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NONLINEAR INTERACTION OF ELECTROMAGNETIC WAVES IN A PLASMA DENSITY GRADIENT*

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ABSTRACT

Two intense electromagnetic waves interact strongly where the local plasma frequency equals their difference frequency, resulting in an irreversible transfer of action from the higher frequency wave to the lower frequency wave. The amount of transfer depends only on the intensities and the density scale length. Successive transfers among a set of waves may produce efficient plasma heating.

Interest in the nonlinear interaction between coherent electromagnetic waves arises from the possibility of exciting longitudinal plasma modes in an underdense $(\omega_p \ll \omega)$ plasma, by resonance with the difference frequency of two lasers,[1] thereby heating the plasma upon damping of the longitudinal modes. This process has been studied by Rosenbluth and Liu[2] for an inhomogeneous plasma, but neglecting the reaction of the longitudinal mode on the transverse waves; and by Cohen, Kaufman, and Watson,[3] including the reaction and allowing for a cascade, but for a homogeneous plasma.

The present paper treats the transfer of energy between two transverse waves (of frequencies ω_0, ω_1 with $\omega_0 > \omega_1$) in a plasma

density gradient. The mechanism of the transfer is the resonant excitation of an electron longitudinal mode at the beat frequency $\Omega \equiv \omega_0 - \omega_1$ and beat wave number $K \equiv k_0 + k_1$ (for the optimum case of opposed lasers, which we consider for definiteness). The excitation occurs over a zone of thickness $h \sim (\nu/\omega_p)L$ about the surface where $\omega_p = \Omega$; L is the density scale length and ν the longitudinal damping rate.

We stress two important conclusions:[4] (1) The dominant effect of the process is the transfer of action ΔJ from the higher frequency (ω_0) wave to the lower frequency (ω_1) wave, transverse action being conserved. Accordingly, the energy loss $\omega_0 \Delta J$ of the ω_0-wave is partitioned, with $\omega_1 \Delta J$ going to the ω_1-wave, and $\Omega \Delta J$ being irreversibly deposited in the plasma. The maximum heating efficiency is thus Ω/ω_0. [That this ratio is low for an underdense plasma led us in Ref. 3 to suggest cascading; we return to this below.] (2) The total amount of action transfer depends on the input power and on the density scale, but is <u>independent</u> of the damping rate[5] ν (as long as WKB-conditions are satisfied: $h \gg k^{-1}$). There is thus no need to be concerned with the damping mechanism, be it collisional, Landau, or nonlinear.

Our formulation of the interaction is in terms of the local longitudinal dielectric function, and thus is quite model-independent. For simplicity of presentation, we ignore ion dynamics, but its inclusion is straightforward. As a by-product of the calculation, we obtain the exponential spatial growth of Raman back-scattering instability; our result is identical to that of Liu and Rosenbluth,[6] although our basic assumptions are somewhat antithetical to theirs.

After treating the problem of two opposed lasers, we consider using additional lasers to cascade the action to still lower frequencies, with each step providing an incremental efficiency $\sim\Omega/\omega$. We find that this induced cascading, with alternate laser directions (see Fig. 1), appears feasible, in that the intensities required are below the effective Raman instability threshold, as determined by Mostrom et al.[7] On the other hand, self-induced cascading,[3] which requires two equally intense parallel lasers, is effective only for intensities well above this threshold.[8]

For simplicity, we treat the case of one-dimensional spatial variation (density gradient, propagation, and amplitude modulation all along z), polarization of the transverse waves along x, and steady-state amplitudes (corresponding to intensities below the absolute instability threshold[9]). The dimensionless vector potential $a(z,t) \equiv eA_x(z,t)/mc^2$ satisfies the nonlinear wave equation[3] $[\nabla^2 - c^{-2}[(\partial^2/\partial t^2) + \omega_p{}^2(z)]]a = -a\nabla^2\psi$, where the term in the dimensionless scalar potential $\psi(z,t) \equiv e\phi(z,t)/mc^2$ is the nonlinear part of the transverse current density. The vector potential is expressed in terms of the amplitudes of the two opposed transverse waves: $a(z,t) \equiv a_0(z)\exp[-i\omega_0 t + i\int^z k_0(z')dz'] + a_1(z)\exp[-i\omega_1 t - i\int^z k_1(z')dz'] + \text{c.c.}$, where $k_\ell{}^2(z)c^2 \equiv \omega_\ell{}^2 - \omega_p{}^2(z)$, $\ell = 0,1$. Upon substituting into the wave equation, we obtain the coupled set

$$D_0 a_0 \equiv \left\{\frac{\partial}{\partial t} + c_0\left(\frac{\partial}{\partial z} + \frac{\partial \ell n k_0^{\frac{1}{2}}}{\partial z}\right)\right\} a_0 = (K^2c^2/2i\omega_0)a_1\psi_B ,$$
$$D_1 a_1 \equiv \left\{\frac{\partial}{\partial t} - c_1\left(\frac{\partial}{\partial z} + \frac{\partial \ell n k_1^{\frac{1}{2}}}{\partial z}\right)\right\} a_1 = (K^2c^2/2i\omega_1)a_0\psi_B^* ; \qquad (1)$$

where the first two terms are the convective derivatives ($\partial/\partial t$ vanishes here, $c_\ell \equiv k_\ell c^2/\omega_\ell$ is the group velocity), the third term produces the WKB-variation $a_\ell \sim k_\ell^{-\frac{1}{2}}$, and the coupling involves the local Fourier amplitude $\psi_B \equiv \psi(\Omega,K;\, z)$ of the scalar potential at the beat frequency and wave number. On the left side of (1), we have neglected second derivatives of a_ℓ; on the right, we have kept only the potentially resonant terms.

To determine ψ_B, we note[10,3] that the Lorentz force is equivalent to a ponderomotive potential $\psi_{pon}(z,t) = \frac{1}{2}a^2(z,t)$, so that its Fourier amplitude is $\psi_{pon}(\Omega,K;\, z) = a_0(z)a_1^*(z)$. The local longitudinal response of the electron plasma to this potential is then given by $\psi_B = \psi_{pon}(\Omega,K;\, z)[\epsilon^{-1}(\Omega,K;\, z) - 1]$, in terms of the local dielectric function. With these expressions substituted into (1), we obtain

$$D_0 a_0 = (K^2c^2/2i\omega_0)|a_1|^2\, a_0(\epsilon^{-1} - 1)$$
$$D_1 a_1 = (K^2c^2/2i\omega_1)|a_0|^2\, a_1(\epsilon^{-1*} - 1) . \qquad (2)$$

It is now convenient to introduce the action flux density for each transverse wave. Since the wave energy density is $W_\ell = \omega_\ell{}^2|a_\ell|^2(mc/e)^2/2\pi$, the (absolute) action flux density is $c_\ell W_\ell/\omega_\ell = (k_\ell/2\pi)|a_\ell|^2(mc^2/e)^2$. In our natural units we thus define $J_\ell(z) \equiv (k_\ell/2\pi)|a_\ell|^2$, and convert Eqs. (2) to

$$dJ_0/dz = dJ_1/dz = \bar{8}\pi J_0 J_1 \,\mathrm{Im}\, \epsilon^{-1} , \qquad (3)$$

where $\bar{8} \equiv 2K^2/k_0k_1 \to 8$ for $\Omega \ll \omega_0$. Noting that $\mathrm{Im}\,\epsilon^{-1} < 0$, we see that the ω_0-wave loses action flux as it propagates to the right (increasing z), while the ω_1-wave increases action flux as it propagates to the left. The invariance of the signed action flux $\tilde{J} \equiv J_0 - J_1$ represents action conservation. The irreversible

dissipation of energy by nonlinear coupling follows from (3): $d(\omega_0 J_0 - \omega_1 J_1)/dz = \bar{8}\pi\Omega J_0 J_1 \operatorname{Im} \epsilon^{-1}$, the left side being the divergence of the (signed) energy flux density.

To solve Eq. (3), convert it to $d\,\ell n(J_0/J_1)/dz = -\bar{8}\pi\tilde{J} \operatorname{Im} \epsilon^{-1}(\Omega,K;\, z)$ and integrate across the resonant zone $z \sim z_R$ (where $\epsilon \approx 0$), obtaining $\Delta \ell n(J_0/J_1) = \bar{8}\pi\tilde{J} \int dz \operatorname{Im} \epsilon^{-1}(\Omega,K;\, z)$, with $\Delta f \equiv f(z < z_R) - f(z > z_R)$. To evaluate the integral, consider the limit $\operatorname{Im} \epsilon \to 0+$ (representing weak damping $\nu \ll \omega_p$, i.e., narrow resonance zone $h \ll L$). Then $\operatorname{Im} \epsilon^{-1} \to -\pi\, \delta[\epsilon(\Omega,K;\, z)]$, and $\int dz \operatorname{Im} \epsilon^{-1} \to -\pi |\partial\epsilon(\Omega,K;\, z)/\partial z|^{-1} \equiv -\pi L$ (defining the scale length L precisely[11]). We finally obtain for the action transfer ΔJ the formula:

$$\left(1 - \frac{\Delta J}{J_0^{in}}\right)\left(1 + \frac{\Delta J}{J_1^{in}}\right) = \exp[\bar{8}\pi^2 L(J_0^{in} - J_1^{in} - \Delta J)] \,, \qquad (4)$$

where $J_0^{in} \equiv J_0(z < z_R)$ and $J_1^{in} \equiv J_1(z > z_R)$ are the input action flux densities. This transcendental relation yields ΔJ as a function of J_0^{in}, J_1^{in}, and L, and is independent of the dissipative mechanism and magnitude. All that is required of the dissipation is that it be not too large[12] ($\nu \ll \omega_p$) and not too small[13] $[\nu/\omega_p \gg (k_0 L)^{-1}]$.

Equation (4) can be converted to the formula

$$\bar{J}_0 = (1 - R - \rho)^{-1}\, \ell n[(1 - R)(\rho + R)/\rho] \qquad (5)$$

for the dimensionless input action $\bar{J}_0 \equiv \bar{8}\pi^2 L J_0^{in}$ needed to produce a relative action transfer $R \equiv \Delta J/J_0^{in}$ for given input ratio $\rho \equiv J_1^{in}/J_0^{in}$. This relation is plotted in Fig. 2. The relation between $\bar{J}_0$ and input power density (in units of $10^{12}\,\mathrm{W/cm^2}$) is $P_0^{in} = (2/3)\bar{J}_0 L_{cm}^{-1}(\omega_0/\bar{\omega})$, where $\bar{\omega} \equiv 1.8 \times 10^{14}\ \mathrm{sec}^{-1}$ is the

frequency of a CO_2 laser.

To illustrate the use of Fig. 2, we see that with $\bar{J}_0 \approx 6$, $L = 10$ cm (so that $P_0^{in} \approx 4 \times 10^{11}\ \mathrm{W/cm^2}$), and $\rho = 0.1$ (so that $P_1^{in} \approx 4 \times 10^{10}\,\mathrm{W/cm^2}$), a fraction $R \approx 0.8$ of the action of the ω_0-wave is transferred to the ω_1-wave. The heating efficiency is $\approx(\Omega/\omega_0)R \approx 8\%$ for $\Omega/\omega_0 \approx 0.1$.

A number of special cases are of interest:

(i) For $\rho \ll 1$, $R \ll 1$, Eq. (4) yields $J_1^{out} = J_1^{in} \exp \bar{J}_0$, corresponding to the exponential growth of a small-amplitude wave in the Raman back-scattering instability. The exponent is $\bar{J}_0 = (3/2)P_0^{in}(10^{12}\ \mathrm{W/cm^2})L_{cm}$ for $\omega_0 = \bar{\omega}$, in exact agreement with Liu and Rosenbluth.[6] While those authors neglect dissipation but include convection of the longitudinal mode, our approach ignores convection relative to dissipation. A more general study of the instability by DuBois and Williams,[9] including both dissipation and convection parameters for the longitudinal mode, again yields this result, now independent of both parameters.

(ii) For $\rho \gg 1$, Eq. (4) yields $J_0^{out} = J_0^{in} \exp(-\bar{J}_1)$. Here the ω_1-wave produce an exponential attenuation of the ω_0-wave, just the opposite of case (i).

(iii) For $\rho \sim O(1)$, $\bar{J}_0 \ll 1$, Eq. (5) yields $R = \bar{J}_1$.

(iv) For $\rho < 1$, $R = 1 - \rho$, (5) yields $\bar{J}_0 = \rho^{-1} - 1$. In this special case $J_1^{out} = J_0^{in}$ and $J_0^{out} = J_1^{in}$; i.e., there is an exchange of actions. For example, choose $\rho = 0.1$: $\bar{J}_0 = 9$ and $\bar{J}_1 = 0.9$ are the inputs, while 0.9 and 9 are the respective outputs.

The last example (iv) is typical of useful orders of magnitude for a study of a cascade arrangement (Fig. 1). Suppose we have

available four lasers with $\omega_0 \approx 1.8 \times 10^{14}$ (CO_2), $\omega_1 \approx 1.6 \times 10^{14}$, $\omega_2 \approx 1.5 \times 10^{14}$, $\omega_3 \approx 1.2 \times 10^{14}$, so that the successive beat frequencies are $\Omega \approx 2 \times 10^{13}$, 1×10^{13}, 3×10^{13}. With the parameters of example (iv) for the first two lasers, and $L \sim 10$ cm, we need $P_0^{in} \sim 6 \times 10^{11}$ W/cm, $P_1^{in} \sim 6 \times 10^{10}$ W/cm^2. These waves interact in the zone at $\omega_p \approx 2 \times 10^{13}$ sec^{-1}, whereupon now $P_0 \sim 6 \times 10^{10}$, $P_1 \sim 6 \times 10^{11}$. The exchange is repeated between the ω_1- and ω_2-lasers at $\omega_p \approx 1 \times 10^{13}$ sec^{-1}; choosing $P_2^{in} \sim 6 \times 10^{10}$, the ω_2-wave extracts most of the power from the ω_1-wave, producing $P_2 \sim 6 \times 10^{11}$, $P_1 \sim 6 \times 10^{10}$. Thus the ω_1-wave has acted as a catalyst for transferring action from ω_0 to ω_2. The process can be repeated with ω_3 in an obvious way. The heating efficiencies of the successive steps are roughly 9%, 5%, 16%, with a total efficiency of about 30%.

The study of Mostrom et al.[7] has shown that nonlinear attenuation due to Raman side- and back-scattering is effective over a distance $< L$ when $\bar{J}_0 > 35$. The parameters chosen here are below this threshold, which is extremely sharp. The reason is that the transfer mechanism for the instability is identical to that for coherent interaction, but the former starts from small-amplitude noise.

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FOOTNOTES AND REFERENCES

* Work supported by the U. S. Atomic Energy Commission, the U. S. Air Force, and the National Science Foundation.

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5. This remarkable fact was pointed out to us by W. Kunkel.
6. C. S. Liu and M. N. Rosenbluth, Parametric Backscattering Instabilities, C003237-11, Eq. (10). See also D. Forslund, J. Kindel, and E. Lindman, Phys. Rev. Letters 30, 739 (1973).
7. M. Mostrom, A. Kaufman, and D. Nicholson, Bull. Am. Phys. Soc. 17, 1065 (1972).
8. When the interacting waves are parallel [required in order to maintain resonance in the self-induced cascade (see Ref. 3 for details)], the quantity $\bar{8}$ appearing in Eq. (3) has the value $\sim 2(\omega_p/\omega_0)^2 \ll 1$, rather than 8. Accordingly, the laser intensity required for the parallel case is greater by the ratio $4(\omega_0/\omega_p)^2$, thereby easily exceeding the critical value 35 (Ref. 7) for instability. K. Watson has suggested the possibility of avoiding the instability by rapid pulsing; this is now under study.
9. D. DuBois and E. Williams, Parametric Instabilities of Damped Modes in Inhomogeneous Plasmas, CU #1005.

10. G. Schmidt, Physics of High Temperature Plasmas (Academic Press, 1966), Sec. 2-9.

11. For a cold-plasma model $[\epsilon = 1 - \omega_p^2(z)/\omega^2]$, $L = |d\ \ell n\ n/dz|^{-1}$.

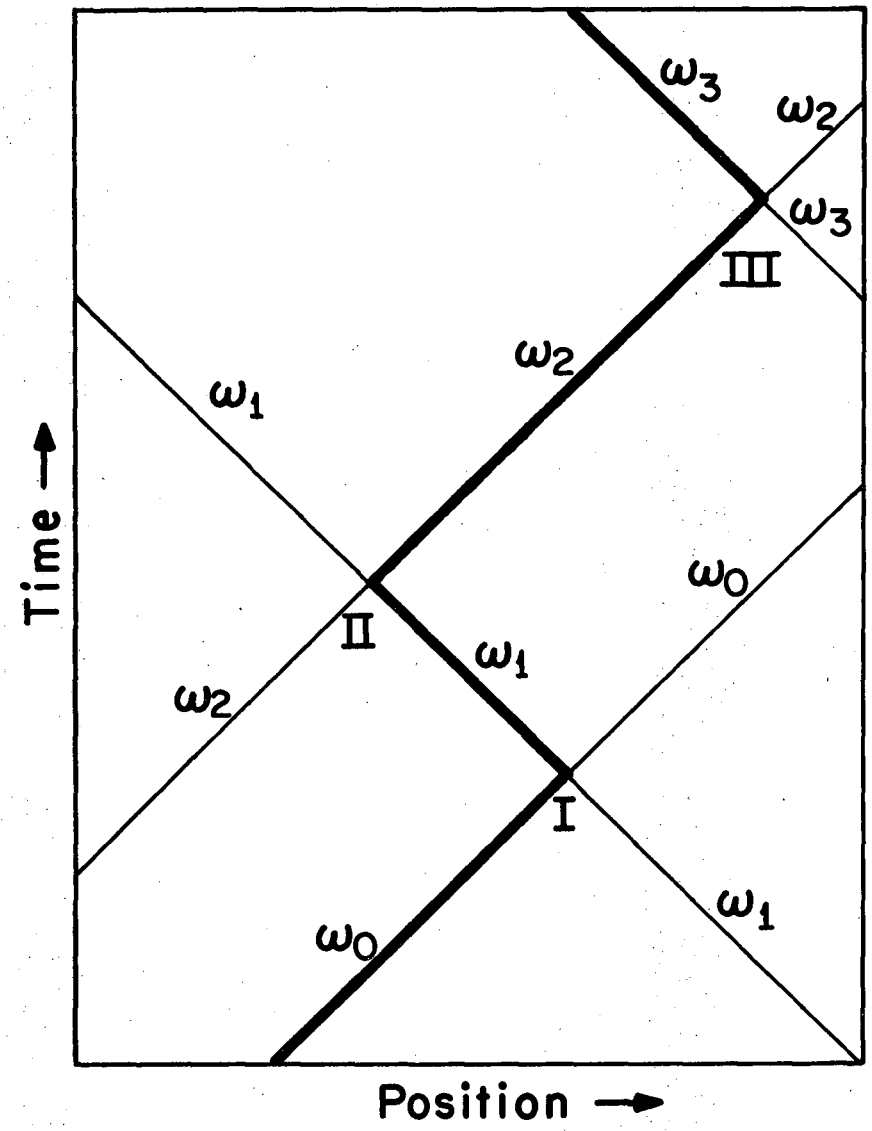
12. Model calculations with $\nu \sim \omega_p$ indicate that (4) retains qualitative validity, within a factor of two in the exponent.

13. C. Oberman brought the need for such a condition to our attention.

FIGURE CAPTIONS

Fig. 1. Schematic space-time plot of a three-step cascade, using four lasers, at frequencies $\omega_0 > \omega_1 > \omega_2 > \omega_3$, propagating in alternate directions. Intensity is represented by line thickness. Each line represents a continuous family of parallel lines, corresponding to steady-state intensities. At each resonance of a difference frequency with the local plasma frequency $(\omega_p^{I} = \omega_0 - \omega_1,\ \omega_p^{II} = \omega_1 - \omega_2,\ \omega_p^{III} = \omega_2 - \omega_3)$, most of the action of the higher frequency wave is transferred to the lower frequency wave of the interacting pair. A small fraction $(\lesssim \omega_p/\omega)$ of the wave energy is deposited locally in the plasma at each transfer.

Fig. 2. Relative action transfer $R \equiv \Delta J/J_0^{in}$ as a function of $\bar{J}_0$, the dimensionless action input in the ω_0-wave, for representative values of input ratio $\rho \equiv J_1^{in}/J_0^{in}$.



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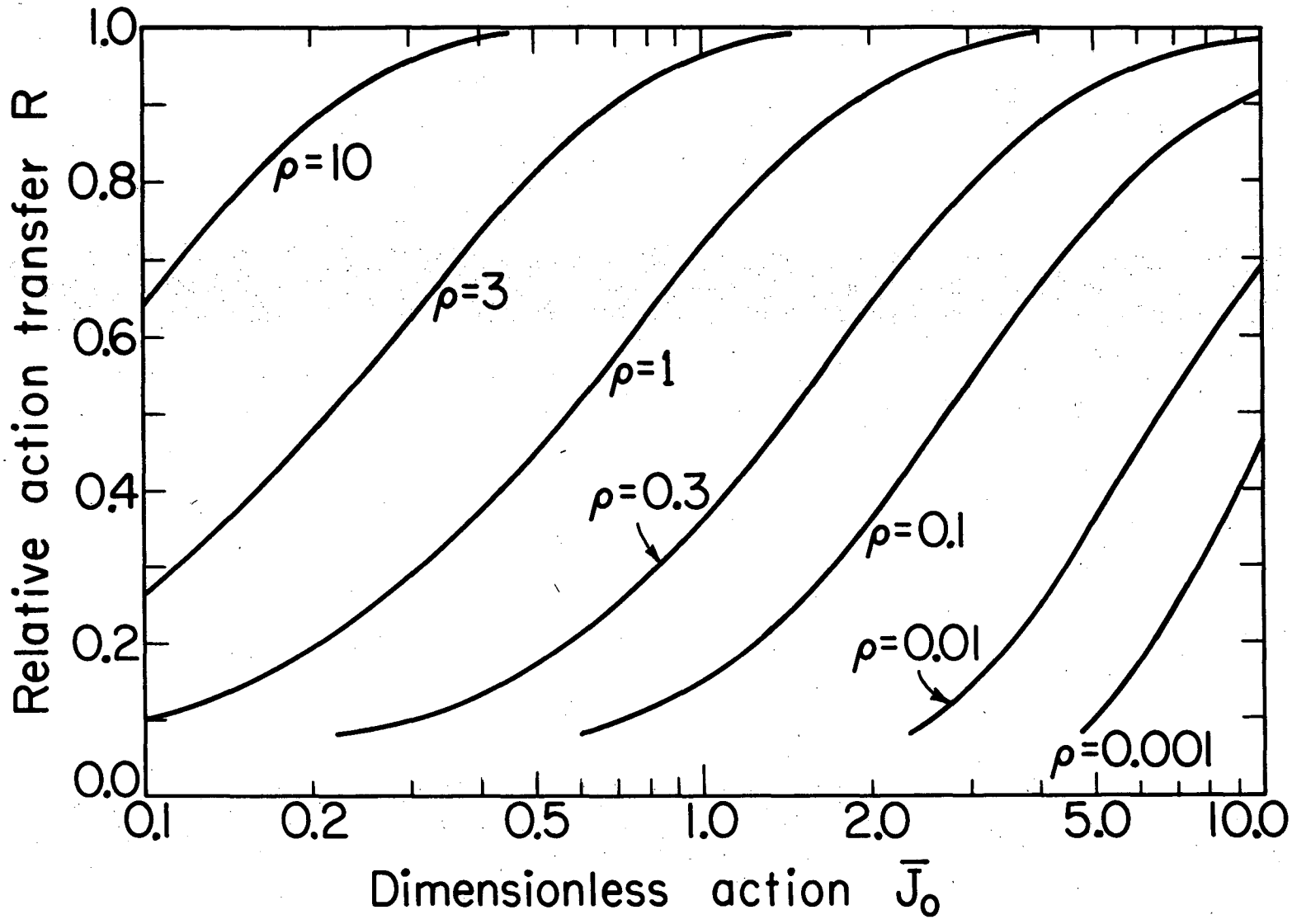
1.0
0.8
0.6
0.4
0.2
0.0
Relative action transfer R
ρ=10
ρ=3
ρ=1
ρ=0.3
ρ=0.1
ρ=0.01
ρ=0.001
0.1
0.2
0.5
1.0
2.0
5.0
10.0
Dimensionless action $\bar{J}_0$

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