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Ian F. Davenport and C. Judson King

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THE ONSET OF NATURAL CONVECTION FROM TIME-DEPENDENT PROFILES

by

Ian F. Davenport and C. Judson King

Department of Chemical Engineering
and Lawrence Berkeley Laboratory

University of California
Berkeley, California 94720, U.S.A.

ABSTRACT

Experiments were conducted on the initiation of natural convection in deep pools (penetration depth is small compared with the fluid depth). Fluids covering a wide range of Prandtl number were heated from below. A constant value of Rayleigh number based upon density boundary layer thickness (Ra_ℓ) correlates the effects of surface boundary condition and fluid depth. These effects and those of Prandtl number and wall spacing can be interpreted in terms of growth rate - Amplification theory; however a constant value of the Peclet number at the onset of convection was found to correlate the data better than did a constant value of the amplification factor. Theoretical reasons for this result are presented.

INTRODUCTION

The reliable prediction of conditions at the onset of convection from time-dependent driving forces is of importance to the engineer in designing commercial equipment and to the mathematician and physicist in understanding natural phenomena. The results of linear stability theory in this area (3) have generally predicted critical Rayleigh numbers which have been approximately two orders of magnitude below those observed. The apparent reason for this discrepancy is that linear stability theory predicts the condition at which an infinitesimal disturbance will begin to grow in amplitude, whereas convection is first observed when the amplitude has reached such a finite size as to produce a "measurable" influence on the transport process. A growth period is required, as it has been suggested (5), for the necessary amplification of the convection motion, and, in a situation where the driving force for convection is changing with time, such as diffusion into a semi-infinite medium, the net result is that convection is first observed at a much higher Rayleigh number than that corresponding to first growth.

Although this mechanistic picture appears reasonable, especially in light of the advanced work of Gebhart and co-workers (7) in a similar study of vertical wall natural convection, the means to predict the first observable convection is clouded; i.e., reported incipient convection conditions occur over a range of amplification factor from 10^1 to 10^6 (4), and theoretical predictions are available only for a linear increase over time or a step change in surface temperature. The purpose of this paper is to present an analysis aimed at improving the means of predicting the onset of convection from time-dependent driving forces at solid-liquid surfaces.

THEORY

The stability of a non-linear density profile $\rho(z)$ can be defined (3) in terms of a Rayleigh number, Ra_ℓ, by assuming the profile to consist of two linear segments, where

$$\ell = \int_0^\infty \frac{\rho(z)}{\rho(0)}\, dz \qquad -(1)$$

$$\text{and } Ra_\ell = \frac{\rho g \beta (T_s - T_B)\ell^3}{\mu\alpha} \qquad -(2)$$

For a time-dependent profile,

$$\ell = K\sqrt{\alpha t} \qquad -(3)$$

The constant of proportionality (K) in Equation (3) will be dependent upon the shape of the profile, which in turn will depend upon the behavior of the surface temperature (T_s) with time. One means of identifying the shape of the density curve is through

$$S = \frac{\int_0^t T_s\,(\theta)d\theta}{T_s t} \qquad -(4)$$

S is 1.0 for a step change and 0.5 for a linear change in surface temperature with time. This is the simplest parameter that can describe the surface temperature curve. It has the drawback that several curve shapes can be described by a given value of S. The relationship between K and S was found by assuming $T_s = t^n$.

The time-dependent Rayleigh number is found from Equations (2) and (3):

$$Ra_t = \frac{\rho g \beta (T_s - T_B)\alpha^{1/2}\, t^{3/2}}{\mu} \qquad -(5)$$

EXPERIMENTATION

(1) Effect of Prandtl number on Ra_t. The apparatus to measure Ra_t, and its variation with Prandtl number, was designed to produce a vertical, one-dimensional temperature profile in the liquid so as to conform strictly with the theoretical models. A cylinder (9 cm i.d., 4.5 cm height), made of Plexiglass which had similar thermal properties as the liquids tested, was attached to a variable 750 watt plate heater. The surface of the heater was carefully designed to be isothermal. The assembly was fitted to a leveling table so that the heating surface was horizontal. A 1 mil copper-Constantan thermocouple was placed on the surface of the isothermal heating plate, being insulated from it by a thin sheet of mica. The temperature response was recorded over ranges of 50μV to 2mV.

A run was begun by allowing the test liquid to come to thermal equilibrium with the surroundings, as indicated by the change in thermocouple response. A DC heating power supply was adjusted to give an initial high power input, followed by a constant power level to produce a linear increase of the heating surface temperature with time. When the test fluid was a liquid, convection initiation was observed by the first deviation from linearity of the surface temperature with time. However, with air, a thin thermocouple was suspended about 1 mm above the surface, and this detected the onset of convection much better than the thermocouple on the air-solid surface.

The variation in Prandtl number came from testing fluids which

varied in viscosity. These fluids were air (Pr = 0.7), methanol (Pr = 7.7), n-decane (Pr = 13.3), n-undecane (Pr = 17), n-butanol (Pr = 43), n-hexanol (Pr = 64), n-octanol (Pr = 108), silicone oils 50 cs (Pr = 450), 100 cs (Pr = 890), and 1,000 cs (Pr = 8,500).

For a particular fluid, the conditions at the onset of convection were identified by the temperature elevation, ΔT_c, and the period of conduction, t_c, prior to observable convection. These critical conditions, where ΔT_c for a fluid varied by almost a factor of 10, showed that ΔT_c varied as $t_c^{-3/2}$ for each fluid, as required for correlation by the time-dependent Rayleigh number. The experimental results are plotted in Figure 1 and tabulated elsewhere (4).

Predicted relationships between Ra_t and Pr calculated by Foster (5) using Amplification Theory with constant amplification factors of 10^1 and 10^8 are also given in Figure 1. The data show that the apparent amplification factor corresponding to onset decreases as Pr increases. Foster (6) published experimental results for heating water in a similar apparatus to that used in this report. With Pr of 6.6, he correlated his results with an amplification factor of 10^3 to 10^5, which agrees well with the present results.

Onat and Grigull (9) report values of Ra_t which are lower by approximately a factor of five than the present results, over a range of Pr from 7 to 4,400. Their experiments were carried out in a similar manner to the present experiments and those of Foster (6). The lower values of Ra_t could be due to high residual velocity resulting from a short period of rest between runs. Such an effect was noted in our preliminary investigations. In the current experiments

the liquid was kept as isothermal and motionless as possible as a starting condition, in an attempt to produce reproducible and low amplitude pre-convection fluid disturbances. Consequently, rest periods between runs were up to 1 1/2 hours for a run lasting 20 seconds to 150 seconds. Runs with longer conduction times before instability were left longer to rest between runs.

(2) Effect of Wall Spacing on Ra_t. As the ratio of fluid width (D) to depth (H) decreases, an influence of the walls on the incipient convection can be expected.

Experiments in the present study observed the effect on Ra_t of $D^2/\alpha t$, the ratio of the square of the fluid width to the square of the penetration depth. Data were obtained by temporarily placing a Plexiglas block with a well finished face on the heating surface of the large vessel used for the Ra_t-vs-Pr experiments. Holes of 5.7 mm and 8.7 mm were drilled into the block to contain the fluid. The results are shown in Figure 2. It can be seen that wall effects are negligible for n-octanol (Pr = 108) and n-decane (Pr = 13), provided $\frac{\alpha t}{D^2} < 0.1$. For greater values of $\frac{\alpha t}{D^2}$, Ra_t rises sharply.

(3) Effect of a Vibrating Heating Surface on Ra_t. Experimental data were obtained by mounting the apparatus used to determine Ra_t vs. Pr on top of a vertical sinusoidal vibrator described elsewhere (4). The frequency (f) range was 10 to 10,000 cps, and the amplitude (A) range was 10^{-7} to 10^{-1} inches, but these variables were limited in their independence so that A^2f^3 varied by approximately 1,000 at most. Different bulk fluid motions were produced by the upper surface of the liquid being either flush with the vessel lid or free of it. The two different wave forms produced in the liquid with free and fixed upper surfaces did not produce results which were significantly

different from each other. The exception came at low frequencies and high amplitudes, where the liquid surface was broken for the free surface case, which lowered Ra_t significantly.

The results tabulated in ref. (4) showed that, as the forcing vibration decreased in frequency and increased in amplitude, the onset of convection occurred earlier. The greatest destabilizing effect occurred at approximately 60 cps and 0.01 inches amplitude, where the value of Ra_t was lowered by 40% from the value obtained under "vibration-free" condition discussed in Section (1) above.

(4) Effect of Fluid Depth on Ra_t. The transition from the deep-pool (semi-infinite medium) to the shallow-pool behavior was observed experimentally by varying the fluid depth and the heating rate. The fluid depth was regulated by supporting an aluminum cylinder, 5 cm high and slightly less than 9 cm wide, with its base parallel to the heating surface on a Plexiglass annulus of known height attached to the heating surface. Aluminum was selected to maintain the temperature of the upper surface of the liquid constant. A thermocouple attached to the base of the metal cylinder showed that the upper liquid surface temperature varied by less than 5% of the total temperature drop across the fluid.

The correlating parameter to show the change in critical Rayleigh number, based on either fluid depth, Ra_H, or penetration depth, Ra_t, at shallow depths and constant Pr is $\frac{\alpha t}{H^2}$. The results have been plotted on Figure 3 for three fluids and for liquid depths as small as 2.05 mm. The transition from the deep-pool mechanism to the shallow-pool mechanism is clearly seen in Figure 3. The shallow-pool data converge on an asymptote which is independent

of Pr and heating rate and dependent on fluid depth (Ra_H = 1,700). The deep pool asymptotes are independent of H but are dependent on Pr, as reported in Figure 1.

There are no previously published shallow-pool experimental results for constant temperature at the upper surface. However, some shallow-pool experimental results for an adiabatic upper surface have been reported by Soberman (10). His values of Ra_H are below the current results and lower than what was theoretically predicted by Currie (3) using linear stability analysis. Foster (6) reported values for convection onset in shallow pools with a linear increase in the lower surface temperature with time. The liquid at the upper surface was open to the air and so was closer to an insulated surface than to an isothermal surface. The critical Rayleigh numbers from Foster are estimated as Ra_t = 10,800 - 12,700 at $\frac{\alpha t}{H^2}$ = 1.8, as shown in Figure 3.

(5) Effect of the Shape of the Density Profile in the Fluid on Ra_t.

The apparatus used to investigate the influence of density profile shape on Ra_t was similar to that used to determine Ra_t vs. Pr, with the exception that the heating at the surface in this case was produced by a thermoelectric heat pump rather than by a plate heater. The variation of the heated surface temperature with time was controlled by monitoring the current to the heat pump. In this way, curve shapes ranging from a step function to a linear time increase were produced.

The results for n-octanol have been plotted in Figure 4. They show that Ra_t decreases as S increases in the range $0.5 < S < 0.9$. Similar dependence of Ra_t on S has been found for n-undecane,

n-butanol, n-hexanol, and n-octanol at a free surface; these results have been reported by Davenport and King (4).

Discussion

Previously published convection initiation data from time-dependent density profiles (1,2,6,8,11) have been reported in the form of Ra_t, except for Spangenberg and Rowland (11), who employed a value of Rayleigh number based on penetration depth somewhat similar to Equation (2). The disadvantage of Ra_t as a correlating parameter is that its value is dependent upon the thermal boundary conditions of the experiment. A more general presentation is in the form of Ra_ℓ, which overcomes this limitation. Consequently, the Ra_t data in Figures 3 and 4 have been transformed to the Ra_ℓ form, using Equations (1), (2) and (3).

The lack of dependence of Ra_ℓ upon the density profile shape (S) can be seen in Figure 5. The scatter in Ra_ℓ at high values of S is probably due to the assumed relationship between the length ratio (K) and the shape factor (S), mentioned earlier. The weakness of the general application of the relationship stems from one value of S describing many density shape profiles. Notwithstanding this weakness, Figure 5 shows S to be adequate as an approximation, and it is easy to compute.

The difference between conditions at observed incipient convection and at first growth can be seen in Figure 6 , where the linear stability results of Currie (3), applicable to this physical situation, have been plotted to show the conditions at first "possible" convection motion. The apparent reason for this difference is slow growth rates of the convection motion until Ra_ℓ is of the order of 1,000, whereupon the growth rate increases

rapidly. The sudden increase in growth rate can be seen from the velocity amplification ratios calculated by Foster (6) as a function of Ra_t for a linear change in surface temperature with time and plotted on Figure 1. In this case, Ra_ℓ is 3.4 times Ra_t. For the high Pr limit, the average convection velocity is predicted from Figure 1 to be amplified by a factor of 10 for Ra_ℓ between 0 and 850, while the second and third decade amplifications occur for Ra_ℓ between 850 to 1550, and 1550 to 2100 respectively. Although the amplification time will be dependent upon the initial velocity distribution in the pre-convective fluid, the growth rate is so steep around Ra_ℓ of 1,500 (depending upon the Prandtl number) that experimentally observed Rayleigh numbers fall within a narrow range. Obviously this is of great computational value to the design engineer.

The increase in Ra_t (and Ra_ℓ) with decreasing Prandtl number for deep pool conditions as shown in Figure 1 is qualitatively in agreement with Foster's (6) theoretical predictions. However, the amplification factor corresponding to observable convection appears to increase as the Prandtl number decreases. Two explanations for this seem possible.

(1) An increase in the magnitude of the convective velocity at the onset of convection with decreasing viscosity and Prandtl number of the fluid.

(2) The motion in the viscous fluid before heating is begun is greater than in the non-visious fluid. Both (1) and (2) would account for a lower amplification factor for observable convection.

The importance of point (1) above can be seen when the components of the actual heat flux are analyzed; i.e.

$$q = -\alpha \frac{\partial(\rho C_p T)}{\partial z} + u(\rho C_p T) \qquad (6)$$

The onset of convection will be observed when the convection flux is first comparable to the conduction flux; i.e., the Peclet number $\frac{u_c \ell_c}{\alpha}$ or $\frac{u_c t_c^{1/2}}{\alpha^{1/2}}$ is O(1) where u_c and ℓ_c are convection velocity and penetration depth respectively at the onset of observable convection, and α is the thermal diffusivity. In the present experiments, it was observed that, as the Prandtl number decreased, the critical penetration depth generally decreased, as manifested by shorter conduction periods (t_c). N-decane (Pr = 13.7) showed convection in the range of 26 to 118 seconds, while 1000 cs silicone oil (Pr = 8,500) showed convection in the range of 276 to 617 seconds. The thermal diffusivity of liquids tested did not vary appreciably from $10^{-3} cm^2/sec$. If the onset of convection occurs at a constant value of the Peclet number, the convection velocity u_c required should increase as the penetration depth ℓ_c decreases. Indeed, a closer inspection of the data showed that Ra_ℓ was up to 20% higher for the shortest conduction periods (t_c) of each fluid tested. Interestingly, Foster's (6) datum point for water which was at much longer conduction times than the organic fluids with similar Prandtl number in the present experimental results is at a slightly lower amplification factor, as would be expected if a lower u_c is required.

The significance of the diffusivity in the Peclet number can be realized by comparing critical Ra_t values for heat and mass transfer systems, at equivalent Prandtl-Schmidt numbers (4). The essential difference between the two systems is the value of thermal ($10^{-3} cm^2/sec$)

and mass (10^{-5} cm^2/sec) diffusivity, and to a lesser extent the slightly shorter critical conduction times in mass transfer systems. If the Peclet number at observable convection is the same in both systems, the critical convective velocity u_c required for the mass transfer system should be approximately an order of magnitude above that required for the heat transfer system. Experimentally, this prediction is observed, with Ra_t in the mass transfer systems approximately 2 to 3 times higher than in the heat transfer systems.

The influence of the pre-convection fluid motion on Ra_t was illustrated with the vibration results. Intense fluid vibration only decreased Ra_ℓ to a value around 1,000. Here, once again, the results show that a major portion of the amplification time is spent during a low growth rate period and that these growth rates increase rapidly as Ra_ℓ approaches 1,000. The vibration either provided a larger initial velocity disturbance or a faster amplification process. Unfortunately, not enough data are available to distinguish which factor dominated.

The effect of fluid depth on Ra_ℓ can be seen from Figure 6. The amplification process appears to be virtually independent of the fluid depth for high Prandtl number fluids, whereas for low Prandtl number fluids, Ra_ℓ decreases as the penetration depth approaches the fluid depth. Both the thermal and momentum boundary layers in the low Prandtl number fluid are being confined to the fluid depth at these shallow pool conditions which removes the dependence of the amplification process on the Prandtl number. The behavior is then similar to a high Prandtl number fluid.

Finally, the results of the narrow fluid width and depth experi-

ments show that the amplification process considers the fluid as semi-infinite, as shown by the constant value of Ra_ℓ for a given fluid, provided the dimensions of the fluid are greater than approximately twice the penetration depth at the onset of convection.

Conclusions

The analysis has shown that Ra_ℓ corresponding to first observable convection occurs at $O(10^3)$ for high Prandtl number fluids. This observation is consistent with the amplification model, which predicts that growth rates should increase rapidly as Ra_ℓ approaches $O(10^3)$ for high Prandtl number fluids. However, a physical interpretation of the results showed that although the amplification factor at observable convection can vary by orders of magnitude, a constant value of the Peclet number $\frac{u_c t_c^{1/2}}{\alpha^{1/2}}$ intercepts onset conditions over a wide range of conditions.

As the Prandtl number decreases, the experimentally observed increase in the critical value of Ra_ℓ is in qualitative agreement with the amplification model predictions.

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NOMENCLATURE

C_p	heat capacity at constant pressure (cal/g°C)
D	fluid width (cm)
g	gravity constant (cm/sec^2)
H	fluid depth (cm)
k	thermal conductivity (cal/cm sec °C)
ℓ	density boundary layer thickness using the segment approximation $= 2\int_0^\infty \left(\frac{T-T_b}{T_s-T_b}\right) dz$ (cm)
Pr	Prandtl number $= \frac{C_p \mu}{k}$
q	vertical heat flux
Ra_H	Rayleigh number based on fluid depth H $= \frac{\Delta\rho g H^3}{\mu\alpha}$
Ra_ℓ	Rayleigh number based on effective density boundary layer thickness $= \frac{\Delta\rho g \ell^3}{\mu\alpha}$
Ra_t	Rayleigh number based on conduction time $= \frac{\Delta\rho g \alpha^{1/2} t^{3/2}}{\mu}$
S	surface temperature shape factor
Sc	Schmidt number $= \frac{\mu}{\rho \mathcal{D}}$
t	conduction time prior to convection onset (sec)
T	temperature (°C)
ΔT	temperature difference between bulk and surface conditions
u	representative convection velocity
w	amplification factor

z	vertical coordinate (cm)

Greek Letters

α	thermal diffusivity (cm^2/sec)
β	coefficient of thermal expansion ($^oC^{-1}$)
μ	viscosity (poise)
ρ	density (g/cc)
$\Delta\rho$	difference in density between surface liquid and bulk liquid (g/cc)

Subscripts

b	bulk
s	surface
c	value at observable convection

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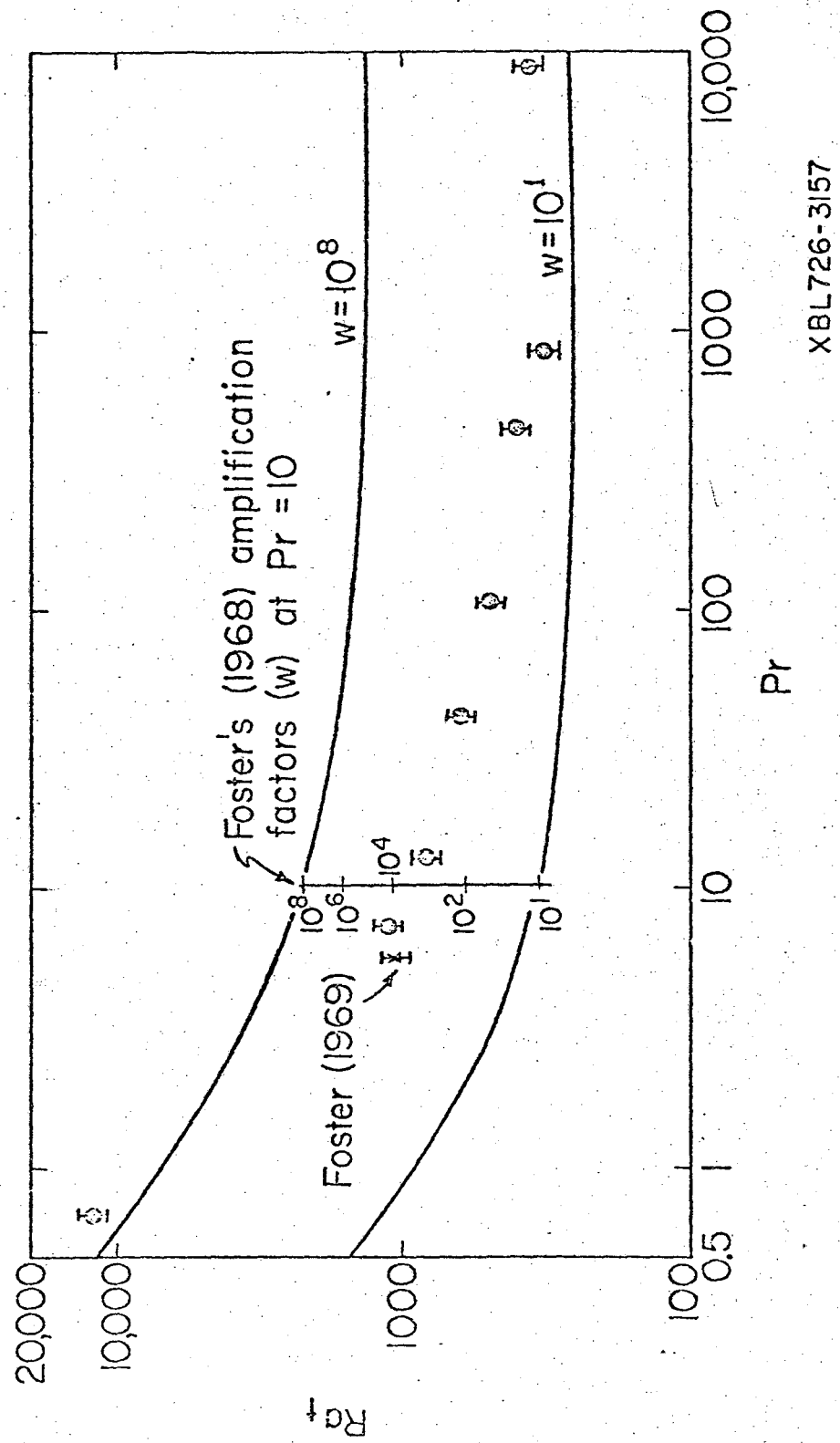

Fig. 1 Experimental Values of Ra_t versus Pr for a Linear Temperature Decay at a Fixed Surface in a Deep Pool.

⊖ Experimental Data
—Foster's (1968) theoretical curve for constant Amplification Factor of 10^1 and 10^8.

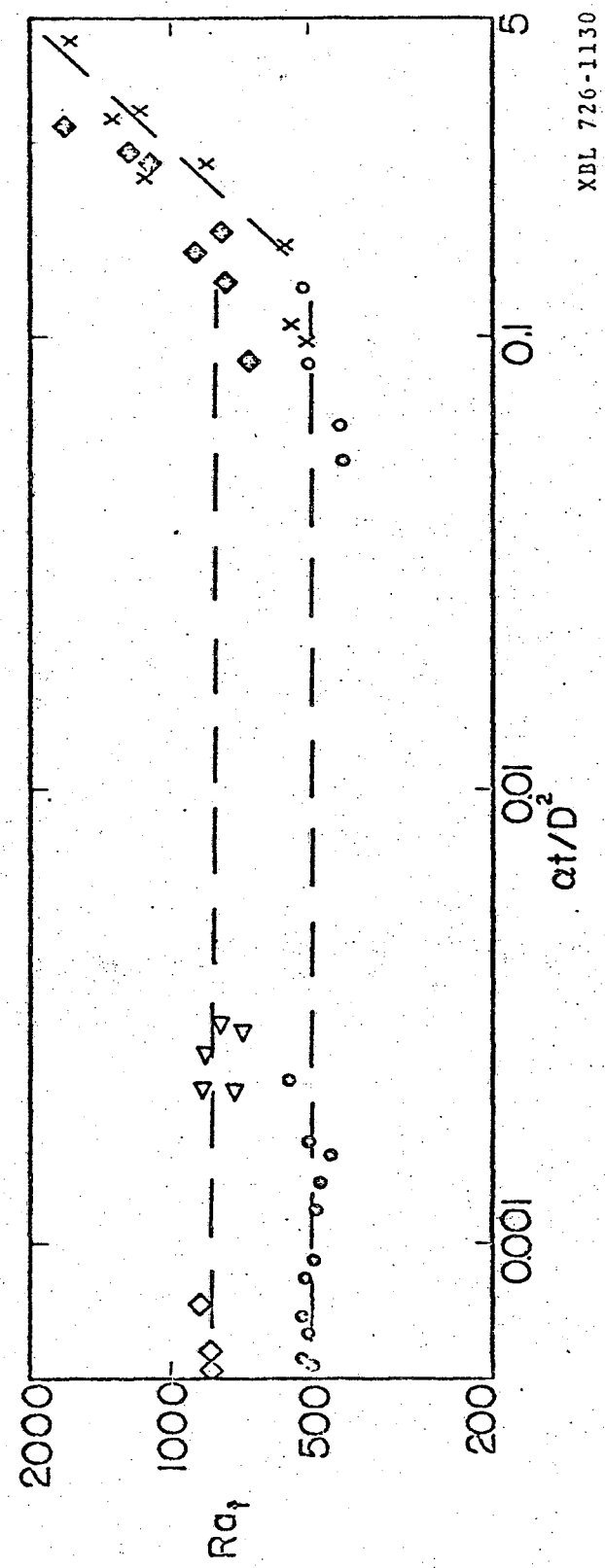

Fig. 2 Experimental Values of Ra_t versus Width Factor ($\alpha t/D^2$) for the Fixed-Surface, Deep-Pool Case.

n-Octanol: ○ D = 8.9 cm ⊙ D = 0.87 cm X D = 0.57 cm

n-Decane: ◇ D = 8.9 cm ◁ D = 5.9 cm ◇ D = 0.57 cm

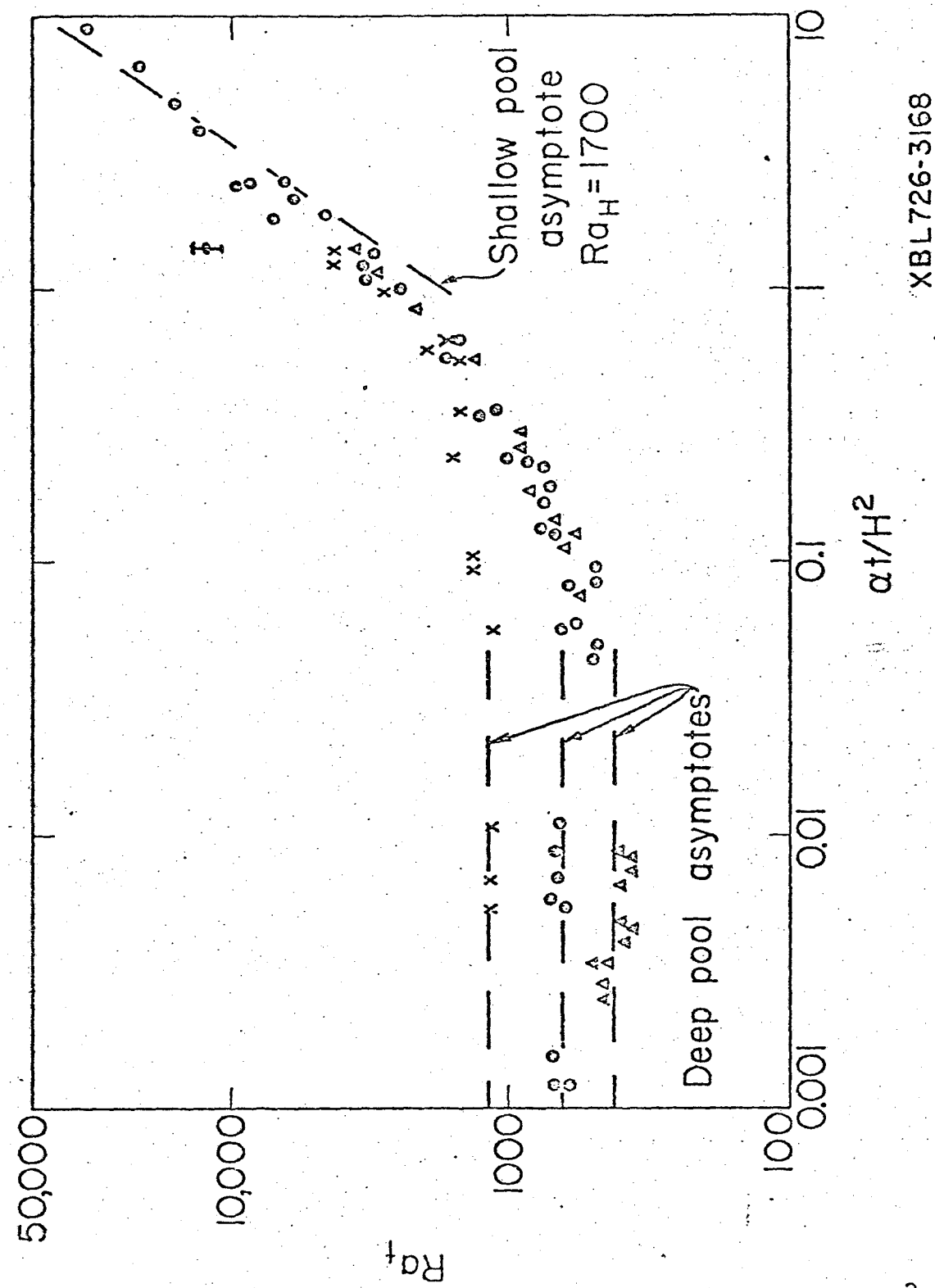

Fig. 3 Experimental Values of Ra_t versus Depth Factor ($\alpha t/H^2$) Showing the Transition from Deep- to Shallow-Pool Conditions for the Fixed-Surface Case.

X Methanol, Pr = 7.6, ○ Butanol, Pr = 43, Δ Silicone Oil 50 cs Pr = 450. ⊥ Foster's (1969) datum point for water.

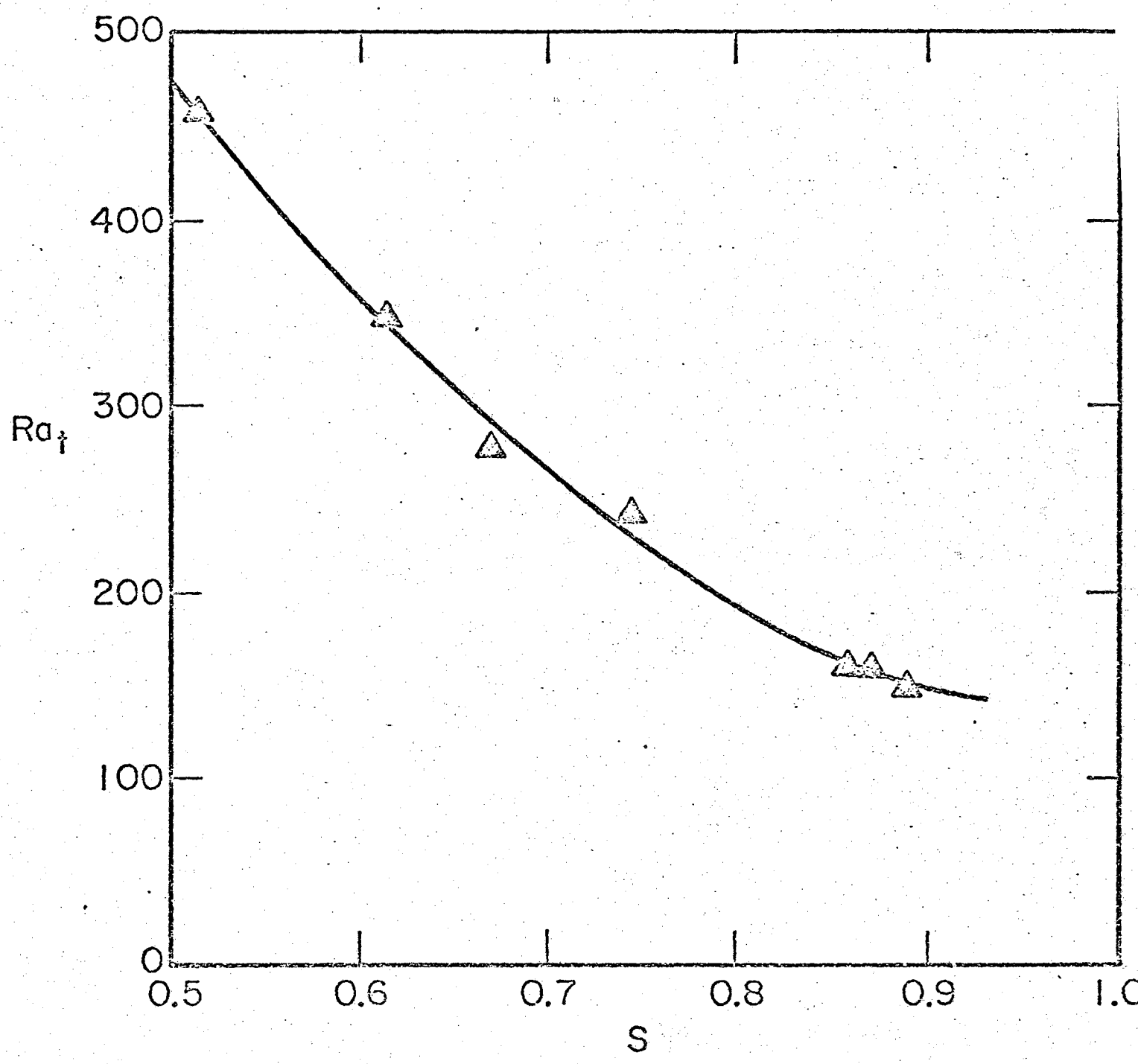

Fig. 4 Ra_t versus Shape Factor (S) for n-Octanol in a Deep Pool.

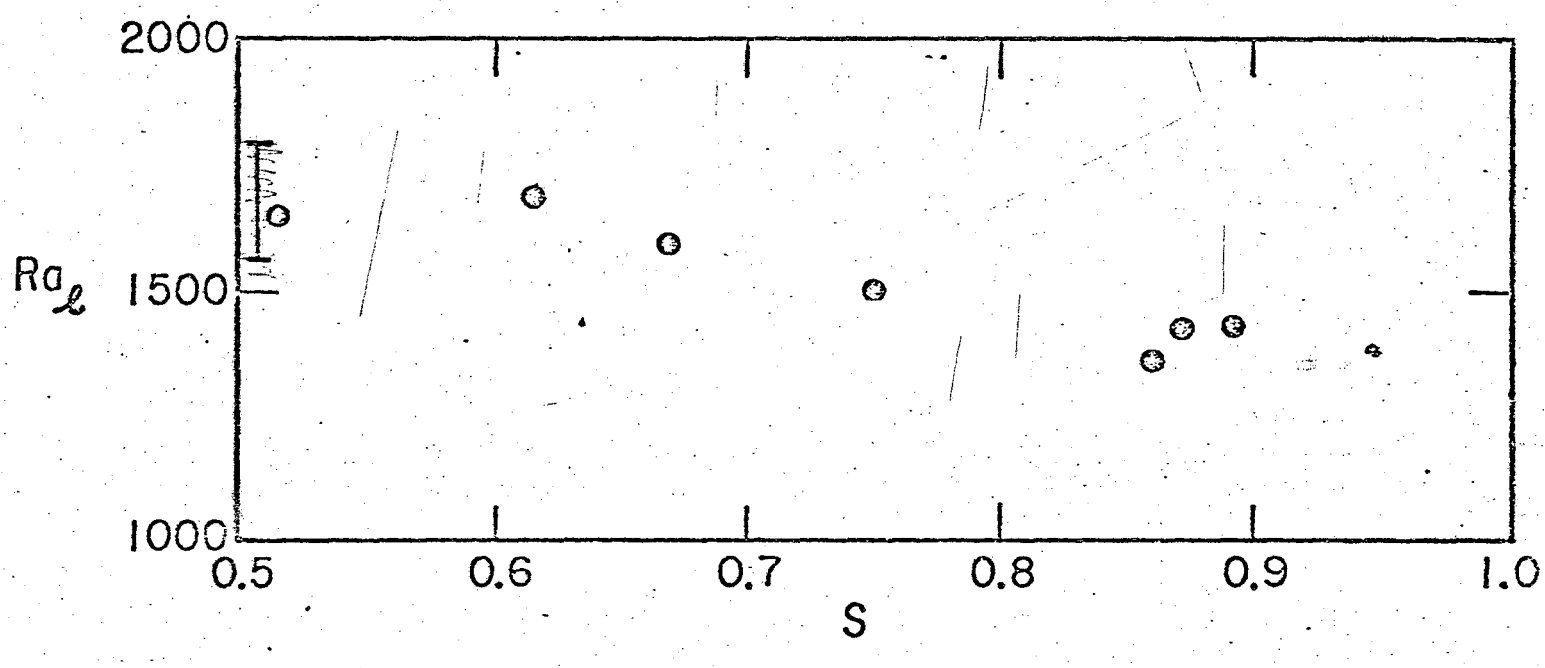

Fig. 5 Experimental Values of Ra_ℓ for n-Octanol (Pr = 108)

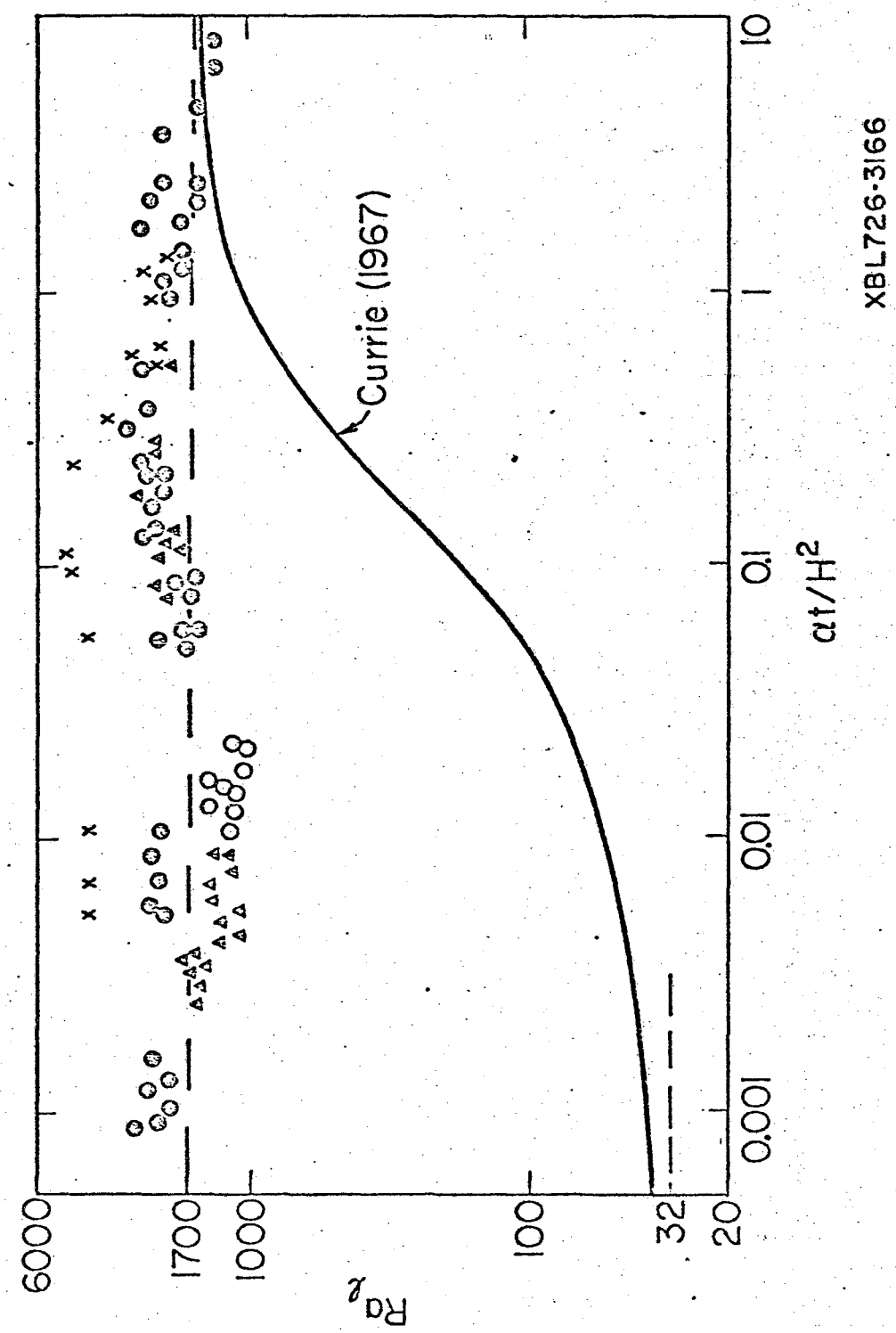

Fig. 6 Experimental Values of Ra_0 as a Function of the Depth Factor $(\alpha t/H^2)$

○ n-Butanol, Pr = 43
▲ Silicone Oil 50 cs, Pr = 465
● Silicone Oil 1000 cs, Pr = 8500
X Methanol, Pr = 7.6

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