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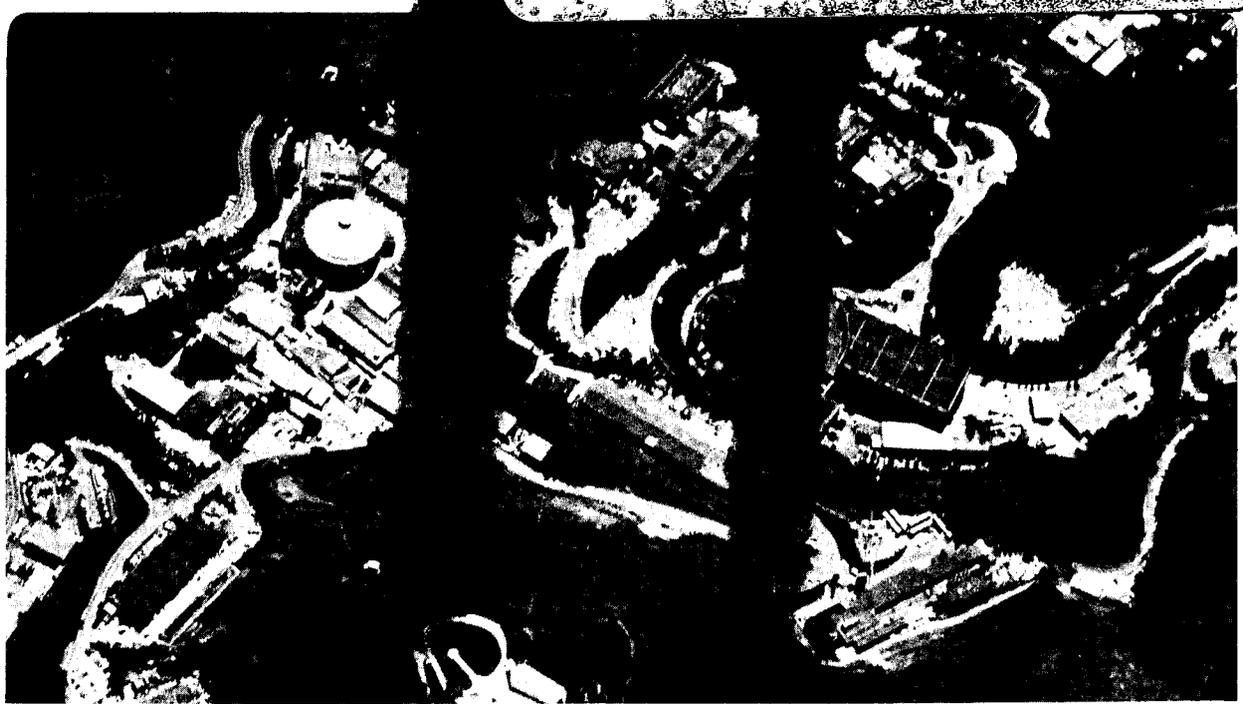
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GHOST CREATING GAUGES IN YANG-MILLS THEORY

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## INTRODUCTION

GHOST CREATING GAUGES IN YANG-MILLS THEORY<sup>1</sup>

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We study the ghost antighost symmetry of the extended BRS equations, discuss the geometrical interpretation of the formalism and define a new class of gauges in which the ghost number is only conserved modulo two.

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The extended BRS equations [1-4] which govern the unitarity [1,5] and renormalizability [1,2,6,7] of Yang-Mills theory are  $Sp(2)$  symmetric in the ghost antighost fields [8,9,10]. On the other hand, the familiar Faddeev Popov gauges [11] break this symmetry and the antighost plays no role [12] in the classification of anomalies [13,14,15].

The purpose of this note is to analyze this situation in some detail, and to emphasize the geometrical interpretation of the formalism. In the first section, we shall study the  $Sp(2) \times$  BRS semi-direct algebra and its eventual decontraction to  $Osp(1/2)$ .

In section 2, we discuss the Curci Ferrari gauges [2] which allow for a controlled breaking of the ghost-antighost symmetry [7] and relate them to the one parameter family of parallel transports that Cartan [16] has defined on a Lie group.

In section 3, we generalize these gauges by allowing the creation of ghost pairs. The theory remains unitary and renormalizable. Nevertheless, these gauges offer the possibility of occurrence, in perturbation theory, of anomalies with ghost number 3 considered recently by Faddeev [17] and Zumino [18].

In the last section we show that the  $Osp(1/2)$  group decontraction discussed in the first section leads to the massive Curci-Ferrari gauges [2] and we explain algebraically why these gauges break unitarity.

This work is a complement to our earlier detailed study with

Laurent Baulieu of the renormalizability of the extended BRS invariance entitled "The principle of BRS symmetry" [7] and we shall use the same notations. It can however be read independently.

### 1. THE Sp(2) $\times$ BRS ALGEBRA

Let  $A_\mu^a$  denote the Yang-Mills field,  $c^a$  the scalar anticommuting Faddeev Popov ghost, and  $\bar{c}^a$  the antighost, all valued in the adjoint representation of the Lie group G. The extended BRS equations [2,3,4,7] can be written:

$$\begin{aligned} sA_\mu &= D_\mu c & \bar{s}A_\mu &= D_\mu \bar{c} \\ sc &= -\frac{1}{2} [c,c] & \bar{s}\bar{c} &= -\frac{1}{2} [\bar{c},\bar{c}] \\ s\bar{c} + \bar{s}c + [c,\bar{c}] &= 0 \end{aligned} \quad (1.1)$$

Let us define a composite connection form

$$\bar{A} = A_\mu dx^\mu + c + \bar{c} \quad (1.2a)$$

and a composite differential operator:

$$\bar{d} = dx^\mu \partial_\mu + s + \bar{s} \quad (1.2b)$$

The BRS equations (1.1) can be rewritten as Maurer Cartan equations [19]

$$\bar{F} = F \quad (1.3)$$

where

$$\begin{aligned} \bar{F} &= \bar{d}\bar{A} + \frac{1}{2} [\bar{A},\bar{A}] \\ F &= dA + \frac{1}{2} [A,A] \\ &= \frac{1}{2} (\partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]) dx^\mu dx^\nu \end{aligned}$$

To define  $s\bar{c}$  in equ. (1.1), one introduces an auxiliary field  $b^a$  of dimension 2 [5,6] such that [20]:

$$\begin{aligned} s\bar{c} &= b - \frac{1}{2} [\bar{c},c] \\ \bar{s}c &= -b - \frac{1}{2} [\bar{c},c] \end{aligned} \quad (1.4)$$

Using (1.1), one may verify that

$$\begin{aligned} s^2 A_\mu &= (s\bar{s} + \bar{s}s) A_\mu = \bar{s}^2 A_\mu = 0 \\ s^2 c &= \bar{s}^2 \bar{c} = 0 \end{aligned} \quad (1.5)$$

Imposing

$$s^2 = s\bar{s} + \bar{s}s = \bar{s}^2 = 0 \quad (1.6)$$

on all fields yields the variation of  $b$  [20]:

$$\begin{aligned} sb &= -\frac{1}{2} [c,b] - \frac{1}{8} [[c,c],\bar{c}] \\ \bar{s}b &= -\frac{1}{2} [\bar{c},b] + \frac{1}{8} [[\bar{c},\bar{c}],c] \end{aligned} \quad (1.7)$$

These well known equations are explicitly symmetric under the exchange of  $c$  and  $\bar{c}$ ,  $s$  and  $\bar{s}$  and the reversal of the sign of  $b$ .

It is often more convenient [4-7] to define the variable

$$b' = b - \frac{1}{2} [\bar{c},c]$$

and rewrite the auxiliary equations 1.4 - 1.7 as

$$\begin{aligned} s\bar{c} &= b' \\ \bar{s}c &= -b' - [c,\bar{c}] \end{aligned} \quad (1.4a)$$

$$\begin{aligned} sb' &= 0 \\ \bar{s}b' &= -[c,b'] \end{aligned} \quad (1.7a)$$

However, the  $c\bar{c}$  symmetry of the equations is less manifest.

Let us now consider a continuous group Sp(2) generated by 3 operators  $\sigma^0, \sigma^+, \sigma^-$ . We define  $A_\mu$  and  $b$  as Sp(2) singlets

$$\sigma^i A_\mu = \sigma^i b = 0 \quad i = +, -, 0 \quad (1.8)$$

and  $(c, \bar{c})$  as a doublet

$$\begin{aligned} \sigma^- c &= \bar{c}, \quad \sigma^0 c = c, \quad \sigma^+ c = 0 \\ \sigma^- \bar{c} &= 0, \quad \sigma^0 \bar{c} = -\bar{c}, \quad \sigma^+ \bar{c} = c \end{aligned} \quad (1.9)$$

It is easy to verify that the  $\sigma$  operators, acting on  $A_\mu, c, \bar{c}, b$  represent the Sp(2) algebra

$$\begin{aligned} [\sigma^+, \sigma^-] &= \sigma^0 \\ [\sigma^0, \sigma^\pm] &= \pm 2\sigma^\pm \end{aligned} \quad (1.10)$$

It may also be checked that the  $s\bar{s}$  operators themselves form an Sp(2) doublet

$$\begin{aligned} [\sigma^-, s] &= \bar{s}, \quad [\sigma^0, s] = s, \quad [\sigma^+, s] = 0 \\ [\sigma^-, \bar{s}] &= 0, \quad [\sigma^0, \bar{s}] = -\bar{s}, \quad [\sigma^+, \bar{s}] = s \end{aligned} \quad (1.11)$$

A great emphasis has been put on this Sp(2) symmetry in the BRS superspace of Bonora and Tonin [8,21] Delbourgo and Jarvis [9].

In this formalism, the Sp(2) group is mixed with the Lorentz group to produce an OSp(4/2) Lorentz supergroup acting on a 6 dimensional superspace  $(x^\mu/\theta, \bar{\theta})$ . Then, considering the super Poincaré extension IOSp(4/2), they identify the BRS operators  $s$  and  $\bar{s}$  with translations in the  $\bar{\theta}, \theta$  directions. However, this supermachinery is extremely inefficient because the 8 superrotations of the OSp(4/2) Lorentz supergroup break the Maurer Cartan equation (1.3) and therefore

are outlawed in the formalism. It is not possible to restore the symmetry off-shell because unfortunately there exist no superspace action which imply the Maurer Cartan equations as classical equations of motion such that the theory could be quantized in superspace off-shell and off BRS. The best one can do [22] is to obtain the spontaneous fibration of the supermanifold [23] as rheonomy conditions using the group manifold formalism of Regge and Ne'eman. [24]. It is not known, however, how to quantize that formalism.

A more economical scheme to unify the Sp(2) and BRS algebra is implicit in the early work of Curci and Ferrari [2]. The algebra defined by eqs. (1,4,7,10,11) is a semi-direct product Sp(2)  $\times$  BRS and can be viewed as the Wigner Inonü contraction of the simple superalgebra OSp(1/2) [25]. It is indeed possible to decontract the algebra and preserve the Maurer Cartan equations (1.1,1.3). We simply modify the variation of the auxiliary field  $b$  into

$$\begin{aligned} sb &= m^2 c - \frac{1}{2} [c, b] - \frac{1}{8} [[c, c], \bar{c}] \\ \bar{s}b &= m^2 \bar{c} - \frac{1}{2} [\bar{c}, b] + \frac{1}{8} [[\bar{c}, \bar{c}], c] \end{aligned} \quad (1.12)$$

In this way, on the fields  $A_\mu, c, \bar{c}$ , the operators  $\sigma^i, s, \bar{s}$  represent OSp(1/2):

$$s^2 = m^2 \sigma^+, \quad \bar{s}^2 = -m^2 \sigma^-, \quad s\bar{s} + \bar{s}s = -m^2 \sigma^0 \quad (1.13)$$

Equations (1,4,12) define a non-linear representation of the OSp(1/2) superalgebra and the limit of vanishing  $m^2$  corresponds to a Wigner Inonü

contraction.

The action of the algebra on the auxiliary field  $b$  is however, anomalous. We have as expected

$$\begin{aligned} s^2 b &= \sigma^+ b = 0 \\ \bar{s}^2 b &= -\sigma^- b = 0 \end{aligned} \quad (1.14)$$

however:

$$(s\bar{s} + \bar{s}s) b = m^2 [c, \bar{c}] \neq \sigma^0 b = 0 \quad (1.15)$$

In section 4, by explicit construction of the Faddeev-Popov Lagrangian, we shall confirm that  $m^2$  is the mass parameter of Curci and Ferrari.

Meanwhile, we set  $m^2 = 0$ .

## 2. QUANTIZED YANG-MILLS THEORY WITH CURVATURE AND TORSION

The most general local polynomial in the fields  $A_\mu$ ,  $c, \bar{c}$ .  $b$  which is

- i) of dimension 4
- ii) globally Lorentz and group invariant
- iii) BRS and anti BRS invariant
- iv) of ghost number zero can be written [7]

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} F_{\mu\nu}^2 \\ &- \frac{1}{2} s\bar{s} (A_\mu^2 - (1+\beta)\lambda \bar{c}c) \\ &- \frac{1}{2} \beta \lambda s(\bar{c}s\bar{c}) \end{aligned} \quad (2.1)$$

Expanding this Lagrangian and eliminating the auxiliary field  $b$ , we obtain by (1.1,4,7)

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2\lambda} (\partial_\mu A_\mu)^2 \\ &+ \frac{1}{2} (\partial_\mu \bar{c} D_\mu c + D_\mu \bar{c} \partial_\mu c) \\ &- \frac{1}{2} \beta f_{abc} \partial_\mu A_\mu^a \bar{c}^b c^c \\ &- \frac{1}{16} \lambda (1-\beta^2) f_{abe} f_{cde} \bar{c}^a \bar{c}^b c^c c^d \end{aligned} \quad (2.2)$$

$\lambda$  is the usual gauge parameter,  $\beta$  controls the  $c\bar{c}$  asymmetry. When  $\beta = \pm 1$ , we recover the usual Faddeev-Popov gauges, when  $\beta = 0$ , the Lagrangian is explicitly  $c\bar{c}$  symmetric.

Substituting the equation of motion of  $b$  in (1.4) we obtain on shell

$$\begin{aligned} s\bar{c} &= -\frac{1}{\lambda} \partial_\mu A_\mu - \frac{1+\beta}{2} [\bar{c}, c] \\ \bar{s}c &= \frac{1}{\lambda} \partial_\mu A_\mu - \frac{1-\beta}{2} [\bar{c}, c] \end{aligned} \quad (2.3)$$

Remarkably, the  $\beta$  parameter is related to the geometry of the gauge group. Consider with Cartan [16] a Lie group  $G$  with generic element  $g$  and Lie algebra generators  $\lambda_a$ . The 1-forms  $\omega^a$  such that:

$$\omega = \omega^a \lambda_a = g^{-1} dg \quad (2.4)$$

define a moving frame. A parallel transport is defined by specifying, together with the  $\omega^a$  a set of connection forms  $\omega^a_b$ .

The one parameter family of transport

$$\omega^a_b = -\frac{1-\beta}{2} f^a_{bc} \omega^c \iff \pi^a_b = -\frac{1+\beta}{2} f^a_{bc} \pi^c \quad (2.5)$$

is particularly interesting. When  $\beta = 1$ , the connection vanishes and the left invariant moving frame  $\omega^a$  is parallel. On the contrary, when  $\beta = -1$ , it is easy to check that the right invariant moving frame

$$\underline{\pi} = \pi^a \lambda_a - dg g^{-1} - g \omega g^{-1} \quad (2.6)$$

is now parallel. At last, when  $\beta = 0$ , the geometry is Riemannian.

If we define à la Cartan the curvature and torsion 2-forms

$$\begin{aligned} T^a &= D\omega^a = d\omega^a + \omega^a_b \omega^b \\ R^a_b &= d\omega^a_b + \omega^a_c \omega^c_b \end{aligned} \quad (2.7)$$

and expand them over the  $\omega^a$

$$\begin{aligned} T^a &= \frac{1}{2} T^a_{bc} \omega^b \omega^c \\ R^a_b &= \frac{1}{2} R^a_{bcd} \omega^c \omega^d \end{aligned} \quad (2.8)$$

We find

$$\begin{aligned} T^a_{bc} &= -\beta f^a_{bc} \\ R^a_{bcd} &= \frac{1-\beta^2}{4} f^a_{be} f^e_{cd} \end{aligned} \quad (2.9)$$

The Riemannian connection is particularly useful if the group is not semi simple and  $f^a_{ab} \neq 0$ . In this case, indeed, the left invariant Haar measure

$$\Omega = \Pi \omega^a \quad (2.10)$$

is not invariant under the right translation generated by Lie derivative along a left invariant vector field  $D_a$ :

$$\begin{aligned} D_a \lrcorner \omega^b &= 0 \\ \Rightarrow \mathcal{L}_{D_b} \Omega &= f^a_{ab} \Omega \end{aligned} \quad (2.11)$$

This property is reflected in the fact that the Jacobian of the BRS transformation of the Yang-Mills field does not vanish

$$\text{Tr} \left( \frac{\partial s A^a_\mu}{\partial A^b_\nu} \right) = \delta^\mu_\nu f^a_{abc} \quad (2.12)$$

On the other hand, by definition of curvature, if  $\omega^a$  is parallel transported around an elementary cycle of base  $D_c, D_d$ , it will rotate and translate by an amount

$$\delta_{cd} \omega^a = R^a_{bcd} \omega^b + T^a_{cd} \quad (2.13)$$

Therefore, in the Riemannian case, the transport of the volume element over the group is path independent:

$$\delta_{cd} \Omega = R^a_{acd} = f^a_{ab} f^b_{cd} = 0 \quad (2.14)$$

This property, which holds even when  $f^a_{ab}$  does not vanish is reflected in the fact that a gauge transformation of  $A_\mu$  with gauge parameter  $[\bar{c}, c]$ :

$$\delta A_\mu = D_\mu [\bar{c}] \quad (2.15)$$

has vanishing Jacobian for an arbitrary Lie group.

Substituting (2.9) in (2.2) we obtain a geometrical form of the Curci-Ferrari Lagrangian:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2} F_{\mu\nu}^2 - \frac{1}{2\lambda} (\partial_\mu A_\mu)^2 \\ &+ \frac{1}{2} (\partial_\mu \bar{c} D_\mu c + D_\mu \bar{c} \partial_\mu c) \\ &- \frac{1}{2} T_{abc} \partial_\mu A_\mu^a \bar{c}^b c^c \\ &- \frac{1}{2} R_{abcd} \bar{c}^a \bar{c}^b c^c c^d \end{aligned} \quad (2.16)$$

which is reminiscent of the super  $\sigma$ -model [26].

Consider now the multiplication map on the Lie group

$$\begin{aligned} G \times G &\rightarrow G \\ \mu_L(x, y) &= xy \end{aligned} \quad (2.17)$$

If we pull back by this map the left invariant form  $\omega_{xy}$

onto  $(G \times G)^*$ , we obtain

$$\begin{aligned} \mu_L^*(\omega_{xy}) &= (xy)^{-1} d(xy) \\ &= y^{-1} x^{-1} dx y + y^{-1} dy \end{aligned} \quad (2.18)$$

Let us call  $\rho$  and  $\bar{\rho}$  the components

$$\begin{aligned} \rho &= y^{-1} x^{-1} dx y \\ \bar{\rho} &= y^{-1} dy \end{aligned} \quad (2.19)$$

It is immediate to verify that

$$\begin{aligned} d_x \rho + \rho \rho &= 0 & d_x \bar{\rho} &= 0 \\ d_y \bar{\rho} + \bar{\rho} \bar{\rho} &= 0 & d_y \rho &= -[\rho, \bar{\rho}] \end{aligned} \quad (2.20)$$

On the other hand, if we pull back the right invariant form  $\Pi_{xy}$

or if we use the reversed map  $(x,y) \rightarrow yx$

$$\mu_R^*(\omega_{yx}) = x^{-1} dx + x^{-1} y^{-1} dy x \quad (2.21)$$

The mixed equation in (2.20) is replaced by

$$\begin{aligned} d_x \bar{\rho} &= -[\rho, \bar{\rho}] \\ d_y \rho &= 0 \end{aligned} \quad (2.20a)$$

Comparing with (2.3), we may identify, when  $\beta = \pm 1$ , the operator  $s, \bar{s}$  with  $d_x, d_y$  and associate the gauge choices with left  $\mu_L$  and right  $\mu_R$  multiplication map.

### 3. GHOST CREATING GAUGES

In the non Riemannian case  $\beta \neq 0$ , the Lagrangian (2.1) explicitly breaks the  $Sp(2)$  symmetry of the BRS equations:

$$[\sigma^+ + \sigma^-] \mathcal{L} = \beta(sc + \bar{s}\bar{c})s\bar{c} \quad (3.1)$$

This suggests the idea of adding a term of this form to the Lagrangian and to consider the 3 parameter gauges:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2} F_{\mu\nu}^2 - \frac{1}{2} s\bar{s}(A_\mu^2 - \lambda(1+\beta)\bar{c}c) \\ &\quad - \frac{1}{2} \beta \lambda s(\bar{c}s\bar{c}) \\ &\quad + \frac{1}{2} \lambda \gamma s\bar{c}(sc + \bar{s}\bar{c}) \end{aligned} \quad (3.2)$$

Of course, the new parameter  $\gamma$  breaks ghost conservation, but this should not jeopardize unitarity because  $\gamma$  multiplies an  $s$  exact operator. Moreover, (3.2) is the most general Lagrangian which violates ghost conservation but yet preserves BRS and anti BRS symmetry. The new model is therefore automatically renormalizable.

Developing (3.2) and eliminating the  $b$  field we find:

$$\begin{aligned} \mathcal{L}' &= -\frac{1}{2} F_{\mu\nu}^2 - \frac{1}{2\lambda} (\partial_\mu A_\mu)^2 \\ &\quad + \frac{1}{2} (D_\mu \bar{c} \partial_\mu c + \partial_\mu \bar{c} D_\mu c) \\ &\quad - \frac{\beta}{2} \partial_\mu A_\mu [\bar{c}, c] \\ &\quad - \frac{\gamma}{4} \partial_\mu A_\mu ([\bar{c}, \bar{c}] + [c, c]) \\ &\quad - \lambda(1-\beta^2 + \gamma^2)/16 [\bar{c}\bar{c}][cc] \end{aligned} \quad (3.3)$$

Evaluating explicitly the one loop counterterms we find that all Ward identities are satisfied and that the gauge parameters are renormalized in the notations of [7] in the following way:

$$Z_\lambda = Z_3 - \frac{Cg^2}{(4\pi)^2 \epsilon} \frac{1 - \beta^2 + \gamma^2}{2}$$

$$Z_\beta = Z_\gamma = 1 \quad (3.4)$$

Thus, we find the remarkable result that the geometrical parameters  $\beta$  and  $\gamma$  are not renormalized at the one-loop level. We have found no Ward identity explaining this stability. (eq. 5.43 of ref. [7] is wrong.)

The gauges  $1 - \beta^2 + \gamma^2 = 0$  are particularly interesting. There, the longitudinal part of the gluon propagator is finite, a well known result in the Faddeev-Popov gauge ( $\beta=1, \gamma=0$ ); furthermore, the 4-ghosts contact term is absent.

Restricting again, the choice ( $\lambda=1, \beta=0, \gamma^2=-1$ ) is especially compelling.

The Lagrangian:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} (\partial_\mu A_\mu)^2$$

$$+ \frac{1}{2} (D_\mu \bar{c} \partial_\mu c + \partial_\mu \bar{c} D_\mu c)$$

$$- \frac{1}{4} \partial_\mu A_\mu ([\bar{c}, \bar{c}] + [c, c]) \quad (3.5)$$

is the Feynman gauge, the  $Sp(2)$  symmetry is manifest and there are no 4-ghost interactions.

Unitarity is maintained despite the creation of ghosts for the following reason. At the tree level, pair of ghosts are only emitted by non-physical longitudinal photons. Consider now a pair of ghosts,  $c^a c^b$  coupled to an arbitrary physical state. Since the S-matrix element must be anti-symmetrized in  $c^a c^b$ ,  $[c^a, c^b]$  can be thought of as emitted by an operator  $sc$ . The BRS operator commutes with the S-matrix

and annihilates the physical states. Therefore, the whole S-matrix element vanishes.

#### 4. ANOMALIES

Consider again the composite gauge field

$$\tilde{A} = A_\mu dx^\mu + c \quad (4.1)$$

satisfying the Maurer Cartan BRS equation [19]:

$$\tilde{F} = F \quad (4.2)$$

Construct in  $d$  dimensions the Chern Simons form  $\omega$  of degree  $d+g$  satisfying

$$d\omega(\tilde{A}) = \text{Tr}(\tilde{F}^n) = \text{Tr}(F^n) \quad 2n = d+g+1 \quad (4.3)$$

If, using (4.1), we develop  $\omega$  in power of  $c$ , the term of degree  $g$ :

$$\omega_d^g(A, c) = \frac{1}{g!} c \frac{\delta}{\delta \tilde{A}} \omega(\tilde{A}) \Big|_{\tilde{A}=A} \quad (4.4)$$

satisfies the Wess-Zumino equation [13,27]:

$$s\omega_d^g = 0 \quad (4.5)$$

$\omega_d^1$  is the generating functional of the BRS transform of the anomalous diagrams of the theory [13,14,15]. Moreover, the reciprocal holds [12]: Any local polynomial in the fields  $(A_\mu, c, \bar{c}, b)$  and the source operators which solves the Wess-Zumino equation (4.5) non-trivially ( $s\omega \neq 0$ ) can be gauge transformed to the form (4.4).

This reciprocal implies that:

- 1) The antighosts play no role in the classification of the anomalies.

ii) The gauge equivalence classes of anomalies are put in one to one correspondence with the cohomology classes of the classical principal fiber bundle  $P$  by identifying  $\bar{A}$  as an Ehresman connection on  $P$  [19].

The usual anomalies which occur in perturbation theory have ghost number 1 and generate the BRS transform of the anomalous diagrams with external gluons. However, the construction (4.1,4.4) also provides solutions with ghost number 0,2,3... and one would like to interpret these geometrical objects in the quantized field formalism.

The  $g=0$  "anomalies" are just topological mass terms [28] and occur in odd dimension.

It has been shown [17] by Faddeev that the  $\omega_3^2$   $g=2$  anomaly induces a Schwinger term in the commutator of constraints in Yang-Mills model.

At last, the  $g=3$  anomaly could be viewed as the breaking of the Jacobi identity around an inconsistent magnetic monopole not satisfying the Dirac quantization condition [18].

However, in the usual gauges, the  $g=3$  anomaly cannot occur in perturbation since the  $g=2$  diagram vanishes. In our new gauges, corresponding to a Weyl invariant symmetric tensor of degree  $n$ :  $2n = d + 4$ , there corresponds an anomalous  $g=3$  term  $\omega_d^3 = d_{abcd} \dots c^a [c, c]_{F\bar{F}}^{bc} \dots$

The corresponding diagram has  $3 + d/2$  external particles: a longitudinal gluon, a pair of ghosts and  $d/2$  transverse gluons. The diagram converges in  $d=4$  and  $d=6$  but is linearly divergent in  $d=8$ . Since there is no direct ghost-fermion coupling, the potentially dangerous diagrams only occur at 2-loops and presumably can be regulated

in a gauge invariant way. Therefore, it seems that these anomalies, although possible, are not generated.

The question remains open in supergravity.

## 5. CURCI FERRARI GAUGES

Let us now return to the  $OSp(4/2)$  simple superalgebra defined in section 1, eq. 12-13.

If  $m^2 \neq 0$ , the Lagrangians (2.1) or (3.2) are no longer BRS invariant. Rather we must consider the Lagrangian:

$$= - \frac{1}{2} F_{\mu\nu}^2 - \frac{1}{2} (s\bar{s} - m^2)(A_\mu^2 - \lambda\bar{c}c) \quad (5.1)$$

Expanding this equation using 1.1, 1.4 and 1.11 rather than 1.7, we obtain in addition to the Lagrangian 2.2 the Curci Ferrari mass term [2]:

$$\mathcal{L}_m = - \frac{m^2}{2} A_\mu^2 - 2\lambda m^2 \bar{c}c \quad (5.2)$$

In the Feynmann gauge  $\lambda=1$  the masses of the ghost and gluon are equal. These gauges provide an infrared regularization of Yang-Mills theory. However, Curci and Ferrari have shown [2] that the theory is unitary only in the  $m^2 = 0$  limit.

In our formalism, we may understand the breaking of unitarity algebraically. In the usual construction, the physical states are defined as cohomology classes of the BRS operator:

$$s | \text{phys} \rangle = 0 \quad (5.3a)$$

$$| \text{phys} \rangle = | \text{phys} \rangle + s | \Lambda \rangle \quad (5.3b)$$

This construction is meaningful as long as

$$s^2 = 0 \tag{5.4}$$

In the massive scheme, we have:

$$s^2 = m^2 \sigma^+$$

and equations 5.3a and b become incompatible since any state  $|\Lambda\rangle$  with ghost number  $-1$  belongs at least to a triplet and  $s^2|\Lambda\rangle \neq 0$ .

It follows that the BRS operator can no longer be used to remove the longitudinal degrees of freedom and the formalism collapses.

#### CONCLUSION

The new gauges considered in this note provide a systematic generalization of several earlier studies [2,7,10,21]. We have found a renormalizable and unitary gauge with continuous  $Sp(2)$   $c\bar{c}$  invariance and related it to the Riemannian connection on a Lie group [16].

Then, abandoning ghost conservation, we have constructed a Feynman gauge where ghost and antighosts pairs are emitted by longitudinal photons, the  $Sp(2)$  symmetry is explicit and the 4-ghost interaction is absent. In these gauges, ghost number 3 anomalies are a priori possible in supergravity.

The ghost creating gauges, and the relation to the geometry of the Lie groups, are certainly amusing and curious. We hope that they will prove useful in future models.

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