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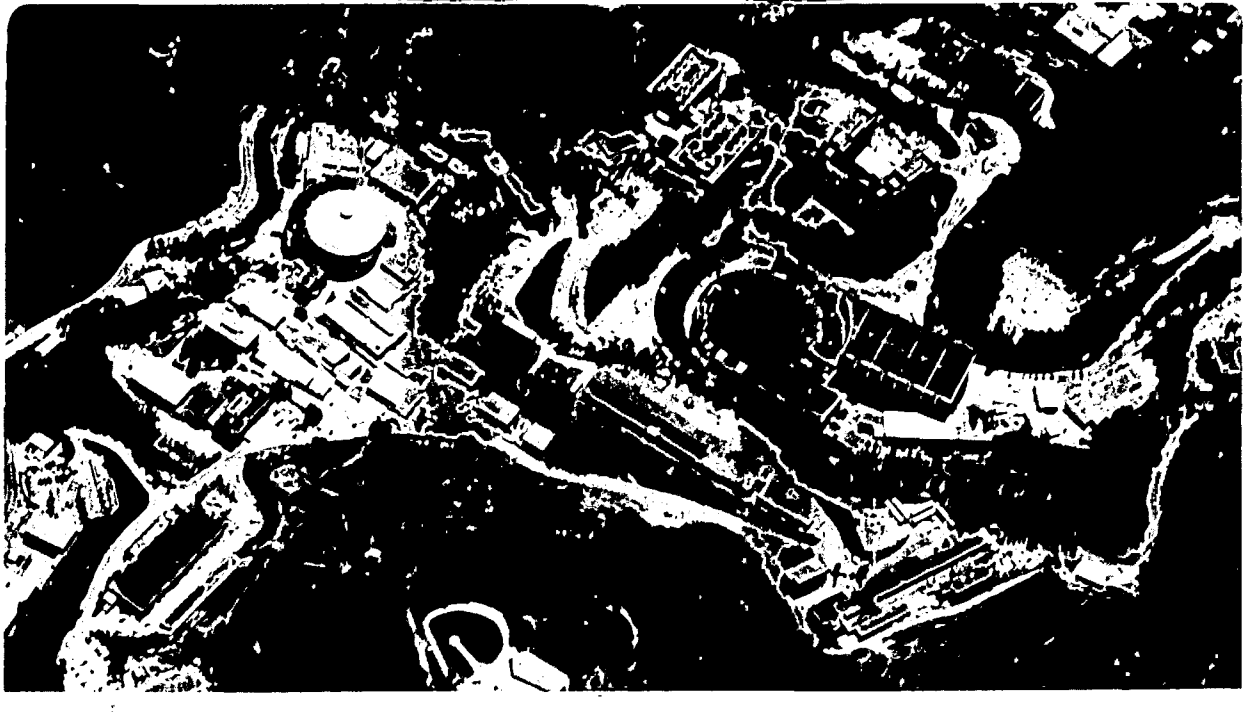

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SUPERSYMMETRY ANOMALY IN TWO DIMENSIONS [1]

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Abstract

In two dimensions we obtain a supersymmetric extension of a gauge anomaly, and find that this is the origin of a supersymmetry anomaly in the Wess-Zumino gauge. We also obtain an effective action whose variations give rise to the gauge and supersymmetry anomalies.

1 Introduction

A gauge anomaly (a chiral anomaly) has been important in two kinds of applications [1]. The first is to get phenomenological interactions when the anomalous symmetry is not a dynamical symmetry [2,3]. The second is as a criterion for model building of unified theories by using the fact that the anomaly should be cancelled in order to have unitarity and renormalizability [4,5]. Recently a gravitational anomaly was found and the second type of application has become important for finding models which unify all the interactions including the gravity [6,7]. In this context it is important to understand a supersymmetry anomaly [8-12]. We will study a two-dimensional supersymmetric Yang-Mills gauge theory in which the supersymmetry is a rigid symmetry [13].

We can think of two kinds of supersymmetry anomalies. The first is an anomaly of a supersymmetry transformation in a superfield formulation without fixing a specific gauge. We will call these a genuine supersymmetry transformation and a genuine supersymmetry anomaly respectively. The second is an anomaly of a supersymmetry transformation in the Wess-Zumino gauge which is composed of two steps of transformations, i.e. a genuine supersymmetry transformation and a restoring supersymmetric gauge transformation. We will call this anomaly a supersymmetry anomaly in the Wess-Zumino gauge.

We find a supersymmetric extension of a gauge anomaly which we will call a supersymmetric gauge anomaly. This anomaly is then used to obtain a gauge anomaly and a supersymmetry anomaly in the Wess-Zumino gauge, which satisfy the mixed consistency conditions. In this derivation it is transparent that the supersymmetry anomaly in the Wess-Zumino gauge originates only from a restoring supersymmetric gauge transformation, not from a genuine supersymmetry transformation. This indicates that there is no genuine supersymmetry anomaly [9]. This situation can be guessed from the fact that the genuine supersymmetry transformation is a rigid transformation. This also shows that when the gauge anomaly is cancelled, the supersymmetry anomaly in the Wess-Zumino gauge is also cancelled automatically.

Furthermore, we obtain the supersymmetric extension of the Wess-Zumino term following Wess and Zumino's original method in superspace [3,14]. We modify this extension such that it depends only on a vector multiplet. This extended Wess-Zumino term's gauge and supersymmetry variations give rise to the gauge and supersymmetry anomalies respectively.

In section 2 we present two-dimensional superfields and their supersymmetry and gauge transformations. In section 3 we obtain a supersymmetric gauge anomaly and gauge and supersymmetry anomalies in the Wess-Zumino gauge. In section 4 we obtain the supersymmetric extension of the Wess-Zumino term.

2 Two-dimensional superspace and superfields

In two-dimensional superspace we have two real space-time coordinates x^0, x^1 and two real spinorial coordinates θ_1, θ_2. The conventions which we will use are given by

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}, \qquad -\eta^{00} = \eta^{11} = 1, \quad \eta^{01} = \eta^{10} = 0,$$

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^5 = \gamma^0\gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.1}$$

The rest of our conventions and their properties are given in Appendix A.

A scalar superfield is given by

$$S = A + i\overline{\theta}\psi + \frac{i}{2}\overline{\theta}\theta F\,, \tag{2.2}$$

where $\overline{\theta}_a = \theta_b\gamma^0_{ba}$ is a linear combination of θ_a's and is not independent of the θ_a's. The supersymmetry transformation of S is given by using a generator

$$Q_a = -\frac{\partial}{\partial\overline{\theta}_a} + i\gamma^\mu_{ab}\theta_b\partial_\mu \tag{2.3}$$

as

$$\delta_\alpha S = [S, \overline{\alpha}Q]:$$

$$\begin{cases} \delta A = i\overline{\alpha}\psi \\ \delta\psi = \partial_\mu A\gamma^\mu\alpha + F\alpha \\ \delta F = i\overline{\alpha}\gamma\cdot\partial\psi\,. \end{cases} \tag{2.4}$$

A spinor superfield or a vector multiplet V_a, which is real and contains a gauge field A_μ as one component field, is given by [13]

$$V_a = \xi_a + \gamma^\mu_{ab}\theta_b A_\mu + \gamma^5_{ab}\theta_b M + \theta_a N + \frac{i}{2}\overline{\theta}\theta\zeta_a\,. \tag{2.5}$$

Its supersymmetry transformation is given by

$$\delta_\alpha V_a = [V_a, \overline{\alpha}Q]:$$

$$\begin{cases} \delta\xi = \gamma^\mu\alpha A_\mu + \gamma^5\alpha M + \alpha N \\ \delta A_\mu = \frac{i}{2}\overline{\alpha}\gamma_\nu\gamma_\mu\partial^\nu\xi - \frac{i}{2}\overline{\alpha}\gamma_\mu\zeta \\ \delta M = -\frac{i}{2}\overline{\alpha}\gamma^5\gamma\cdot\partial\xi - \frac{i}{2}\overline{\alpha}\gamma^5\zeta \\ \delta N = \frac{i}{2}\overline{\alpha}\gamma\cdot\partial\xi - \frac{i}{2}\overline{\alpha}\zeta \\ \delta\zeta = -\gamma^\nu\gamma^\mu\alpha\partial_\mu A_\nu - \gamma^5\gamma^\mu\alpha\partial_\mu M - \gamma^\mu\alpha\partial_\mu N\,. \end{cases} \tag{2.6}$$

In order to have a gauge structure, we let a set of scalar superfields form a representation of a gauge group such that $S = \{S_i\}$ transforms under a finite gauge transformation as

$$S' = e^{-\Lambda}S \tag{2.7}$$

or under an infinitesimal transformation as

$$\delta_\Lambda S = -\Lambda S\,, \tag{2.8}$$

where $\Lambda = \Lambda_i T_i$, Λ_i's are real scalar superfields which are supersymmetric gauge transformation parameters, i.e., $\Lambda_i = a_i + i\overline{\theta}\chi_i + \frac{i}{2}\overline{\theta}\theta f_i$ and T_i's are anti-hermitian gauge group generators which satisfy $[T_i, T_j] = f_{ijk}T_k$.

We gauge covariantize

$$D_a S = (-\frac{\partial}{\partial\overline{\theta}} - i\gamma^\mu\theta\partial_\mu)_a S \tag{2.9}$$

to

$$\nabla_a S = (D_a - iV_a)S = -i(iD_a + V_a)S \ , \tag{2.10}$$

by requiring $V_a = V_{ai}T_i$ to transform under a gauge transformation as $(iD_a + V_a)' = e^{-\Lambda}(iD_a + V_a)e^{\Lambda}$ in order to have $(\nabla_a S)' = e^{-\Lambda}(\nabla_a S)$. That is, under a finite transformation

$$V_a' = (e^{-\Lambda} iD_a e^{\Lambda}) + e^{-\Lambda} V_a e^{\Lambda} \tag{2.11}$$

or under an infinitesimal trasformation

$$\delta_\Lambda V_a = iD_a\Lambda + [V_a, \Lambda] \ . \tag{2.12}$$

In terms of the component fields, (2.12) becomes

$$\begin{cases} \delta\xi = \chi + [\xi, a] \\ \delta A_\mu = \partial_\mu a + [A_\mu, a] + \frac{i}{2}(\overline{\xi}\gamma_\mu\chi + \overline{\chi}\gamma_\mu\xi) \\ \delta M = [M, a] + \frac{i}{2}(\overline{\xi}\gamma^5\chi + \overline{\chi}\gamma^5\xi) \\ \delta N = f + [N, a] + \frac{i}{2}(-\overline{\xi}\chi + \overline{\chi}\xi) \\ \delta\varsigma = -\gamma\cdot\partial\chi + [\varsigma, a] + [\xi, f] - [A_\mu, \gamma^\mu\chi] - [M, \gamma^5\chi] - [N, \chi] \ . \end{cases} \tag{2.13}$$

When we have the gauge symmetry (2.12) or (2.13), we can choose the Wess-Zumino gauge in which $\xi = 0$, $N = 0$ in the following way. Let us start with $\xi = 0$, $N = 0$, then we have the following transformations of ξ and N.

Genuine supersymmetry transformation for ξ and N :

$$\begin{cases} \delta\xi = \gamma^\mu\alpha A_\mu + \gamma^5\alpha M \\ \delta N = -\frac{i}{2}\overline{\alpha}\varsigma \ . \end{cases} \tag{2.14}$$

Supersymmetric gauge transformation for ξ and N :

$$\begin{cases} \delta\xi = \chi \\ \delta N = f \ . \end{cases} \tag{2.15}$$

As we see in (2.14), even though we start with $\xi = 0$, $N = 0$, these component fields become non-zero after a genuine supersymmetry transformation. But we can come back to $\xi = 0$, $N = 0$ by performing a restoring gauge transformation which is given by the following gauge transformation parameter Λ_{RG} as can be seen in (2.15).

$$\delta_{RG} V = iD\Lambda_{RG} + [V, \Lambda_{RG}]$$

with Λ_{RG} :

$$\begin{cases} a = 0 \\ \chi = -\gamma^\mu\alpha A_\mu - \gamma^5\alpha M \\ f = \frac{i}{2}\overline{\alpha}\lambda \end{cases} \tag{2.16}$$

where

$$\lambda = \varsigma + \gamma\cdot\partial\xi \ .$$

Therefore the supersymmetry and gauge transformations in the Wess-Zumino gauge are given by

$$\begin{cases} \delta_{S(WZ)} = \delta_{GEN.\,SUSY} + \delta_{RG} \\ \delta_{G(WZ)} = \delta_{SUP.\,GAUGE} \qquad \text{with} \qquad \Lambda_G : \quad a = a,\ \chi = 0,\ f = 0\,, \end{cases} \tag{2.17}$$

where $\delta_{GEN.\,SUSY}$, δ_{RG} and $\delta_{SUP.\,GAUGE}$ mean genuine supersymmetry, restoring gauge and supersymmetric gauge transformations respectively. Afterwards, we will write $\delta_{S(WZ)}$ and $\delta_{G(WZ)}$ simply as δ_S and δ_G. Under these transformations, the component fields A_μ, M, λ in the Wess-Zumino gauge transform as

$$\begin{cases} \delta_S A_\mu = -\frac{i}{2}\overline{\alpha}\gamma_\mu\lambda \\ \delta_S M = -\frac{i}{2}\overline{\alpha}\gamma^5\lambda \\ \delta_S\lambda = \gamma^\mu\gamma^\nu\alpha F_{\mu\nu} + 2\gamma^\mu\gamma^5\alpha(\partial_\mu M + [A_\mu, M])\,, \end{cases} \tag{2.18}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + A_\mu A_\nu - A_\nu A_\mu$,

$$\begin{cases} \delta_G A_\mu = \partial_\mu a + [A_\mu, a] \\ \delta_G M = \qquad [M, a] \\ \delta_G\lambda = \qquad [\lambda, a]\,. \end{cases} \tag{2.19}$$

Note that in two dimensions the Wess-Zumino gauge has a pseudo-scalar field M as well as A_μ and λ, in contrast to the four-dimensional case in which there is no M [13].

For reference we write down the following supersymmetry and gauge transformations of a scalar multiplet in the Wess-Zumino gauge.

$$\begin{cases} \delta_S\phi = i\overline{\alpha}\psi \\ \delta_S\psi = \gamma^\mu\alpha D_\mu\phi + \gamma^5\alpha M\phi + \alpha F \\ \delta_S F = i\overline{\alpha}\gamma^\mu D_\mu\psi + i\overline{\alpha}\gamma^5 M\psi - \frac{i}{2}\overline{\alpha}\lambda\phi \end{cases} \tag{2.20}$$

where

$$D_\mu = \partial_\mu + A_\mu\,,$$

$$\begin{cases} \delta_G\phi = -a\phi \\ \delta_G\psi = -a\psi \\ \delta_G F = -aF\,. \end{cases} \tag{2.21}$$

3 Anomalies

First let us find a supersymmetric extension of a gauge anomaly which we will call a supersymmetric gauge anomaly. A vector multiplet given by (2.5) gives rise to

$$\overline{D}\gamma^5 V = \overline{D}_a \gamma^5_{ab} V_b = 2M - i\overline{\theta}\gamma^5\lambda + i\overline{\theta}\theta\varepsilon^{\mu\nu}\partial_\mu A_\nu \,. \tag{3.1}$$

Then with $\Lambda = a + i\overline{\theta}\chi + \frac{i}{2}\overline{\theta}\theta f$, we have

$$Tr(\Lambda\overline{D}\gamma^5 V)|_{\overline{\theta}\theta} = i\overline{\theta}\theta\, Tr(a\,\varepsilon_{\mu\nu}\partial^\mu A^\nu + \frac{i}{2}\overline{\chi}\gamma^5\lambda + fM) \,. \tag{3.2}$$

Since the first term on the right hand side of (3.2) is just an ordinary non-supersymmetric gauge anomaly, it seems plausible that (3.2) is a supersymmetric gauge anomaly. In order to confirm this we should show that

$$\triangle(\Lambda) = Tr\int d^2x\, d\overline{\theta}\, d\theta\, (\Lambda\overline{D}\gamma^5 V) \tag{3.3}$$

satisfies the consistency condition

$$\delta_{\Lambda_2}\triangle(\Lambda_1) - \delta_{\Lambda_1}\triangle(\Lambda_2) = \triangle([\Lambda_2, \Lambda_1]) \,, \tag{3.4}$$

where $\delta_\Lambda V_a$ is given in (2.12).

Let us show this.

$$\begin{aligned}\delta_{\Lambda_2}(\Lambda_1\overline{D}\gamma^5 V) &= \Lambda_1\overline{D}\gamma^5(iD\Lambda_2 + [V, \Lambda_2]) \\ &= \Lambda_1(\overline{D}U)\Lambda_2 - \Lambda_1 U(\overline{D}\Lambda_2) - \Lambda_1(\overline{D}\Lambda_2)U - \Lambda_1\Lambda_2(\overline{D}U) \,,\end{aligned} \tag{3.5}$$

where $U \equiv \gamma^5 V$, and $\overline{D}\gamma^5 D = 0$ was used.

From (3.5) we have

$$Tr\{\delta_{\Lambda_2}(\Lambda_1\overline{D}\gamma^5 V) - \delta_{\Lambda_1}(\Lambda_2\overline{D}\gamma^5 V)\} = Tr\{[\Lambda_2, \Lambda_1]\overline{D}\gamma^5 V + \overline{D}([\Lambda_2, \Lambda_1]\gamma^5 V)\} \,. \tag{3.6}$$

Since the second term on the right hand side of (3.6) is a total derivative, it vanishes under the integration $\int d^2x\, d\overline{\theta}\, d\theta$. Thus by taking $\int d^2x\, d\overline{\theta}\, d\theta$ on both sides of (3.6) we obtain (3.4). Of course, (3.4) is also satisfied with an arbitrary normalization factor in (3.3). We can show the above in a more elegant, but equivalent way which is given in Appendix B.

The supersymmetry and gauge transformations in the Wess-Zumino gauge, i.e., (2.18) and (2.19) satisfy the algebra:

$$\begin{cases} [\delta_S(\beta),\ \delta_S(\alpha)] = \delta_G(-2i(\overline{\alpha}\gamma^\mu\beta)A_\mu - 2i(\overline{\alpha}\gamma^5\beta)M) + 2(\overline{\alpha}\gamma^\mu\beta)i\partial_\mu \\ [\delta_G(b),\ \delta_G(a)] = \delta_G([b,\ a]) \\ [\delta_G(a),\ \delta_S(\alpha)] = 0 \,. \end{cases} \tag{3.7}$$

Then a supersymmetry anomaly $\triangle_S(\alpha)$ and a gauge anomaly $\triangle_G(a)$ in the Wess-Zumino gauge satisfy the consistency conditions:

$$\begin{cases} \delta_S(\beta)\triangle_S(\alpha) - \delta_S(\alpha)\triangle_S(\beta) = \triangle_G(-2i(\overline{\alpha}\gamma^\mu\beta)A_\mu - 2i(\overline{\alpha}\gamma^5\beta)M) \\ \delta_G(b)\triangle_G(a) - \delta_G(a)\triangle_G(b) = \triangle_G([b,\ a]) \\ \delta_G(a)\triangle_S(\alpha) - \delta_S(\alpha)\triangle_G(a) = 0 \,. \end{cases} \tag{3.8}$$

The term $2(\overline{\alpha}\gamma^\mu\beta)i\partial_\mu$ in (3.7) did not contribute to (3.8), since the vacuum functional is invariant under translation if we impose the condition that a surface integral vanishes.

The interesting thing is that we can obtain $\triangle_S(\alpha)$, $\triangle_G(a)$ which satisfy (3.8) by using the supersymmetric gauge anomaly (3.3) in the following way. Let us rewrite (3.3) with an arbitrary normalization factor as

$$\begin{aligned}\triangle(\Lambda) &= -ic\,Tr\int d^2x\,d^2\theta\,(\Lambda\overline{D}\gamma^5 V)\\ &= c\,Tr\int d^2x\,(a\,\varepsilon^{\mu\nu}\partial_\mu A_\nu + \frac{i}{2}\overline{\chi}\gamma^5\lambda + fM)\end{aligned} \tag{3.9}$$

where $\int d^2\theta \equiv -\frac{1}{4}\int d\overline{\theta}\,d\theta$ such that $\int d^2\theta\,\overline{\theta}\theta = 1$.

At first we obtain $\triangle_G(a)$ from (3.9) by taking $a = a$, $\chi = 0$, $f = 0$ since δ_G in (2.17) or (2.19) was given by this assignment of Λ, i.e., Λ_G. Next, in order to obtain $\triangle_S(\alpha)$ we observe that δ_S in (2.17) or (2.18) is composed of two steps, i.e., $\delta_{GEN.\,SUSY}$ and δ_{RG} . But we expect that the $\delta_{GEN.\,SUSY}$ step will not produce any anomaly since this transformation is a rigid transformation. Then we expect that $\triangle(\Lambda)$ with $\Lambda = \Lambda_{RG}$ in (2.16) will give rise to $\triangle_S(\alpha)$ [9]. That is, we expect the following to be the solution of (3.8).

$$\begin{aligned}\triangle_G(a) &= \triangle(\Lambda_G:\ a = a,\ \chi = 0,\ f = 0)\\ &= c\ Tr\int d^2x\,a\,\varepsilon^{\mu\nu}\partial_\mu A_\nu\ ,\end{aligned} \tag{3.10}$$

$$\begin{aligned}\triangle_S(\alpha) &= \triangle(\Lambda_{RG}:\ a = 0,\ \chi = -\gamma^\mu\alpha A_\mu - \gamma^5\alpha M,\ f = \frac{i}{2}\overline{\alpha}\lambda)\\ &= ic\ Tr\int d^2x(\frac{1}{2}A_\mu\overline{\alpha}\gamma^\mu\gamma^5\lambda + M\overline{\alpha}\lambda)\ .\end{aligned} \tag{3.11}$$

We have confirmed that these $\triangle_G(a)$ and $\triangle_S(\alpha)$ satisfy (3.8) by explicit application of (2.18) and (2.19). Therefore we have found that the supersymmetry anomaly in the Wess-Zumino gauge originates from the supersymmetric gauge anomaly. This indicates that there is no genuine supersymmetry

anomaly. This also shows that when the gauge anomaly is cancelled, the supersymmetry anomaly in the Wess-Zumino gauge is also cancelled automatically.

4 SUSY extension of the Wess-Zumino term

We will find a supersymmetric extension of the Wess-Zumino term which depends only on component fields of a vector multiplet. This vacuum functional gives rise to gauge and supersymmetry anomalies by gauge and supersymmetry variations respectively. In order to understand the derivation better, let us review briefly the familiar non-supersymmetric gauge theory case [1,3].

An effective action can be obtained by solving an anomaly equation

$$\delta_a W = \int d^2x \, a_i \, G_i \, , \tag{4.1}$$

where G_i's are anomalies. Wess and Zumino solved this equation and obtained the solution

$$W[A,\pi] = \int d^2x \int_0^1 dt \, \pi_i G_i(A(t))(x) \, , \tag{4.2}$$

where the π_i's are a set of fields which transforms as

$$e^{\pi'} = e^{\pi} e^{a} \tag{4.3}$$

and

$$A_\mu(t) = e^{t\pi} A_\mu e^{-t\pi} + e^{t\pi} \partial_\mu e^{-t\pi} \, , \tag{4.4}$$

where

$$a = a_i T_i, \; \pi = \pi_i T_i, \; A_\mu = A_{\mu i} T_i \, .$$

Any gauge invariant functional can be added in (4.2), so (4.2) is a particular solution and is called the Wess-Zumino term.

Now we are interested in having a solution $W[A]$ as a functional of only A_μ without the independent π. This can be achieved by replacing the independent π by a function $\pi(A)$ which transforms as (4.3) when A_μ transforms as a gauge field, if we can find such a function [15].

In the Abelian case we find such a $\pi(A)$ easily as

$$\pi(A) = \frac{1}{\Box} \partial_\mu A^\mu \, , \quad \text{where } \Box \equiv \partial_\mu \partial^\mu \tag{4.5}$$

$$\text{since} \qquad \delta_a A^\mu = \partial^\mu a \, , \quad \delta_a \pi(A) = \frac{1}{\Box} \partial_\mu (\partial^\mu a) = a \, .$$

Then

$$\begin{aligned} W[A] &= \int d^2x \int dt \, (\frac{1}{\Box} \partial_\mu A^\mu) G(A(t)) \\ &= \int d^2x \int_0^1 dt \, (\frac{1}{\Box} \partial_\mu A^\mu) \frac{i}{2\pi} \partial_\nu A_\lambda(t) \varepsilon^{\nu\lambda} \\ &= \frac{i}{2\pi} \int d^2x \int_0^1 dt \, (\frac{1}{\Box} \partial_\mu A^\mu) \partial_\nu (A_\lambda - t \partial_\lambda \pi(A)) \varepsilon^{\nu\lambda} \\ &= \frac{i}{2\pi} \int d^2x \, (\frac{1}{\Box} \partial_\mu A^\mu) \partial_\nu A_\lambda \varepsilon^{\nu\lambda} \, , \end{aligned} \tag{4.6}$$

for the quantum effect of a left-handed chiral fermion. We use the convention $\varepsilon^{10} = -\varepsilon^{01} = 1, \; \varepsilon^{+-} = -\varepsilon^{-+} = -1$. Our conventions are summarized in Appendix A.

In the light-cone coordinates, let us use the anomaly in the form $-(i/\pi)\partial_+ A_-$ which is equivalent to $(i/2\pi)\partial_\nu A_\lambda \varepsilon^{\nu\lambda}$ in (4.6), since they differ by a variation of a local functional $(i/2\pi) \int d^2x \, A_\mu A^\mu$. (∂_μ) in the coordinate space corresponds to $(-ip_\mu)$ in the momentum space, since we will take external momenta as out-going. Then $W[A]$ can be written as

$$W[A] = \frac{i}{2\pi} \int \frac{d^2p}{(2\pi)^2} \frac{p_+}{p_-} A_-(p) A_-(-p) \, , \tag{4.7}$$

after adding an appropriate local functional to (4.6) which is allowed since $W[A]$ is ambiguous by a local functional.

In the non-Abelian case we can get $\pi(A)$ which transforms as (4.3) by inverting

$$A_\mu = e^{-\pi}\partial_\mu e^{\pi} . \tag{4.8}$$

This inversion can be done as a power series of A_μ and the lowest order term has the same form as (4.5). Note that even though we are inverting the pure gauge form (4.8), $\pi(A)$ obtained by this procedure transforms as (4.3) for a general A_μ . That is, (4.8) is just a guide for obtaining $\pi(A)$ for a general A_μ [15]. Using this $\pi(A)$ we can obtain $W[A]$ as a power series of A_μ which starts with the lowest order term similar to (4.6) or (4.7) as

$$W[A] = c' Tr \int \frac{d^2p}{(2\pi)^2}\frac{p_+}{p_-}A_-(p)A_-(-p) \quad + O(A^3) . \tag{4.9}$$

We can use the above procedure to get a supersymmetric extension of the Wess-Zumino term which gives rise to $\triangle_G(a)$, $\triangle_S(\alpha)$ in (3.10), (3.11) by $\delta_G(a)$, $\delta_S(\alpha)$ in (2.18), (2.19) respectively [14]. First we will treat the Abelian case in detail.

The consistency conditions for the Abelian case are the same as (3.8) except that the second condition is replaced by

$$\delta_G(b)\triangle_G(a) - \delta_G(a)\triangle_G(b) = 0 . \tag{4.10}$$

The solution of the consistency conditions is given, in analogy with (3.10)

and (3.11), by

$$\triangle_G(a) = i\int d^2x\, a\, \epsilon^{\mu\nu}\partial_\mu A_\nu , \tag{4.11}$$

$$\triangle_S(\alpha) = -\int d^2x\,(\frac{1}{2}A_\mu\overline{\alpha}\gamma^\mu\gamma^5\lambda + M\overline{\alpha}\lambda) , \tag{4.12}$$

which can be obtained from a supersymmetric Abelian gauge anomaly

$$\begin{aligned}\triangle_{Abel.}(\Lambda) &= \int d^2x\, d^2\theta\,(\Lambda\overline{D}\gamma^5 V) \\ &= i\int d^2x\,(a\epsilon^{\mu\nu}\partial_\mu A_\nu + \frac{i}{2}\overline{\chi}\gamma^5\lambda + fM)\end{aligned} \tag{4.13}$$

through the same procedure as that used in section 3 for the non-Abelian case. In (4.13) we take such a normalization factor for convenience.

In the present two-dimensional supersymmetric case, (4.2) is replaced by

$$W[V,\Pi] = \int d^2x\, d^2\theta \int_0^1 dt\, \Pi_i(\overline{D}\gamma^5 V_i(t)) \tag{4.14}$$

where the Π_i's transform under a supersymmetric gauge transformation as

$$e^{\Pi'} = e^{\Pi}\, e^{\Lambda} \tag{4.15}$$

and

$$V_a(t) = e^{t\Pi}V_a e^{-t\Pi} + e^{t\Pi}(iD_a)e^{-t\Pi} . \tag{4.16}$$

The above formulas (4.14), (4.15) and (4.16) are also valid for the non-Abelian case where $\Pi = \Pi_i T_i$, $\Lambda = \Lambda_i T_i$ are Lie algebra valued scalar superfields and $V_a = V_{ai}T_i$. The formulas for the Abelian case are simply given by omitting the sum over the subscript i in (4.14) and using the Abelian nature of Π, Λ and V_a.

In the Abelian case the gauge transformation given in (2.12) becomes

$$\delta V_a = iD_a\Lambda \,. \tag{4.17}$$

Then we find easily that

$$\Pi(V) = -i\frac{1}{\overline{D}D}\overline{D}V \tag{4.18}$$

transforms as (4.15) which is the same as $\Pi' = \Pi + \Lambda$ in the Abelian case. The expression (4.18) means

$$\begin{aligned}\Pi(V) &= -i\frac{1}{\overline{D}_aD_a}\overline{D}_bV_b \\ &= -i\frac{1}{(\overline{D}_aD_a)(\overline{D}_cD_c)}(\overline{D}_dD_d)(\overline{D}_bV_b) \\ &= \frac{i}{4}\frac{1}{\Box}(\overline{D}_dD_d)(\overline{D}_bV_b)\end{aligned} \tag{4.19}$$

since

$$(\overline{D}D)^2 = (\overline{D}_aD_a)(\overline{D}_cD_c) = -4\partial_\mu\partial^\mu = -4\Box \,.$$

Then from (4.14) we get $W[V]$ as a functional of V_a only

$$\begin{aligned}W[V] &= -i\int d^2xd^2\theta\int_0^1 dt(\frac{1}{\overline{D}D}\overline{D}V)(\overline{D}\gamma^5V(t)) \\ &= -i\int d^2xd^2\theta\int_0^1 dt(\frac{1}{\overline{D}D}\overline{D}V)\{\overline{D}\gamma^5(V - tiD\Pi)\} \\ &= -i\int d^2xd^2\theta(\frac{1}{\overline{D}D}\overline{D}V)(\overline{D}\gamma^5V)\end{aligned} \tag{4.20}$$

since $\quad \overline{D}\gamma^5D = 0 \,.$

Let us express (4.20) in terms of component fields in the Wess-Zumino gauge.

$$V = \gamma^\mu\theta A_\mu + \gamma^5\theta M + \frac{i}{2}\overline{\theta}\theta\lambda$$

$$\overline{D}V = -i\overline{\theta}\lambda + i\overline{\theta}\theta\partial_\mu A^\mu$$

$$\frac{1}{\overline{D}D}\overline{D}V = i\frac{\partial_\mu A^\mu}{\Box} + i\overline{\theta}(-\frac{i}{2}\frac{\gamma\cdot\partial\lambda}{\Box}) \tag{4.21}$$

and

$$\overline{D}\gamma^5V = 2M + i\overline{\theta}(-\gamma^5\lambda) + \frac{i}{2}\overline{\theta}\theta(2\varepsilon^{\mu\nu}\partial_\mu A_\nu) \,. \tag{4.22}$$

Using (4.21) and (4.22) we get

$$W[V] = i\varepsilon^{\mu\nu}\int d^2x(\frac{\partial_\lambda A^\lambda}{\Box}\partial_\mu A_\nu - \frac{i}{4}\overline{\lambda}\gamma_\nu\frac{\partial_\mu\lambda}{\Box}) \,. \tag{4.23}$$

We have checked explicitly that the variations of (4.23) give the anomalies (4.11) and (4.12).

When we use the light-cone coordinates in (4.23), the terms from ε^{+-} and ε^{-+} are equivalent to each other, since their variations give rise to anomalies which differ by variations of a local functional. Therefore we can replace (4.23) by twice the ε^{+-} term in (4.23). Then we have $W[V]$ as

$$W[V] = i\int\frac{d^2p}{(2\pi)^2}[\frac{p_+}{p_-}A_-(-p)A_-(p) - \frac{1}{4}\frac{1}{p_-}\overline{\lambda}(-p)\gamma_-\lambda(p)] \tag{4.24}$$

by adding an appropriate local functional. Variations of (4.24) give rise to anomalies of the form

$$\begin{cases}\triangle_G(a) = -2i\int d^2x\, a\,\partial_+A_- \\ \triangle_S(\alpha) = \int d^2x\,(A_+\overline{\alpha}\gamma_-\lambda - M\overline{\alpha}\lambda)\end{cases} \tag{4.25}$$

which are equivalent to (4.11) and (4.12) since they differ by variations of a local functional.

In the non-Abelian case, in order to get a $W[V]$ depending only on V_a from (4.14), we need a function $\Pi(V)$ which transforms as (4.15) when V_a

transforms as (2.12). This can be obtained by inverting

$$V_a = e^{-\Pi} i D_a e^{\Pi} \tag{4.26}$$

in analogy with (4.8) in the non-supersymmetric non-Abelian case. $\Pi(V)$ can be expanded in a power series of V and the lowest order term has the same form as (4.18) in the Abelian case. Using this $\Pi(V)$ we can get $W[V]$ as a power series of V which starts with the lowest order term similar to (4.23) or (4.24) as

$$W[V] = c'' Tr \int \frac{d^2p}{(2\pi)^2} [\frac{p_+}{p_-} A_-(-p) A_-(p) - \frac{1}{4}\frac{1}{p_-} \overline{\lambda}(-p)\gamma_- \lambda(p)] \quad + O(V^3) \ . \tag{4.27}$$

5 Conclusion

In two dimensions we have shown that the origin of the supersymmetry anomaly in the Wess-Zumino gauge is the supersymmetric gauge anomaly. This indicates that there is no genuine supersymmetry anomaly. This also shows that when the gauge anomaly is cancelled, the supersymmetry anomaly in the Wess-Zumino gauge is also cancelled automatically. We expect that the situation is the same in other supersymmetric gauge theories which have superfield formulations. However, in a theory which has no superfield formulation, a different analysis may be necessary. We have also obtained the supersymmetric extension of the Wess-Zumino term in the form which depends only on the external vector multiplet.

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Appendix A

Let us summarize our conventions and their properties. We use μ, ν, λ, $\cdots$ for space-time indices, and a, b, c, $\cdots$ for spinorial indices.

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}, \qquad -\eta^{00} = \eta^{11} = 1, \quad \eta^{01} = \eta^{10} = 0,$$

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^5 = \gamma^0\gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$\varepsilon^{10} = -\varepsilon^{01} = 1, \quad \varepsilon^{00} = \varepsilon^{11} = 0.$$

$$\gamma^\mu\gamma^5 = \varepsilon^{\mu\nu}\gamma_\nu\,, \quad \gamma^\mu\gamma^\nu = \eta^{\mu\nu} - \varepsilon^{\mu\nu}\gamma^5\,, \quad \gamma^\mu\gamma^\nu\gamma^5 = \eta^{\mu\nu}\gamma^5 - \varepsilon^{\mu\nu}$$

$$\gamma^\mu\gamma^\nu\gamma^\rho = \eta^{\mu\nu}\gamma^\rho + \eta^{\nu\rho}\gamma^\mu - \eta^{\rho\mu}\gamma^\nu\,,$$

$$\varepsilon^{\mu\nu}\varepsilon^{\rho\lambda} = -(\eta^{\mu\rho}\eta^{\nu\lambda} - \eta^{\mu\lambda}\eta^{\nu\rho})\,.$$

$$x^\pm = \frac{1}{\sqrt{2}}(\pm x^0 + x^1)\,, \; x_\pm = \frac{1}{\sqrt{2}}(\pm x_0 + x_1)\,,$$

$$x_- = x^+\,, \; x_+ = x^-\,.$$

$$\eta^{+-} = \eta^{-+} = \eta_{+-} = \eta_{-+} = 1\,, \text{ other } \eta\text{'s} = 0\,.$$

$$a^\mu b_\mu = a^+ b_+ + a^- b_- = a_+ b_- + a_- b_+\,,$$

$$a^\mu a_\mu = 2a_+ a_-\,.$$

$$\varepsilon^{+-} = -\varepsilon^{-+} = -\varepsilon_{+-} = \varepsilon_{-+} = -1\,, \text{ other } \varepsilon\text{'s} = 0\,.$$

$$\gamma^+ = \frac{1}{\sqrt{2}}(\gamma^0 + \gamma^1) = \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}, \; \gamma^- = \frac{1}{\sqrt{2}}(-\gamma^0 + \gamma^1) = \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix},$$

$$\gamma^+\gamma^+ = \gamma^-\gamma^- = 0\,, \quad \gamma^+\gamma^- + \gamma^-\gamma^+ = 2\,.$$

$$\overline{\theta}_a = \theta_b\gamma^0_{ba} = -\varepsilon_{ab}\theta_b \;\; (\gamma^0_{ab} = \varepsilon_{ab}\,,\; \varepsilon_{12} = 1)\,,\; \theta_a = -\overline{\theta}_b\gamma^0_{ba} = \varepsilon_{ab}\overline{\theta}_b\,.$$

$$\theta_a = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} \overline{\theta}_2 \\ -\overline{\theta}_1 \end{pmatrix}, \; \overline{\theta}_a = \begin{pmatrix} \overline{\theta}_1 \\ \overline{\theta}_2 \end{pmatrix} = \begin{pmatrix} -\theta_2 \\ \theta_1 \end{pmatrix}.$$

$$\frac{\partial}{\partial\theta_a}\overline{\theta}_b = \gamma^0_{ab}\,, \quad \frac{\partial}{\partial\overline{\theta}_a}\theta_b = -\gamma^0_{ab}\,,$$

$$\frac{\partial}{\partial\theta_a} = -\frac{\partial}{\partial\overline{\theta}_b}\gamma^0_{ba}\,, \quad \frac{\partial}{\partial\overline{\theta}_a} = \frac{\partial}{\partial\theta_b}\gamma^0_{ba}\,.$$

$$\overline{\theta}_a\theta_b = \frac{1}{2}\overline{\theta}\theta\delta_{ab} \quad (\overline{\theta}\theta = \overline{\theta}_a\theta_a)\,,$$

$$\frac{\partial}{\partial\overline{\theta}_a}(\overline{\theta}\theta) = 2\theta_a\,, \; \frac{\partial}{\partial\theta_a}(\overline{\theta}\theta) = -2\overline{\theta}_a\,.$$

$$D_a = -\frac{\partial}{\partial\overline{\theta}_a} - i\gamma^\mu_{ab}\theta_b\partial_\mu\,, \; \overline{D}_a = D_b\gamma^0_{ba} = \frac{\partial}{\partial\theta_a} + i\overline{\theta}_b\gamma^\mu_{ba}\partial_\mu\,,$$

$$Q_a = -\frac{\partial}{\partial\overline{\theta}_a} + i\gamma^\mu_{ab}\theta_b\partial_\mu\,, \; \overline{Q}_a = Q_b\gamma^0_{ba} = \frac{\partial}{\partial\theta_a} - i\overline{\theta}_b\gamma^0_{ba}\partial_\mu\,.$$

Fierz rearrangement:

$$(\overline{\alpha}\psi)\beta_a = -\frac{1}{2}\{(\overline{\alpha}\beta)\psi_a + (\overline{\alpha}\gamma^5\beta)(\gamma^5\psi)_a + (\overline{\alpha}\gamma_\mu\beta)(\gamma^\mu\psi)_a\}\ .$$

$$\overline{\alpha}\gamma_A\beta = \overline{\beta}\ \tilde{\gamma}_A\ \alpha \quad (\ \alpha, \beta \text{ are real spinors }) \ :$$

Using $(\gamma^\mu)^T = \gamma^0\gamma^\mu\gamma^0$, $(\gamma^5)^T = \gamma^0\gamma^5\gamma^0 = \gamma^5$ (where superscript T means *Transpose*),

$$\tilde{1} = 1\ , \quad \tilde{\gamma^\mu} = -\gamma^\mu\ , \quad \tilde{\gamma^5} = -\gamma^5\ , \quad (\widetilde{\gamma^5\gamma^\mu}) = -(\gamma^5\gamma^\mu)\ ,$$

$$(\widetilde{\gamma^\mu\gamma^\nu}) = (\gamma^\nu\gamma^\mu)\ , \quad (\widetilde{\gamma^5\gamma^\mu\gamma^\nu}) = -(\gamma^5\gamma^\nu\gamma^\mu)\ .$$

We take external momenta as out-going, therefore (∂_μ) in the coordinate space corresponds to $(-ip_\mu)$ in the momentum space.

Appendix B

Let us show in another way that (3.3) satisfies (3.4). Here we treat Λ as a ghost and we take the following BRS transformation.

$$\begin{cases} S\Lambda = -\Lambda^2 \\ SV_a = -iD_a\Lambda - \Lambda V_a - V_a\Lambda\ . \end{cases} \tag{B.1}$$

Then (3.4) can be expressed simply as [1]

$$S\triangle(\Lambda) = 0\ . \tag{B.2}$$

Let us show that (3.3), i.e.,

$$\triangle(\Lambda) = Tr\int d^2x\, d\bar{\theta}\, d\theta\, (\Lambda\overline{D}\gamma^5 V) \tag{B.3}$$

satisfies $(B.2)$. In the following expression, every term is to have Tr in front, i.e., we omit Tr in front of every term for notational simplicity.

$$\begin{aligned} S(\Lambda\overline{D}\gamma^5 V) &= (S\Lambda)\overline{D}\gamma^5 V + (-1)^2\Lambda\overline{D}\gamma^5(SV) \\ &= (-\Lambda^2)\overline{D}\gamma^5 V + \Lambda\overline{D}\gamma^5(-iD\Lambda - \Lambda V - V\Lambda) \\ &= -\Lambda^2\overline{D}U - \Lambda\overline{D}(\Lambda U) - \Lambda\overline{D}(U\Lambda) \text{ (where } U \equiv \gamma^5 V\ , \text{ and } \overline{D}\gamma^5 D = 0 \text{ was used)} \\ &= -\Lambda^2\overline{D}U - \Lambda(\overline{D}\Lambda)U + \Lambda\Lambda(\overline{D}U) - \Lambda(\overline{D}U)\Lambda + \Lambda U(\overline{D}\Lambda) \\ &= (\overline{D}\Lambda)\Lambda U - \Lambda(\overline{D}\Lambda)U + \Lambda\Lambda(\overline{D}U) \\ &= \overline{D}(\Lambda^2 U)\ . \end{aligned}$$

That is,

$$S\triangle(\Lambda) = \int d^2x\, d\bar{\theta}\, d\theta\, \overline{D}\{Tr(\Lambda^2 U)\} = 0\ ,$$

since the integrand is a total derivative.

Appendix C Three-dimensional superspace

We summarize the three-dimensional supersymmetry for reference because of its similarity to the two-dimensional case. The three-dimensional superspace is described by three real space-time coordinates x^0, x^1, x^2 and two real spinorial coordinates θ_1, θ_2. Therefore the structure of the spinorial coordinates is the same as the two-dimensional one. Q_a and D_a have the same forms as (2.3) and (2.9),

$$Q_a = -\frac{\partial}{\partial\overline{\theta}_a} + i\gamma^\mu_{ab}\theta_b\partial_\mu \ , \ D_a = -\frac{\partial}{\partial\overline{\theta}_a} - i\gamma^\mu_{ab}\theta_b\partial_\mu \ , \ \mu = 0,\, 1,\, 2 \ .$$

Our conventions and their properties are as follows.

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}, \qquad -\eta^{00} = \eta^{11} = \eta^{22} = 1,$$

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^2 == \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$\varepsilon^{012} = \varepsilon^{120} = -\varepsilon^{102} = 1, \ etc.$$

$$[\,\gamma^\mu\,,\,\gamma^\nu\,] = 2\varepsilon^{\mu\nu\lambda}\gamma_\lambda$$

$$\gamma^\mu\gamma^\nu = \eta^{\mu\nu} + \varepsilon^{\mu\nu\lambda}\gamma_\lambda \ , \quad Tr(\gamma^\mu\gamma^\nu\gamma^\lambda) = 2\varepsilon^{\mu\nu\lambda} \ ,$$

$$\varepsilon^{\lambda\alpha\beta}\varepsilon_{\lambda\mu\nu} = -\delta^\alpha_\mu\,\delta^\beta_\nu + \delta^\alpha_\nu\,\delta^\beta_\mu \ .$$

A vector multiplet is given by

$$V_a = \xi_a + \gamma^\mu_{ab}\theta_b A_\mu + \theta_a N + \frac{i}{2}\overline{\theta}\theta\varsigma_a \ .$$

Genuine supersymmetry transformation : $\delta_\alpha V_a = [V_a, \overline{\alpha}Q]$

$$\begin{cases} \delta\xi = \gamma^\mu\alpha A_\mu + \alpha N \\ \delta A_\mu = \frac{i}{2}\varepsilon_{\mu\nu\lambda}\overline{\alpha}\gamma^\nu\partial^\lambda\xi + \frac{i}{2}\overline{\alpha}\partial_\mu\xi - \frac{i}{2}\overline{\alpha}\gamma_\mu\varsigma \\ \delta N = \frac{i}{2}\overline{\alpha}\gamma\cdot\partial\xi - \frac{i}{2}\overline{\alpha}\varsigma \\ \delta\varsigma = \varepsilon_{\mu\nu\lambda}\gamma^\lambda\alpha\partial^\mu A^\nu - \alpha\partial_\mu A^\mu - \gamma^\mu\alpha\partial_\mu N \ . \end{cases}$$

Supersymmetric gauge transformation : $\delta_\Lambda V_a = iD_a\Lambda + [V_a, \Lambda]$

$$\begin{cases} \delta\xi = \ \chi + \quad [\xi, a] \\ \delta A_\mu = \partial_\mu a + [A_\mu, a] + \frac{i}{2}(\overline{\xi}\gamma_\mu\chi + \overline{\chi}\gamma_\mu\xi) \\ \delta N = \ f + \quad [N, a] + \frac{i}{2}(-\overline{\xi}\chi + \overline{\chi}\xi) \\ \delta\varsigma = -\gamma\cdot\partial\chi + [\varsigma, a] + [\xi, f] - [A_\mu, \gamma^\mu\chi] - [N, \chi] \ . \end{cases}$$

In the above $V_a = V_{ai}T_i$, $\Lambda = \Lambda_i T_i$ (T_i's are anti-hermitian), $\Lambda_i = a_i + i\overline{\theta}\chi_i + \frac{i}{2}\overline{\theta}\theta f_i$.

In the Wess-Zumino gauge, $\xi = 0$, $N = 0$, $\lambda = \varsigma$,

$$\begin{cases} \delta_{S(WZ)} = \delta_{GEN.\,SUSY} + \delta_{RG} \\ \qquad\qquad \text{with } \Lambda_{RG} : \ a = 0 \ , \ \chi = -\gamma^\mu\alpha A_\mu \ , \ f = \frac{i}{2}\overline{\alpha}\lambda \ , \\ \delta_{G(WZ)} = \delta_{SUP.\,GAUGE} \\ \qquad\qquad \text{with } \Lambda_G : \ a = a \ , \ \chi = 0 \ , \ f = 0 \ . \end{cases}$$

$$\begin{cases} \delta_S(\alpha)A_\mu = -\frac{i}{2}\overline{\alpha}\gamma_\mu\lambda \\ \delta_S(\alpha)\lambda = \gamma^\mu\gamma^\nu\alpha F_{\mu\nu} \ , \end{cases}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + A_\mu A_\nu - A_\nu A_\mu$,

$$\begin{cases} \delta_G(a)A_\mu = \partial_\mu a + [A_\mu, a] \\ \delta_G(a)\lambda = \qquad [\lambda, a] \ , \end{cases}$$

where δ_S, δ_G mean $\delta_{S(WZ)}$, $\delta_{G(WZ)}$ respectively. They satisfy the following algebra.

$$\begin{cases} [\delta_S(\beta) \ , \ \delta_S(\alpha)] = \delta_G(-2i\overline{\alpha}\gamma^\mu\beta A_\mu) + 2(\overline{\alpha}\gamma^\mu\beta)i\partial_\mu \\ [\delta_G(b) \ , \ \delta_G(a)] = \delta_G([b \ , \ a]) \\ [\delta_G(a) \ , \ \delta_S(\alpha)] = 0 \ . \end{cases}$$

An interesting feature of the three-dimensional gauge theory is that there is a gauge invariant topological mass term [16]. In the three-dimensional supersymmetric gauge theory, we have the following supersymmetric topological mass term [17].

$$W = Tr \int d^3x \, \{\lambda_a \gamma^0_{ab} \lambda_b + 2i\varepsilon^{\mu\nu\lambda}(A_\mu F_{\nu\lambda} - \frac{2}{3}A_\mu A_\nu A_\lambda)\} \ .$$

Under $\delta_G(a)$ and $\delta_S(\alpha)$,

$$\begin{cases} \delta_G(a)W = 4i \, Tr \int d^3x \, \varepsilon^{\mu\nu\lambda}\partial_\mu(a\partial_\nu A_\lambda) \\ \delta_S(\alpha)W = -2 \, Tr \int d^3x \, \varepsilon^{\mu\nu\lambda}\partial_\mu(A_\nu(\overline{\alpha}\gamma_\lambda\lambda)) \ . \end{cases}$$

Therefore when we assume that a surface integral is zero, W is invariant under $\delta_G(a)$ and $\delta_S(\alpha)$.

References

[1] B. Zumino, in Relativity, Groups, and Topology II, Les Houches 1983, ed. B.S. DeWitt and R. Stora (North-Holland, 1984)

[2] J. Bell and R. Jackiw, Nuovo Cim. 60A (1969) 47

[3] J. Wess and B. Zumino, Phys. Lett. 37B (1971) 95

[4] H. Georgi and S.L. Glashow, Phys. Rev. D6 (1972) 429

[5] D.J. Gross and R. Jackiw, Phys. Rev. D6 (1972) 477

[6] L. Alvarez-Gaumé and E. Witten, Nucl. Phys. B234 (1984) 269

[7] W.A. Bardeen and B. Zumino, Nucl. Phys. B244 (1984) 421

[8] G. Girardi, R. Grimm and R. Stora, Phys. Lett. 156B (1985) 203

[9] B. Zumino, LBL Preprint LBL-19794 (1985)

[10] P. Howe and P. West, Caltech Preprint CALT-68-1248 (1985)

[11] P. Rossi, Zürich Preprint Print-85-0156 (1985)

[12] H. Itoyama, V.P. Nair and H. Ren, Princeton Preprint (1985)

[13] S. Ferrara, Lett. Nuovo Cim. 13 (1975) 629

[14] T.E. Clark and S.T. Love, Phys. Lett. 138B (1984) 289

[15] D.S. Hwang, LBL Preprint LBL-20134 (1985)

[16] S. Deser, R. Jackiw and S. Templeton, Ann. Phys. (N.Y.) 140 (1982) 372

[17] S.J. Gates, Jr., M.T. Grisaru, M. Roček and W. Siegel, Superspace (Benjamin-Cummings, 1983)

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