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Abstract:

We present a calculation of the macroscopic contribution to the dipole moment of a deformed (axially symmetric but reflection asymmetric) nucleus, using the Droplet Model. In addition to the familiar charge redistribution effect (which is worked out including a correction for the relative neutron excess (N-Z)/A and a second correction of relative order $A^{-1/3}$), the Droplet Model predicts a contribution from the presence of a neutron skin. This contribution turns out to be of the same order as the redistribution effect and of opposite sign. The result is a reduction of the macroscopic contribution to the dipole moment to small or even negative values, as compared with earlier estimates. All the formulae are worked out for a nuclear shape specified by an arbitrary number of Legendre Polynomials.

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1. Introduction

In two recent papers[1]) Leander has examined a number of experiments on isotopes in the Ra-Th region indicating unusually fast E1 transitions. These transitions appear to be collective and associated with ground-state electric dipole moments. Indeed, it was noted earlier[2,3]) that the Coulomb repulsion of the protons can induce such moments in deformed nuclei that have odd-parity components in their intrinsic deformation in addition to the usual even-parity components.

Both theory[4]) and experiment[5]) support the presence of octupole-deformed intrinsic equilibrium shapes in this region of nuclei, and Leander has made pioneering attempts to estimate what fraction of the observed electric dipole moment might be associated with the particular orbital configurations occupied in each nucleus (representing the microscopic or shell effects) and what fraction might be due to the octupole-induced polarization of the bulk nuclear matter (corresponding to a macroscopic or Liquid Drop Model effect).

In the present paper we address ourselves to the calculation of this macroscopic contribution to the nuclear dipole moment of a deformed nucleus. We use the nuclear Droplet Model, formulated as a systematic improvement of the Liquid Drop Model. A guiding theme of the Droplet Model is that the requirement to work consistently to a definite order in the relevant expansion parameters makes it necessary to include two new sets of degrees of freedom specifying the nuclear configuration: the nucleon density non-uniformities and the neutron skin[6,7]). The inclusion of the former leads to a redistribution of the proton and neutron densities under the influence of the electrostatic forces, and this induces a slight dipole moment in deformed,

reflection asymmetric nuclei. This is the effect that had been studied already in the early papers of Strutinsky[2]) and Bohr and Mottelson[3]) in connection with the dipole moment and in refs.[8,9,10]) in connection with fission. The inclusion of the neutron skin degree of freedom also leads to a slight separation of the centers of mass of the neutrons and protons, resulting in another contribution to the dipole moment. This influence of the neutron skin on the dipole moment does not appear to have been studied before. As we shall see, the effect is, in most cases of interest, of a magnitude comparable with the charge-redistribution effect (and of opposite sign!). This means that a meaningful estimate of the macroscopic part of the nuclear dipole moment cannot be made without taking into account the neutron skin contribution in addition to the charge redistribution effect.

In the next section, we present the Droplet Model formula for the dipole moment of a deformed nucleus and we illustrate the numerical values on the examples of ^{222}Th and ^{226}Ra. In the Appendix we sketch the derivation of this formula.

2. The Droplet Model formula for the dipole moment of a deformed nucleus

The shell-effect averaged density distributions of the neutrons and protons in a nucleus may be specified by two smooth functions of position. According to the Droplet Model, these functions may be imagined as resulting from applying a standard diffuseness prescription to two sharp-surfaced generating distributions, $n(\vec{r})$ and $p(\vec{r})$, say, vanishing abruptly outside two effective sharp surfaces Σ_n and Σ_p, respectively. The (small) normal distance from a point on the surface Σ_p to the surface Σ_n is referred to as the neutron skin t (a function of position on the surface). The location of a mean surface Σ may be defined by an appropriate weighting of the locations of Σ_p and Σ_n with the surface values of p and n, respectively, and this surface is used to specify the nuclear shape. The degrees of freedom in the Droplet Model are thus the shape of the surface Σ, the neutron skin function t and the deviations of the generating densities from their respective average values. (These deviations will be denoted by $\tilde{n}$ and $\tilde{p}$.) According to the Droplet Model, the macroscopic nuclear energy is a definite function of these degrees of freedom and consists of a volume energy, a surface-layer energy and an electrostatic energy. Minimization of the total energy with respect to variations of t, $\tilde{n}$ and $\tilde{p}$ for a specified Σ gives algebraic Droplet Model expressions for the equilibrium values of these quantities. Substitution into the energy expression leads to the shape-dependent Droplet Model mass formula. On the other hand, with t, $\tilde{n}$ and $\tilde{p}$ determined, the dipole moment associated with any given shape Σ may also be written down algebraically. As shown in the Appendix, this dipole moment D, in units of the proton charge e, (i.e., $d \equiv D/e$) turns out to be given by the following expression:

$$d = C_r \sum_{\ell=2}^{\infty} \frac{12(\ell - 1)(\ell + 1)(8\ell + 9)}{5(2\ell + 1)^2(2\ell + 3)^2} \alpha_\ell \alpha_{\ell+1}$$

$$- C_s \sum_{\ell=2}^{\infty} \frac{(\ell - 1)(\ell + 1)(\ell + 3)}{(2\ell + 1)(2\ell + 3)} \alpha_\ell \alpha_{\ell+1}$$

$$+ \text{ terms of higher order in the } \alpha_\ell\text{'s} \quad . \tag{1}$$

Here α_ℓ are the coefficients specifying the usual Legendre Polynomial expansion of the radius vector $R(\theta)$, describing the surface Σ according to

$$R(\theta) = R_0 \left[1 + \sum_{\ell=2}^{\infty} \alpha_\ell P_\ell(\cos\theta) \right] \quad . \tag{2}$$

The coefficients C_r and C_s are definite functions, given below, of the neutron and proton numbers N,Z and of the equivalent radius R_0 of the nucleus in question, as well as of the relevant parameters specifying the volume energy per particle and the surface tension coefficient in the Droplet Model. The first term in eq.(1) describes essentially the "lightning rod" effect, according to which the electric charge, when allowed to redistribute itself, tends to move towards regions of the surface with large curvature. The second term describes the neutron skin effect according to which, even in the absence of any non-uniformities either in the densities n,p or the neutron skin thickness t, a dipole moment may appear (for non-zero t) because of the geometrical fact that the center of gravity of a uniform thin neutron layer around the surface Σ does not coincide, in general, with the center of gravity of the volume enclosed by Σ.

The estimates of the dipole moment according to refs.[2,3]), in which the neutron skin contribution is disregarded and the redistribution effect is calculated to leading order only, correspond to taking $C_s = 0$ and $C_r = AZe^2/8J$, where A is the mass number and J is the symmetry energy coefficient (≈33 MeV). As shown in the Appendix, a more accurate treatment, following from a consistent application of the Droplet Model, leads to

$$C_r = (AZe^2/8)\left[\frac{1}{J} + \frac{6L}{JK} I + \frac{15}{8Q} A^{-1/3}\right] \quad , \tag{3}$$

$$C_s = (3NZ/A)\bar{t} = (2NZ/A)(I - \bar{\delta})R_0 \quad , \tag{4}$$

where

$$\bar{\delta} = \frac{I + (3c_1/16Q)ZA^{-2/3}}{1 + (9J/4Q)A^{-1/3}} \quad . \tag{5}$$

In the above, I is the overall relative neutron excess, (N-Z)/A, $\bar{\delta}$ is the average relative neutron excess in the bulk [i.e. the average of (n - p)/(n + p) in the bulk], whose Droplet Model equilibrium value is given by eq.(5), and $\bar{t}$ is the average value of the neutron skin thickness, related to I and $\bar{\delta}$ by the 'geometrical' expression

$$\bar{t} = (2/3)(I - \bar{\delta})R_0 \quad . \tag{6}$$

The three Droplet Model coefficients appearing in eq.(3) are the compressibility coefficient K (≈240 MeV), the density symmetry coefficient L

($\approx$0-100 MeV) and the effective neutron-skin stiffness coefficient Q ($\approx$30 MeV). In eq.(5), c_1 is the Coulomb energy coefficient, given by $c_1 = (3/5)e^2/r_0$, where $r_0 = R_0/A^{1/3}$.

We note that, compared to the older estimates of the dipole moment, eqs.(3),(4) introduce three new contributions: the neutron skin term (eq.(4)), a neutron-excess correction (the term in eq.(3) proportional to I) and the last term in eq.(3), which is a charge-redistribution contribution associated with the non-uniformity of the neutron skin.

In order to illustrate the relative magnitudes of the various contributions, we present below numerical results based on using the following set of recent Droplet Model parameters[11]): J = 32.5 MeV, K = 240 MeV, L = 0, Q = 29.4 Mev, r_0 = 1.16 fm ($\therefore c_1$ = 0.744816 MeV). We use eqs.(1)-(5) to calculate the macroscopic dipole moments for ^{222}Th and ^{226}Ra, for which calculated ground-state deformations are given in ref.[4]) as $\beta_2 = 0.114$, $\beta_3 = 0.096$, $\beta_4 = 0.0678$ and $\beta_2 = 0.159$, $\beta_3 = 0.083$, $\beta_4 = 0.0872$, respectively. (The deformation parameters β_ℓ are related to α_ℓ by $\alpha_\ell = \sqrt{(2\ell + 1)/4\pi}\ \beta_\ell$). The result of this calculation is as follows:

$$d = 0.1564 + 0.0 + 0.0548 - 0.1880 = 0.0232 \text{ fm} \tag{7}$$

for ^{222}Th, and

$$d = 0.1832 + 0.0 + 0.0624 - 0.2620 = -0.0164 \text{ fm} \tag{8}$$

for ^{226}Ra.

The first three contributions to d in these equations correspond to the three terms in eq.(3) and the last (negative) contribution corresponds to the neutron-skin effect represented by eq.(4).

The most striking feature of these results is the importance of the neutron-skin contribution which, in the above examples, tends to wipe out the redistribution contribution, leaving a final dipole moment several times smaller than would be given by the simplest redistribution calculation (i.e. by the first term in eq.(3)). This would be bad news for interpretations of the estimated empirical values of the dipole moments that rely on a theory without the neutron-skin term[1]). But how sensitive is this conclusion to possible changes of the Droplet Model parameters, some of which are not determined accurately? We have repeated the calculation with an older set of Droplet Model parameters [J = 36.8 MeV, K = 240 MeV, L = 100 MeV, Q = 17 MeV, r_0 = 1.18 fm[7])] with the following result:

$$d = 0.1381 + 0.0653 + 0.0926 - 0.2996 = -0.0036 \text{ fm} \tag{9}$$

for ^{222}Th, and

$$d = 0.1618 + 0.0895 + 0.1078 - 0.4161 = -0.0570 \text{ fm} \tag{10}$$

for ^{226}Ra.

The neutron-skin term is even larger now (because of the smaller value of the neutron-skin stiffness coefficient Q), resulting in small negative dipole moments in both Th and Ra. But, conversely, could Q be even larger than the

value assumed in the more recent fit (29.4 MeV)? The extreme assumption of an infinite Q (no neutron skin at all) would give

$$C_r = (AZe^2/8)\left[\frac{1}{J} + \frac{6IL}{KJ}\right] \quad , \tag{11}$$

$$C_s = 0 \quad .$$

With L = 0, this would reduce to the old redistribution calculation (first terms in eqs.(7,8)). With L = 100 the result would be even bigger, by a factor of $1 + (6IL/K) \approx 1 + (6 \times 0.2 \times 100/240) \approx 1.5$. But is it reasonable to take $Q = \infty$? The answer would seem to be rather definitely NO, on several counts. First, fits of the Droplet Model mass formula to nuclear masses and fission barriers give invariably a finite Q, with typical values indicated above. (The finiteness of Q is directly responsible for the surface symmetry energy in the mass formula. The existence of this term seems today beyond doubt, although its precise value is still subject to substantial uncertainty.) Second, the analysis of the systematics of nuclear radii, especially of the isotope and isotone shifts, seems to point unambiguously to a finite Q, with a value in general agreement with that deduced from fits to nuclear masses[12]). (Further unpublished studies along those lines indicate that a large value of Q tends to be associated with a small value of L, so that the choice $Q = \infty$, L = 100 would be particularly inappropriate.) Third, the extensive studies of the trend with mass number A of the giant dipole resonance energies, which have established with considerable precision a behaviour intermediate between $A^{-1/3}$ and $A^{-1/6}$, point directly to the participation of the Goldhaber-Teller mode in dipole oscillations. This

mode, an overall oscillation of protons against neutrons, involves the neutron skin degree of freedom and the associated effective stiffness coefficient Q. The mere presence of the $A^{-1/6}$ component in the trend of the resonance energies attests to the finiteness of Q, and a more quantitative theory leads to a value consistent, once more, with the values deduced from fits to nuclear masses.[13]) Lastly, Thomas-Fermi or Hartree-Fock calculations with effective (Seyler-Blanchard or Skyrme) interactions, applied to a study of the nuclear surface, give finite values of Q, ranging from ~17 MeV to ~60 MeV[7,14]).

It would seem from all this that a finite value of Q, of the general order of 30 MeV (with an uncertainty of probably less than a factor of two) has to be accepted at this time. This implies a considerable reduction from the previously estimated macroscopic contribution to the dipole moments of deformed nuclei. Any apparent disagreement with experimental estimates which this may cause[1]) could be an indication of several possible shortcomings in the analysis of dipole moments, among which the following should be investigated. First the assumptions underlying the experimental estimates themselves should be scrutinized. Second, the difficult question of estimating the microscopic (individual particle) contribution to the dipole moment should be examined further. Third, the reliability and precision of the estimated values of α_2, α_3, α_4 (and perhaps also α_5 and α_6) should be investigated: because of the cancellations of relatively large contributions and because of the growing sensitivity of the neutron-skin term to higher multipoles (see eq.(1)), a fairly accurate determination of the nuclear shape is necessary for a reliable estimate of the macroscopic dipole moment.

Without being able at this time to make a closer analysis of the relation of the macroscopic theory to the experimental determinations of the dipole

moments of nuclei, we hope that the relatively accurate Droplet Model expression for the macroscopic part, eq.(1), will be of help in improving our understanding of this aspect of nuclear structure.

Appendix

1. Effect of surface diffuseness on the dipole moment

As described in sec. 2, the Droplet Model neutron and proton densities are obtained by applying a diffuseness prescription to sharp-surfaced generating densities $n(\vec{r})$, $p(\vec{r})$ bounded by effective sharp surfaces Σ_n and Σ_p. Assuming the diffuseness prescription to be in the form of a folding (convolution) operation with a finite-ranged spherically symmetric function, the discussion of the effect of the diffuseness on the dipole moment may be trivially disposed of: the center of gravity of each infinitesimal element of the densities n or p is unaffected by the convolution, and hence the centers of gravity of the whole distributions are also unaffected and the dipole moment remains unchanged. [Note that the result is very general: the folding functions may be different for the neutrons and protons, their range need not be small compared to the nuclear size and the independence on the folding operation extends to arbitrary multipole moments[15])]. It follows that the results derived below for the sharp generating densities n,p continue to hold exactly for diffuse distributions obtained by a folding operation.

2. Expression for the dipole moment

Denoting the center of mass location of the protons and neutrons by $\vec{r}_p$ and $\vec{r}_n$, and of the nucleus as a whole by $\vec{r}_A$, the dipole moment may be written as

$$\vec{D} = Ze(\vec{r}_p - \vec{r}_A) = Ze\left(\vec{r}_p - \frac{Z\vec{r}_p + N\vec{r}_n}{A}\right)$$

$$= \frac{ZNe}{A}(\vec{r}_p - \vec{r}_n) = \frac{ZNe}{A}\left(\frac{1}{Z}\int_p p\vec{r} - \frac{1}{N}\int_n n\vec{r}\right) \quad , \tag{A.1}$$

where the integrals are over the volumes V_p, V_n enclosed by Σ_p and Σ_n, respectively. Writing $p = \bar{p} + \tilde{p}$, $n = \bar{n} + \tilde{n}$, where $\bar{p}, \bar{n}$ are the volume averages of p,n, we find for $\vec{d}$ ($\equiv \vec{D}/e$) the expression

$$\vec{d} = \frac{NZ}{A}\left[\frac{1}{Z}\int_p (\bar{p} + \tilde{p})\vec{r} - \frac{1}{N}\int_n (\bar{n} + \tilde{n})\vec{r}\right]$$

$$= \frac{NZ}{A}\left[\left(\frac{1}{Z}\int_p \tilde{p}\,\vec{r} - \frac{1}{N}\int_n \tilde{n}\,\vec{r}\right) + \left(\frac{1}{V_p}\int_p \vec{r} - \frac{1}{V_n}\int_n \vec{r}\right)\right] \quad , \tag{A.2}$$

Writing $V_n = V_p + \Delta$, where Δ is the volume of the neutron skin, assumed small, we may rewrite the last term as

$$\frac{NZ}{A}\,\frac{1}{V_p}\int_p \vec{r} - \frac{1}{V_p}\left(1 - \frac{\Delta}{V_p}\right)\left(\int_p \vec{r} + \oint_p t\vec{r}\right)$$

$$\approx \frac{NZ}{A}\cdot\frac{\Delta}{V_p}\left(\frac{1}{V_p}\int_p \vec{r} - \frac{1}{\Delta}\oint_p t\vec{r}\right) \quad , \tag{A.3}$$

where the surface integrals are over the surface Σ_p.

Inserting in eq.(A.2), we find

$$\vec{d} = \vec{d}_{volume} + \vec{d}_{skin} \quad , \tag{A.4}$$

where

$$\vec{d}_{volume} \approx \frac{NZ}{A} \int \left(\frac{\tilde{p}}{Z} - \frac{\tilde{n}}{N}\right)\vec{r} \quad , \tag{A.5}$$

$$\vec{d}_{skin} \approx \frac{NZ}{A}\frac{\Delta}{V}(\vec{r}_A - \vec{r}_t) \quad , \tag{A.6}$$

where $\vec{r}_t$ is the location of the center of mass of the neutron skin and V is the volume inside the mean surface Σ. Note that we have replaced integrals over the proton volume by integrals over the mean volume. This is permitted in the above expression, since we are working consistently to first order in the small quantities $\tilde{p},\tilde{n},t$(or Δ), without cross terms between them.

3. <u>Droplet Model expressions for $\tilde{p},\tilde{n}$</u>

According to the Droplet Model[6,7]) the density non-uniformities $\tilde{p},\tilde{n}$ follow from making stationary the bulk energy (consisting of nuclear and electrostatic contributions). The presence of the neutron skin or of an overall compression or dilatation of the nucleus plays no role to the relevant order in small quantities. It follows that expressions for $\tilde{p}$ and $\tilde{n}$ may be thought of as derived in an idealized situation where N neutrons and Z protons are put in a common standard volume $4\pi R_0^3/3$, and the bulk energy is minimized. We shall go over this calculation once again, rather than simply use the results of refs.[6,7]), in order to include a small refinement in the expressions for $\tilde{n}$ and $\tilde{p}$ proportional to the relative neutron excess I. This refinement was not relevant in the context of the Droplet Model mass formula where I, as well as $\tilde{p},\tilde{n},t$, were consistently treated as small quantities. The present derivation considers the smallness of I as a separate

assumption that may or may not be introduced at a later stage of the calculation.

The bulk energy to be minimized may be written as

$$E = \int (\eta + \frac{1}{2} epv) \quad , \tag{A.7}$$

where η is the nuclear energy density in the bulk, v is the electric potential due to the protons and the integral is over the standard volume. Introducing the total and differential densities

$$\rho = n + p \quad , \tag{A.8}$$

$$\omega = n - p \quad , \tag{A.9}$$

we may write

$$E = \int [\eta(\rho,\omega) + \frac{1}{4} e(\rho - \omega)v] \quad . \tag{A.10}$$

The stationarity of E for variations of ρ and ω that conserve the particle numbers requires that

$$\frac{\partial\eta}{\partial\rho} + \frac{1}{2} ev = \text{constant} \quad , \tag{A.11}$$

$$\frac{\partial\eta}{\partial\omega} - \frac{1}{2} ev = \text{constant} \quad . \tag{A.12}$$

Assume now a Taylor expansion for η in the form

$$\eta = \eta_0 + \eta_\rho(\rho - \bar{\rho}) + \eta_\omega(\omega - \bar{\omega}) + \frac{1}{2}\eta_{\rho\rho}(\rho - \bar{\rho})^2 + \eta_{\rho\omega}(\rho - \bar{\rho})(\omega - \bar{\omega}) + \frac{1}{2}\eta_{\omega\omega}(\omega - \bar{\omega})^2 \quad , \tag{A.13}$$

where $\bar{\rho} = \bar{n} + \bar{p} = \rho_0$ is the standard nuclear matter density and $\bar{\omega} = \bar{n} - \bar{p} = \bar{\rho}_0 I$. The quantities η_ρ, $\eta_{\rho\rho}$, $\eta_{\rho\omega}$ etc. are repeated derivatives of η with respect to ρ and ω, evaluated at $\rho = \bar{\rho}$, $\omega = \bar{\omega}$. Inserting eq.(A.13) into eqs.(A.11),(A.12) and subtracting from the latter their average values, we find

$$\eta_{\omega\omega}\tilde{\omega} + \eta_{\rho\omega}\tilde{\rho} = \frac{1}{2}e\tilde{v} \quad ,$$

$$\eta_{\rho\rho}\tilde{\rho} + \eta_{\rho\omega}\tilde{\omega} = -\frac{1}{2}e\tilde{v} \quad ,$$

where $\tilde{\rho} = \tilde{n} + \tilde{p}$, $\tilde{\omega} = \tilde{n} - \tilde{p}$ and $\tilde{v}$ is the deviation of v from its average value $\bar{v}$. Solving for $\tilde{n}, \tilde{p}$ we find

$$\tilde{n} = -\frac{e\tilde{v}}{4}(\eta_{\omega\omega} - \eta_{\rho\rho})/(\eta_{\rho\rho}\eta_{\omega\omega} - \eta_{\rho\omega}^2) \quad , \tag{A.14}$$

$$\tilde{p} = -\frac{e\tilde{v}}{4}(\eta_{\omega\omega} + \eta_{\rho\rho} + 2\eta_{\rho\omega})/(\eta_{\rho\rho}\eta_{\omega\omega} - \eta_{\rho\omega}^2) \quad . \tag{A.15}$$

4. <u>The volume redistribution contribution to the dipole moment</u>

Using the above result, eq.(A.5) now becomes:

$$\vec{d}_{vol} = \frac{NZ}{A}\int\left(\frac{\tilde{p}}{Z} - \frac{\tilde{n}}{N}\right)\vec{r} = \frac{1}{2}\int[(1 + I)\tilde{p} - (1 - I)\tilde{n}]\vec{r}$$

$$= \frac{1}{4}\,\frac{\eta_{\rho\rho} + \eta_{\rho\omega} + I(\eta_{\omega\omega} + \eta_{\rho\omega})}{\eta_{\rho\rho}\eta_{\omega\omega} - \eta_{\rho\omega}^{2}}\left(\int - e\tilde{v}\,\vec{r}\right) \quad . \tag{A.16}$$

In order to relate the various second derivatives of η to more familiar coefficients, we write the volume energy density in the canonical Droplet Model form, i.e., as the density times the energy per particle expressed in terms of the relative neutron excess δ, where $\delta = \omega/\rho$, and of the dilatation factor ε, where $\varepsilon = (\rho_0 - \rho)/3\rho_0$. Thus

$$\eta(\rho,\omega) = \rho\left[-a_1 + J\left(\frac{\omega}{\rho}\right)^2 + \frac{K}{2}\left(\frac{\rho_0 - \rho}{3\rho_0}\right)^2 - L\left(\frac{\rho_0 - \rho}{3\rho_0}\right)\left(\frac{\omega}{\rho}\right) + \frac{1}{2}M\left(\frac{\omega}{\rho}\right)^4\right] \quad ,$$

where a_1,J,K,L,M are Droplet Model parameters. (A.17)

Evaluating the second derivatives at $\rho = \rho_0$, $\omega = \rho_0 I$, we find

$$\eta_{\rho\rho} = -\frac{2I^2}{\rho_0}(J - \frac{L}{3}) + \frac{K}{9\rho_0} + \frac{6MI^4}{\rho_0} \approx \frac{K}{9\rho_0} + \text{terms of higher order in I} \tag{A.18}$$

$$\eta_{\omega\omega} = \frac{2J}{\rho_0} + \frac{6MI^2}{\rho_0} \approx \frac{2J}{\rho_0} + \ldots \tag{A.19}$$

$$\eta_{\rho\omega} = -\frac{2I}{\rho_0}(J - \frac{L}{3}) - \frac{6MI^3}{\rho_0} \approx -\frac{2I}{\rho_0}(J - \frac{L}{3}) + \ldots \tag{A.20}$$

If at this stage one decides to retain, in the expressions for $\tilde{n}$ and $\tilde{p}$, terms linear in I, one finds

$$\tilde{n} = \frac{1}{8}\, e\tilde{v}\, \rho_0 \left(\frac{18}{K} - \frac{1}{J} + \text{terms in } I^2 \text{ and higher}\right) \quad , \tag{A.21}$$

$$\tilde{p} = \frac{1}{8}\, e\tilde{v}\, \rho_0 \left[\frac{18}{K} + \frac{1}{J} - \frac{36}{K}\left(1 - \frac{L}{3J}\right) I + \text{terms in } I^2 \text{ and higher}\right] \quad . \tag{A.22}$$

In the last equation, the term linear in I is the refinement that we choose to keep in the present context, but which would not contribute to the standard Droplet Model <u>mass formula</u>. Eq.(A.16) now becomes

$$\vec{d}_{vol} = \left[\frac{1}{\eta_{\omega\omega}} + \frac{\eta_{\rho\omega}}{\eta_{\omega\omega}\eta_{\rho\rho}} + \frac{I}{\eta_{\rho\rho}} + \text{higher powers of } I\right]\left(\int - e\tilde{v}\,\vec{r}\right)$$

$$\approx (AZe^2/8)\left(\frac{1}{J} + \frac{6L}{KJ}\, I\right) \overline{\phi\vec{\zeta}} \quad , \tag{A.23}$$

where ϕ stands for $(-\tilde{v})/(Ze/R_0)$ (i.e., for the negative of the non-uniformity of the electric potential in units of Ze/R_0), $\vec{\zeta}$ stands for $\vec{r}/R_0$, and the bar denotes the volume average of $\phi\vec{\zeta}$. Even for heavy nuclei, the value of I is only a little over 0.2, so the neglect of higher powers of I may not be a serious approximation, but the term in I could be significant.

5. <u>The neutron skin contribution to the dipole moment</u>

From the definition of $\vec{d}_{skin}$ we have

$$\vec{d}_{skin} = (NZ/A)(\Delta/V)\left(\vec{r}_A - \frac{\oint t\vec{r}}{\oint t}\right)$$

$$= (NZ/A)(\Delta/V)\left(\vec{r}_A - \frac{\oint (\bar{t} + \tilde{t})\vec{r}}{S\bar{t}}\right) \quad , \tag{A.24}$$

where $\bar{t}$ is the average skin thickness, $\tilde{t}$ is the deviation of t from $\bar{t}$ and S stands for the area of the surface Σ. It follows that

$$\vec{d}_{skin} = \vec{d}_{surface} + \vec{d}_{surface\ redistribution} \quad ,$$

where

$$\vec{d}_{surf} = (NZ/A)(\Delta/V)(\vec{r}_A - \vec{r}_s) \quad , \tag{A.25}$$

$$\vec{d}_{surf\ red} = -(NZ/A)(\Delta/V\bar{t})\langle\tilde{t}\ \vec{r}\rangle \quad . \tag{A.26}$$

Here $\vec{r}_s$ locates the center of gravity of an infinitesimal <u>uniform</u> layer on the surface Σ and $\langle\tilde{t}\ \vec{r}\rangle$ stands for the surface average of $\tilde{t}\ \vec{r}$.

Since $V_n = N/\bar{n}$, $V_p = Z/\bar{p}$ and $V = A/(\bar{n} + \bar{p})$, the quantity Δ/V is given by

$$\frac{\Delta}{V} = \frac{V_n - V_p}{V} = \frac{N/A}{\bar{n}/(\bar{n} + \bar{p})} - \frac{Z/A}{\bar{p}/(\bar{n} + \bar{p})}$$

$$= \frac{2(I - \bar{\delta})}{1 - \bar{\delta}^2} \approx 2(I - \bar{\delta}) \quad , \tag{A.27}$$

where $\bar{\delta} = (\bar{n} - \bar{p})/(\bar{n} + \bar{p})$. (This definition of $\bar{\delta}$ is, strictly speaking, different from the standard Droplet Model definition, namely $\bar{\delta} = \overline{(n - p)/(n + p)}$. The difference is readily shown to be of higher order in $\tilde{n},\tilde{p}$.) Using also the relation $\Delta/\bar{t} = S$, so that $\Delta/V\bar{t} = 3B_s/R_0$, where B_s is the area of the shape in question in units of the area of the standard sphere, we find

$$\vec{d}_{surf} = (2NZ/A)(I - \bar{\delta})R_0(\bar{\vec{\zeta}} - \langle\vec{\zeta}\rangle) \quad , \tag{A.28}$$

$$\vec{d}_{surf\ red} = -(3NZ/A)B_s\langle\tilde{t}\ \vec{\zeta}\rangle \quad . \tag{A.29}$$

According to the Droplet Model[6]) the non-uniformity of the neutron skin is proportional to the non-uniformity on the surface of the electric potential

$$\tilde{t} = \frac{3}{8}\frac{er_0}{Q}(\tilde{v}_s - \langle\tilde{v}_s\rangle) \quad , \tag{A.30}$$

or, using the notation of the previous section,

$$\tilde{t} = -\frac{3Ze^2}{8Q}A^{-1/3}(\phi - \langle\phi\rangle) \quad . \tag{A.31}$$

(It is understood that ϕ in this equation refers to its value on the surface.) Using the approximation $NZ/A = (A/4)(1 - I^2) \approx A/4$, eq.(A.29) becomes

$$\vec{d}_{\text{surf red}} = (9AZe^2/32Q)A^{-1/3}B_s(\langle\phi\vec{\zeta}\rangle - \langle\phi\rangle\langle\vec{\zeta}\rangle) \quad .$$

It will be shown in the next section that the following remarkable relation holds between the volume and surface moments of ϕ for a slightly distorted sphere

$$\langle\phi\vec{\zeta}\rangle - \langle\phi\rangle\langle\vec{\zeta}\rangle = \frac{5}{6}\overline{\phi\vec{\zeta}} \quad ,$$

and also that $B_s = 1$ to the required order in the distortion parameters. It follows that

$$\vec{d}_{\text{surf red}} = (AZe^2/8)(15/8Q)A^{-1/3}\overline{\phi\vec{\zeta}} \quad ,$$

and this part of the neutron skin contribution can therefore be combined with the volume redistribution term, eq.(A.21). This leads to eq.(3) for the coefficient C_r.

6. Evaluation of integrals

In order to evaluate the dipole moment we need altogether two volume integrals, $\overline{\vec{\zeta}}$ and $\overline{\phi\vec{\zeta}}$, and three surface integrals, $\langle\vec{\zeta}\rangle$, $\langle\phi\rangle$ and $\langle\phi\vec{\zeta}\rangle$ for a shape (assumed axially symmetric) specified by eq.(2). We shall work these out to the lowest non-vanishing order in the expansion coefficients α_ℓ. As we shall see, this means terms of second order and, because a dipole moment can couple spherical harmonics differing only by $\Delta\ell = \pm 1$ (the parity requirement having eliminated $\Delta\ell = 0$) the surviving terms are all of the form $\alpha_\ell\alpha_{\ell+1}$. Since the sphere is a shape for which the volume, the surface energy and the Coulomb energy are stationary with respect to the α_ℓ's and, since by a straightforward symmetry argument, terms in $\alpha_\ell\alpha_{\ell+1}$ do not appear in those expressions, there is no need to introduce a volume normalisation factor in eq.(2), and it is also an adequate approximation to put $B_s \approx 1$, $B_c \approx 1$. (B_c is the Coulomb energy of the shape in units of its value for a sphere.)

With the shape assumed axially symmetric about the z-axis, the only surviving component of the dipole moment $\vec{d}$ is along this axis and will be denoted by d. We may also drop the vector symbol over ζ, so that the five integrals to be evaluated are $\overline{\zeta}$, $\overline{\phi\zeta}$, $\langle\zeta\rangle$, $\langle\phi\rangle$ and $\langle\phi\zeta\rangle$, where ζ now stands for z/R_0.

The center of mass of a slightly deformed sphere, as given in ref.[16]), leads to

$$\bar{\zeta} = \sum_{\ell=2}^{\infty} \frac{9(\ell + 1)}{(2\ell + 1)(2\ell + 3)} \alpha_\ell \alpha_{\ell+1} \quad . \tag{A.35}$$

The surface average of ζ is given by

$$\langle\zeta\rangle = R_0^{-1} \int_0^{\pi} 2\pi R^2 \sin\theta \cos\theta \sqrt{R^2 d\theta^2 + dR^2} \Big/ \int_0^{\pi} 2\pi R \sin\theta \sqrt{R^2 d\theta^2 + dR^2}$$

$$\approx \frac{1}{2} \int_{-1}^{1} d\mu F^3 P_1(\mu) \sqrt{1 + F^{-2}(dF/d\theta)^2}$$

$$\approx \frac{1}{2} \int_{-1}^{1} d\mu P_1 F^3 [1 + \frac{1}{2}(1 - \mu^2) F'^2]$$

$$\approx \frac{1}{2} \int_{-1}^{1} d\mu P_1 [\frac{1}{2}(1 - \mu^2) F'^2 + 3(F - 1)^2] \quad ,$$

where $F = 1 + \sum_{\ell=2}^{\infty} \alpha_\ell P_\ell$, $\mu = \cos\theta$, $F' = dF/d\mu$.

Using the relations

$$(1 - \mu^2) P_\ell' = \frac{\ell(\ell + 1)}{2\ell + 1} (P_{\ell-1} - P_{\ell+1}) \quad , \tag{A.36}$$

$$P_1 P_\ell = \frac{\ell + 1}{2\ell + 1} P_{\ell+1} + \frac{\ell}{2\ell + 1} P_{\ell-1} \quad , \tag{A.37}$$

and

$$\int_{-1}^{1} P_1 P_\ell P_{\ell+1} = \frac{2(\ell + 1)}{(2\ell + 1)(2\ell + 3)} \quad . \tag{A.38}$$

if the "triangle relation" is satisfied by $(1,\ell,\ell+1)$, and zero otherwise, we find

$$\langle\zeta\rangle = \sum_{\ell=2}^{\infty} \frac{(\ell + 1)(\ell^2 + 2\ell + 6)}{(2\ell + 1)(2\ell + 3)} \alpha_\ell \alpha_{\ell+1} \quad . \tag{A.39}$$

From refs.[2,10]) we may deduce that the electric potential non-uniformity function ϕ at a point specified by r,θ may be written as

$$\phi = -\frac{3}{10} + \frac{1}{2}\left(\frac{r}{R_0}\right)^2 - \sum_{\ell=2}^{\infty} \frac{3}{2\ell + 1} \left(\frac{r}{R_0}\right)^{\ell} \alpha_\ell P_\ell \quad . \tag{A.40}$$

Since, in the dipole moment expression, the surface average $\langle\phi\rangle$ enters multiplied by $\langle\zeta\rangle$, which is of order $\alpha_\ell \alpha_{\ell+1}$ (see eq.(A.32)), we need $\langle\phi\rangle$ only to lowest order (i.e. for $\alpha_\ell = 0$). It follows that

$$\langle\phi\rangle = \frac{1}{5} \quad , \tag{A.41}$$

to sufficient accuracy.

The volume average $\overline{\phi\zeta}$ is given by

$$\overline{\phi\zeta} = R_0^{-1} \left(\frac{4}{3}\pi R_0^3\right)^{-1} \int_{\theta=0}^{\pi} \int_{r=0}^{R} dr\, d\theta\, \phi\, 2\pi r^3 \sin\theta \cos\theta$$

$$= \frac{3}{2} \int_{-1}^{1} d\mu P_1 \left[-\frac{3}{40} F^4 + \frac{1}{12} F^6 - \sum_{\ell=2}^{\infty} \frac{3}{(2\ell + 1)(\ell + 4)} F^{\ell+4} \alpha_\ell P_\ell \right]$$

Proceeding as before, we find

$$\overline{\phi\zeta} = \sum_{\ell=2}^{\infty} \frac{12(\ell - 1)(\ell + 1)(8\ell + 9)}{5(2\ell + 1)^2(2\ell + 3)^2} \alpha_\ell \alpha_{\ell+1} \quad . \tag{A.42}$$

The evaluation of $\langle\phi\zeta\rangle$ proceeds similarly and we find

$$\langle\phi\zeta\rangle = \sum_{\ell=2}^{\infty} \frac{(\ell + 1)(4\ell^4 + 16\ell^3 + 123\ell^2 + 64\ell - 72)}{5(2\ell + 1)^2(2\ell + 3)^2} \alpha_\ell \alpha_{\ell+1} \quad . \tag{A.43}$$

Using eqs.(A.39),(A.41) and (A.43) we now find

$$\langle\phi\zeta\rangle - \langle\phi\rangle\langle\zeta\rangle =$$

$$= \sum_{\ell=2}^{\infty} \left[\frac{(\ell + 1)(4\ell^4 + 16\ell^3 + 123\ell^2 + 64\ell - 72)}{5(2\ell + 1)^2(2\ell + 3)^2} - \frac{(\ell + 1)(\ell^2 + 2\ell + 6)}{5(2\ell + 1)(2\ell + 3)} \right] \alpha_\ell \alpha_{\ell+1} \quad ,$$

$$= \sum_{\ell=2}^{\infty} \frac{2(\ell - 1)(\ell + 1)(8\ell + 9)}{(2\ell + 1)^2(2\ell + 3)^2} \alpha_\ell \alpha_{\ell+1} = \frac{5}{6} \overline{\phi\zeta} \quad . \tag{A.44}$$

We suspect that there is a way of proving this relation that is more general than the explicit working out of the expansions for the separate terms in the equation, and which relies instead on the general properties of the electric potential for a slightly distorted sphere. We have not succeeded so far in finding such a proof.

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References

1. G.A. Leander, Proc. Fifth Int. Symp. on Capture Gamma-Ray Spectroscopy and Related Topics, Sept. 1984, S. Raman editor; Proc. Niels Bohr Centennial Conf. on "Nuclear Structure 1985." Copenhagen, Denmark, May 1985, eds. R.A. Broglia, G.B. Hagemann and B. Herskind.
2. V. Strutinsky, Atomnaya Energiya 4 (1956) 150; J. Nucl. Energy 4 (1957) 523
3. A. Bohr and B.R. Mottelson, Nucl. Phys. 4 (1957) 529; 9 (1959) 687
4. W. Nazarewicz, et al., Nucl. Phys. A429 (1984) 269-295
5. G.A. Leander and R.K. Sheline, Nucl. Phys. A413 (1984) 375
6. W.D. Myers and W.J. Swiatecki, Ann. Phys. (N.Y.) 84 (1974) 186-210
7. W.D. Myers, Droplet Model of Atomic Nuclei (IFI/Plenum Data, New York, 1977)
8. R.D. Present, Phys. Rev. 72 (1947) 7
9. W.J. Swiatecki, Phys. Rev. 83 (1951) 178; J. de Physique, suppl. nr. 8-9, 33 (1972) C5-45
10. R.W. Hasse, Pramāna 11 (1978) 441-55
11. P. Möller, et al., Proc. 7th Int. Conf. Atomic Masses (AMCO-7) (1984) 457-465
12. W.D. Myers and K.-H. Schmidt, Nucl. Phys A410 (1983) 61-73
13. W.D. Myers, et al., Phys. Rev. C15 (1977) 2032
14. K. Kolehmainen, et al., Lawrence Berkeley Laboratory report LBL-18513 (1984)
15. W.D. Myers, Nukleonika 21 (1976) 3
16. W.J. Swiatecki, Phys. Rev. 104 (1956) 993

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