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# NUMBER-DISPLACEMENT DEGREES OF FREEDOM IN NUCLEAR ROTATIONAL THEORY*

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Abstract:

A new degree of freedom associated with shifting of particles between the even-parity orbitals system and the odd-parity orbitals system is introduced in the study of nuclear rotational problem. It is found that this new mode contributes significantly to the deviation of the I(I + 1) rule with correction terms comparable to the Coriolis-anti-pairing effect.

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In theoretical calculations of nuclear rotational spectrum, several specific collective degrees of freedom have been considered as influencing deviations from the I(I + 1) energy spacings. There are the proton and neutron pairing correlations involved in the Coriolis anti-pairing (CAP) effect. There

are the beta- and gamma-vibrational degrees of freedom involved in shape change effects. There is also a fourth-order cranking effect that may be expressed in terms of an additional collective degree of freedom.

The total energy E of a deformed rotating system may be expressed in terms of these collective degrees of freedom as[1]

$$E = \sum_i \frac{1}{2} C_i (X_i - X_{io})^2 + \frac{I(I+1)}{2J(X_i)} \tag{1}$$

where $X_i$ (i = 1, 2, 3, . . .) describes the various degrees of freedom, $C_i$ is the spring constant associated with the i-th degree of freedom and $J(X_i)$ is the moment-of-inertia. The rotational solution may be obtained by minimizing the total energy of Eq. (1) with respect to each of $X_i$ at a given angular momentum I.

In this note we shall identify a new degree of freedom which will affect both the moment-of-inertia and the potential energy of Eq. (1). We first express the moment-of-inertia J in terms of the cranking formula of Inglis[2] and Belyaev[3]

$$J = 2\hbar^2 \sum_{\Omega_\alpha > 0} \frac{\langle \alpha' | j_x | \alpha \rangle}{E_{\alpha'} + E_\alpha} (U_{\alpha'} V_\alpha - U_\alpha V_{\alpha'})^2 \tag{2}$$

where $|\alpha\rangle$ is the deformed single particle state with $\alpha$ denoting the appropriate quantum numbers, $\Omega_\alpha$ is the projection of the angular momentum on the symmetry axis, $E_\alpha$ is the quasi-particle energy and $U_\alpha$, $V_\alpha$ are the probability coefficients

$$\left.\begin{matrix} U_\alpha^2 \\ V_\alpha^2 \end{matrix}\right\} = \frac{1}{2} \left[ 1 \pm \frac{\varepsilon_\alpha - \lambda}{\sqrt{(\varepsilon_\alpha - \lambda)^2 + \nu^2}} \right] \tag{3}$$

where $\varepsilon_\alpha$ is the single particle energy, $\nu$ is the pairing correlation parameter and $\lambda$, the chemical potential. In usual practice, the chemical potential takes

two values, $\lambda_p$ for the proton system and $\lambda_n$ for the neutron system. What interests us here is the fact that the even-parity orbital systems and odd-parity orbital systems make separable contributions to the moment-of-inertia, since the $j_x$ operator has even parity. Thus, it is possible that the nuclear system could change its moment-of-inertia by shifting nucleons between the two orbital systems of opposite parity. In other words, the nuclear system might assume unequal chemical potentials ($\lambda_+$ and $\lambda_-$) for the different-parity orbital systems in order to minimize the total energy for a given angular momentum.

The potential energy term corresponding to the separation of $\lambda_+$ and $\lambda_-$ is easily derived in the limit of the continuous model, where the discrete distributions of Nilsson orbital energies are replaced by a density function $\rho$. Let $\rho_+$ and $\rho_-$ be the number of Nilsson orbitals per energy interval for positive parity and negative parity, respectively. The number of nucleons transferred for a given change of chemical potential $\lambda$ is by definition of $\rho$

$$dn = 2\rho d\lambda \quad , \tag{4}$$

noting that Nilsson orbitals are doubly degenerate. Thus, conservation of particle number gives the condition

$$\rho_+ d\lambda_+ = -\rho_- d\lambda_- \tag{5}$$

The new degree of freedom will be characterized by $\sigma$, the difference of the chemical potentials.

$$\sigma \equiv \lambda_+ - \lambda_- \quad . \tag{6}$$

The fundamental definition of the chemical potential is

$$\lambda_{\pm} = \frac{dE}{dn_{\pm}} \quad .$$

Integrating in the continuous model, using Eqs. (4) - (6), we obtain

$$\begin{aligned} E &= \int \lambda_+ \, dn_+ + \int \lambda_- \, dn_- = \int (\lambda_+ - \lambda_-) \, dn_+ \\ &= 2\rho_+ \int \sigma d\lambda_+ = \frac{2\rho_+\rho_-}{\rho_+ + \rho_-} \int \sigma d\sigma \\ &= \frac{2\rho_+\rho_-}{\rho_+ + \rho_-} \, \frac{1}{2} \sigma^2 = \frac{1}{2} C_\sigma \, \sigma^2 \end{aligned} \tag{7}$$

In the continuous model with the constant G pairing assumption the pairing energy terms proportional to G will not be altered. We see that the restoring potential against non-zero $\sigma$ is harmonic and dependent solely on level densities. The desired spring constant $C_\sigma$ associated with the number displacement degree of freedom is thus

$$C_\sigma = \frac{2\rho_+\rho_-}{\rho_+ + \rho_-} \tag{8}$$

The effect of the new degree of freedom can be estimated by calculating its contribution to the correction term $BI^2(I + 1)^2$ in an expansion of the rotational energy.[1,4,5] The general form of the coefficient B is given by[1]

$$B = -\sum_i \frac{1}{8C_i J^4} \left(\frac{\partial J}{\partial X_i}\right)^2 \tag{9}$$

Eq. (9) shows that each degree of freedom makes additive contribution which is inversely proportional to the spring constant $C_i$ and directly proportional to the square of the corresponding partial derivative of the moment-of-inertia.

We have calculated the various derivatives of moments-of-inertia with respect to chemical potentials $\lambda_+$ and $\lambda_-$ for protons and neutrons for several nuclei in the rare earth region. The calculations were made with the new Nilsson et al.[6] method with diagonalization over the space $0 \leqslant N \leqslant 12$ and with both quadrupole ($\varepsilon$) and hexadecapole ($\varepsilon_4$) deformation taken into account. The pairing force strengths, number of orbitals included in the BCS solution, and potential parameters $\kappa$ and $\mu$ are as given by Nilsson et al.[6] Table I gives for three nuclei the deformations assumed and the calculated inertial derivatives. It can be shown from Eqs. (5) and (6) that $\frac{\partial J}{\partial \sigma} = [\rho_- \frac{\partial J}{\partial \lambda_+} - \rho_+ \frac{\partial J}{\partial \lambda_-}]/\rho$. The trends of these derivatives are understandable from inspection of Fig. 1 of Ref. 1, where the contributions of various j shells to the moment-of-inertia are graphed. One expects large positive derivatives in the beginning of the deformed region (i.e., $^{152}$Sm), especially from $i_{13/2}$ neutron orbitals, which are just beginning to fill. These derivatives decrease around mid-shell.

For the estimate of the spring constants in the $\sigma$ mode we have used for total level density $\rho$ a Strutinsky-type smeared average over the new Nilsson level diagram at $\varepsilon = 0.25$ and $\varepsilon_4 = 0$ with a smearing constant of $0.4\ \hbar\omega_o$. The ratios $\rho_+/\rho_-$ were roughly estimated from the total of shell model orbitals in the 50-82 shell for protons and the 82-126 shell for neutrons. That is, for protons $\rho_+/\rho_- = 10/6$ and for neutrons $\rho_+/\rho_- = 7/15$. With these level density values were calculated the spring constants according to Eq. (8). In turn were calculated the contributions to the $BI^2(I + 1)^2$ term according to Eq. (9) where the moments-of-inertia were taken from the experimental $2^+$ energy levels. These values are summarized in Table II along with comparison values of the total CAP contribution according to Marshalek[7] and the experimental B value.

Including these new degrees of freedom ($\sigma$-effect) actually makes worse the disagreement with experiment in Refs. 1 and 7, where the CAP contribution ($B_\Delta$) alone often exceeds experimental. However, the disagreement of cranking model calculations with experiment is a much broader question which we do not address here.

It is evident that the $\sigma$ effect contributions due to the number displacement degree of freedom are very significant and cannot be neglected. Of course, there should be made more realistic calculations of the spring constant $C_\sigma$ without using the continuous model, but we do not expect a qualitative change from the estimates here. The $\sigma$-degrees of freedom may alter the role of subshell irregularities in the Coriolis anti-pairing effect.

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## FOOTNOTES AND REFERENCES

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†On leave from Yale University.

1. C. W. Ma and J. O. Rasmussen, Phys. Rev. C2, 798 (1970).
2. D. R. Inglis, Phys. Rev. 96, 1059 (1954).
3. S. T. Belyaev, Kgl. Danske Videnskab Selskab, Mat.-Fys. Medd. 31, No. 11 (1959).
4. A. Bohr and B. R. Mottelson, Mat. Fys. Medd. Dan. Vid. Selsk. (1953).
5. V. G. Soloviev, Theory of Complex Nuclei, Science Press, Moscow (1971); (in Russian) (English translation, Pergamon Press, in progress).
6. S. G. Nilsson et al., Nucl. Phys. A131, 1 (1969).
7. E. R. Marshalek, Phys. Rev. 158, 993 (1967).

Table I. Derivatives of Cranking Moment-of-Inertia with Respect to Chemical Potentials (All Units in $MeV^{-2}$).

| Nucleus | $\varepsilon$ | $\varepsilon_4$ | $\frac{\partial 2J}{\partial \lambda_{+p}}$ | $\frac{\partial 2J}{\partial \lambda_{-p}}$ | $\frac{\partial 2J}{\partial \sigma_p}$ | $\frac{\partial 2J}{\partial \lambda_{+n}}$ | $\frac{\partial 2J}{\partial \lambda_{-n}}$ | $\frac{\partial 2J}{\partial \sigma_n}$ |
|---|---|---|---|---|---|---|---|---|
| $^{152}Sm$ | 0.202 | -0.036 | 7.42 | 22.80 | -11.5 | 44.32 | 19.40 | 24.0 |
| $^{162}Dy$ | 0.256 | -0.006 | 5.46 | 17.97 | - 9.17 | 37.16 | 17.38 | 19.8 |
| $^{178}Hf$ | 0.250 | 0.052 | 0.30 | 8.83 | - 5.40 | 25.65 | 5.66 | 15.7 |

Table II. Number-Displacement Effect Calculations and Comparison of CAP Values.

| Nucleus | $\rho_p$ ($MeV^{-1}$) | $\rho_n$ ($MeV^{-1}$) | $C_{\sigma p}$ ($MeV^{-1}$) | $C_{\sigma n}$ ($MeV^{-1}$) | $B_{\sigma p}$ ($MeV^{-1}$) | $B_{\sigma n}$ (eV) | $B_{exp}$ (eV) | $B_{\Delta}$ Marshalek[7] (eV) |
|---|---|---|---|---|---|---|---|---|
| $^{152}Sm$ | 2.32 | 3.40 | 1.09 | 1.48 | 10.2 | 33.1 | 193 | 33.7 |
| $^{162}Dy$ | 2.20 | 3.03 | 1.03 | 1.32 | 1.33 | 4.86 | 10.8 | 13.1 |
| $^{178}Hf$ | 2.68 | 2.88 | 1.26 | 1.25 | 0.67 | 5.79 | 13.1 | 19.4 |

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