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WITH DERIVATIVE COUPLINGS

M.K. Gaillard

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The Effective One-Loop Scalar Lagrangian with Derivative Couplings*

Mary K. Gaillard

Lawrence Berkeley Laboratory
and
Department of Physics
University of California
Berkeley, California 94720, U.S.A.

Abstract

The one-loop effective scalar lagrangian is given in an explicitly field redefinition invariant form for a general model with derivative scalar self-couplings and scalar couplings to spin 1/2 fermions. Applications are given for the non-linear σ-model and the leading N (N = number of chiral multiplets) contribution to the quadratically divergent terms in no-scale supergravity models of the type that may arise from compactification of superstring theories. The formalism is easily generalized to the case of background gauge fields.

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1 Introduction

Effective field theories with non-derivative couplings are playing an increasingly prominent role in elementary particle phenomenology, particularly in the context of $N = 1$ supergravity, where the starting point is the effective tree lagrangian [1,2] for $N = 1$ supergravity coupled to $N = 1$ supersymmetric matter. Phenomenological analyses [3,4] have generally been performed by using this lagrangian to define an effective renormalizable theory near the Planck scale κ^{-1}. One takes the limit $\kappa \to 0$ after appropriate shifts of those fields that acquire vacuum expectation values (vevs) of order or greater than the Planck mass $m_P = \kappa^{-1} = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}$ GeV. The usual renormalization group methods are then used to study [4] the theory at lower energies. This procedure is sufficient as long as loop corrections to the tree potential play no signficiant role near the Planck scale.

An obvious counter-example is the one-loop effective potential at finite temperatures near the Planck scale, $T \sim m_P$, that may play an essential role in determining the initial conditions for a suitable inflationary scenario. It has been shown [5] that the scalar loop contribution to the effective potential in a theory with non-derivative couplings is obtained from the second covariant derivative of the tree potential, where here covariance means with respect to general transformations among scalar fields, as will be seen explicitly in Section 2. This result, first applied to the study of finite temperature potentials [5], was subsequently [6] used to determine the one-loop effective zero-temperature potential in "no-scale" models [7] of $N = 1$ supergravity. As these models have the feature that some vevs are left unspecified at tree level, radiative corrections near the Planck scale can again be important. Indeed it was shown [6] that, in the absence of nonperturbative effects, one loop corrections leave the vacuum unchanged in a class of models obtained [8] from compactification of the zero-slope limit of 10-dimensional superstring theories. On the other hand, for the effective tree potential obtained [9] in the presence of conjectured non-perturbative effects that determine two otherwise undetermined vevs at tree level, one finds [6,10] that at one loop an additional vev is determined, fixing the gravitino mass within a few orders of magnitude of the Planck mass.

The purpose of this paper is to extend the methods of Ref. 5 to the full one-loop effective scalar lagrangian including derivative terms. Here we discuss only contributions from scalar and spin-1/2 fermion loops. A full calculation of the one-loop effective lagrangian for $N = 1$ supergravity requires, of course, the inclusion

of higher spin loops. In Section 2 we give the scalar loop contribution. This is a generalization to the case of non-derivative couplings of previous derivations [11,12] of the effective one-loop scalar lagrangian in renormalizable models. In Section 3 we apply this result to the non-linear σ-model and give the explicit result including terms with two derivatives. The covariant multi-loop expansion for the non-linear σ-model [13], or for more general models posessing a non-linear symmetry [14], has been studied previously. The main difference here is that we include a non-trivial potential (a chiral symmetry breaking "σ-term") and show that the result obtained from the non-linear model is reproduced by an appropriate limit of the linear σ-model.

An additional difference is that we display the effective one-loop lagrangian as an integral over four-momentum of a function of the momentum and of covariant functions of the fields and of their covariant derivatives. In the context of theories with derivative scalar couplings , "covariance" is meant with respect to general transformations among the scalar fields. As discussed in Section 4, the methods presented here are equally applicable to scalar and spin-1/2 fermion loop contributions to the effective lagrangian for higher spin fields, in which case "covariance" is meant with respect to gauge transformations. In addition to assuring an invariant (under field redefinition and/or gauge transformations) result, this explicitly invariant formalism considerably simplifies application to the case of a specific theory.

In Section 4 we give the spin-1/2 fermion-loop contribution to the effective scalar lagrangian in a general model where derivative couplings arise through multiplication of the canonical fermion kinetic energy by a matrix-valued function $Z(\phi)$ of the scalar fields. The immediate purpose of presenting this result is to show that this contribution contains no quadratic divergences, since for a general simply supersymmetric supergravity theory with N chiral multiplets, only spin-1/2 fermions and scalars contribute to the leading N radiative corrections that we discuss in Section 5. As an aside we point out in Section 4 that the explicitly invariant formalism developed here is readily adapted to the evaluation of the fermion (or scalar) loop contribution to effective lagrangians for higher spin fields and evaluate the QED β-function as an example.

Section 6 summarizes the results.

2 The Scalar Loop Contribution

In theories with no derivative couplings, the scalar lagrangian can be written in the form

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - V(\phi) \tag{2.1}$$

where $\phi_i (i = 1, ..., N)$ are N real scalar fields. In this case the one-loop correction to the effective scalar lagrangian is given by

$$\Delta\mathcal{L} = i \ln \det{}^{1/2}\Delta^{-1} = i\frac{1}{2}\,\mathrm{Tr}\ln\Delta^{-1} \tag{2.2}$$

where Δ is the scalar propagator in the presence of a background scalar field, and can be obtained as the second scalar functional derivative of the action [15]:

$$S = \int dx\mathcal{L}, \tag{2.3}$$

$$(\Delta^{-1}(x,y))^{ij} = \frac{\delta^2}{\delta\phi_i(x)\delta\phi_j(y)}S. \tag{2.4}$$

Here we consider a more general scalar lagrangian of the form

$$\mathcal{L} = \frac{1}{2}Z^{ij}(\phi)\partial_\mu\phi_i\partial^\mu\phi_j - V(\phi). \tag{2.5}$$

In this case, the one-loop correction to the effective scalar lagrangian is modified according to [16,5]:

$$\Delta\mathcal{L} = i \ln \det{}^{1/2}Z^{-1}\det{}^{1/2}\Delta^{-1} = \frac{i}{2}\,\mathrm{Tr}\ln(Z^{-1}\Delta^{-1}), \tag{2.6}$$

where in this case the inverse scalar propagator Δ is obtained as the second covariant scalar derivative [13,14,5] of the action:

$$(\Delta^{-1}(x,y))^{ij} = \widetilde{D}^j(y)\widetilde{D}^i(x)S, \tag{2.7}$$

where in (2.7) the covariant functional derivative $\widetilde{D}$ is related to the ordinary functional dervative δ according to:

$$
\begin{aligned}
\widetilde{D}^i(x)S[\phi] &\equiv \frac{\delta S[\phi]}{\delta\phi_i(x)}, \\
\widetilde{D}^i(x)S^j[\phi] &\equiv \frac{\delta S^j[\phi]}{\delta\phi_i(x)} - \Gamma^{ij}_l(\phi(x))S^l[\phi], \\
\widetilde{D}^i S_j[\phi] &\equiv \frac{\delta S_j[\phi]}{\delta\phi_i(x)} + \Gamma^{il}_j(\phi(x))S_l[\phi], \\
&\text{etc.}
\end{aligned} \tag{2.8}
$$

where the Christoffel symbol Γ^{jk}_i is defined in terms of the scalar metric Z^{ij}:

$$
\Gamma^{jk}_i = \Gamma^{kj}_i = Z^{-1}_{im}\Gamma^{mkj}, \tag{2.9}
$$

$$
\Gamma^{mij} = \frac{1}{2}\left(\frac{\partial Z^{mj}}{\partial\phi_i} + \frac{\partial Z^{mi}}{\partial\phi_j} - \frac{\partial Z^{ij}}{\partial\phi_m}\right). \tag{2.10}
$$

(throughout upper and lower indices $i, j...$, indicate, respectively, contravariant $(\frac{\partial}{\partial\phi_i})$ and covariant $(\delta\phi_i)$ quantities under scalar field redefinitions.)

To evaluate the inverse propagator (2.7), we first take the functional derivative of the action to obtain:

$$
\begin{aligned}
-\widetilde{D}^i(x)S[\phi] &= Z^{ik}(\phi(x))[\partial^\mu_x\delta^m_k + \Gamma^{jm}_k(\phi(x))\partial^\mu\phi_j(x)]\partial^\mu\phi_m(x) + \frac{\partial}{\partial\phi_i(x)}V(\phi(x)) \\
&\equiv Z^{il}(\phi(x))d_\mu(x)^k_l\partial^\mu\phi_k(x) + D^iV(\phi(x))
\end{aligned} \tag{2.11}
$$

where D^i is the ordinary covariant derivative:

$$
\begin{aligned}
D^iV(\phi) &= \frac{\partial}{\partial\phi_i}V(\phi), \\
D^iV^j(\phi) &= \frac{\partial}{\partial\phi_i}V^j - \Gamma^{ij}_l(\phi)V^l(\phi), \\
D^iV_j(\phi) &= \frac{\partial}{\partial\phi_i}V_j(\phi) + \Gamma^{il}_j(\phi)V_l(\phi), \\
&\text{etc.,}
\end{aligned} \tag{2.12}
$$

and we have introduced the matrix-valued covariant 4-derivative:

$$
d_\mu(x)^m_k \equiv \partial^{(x)}_\mu\delta^m_k + \Gamma^{jm}_k\partial_\mu\phi_j. \tag{2.13}
$$

We then obtain, after some algebra, the following expression for the inverse propagator (2.7):

$$
\begin{aligned}
\Delta^{-1}(x,y)^{ij} &= D^j(y)D^i(x)S[\phi] = \frac{\delta^2 S[\phi]}{\delta\phi_j(y)\delta\phi_i(x)} - \Gamma^{ij}_l(\phi(y))\frac{\delta S[\phi]}{\delta\phi_l(x)} \\
&= -Z^{ip}(x)\{U^j_p(x) + (d(x)^2)^j_p + R^j_p(x)\}\delta(x-y) \\
&= \Delta^{-1}(y,x)^{ji}
\end{aligned} \tag{2.14}
$$

where it is to be understood that $Z(x)$ and $U(x)$ depend on x only through $\phi(x)$, and

$$
U^j_p = Z^{-1}_{pi}D^iD^jV \tag{2.15}
$$

is the scalar mass matrix [5] as a function of the background field ϕ. The matrix $R(x)$ in (2.14) is defined as

$$
R^j_p(x) = \partial_\mu\phi_r(x)\partial^\mu\phi_q(x)R^{rqj}_p(x), \tag{2.16}
$$

where $R^{rqj}_p(x) = R^{rqj}_p(\phi(x))$ is the curvature tensor:

$$
R^{rqj}_p = -R^{rjq}_p = \frac{\partial}{\partial\phi_j}\Gamma^{rq}_p + \Gamma^{lj}_p\Gamma^{rq}_l - \frac{\partial}{\partial\phi_q}\Gamma^{jr}_p - \Gamma^{ql}_p\Gamma^{jr}_l \equiv Z^{-1}_{pl}R^{lrqj} \tag{2.17}
$$

and vanishes identically if the scalar metric Z^{ij} can be expressed as

$$
Z^{ij} = \frac{\partial\chi_\alpha}{\partial\phi_i}\frac{\partial\chi_\alpha}{\partial\phi_j}. \tag{2.18}
$$

The condition (2.18) simply means that the kinetic energy term in the lagrangian (2.5) can be reduced to canonical form by a suitable field redefinition $\phi_i \to \chi_\alpha(\phi)$:

$$
\partial_\mu\phi_i\partial^\mu\phi_j Z^{ij} = \partial_\mu\chi_\alpha\partial^\mu\chi_\alpha. \tag{2.19}
$$

Returning to the general case $R \neq 0$, the quantity we wish to evaluate is $\mathrm{Tr}\ln A$ where (see Eq. (2.6))

$$
\begin{aligned}
A_i^j(x,y) &= -Z_{ik}^{-1}(x)\Delta^{-1}(x,y)^{kj} \\
&= -\{(d(x)^2)_i^j + U_i^j(x) + R_i^j(x)\}\delta(x-y). \qquad (2.20)
\end{aligned}
$$

We introduce the notation

$$
\gamma_\mu(x)_i^j \equiv \Gamma_i^{kj}[\phi(x)]\partial_\mu\phi_k(x) \qquad (2.21)
$$

and express the matrix-valued covariant 4-derivative as:

$$
d_\mu(x) = \partial_\mu^{(x)} + \gamma_\mu(x). \qquad (2.22)
$$

Introducing a further matrix-valued function of the scalar fields:

$$
H(x) = U(x) + R(x), \qquad (2.23)
$$

we follow Ref. 11 and express A, Eq. (2.20), in terms of its Fourier transform:

$$
\begin{aligned}
-A(x,y) &= \{[\partial_x + \gamma(x)]^2 + H(x)\}\delta(x-y) \\
&= \{[\partial_y - \gamma(y)]^2 + H(y)\}\delta(y-x) \\
&= \int \frac{d^4p}{(2\pi)^4} e^{-ipx}\{[ip_\mu - \gamma_\mu(-\frac{i\partial}{\partial p})]^2 + H(-\frac{i\partial}{\partial p})\}e^{ipy}. \qquad (2.24)
\end{aligned}
$$

Inserting this expression into Eqs. (2.6), (2.20) gives [11]

$$
\begin{aligned}
\mathrm{Tr}\ln A &= \int d^4x \int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\ln \\
&\left\{-\left[ip_\mu - \gamma_\mu\left(x - i\frac{\partial}{\partial p}\right)\right]^2 - H\left(x - i\frac{\partial}{\partial p}\right)\right\} \qquad (2.25) \\
&= -2i\int d^4x\,\mathcal{L}_{eff}^{(1)}(x).
\end{aligned}
$$

The matrix-valued, functions γ_μ and H may be expanded [11] as a Taylor series in $\frac{\partial}{\partial p}$, e.g.:

$$
\begin{aligned}
\overline{H} &\equiv H(x - i\frac{\partial}{\partial p}) = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}[\partial_{\mu_1}...\partial_{\mu_n}H(x)]\frac{\partial^n}{\partial p_{\mu_1}...\partial p_{\mu_n}} \\
&= e^{-i\partial_x\cdot\frac{\partial}{\partial p}}H(x)e^{i\partial_x\cdot\frac{\partial}{\partial p}}. \qquad (2.26)
\end{aligned}
$$

Then following, for example, the methods of Ref. 11, the effective one-loop lagrangian $\mathcal{L}_{eff}^{(1)}$ can be constructed as a function of the scalar fields and their derivatives.

However, it is possible to simplify calculations and cast the result in manifestly covariant form by using the identity:

$$
\int d^4x \int d^4p\,\mathrm{Tr}\,g(i\frac{\partial}{\partial p},\partial_x)F(p,x-i\frac{\partial}{\partial p})g^{-1}(i\frac{\partial}{\partial p},\partial_x) = \int d^4x\int d^4pF(p,x-i\frac{\partial}{\partial p}) \qquad (2.27)
$$

where F and g are matrix-valued functions of their arguments with

$$
g(0,0) = 1. \qquad (2.28)
$$

The identity (2.27) holds because in the expansion of g^{-1} around vanishing arguments the derivatives (which unless otherwise specified operate on everything to their right) give no contribution. Similarly, since by integration by parts the derivatives in the expansion of g can be made to operate to left, they again give no contribution. It follows from the identity (2.27) that

$$
\int d^4x\int d^4p\,\mathrm{Tr}\ln A(p,x-i\frac{\partial}{\partial p}) = \int d^4x\int d^4p\,\mathrm{Tr}\ln B(p,x-i\frac{\partial}{\partial p}) \qquad (2.29)
$$

where

$$
B = g(i\frac{\partial}{\partial p},\partial_x)Ag^{-1}(i\frac{\partial}{\partial p},\partial_x). \qquad (2.30)
$$

Let us now choose

$$
g \equiv e^{-i(\partial_x+\gamma(x))\cdot\frac{\partial}{\partial p}}e^{i\partial_x\cdot\frac{\partial}{\partial p}} = e^{-id(x)\cdot\frac{\partial}{\partial p}}e^{i\partial_x\cdot\frac{\partial}{\partial p}} \qquad (2.31)
$$

which is not precisely of the form $g(i/\partial p, \partial_x)$ since there is an explicit x-dependence through $\gamma(x)$. However, it is easy to see that the identities (2.27) and (2.29) still hold with this choice. We use the further identities:

$$e^{i\frac{\partial}{\partial p}\cdot\partial_x} i p_\mu e^{-i\frac{\partial}{\partial p}\cdot\partial_x} = i p_\mu - \partial_\mu^{(x)} \tag{2.32}$$

and

$$g\gamma_\mu(x - i\frac{\partial}{\partial p})g^{-1} = e^{-id(x)\cdot\frac{\partial}{\partial p}}\gamma_\mu(x)e^{id(x)\cdot\frac{\partial}{\partial p}} \tag{2.33}$$

to write

$$g[ip_\mu - \gamma_\mu(x - i\frac{\partial}{\partial p})]g^{-1} = e^{-id\cdot\frac{\partial}{\partial p}}(ip_\mu - d_\mu)e^{id\cdot\frac{\partial}{\partial p}} = ip_\mu + i\tilde{G}_{\nu\mu}\frac{\partial}{\partial p_\nu} \tag{2.34}$$

where we have introduced the covariant matrix-valued Lorentz tensor

$$\begin{aligned} G_{\mu\nu}(x)_i^j &\equiv [d_\mu(x), d_\nu(x)]_i^j = (\partial_\mu\gamma_\nu(x) - \partial_\nu\gamma_\mu(x) + [\gamma_\mu(x), \gamma_\nu(x)])_i^j \\ &= \partial_\mu\phi_l(x)\partial_\nu\phi_k R_i^{jkl}[\phi(x)], \end{aligned} \tag{2.35}$$

and the last term in (2.34) is obtained by expanding the exponentials to give

$$\tilde{G}_{\nu\mu} = \sum_{n=0}^{\infty}(-i)^n\frac{n+1}{(n+2)!}(D_{\mu_1}...D_{\mu_n}G_{\nu\mu}(x))\frac{\partial^n}{\partial p_{\mu_1}...\partial p_{\mu_n}}, \tag{2.36}$$

where

$$D_\mu \equiv \partial_\mu\phi_i D^i, \tag{2.37}$$

with the covariant derivative D^i defined in Eq. (2.12), is the general covariant 4-derivative understood as operating on an arbitrary tensor-valued function of the scalar fields. In the particular case of a matrix-valued function of the type G_i^j:

$$D_\mu G \equiv [d_\mu, G]. \tag{2.38}$$

Finally we introduce

$$\begin{aligned} \hat{H} &\equiv e^{-id\cdot\frac{\partial}{\partial p}}H(x)e^{id\cdot\frac{\partial}{\partial p}} = g\overline{H}g^{-1} \\ &= \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}(D_{\mu_1}...D_{\mu_n}H(x))\frac{\partial^n}{\partial p_{\mu_1}...\partial p_{\mu_n}} \end{aligned} \tag{2.39}$$

where $\overline{H}$ has been defined in Eq. (2.26).

Collecting the above results, we obtain from Eqs. (2.29-30):

$$B(p, x - i\frac{\partial}{\partial p}) = -\left(ip_\mu + i\tilde{G}_{\nu\mu}\frac{\partial}{\partial p_\nu}\right)^2 - \hat{H} \tag{2.40}$$

and the one-loop effective lagrangian is given from (2.25) as:

$$\mathcal{L}_{eff}^{(1)} = \frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\ln\left[(p_\mu + \tilde{G}_{\nu\mu}\frac{\partial}{\partial p_\nu})^2 - \hat{H}\right]. \tag{2.41}$$

The result (2.41) may be expanded, using methods similar to those of Ref. 11, to arbitrary order in the scalar derivatives. It is manifestly invariant under general transformations among the scalar fields as it depends only on the covariant functions $G_{\mu\nu}, U$ [Eq. (2.15)] and R [Eq. (2.16)]. In the case of zero "curvature" [Eqs. (2.18-19)]

$$G_{\mu\nu} = R = 0, \qquad H = U \tag{2.42}$$

the result is precisely that of Ref. 11 with 4-derivatives of the mass matrix U replaced by covariant 4-derivatives; in this case all derivative couplings in the one-loop contribution (2.41) to the effective lagrangian are finite. As mentioned above this simply reflects the fact that one is using a non-linear formulation of a renormalizable (if U has dimension ≤ 4) scalar field theory.

To evaluate the integral (2.41), we introduce a renormalization scale μ and regularize the integral by making two subtractions:

$$\mathcal{L}_{eff}^{(1)} \to \mathcal{L}_{eff}(0) + \mathcal{L}_{eff}(2\mu^2) - 2\mathcal{L}_{eff}(\mu^2) \equiv \mathcal{L}_{Reg}, \tag{2.43}$$

where

$$\mathcal{L}_{eff}(\mu^2) \equiv \frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\ln(B - \mu^2), \tag{2.44}$$

so that:

$$\mathcal{L}_{Reg} = \frac{i}{2}\mu^2 \int_0^1 d\lambda \int \frac{d^4p}{(2\pi)^4}\left[\frac{1}{B-\lambda\mu^2} - \frac{1}{B-(1+\lambda)\mu^2}\right]$$
$$= -\frac{i}{2}\mu^4 \int_0^1 d\eta \int_0^1 d\lambda \left(\frac{1}{B-\lambda\mu^2-\eta\mu^2}\right)^2 \frac{d^4p}{(2\pi)^4}. \tag{2.45}$$

One reason for this procedure is that we do not wish to lose the quadratically divergent contributions, since in many applications the cut-off μ^2 is physically meaningful; applying dimensional regularization to the once-subtracted integral as in Ref. 11 would eliminate these terms. Using the prescription (2.43) rather than simply cutting off the integral at $\mid p \mid^2 = \mu^2$ has the advantage that decoupling is explicitly satisfied in the sense that (as will be seen below) contributions from massive internal particles ($U \gg \mu^2$) vanish up to scalar field independent terms. This will be particularly important in our discussion of the σ-model. Finally, as expressed in the last term in (2.45) the integral over d^4p is at most logarithmically divergent. It can be evaluated, for example, using dimensional regularization; then the remaining divergence in $1/\epsilon$ is field independent, as are finite corrections to replacements of the form

$$p_\mu p_\nu \to \frac{1}{4} g_{\mu\nu} p^2 \tag{2.46}$$

that we will use below. (Note that in supersymmetric theories all the field-independent terms that we will drop vanish in the supertrace.)

In the applications given below we will consider only terms up to second order in derivatives of the scalar fields. In this case the term in $\hat{G}_{\mu\nu}$ in Eq. (2.41) will not contribute, as can be seen by writing:

$$\frac{1}{-B+\lambda\mu^2} = \frac{1}{-p^2+\hat{H}} + \frac{1}{-p^2+\hat{H}} p^\mu G_{\nu\mu} \frac{\partial}{\partial p_\nu} \frac{1}{-p^2+\hat{H}} + O(\partial^3) \tag{2.47}$$

since

$$\tilde{G}_{\mu\nu} = \frac{1}{2} G_{\mu\nu} + O(\partial^3) \tag{2.48}$$

and $G_{\mu\nu}$ is already of second order in derivatives. The second term in (2.47) vanishes upon symmetric integration when inserted in, *e.g.*, Eq. (2.45), since

$$\frac{\partial}{\partial p_\nu} \frac{1}{-p^2+\hat{H}} = \frac{2p^\nu}{(-p^2+\hat{H})^2}. \tag{2.49}$$

The resulting integral is therefore very similar to the case previously considered for a renormalizable theory, Eqs. (2.42), the only difference being that U is here a matrix, and the integral cannot be completely performed without specifying U since $[U, \partial_\mu U] \neq 0$ in general. For $\mu^2 \gg U$ one obtains in this case (after a Wick rotation in the p_0 plane):

$$\mathcal{L}_{Reg}^{(Renorm)} = -\frac{1}{2(4\pi)^2} \mathrm{Tr}\left\{2\mu^2 U \ln 2 + (\frac{1}{2}U^2 + \frac{1}{6}\partial^2 U)\ln(2U/\mu^2) - \frac{3}{4}U^2\right\}$$
$$-\frac{1}{2}\int_0^1 \mu^2 d\lambda \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}$$
$$\left[\frac{p^2}{(p^2+U+\lambda\mu^2)^4}\partial_\mu U \frac{1}{(p^2+U+\lambda\mu^2)}\partial^\mu U\right] + 0(\partial^4). \tag{2.50}$$

Note that, aside from the coefficients of $\ln\mu^2$, the numerical coefficients in (2.50) are specific to our regularization prescription, Eqs. (2.43-45). In the case of a single scalar field, the remaining integral in (2.50) can be evaluated, giving a finite result, in agreement (up to a total derivative) with Ref. 11, for the second order derivative term.

To generalize the result (2.50) to the case of nonderivative couplings we need only make the replacements

$$U \to H = U + R$$
$$\partial_\mu U \to D_\mu H. \tag{2.51}$$

However, since R [Eq. (2.16)] is second order in derivatives, we need only retain the terms linear in R in those terms of (2.50) that contain no explicit derivatives. We use:

$$\mathrm{Tr}\, F(H) = \mathrm{Tr}\, F(U) + \mathrm{Tr}\, R F'(U) + 0(R^2) \tag{2.52}$$

where F is an arbitrary function of H, to obtain

$$\mathcal{L}_{Reg} = -\frac{1}{2(4\pi)^2}\mathrm{Tr}\left\{2\mu^2(U+R)\ln 2 + (\frac{1}{2}U^2 + RU + \frac{1}{6}D^2U)\ln(2U/\mu^2) - \frac{3}{4}U^2 - UR\right\}$$
$$-\frac{1}{2}\int_0^1 \mu^2 d\lambda \int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\frac{p^2}{(p^2+U+\lambda\mu^2)^4}D_\mu U\frac{1}{(p^2+U+\lambda\mu^2)}D^\mu U\right]$$
$$+O(\partial^4) + O(U/\mu^2). \tag{2.53}$$

Note that the only quadratically divergent derivative term is proportional to the "curvature" R; $R = 0$ for a renormalizable theory.

3 The Non Linear σ-model

In this section the above result will be applied to the non linear σ-model defined by the tree Lagrangian

$$\mathcal{L}_\sigma = \frac{1}{2}\partial_\mu\pi_i\partial^\mu\pi_j Z^{ij} - V(\pi), \quad i = 1, \quad \cdots, \quad N \tag{3.1}$$

with the potential

$$V(\pi) = c\sqrt{F^2 - \pi^2} \equiv -uF^2 \tag{3.2}$$

and the scalar metric ($\pi_i \equiv \pi^i$)

$$\begin{aligned} Z^{ij} &= \delta^{ij} + \frac{\pi^i\pi^j}{F^2 - \pi^2}, \\ Z^{-1}_{ij} &= \delta_{ij} - \frac{\pi_i\pi_j}{F^2}. \end{aligned} \tag{3.3}$$

The curvature R is determined from the definitions (2.16-17) and (2.9-10) as:

$$R^j_i = \frac{1}{F^2}\left(\delta^j_i\partial_\mu\pi_k\partial^\mu\pi_l Z^{kl} - \partial_\mu\pi_k\partial^\mu\pi_i Z^{jk}\right) \tag{3.4}$$

and the scalar mass matrix is

$$U^j_i = Z^{-1}_{ik}D^kD^jV = u\delta^j_i \tag{3.5}$$

where u is the negative of the potential, Eq. (3.2). Since U is proportional to the unit matrix, the last term in (2.53) can be integrated immediately. Using, from (3.4),

$$\mathrm{Tr}\, R = \frac{1}{F^2}(N-1)\partial_\mu\pi_i\partial^\mu\pi_j Z^{ij} \tag{3.6}$$

and

$$D_\mu U = \partial_\mu U + [\gamma_\mu, U] = \partial_\mu U \tag{3.7}$$

we obtain the effective one loop Lagrangian

$$\mathcal{L}^\sigma_{Reg} = -\frac{1}{2(4\pi)^2}\left\{N\left[2\mu^2 u\ln 2 + \frac{1}{2}u^2\ln(2u/e^{3/2}\mu^2) - \frac{1}{12}\frac{\partial_\mu u\partial^\mu u}{u}\right]\right.$$
$$\left.+\frac{N-1}{F^2}\partial^\mu\pi_i\partial^\mu\pi_j Z^{ij}[2\mu^2\ln 2 + u\ln(2u/e\mu^2)]\right\}$$
$$+0(\partial^4) + 0(u/\mu^2) + \text{total derivatives}. \tag{3.8}$$

As expected, the quadratically divergent derivative term is chiral invariant, while soft symmetry breaking $u \neq 0$ generates a logarthmically divergent derivative coupling at one loop.

As a check of the formalism developed in Section 2, we show that the result (3.8) can be obtained as an appropriately taken limit of the linear σ-model. We start from the Lagrangian

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi^i - c\sigma - \frac{g}{4}(\phi^2 - F^2)^2 \\ i &= (0, \cdots, N), \quad \phi_0 = \sigma, \quad \phi_{i>0} = \pi_i. \end{aligned} \tag{3.9}$$

In the limit $g \to \infty$, the field σ is fixed at its ground state value

$$\sigma^2 = F^2 - \pi^2, \tag{3.10}$$

such that the coefficient of g vanishes; substituting (3.10) into the Lagrangian (3.9), one obtains the non-linear model of Eqs. (3.1-3). One expects that for finite but large g, specifically $g\phi^2 \gg \mu^2$, the linear model should mimic the non-linear model

for the effective theory at scales $\phi^2 \lesssim \mu^2$. We show here that this is indeed the case for the one-loop effective Lagrangian when the limit of large g is suitably defined.

Consider first the case of a constant background field: $< \partial_\mu \phi_i >= 0$. Then the effective Lagrangian (3.8) reduces to the effective one-loop potential [17]

$$V_{eff}^{(1)} = \frac{N}{2(4\pi)^2} f(u, \mu^2) \tag{3.11}$$

where u is defined in Eq. (3.2) and [*cf.* Eq. (2.45)]

$$\begin{aligned} \frac{1}{2(4\pi)^2}\, f(m^2, \mu^2) &= -\frac{i}{2}\mu^4 \int_0^1 d\lambda \int_0^1 d\eta \frac{1}{(-p^2 + m^2 + \lambda\mu^2 + \eta\mu^2)^2} \frac{d^4p}{(2\pi)^4} \\ f(m^2, \mu^2) &\xrightarrow{\mu^2 \gg m^2} 2\mu^2 m^2 \ln 2 + \frac{1}{2} m^2 \ln(2m^2/e^{3/2}\mu^2) + \text{const.} \\ f(m^2, \mu^2) &\xrightarrow{m^2 \gg \mu^2} 0 \quad + \quad \text{const.} \end{aligned} \tag{3.12}$$

where "constant" means independent of m^2. For the Lagrangian (3.9) the scalar mass matrix (2.15) is

$$U_i^j = \delta_{ik} \frac{\partial V}{\partial \phi_k \partial \phi_j} = g[\delta_i^j(\phi^2 - F^2) + 2\phi_i \phi^j] \tag{3.13}$$

which has N degenerate eigenvalues

$$m^2 = g(\phi^2 - F^2) \tag{3.14}$$

and a non-degenerate eigenvalue

$$m_0^2 = g(3\phi^2 - F^2). \tag{3.15}$$

The one loop correction to the effective potential is given in this case by:

$$V_{eff}^{(1)} = \frac{1}{2(4\pi)^2} \left\{ N f(m^2, \mu^2) + f(m_0^2, \mu^2) \right\}. \tag{3.16}$$

As we take the limit $g \to \infty$, $\phi^2 \to F^2$, we obtain

$$m_0^2 \to 2gF^2 \to \infty \tag{3.17}$$

so the σ-loop contribution drops out of (3.16) for

$$g \gg \mu^2/2F^2 \tag{3.18}$$

which defines the scale μ below which the non-linear model gives the correct effective theory. However, the pion mass m^2, Eq. (3.14) is undefined in this limit. Therefore we must determine the way in which ϕ^2 varies with g as $g \to \infty$. To do this we make a change of variables

$$\begin{aligned} \sigma &= \rho \cos\theta/F, \quad \pi_i = \rho \tfrac{\theta_i}{\theta} \sin\theta/F \\ \theta^2 &= \Sigma\theta_i^2, \qquad \rho^2 = \phi^2 = \sigma^2 + \pi^2. \end{aligned} \tag{3.19}$$

In this formulation the θ_i are the Goldstone modes. The part of the potential that grows with g contains only ρ; exciting the Goldstone modes costs only a finite (for $c \neq 0$ in (3.9)) amount of energy as $g \to \infty$. We therefore fix ρ at its ground state value with the θ_i free (in other words we introduce sources for the θ_i but not for ρ), giving the condition

$$\begin{aligned} 0 &= \frac{\partial\sigma}{\partial\rho}\frac{\partial V}{\partial\sigma} + \frac{\partial\pi_i}{\partial\rho}\frac{\partial V}{\partial\pi_i} \\ &= \frac{1}{\rho}\left(c\sigma + \phi^2 g(\phi^2 - F^2)\right) \end{aligned} \tag{3.20}$$

which is solved by:

$$c\sigma + \phi^2 g(\phi^2 - F^2) = 0, \tag{3.21}$$

$$\begin{aligned} m^2 &= g(\phi^2 - F^2) = -\frac{c\sigma}{\phi^2} = -\frac{c\sqrt{F^2 - \pi^2}}{F^2} + 0(g^{-1}) \\ &= u + 0(g^{-1}). \end{aligned} \tag{3.22}$$

Substituting this result in (3.16) and taking the limit $m_0^2 \gg \mu^2 \gg m^2$, one recovers the result for the non-linear case, Eq. (3.11).

In the case of a non-constant background field, we minimize the action with respect to the constrained field ρ, with the θ_i and their derivatives as free variables:

$$0 = \frac{\delta S}{\delta \rho} = \frac{\partial \sigma}{\partial \rho}\frac{\partial S}{\partial \sigma} + \frac{\partial \pi_i}{\partial \rho}\frac{\partial S}{\partial \pi_i}$$
$$= g\rho(\rho^2 - F^2) + \frac{c\sigma}{\rho} + \frac{1}{\rho}\phi_i\partial^2\phi_i, \tag{3.23}$$

giving

$$m^2 = g(\rho^2 - F^2) = -\frac{c\sigma}{\rho^2} - \frac{1}{\rho^2}\phi_i\partial^2\phi_i$$
$$= u - \frac{1}{F^2}\phi_i\partial^2\phi_i + 0(g^{-1}); \tag{3.24}$$

we have

$$\phi_i\partial^2\phi_i = \frac{1}{2}\partial^2\rho^2 - \partial_\mu\phi_i\partial^\mu\phi_i$$
$$= Z^{ij}\partial_\mu\pi_i\partial^\mu\pi_j + 0(g^{-1}), \tag{3.25}$$

so finally

$$m^2 = u - \frac{1}{F^2}Z^{ij}\partial_\mu\pi_i\partial^\mu\pi_j + 0(g^{-1}). \tag{3.26}$$

Substituting this result in (3.16), and taking the limits as before, we obtain a contribution

$$\Delta\mathcal{L}^{(1)}_{eff} = -\frac{N}{2(4\pi)^2}\left\{2\mu^2 u\ln 2 + \frac{1}{2}u^2\ln(2u/e^{3/2}\mu^2)\right.$$
$$\left. + \frac{1}{F^2}Z^{ij}\partial_\mu\pi_i\partial^\mu\pi_j[2\mu^2\ln 2 + u\ln(2u/e\mu^2)]\right\}, \tag{3.27}$$

which differs from the result (3.8) for the non-linear σ-model by the expression:

$$\Delta\mathcal{L}' = \frac{1}{2}\frac{1}{(4\pi)^2}\left\{\frac{N}{12}\frac{\partial_\mu u\partial^\mu u}{u} + \frac{1}{F^2}Z^{ij}\partial_\mu\pi_i\partial^\mu\pi_j[2\mu^2\ln 2 + u\ln(2u/e\mu^2)]\right\}. \tag{3.28}$$

However, in order to extract the full effective Lagrangian up to second order in derivatives of π_i, we must include the derivative terms in the expansion [*cf.* Eq.

(2.26)] of $U(x - i\frac{\partial}{\partial p})$; in other words we must go through the steps leading up to (2.50), except that we must take the limit $g \to \infty$ before the limit $\mu^2 \to \infty$.

The term $\mathrm{Tr}[\partial^2 U\ln(2U/\mu^2)]$ was obtained from [*cf.* Eq. (2.45)]:

$$+\frac{1}{2(4\pi)^2}\frac{1}{6}\mathrm{Tr}\left\{\partial^2 U\int_0^1 \mu^2 d\lambda\left(\frac{1}{p^2 + U + \lambda\mu^2} - (\lambda \to \lambda + 1)\right)\right\} \tag{3.29}$$

which may be evaluated by expressing the matrix U, Eq. (3.13), in terms of projection operators:

$$P^j_i = \delta^j_i - \phi_i\phi^j/\phi^2 \quad , \quad K^j_i = \phi_i\phi^j/\phi^2,$$
$$PK = KP = 0\,, \quad P^2 = P, \quad K^2 = K, \quad P + K = 1,$$
$$U = m^2 P + m_0^2 K. \tag{3.30}$$

Then the expression (3.29) takes the form

$$+\frac{1}{2(4\pi)^2}\frac{1}{6}\mathrm{Tr}\left\{\partial^2 U\int_0^1 \mu^2 d\lambda\left[\frac{P}{p^2 + m^2 + \lambda\mu^2} + \frac{K}{p^2 + m_0^2 + \lambda\mu^2} - (\lambda \to \lambda + 1)\right]\right\}$$
$$= -\frac{1}{2(4\pi)^2}\frac{1}{6}\left\{\mathrm{Tr}(\partial^2 UP)\ln\left[\frac{m^2(m^2 + 2\mu^2)}{(m^2 + \mu^2)^2}\right]\right.$$
$$\left. + \mathrm{Tr}(\partial^2 UK)\ln\left[\frac{m_0^2(m_0^2 + 2\mu^2)}{(m_0^2 + \mu^2)^2}\right]\right\} \tag{3.31}$$

with

$$\partial^2 U^j_i = 2g[\delta^j_i\partial_\mu\chi^\mu + \partial^2\phi_i\phi^j + \partial^2\phi^j\phi_i + 2\partial_\mu\phi_i\partial^\mu\phi^j] \tag{3.32}$$

and

$$\mathrm{Tr}(\partial^2 UP) = 2gN\partial_\mu\chi^\mu + 4g\left[\partial_\mu\phi_i\partial^\mu\phi^i - \frac{\chi_\mu\chi^\mu}{\phi^2}\right] \tag{3.33a}$$

$$\mathrm{Tr}(\partial^2 UK) = 2g\left[3\partial^\mu\chi_\mu - 2\partial_\mu\phi_i\partial^\mu\phi^i + 2\frac{\chi_\mu\chi^\mu}{\phi^2}\right] \tag{3.33b}$$

where

$$\chi_\mu \equiv \phi_i \partial_\mu \phi_i = \frac{1}{2}\partial_\mu \phi^2. \tag{3.34}$$

Since we are neglecting terms of higher than second order in field derivatives, we may use the condition (3.22) in taking the limit $g \to \infty$ in the above expressions. First we note that

$$\ln\left[\frac{m_0^2(m_0^2+2\mu^2)}{(m_0^2+\mu^2)^2}\right] \xrightarrow{m_0^2 \gg \mu^2} 0[(\mu^2/m_0^2)^2] = 0(g^{-2}) \tag{3.35}$$

as $g \to \infty$. Since $\mathrm{Tr}(\partial^2 UK)$, Eq. (3.33b) is $0(g)$, the last term in (3.31) gives no contribution in the limit. To extract the divergent and finite terms we have to retain terms of order g^{-1}. Solving Eq. (3.22) to this order gives:

$$\begin{aligned}
\rho^2 &= \phi^2 = F^2 + u/g + 0(g^{-2}) \\
\chi_\mu &= \frac{1}{2}\partial_\mu \rho^2 = \frac{1}{2g}\partial_\mu u + 0(g^{-2}) \\
\sigma &= \sqrt{\rho^2 - \pi^2} = \sqrt{F^2 - \pi^2} - \frac{c}{2gF^2} + 0(g^{-2}) \\
\partial_\mu \sigma &= \partial_\mu \sqrt{F^2 - \pi^2} + 0(g^{-2}) \\
\partial_\mu \phi_i \partial^\mu \phi^i &= Z^{ij}\partial_\mu \pi_i \partial^\mu \pi_j + 0(g^{-2})
\end{aligned} \tag{3.36}$$

Inserting these results in (3.33a) gives for (3.31) the expression:

$$\begin{aligned}
&-\frac{1}{2(4\pi)^2}\frac{1}{6}\left\{N\partial^2 u \ln(2u/\mu^2) + 4g\partial_\mu \pi_i \partial^\mu \pi_j Z^{ij} \ln\left[\frac{m^2(m^2+2\mu^2)}{(m^2+\mu^2)^2}\right]\right\} \\
&+ 0(g^{-1}) + 0(u/\mu^2) + 0(\partial^4).
\end{aligned} \tag{3.37}$$

The last term in (2.50) may be evaluated in a similar way using

$$\partial_\mu U_i^j = 2g[\chi_\mu (K+P)_i^j + \partial_\mu \phi_i \phi^j + \phi_i \partial_\mu \phi^j]. \tag{3.38}$$

One finds, after some algebra, the expression:

$$\begin{aligned}
&-\frac{4g^2}{2}\int_0^1 \mu^2 d\lambda \int \frac{d^4p}{(2\pi)^4} p^2 \left\{\chi_\mu \chi^\mu \left[\frac{N}{(p^2+m^2+\lambda\mu^2)^5} + \frac{9}{(p^2+m_0^2+\lambda\mu^2)^5}\right]\right. \\
&\left. +(\phi^2 \partial_\mu \phi_i \partial^\mu \phi_i - \chi_\mu \chi^\mu)\left[\frac{1}{(p^2+m^2+\lambda\mu^2)^4}\frac{1}{(p^2+m_0^2+\lambda\mu^2)} + (m^2 \leftrightarrow m_0^2)\right] - (\lambda \to \lambda+1)\right\} \\
&\xrightarrow{g\to\infty} -\frac{1}{2}\int_0^1 \mu^2 d\lambda \int \frac{d^4p}{(2\pi)^4} p^2 \left\{\frac{N\partial_\mu u \partial^\mu u}{(p^2+u+\lambda\mu^2)^5} + \frac{\partial_\mu \pi_i \partial^\mu \pi_j Z^{ij}}{F^2} \times\right. \\
&\left.\left(\frac{2gF^2}{(p^2+u+\lambda\mu^2)^4} - \frac{1}{(p^2+u+\lambda\mu^2)^3}\right) - (\lambda \to \lambda+1)\right\} + 0(g^{-1}) \\
&= -\frac{1}{2(4\pi)^2}\left\{\frac{1}{12}N\partial_\mu u \partial^\mu u/u - \frac{\partial_\mu \pi_i \partial^\mu \pi_j Z^{ij}}{F^2} \times\right. \\
&\left.\left[\frac{2gF^2}{3}\ln\frac{m^2(m^2+2\mu^2)}{(m^2+\mu^2)^2} + 2\mu^2 \ln 2 + u\ln(2u/e\mu^2)\right]\right\} + 0(g^{-1}) + 0(u/\mu^2).
\end{aligned} \tag{3.39}$$

One sees that the divergent term as $g \to \infty$ cancels in the sum of (3.37) and (3.39), and that their contribution gives precisely (3.28) up to a total derivative:

$$\partial^2 u \ln(2u/\mu^2) = -\partial_\mu u \partial^\mu u/u + \partial_\mu[\partial^\mu u \ln(2u/\mu^2)]. \tag{3.40}$$

4 The Fermion Loop Contribution

We consider a (Dirac) fermion Lagrangian of the form:

$$\mathcal{L} = -\overline{\psi}(-iZ(\phi)\partial\!\!\!/ + B(\phi))\psi \tag{4.1}$$

where $Z(\phi)$ is a scalar field dependent matrix in the space of internal fermion quantum numbers, and $B(\phi)$ is a field dependent matrix in both internal and Dirac spaces.

As before, the inverse propagator is given by the second derivative of the action:

$$S = \int d^4x \mathcal{L} = -\int d^4x \overline{\psi}\left(-iZ(\phi)\partial\!\!\!/ + B(\phi)\right)\psi \tag{4.2}$$

$$\frac{\delta S}{\delta \overline{\psi}(x)^B} = -(-iZ(x)\partial\!\!\!/_x - B(x))^A_B \psi_A \tag{4.3}$$

giving:

$$\Delta_B^{-1A}(x,y) = \frac{\delta^2 S}{\delta \psi_A(y)\delta\overline{\psi}^B(x)} = -(-iZ(x)\partial\!\!\!/_x + B(x))^A_B \delta(x-y)$$

$$\equiv -[Z(x)(-i\partial\!\!\!/_x + C(x))]^A_B\,\delta(x-y),$$

$$C \equiv Z^{-1}B. \tag{4.4}$$

For Dirac fermion loops the one-loop contribution to the effective Lagrangian is:

$$\int d^4x\mathcal{L}_1 = -i\,\mathrm{Tr}\ln Z^{-1}\Delta^{-1} = -i\,\mathrm{Tr}\ln(i\partial\!\!\!/ - C). \tag{4.5}$$

As in Section 2 we write:

$$(-i\partial_x + C(x))\delta(x-y) = (i\partial_y + C(y))\delta(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}(-p\!\!\!/ + C(-i\frac{\partial}{\partial p}))e^{ipy} \tag{4.6}$$

and obtain:

$$\int d^4x\mathcal{L}_1(x) = -i\int d^4x \int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\ln\left(p\!\!\!/ - C(x - i\frac{\partial}{\partial p})\right). \tag{4.7}$$

In the following we take the field-dependent matrix C of the form:

$$C(x) \equiv C(\phi(x)) = A\!\!\!/(x) + M(x). \tag{4.8}$$

We use the identities

$$\mathrm{Tr}\ln AB = \mathrm{Tr}\ln A + \mathrm{Tr}\ln B$$

$$\mathrm{Tr}\,F(\gamma_\mu) = \mathrm{Tr}\,F(-\gamma_\mu) \tag{4.9}$$

where γ_μ is a Dirac matrix, to write

$$\mathrm{Tr}\ln(p\!\!\!/ - \overline{A\!\!\!/} - \overline{M}) = \frac{1}{2}\mathrm{Tr}\ln(-(p\!\!\!/ - \overline{A\!\!\!/})^2 + \overline{M}^2 + [p\!\!\!/ - \overline{A\!\!\!/}, \overline{M}]), \tag{4.10}$$

where we have defined

$$\overline{A}_\mu = A_\mu(x - i\frac{\partial}{\partial p}) = e^{-i\partial_x\cdot\frac{\partial}{\partial p}}A_\mu(x)e^{i\partial_x\cdot\frac{\partial}{\partial p}} \tag{4.11}$$

and similarly for $\overline{M} = M(x - i\frac{\partial}{\partial p})$. Next we note that

$$\begin{aligned}
[p\!\!\!/, \overline{M}] &= i\overline{\partial\!\!\!/ M} \equiv e^{-i\partial_x\cdot\frac{\partial}{\partial p}}\partial\!\!\!/ M(x)e^{i\partial_x\cdot\frac{\partial}{\partial p}} \\
[p\!\!\!/ - \overline{A\!\!\!/}, \overline{M}] &= i(\overline{\partial\!\!\!/ M} + i[\overline{A\!\!\!/}, \overline{M}]) \\
&\equiv i\overline{D\!\!\!/_\mu M}
\end{aligned} \tag{4.12}$$

where we define D_μ as a covariant four-derivative operator; for a matrix valued operator $M(x)$:

$$\begin{aligned}
D_\mu(x)M(x) &= [d_\mu(x), M(x)] \\
d_\mu &= \partial_\mu + iA_\mu.
\end{aligned} \tag{4.13}$$

Finally, we follow the discussions of Sect. 2, Eqs. (2.27-41), to write the effective one loop Lagrangian in the manifestly invariant form:

$$\mathcal{L}_1^F = -\frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\ln(-(p\!\!\!/ + \tilde{G\!\!\!/})^2 + \hat{M}^2 + (\hat{D\!\!\!/M})) \tag{4.14}$$

where

$$\begin{aligned}
\hat{M} &= e^{-id\cdot\frac{\partial}{\partial p}}M(x)e^{id\cdot\frac{\partial}{\partial p}} = g\overline{M}g^{-1} \\
&= \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}(D_{\mu_1}\cdots D_{\mu_n}M)\frac{\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}}
\end{aligned} \tag{4.15}$$

and similarly for $(\hat{D\!\!\!/M})$, and

$$\begin{aligned}
\tilde{G}_\mu &= \tilde{G}_{\nu\mu}\frac{\partial}{\partial p_\nu} \\
\tilde{G}_{\mu\nu} &= \sum_{n=0}^{\infty}(-i)^n\frac{n+1}{(n+2)!}(D_{\mu_1}\cdots D_{\mu_n}G_{\mu\nu})\frac{\partial^n}{\partial_{p_{\mu_1}}\cdots\partial_{p_{\mu_n}}} \\
G_{\mu\nu} &= [d_\mu, d_\nu].
\end{aligned} \tag{4.16}$$

As before the expression (4.14) depends only on the covariant quantities M and $G_{\mu\nu}$ and their covariant derivatives. We further explicate the expression (4.14) by writing:

$$(p\!\!\!/ + \tilde{G\!\!\!/})^2 = p^2 + \tilde{G}^2 + \{p\!\!\!/, \tilde{G\!\!\!/}\} \tag{4.17}$$

and

$$\{p\!\!\!/, \tilde{G\!\!\!/}\} = 2p^\mu\tilde{G}_\mu + [\tilde{G}^\mu, p_\mu] + i\sigma^{\nu\mu}[\tilde{G}_\nu, p_\mu]. \tag{4.18}$$

From the definition (4.16) of $\tilde{G}_\mu$ we obtain

$$[\tilde{G}_\nu, p_\mu] = \tilde{G}_{\mu\nu} - ig_{\mu\nu} \tag{4.19}$$

where we define

$$\begin{aligned}
g_{\mu\nu} = &\sum_{n=0}^{\infty}(-i)^n\frac{(n+1)(n+2)}{(n+3)!} \\
&\times (D_{\mu_1}\cdots D_{\mu_n})D_\mu G_{\sigma\nu}\frac{\partial}{\partial p_\sigma}\frac{\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}}.
\end{aligned} \tag{4.20}$$

Substituting these relations in (4.14) we obtain our final result for the fermion loop contribution to the effective Lagrangian:

$$\mathcal{L}_1^F = -\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\ln\left(-p^2 - \tilde{G}^2 - 2p^\mu\tilde{G}_\mu + ig^\mu_\mu - i\sigma^{\mu\nu}(\tilde{G}_{\mu\nu} - ig_{\mu\nu}) + \hat{M}^2 + \hat{D\!\!\!/M}\right) \tag{4.21}$$

with $\tilde{G}_\mu$, $\tilde{G}_{\mu\nu}$ and $g_{\mu\nu}$ defined by Eqs. (4.16) and (4.20). While the expression (4.21) may at first sight appear somewhat cumbersome, it should become immediately clear in the application to the QED β-function, to be given below, that it permits considerable simplification of calculations with respect to a straightforward expansion of (4.7) or (4.10) that involve the non-covariant field operator $A_\mu(x)$, producing a number of terms that must cancel to give the necessarily invariant result.

It is clear that the above discussion holds with $C(x)$ in Eq. (4.8) as any function of background bose fields. As a first application to provide a check of the invariant formalism, we consider the case of a single Dirac field with the Lagrangian (4.1) taken to be the fermion part of the QED Lagrangian.

$$\begin{aligned} Z(\phi) &= 1 \\ B(\phi) &= C(\phi) = e\not{A} + m. \end{aligned} \tag{4.22}$$

In other words in Eq. (4.8) $A_\mu(\phi) \to eA_\mu$, is the background electromagnetic field and $M(\phi) \to m$ is the fermion mass. Then

$$\begin{aligned} D_\mu M &= 0 \\ G_{\mu\nu} &= ieF_{\mu\nu} \\ F_{\mu\nu} &\doteq \partial_\mu A_\nu - \partial_\nu A_\mu. \end{aligned} \tag{4.23}$$

We will use the above formalism to calculate the QED β-function. This is determined by the one-loop fermion contribution to the renormalization of the photon kinetic energy term:

$$\mathcal{L}_\gamma^{K.E.} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{4.24}$$

so that upon substitution of (4.22-23) in the effective one-loop Lagrangian (4.21), we need only retain terms up to second order in the electromagnetic field strength $F_{\mu\nu}$, and we may drop derivatives of F. From the definitions (4.16) and (4.20):

$$\begin{aligned} \tilde{G}_\mu &\to \frac{ie}{2}F_{\nu\mu}\frac{\partial}{\partial p_\nu} \\ \tilde{G}_{\mu\nu} &\to \frac{ie}{2}F_{\mu\nu} \\ g_{\mu\nu} &\to 0 \end{aligned} \tag{4.25}$$

and the effective Lagrangian (4.21) reduces to

$$\mathcal{L}_1^F = -\frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\ln\left(-p^2 - iep^\mu F_{\nu\mu}\frac{\partial}{\partial p_\nu} + \frac{e^2}{4}F^{\mu\nu}F_{\mu\rho}\frac{\partial^2}{\partial p_\nu \partial p_\rho} - \frac{e}{2}\sigma_{\mu\nu}F^{\mu\nu} + m^2\right)$$

$$\equiv -\frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\ln \not{A}. \tag{4.26}$$

The β-function can be extracted from the once-subtracted result:

$$\begin{aligned} \mathcal{L}_1^{Reg} &= \mathcal{L}_1^F(0) - \mathcal{L}_1^F(\mu^2) = \mu^2\frac{i}{2}\int_0^1 d\lambda \int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left(\frac{1}{\not{A} + \lambda\mu^2}\right), \\ \mathcal{L}(\mu^2) &= \mathcal{L}(\not{A} + \mu^2). \end{aligned} \tag{4.27}$$

Because of the antisymmetry of $F_{\mu\nu}$, the only terms in (4.26) that survive symmetric integration over d^4p and the Dirac trace in the expansion of (4.27) are

$$\begin{aligned} \mathcal{L}_1^{Reg} &= \mu^2\frac{i}{2}\int_0^1 d\lambda \int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left\{\frac{1}{-p^2+\overline{m}^2} - \frac{1}{-p^2+\overline{m}^2}\frac{e^2}{4}F^{\mu\nu}F_{\mu\rho}\frac{\partial^2}{\partial p_\nu \partial p_\mu}\frac{1}{-p^2+\overline{m}^2}\right. \\ &\left. + \frac{e^2}{4}\frac{1}{-p^2+\overline{m}^2}\sigma_{\mu\nu}F^{\mu\nu}\frac{1}{-p^2+\overline{m}^2}\sigma_{\rho\sigma}F^{\rho\sigma}\frac{1}{-p^2+\overline{m}^2}\right\} + 0(F^4) \\ &+ \text{ derivatives}, \end{aligned} \tag{4.28}$$

where

$$\overline{m}^2 = m^2 + \lambda\mu^2.$$

The first term in (4.28) is the usual Coleman-Weinberg [17] contribution. After a Wick rotation in the p_0 plane, the second term in (4.28) gives a contribution

$$\begin{aligned} & e^2F_{\mu\nu}F^{\mu\nu}\int_0^1 \mu^2 d\lambda \int \frac{d^4p}{(2\pi)^4}\frac{(m^2+\lambda\mu^2)}{(p^2+m^2+\lambda\mu^2)^4} \\ &= \frac{e^2}{6(4\pi)^2}F_{\mu\nu}F^{\mu\nu}\ln\mu^2/m^2 \qquad + 0(m^2/\mu^2), \end{aligned} \tag{4.29}$$

and the third term gives

$$\begin{aligned} & -e^2F_{\mu\nu}F^{\mu\nu}\int_0^1 \mu^2 d\lambda \int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2+m^2+\lambda\mu^2)^3} \\ &= -\frac{e^2}{2(4\pi)^2}F_{\mu\nu}F^{\mu\nu}\ln\mu^2/m^2, \end{aligned} \tag{4.30}$$

and

$$\mathcal{L}_1^{Reg} = -\frac{e^2}{3(4\pi)^2}F_{\mu\nu}F^{\mu\nu}\ln\mu^2/m^2. \tag{4.31}$$

The A_μ-dependent part of the full effective one loop Lagrangian is therefore

$$\begin{aligned} \mathcal{L}_{eff}(A) &= \mathcal{L}_{tree}(A,\mu) + \mathcal{L}_1^{Reg} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\left(1 + \frac{4e^2}{3(4\pi)^2}\ln\mu^2/m^2\right) - e(\mu)\overline{\psi}\not{A}\psi \\ &= -\frac{1}{4}F^R_{\mu\nu}F^{\mu\nu}_R - e(\mu)\left(1 - \frac{2\alpha}{3(4\pi)}\ln\mu^2/m^2\right)\overline{\psi}\not{A}_R\psi, \end{aligned} \tag{4.32}$$

where $A_\mu^R = A_\mu(1+\frac{2\alpha}{3(4\pi)}\ln\mu^2/m^2)$ is the renormalized photon field. Renormalization group invariance:

$$\partial\mathcal{L}_{eff}(A)/\partial\ln\mu = 0$$

gives the standard result:

$$\frac{1}{e}\beta_F = \frac{1}{\alpha}\frac{\partial\alpha(\mu)}{\partial\ln\mu^2} = \frac{2\partial e}{e\partial\ln\mu^2} = \frac{\alpha}{3\pi}. \tag{4.33}$$

Note that for a scalar loop, there is only a contribution from the second term in (4.28), giving the result (4.29). Comparing this with (4.31) and taking into account a relative weight factor - 1/2 for a complex scalar loop relative to a fermion loop, we immediately obtain

$$\beta_S = \frac{1}{4}\beta_F. \tag{4.34}$$

Returning now to the fermion loop contribution to the effective scalar Lagrangian, we make the identifications:

$$\begin{aligned} M &= M(\phi) \\ A_\mu &= -i\partial_\mu\phi_i F^i(\phi) \end{aligned} \tag{4.35}$$

for the field-dependent matrices defined in Eq. (4.8). In the following we retain terms only up to second order in derivatives of the scalar fields. Then since $G_{\mu\nu} = 0(\partial^2)$, expansion of (4.27) with a $\mathcal{A}$ defined by (4.21) gives the non-vanishing terms (after Wick rotation)

$$\begin{aligned} \mathcal{L}_1^{Reg} = & -2\int_0^1 \mu^2 d\lambda \int \frac{d^4p}{(2\pi)^4}\left\{\overline{\mathrm{Tr}}\left(\frac{1}{p^2+\hat{M}^2(\phi)+\lambda\mu^2}\right)\right. \\ & + \overline{\mathrm{Tr}}\left(\frac{1}{p^2+M^2(\phi)+\lambda\mu^2}D_\mu M(\phi)\frac{1}{p^2+M^2(\phi)+\lambda\mu^2}\times\right. \\ & \left.\left. D^\mu M(\phi)\frac{1}{p^2+M^2(\phi)+\lambda\mu^2}\right)\right\} \end{aligned} \tag{4.36}$$

where $\overline{\mathrm{Tr}}$ implies a trace over internal indices only. The first term in the integrand of Eq. (4.36) is the same as in the case without derivative couplings, but with $\overline{M}(\phi)$ (see Eq. (4.11)) replaced by $\hat{M}(\phi)$, Eq. (4.15). The expansion of this term is identical (after a second subtraction and with the Dirac fermion weight factor -4) to the expression (2.50) with the replacements

$$\begin{aligned} U(\phi) &\to M^2(\phi) \\ \partial_\mu U(\phi) &\to D_\mu M^2(\phi) = \partial_\mu M^2(\phi) + \left[F^i(\phi)\partial_\mu\phi_i, M^2(\phi)\right]. \end{aligned} \tag{4.37}$$

Note that the derivative terms are at most logarithmically divergent, as is also the case for the second term in (4.36) since it contains two extra propagators relative to the quartically divergent expression (4.14).

5 Locally Supersymmetric Models

Locally supersymmetric models are characterized by N complex scalar fields

$$z_i = (z_{\bar{i}})^\dagger \tag{5.1}$$

with the scalar Lagrangian

$$\mathcal{L}(z) = \mathcal{G}^{i\bar{j}}\partial_\mu z_i \partial^\mu z_{\bar{j}} + V(z, z^\dagger) \tag{5.2}$$

where

$$\mathcal{G}^{i\bar{j}} = \frac{\partial^2\mathcal{G}(z,z^\dagger)}{\partial z_i \partial z_{\bar{j}}} \tag{5.3}$$

is the Kähler metric with the Kähler potential $\mathcal{G}(z,z^\dagger)$ a real function of the scalar fields. In this section we wish to determine only the quadratically divergent contribution which contains at most two derivatives of the scalar fields. The effective Lagrangian up to terms with two derivatives is given in Eq. (2.53); the quadratically divergent part is

$$\mathcal{L}_{eff}^{quad} = -\frac{1}{2(4\pi)^2}\eta\mu^2\,\mathrm{Tr}(U+R) \tag{5.4}$$

where η is a prescription dependent parameter; $\eta = 2\ln 2$ for the double subtraction procedure used in Sect. 2. $\mathrm{Tr}\,U$, Eq. (2.15), determines [5,6] the effective Coleman-Weinberg potential [17] and $\mathrm{Tr}\,R$, Eq. (2.16), contains the derivative terms. With the scalar metric defined by (5.2), the only non-vanishing elements of the scalar metric Z and the Christoffel symbol Γ are:

$$\begin{aligned} Z^{i\bar{j}} &= Z^{\bar{j}i} = \mathcal{G}^{i\bar{j}}, \\ \Gamma^{i\bar{j}\bar{k}} &= (\Gamma^{\bar{i}jk})^* = \frac{\partial^3\mathcal{G}}{\partial z_i \partial z_{\bar{j}}\partial z_{\bar{k}}} \\ \Gamma_{\bar{i}}^{\bar{j}\bar{k}} &= \left(\Gamma_i^{jk}\right)^* = Z_{\bar{i}l}^{-1}\Gamma^{l\bar{j}\bar{k}}. \end{aligned} \tag{5.5}$$

Then the matrix valued function R, Eq. (2.16) has non zero elements determined by:

$$R_i^{jk\bar{m}} = -R_i^{j\bar{m}k} = \left(R_{\bar{i}}^{\bar{j}\bar{k}m}\right)^* = -\left(R_{\bar{i}}^{\bar{j}m\bar{k}}\right)^* = \frac{\partial}{\partial z_{\bar{m}}}\Gamma_i^{jk} \tag{5.6}$$

so that

$$\mathrm{Tr}\, R = -\frac{\partial}{\partial z_{\overline{m}}}\Gamma_i^{ki}\partial_\mu z_k \partial^\mu z_{\overline{m}} + \quad \text{h.c.} \quad . \tag{5.7}$$

We now focus specifically on a class of models [7-9] with a Kähler potential of the form:

$$\mathcal{G}(z, z^\dagger) = -\ln(s+s^*) - 3\ln(t+t^* - k\,|\phi|^2) = \ln|W(z)|^2 \tag{5.8}$$

where $|\phi|^2 = \Sigma_i \phi_i \phi_i^\dagger$ and s, t and the ϕ_i, $i = 1, \cdots, N$ are the $N+2$ complex scalar fields of the theory with s and t gauge singlets. Using the Kähler potential (5.8), we evaluate (5.7) to obtain:

$$\begin{aligned}\mathrm{Tr}\, R &= -\frac{4}{(s+s^*)}\partial_\mu s \partial^\mu s^* - \frac{2}{3}(N+1)\partial_\mu z_a \partial^\mu z_{\overline{b}} \mathcal{G}^{a\overline{b}} \\ &= -\frac{2}{3}N\partial_\mu z_a \partial^\mu z_{\overline{b}} \mathcal{G}^{a\overline{b}} \left\{1 + 0\left(\frac{1}{N}\right)\right\}\end{aligned} \tag{5.9}$$

where a, $b = 0, \cdots, N$, $z_0 = t$, $z_{i>0} = \phi_i$. As discussed in the preceeding section, fermion loops give no quadratically divergent contribution to the derivative terms. Since contributions proportional to N can arise only from scalars or their chiral fermion partners in internal loops, Eq. (5.9) completely determines the leading N quadratically divergent derivative term. Combining this with the previous determination [6] of $\mathrm{Tr}\, U$, we obtain

$$\begin{aligned}\mathcal{L}_{eff}^{(1,N)} = \frac{\partial_\mu s \partial^\mu s^*}{(s+s^*)^2} &+ (1+\epsilon)\partial z_a \partial z_{\overline{b}} \mathcal{G}^{a\overline{b}} - (1+2\epsilon)(\hat{V} + \mathcal{D}) \\ &- (1+3\epsilon)U\end{aligned} \tag{5.10}$$

where

$$\begin{aligned}\epsilon &= \frac{N}{3(4\pi)^2}\eta\mu^2 \\ \hat{V} &= \frac{1}{3k}e^{2G/3}\frac{\overline{W}_i W^i}{(s+s^*)} \quad , \quad W^i = \frac{\partial W}{\partial \phi_i} = \left(\overline{W}_i\right)^* \\ \mathcal{D} &\ \frac{g^2}{2f}D^\alpha D^\alpha \quad , \quad D^\alpha = 3ke^{G/3}\overline{\phi}^i T_i^{\alpha j}\phi_j \\ U &= e^{\mathcal{G}}(s+s^*)\left|\frac{\partial \mathcal{G}}{\partial s}\right|^2 \quad .\end{aligned} \tag{5.11}$$

In writing (5.11) we used the notation

$$\begin{aligned}\overline{\phi}^i &\equiv \phi_{\overline{i}} = (\phi_i)^\dagger \\ G &= -3\ln(t+t^* - k\,|\phi|^2)\end{aligned} \tag{5.12}$$

and we have assumed a superpotential $W(z)$ of the form

$$W(z) = W(\phi_i) + W(s), \tag{5.13}$$

in which case the tree potential is of the form (5.10) with $\epsilon = 0$. We see that the vacuum structure is unchanged by the leading N quadratically divergent radiative corrections.

6 Summary and Conclusions

We have presented expressions for the scalar- and fermion-loop contributions to the effective one loop scalar lagrangian that are explicitly invariant with respect to a redefinition of the scalar field variables. Our results, for the general class of models where derivative couplings arise through multiplication of canonical kinetic energy terms by a matrix-valued function of the scalar fields are given in Eqs. (2.41) and (4.21) for scalar and fermion loops, respectively. We have expanded these expressions up to terms of second order in the derivatives of scalar fields, to obtain more explicit expressions for the effective lagrangian in this order as displayed in Eqs. (2.53) and (4.36), respectively. As discussed in Section 4, the formalism is readily adapted to the case of a background gauge field, in which case the results are expressed in terms of the gauge covariant field strength and its gauge covariant derivatives.

In Section 3 we applied the formalism to calculate the one loop effective lagrangian of the non-linear σ-model including terms of second order in derivatives of the pion fields, and showed that the same result can be obtained as a suitably defined limit of the one loop effective lagrangian of the linear σ-model.

Finally, in Section 5 we applied the formalism to determine the leading N, where N is the number of chiral multiplets, contribution to the one-loop effective scalar lagrangian of "no-scale" locally supersymmetric models. A full treatment of this problem will require a generalization of the formalism to include higher spin loops.

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