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BACKBENDING AND ROTATION ALIGNMENT*

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Abstract

We have examined the Z and N dependence of backbending in the rare-earth region as given by a model based on the rotation alignment of two $i_{13/2}$ neutrons. The important parameters in this model are the rotational moment of inertia, the position of the fermi surface, and the deformation (both quadrupole and hexadecapole). To fix these (and other) parameters most reliably, we propose a comparison with the observed rotation alignment of a single $i_{13/2}$ neutron in the adjacent odd-A nuclei. This comparison is made qualitatively throughout the rare-earth region, and then quantitatively for the two pairs, ^{161}Er, ^{162}Er and ^{171}Hf, ^{172}Hf. Based on these comparisons it seems plausible that all the backbending so far observed in the rare-earth region could be due to such rotation alignment.

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1. Introduction

It is now clear that a rather sudden structural change occurs in the ground-state rotational band of a considerable number of rare-earth even nuclei at high angular momenta[1,2]. Early evidence for such a change came from the population patterns of such bands following (HI,xn) reactions[3], but the conclusive step was the observation of irregularities in the rotational-energy spacings[4]. These irregularities are such that two or three rotational transitions in the region of I $\sim$ 12-20 become lower in energy with increasing I, whereas transitions above and below this I-region have the normal (rotational) monotonic increase in energy with I. If the moment-of-inertia, $\mathfrak{J}$, is plotted against the square of the rotational frequency, ω^2, such a behavior produces a "backbending" curve (larger $\mathfrak{J}$ but smaller values for ω^2). There has recently been considerable interest in determining the nature of the structural change responsible for this behavior.

Rotation of the nucleus introduces Coriolis and centrifugal forces into the nuclear-fixed system. Since the rare-earth nuclei we are considering are clearly rotational nuclei, some consequence of these new forces must be responsible for the observed changes, and suggestions have been made involving each force. Thieburger[5] and others[6,7] have shown that the backbending could be a centrifugal stretching effect. However, such an explanation does not explain the feeding data, nor is the required potential energy versus deformation curve given *a priori* by microscopic nuclear-structure calculations. On the other hand, explanations involving the Coriolis force can explain the feeding data together with the backbending effects, and can be directly related to the *a priori* microscopic structure. Thus, at the present time, it seems more probable that the observed changes are Coriolis effects.

There have been two types of proposals for the detailed way in which the Coriolis force might affect the nuclear structure in order to produce the observed effects. The older one, which predated the experimental observations is that the pairing correlations might be quenched, resulting in normal, rather than superfluid, nuclei above the critical spin value. This "phase transition" was proposed by Mottelson and Valatin[8]) in analogy with the Meissner effect for superconductors in a magnetic field. The other type of explanation[9]) suggests that certain individual nucleons may respond to the Coriolis force prior to the phase transition. It is clear that the Coriolis force tends to align the angular momentum of a particle with that of the rotor, the tendency being stronger the larger the value of j for the particle. In the even rare-earth nuclei, 12 units of angular momentum can be obtained in this way by "rotation alignment" of a single pair of $i_{13/2}$ neutrons. It seems plausible that a band involving one such pair could drop below the normal (completely-paired) ground band at these spin values, and thus cause the backbend. The question then becomes which of these processes is responsible for the observed changes, or is it a mixture of the two, or perhaps neither.

Two types of calculations can give information on this question. The first type would be sufficiently general to include both possibilities, so that a calculation of the competition between the processes would be possible. Until recently, the calculations with sufficiently general rotational Hamiltonians, had unrealistically simple particle spaces, but considerable progress is now being made in this direction[10,11]). The other possibility is to pursue the consequences of the separate models to see how well in accord with the observations these are. The present paper is an attempt to do this for the rotation-alignment model.

Specifically, we have examined the Z and N dependence of backbending in the rare-earth region as expected from this model. Our progress has come in two steps. The first was the observation that the hexadecapole deformation, ε_4, is an important parameter for these calculations, and the second was the realization that the adjacent odd nuclei provide independent measures of the tendency for the $i_{13/2}$ particles to align their angular momentum with the core rotation in a given region. Thus, the parameters entering an even-mass calculation can be extracted from experimental data in the odd nuclei. These considerations have given additional insight into the rotation-alignment picture, and show it to be in reasonable accord with the experimental data.

2. Qualitative Description

The underlying model for the effects we want to estimate here will be that of Stephens and Simon[9]) (SS). A short quantitative description of this model is given in section 3; here we want to consider only the general features. The basic proposal is that certain 2-quasiparticle states in even-even nuclei gain enough Coriolis energy by aligning their angular momenta with the rotation axis so that at high spin values they become the lowest states. The maximum Coriolis energy that can be gained for each particle is approximately:

$$E_{max} \approx 2Ij(\hbar^2/2\mathfrak{J}) \qquad (1)$$

where I is the total angular momentum, j is the particle angular momentum, and $\hbar^2/2\mathfrak{J}$ is the rotational constant. Clearly the high-j particles can gain the most energy; so that, in the rare-earth region, the $i_{13/2}$ neutrons are most favorable. The mathematical mechanism for the decoupling is just a mixing of states by the Coriolis interaction, resulting eventually in a lowest mixed state which is the one aligned with the rotation axis. Thus, effects which tend to increase the Coriolis mixing will enhance the rotational alignment and vice versa. One other feature is important. The rotation-aligned state has its angular momentum nearly perpendicular to the symmetry axis. Thus, the projection of the particle angular momentum on the symmetry axis must be small, i.e. mainly low Ω values are involved.

The single-particle energies going into the SS calculation are taken to be the eigenfunctions of a Nilsson-type calculation[12]). Within the above framework we can estimate how various parameters will effect the rotation alignment. For instance, it is clear that a fermi-surface (λ) near the low-Ω orbitals of the $i_{13/2}$ j-shell is favorable, since, in that case, these essential states lie low in the quasiparticle spectrum. This feature was pointed out in

SS. The effect of ε_4 was not considered by SS, but can be estimated from fig. 1, which shows the Nilsson eigenvalues for the $i_{13/2}$ j-shell as a function of ε_4. The strong bunching of the low Ω orbitals for positive ε_4 values will produce much more closely spaced 2-quasiparticle states, enhancing the mixing and thus the alignment. Negative ε_4 values will have the opposite effect. Since ε_4 is negative in the beginning of the rare earths, and positive near the end, the trend will disfavor alignment in the beginning, and favor it at the end. This runs counter to the λ trend and appears to make a qualitative prediction difficult.

At this point it is useful to remember that most of the parameters entering into the 2-quasiparticle calculation for even nuclei also enter in much the same way into the 1-quasiparticle calculation of the lowest $i_{13/2}$ band in an odd nucleus. Such bands are observed throughout the rare earth region and it seems clear that backbending in the even nuclei should be related to the characteristics of these bands in the adjacent odd nuclei if the rotation-alignment model is correct. We will attempt to make this comparison, first qualitatively, and then in the next section quantitatively.

If we look at the lowest $i_{13/2}$ band in an odd-neutron rare-earth nucleus one characteristic feature of the energies is the presence of a term whose sign alternates as I increases: this term has been called[13]) the "signature" term. This alternation of energies is the beginning of the rotation alignment process, and can be traced back to the Coriolis-induced amplitude of the $\Omega = 1/2$ orbital in the wave function, and further, to its decoupling term. Thus, this signature term is related to the extent of alignment, though it is not a direct measure of it. Therefore, it seems to be of interest to compare the size of this term with the degree of backbending in the adjacent even nuclei. In fig. 2

we have plotted all the information on rotational levels of even nuclei in the rare-earth region from $N \approx 90$ through $Z \approx 76$. A figure similar in this respect has been given by Sorensen[2]). We have plotted in fig. 3 the rotational levels of the lowest $i_{13/2}$ band in the odd-mass nuclei. The effects of the alternating energy term are apparent. The rotation-alignment model would imply some correlation between the size of the alternating energy term and backbending. Comparison of figs. 2 and 3 suggests that this may well be the case, but a more quantitative comparison would be useful.

One of the effects that Coriolis mixing has on the levels of the lowest mixed band is a compression of the band; that is, an increase in the apparent moment of inertia. This is very well documented for the lowest $i_{13/2}$ bands in the rare-earth region, ^{161}Dy being an excellent example[14]) with $(\hbar^2/2\mathcal{J})_{obs} \approx 7$ keV. This compression is a result of the fact that all the levels of the band are lowered, but the higher spin levels are lowered more. This lowering of the levels in the lowest band is exactly what SS suggest causes the 2-quasiparticle band in even nuclei to drop below the ground-state band around $I \sim 16$. It seems clear that the lowering in the 1-quasiparticle system of levels will be closely related to that in the 2-quasiparticle system, and the band compression is an indirect measure of the former of these. Thus a correlation between backbending and the 1-quasiparticle band compression is strongly suggested.

It is not difficult to arrive at a quantitative expression for the compression of the lowest $i_{13/2}$ band in the odd-neutron nuclei. If the band is decoupled (rotation aligned), then the $I = 17/2$ to $13/2$ separation should be just the average $I = 2$ to 0 separation in the two adjacent even nuclei. Thus, the ratio, $6\,E(17/2 \rightarrow 13/2)/\,32\,\bar{E}(2 \rightarrow 0)$, should be

6/32 = 0.188 if the band is decoupled. This ratio should be 1.00 if the band is not mixed at all. This "compression factor" for the odd nuclei has been included in figs. 2 and 3, and we have drawn a rough contour line for a compression factor of 0.45. It is apparent that these numbers correlate rather closely with the size of the alternating energy term. Furthermore, the contour line approximately divides the backbending even nuclei from those that do not seem to backbend, though more data are badly needed in the lower right portions of figs. 2 and 3. We find the correlation between compression factor and backbending, as indicated in fig. 2, quite encouraging, and will now try to understand the trends qualitatively. In the next section these ideas will be tested by direct calculation.

The major trends of the alternating energy term and the compression factor (and perhaps backbending) shown in fig. 3 can probably be explained by consideration of three variables, λ, ε_4, and $\hbar^2/2\mathfrak{J}$ (or ε_2). Sample calculations showing how these variables affect the lowest $i_{13/2}$ band in an odd-mass nucleus are shown in fig. 4. The circled numbers indicate the ε_2, ε_4, and λ values for each calculation on the two partial Nilsson diagrams at the top of fig. 4, and the frames on the lower part of the figure show the resulting calculated bands. The variation of ε_2 (0.20, 0.25, and 0.30) is shown in the sequence 2-1-3. This is due mostly to the effect of $\hbar^2/2\mathfrak{J}$, which is assumed to vary inversely with ε_2^2, but an additional contribution in the same direction comes about because the separation of the Ω states increases approximately linearly with ε_2. Similarly, the sequences 4-1-5 and 6-1-7 show the variation of λ and ε_4. The apparent effect of ε_4 is a little misleading since the position of λ relative to the Ω = 7/2 and 9/2 levels changes considerably in the ε_4 sequence. This causes the effect of ε_4 to appear much weaker than it would if λ remained fixed relative to a given Ω-level.

The effect of each variable is clear in fig. 4: higher λ reduces the alternating energy term and the compression; more positive ε_4 increases them; and larger values of $\hbar^2/2\mathcal{J}$ (smaller ε_2) also increase them. Furthermore, the N and Z dependence of each variable is also reasonably well known: λ increases with N; ε_4 generally increases with both N and Z; and, within the limited region plotted in fig. 2, $\hbar^2/2\mathcal{J}$ generally decreases with N and increases with Z. Thus at low N and Z, λ and $\hbar^2/2\mathcal{J}$ strongly favor decoupling (compression) and the compression factor is at the limit. 0.188, on the lower left edge of fig. 3. As N increases both these variables disfavor decoupling and it is observed to weaken. This trend is opposed by ε_4, but ε_4 is not a strong function of N alone, and the compression factors in fig. 3 show without exception a decrease with increasing N. However, as Z increases, both $\hbar^2/2\mathcal{J}$ and ε_4 favor decoupling and the data bear this out. For constant N, this trend also occurs without exception in fig. 3. Backbending resumes in the even Os nuclei which is consistent with the above trends in the odd nuclei, and with the contour line at 0.45 in fig. 2. However, one should be cautious in this region. A mixed band of levels from the $h_{9/2}$ proton orbital has been seen near the ground state in the odd Re nuclei, with a compression factor of 0.38 (the decoupled limit for j = 9/2 is 0.25). This orbital rather than $i_{13/2}$ could easily be responsible for the backbending in the Os nuclei.

3. Calculations

The above discussion suggests that calculations of the type made by SS for the even nuclei should first be tested against the adjacent odd nuclei, and adjustments of the parameters be made, if necessary, in order to fit these odd nuclei. There are many 1-quasiparticle bands known in the rare-earth region based on the $i_{13/2}$ shell, but as far as we know there have been no attempts to make systematic Coriolis calculations for these bands throughout the region. Thus nothing like a general set of acceptable parameters is known, and the present work is not sufficiently ambitious to undertake this problem. Instead we start with an a priori estimate for the parameters, and an order for varying them until a satisfactory fit of each 1-quasiparticle band is achieved. Then this identical set is used for the 2-quasiparticle states in the adjacent even nuclei. In fact, without a good search program to fix the parameters, this procedure is rather slow and thus far only two sets of nuclei have been done: ^{161,162}Er in a backbending region, and ^{171,172}Hf in a non-backbending region. In the rest of this section we will discuss the Hamiltonian used, the parameters involved, and the results of these calculations.

The calculations are based on the particle-plus-rotor model, and the Hamiltonian used is:

$$H = H_p + H_{rot} = H_p + \frac{\hbar^2}{2\mathcal{J}} \vec{R}^2 \qquad (1)$$

where H_p is the Hamiltonian of the particles in the absence of rotation. Due to the assumption of axial symmetry, $R_3 = 0$.
Using the relation, $\vec{R} = \vec{I} - \vec{J}$, we get:

$$H = H_p + H_c + \frac{\hbar^2}{2\mathcal{J}} [I(I+1) - K^2] + \frac{\hbar^2}{2\mathcal{J}} [\vec{J}^2 - K^2] \quad , \tag{2}$$

where H_c is the Coriolis operator given by:

$$H_c = -\frac{\hbar^2}{2\mathcal{J}} [I_+J_- + I_-J_+] f(U,V) \quad , \tag{3}$$

and $f(U,V)$ is a pairing correction factor. The last term in eq. (2) is the so-called "recoil term", which we include in both the 1- and 2-quasiparticle calculations. The vector $\vec{J}$ is the total particle angular momentum: $\vec{j}$ for the one-particle problem; and $\vec{j}(1) + \vec{j}(2)$ for the two-particle problem. In all cases we use Nilsson wave functions to evaluate H_c and $\vec{J}$. For H_p we use:

$$H_p(K) = \sum_{\Omega} \sqrt{(E(\Omega) - \lambda)^2 + \Delta^2} \quad , \tag{4}$$

where $E(\Omega)$ is the Nilsson solution for the state, Ω, and 2Δ is the paring gap for the nucleus under consideration. The sum contains one (two) term(s) for the 1- (2-) quasiparticle states. The index, K, is just Ω for the one-particle case and $\Omega(1) + \Omega(2)$ for the two-particle case. The pairing factor, $f(U,V)$ is given by

$$f(U,V) = \begin{cases} (U_1U_2 + V_1V_2) & \begin{cases} 1 \text{ qp-1qp} \\ 2 \text{ qp-2qp} \end{cases} \\ (U_1V_2 - V_1U_2) & 0\text{qp} - 2\text{qp} \quad , \end{cases} \tag{5}$$

where:

$$V^2 = \frac{1}{2} \left(1 - \frac{E(\Omega) - \lambda}{H_p(\Omega)}\right), \tag{6}$$

and

$$U^2 = 1 - V^2 \quad . \tag{7}$$

These equations are straightforward, and involve five parameters. Two parameters enter the Nilsson calculation, ε_2 and ε_4; two come from the pairing equations, λ and Δ, and $\hbar^2/2\mathcal{J}$ is the rotational constant.

For the a priori estimates of these parameters we have used values obtained in the following way. For ε_2 we wanted a value that could be easily estimated for any rare-earth nucleus. We eventually decided it was most expedient to tie this quantity to the energies of the 2+ states in the even nuclei according to the formula:

$$\varepsilon_2 = \sqrt{\frac{1225}{E_{2+}\, A^{7/3}}} \, , \tag{8}$$

where E_{2+} is in MeV. This form of the relationship has been shown by Grodzins[15]) to give good agreement with deformations derived from B(E2; 0 → 2) values. For ε_4 we took calculated values from fig. 12a given in ref. [16]). These parameters defined the input for the Nilsson calculation. From the output of that calculation we defined λ as the energy of the last filled level (of either parity); thus it was the same for the odd and even nucleus in each pair. The parameter, Δ, was taken from the Δ_n plot in fig. 2-5 of ref. [13]).

Fixing the rotational constants was more complicated. We first defined this quantity for the even nuclei as $(\hbar^2/2\mathcal{J})_o = E(2 \rightarrow 0)/6$. For an odd nucleus the value could be taken to be the average of the adjacent even nuclei. However, it is well known that this quantity is smaller in odd and odd-odd nuclei due to blocking (and perhaps other) effects. Furthermore, from previous Coriolis fits in the 1-quasiparticle

systems we knew that $(\hbar^2/2\mathfrak{J})_1$ had to be 10-15% smaller than $\overline{(\hbar^2/2\mathfrak{J})}_0$. We settled somewhat arbitrarily on 13%; so that $(\hbar^2/2\mathfrak{J})_1 \equiv \overline{(\hbar^2/2\mathfrak{J})}_0/1.13$. Fixing a value for the 2-quasiparticle states, $(\hbar^2/2\mathfrak{J})_2$, was even more arbitrary. It is known that the odd-odd nuclei have smaller values of $\hbar^2/2\mathfrak{J}$ than the nearby odd nuclei (further blocking, etc., effects). Furthermore, the calculation of SS (and additional ones made to check this point) taking into account some 4-quasiparticle states showed that, below spin 30 or so, inclusion of these states could be roughly simulated by lowering and compressing the high-spin 2-quasiparticle states: that is, by using a lower effective value for $(\hbar^2/2\mathfrak{J})_2$. We decided that approximately doubling the size of the reduction used in the odd nuclei was the best estimate we could make: so that $(\hbar^2/2\mathfrak{J})_2 \equiv (\hbar^2/2\mathfrak{J})_0/1.25$.

The energies of the ground-state band in the even nuclei were still more difficult. The calculations of SS showed that the $i_{13/2}$ Coriolis effects do not cause all of the observed energy deviations in the ground band at the low-spin values. It is not reasonable that they should do so. Thus, instead of using the I(I + 1) relationship as input for this band we used the Harris/VMI expressions[17,18]. This is important, since otherwise the energy of the ground band in the intersection region would be considerably in error. For the parameters $\mathfrak{J}_0$ and C in these expressions we began with the (observed) values given in ref. [18]. However, the Coriolis effects (among the levels of the $i_{13/2}$ orbital) that we explicitly take into account do contribute to these parameters. A few tries on ^{162}Er showed that our <u>input</u> $\mathfrak{J}_0$ had to be reduced 10%, and C had to be about twice its observed value in order to get out of the calculation the observed values. This procedure insures that the ground-band levels will be at about the right place in the intersection region.

For consistency we tried using the Harris/VMI expressions for the quasiparticle states also, but the fits we could get were not improved, the $\mathfrak{J}_0$

and C values of the even nuclei could not be used (the odd nuclei require larger C values), and it was not entirely clear to us how to treat $K \neq 0$ in these expressions. Thus we used the $I(I + 1)$ rotational energies in the quasiparticle states. If the calculations were extended nearer to the edge of the deformed region, this procedure would probably not be satisfactory.

It seems possible to characterize the bands in the odd nuclei roughly by two features; a compression from the input $(\hbar^2/2\mathcal{J})_1$ value and a magnitude for the energy oscillations. It is well known that both these quantities are too large if one does the Coriolis calculations with the a priori parameters described. The Coriolis matrix elements must be reduced, and to do this we chose the form used by SS, which is decreasing $f(U,V)$. We use:

$$f(U,V) = (U_1U_2 + V_1V_2)^n \qquad \begin{cases} 1qp\text{-}1qp \\ 2qp\text{-}2qp \end{cases}, \tag{9}$$

where n is adjusted to fit the odd nucleus. In the off-diagonal matrix elements of $\vec{J}^2$, however, the value of n was kept at one. Our procedure, therefore, was to fit the compression of the 1-quasiparticle band with n, and then fit the energy oscillation by varying ε_4. (We could also have varied λ within small limits to fit these oscillations, but we chose to vary ε_4.) To obtain the fits shown in fig. 5 for ^{161}Er and ^{171}Hf, only these two quantities had to be varied from the a priori input values. The value of n required was 3 in ^{161}Er and 4 in ^{171}Hf, whereas ε_4 was changed from -0.01 to 0.00 for ^{161}Er and from +0.05 to +0.02 for ^{171}Hf. The input values are given in Table 1. Our impression is that this is not a unique set of input parameters. It would be interesting to test other sets that fit the odd nuclei, but we have not at present determined any such sets.

The results for the even nuclei are also shown in fig. 5. They seem to us to be very encouraging. In ^{162}Er, the calculated backbending is not quite strong enough. This is sensitive to the 0-2 quasiparticle matrix elements, the only input quantities for which we have no real previous experience. Also, after the backbend, the calculated $2\mathcal{J}/\hbar^2$ is high. This is sensitive to the input value of $(\hbar^2/2\mathcal{J})_2$, which we certainly do not know within the difference between experiment and calculation in fig. 5 ($\sim$ 7%). For ^{172}Hf, the calculation is above experiment at low spin values because the Coriolis effects we explicitly take into account are somewhat stronger at low spin values than for ^{162}Er (due to pairing effects), but we did not alter the 10% reduction of $\mathcal{J}_o$ going into the calculation. In both cases the calculated value of $(\hbar\omega)^2$ at which the bands intersect appears to be a little low, but there are many uncertainties in the calculations that could produce effects of this size.

It seems to us that in these calculations we have, for the first time, enough knowledge of the Hamiltonian and the input parameters to hope for some kind of detailed agreement with the experiments. That this agreement seems to exist (fig. 5) suggests both that the input parameters are behaving as we have proposed and that the basic ideas may be correct. Although we have pointed out the general trends in the input parameters, and their effect on backbending, it should be noted that for each individual case these parameters are determined from the particular nucleus and its adjacent odd-mass neighbors. This leaves room for individual irregularities, and to see if any observed irregularity constitutes a significant deviation from the model or not requires a careful consideration of the indicated values for the input-parameters, n, ε_2, ε_4, λ, Δ, $(\hbar^2/2\mathcal{J})_2$, $\mathcal{J}_o$ and C.

As an example, consider the cases[19,20]) of ^{168}Yb and ^{170}Yb. Recently ^{170}Yb has been observed to bend up more sharply than ^{168}Yb (see fig. 2), and this has been cited as evidence against the present model. Based on the general trends predicted, or even the contour line in fig. 2, this seems plausible. However, the compression factors for ^{169}Yb and ^{171}Yb (fig. 3) are rather similar, indicating that the 2-quasiparticle states in ^{168}Yb and ^{170}Yb are probably not much different. On the other hand, the 14+ member of the ground band lies $\sim$100 keV higher in ^{170}Yb than in ^{168}Yb, and the 14 $\rightarrow$ 12 transition energy is considerably larger in ^{170}Yb (597 vs. 553 keV). These features of the ground band in ^{170}Yb are clearly reflected in the values of C given in ref. [18]), and are in the direction of producing a more violent intersection with the 2-quasiparticle band and thus a stronger tendency to backbend. Such behavior, then, cannot be taken as a strong argument against the model (at least not prior to detailed calculations). It would be useful in this regard to extend the calculations shown in fig. 5 to many more nuclei - the present Yb cases, the Os nuclei, etc., - and we hope to develop techniques that will make it feasible to do this on a broad scale.

4. Conclusions

One of the models proposed to explain the backbending phenomenon at high spins in the even nuclei involves the unpairing of two $i_{13/2}$ particles and the alignment of the resulting particle angular momentum with the core rotational angular momentum. Such a model has had considerable impact recently in the interpretation of spectra in the odd nuclei[21,22]). The alignment of particle angular momentum with core rotational angular momentum has been found to be the limiting situation for the high-spin states from high-j orbitals in deformed nuclei, and also to provide a possible new interpretation for these states in the "vibrational" nuclei. There is even evidence that the high-spin states from lower-j orbitals in the "vibrational" nuclei behave in this way[23]). This development in the odd nuclei has called attention to the close relationship between the states based on a high-j orbital in the odd and even nuclei, and led to the proposal made here that one should try to correlate the effects of the rotation alignment in the two types of nuclei; i.e. backbending in the even nuclei and compression and energy oscillations in the odd nuclei. We have tried to do that, first qualitatively on a broad scale in section 2, and then quantitatively for two pairs of nuclei in section 3. Our conclusion is that the proposed correlation appears to be quite reasonable.

There seems to be evidence accumulating that the backbending observed in the light Er region is caused by an intersection of the ground band with the rotation-aligned 2-quasiparticle band based on $i_{13/2}$ particles. Data on backbending in the odd nuclei of this region support this[24]), as do recent calculations encompassing both pairing collapse and rotation alignment[10,11]).

What occurs in other regions is not very clear at present, even experimentally. Based on the present work, it seems plausible that all the backbending in the rare-earth region could be due to these $i_{13/2}$ 2-quasiparticle states, though in the Os region it is quite possible that the $h_{9/2}$ proton shell is more important, or even that another process is involved. We have made no estimates outside the rare-earth deformed region. However, in the "vibrational" regions one should be especially cautious in interpreting "backbending" phenomena, since additional effects having this appearance are known to occur. The well-known spectrum of two particles in a j-shell looks somewhat like "backbending", as does the spectrum recently observed in the light Hg isotopes[25,26]) caused by an angular-momentum-induced shape transition (an extreme case of centrifugal stretching). Thus we conclude that there is not a single cause underlying all "bacbending" but rather every region, and sometimes each nucleus within a region, has to be studied carefully and interpreted according to its individual characteristics.

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Table 1. Parameters used in the calculations.

	^{161}Er	^{162}Er	^{171}Hf	^{172}Hf
n	3	3	4	4
ε_2	0.275	0.275	0.281	0.281
ε_4	0.000	0.000	+0.020	+0.020
λ^* (MeV)	1.367	1.367	2.040	2.040
Δ (MeV)	1.006	1.006	0.840	0.840
$(\hbar^2/2\mathcal{J})_1 \times 10^2$ (MeV)	1.65		1.41	
$(\hbar^2/2\mathcal{J})_2 \times 10^2$ (MeV)		1.28		1.19
$\mathcal{J}_o \times 10^2$ (keV $^{-1}$)		2.60		2.80
$C \times 10^{-6}$ (keV^3)		7.36		7.60

*Based on the $\Omega = 1/2$ bandhead as $E = 0$.

Figure Captions

Fig. 1. The dependence of the component levels of the $i_{13/2}$j-shell on ε_4 is shown. The other Nilsson-model parameters are $\varepsilon_2 = 0.25$, $\kappa = 0.0637$, and $\mu = 0.42$, and for the rare-earth region $\hbar\omega_o$ would be about 7.8 MeV.

Fig. 2. Ground-band level energies in doubly-even rare-earth nuclei. The plots give the moment-of-inertia $\mathcal{J}$ versus the square of the rotational frequency ω^2, both quantities derived from the transition energy. In a few cases where more than one possible choice exists, the lowest-energy transition is always used. Tentatively assigned band members are indicated by an omitted dot. The compression factors C and the contour line for C = 0.45 are derived from the $17/2^+ \rightarrow 13/2^+$ level spacings observed in the odd-N nuclei (fig. 3), and from the mean value $\bar{E}(2\rightarrow 0)$ of the 2+ energies in the adjacent even nuclei. The data are taken mainly from a recent compilation by Saethre et al. (Nucl. Phys. A207 (1973) 486). References to more recent results are given below:

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Fig. 3. The $i_{13/2}$ yrast level energies in odd-N rare-earth nuclei. The plots give the apparent $\hbar^2/2\mathcal{J}$ as derived from the transition energy (in units of the mean value of $\hbar^2/2\mathcal{J}$ in the neighboring doubly-even isotopes) versus the square of the spin of the upper level. In this plot an unperturbed rotational band gives a horizontal line (with the ordinate close to one), a band following the equation $E = AI(I+1) + BI^2(I+1)^2$ gives a straight line with the slope B. The compression factor C derived from the $17/2^+ \rightarrow 13/2^+$ transition energy is indicated for each nucleus. In several nuclei with $N \leqslant 99$ only one E2 cascade was observed, which establishes the energy-favored band members with $I = j, j + 2, \ldots$; these points are connected

by a broken line to indicate the absence of the alternate band members. (For illustrative purposes this is also done for several complete bands.) Dots are omitted for tentatively assigned band members. Reference to the original data is made in the list below.

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Fig. 4. Calculated dependence of the $i_{13/2}$ yrast level energies in odd-N rare-earth nuclei on the fermi surface λ and the deformation parameters ε_2 and ε_4. Nilsson model single particle energies (κ = 0.0637, μ = 0.42, $\hbar\omega_o$ = 7.83 MeV, ε_2 = 0.20, 0.25, and 0.30, ε_4 = -0.05, 0, +0.05) and unattenuated Coriolis matrix elements (n = 1) have been used in the calculations. The quantity $\hbar^2/2\mathcal{J}$ was calculated from ε_2 through eq. (8). The pairing gap was Δ = 0.9 MeV, and the three values of λ were 6.513, 6.692, and 6.896 $\hbar\omega_o$. The parameter combinations for the individual yrast bands are indicated in the partial Nilsson diagrams in the upper part of the figure.

Fig. 5. A comparison of experimental (dots) and calculated (lines) properties of levels in the pairs of nuclei ^{161,162}Er and ^{171,172}Hf. The plots are of the same type as those in Figs. 2 and 3. The left side of the figure shows the fits obtained for the lowest $i_{13/2}$ band in the odd nucleus of each pair, and the right side shows the results for the doubly-even nucleus calculated using the same parameters (given in Table 1).

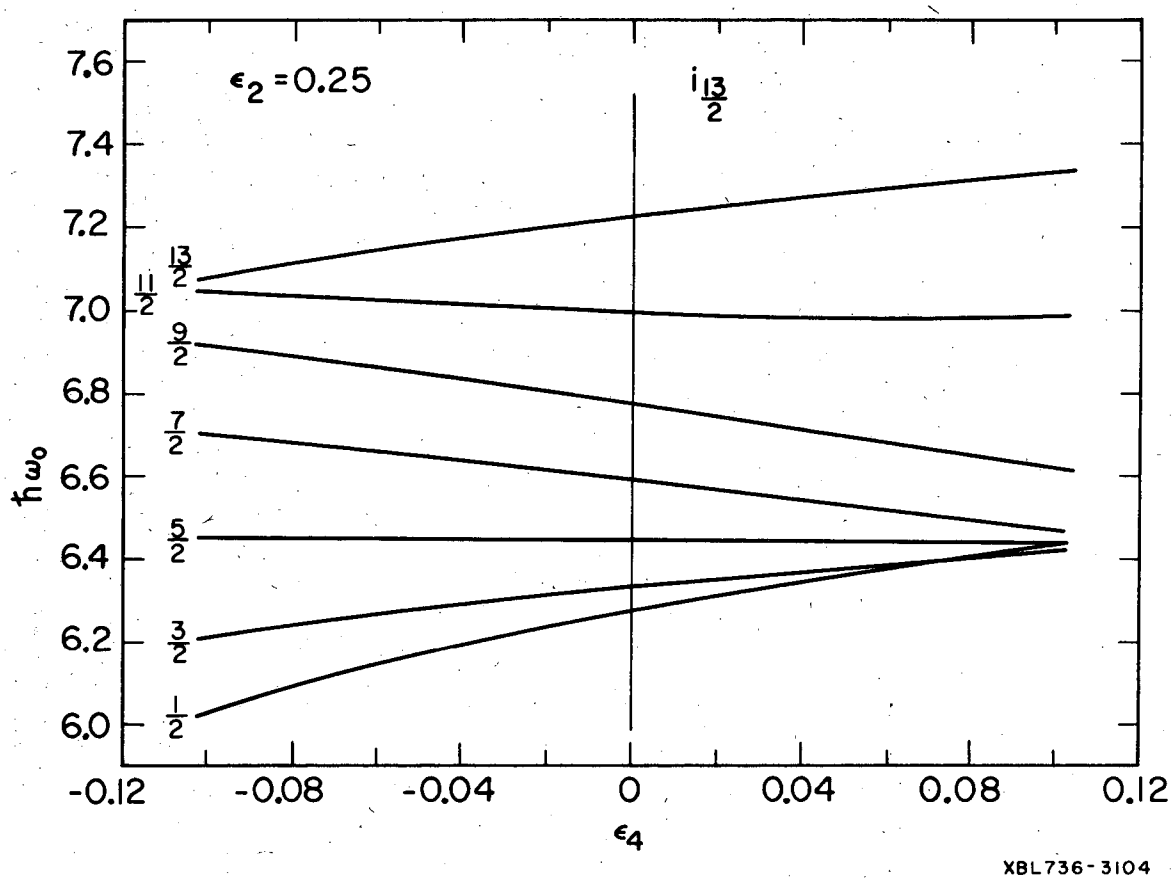

$\epsilon_2 = 0.25$
$i_{\frac{13}{2}}$
7.6
7.4
7.2
7.0
6.8
6.6
6.4
6.2
6.0
$\hbar\omega_0$
$\frac{13}{2}$
$\frac{11}{2}$
$\frac{9}{2}$
$\frac{7}{2}$
$\frac{5}{2}$
$\frac{3}{2}$
$\frac{1}{2}$
-0.12
-0.08
-0.04
0
0.04
0.08
0.12
ϵ_4
XBL736-3104

Fig. 1.

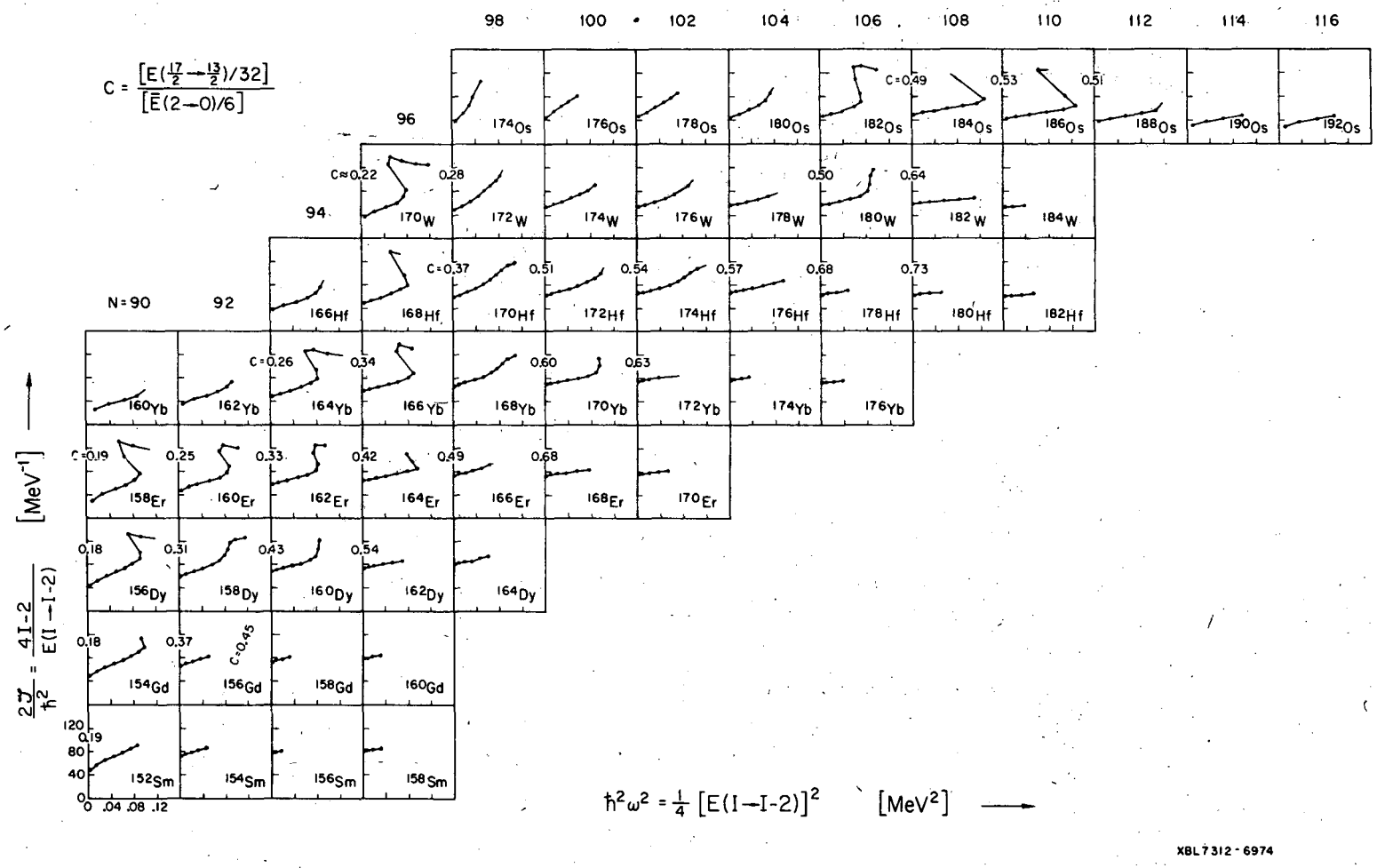

$C = \frac{[E(\frac{17}{2}\rightarrow\frac{13}{2})/32]}{[\bar{E}(2\rightarrow 0)/6]}$
N=90
92
94
96
98
100
102
104
106
108
110
112
114
116
$\frac{2\mathcal{J}}{\hbar^2} = \frac{4I-2}{E(I\rightarrow I-2)}$ [MeV^{-1}]
$\hbar^2\omega^2 = \frac{1}{4}[E(I\rightarrow I-2)]^2$ [MeV^2]
XBL7312-6974

Fig. 2.

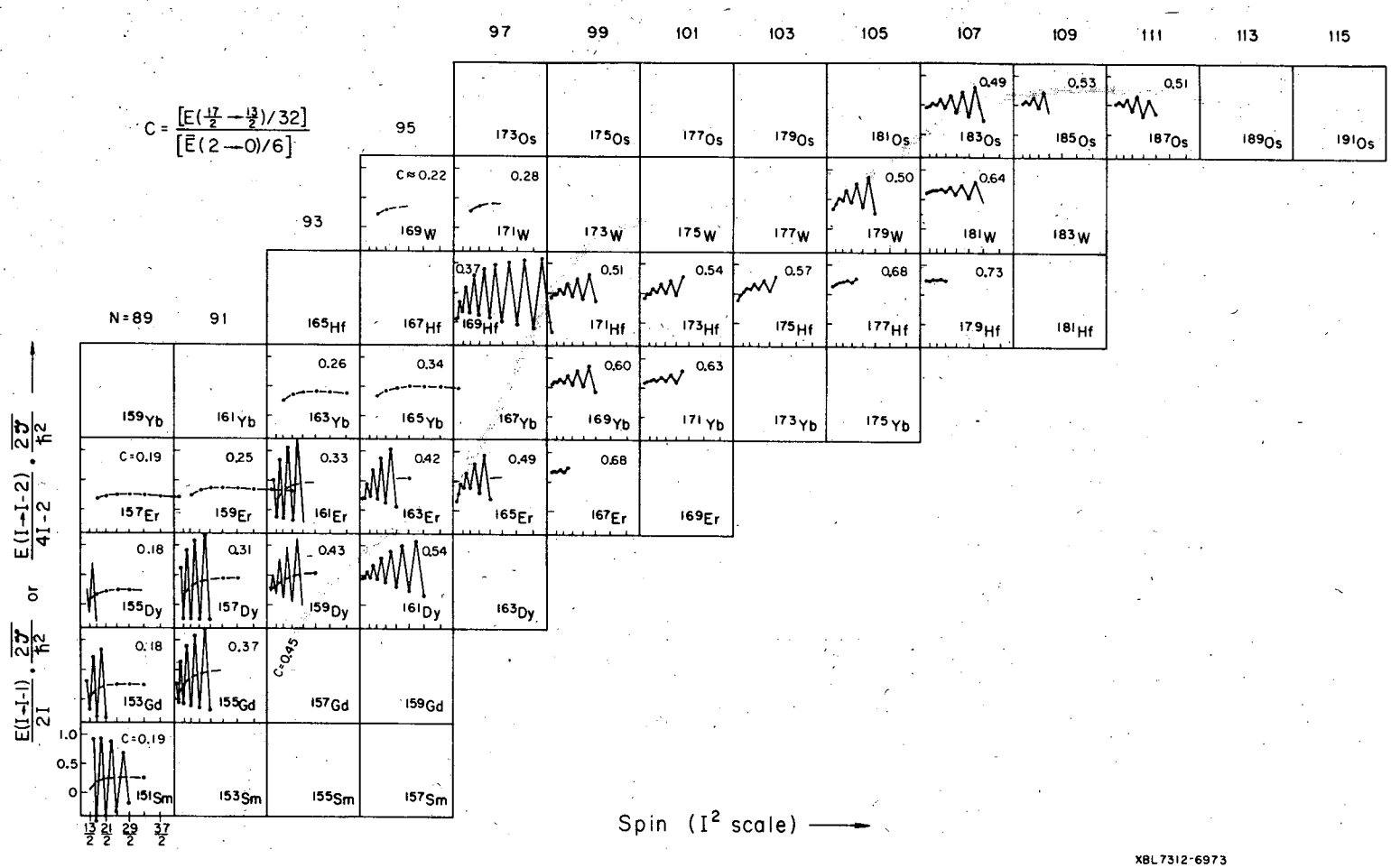
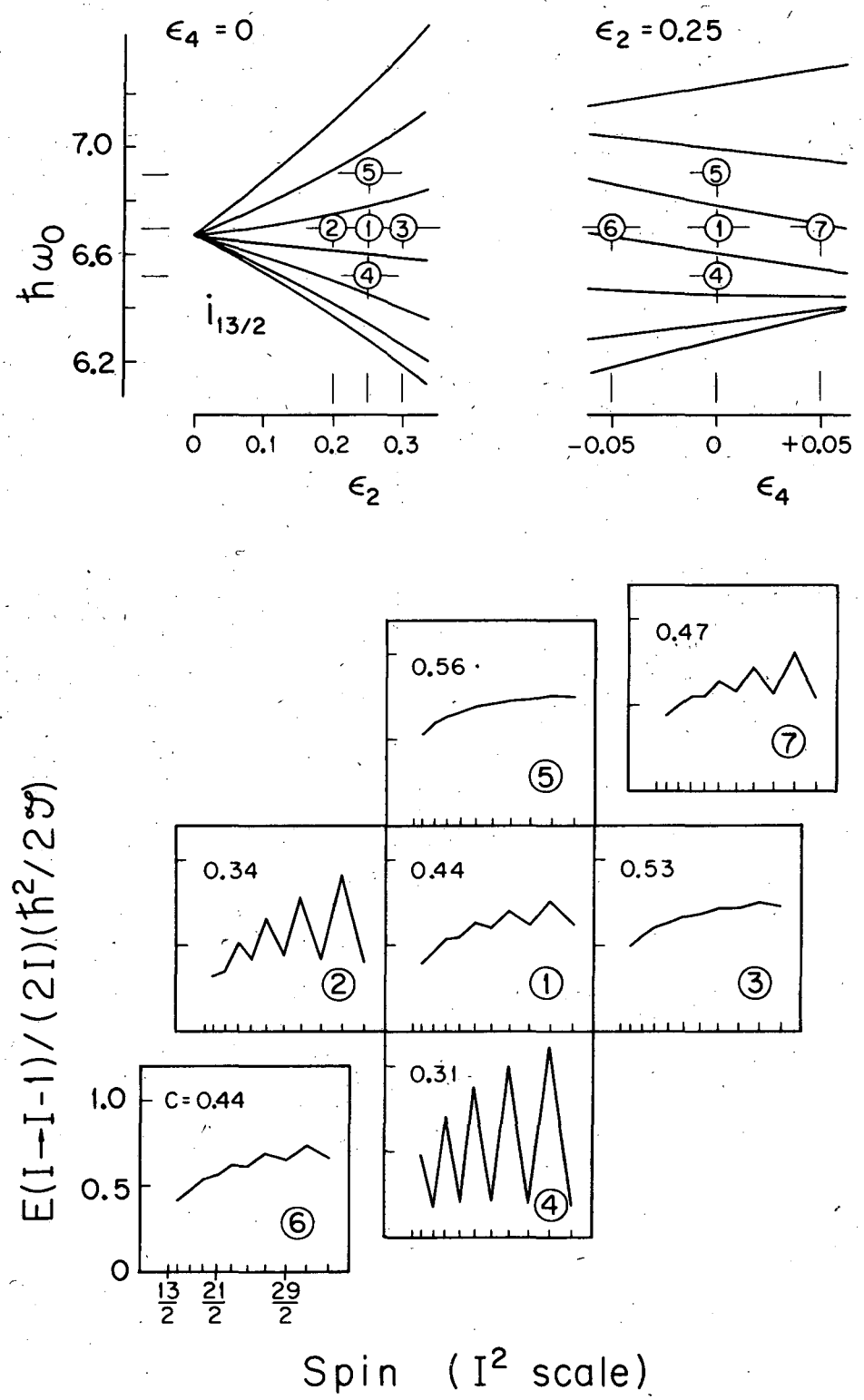
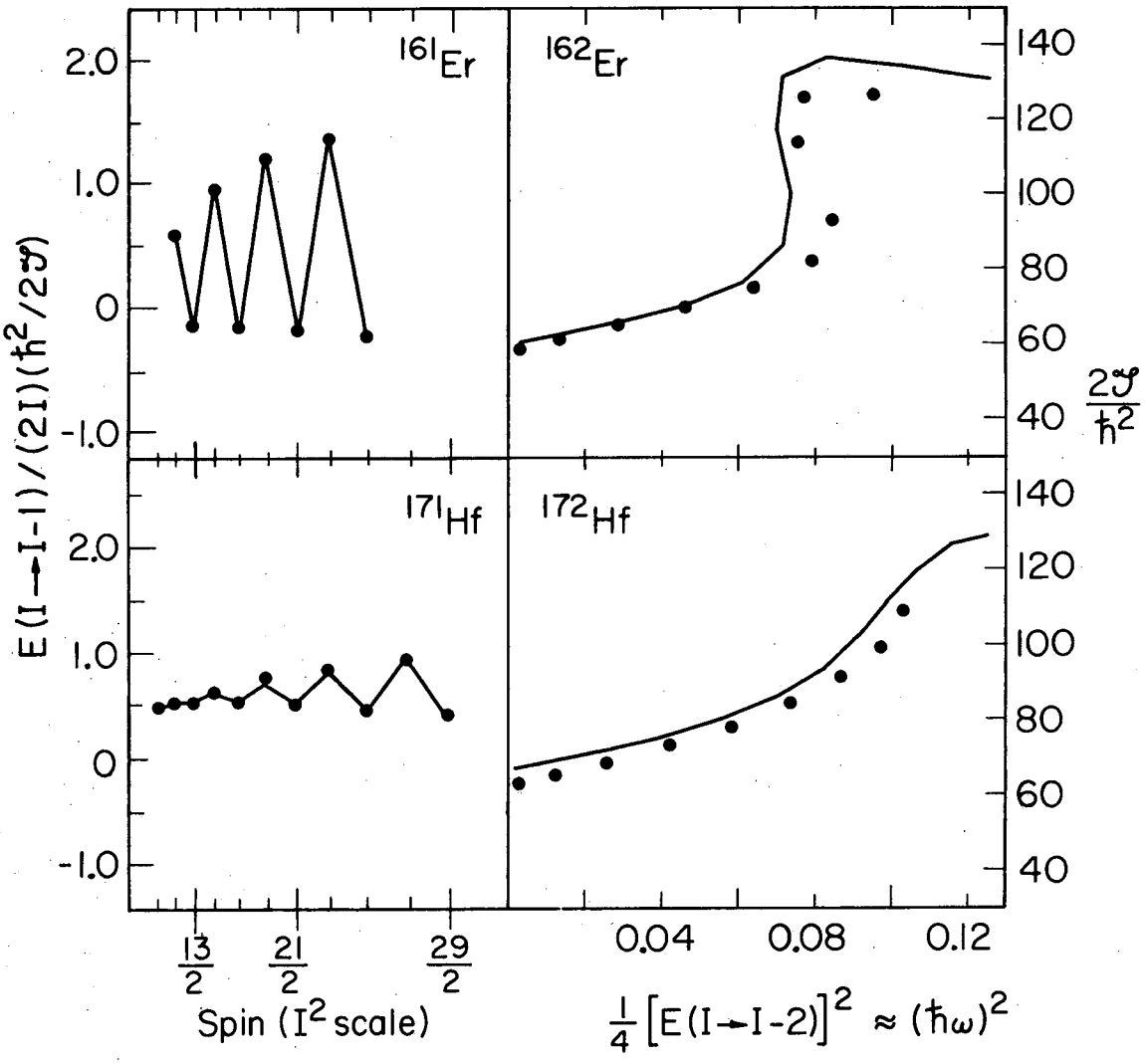

$C = \frac{[E(\frac{17}{2} \to \frac{13}{2})/32]}{[\bar{E}(2 \to 0)/6]}$
N = 89
91
93
95
97
99
101
103
105
107
109
111
113
115
^{159}Yb
^{161}Yb
^{163}Yb
^{165}Yb
^{167}Yb
^{169}Yb
^{171}Yb
^{173}Yb
^{175}Yb
^{157}Er
^{159}Er
^{161}Er
^{163}Er
^{165}Er
^{167}Er
^{169}Er
^{155}Dy
^{157}Dy
^{159}Dy
^{161}Dy
^{163}Dy
^{153}Gd
^{155}Gd
^{157}Gd
^{159}Gd
^{151}Sm
^{153}Sm
^{155}Sm
^{157}Sm
^{165}Hf
^{167}Hf
^{169}Hf
^{171}Hf
^{173}Hf
^{175}Hf
^{177}Hf
^{179}Hf
^{181}Hf
^{169}W
^{171}W
^{173}W
^{175}W
^{177}W
^{179}W
^{181}W
^{183}W
^{173}Os
^{175}Os
^{177}Os
^{179}Os
^{181}Os
^{183}Os
^{185}Os
^{187}Os
^{189}Os
^{191}Os
C ≈ 0.22
0.28
0.50
0.64
0.37
0.51
0.54
0.57
0.68
0.73
0.26
0.34
0.60
0.63
C = 0.19
0.25
0.33
0.42
0.49
0.68
0.18
0.31
0.43
0.54
0.18
0.37
C = 0.45
C = 0.19
0.49
0.53
0.51
1.0
0.5
0
$\frac{13}{2}$ $\frac{21}{2}$ $\frac{29}{2}$ $\frac{37}{2}$
$\frac{E(I \to I-1)}{2I} \cdot \frac{2\mathcal{J}}{\hbar^2}$ or $\frac{E(I \to I-2)}{4I-2} \cdot \frac{2\mathcal{J}}{\hbar^2}$
Spin (I^2 scale)
XBL7312-6973

Fig. 3.



XBL 7312-6975

Fig. 4.



XBL736-3105

Fig. 5.

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