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COOLDOWN OF AN INFINITELY LONG HOLLOW CYLINDER
AND APPLICATION TO THE SSC COOLDOWN

S. Caspi

February 1985

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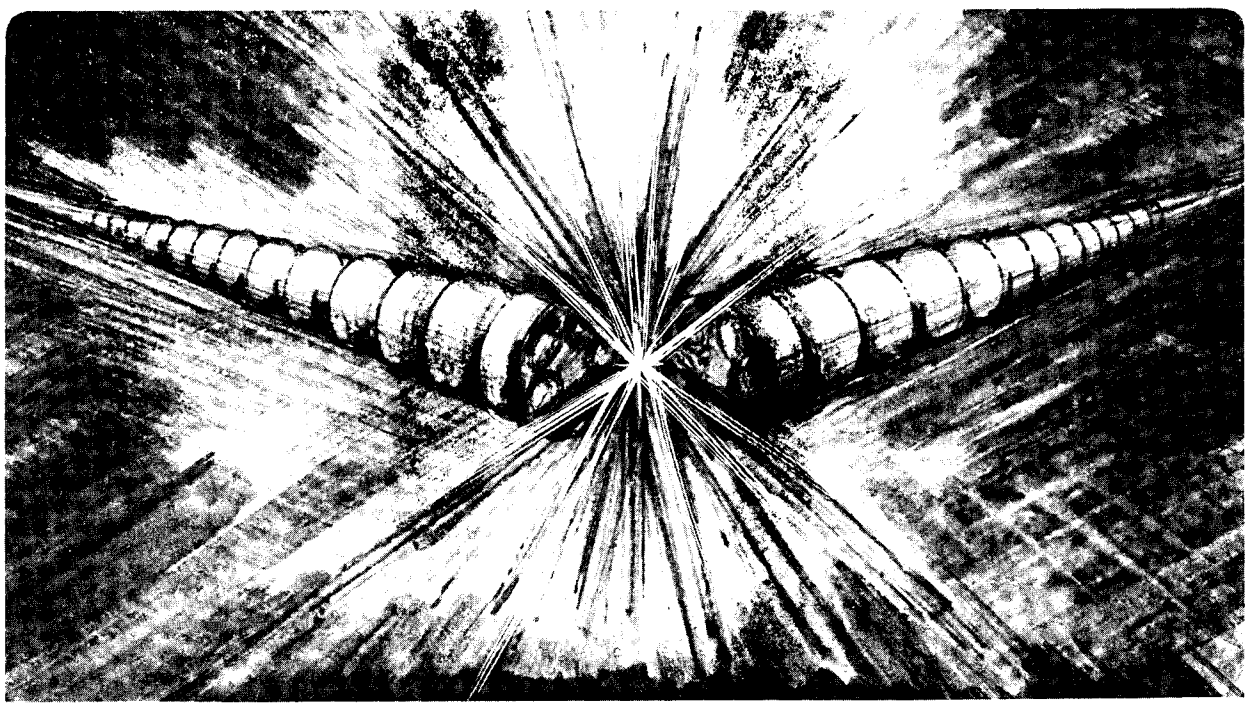

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COOLDOWN OF AN INFINITELY LONG HOLLOW CYLINDER AND APPLICATION TO THE SSC COOLDOWN*

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COOLDOWN OF AN INFINITELY LONG HOLLOW CYLINDER AND APPLICATION TO THE SSC COOLDOWN

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Introduction

This writeup is an attempt to calculate some of the cooldown parameters of a long string of magnets such as the case might be for the SSC. Besides the cooldown time, temperature gradients along a magnet, and in the transverse direction, are influenced by the mass flow of the refrigerant. A number of assumptions and simplifications have been made so that an analytical solution can be obtained.

In Part I of this report we assume a one dimensional model with a finite axial conductivity and infinite transverse conductivity. In Part II, we consider the cooldown in the transverse direction only. A common example for both parts points out the limitation of the assumptions made in Part I and suggests the need for a two dimensional time dependent model $T = T(r,z,t)$.

Part I - Axial Cooldown

Statement of the Problem

Consider an infinite hollow cylinder with radii R_1, R_2 with an assigned initial temperature T_o. A axis-symmetry frame of reference is constantly moving along the cylinder (Z direction) at a velocity U with respect to the cylinder (Fig. 1). The bore ($r \leq R_1$) to the left of the reference frame

($Z \leq 0$) is filled with a fluid at $T=T_f$, which has a heat transfer coefficient h. The bore to the right ($Z > 0$) is empty and therefore adiabatic. The outer surface $r=R_2$ is also adiabatic.

We would like to find out the axial temperature distribution under the following assumption.

- o Properties are constant
- o The transverse time constant is much shorter than the axial one and, therefore, the problem can be reduced to a one dimension case.

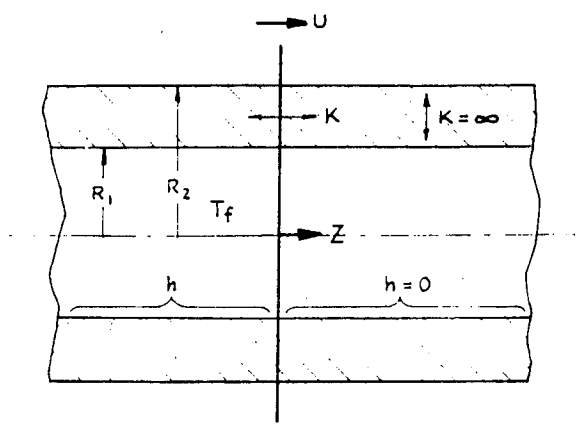

Fig. 1. An infinite hollow cylinder and the moving frame of reference used in Part I.

The solution of the diffusion equation $\frac{\partial T}{\partial t} = \alpha \nabla^2 T$ can be reduced to $-U \frac{\partial T}{\partial Z} = \alpha \nabla^2 T$ for the traveling frame of reference. (See Eckert and Drake "Heat and Mass Transfer", pg. 114). For the one dimensional problem, we can write

$$-U \frac{\partial T}{\partial Z} = \alpha \frac{\partial^2 T}{\partial Z^2} - \frac{Q'}{\rho C_p} \quad ; \quad \alpha = \frac{K}{\rho C_p} \tag{1}$$

Q' = heat flux per unit volume.

At $Z < 0$ $\qquad Q' A \delta z = C \delta z h (T - T_f)$

$$A = \pi (R_2^2 - R_1^2)$$

$$C = 2\pi R_1$$

At $Z > 0$ $\quad Q' = 0 \quad$ or $\quad h = 0$

Define $\quad m^2 = \frac{hC}{KA}$

$$\theta = \frac{T - T_f}{T_o - T_f}$$

so that equation (1) can be rewritten:

$$-\frac{U}{\alpha}\frac{\partial\theta}{\partial Z} = \frac{\partial^2\theta}{\partial Z^2} - m^2\theta \tag{2}$$

Boundary Conditions:

$$\begin{array}{ll} Z \to -\infty & \theta = 0 \\ Z \leq 0 & h = h \\ Z \to +\infty & \theta = 1 \\ Z > 0 & h = 0 \end{array}$$

If we substitute $\theta = e^{-(U/2\alpha)Z} f(Z)$, the solution to equation (2) including the boundary condition is

$$\begin{array}{ll} Z \leq 0 & \theta = \theta_m e^{\left[-\frac{U}{2\alpha} + \sqrt{\left(\frac{U}{2\alpha}\right)^2 + m^2}\right]Z} \\ Z \geq 0 & \theta = 1 - (1 - \theta_m)\, e^{-2\left(\frac{U}{2\alpha}\right)Z} \end{array} \tag{3}$$

where $\quad \theta_m = \frac{T_m - T_f}{T_o - T_f}$ is the temperature at $Z = 0$.

The exponent in the solution for $Z \leq 0$ can be simplified for the case $\left(\frac{U}{2\alpha}\right)^2 \gg m^2$ in the following way:

$$\begin{aligned} \sqrt{\left(\frac{U}{2a}\right)^2 + m^2} - \left(\frac{U}{2\alpha}\right) &= \left(\frac{U}{2\alpha}\right)\left[\sqrt{1 + \left(\frac{m^2}{U/2\alpha}\right)} - 1\right] \\ &\simeq \frac{U}{2\alpha}\left[1 + \frac{1}{2}\left(\frac{m}{U/2\alpha}\right)^2 + \ldots\ldots - 1\right] \\ &\simeq \frac{\alpha}{U} m^2 \end{aligned} \tag{4}$$

Equation (3) can now be rewritten

$$Z \leq 0 \qquad \theta = \theta_m \, e^{\frac{\alpha m^2}{U} Z}$$

$$Z \geq 0 \qquad \theta = 1 - (1 - \theta_m) e^{-\frac{U}{\alpha} Z} \tag{5}$$

Under assumption (4) $\quad e^{-(U/\alpha)Z} \ll e^{\frac{\alpha m^2}{U} Z}$

Relation (5) can be further simplified

$$Z \leq 0 \qquad \theta = \theta_m \, e^{\frac{\alpha m^2}{U} Z}$$

$$Z \geq 0 \qquad \theta = 1 \tag{6}$$

This points out that most of the axial temperature gradient is laying in the convective region $Z \leq 0$.

In order to determine the characteristic length of this gradient, we rewrite (6)

$$Z \leq 0 \qquad \ln\left(\frac{\theta}{\theta_m}\right) = \frac{\alpha}{U} m^2 Z$$

$$\text{or for } Z \geq 0 \qquad Z = \frac{U}{\alpha m^2} \ln\left(\frac{\theta_m}{\theta}\right) \tag{7}$$

The velocity U can be estimated from the following enthalpy balance

$$\dot{m} \, \Delta h_f = \rho A \, U \, \Delta h_t \quad \text{or} \quad U = \frac{\dot{m} \, \Delta h_f}{\rho A \, \Delta h_t} \tag{8}$$

where $\dot{m} \, \Delta h_f \equiv$ rate of enthalpy change per unit volume of the fluid

$\rho \, \Delta h_t \equiv$ enthalpy change of the tube per unit volume.

Introducing (8) into (7), a characteristic length $Z = \lambda$ can be expressed as

$$\lambda = \frac{\dot{m} \, C_{pf}}{h \, 2\pi R_1} \ln\left(\frac{\theta_m}{\theta}\right) \tag{9}$$

C_{pf} = specific heat of the fluid.

Note that expression (9) represents a thermal balance between the heat convected away and the heat radiating by the tube.

Since $U \propto \dot{m}$

$\lambda \propto \dot{m}$

The characteristic time τ is independent of the flow rate and is written as:

$$\tau = \frac{M\,\bar{C}_{pw}}{h\,A}\,\ell n\left(\frac{\theta_m}{\theta}\right) = \frac{\rho S\,\bar{C}_{pw}}{h 2\pi R_1}\,\ell n\left(\frac{\theta_m}{\theta}\right) \qquad (10)$$

$M = \rho V = \rho A \lambda$ of the tube

$A = 2\pi R_1 \lambda$

$S \equiv$ cross section area of the tube annulus.

The time corresponds to the ratio between the total thermal mass and the heat transfer area.

Note that $\dot{m}$ will effect the axial temperature gradient (eq. 9), but not the cooldown time of the characteristic length, which is only effected by the geometry and thermal properties.

<u>Example</u>

For Helium at $80 < T < 300$ K $\qquad C_{pf} = 5.2$ J/gr/K

The heat transfer coefficient $\qquad h \simeq 0.15$ W/cm^2/K

$R_1 = 1.5$ cm

$\theta_m \approx 1$

$$\left.\begin{array}{ll} \theta = 1 & \ell n\left(\frac{1}{0.1}\right) = 2.3 \\ \theta = 0.01 & \ell n\left(\frac{1}{.01}\right) = 4.6 \end{array}\right\} \quad \text{take } \ell n\left(\frac{1}{\theta}\right) \approx 4$$

Use equation (9) $\lambda \simeq 15 \cdot \dot{m}$

If we take $\dot{m}$ = 100 gr/sec, we get λ = 15 m.

To calculate the characteristic time τ (eq. 10) (the time it takes to cooldown the characteristic length λ).

$$A = \pi(R_2^2 - R_1^2) = \pi(5.9^2 - 1.5^2) \simeq 100 \text{ cm}^2$$

For iron $\rho = 7.9$ (gr/cc)

$\bar{C}_{pw} = 0.39$ (J/gr/K)

$$\tau = \frac{7.9 \cdot 1000 \cdot 0.39}{0.15 \cdot \pi \cdot 3} \ln\left(\frac{\theta_m}{\theta}\right) \simeq 15 \text{ min.}$$

and the velocity $U = \frac{\lambda}{\tau} \simeq 1.6$ cm/sec.

Part II - Transverse Cooldown

The cooldown of an infinite hollow cylinder is calculated to estimate the transverse cooldown time of a SSC type magnet. The cylinder, with radii a and b (Fig. 2.), initially at T_0 is put into a medium with constant temperature T_f. The external surface r=b is adiabatic, the heat transfer coefficient at r=a is h and the thermal diffusivity α is assumed constant.

The two dimensional time dependent heat transfer problem is written as

$$\frac{\partial\theta}{\partial t} = \alpha \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial\theta}{\partial r}\right) \tag{1}$$

$$\theta = \frac{T - T_f}{T_0 - T_f}$$

$t = 0$ $\quad \theta = 1$

$t > 0$ $\quad r=a$ $\quad \frac{\partial\theta}{\partial r} = \frac{h}{K}\theta$

$\quad r=b$ $\quad \frac{\partial\theta}{\partial r} = 0$

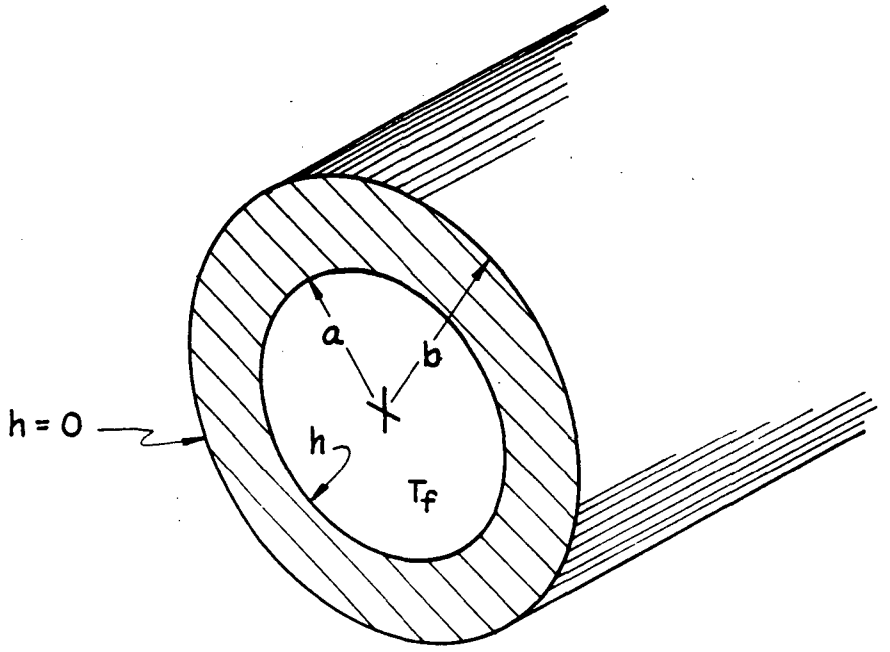

Fig. 2. Cross section of an infinite hollow cylinder used in Part II.

The solution to equation (1) with its boundary conditions can be found in "Conduction of Heat in Solids," Carslow and Jaeger, Pg. 332, or "Analytical Heat Diffusion Theory," A.V. Luikov, pg. 281.

$$(2) \quad \theta = -\pi \sum_{n=1-} e^{-\alpha\lambda_n^2 t} \frac{1}{F(\lambda_n)} J_1^{\,2}(b\lambda_n)\, C_o(r, \lambda_n)$$

$$F(\lambda_n) = [p\lambda_n J_1(a\lambda_n) + J_0(a\lambda_n)]^2 - (p^2\lambda_n^2 + 1)J_1^{\,2}(b\lambda_n)$$

$$C_0(r,\lambda_n) = J_0(r\lambda_n)\,[p\lambda_n Y_1(a\lambda_n) + Y_0(a\lambda_n)]$$
$$-Y_0(r\lambda_n)\,[p\lambda_n J_1(a\lambda_n) + J_0(a\lambda_n)]$$

λ_n are the eign values of

$$p\lambda\,[J_1(a\lambda)Y_1(b\lambda) - J_1(b\lambda)Y_1(a\lambda)]$$
$$+ J_0(a\lambda)Y_1(b\lambda) - J_1(b\lambda)Y_0(a\lambda) = 0$$

$$p = \frac{K}{h} \qquad \alpha = \frac{K}{\rho C_p}$$

Note that for large λ we get $\lambda_n \simeq \frac{n\pi}{b-a}$.

Example

Equation (2) was solved on a computer using the following parameters:

$a = 1.5$ cm

$b = 5.9$ cm

$K = 0.65$ w/cm/K

$h = 0.153$ w/cm^2/K

$\alpha = 0.211$ cm^2/sec

The values of λ_n are:

$\lambda_n = .128, .804, 1.485, 2.183, 2.887, \ldots\ldots$

The solution of θ as a function of r and t is plotted in Fig. 3.

The time constant τ for this geometry is of the order of $\tau \simeq 5$ min. This should be compared with the equivalent axial cooldown characteristic time of $\tau \approx 15$ min. for a one dimensional model with a characteristic length of $\lambda \simeq 15$ m. The assumption, therefore, of infinite conductivity in the transverse direction is not valid in the axial model and the cooldown problem should be solved for the case where $T = T(r,z,t)$.

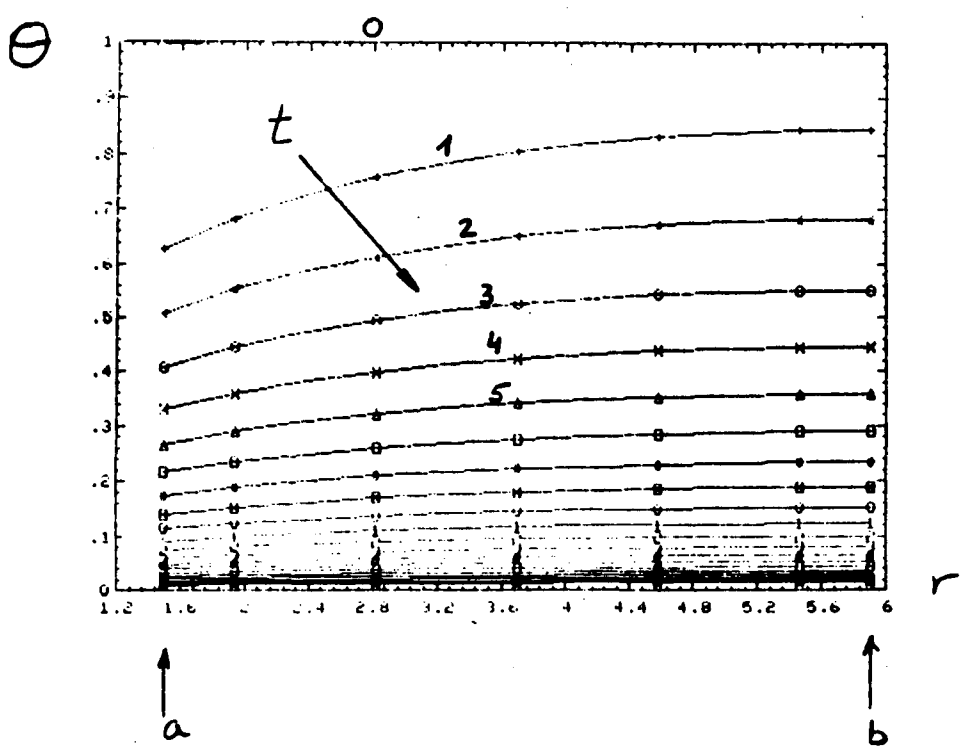

Fig. 3. The temperature across the cylinder at time intervals of 1 min.

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