colour degree of freedom is of no physical consequence, and no quantum numbers

[17]Graphs similar to HR quark-line diagrams were also used by 't Hooft in his investigations of the $1/N$ expansion of $U(N)$ gauge theories. His graphs do not show the chiral structure and spin correlations at vertices [34].

attach to feathers #2 and #3 in leptons and H bosons. In contrast, the spin-chiral structure of leptons and especially gauge bosons is much more complex than that of elementary hadrons. Accordingly it appears useful to devise a shorthand which explicitly shows only the flow of isospin and lepton-generation number along "fermion" and "boson" lines, respectively, and indicates the chiral structure but suppresses the spin complications. The Feynman graph may also be included, mainly to guide the eye and later to allow connection with hadronic shorthand graphs. It will also be useful to distinguish clearly between quark lines (carrying eight flavours), nonhadronic fermion lines (representing isospin only) and nonhadronic boson lines (four lepton generations). Unfortunately, many different conventions have been used for shorthand graphs in past years. This paper will follow the notation of Refs.[21,9] for hadrons; Fig.19 shows the representation for all elementary particles in TPT. Sometimes it is convenient to use an even more condensed notation as in standard Feynman graphs, see Fig.20.

The shorthand graph corresponding to a given complete topology may be obtained in the following way:

1. The Feynman graph is taken over unchanged except for lepton–gauge-boson vertices (Fig.15) where the Feynman line corresponding to the boson is flipped over – then gauge bosons consistently attach to the fermionic side of leptons and H bosons to the bosonic one.

2. The so rearranged Feynman graph is thickened by the Finkelstein lines where τ_J is represented by zig-zag lines (boson lines) and τ_φ by dotted ones (fermion lines). Except at the anomalous $H - V^{n\bar{n}}$ vertex, fermion lines and boson lines never cross each other nor themselves nor the Feynman graph.

3. Chiral switches may be indicated by crosses on fermion or quark lines. Because

of the large number of such switches along fermion lines it is often more convenient to mark by a full square the spots where the handedness (L or R) of a fermion line changes.

4. For the sake of conciseness, kinetic-energy trivial vertices will be suppressed.

6.2 The Feynman Rules

Table I gives the Feynman rules as they can be read off the topologies Figs.10–12, 13(c), 14(c), 15, 16 and 18. The different coupling constants e, f, g and h should not be considered constants at very high energies – this question will be discussed in [II]. In the following subsection, symmetry arguments will be used to establish relations between e, f, g and h.

The following remarks concerning Table I are in order:

1. Fermion lines maintain their handedness except at points marked with a full square. This statement follows from the fact that lepton and gauge-boson propagators as well as gauge-boson and gauge-boson–lepton vertices impose an odd number of switches on a fermion line. A fermion line passing by a single vertex thus maintains its handedness; each further vertex also implies an additional propagator so that the number of chiral switches on a fermion line increases in even steps, staying odd. An exception to this chiral structure is the lepton–H vertex which breaks chiral symmetry. From Fig.16(b) it may be inferred that H bosons can couple to one left-handed and one right-handed gauge boson and so provide a further source for $L - R$ mixing and chiral-symmetry breaking.

2. For simplicity, Faddeev–Popov ghosts and their couplings have been left out. The topologically most natural way of dealing with the gauge degrees of freedom is still being investigated. Neither the present paper nor [II] deal with radiative corrections arising from Yang–Mills loops.

3. The very simple isospin factors are equivalent to those of the more standard representation of non-abelian gauge theory – a circumstance noted, *e.g.*, in Ref.[32].

4. As noted in Sect.3, the conventions about lepton charges and lepton number imply that charge-conjugate spinors should be used for leptons if normal spinors describe quarks. Electroweak amplitudes may be written down in accord with usual conventions by simply "reading" fermion lines in the sense opposite to their arrows. If interference terms with semihadronic amplitudes are present, care must be taken to include the charge-conjugation phases of all operators acting on fermion lines (this has been done in Table I).

6.3 Symmetries

With the help of the Feynman rules it is now easy to discuss the symmetries of the nonhadronic sector of TPT. As explained in Ref.[10], discrete symmetries arise where the two possible orientations of a line, patch or patch boundary are topologically equivalent. The superposition principle of quantum mechanics then allows enlarging some of the discrete symmetries into continuous symmetries.

Using the definitions of $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$ transformations, given in Appendix D of Ref.[9], one easily sees that the rules presented in Table I are invariant under those operations. Chiral symmetry, however, is broken by the lepton–H vertex: Left-handed fermion lines change into right-handed ones, and *vice versa*, so that independent transformations on L and R lines are not possible.

In gauge-boson and gauge-boson–lepton couplings alone, L and R fermion lines

maintain their separate identities. All isospin orientations being equivalent, one may define global $U(2)_R \times U(2)_L$ transformations acting on fermion lines. Furthermore, all boson-line orientations are on equal footing in the $\bar{l}Vl$ vertex and in the lepton propagator so that the coupling constant e is independent of the generation index of the leptons. As this vertex is obtained by the substitution principle from the lepton kinetic energy, one infers $e = g$ and thus obtains *local* gauge symmetry $U(2)_R \times U(2)_L \times \mathcal{P}$, the parity symmetry forcing g_L and g_R to be equal (before renormalization). Within this subset of vertices and propagators, the complete equivalence of all four lepton generations gives rise to a global $U(4)_G$ symmetry.

Adding the lepton–H interaction breaks chiral symmetry and thereby reduces $U(2)_R \times U(2)_L \times \mathcal{P}$ to $U(2)_V \times \mathcal{P}$. In the sense of perturbative QFT the model is then no longer renormalizable ([II] will discuss why this is not a disaster in the context of TPT). Two-loop diagrams will mix V_L with V_R and produce couplings between V_L and right-handed lepton currents, etc.

The restriction of the $H_{GG'}$ spectrum to G and G' with opposite (O, P) content does not sustain the full $U(4)_G$ symmetry deduced above but breaks it down to $U(2)_O \times U(2)_P$ acting on bosonic indices. Before hadrons are added to the dynamics, bosonic O and P orientations are equivalent, *i.e.*, there is an additional discrete symmetry left over from the breaking of $U(4)_G$ (otherwise the surviving local $U(2)_V$ symmetry would degrade into a global one).

The effect of the $H - V^{n\bar{n}}$ couplings is to break the residual gauge group $U(2)_V$ into a local $U(1)$ associated with phase transformations on charged τ_φ lines – the gauge group of electromagnetism. This outcome is not surprising because the desired conservation of electric charge was the criterion used to determine the orientation of nonhadronic τ_J's. The corresponding gauge boson is a superposition of equal amounts of left-handed and right-handed, isosinglet and isotriplet states. As

noted by Chew and Finkelstein [19], the resulting unrenormalized Weinberg angle is 30°.

It should be stressed that the foregoing discussion of topological symmetries, although presented as a sequence of symmetry breakings by considering more and more terms, does not imply any notion of *spontaneous* symmetry breaking. There are neither Goldstone bosons nor a Higgs mechanism in the nonhadronic sector of TPT.

7 Conclusions

The preceding sections established that electroweak particles and interactions can be given a topological representation which is fully compatible with strong-interaction structures. Some particular features emerge which are connected with the relative weakness of the interactions (absence of duality) and the masslessness of elementary nonhadrons (necessity for specific kinetic-energy topologies, cylindrical patches and spin-correlating handles). Motivated by the experimental fact that weak interactions to high accuracy proceed via left-handed currents and by the phenomenological success of the GSW model, elementary singularity-free chiral Yang–Mills couplings were assumed for vector bosons. As a consequence the corresponding surfaces acquired genus $g_4 = 8$. In order to maximize topological Fermi–Bose symmetry, the remaining nonhadron topologies were required to have matching g_4. A set of topological properties was proposed which, among other consequences, led to the exclusion of vertices with dimensionful coupling constants.

The gross features of nonhadron topologies are fairly well determined by those rules, except for the $H\bar{l}l$ vertex. The Feynman rules presented in Sect.6.2 thus appear to be rather definite within the framework of theabove-mentioned assumption; their main consequences for the phenomenology will be discussed in [II].

It is to be expected, however, that some details of the proposed topologies will have to be changed when additional consistency constraints become recognized in future work. The remaining ambiguities are primarily connected with chirality, tangency points, patch-boundary structure and the function of handles. Additional requirements will probably result from a better understanding of photon-hadron vertices.

One aspect deserves particular attention in future investigations: While all features of strong-interaction topologies (except "white" spinor patches) directly reflect themselves in the Feynman rules, the remaining ambiguities in nonhadronic surfaces are not expected to have noticeable impact on Table I. Hence the present proposals are either too complicated – in which case the complexity unavoidably present in Yang–Mills topologies would be hard to understand – or they are subject to more subtle consistency constraints that will not be easy to unravel.

One of the most fundamental questions regards the origin of electroweak topologies. Are they truly elementary, representing a new and different bootstrap level, or can they be thought of as summarizing some as yet unknown dynamical mechanism which generates nonhadrons from complex hadronic dynamics? Whatever the answer to this question, it should at the same time provide a way to compute (at least in principle) the various electroweak coupling constants either through some new bootstrap system or from the fundamental strong-interaction parameters.

Most likely, progress in this question will depend on the link forged between hadrons and nonhadrons by semihadronic topologies. Unfortunately, their development has not yet reached a comparable degree of definiteness – exactly because that dynamical link is still unknown. The presence of an elementary mass scale in that sector of TPT does not justify exclusion of couplings characterized by dimensionful parameters, so many different types of vertices need to be considered. Appendix A demonstrates that the present topological scheme can accomodate couplings to hadrons for all nonhadrons without patently conflicting with known data. It is expected that more definite proposals will emerge as soon as a consistent set of photon-hadron couplings has been developed: The principle that the electric charge be conserved locally must translate into restrictions on the topologies which may then be applied to other cases.

In the meantime one may draw some encouragement from the rather simple semi-

quantitative explanation of the observed lepton mass spectrum which is suggested by the coupling patterns developed in the present paper. TPT thereby achieves a unification of strong, electromagnetic and weak interactions which differs fundamentally from the approach pursued in grand unified theories and promises a wealth of new phenomena in the TeV energy range. A first survey of those processes is the purpose of [II].

A Topologies for Semihadronic Couplings

A.1 Phenomenological Considerations

The Feynman rules in Table I imply that the nonhadronic sector of TPT does not represent a viable theory on its own because of nonrenormalizability. [II] will indicate how semihadronic couplings overcome the problem and also generate a realistic mass spectrum. The goal of this appendix is to make preliminary proposals for corresponding topologies, even though the underlying principles are not yet known. Certain phenomenological facts may temporarily provide guidance in this situation:

1. Nonhadrons appear pointlike up to high q^2 (except for mixing between gauge bosons and nearby 1^- resonances), excluding direct interaction between leptons and light *physical* hadrons. Leptons may, however, have a not too large number of couplings to *elementary* hadrons whose common mass $m_0 \gtrsim 1$ TeV [11] will strongly suppress couplings to light physical hadrons.

2. H bosons and right-handed gauge bosons must be very heavy and should thus interact with heavy hadrons of high multiplicity; on the other hand, most leptons are extremely light. For the sake of renormalizability, the latter should also communicate with high-multiplicity hadrons, but chiral symmetry must be preserved to a very high degree.

3. Neutrinos are several orders of magnitude lighter than their charged partners, if they are not completely massless. Their direct interactions with hadrons thus have to conserve chiral symmetry exactly. It also turns out that charged lepton currents must not directly couple to hadrons for this reason.

No general topological principle governing such couplings has been recognized yet; the most stringent restrictions may well stem from electromagnetic gauge invariance which is needed for keeping the photon massless. Such gauge invariance can be achieved if the photon couples in a noncontractible way to all charged τ lines in elementary hadron currents consisting of a particle-antiparticle pair (or one incoming and one outgoing particle of the same type). Ref.[18] showed which topological questions and phenomenological potential were connected with those couplings in the earlier version of TPT; work is in progress to adapt the earlier proposals to the present, more economical scheme [9].

A.2 Lepton–Hexon and H-Boson–Hexon Couplings

Conservation of lepton-generation numbers implies that leptons and H bosons always couple in pairs to other particles (except for the $H\bar{l}l$ vertex). If one constructed cubic vertices involving a pair of leptons or H bosons and a single hadron, the latter would have to occupy a belt component by itself. There is strong-interaction precedent for single-particle belt components in "naked-cylinder" topologies [10], but the discontinuities of the corresponding amplitudes vanish on the elementary-hadron mass shell, creating a new pole (called "vacuon", see Sect.A.3) with lower mass. Therefore only quartic couplings involving a hadron pair will be considered in this subsection. Higher than quartic couplings are not promising because they would have to be exceedingly complicated in order to be manifestly noncontractible. Furthermore, the discussion will be restricted to hexons because their very high multiplicity will produce the most significant physical effects.

Fig.21 shows a lepton–hexon surface which was constructed so as to couple only neutral lepton currents, treating all four generations alike. One quark in each di-

quark is passive. The second quark associated with JL a undergoes isospin coupling to the leptonic JL c; it is inserted in such a way as to prevent patch α from being "white". The φ sides of the leptons are linked by their τ_φ line while their spins and chiralities are correlated with one quark belonging to JL b; due to the tangency points between the τ lines and the Feynman graph or quark-colour line, the patches β and γ have opposite chiralities – a vector coupling. The SCH is necessary because there would otherwise appear a "white" spinor patch between the hexons' Feynman line and τ_J of b. Along the same lines one may construct a topology with baryons rather than hexons by replacing JL b and its diquark by a single quark; the main properties of both vertices are very similar.

An interesting feature of Fig.21 is the patch δ whose boundary contains a pair of both δ units and Y units, implying a coupling between lepton and quark generations. Once the generation symmetry of leptons is broken [II], a similar breaking of quark-generation symmetry will be induced, although no mixing will occur yet.[18]

H-boson–hadron couplings being less constrained phenomenologically, Fig.21 can easily be modified into a hadron–H-boson topology where no spin exchange occurs between hadrons and nonhadrons (Fig.22). The SCH is again used for avoiding "white" spinor patches. The previously encountered mixed scalar patches with δ and Y units are also present here, and again one may replace the hexons by a baryon pair.

A completely different coupling scheme makes use of the JL's to link baryons or hexons to leptons or H bosons. The frozen patch and patch-boundary oientations of hadronic JL's have to be matched by the nonhadrons so that only λ, ν_λ, H_{24} and H_{34} may get involved in such coupling. If the quarks play a completely passive role, the

[18] Chew has recently constructed a similar topology with H-bosons where quark-generation mixing is induced by the opposite Y-patch-boundary orientations of the latter [36].

isospin coupling must proceed via hadronic τ_J lines. As hadronic and nonhadronic τ_J orientations are frozen in opposite directions, only the lepton with τ_φ orientation c, *i.e.* λ, meets all the requirements. Fig.23 shows a preliminary topology where the leptonic φ units have the same spin and chiral orientations. Chiral symmetry is broken and λ may acquire a big mass through hadronic loop diagrams. [II] will discuss how the lepton mass hierarchy develops from this starting point.

A.3 The Vacuon and Its Couplings to Leptons

Already the first attempts at understanding the lepton mass spectrum in the framework of TPT [20] indicated the need for a special mechanism to lift the degeneracy between μ and τ. Because of absolute conservation of lepton-generation numbers, generation mixing is excluded and τ must undergo some extra coupling in order to become more massive than μ. It was conjectured that a mysterious scalar particle emerging from TPT dynamics at the cylindrical level [37] might have the required properties to accomplish such splitting. Because of its zero quantum numbers it is called *vacuon* or Θ. Its place in TPT and its representation are briefly reviewed below [10] before a proposal is made for its couplings to leptons.

Certain ("naked") cylinder topologies possess a simlar self-reproduction capability in plugs as do (planar) zero-entropy topologies. A new pole corresponding to a single neutral scalar state is thereby created while a specific superposition of zero-entropy states – containing mesons as well as hexons – is eliminated. Being also capable of contractions, the newly created vacuon must still be considered an elementary particle[19] and hence needs its own topological representation differing from that of mesons and hexons. It has not been possible to consistently implement an earlier proposal [37] which employed a pair of φ boundary units. More recently,

a closed Y^-Y^+ portion emerged as a promising candidate (Fig.24).

In order to assure the contractibility of certain graphs with vacuons, Σ must be three-sheeted (at least in hadronic couplings). A tadpole-like Feynman line – not representing any specific particle – encircles the vacuon to prevent quantum number exchange. The strong interactions hence cannot determine the orientations of the τ_J line and of the scalar patch and patch boundary . It is expected that consistency constraints from electroweak couplings will eventually determine them; at the present stage where only Θ–lepton couplings are considered, those orientations are assumed which are most promising with respect to the phenomenology: c, U and O – the orientations ascribed to hadronic JL patches.

When Θ interacts with nonhadrons, no contractions are possible and the coupling is not expected to be independent of the channel. This consideration indicates that the tadpole line is not present in electroweak vertices involving Θ. Under the present assumptions about the topological orientations of Θ one can easily construct the surface shown in Fig.25. It correlates the lepton charge with the τ_J orientation of Θ so that only charged leptons are able to couple. The JL's of Θ and the lepton pair border on the same scalar patch whose orientation is thus fixed at U. That patch has two independent boundary components so that both λ and τ are subject to this coupling.

The proposal of Fig.25 can, of course, not be considered final. In particular, it is unclear whether the vacuon always requires a three-sheeted structure or follows the pattern of normal hadrons. One may also suspect that the colour lines will have to acquire a more substantial role. At the very least, however, the present proposal

[19]The vacuon occupies an intermediate position between hadrons and nonhadrons which led to the suggestion that it might provide an opportunity to bootstrap the nonhadrons [37]. An encouraging property of Θ is that its coupling to hadrons is weak, of the order of the electric charge.

strengthens the case for this unusual coupling and the dynamical scenario presented in [II].

Acknowledgments

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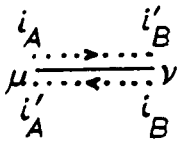
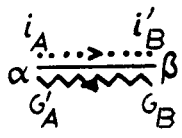
Table I

Nonhadronic Feynman Rules

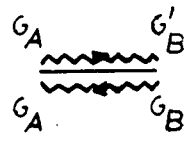
Notation:	$\alpha, \beta, \gamma, \delta$:	4–spinor indices
	μ, ν, ρ, σ:	4–vector polarization indices
	i, (i'):	isospin indices associated with $-$ $(+)$ end of fermion line
	G (G'):	lepton-generation index of $-$ $(+)$ end of boson line
	:	change of handedness $(L \leftrightarrow R)$ on fermion line

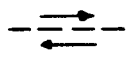


$$i\,\frac{-g^{\mu\nu}+p^\mu p^\nu/p^2}{p^2-i\epsilon}\,\delta_{i'_A i_B}\,\delta_{i'_B i_A}$$

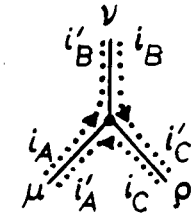


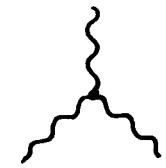
$$\frac{i\,p\cdot\gamma_{\alpha\beta}}{p^2-i\epsilon}\,\delta_{i'_A i_B}\,\delta_{G'_A G_B}$$



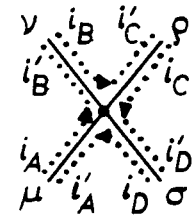


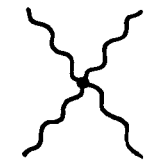
$$\frac{i}{p^2-i\epsilon}\,\delta_{G'_A G_B}\,\delta_{G'_B G_A}$$



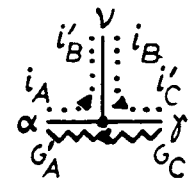


$$-ig\,[(p_A-p_B)_\rho g_{\mu\nu}+\text{cyclic perm.}]$$
$$\times\delta_{i'_A i_C}\,\delta_{i'_C i_B}\,\delta_{i'_B i_A}$$

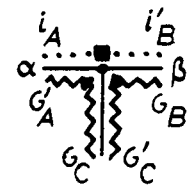




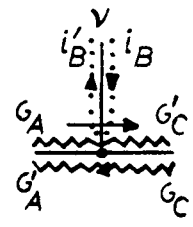
$$+ig^2\,[g_{\mu\nu}g_{\rho\sigma}+g_{\mu\sigma}g_{\nu\rho}-2g_{\mu\rho}g_{\nu\sigma}]$$
$$\times\delta_{i'_A i_D}\,\delta_{i'_D i_C}\,\delta_{i'_C i_B}\,\delta_{i'_B i_A}$$

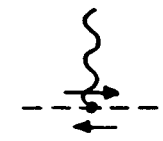


$$-ie\left(\gamma_\nu\tfrac{1\pm\gamma_5}{2}\right)_{\alpha\gamma}\delta_{G'_A G_C}\,\delta_{i'_C i_B}\,\delta_{i'_B i_A}$$

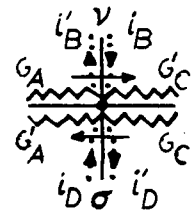


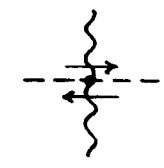
$$-if(-\delta_{\alpha\beta})\,\delta_{i'_B i_A}\,\delta_{G'_A G_C}\,\delta_{G'_C G_B}$$





$$-ih\,\delta_{G'_A G_C}\,\delta_{G'_C G_A}\left(\tfrac{1-\tau_3}{2}\right)_{i'_B i_A}(p_A+p_C)_\nu$$





$$+ih^2\,g_{\nu\sigma}\left(\tfrac{1-\tau_3}{2}\right)_{i'_B i_B}\left(\tfrac{1-\tau_3}{2}\right)_{i'_D i_D}$$
$$\times\delta_{G'_A G_C}\,\delta_{G'_C G_A}$$

Figure Captions

Fig.1: (a) Single-vertex Feynman graph F.
(b) Corresponding Harari–Rosner diagram.
(c) Embedding of F in 2-d planar surface Σ. Particle boundary portions π_i are indicated by braces; the arrow around the Feynman vertex shows the global orientation of Σ.

Fig.2: Connected sum of two TPT single-vertex surfaces.

Fig.3: Surface representing the simplest meson–baryon–antibaryon vertex. Note how the JL induces orientations on feathers #2 and #3.
— — — — : Feynman lines
— · — · — : colour lines
▬▬▬ : junction line
∿∿∿∿ : Finkelstein lines

Fig.4: (a) $\varphi^{\pm}$ boundary units.
(b) $\delta^{\pm}$ boundary units.
(c) $Y^{\pm}$ boundary units.

Fig.5: Hadronic boundary portions. (a) Meson. (b) Baryon. (c) Hexon.

Fig.6: Creation of a gauge hole in a plug. Chiralities are shown as arrows on φ boundary units. They match in the patches α and α', the plug is smooth; the mismatch in the patches β and β' forbids identification and erasure of φ units.

Fig.7: Closed $\varphi^-\varphi^+$ belt portion representing a gauge boson.

Fig.8: Lepton belt portion built from $Y^{\pm}\varphi^{\mp}$.

Fig.9: (a) Belt of H "propagator" with single boundary component. JL's, colour lines and Finkelstein lines are also shown.
(b) Belt of H "propagator" where each H occupies its own boundary component. (In the full surface, lines do not cross.)

Fig.10: Topological representation of quartic Yang–Mills vertex. Σ has been mapped onto the infinite plane; dotted circles with equal numbers are to be identified. Arrows indicate patch-boundary orientation corresponding to left-handed vectors.

Fig.11: Cubic Yang–Mills vertex.

Fig.12: Surfaces representing the kinetic energy of gauge bosons (shown for the case of left-handed vectors).

Fig.13: H-boson kinetic-energy surface.
(a) Structure of feather #1.
(b) Two equivalent representations of feather #2.
(c) The full surface.

Fig.14: Lepton kinetic-energy surfaces.
(a),(b) Rejected proposals.
(c) Adopted proposal with $g_4 = 8$. Greek letters label patches.

Fig.15: Topology for lepton–gauge-boson interaction.

Fig.16: (a) Cubic and (b) quartic couplings between H bosons and $V^{n\bar{n}}$ gauge bosons.

Fig.17: (a) Surface representing four-fermion coupling before insertion of handles.
(b) After insertion of two handles.

Fig.18: Proposal for $H\bar{l}l$ vertex topology based on H kinetic-energy surface.

Fig.19: Shorthand representation for elementary particles in TPT. The circular arrow indicates the surface orientation .
——— : Feynman line;
— — — : quark line;
· · · · · · : nonhadronic fermion line;
∿∿∿∿ : nonhadronic JL (boson line).

Fig.20: Alternative shorthand notation for nonhadrons. The arrows indicate the flow of lepton-generation number.

Fig.21: Proposal for lepton–hexon vectorial coupling.

Fig.22: Proposal for H-boson–hexon coupling.

Fig.23: Candidate topology for scalar coupling between lepton pair $(\lambda^+\lambda^-)$ and hexon pair.

Fig.24: Two-meson–vacuon coupling. A tadpole in the Feynman graph isolates Θ from the other particles.

Fig.25: Proposal for coupling between vacuon and τ and λ leptons.

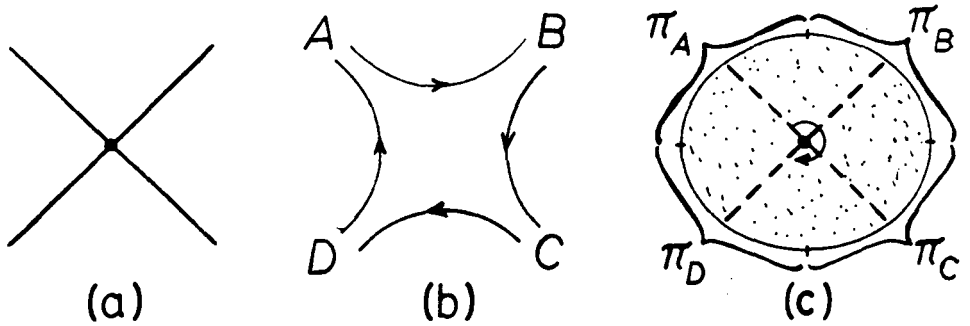


Fig. 1

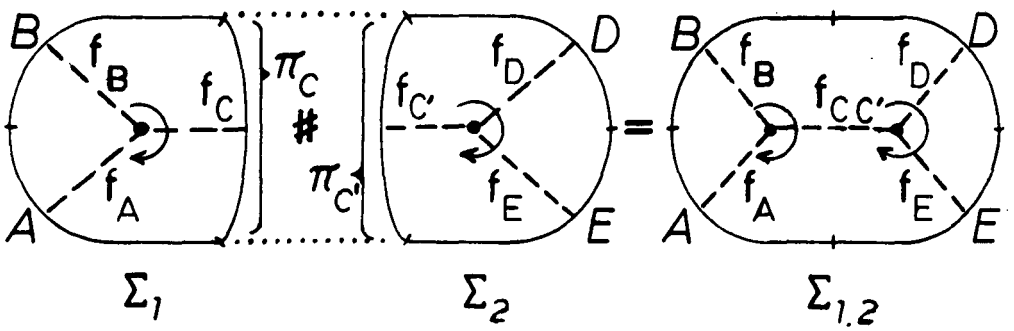
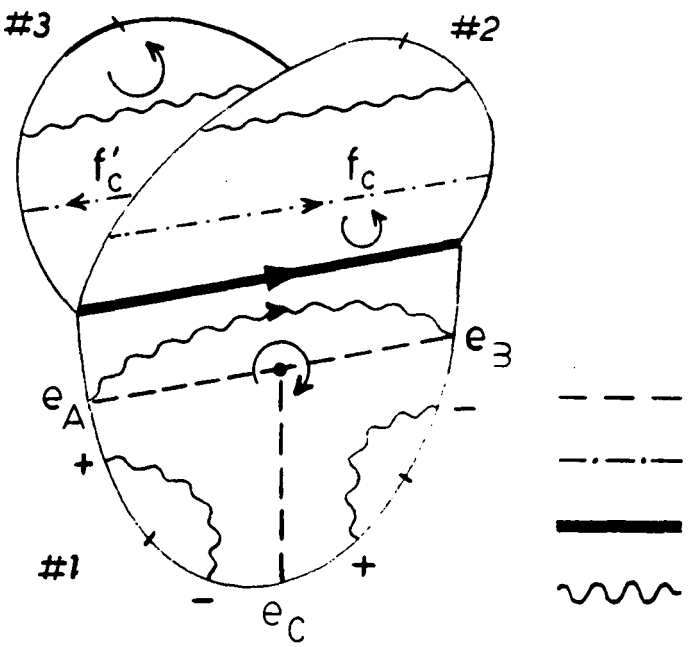


Fig. 2



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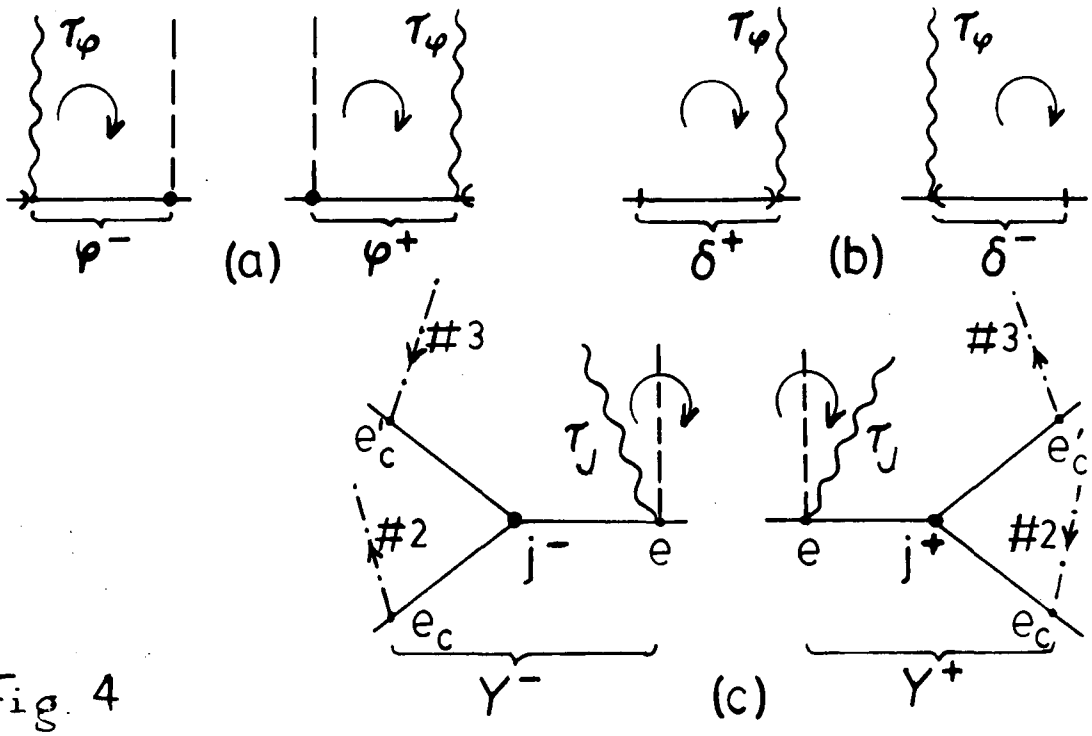

τ_φ
φ^-
(a)
φ^+
δ^+
(b)
δ^-
#3
e_c'
#2
e_c
τ_J
j^-
e
Y^-
(c)
j^+
Y^+

Fig. 4

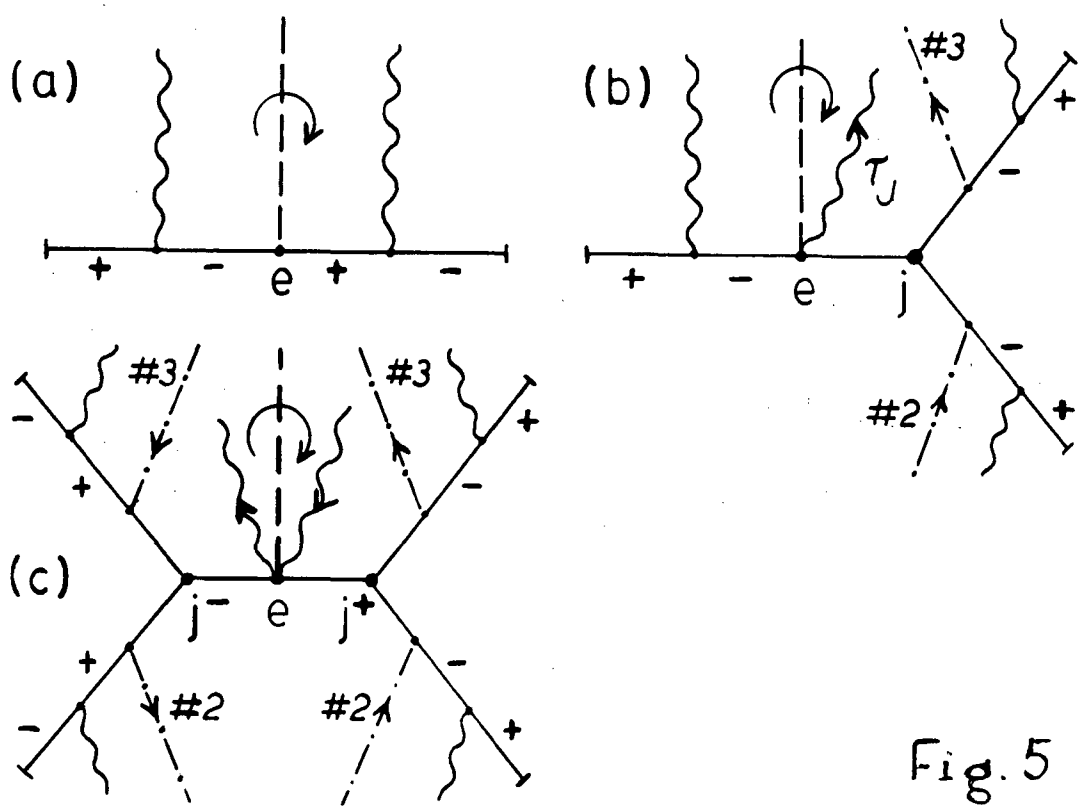
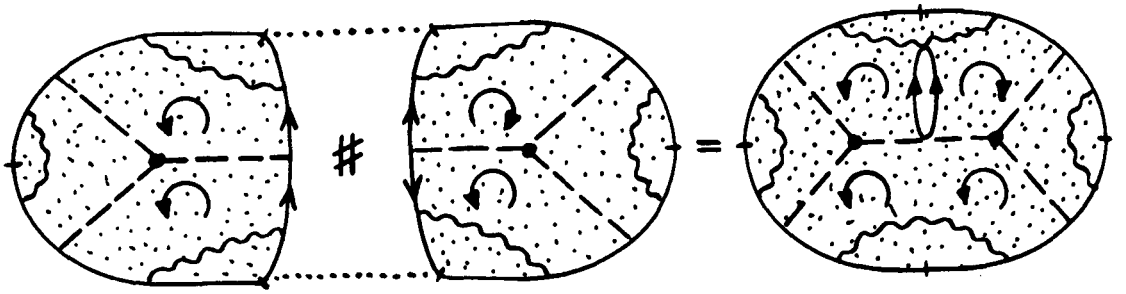

(a)
+
−
e
(b)
#3
τ_J
j
#2
(c)
j^-
j^+

Fig. 5



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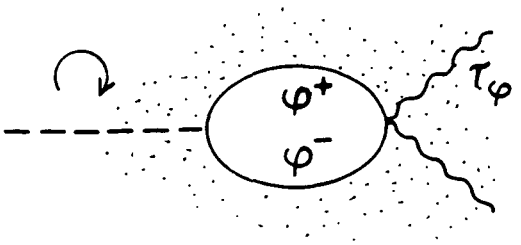

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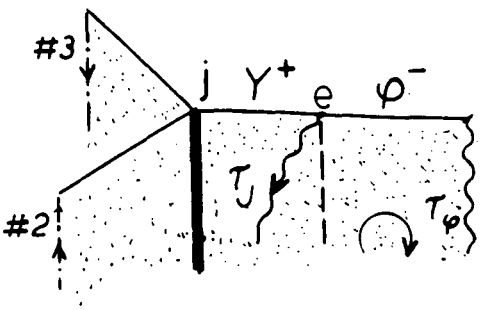

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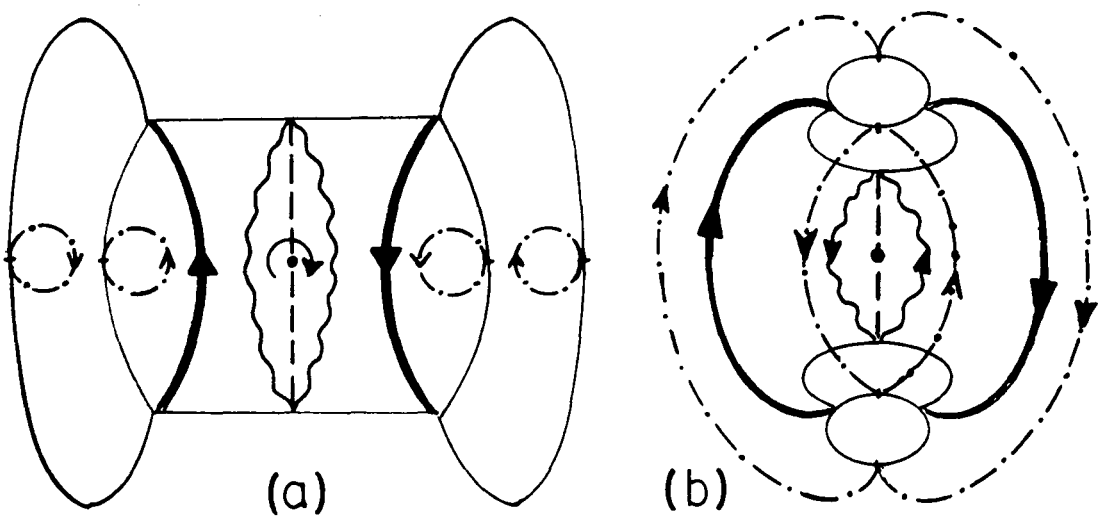

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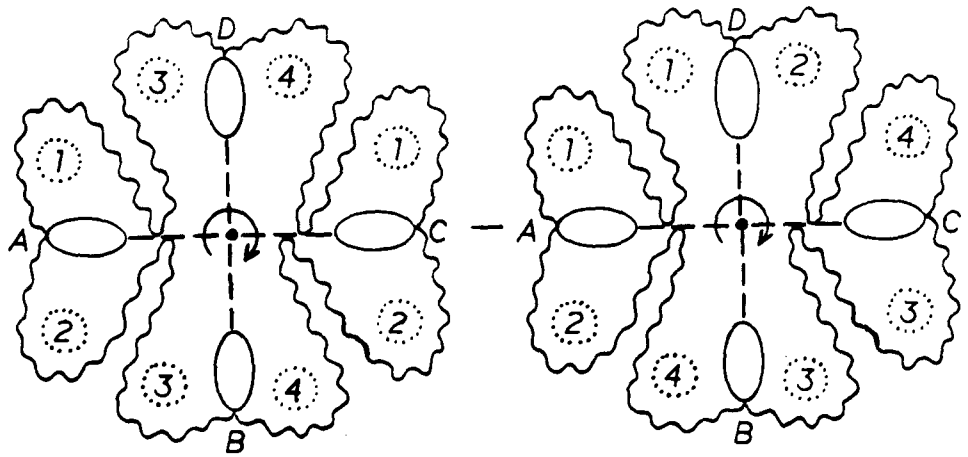

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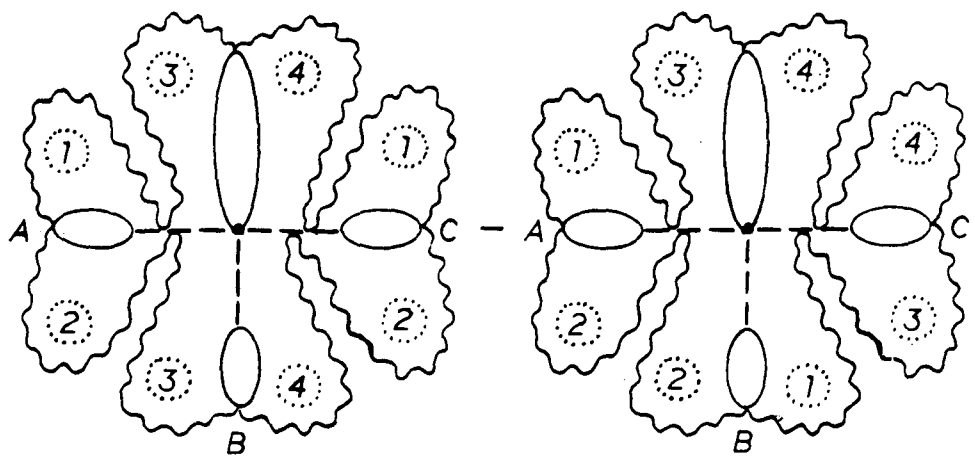

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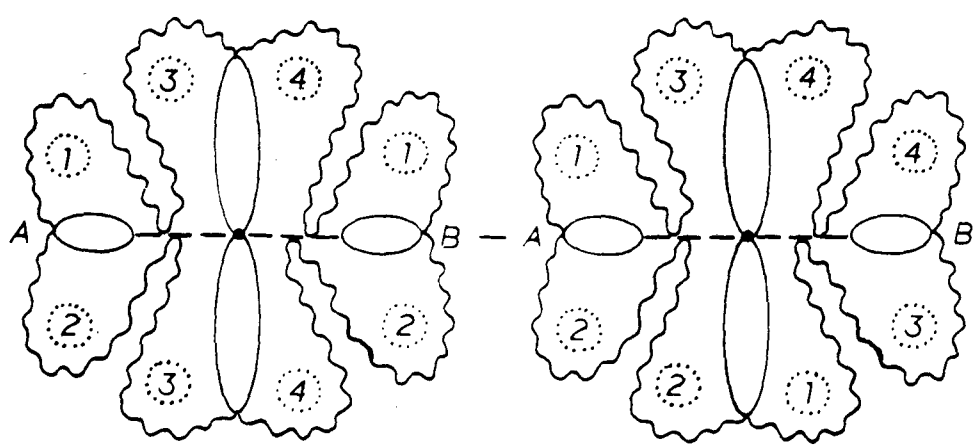

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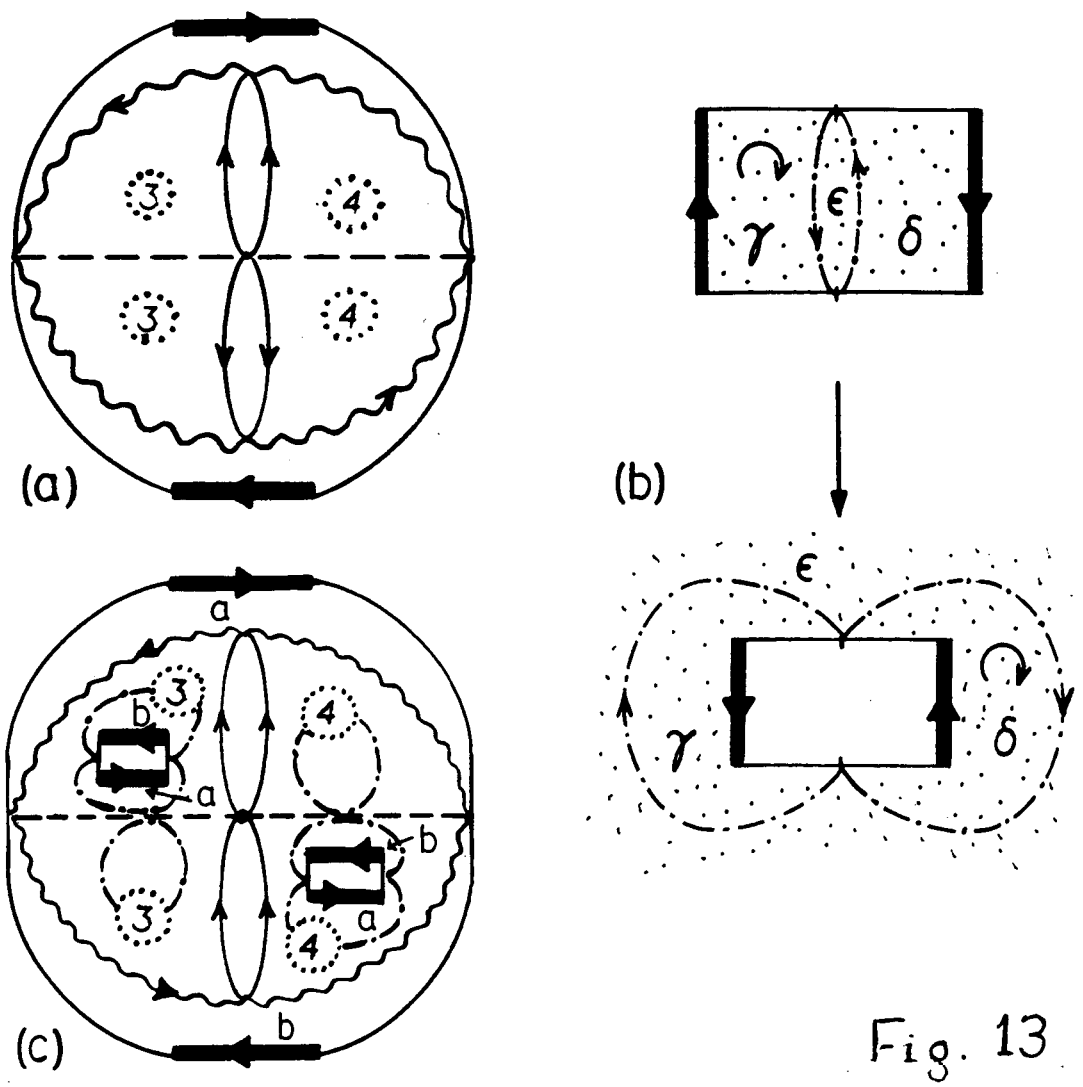
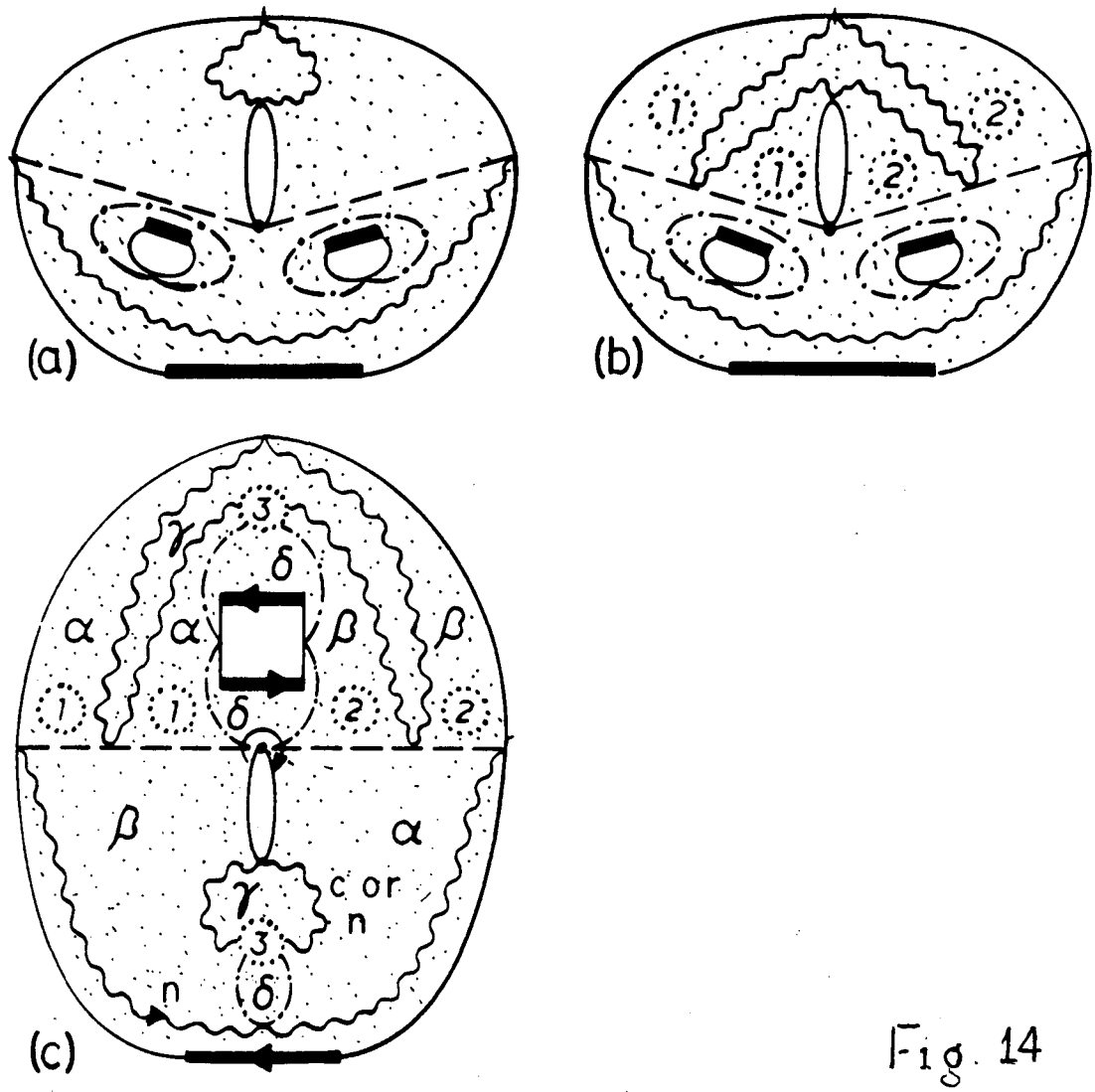


Fig. 13



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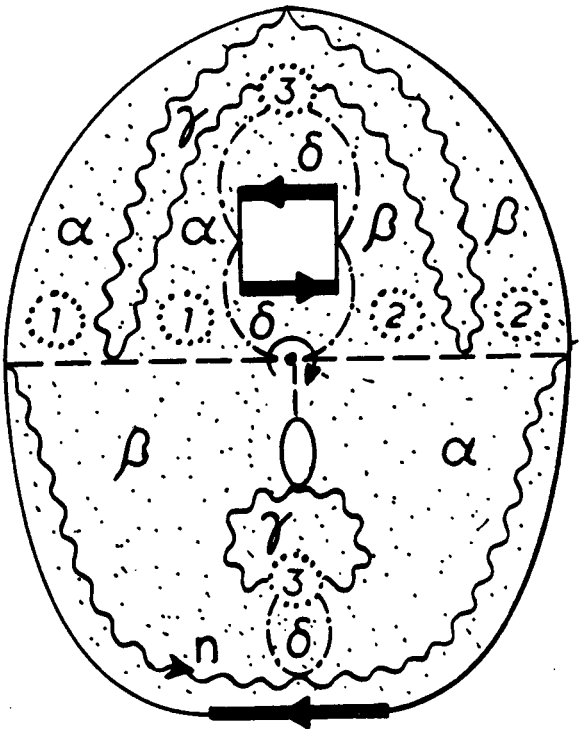

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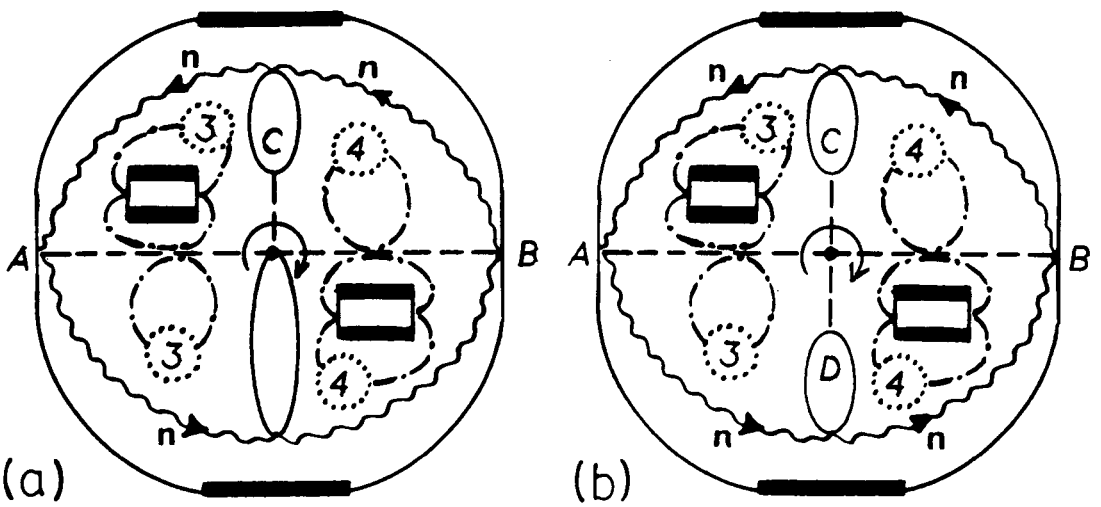

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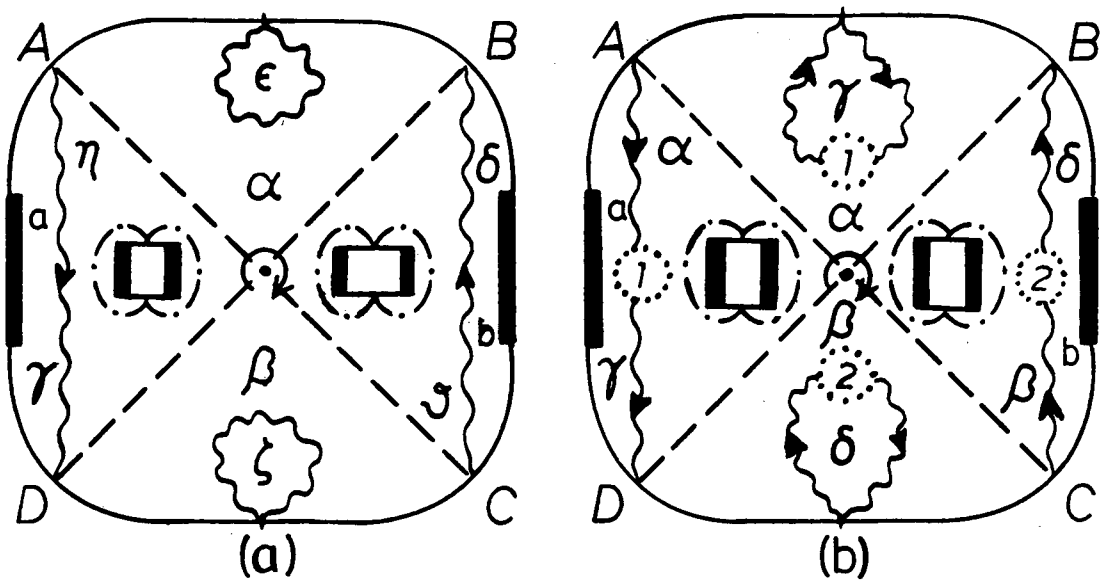

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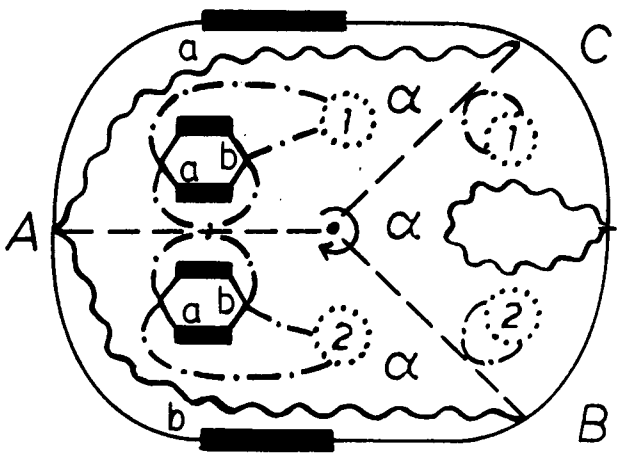

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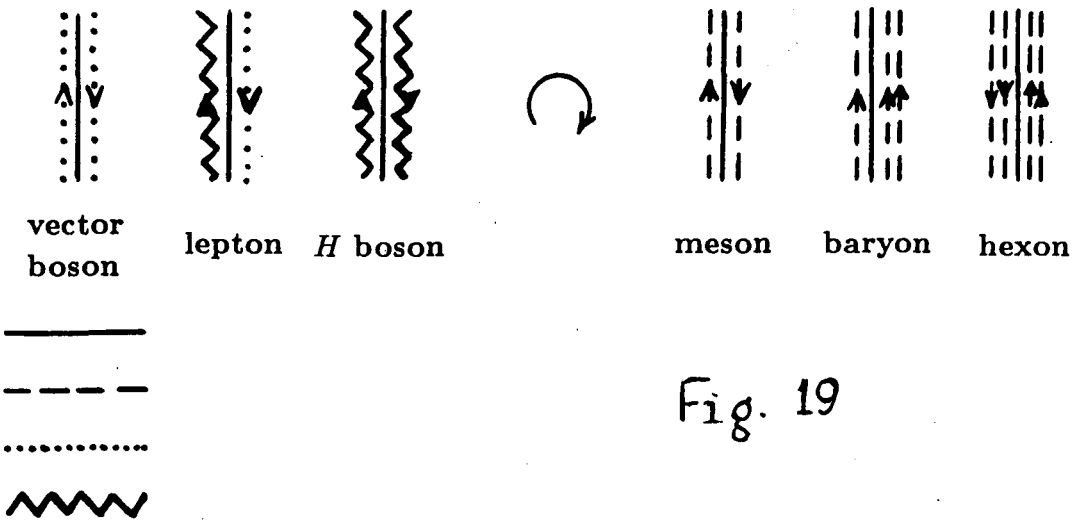

vector boson
lepton
H boson
meson
baryon
hexon

Fig. 19

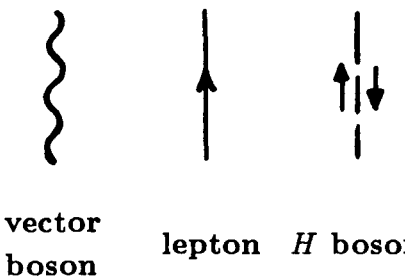
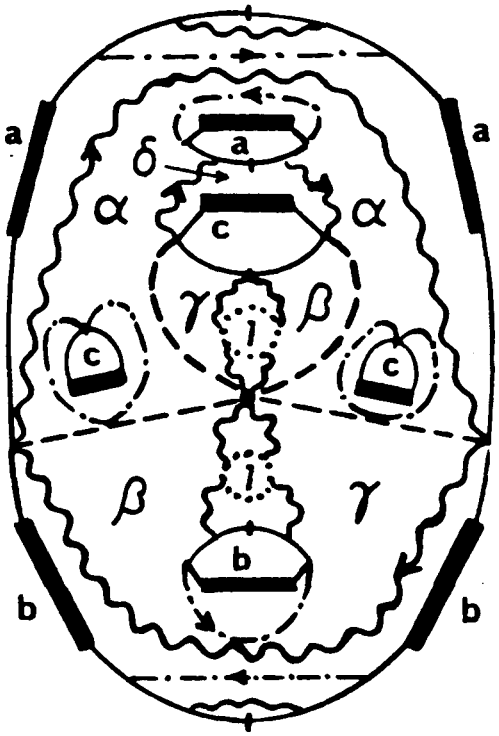

vector boson
lepton
H boson

Fig. 20



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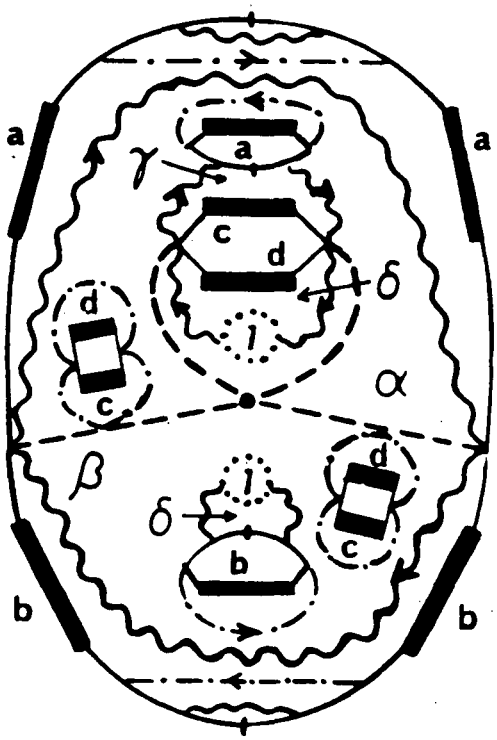

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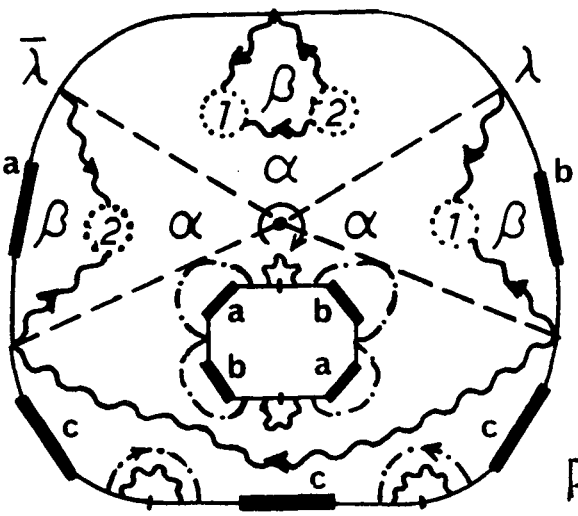

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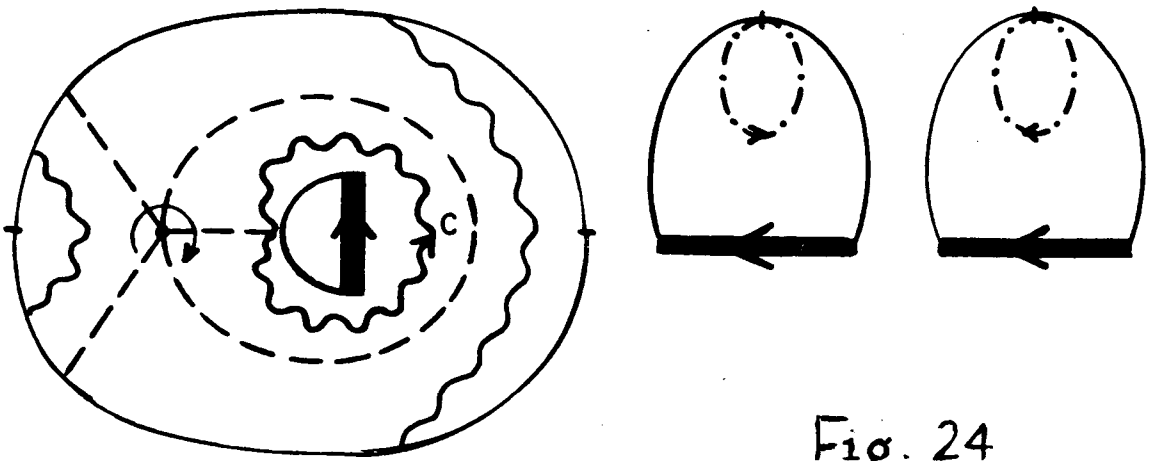

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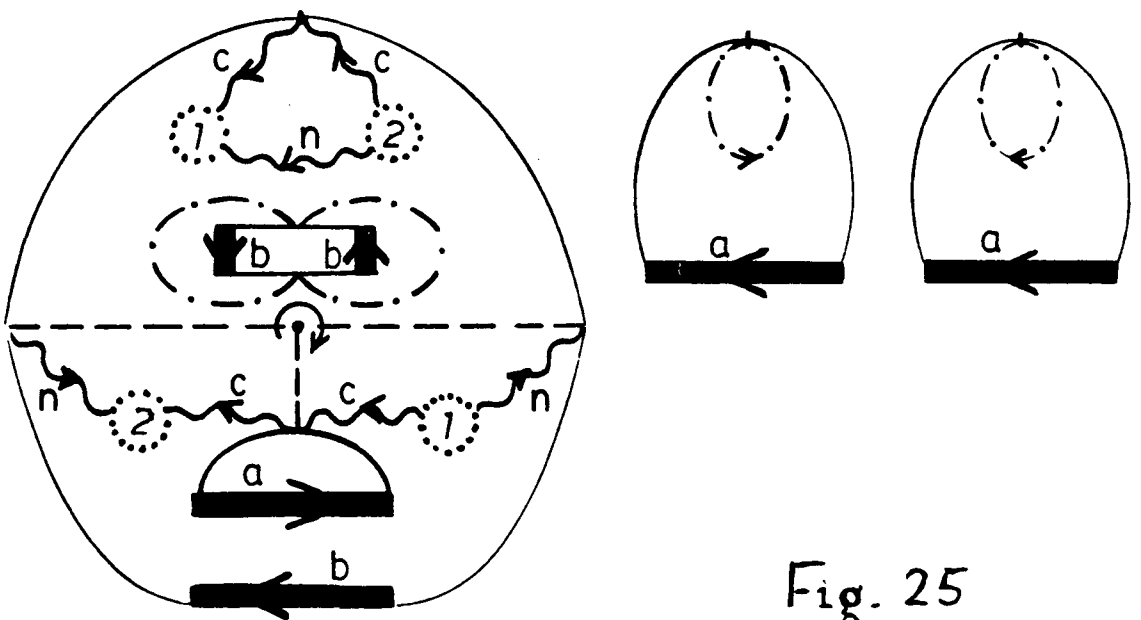

Fig. 25

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LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720