LBL-19519
Preprint c.2

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

RECEIVED
LAWRENCE
BERKELEY LABORATORY
JUL 8 1985
LIBRARY AND
DOCUMENTS SECTION

Submitted to Physics Letters B

FLUCTUATIONS AND THE NUCLEAR MEISSNER EFFECT
IN RAPIDLY ROTATING NUCLEI

L.F. Canto, P. Ring, and J.O. Rasmussen

April 1985

TWO-WEEK LOAN COPY
This is a Library Circulating Copy
which may be borrowed for two weeks.

LBL-19519
c.2

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

LBL-19519

Fluctuations and the Nuclear Meissner Effect in Rapidly Rotating Nuclei.*

L.F.Canto[†], P.Ring[‡] and J.O.Rasmussen

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

*This work was supported in part by the the Brazilian CNPq and the Deutsche Forschungsgemeinschaft and in part by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

[†]Permanent address: Instituto de Fisica, Universidade Federal do Rio de Janeiro, Brasil

[‡]Permanent address: Physikdepartment der Technischen Universität München, West Germany

LBL-19519

Fluctuations and the Nuclear Meissner Effect in Rapidly Rotating Nuclei.*

L.F.Canto†, P.Ring‡ and J.O.Rasmussen

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Abstract:

The phase transition from a superfluid system to a normal fluid system in nuclei under the influence of a strong Coriolis field is investigated by the Generator Coordinate Method (GCM). It allows to take into account fluctuations of the orientation in gauge space connected with the violation of number symmetry in the BCS-approach as well as fluctuations of the gap-parameter connected with a virtual admixture of pairing vibrations in the wave function at the yrast line. The strange behavior of the experimental moments of inertia in the nucleus ^{168}Hf is well reproduced in this theory. The pairing collapse of the neutrons, however, is completely washed out by the fluctuations.

*This work was supported in part by the the Brazilian CNPq and the Deutsche Forschungsgemeinschaft and in part by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

†Permanent address: Instituto de Fisica, Universidade Federal do Rio de Janeiro, Brasil

‡Permanent address: Physikdepartment der Technischen Universität München, West Germany

Superfluid systems are characterized by the fact, that a short range effective interaction produces binding of particles in time-reversed orbits and the formation of a condensate of Bosons.The influence of an external field, which violates time reversal symmetry can break these pairs. In a sufficiently strong field one therefore finds a phase transition from superfluidity to normal fluidity. The best example for such a transition is Meissner-Effect in a superconductor: Under the influence of a sufficiently strong magnetic field superconductivity breaks down.In an infinite system this is a sharp phase transition and the corresponding order parameter, the pairing gap, goes abruptly to zero.

In nuclei Mottelson and Valatin[1]) predicted a similar effect: under the influence of a strong Coriolis field in rapidly rotating nuclei the superfluid behavior of heavy nuclei should disappear, a phase transition to a normal fluid should occur.

Many attempts have been made to see this transition experimentally. Most of them were devoted to search for anomalies in the rotational spectra, which can be measured by (HI,xn)-reactions with great accuracy. In fact one has found such anomalies, as the backbending phenomenon[2]), but it was soon realized, that it could not be attributed to a pairing collapse, but to a sudden alignment of one pair of nucleons with large single particle angular momenta[3]). It turned out, that the moment of inertia, which can be deduced from the spectra is of limited value to characterize a phase transition from superfluid to normal fluid in a rotating system: Only at low spins pairing correlations produce a energy gap in the quasiparticle spectrum. In cases of alignment quasiparticle energies can vanish even when pairing correlations are still strong, because the pairing field is in such a situation no longer

diagonal. One than has nuclear gapless superconductivity[4]) and in such a situation it is difficult to observe the still existing superfluidity from the spectrum. In fact after a few alignments one can assume to a first approximation, that the quasiparticle energies are distributed statistically, a situation, which causes the moment of inertia to be close to the rigid body value. Model calculations[5]) show, that this occurs even for a constant pairing field.

The experimental search for a pairing collapse should therefore concentrate not so much on spectra as on other nuclear properties, which indicate a phase transition more clearly. Of course the most direct evidence for a phase transition should be given by the order parameters themselves, i.e. by the pair-transfer matrix elements $\langle A+2,I|(a^+a^+)_0|A,I\rangle$. So far it seems to be very difficult to measure those at high spins[6]). We therefore have at the moment to rely on theoretical investigations about the possibility of a pairing collapse in rapidly rotating nuclei.

In most of these investigations mean field theory has been used. General considerations show that this approximation is very suitable in the case of large particle numbers or strong interactions, where perturbation theory breaks down. In the case of pairing correlations mean field theory is BCS-theory or HFB-theory. In fact one has found in all self-consistent solutions of the cranked HFB-equations[7-9]) a pairing collapse: In the Rare Earth region usually neutron pairing vanishes between 20 and 30 $\hbar$ and proton pairing vanishes between 40 and 50 $\hbar$. The question is, however, how reliable is the mean field approximation in this context. The total number of particles in such heavy nuclei is certainly large. The crucial quantity is, however not the total particle number, but only the number of particles

participating in the collective process of pairing. This is in fact not a very large number. Considering an average level spacing of 200 to 300 keV one finds only 3 to 5 pairs of particles in an energy interval of a pairing energy of 1 MeV. This is a rather small number and one therefore has to be very careful in drawing conclusions from the mean field approximation. In a way such an approximation shows already for small particle numbers features of the infinite system, where it becomes eventually exact. There are a number of investigations of the transition from the superfluid to the normal phase in exactly soluble models[10-12]), in which the mean field approximation seems to work relatively well. One has to bear in mind, however, that these models usually show a large degeneracy at the Fermi surface not observed in realistic nuclei. They are therefore not very helpful in the present context.

If one wants to go beyond the mean field approximation, one has to take into account fluctuations. One way to do this is the Random Phase Approximation(RPA). It is well known, however that fluctuations can become very large in the region of the phase transition. In fact the RPA, which is a theory for small amplitudes around the mean field theory, breaks down exactly at the point where the mean field shows the sharp phase transition. We therefore use in this paper a method, which goes essentially beyond the mean field approximation, the Generator Coordinate Method. It yields in many cases the exact solution[13]) and it has the general advantage to be based on a variational principle. In particular it contains the mean field and the random phase approximation as limiting cases. The conclusions we can draw from such a calculation are therefore clearly more general. The disadvantage of the method is, that it is numerically rather demanding and thus requires a considerable computational effort.

Certainly we are not able to solve the realistic manybody problem exactly: We include fluctuations of large amplitude, but we cannot include all kind of fluctuations. Since we are dealing with a very special type of collective motion, namely the pairing degree of freedom, we restrict ourselves to fluctuations in the order parameter, the gap. In addition we have to take into account the fact that phase transitions are connected with violations of symmetries: A BCS-wavefunction is a superposition of different particle numbers, i.e. it has a definite orientation in gauge space just as a deformed shape has an orientation in the three-dimensional ordinary space. In addition to the fluctuations in the gap, we therefore have to take into account fluctuations around this fixed orientation. This can be done in a rather simple way by the GCM method: The wavefunction is chosen to be a superposition of HFB-functions with different orientations in gauge space. Symmetry considerations determine the corresponding weight function completely: One ends up with a projection onto good particle number[14,15]).

For our numerical applications we start from the Baranger-Kumar Hamiltonian[16])

$$H = \varepsilon - \frac{\chi}{2} Q^{+}Q - G_p S_p^{+} S_p - G_n G_n^{+} S_n \tag{1}$$

which has proven to be very successful for a microscopic investigation of the interplay between quadrupole, pairing and rotational degrees of freedom in heavy deformed nuclei. We use the parameters of ref. 16. Only the strength parameters G_p and G_n of the pairing force are adjusted to the experimental gap parameters at spin zero. In the nucleus ^{168}Hf, which we investigate in the following, this means $G_p = 25.92/A$ (MeV) and $G_n = 22.4/A$ (MeV), which are very close to the values of ref. 16.

In the spirit of the cranking model we minimize the expectation value of the Hamiltonian in the rotating frame

$$\delta\langle\Psi|H - \omega J_x|\Psi\rangle = 0 \tag{2}$$

The angular velocity ω is determined by the constraint $\langle\Psi|J_x|\Psi\rangle = \sqrt{I(I+1)}$. For the variational functions we use the GCM-ansatz

$$|\Psi\rangle = \int_0^\infty d\Delta\ f(\Delta)P^N|\Phi(\Delta)\rangle \tag{3}$$

P^n is a projector onto good particle number and the GCM-basis states $|\Phi(\Delta)\rangle$ are generalized Slater determinants (HFB-functions) in a rotating superfluid mean field. They are found as eigenstates of the generalized single particle Hamiltonian

$$H_\omega = \varepsilon - \beta Q - \lambda_p N_p - \lambda_n N_n - \omega J_x$$

$$- \delta_p(S_p^+ + S_p) - \delta_n(S_n^+ + S_n) \tag{4}$$

ε are the spherical single particle energies of the Hamiltonian (1), and λ_t are chemical potentials determined in each case by the constraint on the particle numbers $\langle\Phi|N_\tau|\Phi\rangle = Z$ or N. Q is the quadrupole operator and β is the deformation parameter. Since we are interested only in the pairing degree of freedom, and since the nucleus ^{168}Hf, which we investigate in the following is relatively stiff against shape changes, we keep β fixed at the mean field value for spin zero ($\beta = 0.274$). The operators S_τ^+ create

Cooper-pairs for protons and neutrons. G_τ are pairing parameters for the corresponding pairing fields. Our generator coordinate is the gap parameter for neutrons

$$\Delta = G_n \langle\Phi|S_n^+|\Phi\rangle \tag{5}$$

the "pairing deformation" of the wave function, rather than the "pairing deformation" δ of the potential. For the sake of simplicity we do not investigate in the following the pairing properties of protons and keep δ_p at the experimental value $\delta_p = 0.85$ (MeV) for all angular momenta.

Our GCM wave function $|\Psi\rangle$ is therefore a superposition of number projected mean field functions $|\Phi(\Delta)\rangle$ and the Generator Coordinate is the pairing deformation of neutrons. The variation with respect to the weight function $f(\Delta)$ yields the well known Griffin-Hill-Wheeler equation[17,18]), which is solved by discretization. As the kernels of this integral equation are highly singular special care has to be taken[13]). We insured convergence with respect to the discretization and with respect to the cut off parameter of the corresponding zero eigenvalues of the norm. More details of this calculation will be given in a separate paper[19]).

Recent experiments in Daresbury[20]) have found for a number of nuclei in the Hf-region a very constant value for the moment of inertia above spin 20 $\hbar$, close to the rigid body value. It has been suggested that this behavior signifies for a sudden pairing collapse at spin 20 $\hbar$. In this letter we will concentrate on the case ^{168}Hf. In Fig.1 we compare moments of inertia obtained in four different theories with the experimental values. The lower part shows mean field theories, namely simple HFB and number projected HFB, the upper part shows two versions of the GCM method. In one case we used

simple HFB-functions as generating functions; in the second case we used number projected HFB-functions, as in eq.(3)

We find in all theories more or less the same behavior: a steep increase of the moment of inertia for angular momenta between 10 and 20 $\hbar$ and rather constant values above 20 $\hbar$. The increase between 10 and 20 $\hbar$ is connected with the sudden alignment of a pair of neutrons in the j=13/2 orbit. The experimental increase is somewhat steeper than the theoretical one. We understand this as a shortcoming of the Cranking approximation used in all four theories. It is well known that this approximation without angular momentum projection washes out level crossing phenomena, but we do not concentrate on this problem in this paper. We are interested in the region above 20 $\hbar$. Here all four theories show rather constant values for the moment of inertia, in agreement with the experimental data. Only the simple HFB-approach shows a small kink close to spin 22 $\hbar$. It has its origin in the sudden pairing collapse in this theory at this point. In the other three theories this kink is washed out.

In Fig.2 we show the GCM-"wavefunctions". Since the GCM-basis states $|\Phi(\Delta)\rangle$ and $P^N|\Phi(\Delta)\rangle$ are neither orthogonal nor linear independent, the weight-functions $f(\Delta)$ in eq.(3 are not uniquely determined by the Hill-Wheeler equation[13]). In particular they depend on the discretization procedure. We therefore show in Fig.2 the "covariant components" of the GCM representation, the overlap integrals

$$\varphi(\Delta) = \langle\Phi(\Delta)|\Psi\rangle \tag{6}$$

They measure the probability to find a basis state $|\Phi(\Delta)\rangle$ or $P^N|\Phi(\Delta)\rangle$ in

the GCM-function $|\Psi\rangle$. We also plot the collective potential in the rotating system

$$V(\Delta) = \langle\Phi(\Delta)|P^N(H - \omega J_x)P^N|\Phi(\Delta)\rangle \tag{7}$$

It is given by the diagonal element of the Hamilton kernel in the Hill-Wheeler equation. Using the parameter Δ as a classical coordinate this quantity is just the potential energy for a classical Hamilton function describing the collective motion in the pairing degree of freedom[21,22]). The Griffin-Hill-Wheeler equation contains in addition off-diagonal elements, which produce the kinetic energy and correction to the potential, and it provides a fully quantum mechanical description. In a simplified picture we can visualize the quantity $\varphi(\Delta)$ as being the wave function of the zero point oscillation in the potential $V(\Delta)$. In the simple mean field theory we have only one generalized Slater determinant (eventually projected onto good particle number). According to the variational principle it is the function, which corresponds to the minimum in the potential $V(\Delta)$. We see that the GCM theory changes this picture considerably. Now the wavefunction contains contributions with rather different gap parameters. Note that the maxima in wavefunctions generally do not occur at the same Δ as the potential minimum. Such a result implies a non constant inertial parameter in the pairing coordinate.

Two angular velocities are considered in Fig.2. At $\omega = 0.0$ both energy surfaces (with and without number projection) show well pronounced minima at $\Delta \approx 1$ (MeV). The number projected energy is lowered, because the spurious energy coming from the symmetry violation is removed. The GCM wave

functions are rather similar in both cases. At $\omega = 0.4$ (MeV) the situation is very different. At this point we are already beyond the pairing collapse in HFB-theory, i.e. the unprojected energy surface shows a minimum at $\Delta = 0.0$. The number projected wave functions contain more correlations and the corresponding energy surface has a minimum at $\Delta \approx 0.5$ (MeV). That means the pairing collapse is considerably smeared out now, a fact which has been seen already in earlier investigations[9,5,23]). Because of the different behavior of the energy surface, we also find differences in the GCM-wavefunctions. Without number projection it has its maximum at zero gap; with number projection the maximum is shifted to finite values of Δ. In both cases we observe again considerable fluctuations.

There has been a much discussion about how to define the gap parameter in theories going beyond the HFB-approach. We use the definition

$$\tilde{\Delta} = G \sqrt{\langle S^{+} S \rangle} \tag{8}$$

In HFB-theory the above expression gives the pairing deformation of the wavefunction, usually called the gap parameter. In the extended theories it measures the energy of the pairing correlations (the so called exchange term $G\, v^4$ is, of course, neglected in all our calculations, i.e. $\tilde{\Delta}$ vanishes in the case $\Delta = 0$). It is evident that this gain in energy by pairing is partially compensated by the energy we have to add to the system in order to scatter the particles around the Fermi surface. The so called correlation energy, which is the difference in total energy between a system with pairing and without pairing is therefore considerably smaller. We believe that the crucial quantity for measuring pairing correlations is only that part of the

energy gained by the pairing. In all our calculations the effective gap parameter is delivered through eq.(8).

In Fig.3 we show this effective gap parameter for different theories as a function of the angular momentum: In HFB-theory we observe a pairing collapse at $I = 22\ \hbar$. In all theories which go beyond this simple mean field approximation this sharp phase transition is smeared out. Surprisingly, there is little difference between the method of number projection before variation, which takes into account fluctuations in the gauge angle but neglects the virtual admixtures of pairing vibrations, and the full GCM-theory with number projection. This result seems to indicate that the most important fluctuations for a proper description of pairing in nuclei are those treated in the symmetry conserving mean field theory. Additional correlations coming from virtual admixtures of pairing vibrations seem to play only a minor role.

Summarizing the results of these investigations, we must conclude that the sharp pairing collapse found in many HFB-calculations for high angular velocities is completely washed out if one includes fluctuations. In particular we see that the surprisingly constant moments of inertia near rigid body values in several nuclei in the Hf region for spins above 20 $\hbar$ by no means are any indication for a pairing collapse. The absence of a sharp phase transition was shown already earlier by number-projected HFB theory[5]). From the present work this absence is not changed, when we go a step further and include in addition fluctuations caused by pairing vibrations in the framework of a Generator Coordinate Method.

Two of us (L.F.C. and P.R.) want to express their gratitude to the Nuclear Science Division for the hospitality extended to them during their stay at the Lawrence Berkeley Laboratory. We also acknowledge helpful discussions with J.L.Egido, R.R.Hilton, H.J.Mang and N.Onishi.

This work was supported in part by the the Brazilian CNPq and the Deutsche Forschungsgemeinschaft and in part by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

Figure Captions:

Fig.1 Moments of inertia at the yrast line in ^{168}Hf. The experimental values of ref. 20 (full triangles) are compared in the lower part with mean field approximations and in the upper part with calculations based on the GCM method. For the dashed lines no number projection has been used. Full lines represent calculations with number projection. No core moment-of-inertia has been used.

Fig.2 GCM-"wavefunctions" $\varphi(\Delta)$ and energy surfaces for two angular velocities in the nucleus ^{168}Hf. Both quantities are given as function of the generator coordinate Δ, the pairing deformation of the underlying unprojected HFB-function defined in eq.(5). The "wave functions" are defined in eq.(6). The energy surfaces $V(\Delta)$ correspond to the rotating frame. Dashed lines correspond to calculations without number projection. Full lines include exact number projection.

Fig.3 Effective gap parameters defined in eq.(8) as a function of the angular momentum I. At I=0 all theories show the experimental values of Δ. This has been achieved by a minor adjustment of the strength parameter G_n in each calculation. Dashed lines correspond to a calculation without number projection; full lines include exact number projection.

References:

[1] B.R.Mottelson and J.G.Valatin; Phys.Rev.Lett. 5 (1960) 511

[2] A.Johnson, H.Ryde, and J.Sztarkier; Phys.Lett. 34B (1971) 605

[3] F.S.Stephens and R.S.Simon; Nucl.Phys. A183 (1972) 257

[4] A.Goswami, L.Lin, and G.Struble, Phys.Lett. 25B (1967) 451

[5] U.Mutz and P.Ring; J.Phys. G10 (1984) L39

[6] M.W. Guidry, T.L. Nichols, R.E. Neese, J.O. Rasmussen, L.T. Oliveira, and R. Donangelo; Nucl. Phys. A361 (1981) 275

J.O. Rasmussen, M.W. Guidry, and L.F. Canto; Preprint LBL-19125 (1985)

[7] P.Ring, R.Beck, and H.J.Mang; Z.Phys. 231 (1970) 10

[8] A.L. Goodman; Nucl. Phys. A256 (1976) 113

[9] J.L.Egido and P.Ring; Nucl.Phys. A388 (1982) 19

[10] S.Y.Chu, E.R.Marshalek, P.Ring, I.Krumlinde, and J.O.Rasmussen; Phys.Rev. C12 (1975) 1017

[11] O.Civitarese, A.Plastino, and A.Faessler; Z.Phys. A313 (1983) 197

[12] W.Nazarewicz, J.Dudek, and Z.Szymanski; Nucl.Phys. A436 (1985) 139

[13] P.Ring and P.Schuck; The Nuclear Manybody Problem (Springer Verlag, New York 1980)

[14] K.Dietrich, H.J.Mang, and J.H.Pradal; Phys.Rev. 135 (1964) B22

[15] H.J.Mang, J.O.Rasmussen, and M.Rho; Phys.Rev. 141 (1966) 941

[16] B.Baranger and K.Kumar; Nucl.Phys. A110 (1968) 490

[17] D.L.Hill and J.A.Wheeler; Phys.Rev. 89 (1953) 1102

[18] J.J. Griffin and J.A. Wheeler; Phys. Rev. 108 (1957) 311

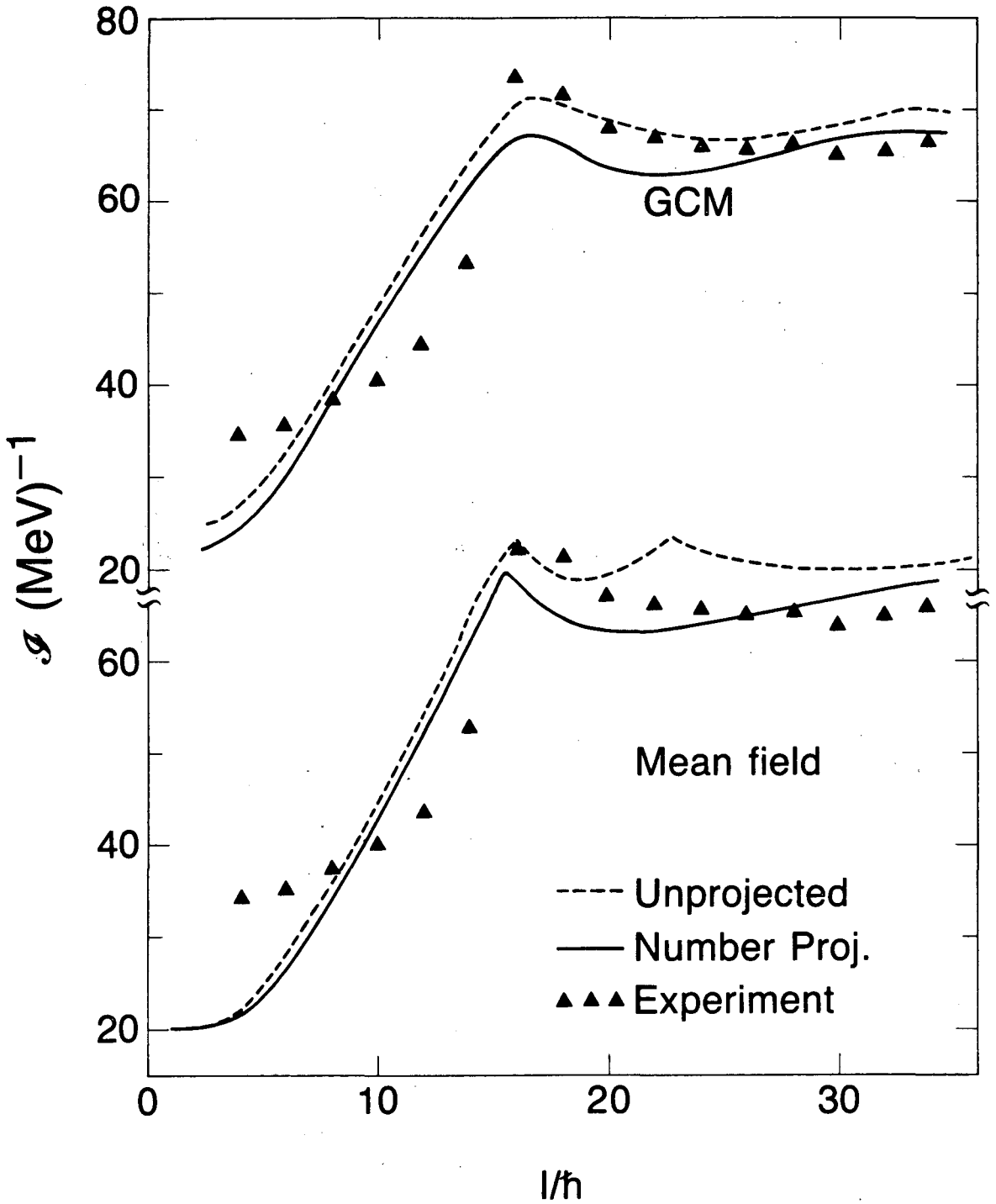
[19] L.F.Canto and P.Ring to be published

[20] R.Chapman, J.C.Lisle, J.N.Mo, E.Paul, A.Simcock, J.C.Willmott, and J.R.Leslie; Phys.Rev.Lett. 51 (1983) 2265

[21] M.Baranger and M.Veneroni; Ann.Phys.(New York) 114 (1978) 123

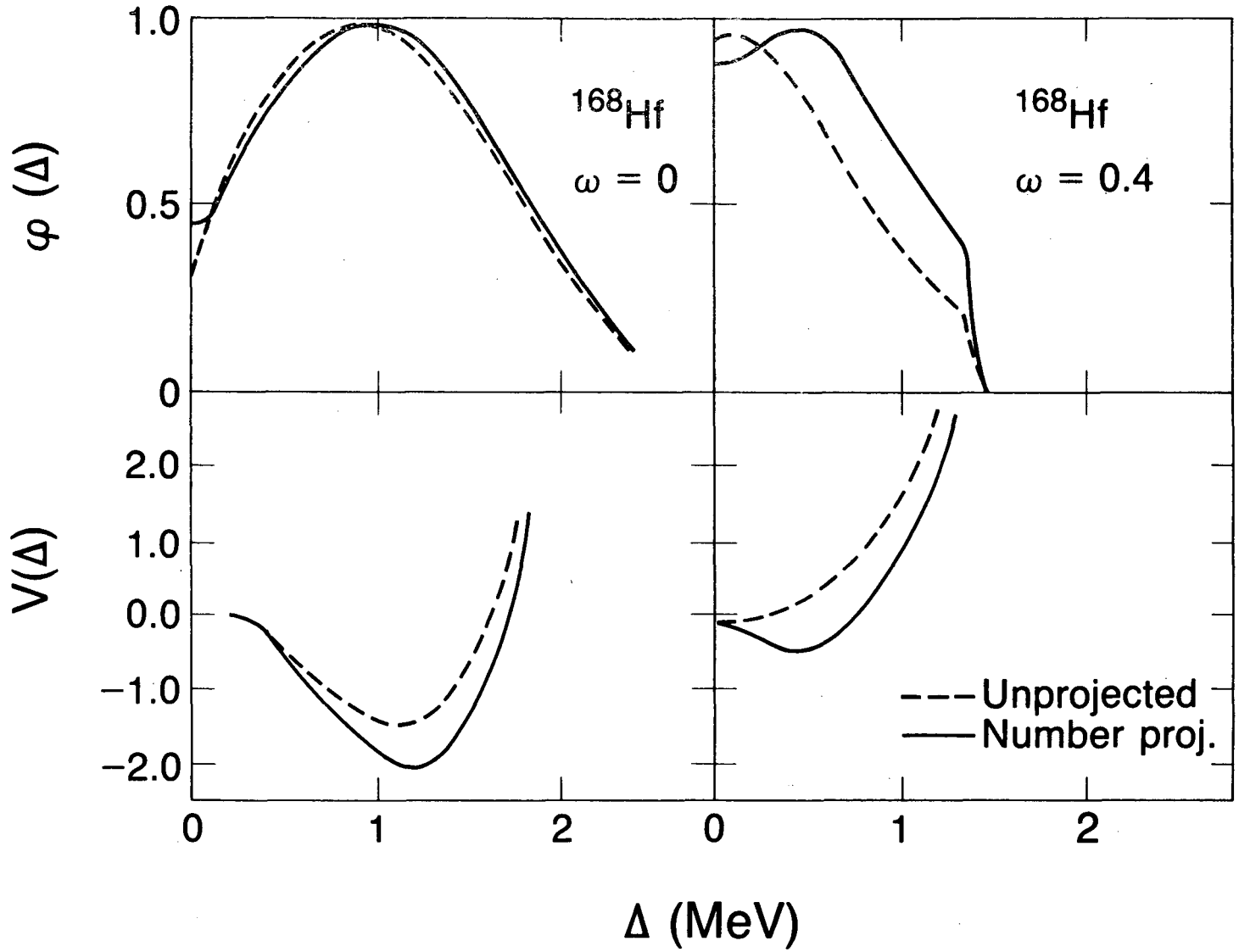
[22] F.Villars; Nucl.Phys. A285 (1977) 269

[23] A.Faessler, K.R.Sandya Devi, F.Gruemmer, K.W.Schmid, and R.R.Hilton; Nucl.Phys. A256 (1976) 106



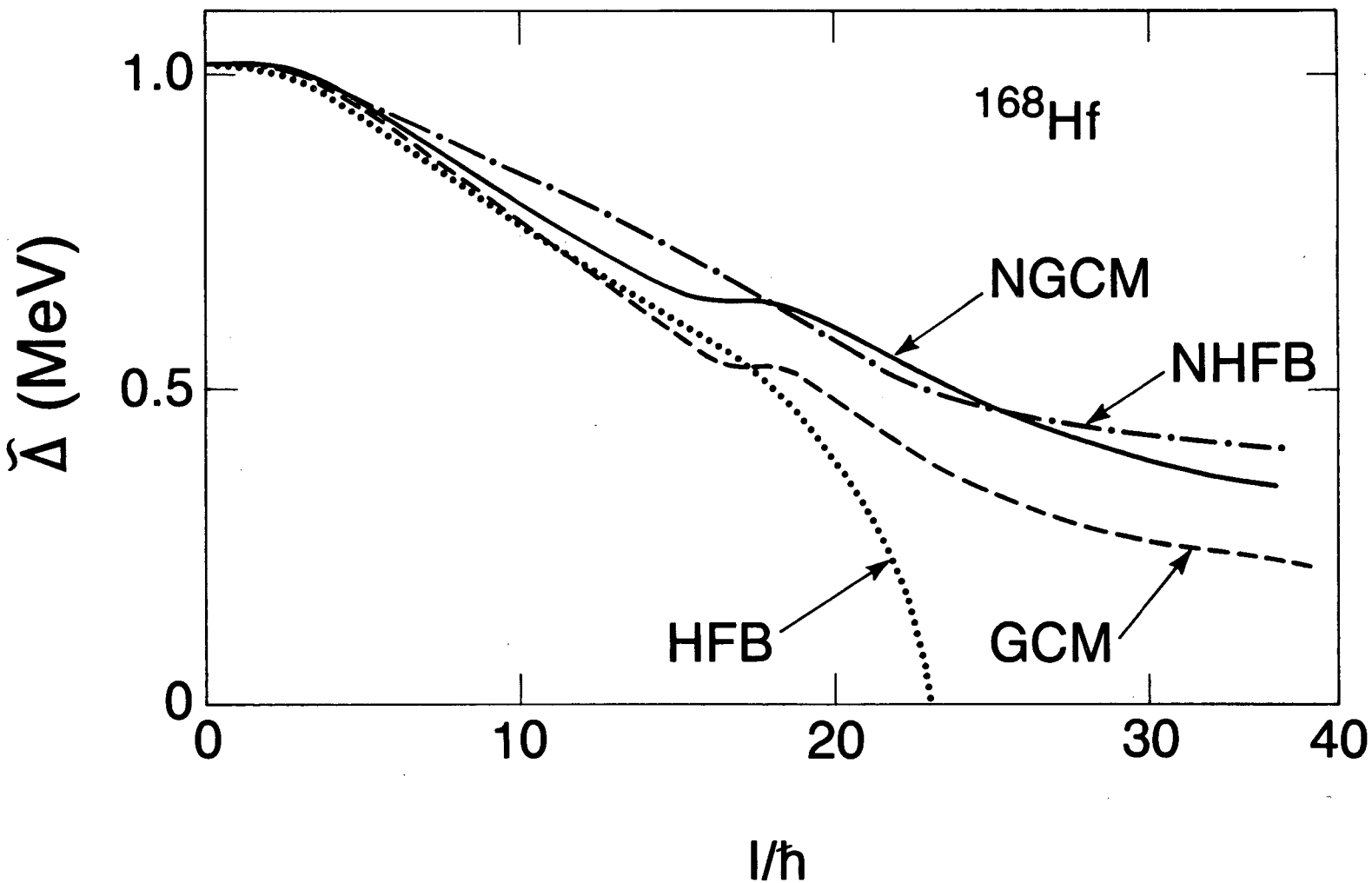
XBL 854-11061

Fig. 1



XBL 854-11066

Fig. 2



XBL 854-11060

Fig. 3

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720