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HALL CONDUCTIVITY AND LORENTZ FORCE LAW FOR TWO-BAND SEMICONDUCTORS

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ABSTRACT

The intraband and interband contributions to the Hall conductivity are calculated for a two-band semiconductor. The interband contributions to the conductivity include a residual term at frequencies less than the band gap. The residual conductivity can be related to a spin-orbit component of the Lorentz force law, analogous to the relativistic case for electrons and positrons. The conductivity is shown to satisfy the proper sum rule.

Introduction

The free-carrier Hall effect and its related high frequency counterpart, free-carrier Faraday rotation, are regularly used in semiconductors to determine free-carrier concentrations and effective masses. The Hall effect is particularly useful since it normally provides a direct measure of the free-carrier concentration provided that the magnetic field is sufficiently large to satisfy the condition $\omega_c\tau >> 1$, where ω_c is the cyclotron frequency and τ the appropriate lifetime. This "high-field limit" has been demonstrated experimentally and theoretically to give correct results over an extremely wide range of physical conditions, provided the carriers are able to execute closed orbits as would be the case for closed and simply connected Fermi surfaces.[1] The theoretical treatments of the free-carrier Hall effect normally do not include relativistic corrections such as spin-orbit corrections

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to the velocity. In most cases such effects are negligible. However, in small gap semiconductors the spin-orbit correction to the Lorentz force law can be appreciable.

A direct calculation of the Hall conductivity $\sigma_{xy}(\omega)$ in a simple two-band model shows that interband terms are present which arise from the unequal blockage of left-circularly-polarized (lcp) and right-circularly- polarized (rcp) transitions by the free-carrier population. This is the magnetic analog of the Burstein-Moss effect.[2] The use of a simple two-band model allows these terms to be evaluated directly and to show that the residual conductivity at low frequencies can be described phenomenologically by a spin-orbit term in the Lorentz force law.

The relativistic band model and quantum-mechanical calculation of the Hall conductivity are discussed in Section I. A semiclassical calculation is given in Section II together with a discussion of the phenomenology. Relevant experimental results for the Hall constant, Faraday rotation and magneto-plasma dielectric anomalies are discussed in Section III.

I. Hall Conductivity: Quantum-Mechanical

The two-band model is illustrated in figure 1. In this model, the one-electron energy levels in the presence of magnetic field can be calculated by using a $\vec{k}\cdot\vec{p}$ formulation based on the Luttinger-Kohn representation.[3] The momentum operator in this representation is of the form:

$$\vec{\pi} = \vec{p} + \frac{1}{4m_0c^2}\vec{\sigma}\times\vec{\nabla}V + \frac{e\vec{A}}{c} \qquad 1)$$

where the spin-orbit interaction is included in the second term and the vector potential $\vec{A}$ includes both the static magnetic field and the radiation field. For the purposes of the calculations that follow, the magnetic susceptibility is taken to be unity and only electric-dipole transitions are considered. Errors introduced by these approximations are neglible for narrow gap semiconductors.

The eigenvalues for relativistic electrons in the presence of a static magnetic field have been derived by Johnson and Lippmann[4] using a cylindrical gauge. This gauge is particularly useful in deriving selection rules for transitions between Landau levels.

For relativistic electrons, the g-factor is related to the effective mass by $g=2m_0/m$ where m_0 is the "free-space" mass of the electron, g is the energy dependent g-factor and m the energy dependent mass. This equivalence leads to spin-splittings in a magnetic field that are equal to the cyclotron frequency for all values of kinetic energy. Therefore, the lowest level in the conduction band Landau ladder is a non-degenerate "spin-down" level and all higher levels are doubly degenerate. The electronic states in the "valence" band are a mirror image of the "conduction" band levels. Cohen and Blount[5] have shown that the same energy level system is applicable in periodic solids provided the free-space electron effective mass and g-factor are replaced by an appropriate effective mass m^* and effective g-factor g^* and provided that the contributions from the free-space electronic masses and g-factors can be neglected. The neglect of the free-space electronic mass and g-factor is a good approximation in narrow-gap semiconductors.

In the one-electron effective mass approximation the conduction band energy levels are given by:[6]

$$\mathcal{E}_c(n, k_z) = \frac{\mathcal{E}_g}{2} + (2n+1\pm1)\frac{\hbar\omega_c}{2} + \frac{\hbar^2 k_z^2}{2} \qquad 2)$$

where $\mathcal{E}_g$ is the energy gap, $\omega_c = eH/m^*c$ is the cyclotron frequency, m^* is the energy dependent effective mass, n is the Landau-index and k_z is the wavevector for motion along the z axis. The magnetic field $\vec{H}$ is also along the z axis. The valence levels are the negative of the conduction levels. The ±1 refers to the "spin-up", "spin-down" splitting of a given Landau level of index n. This splitting is equal to the cyclotron frequency in the two-band model.

The labels α,ß are used for the "spin-up" and "spin-down" eigenvalues of the spin operator since these states are spin-orbit mixtures of the spin-up and spin-down components of the spin angular momentum. The α and ß satisfy time reversal symmetry in the absence of a magnetic field (i.e., all states are two-fold degenerate) and therefore satisfy the rule for Kramer's conjugation.[3]

The frequency dependent Hall conductivity $\sigma_{xy}(\omega)$ has both intraband (cyclotron resonance) and interband contributions. The real part of $\sigma_{xy}(\omega)$ can be written:[7]

$$\sigma_{xy}^{(1)}(\omega) = \frac{M}{V}\frac{e^2}{\hbar m_0^2}\sum_{i<f}\left\{\frac{|\langle f|\Pi_-|i\rangle|^2}{(\omega_{fi}^2-\omega^2)} - \frac{|\langle f|\Pi_+|i\rangle|^2}{(\omega_{fi}^2-\omega^2)}\right\}F_{if}(\mathcal{E},T) \qquad 3)$$

where M is the number of equivalent band-pairs (four in the case of the lead salts), V is the volume and the final state (f) is restricted to be higher in

energy than the initial state (i) in the sum-over-states. At finite temperatures it is necessary to include the full Fermi function $F_{if}(\mathcal{E},T)$ which gives the difference in probability for occupation of initial and final states. At low temperatures such that $kT \ll \hbar\omega_c$ all states below the Fermi level $\mathcal{E}_f$ are occupied and those above are empty.

The selection rules for the $\langle f|\pi_\pm|i\rangle$ matrix elements are given by:[6]

$$\langle f|\pi_+|i\rangle = \frac{\hbar m_0}{m^*}\left[(n+1)/\lambda^2\right]^{1/2}\delta_{n'+1,n}\,\delta(k_z'-k_z) \qquad 4)$$

for intraband (cyclotron resonance) and:

$$\langle f|\pi_\pm|i\rangle = \pi_{cv}^0\,\delta_{n',n}\,\delta_{\sigma'\pm1,\sigma}\,\delta(k_z'-k_z) \qquad 5)$$

for interband transitions. The unit of magnetic length λ is equal to $(\hbar c/eH)^{1/2}$. These selection rules are for electric-dipole allowed transitions. All other matrix elements are of higher order and may be neglected.

With the selection rules given above, Equation (3) can be evaluated directly:

$$\sigma_{xy}^{(1)}(\omega) = -(n_\alpha+n_\beta)\frac{e^2\lambda^2\omega_c^2}{\hbar\,(\omega_c^2-\omega^2)} + (n_\alpha-n_\beta)\frac{e^2}{\hbar m_0^2}\,\frac{|\pi_{cv}^0|^2}{(\omega_g'^2-\omega^2)} \qquad 6)$$

where n_α and n_β are the carrier densities in the α and β sub-bands and π_{cv}^0 is the band-edge momentum matrix element. The sum-over-states gives contributions only in the vicinity of the Burstein-Moss energy gap which is defined by $\mathcal{E}_g' = \mathcal{E}_g + 2\mathcal{E}_f$ where $\mathcal{E}_f$ is the Fermi energy measured

relative to the conduction-band edge. Thus, the second term in Equation (6) is the magnetic analog of the Burstein-Moss effect. Both effects are due to partially occupied bands. The Burstein-Moss shift arises from blockage of interband transitions which contribute terms proportional to $(n_\alpha + n_\beta)$ to the diagonal component of conductivity σ_{xx}. The effect calculated here appears in σ_{xy} and is proportional to $(n_\alpha - n_\beta)$.

The appearance of an interband term in the Hall conductivity arising from band population effects is somewhat unusual since it given rise to a residual conductivity at frequencies less than the band gap. This conductivity affects both the real part of the dielectric constant and the d.c. Hall constant. Because of this seemingly "unphysical" result, past work has tended to argue that the residual term at zero frequency should be cancelled by other interband transitions[8] or by magnetic resonance transitions at lower frequency.[9] Bennett and Stern[7] have derived the appropriate sum rule for σ_{xy} (Equation (71) in reference (7)) and have shown that the sum rule gives the correct value for the Hall constant for intraband transitions (cyclotron resonance). The calculation given in Equation (6) satisfies their sum rule and includes all the interband terms that would appear for a two-band semiconductor.

The physical significance of this interband contribution to the low frequency conductivity can be made evident by rewriting Equation (6) with $\omega = 0$ in an alternate form:

$$\sigma_{xy}^{(1)}(0) = \frac{n_0 e c}{H} \left[1 + \frac{\beta^* H}{\varepsilon_g} \langle \sigma_z \rangle \right] \qquad 7)$$

where β^* is the effective Bohr magneton ($\beta^* = \hbar e/2m^* c$) and $\langle\sigma_z\rangle$ is the expectation value of the z component of "spin" angular momentum $\langle\sigma_z\rangle = (n_\alpha - n_\beta)/n_o$. In going from Equation (6) to Equation (7) the momentum matrix element is replaced according to:

$$\frac{1}{m^*} = \frac{2\,|\Pi^o_{cv}|^2}{m_o^2\,\mathcal{E}_g} \qquad 8)$$

which follows from $\vec{k}.\vec{p}$ theory for a two-band model.

The Hall conductivity given in Equation (7) follows directly if the Lorentz force law is written:

$$\frac{\vec{F}}{q} = \frac{\vec{v}\times\vec{H}}{c} + \frac{\beta^*}{\mathcal{E}_g}\,\vec{\sigma}\times\vec{E}\times\vec{H} + \vec{E} \qquad 9)$$

where q is the charge, $\vec{v}$ is the velocity neglecting spin-orbit effects and $\vec{\sigma}$ is the "spin" of the charged particle.

The form of Equation (9) is the same as for relativistic electrons with the velocity operator $\vec{\pi}/m_o = \vec{v} + (\hbar/4m_o^2c^2)\vec{\sigma}\times\vec{\nabla}V$. Therefore, the interband term which appears in Equation (7) is the semiconductor analog to the case for relativistic electrons. The analogy is effected with the Bohr magneton β_o replaced by β^* and $2m_oc^2$ replaced by $\mathcal{E}_g$. The two-band analogy with relativistic electrons and positrons has been frequently noted.[10] The effect discussed here is another manifestation of this analogy.

II. Hall Conductivity: Semi-Classical

The calculation carried out in Section I did not include the effects of temperature or finite relaxation times and therefore is limited to intermediate values of the magnetic field such that the conditions $kT \ll \hbar\omega_c$ and $\omega_c\tau \gg 1$ are both satisfied. The effects of temperature, relaxation time and non-parabolicity can be treated by an extension of the usual semiclassical calculation for the Hall conductivity where it is assumed that a single relaxation time τ exists which may depend on magnetic field and that the non-parabolicity is included through a field dependent effective mass. To avoid undue complications, the low-frequency limit (Hall effect) and high-frequency case ($\omega_c \ll \omega \ll \omega_g$) are treated separately.

A. Hall Effect

The d.c. Hall effect is normally measured in a configuration that gives values for the ρ_{xx} and ρ_{xy} components of the resistivity tensor. Therefore it is necessary to invert the measured resistivity tensor for comparison with the calculations:[7]

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} \qquad 10)$$

$$\sigma_{xy} = \frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2} \qquad 11)$$

The semiclassical equations for the real part of the free-carrier Hall conductivity σ_{xy} and the transverse conductivity σ_{xx} are given by:[7]

$$\sigma_{xx} = \frac{\sigma_o}{[1 + (\omega_c\tau)^2]} \qquad 12)$$

$$\sigma_{xy} = \frac{\sigma_o (\omega_c\tau)}{[1 + (\omega_c\tau)^2]} \qquad 13)$$

where $\sigma_o = n_o e^2\tau/m^*$. It is convenient to rewrite Equations (12) and (13) in another form for inclusion of the interband contributions and for analyses of experimental data to follow in Section III.

$$\sigma_{xx} = \frac{n_o ec}{H}\frac{(\omega_c\tau)}{[1+(\omega_c\tau)^2]} = \frac{n_o ec}{H} f(\omega_c\tau) \qquad 12a)$$

$$\sigma_{xy} = \frac{n_o ec}{H}\left[1+\frac{\beta^* H}{\mathcal{E}_g}\langle\sigma_z\rangle\right]\frac{(\omega_c\tau)^2}{[1+(\omega_c\tau)^2]} = \frac{n_o ec}{H}\left[1+\frac{\beta^* H}{\mathcal{E}_g}\langle\sigma_z\rangle\right]\frac{(\omega_c\tau)\, f(\omega_c\tau)}{[1+(\omega_c\tau)^2]} \qquad 13a)$$

where $f(\omega_c\tau) = (\omega_c\tau)/[1+(\omega_c\tau)^2]$. The inclusion of the interband term follows after Equation (7).

The diagonal component of the conductivity only includes terms proportional to $(n_\alpha + n_\beta)$ in lowest order while σ_{xy} is dominated by effects proportional to $(n_\alpha - n_\beta)$. From Equations (12a) and (13a) it follows that the interband spin-orbit term in σ_{xy} may be obtained from an analysis of experimental values for the magnetic field dependent conductivities σ_{xx} and σ_{xy} by taking the ratio:

$$\frac{\sigma_{xy}}{\sigma_{xx}} = (\omega_c\tau)\left[1 + \frac{\beta^* H}{\mathcal{E}_g}\langle\sigma_z\rangle\right] \qquad 14)$$

and by using the measured values for $\omega_c\tau$. To carry out this analysis it is necessary to invert the measured values of ρ_{xx} and ρ_{xy} according to Equations (10) and (11).

The temperature and magnetic field dependences of the spin-orbit correction to the d.c. Hall constant differ from the corresponding dependences for the usual free-carrier Hall constant and therefore provide signatures for the effect. At intermediate magnetic fields such that $1 \ll \omega_c\tau$ and $\beta^*H \ll \mathcal{E}_f$, the expectation value for $\langle\sigma_z\rangle$ can be approximated by:

$$\langle\sigma_z\rangle = \frac{(n_\alpha - n_\beta)}{n_0} = \frac{3}{2}\frac{\Delta\mathcal{E}_f}{\mathcal{E}_f} = 3\frac{\beta^* H}{\mathcal{E}_f} \qquad 15)$$

where the "spillage from sub-band α to sub-band β is proportional to the splitting between sub-bands ($\hbar\omega_c = 2\beta^*H$). Non-parabolicity is taken into account by using the magnetic field or energy dependent effective mass m*. The d.c. Hall conductivity in this regime is therefore given by:

$$\sigma_{xy}^{(1)}(0) = \frac{n_0 e c}{H}\left[1 + \frac{3(\beta^* H)^2}{\mathcal{E}_f \mathcal{E}_g}\right] \qquad 16)$$

It is seen from the form of Equation (16) that de Haas-Van Alphen (dHvA) type oscillations will result from the spin-orbit contribution. The dHvA contribution to σ_{xy} has been calculated and observed in the low temperature Faraday rotation for "n"-type PbS.[12] The paramagnetic temperature dependence resulting from Equation (16) also has been calculated and observed for Faraday relation[13] but has not been reported for the d.c. Hall effect.

B. Faraday Rotation

Faraday rotation provides a convenient probe for determination of the frequency dependent Hall conductivity in semiconductors. In regions of low absorption, the Faraday rotation is given by:[8]

$$n(\omega)\,\theta(\omega) = \frac{2\pi}{c}\,\sigma_{xy}^{(1)}(\omega) \qquad 17)$$

where $n(\omega)$ is the frequency dependent index of refraction, $\theta(\omega)$ is the rotation of the plane of polarization per unit path and $\sigma_{xy}(\omega)$ is the real part of the frequency dependent Hall conductivity. For a two-band semiconductor with carriers in the conduction band, the rotation is given by:

$$n(\omega)\,\theta(\omega) = -\frac{n_0 e c}{H}\left[\frac{\omega_c^2}{\omega^2} - \frac{\beta^* H}{\mathcal{E}_g}\langle\sigma_z\rangle\frac{\omega_g^2}{(\omega_g^2-\omega^2)}\right] \qquad 18)$$

where the high frequency limit $1 \ll (\omega-\omega_c)\tau$ has been used for the free carrier (cyclotron resonance) contribution and the low frequency limit $\omega \ll \omega_g$ has been used for the interband contribution.

C. Dielectric Anomalies

The presence of a residual conductivity for frequencies less than the band gap also modifies the dielectric "constant" ε_∞ which is used in calculating the position of dielectric anomalies in the Faraday geometry (propagation parallel to the magnetic field). Ignoring the effect of partially occupied bands on $\sigma_{xx}(\omega)$, we can write:[6]

$$\epsilon_{l,r}(\omega) = \epsilon_\infty(\omega) \mp \frac{4\pi}{\omega}\,\sigma_{xy}^{(1)}(\omega) \qquad 19)$$

where $\varepsilon_{lr}(\omega)$ is the real part of the frequency dependent dielectric "constant" for lcp and rcp, $\varepsilon_{\infty}(\omega)$ is the total interband contribution to the diagonal component of the dielectric "constant" for frequencies less than the band gap and $\sigma_{xy}(\omega)$ is given by Equation (18). The effect of the spin-orbit term has been calculated and observed for the splitting of the lcp and rcp magneto-plasma edges in "n" type PbS.[12]

III. Hall Conductivity: Experimental

The calculations carried out in Sections I and II apply strictly only to the case of two-band semiconductors with spherical Fermi surfaces. There are several narrow-gap semiconductor compounds and alloys that approximate this model which may be used to test the validity of the calculations.

The binary compounds PbS, PbSe and PbTe as well as the "zero-gap" ternary alloy $Pb_x Sn_{1-x} Te$ provide the closest analog to the idealized model. These IV-VI semiconductors have "mirror-type" band edges located at L-point of the Brillouin zone with approximately equal effective masses in the valence and conduction bands. Also, the effective g-factors are approximately equal in magnitude but opposite in sign and satisfy the relation $g^* \simeq 2m_o/m^*$. A recent review of the band structure, band parameters and energy levels in magnetic field has been given by Bauer.[14]

The III-V and II-VI semiconductors with direct band gaps at the Γ-point of the Brillouin zone have a more complicated valence band structure with degenerate "light" and "heavy" hole bands at the band edge. However, for very small energy gaps the conduction band and "light" hole valence band provide a good approximation to the two-band model[15] with the important distinction that $g^* = - m_o/m^*$, i.e. the spin splitting in a magnetic field is one-half the cyclotron frequency and the g-factor for conduction electrons is negative. The "zero-gap" ternary alloy $Hg_x Cd_{1-x} Te$ is the best studied narrow-gap semiconductor with this band structure.[16]

A. d.c. Hall Effect

Anomalies in the low temperature and high field Hall effect have been reported for "n"-type PbTe[17], $Pb_xSn_{1-x}Te$[17] and $Hg_xCd_{1-x}Te$.[18] Typically, the measured Hall constant R_H exhibits dHvA type oscillations at moderate fields and a decrease with increasing field in the quantum limit when only the lowest Landau level is occupied.

The high field measurements reported to date on narrow-gap semiconductors have been carried out in the "bar" geometry rather than Corbino disc geometry. Therefore, resistivity components ρ_{xx} and ρ_{xy} are measured and the conductivity components σ_{xx} and σ_{xy} must be calculated. The published data for PbTe and $Pb_xSn_{1-x}Te$ are not sufficiently complete to carry out the inversion. More complete data extending to 30T has been published for $Hg_{0.8}Cd_{0.2}Te$ which does permit the conductivities to be calculated.

The data of De Vos and Herlach[18] for the Hall constant R_H and the transverse resistivity ρ_{xx} have been analyzed using Equations (10) and (11) for the inversion and assuming that the conductivity components σ_{xx} and σ_{xy} are correctly described by Equations (12a) and (13a). It was necessary to normalize the values for ρ_{xx} since De Vos and Herlach only report relative values. The normalization was done by choosing $\omega_c\tau = 1$ at 5T. This choice is not arbitrary since the function $f(\omega_c\tau)$ has a maximum value one-half when $\omega_c\tau=1$. Such a maximum was observed in a plot of ρ_{xx} H obtained from this analysis.

The results of the analysis are presented in Figure 2 together with the

calculated values for the spin-orbit parameter η.

$$\eta = \frac{\beta^* H}{\mathcal{E}_g} \langle \sigma_z \rangle \qquad 20)$$

The values for η were calculated using measured values for the g-factor to 15T,[16] extrapolated values to 30T and by taking $\langle\sigma_z\rangle = 1$. The energy gap was taken to be 70meV as reported for this sample.[18] The values for η calculated in his manner agree in magnitude and sign (negative) with the values derived from the measurements of De Vos and Herlach. However, the correction introduced by η is only of the order 50% and it would be desirable to repeat the experiments in $Pb_xSn_{1-x}Te$ where the effects are expected to be much larger.

It is interesting to note that the values of $\omega_c\tau$ derived from this analysis decrease with increasing magnetic field in the quantum limit. This is not too surprising since the effective mass increases with magnetic field and τ is expected to decrease with field due to the increase in the DOS at the Fermi level with increasing field.

B. Free-Carrier Faraday Rotation

The free-carrier Faraday rotation has been measured and anomalous low frequency residues noted in PbS[13], PbSe[9] and PbTe[19]. The measured values of the residual Hall conductivity in PbS were shown to agree with values calculated along the lines of Equations (6) and (17) using known band parameters. The values measured for the residual rotation in PbSe and PbTe are of the correct magnitude and sign to be attributed to the spin-orbit parameter η. However, the authors attribute the residual rotation to a high

frequency polarization current proportional to the paramagnetic susceptibility. They suggest that this term disappears for frequencies below the spin-resonance transition.

The possibility has been investigated that either magnetic-dipole or electric-dipole induced spin resonance transitions could cancel the interband contributions to the d.c Hall conductivity. The magnitudes are too small to provide such cancellation.

The oscillatory dHvA effect also has been observed in the Faraday rotation for PbS.[12] The measured period of the oscillations was in good agreement with the value calculated using the carrier concentration determined by Hall effect measurements.

C Dielectric Anomalies

Dielectric anomalies occur when the index of refraction switches from real to imaginary at a plasma frequency. The resulting step in the reflectivity is called the plasma edge. The separation of the lcp and rcp free-carrier plasma edges in semiconductors is just equal to the cyclotron frequency ω_c if the background dielectric constant is the same for lcp and rcp[6]. The measured separation of the magneto-plasma edges in PbS has been measured[12] and shown to be smaller that the cyclotron frequency. The difference is accounted for by including the second term of Equation (19) in the analysis.

IV. Summary and Conclusions

The frequency dependent Hall conductivity $\sigma_{xy}(\omega)$ has been calculated for a two-band model with energy levels analogous to those for relativistic electrons and positrons. The real part of $\sigma_{xy}(\omega)$ was shown to have a residue at low frequencies which arises from the unequal blockage of left-circularly-polarized (lcp) and right-circularly-polarized (rcp) transitions in partially occupied bands. The value of the residual term was shown to be the same as would be obtained by including the relativistic correction to the velocity operator in the Lorentz force law with the Bohr magneton and energy gap ($2m_oc^2$) replaced by their solid state analogs. The calculated conductivity satisfies the appropriate sum rule. The effects of finite temperature and relaxation time were included in a semi-classical calculation of the Hall conductivity and transverse conductivity σ_{xx} to allow comparison with experiments.

The effects of the spin-orbit parameter η in the Lorentz force law were shown to provide good agreement with experiments which measured the Faraday rotation in PbS, PbSe and PbTe as well as experimental measurements of the magneto-plasma dielectric anomalies in PbS.

The appearance of the spin-orbit parameter in the Lorentz force is less well supported by experimental studies of the d.c. Hall effect. Anomalies at high fields have been noted for PbTe, $Pb_xSn_{1-x}Te$ and $Hg_xCd_{1-x}Te$ however, the published results are not sufficiently complete to carry out an unambiguous analysis. Experiments on $Hg_{0.8}Cd_{0.2}Te$ carried out in fields extending to 30T at 4.2K were analyzed to show that the high field behavior is consistent with a spin-orbit correction term of the magnitude and sign

calculated using known experimental parameters.

The addition of a spin-orbit correction term to the Lorentz force law in periodic solids has a number of implications that need to be explored and verified. A critical experiment is the careful measurement of the high field Hall conductivity and transverse conductivity in PbSe, PbTe and $Pb_xSn_{1-x}Te$ since these systems most closely approximate the conditions for the simple two-band model. Measurements in "n"-type PbSe would be particularly valuable since the valance and conduction bands are nearly spherical.

Also, theoretical calculations and experiments are indicated for systems that cannot be described by a simple two-band model. An obvious extension is the case of the Quantum Hall Effect in quasi-two-demensional systems with large spin-orbit interactions such as PbTe or $Pb_xSn_{1-x}Te$.

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Figure Captions

Figure 1: Landau levels in the presence of a magnetic field. The levels are separated by $\hbar\omega_c = 2\beta^*H$. The intraband (cyclotron resonance) transitions are for lcp radiation. The interband transitions are lcp for the valence band "spin-down" to conduction band "spin-up" levels and rcp for the other sub-band pair. All states in the conduction band are occupied for energies less than the Fermi energy $\mathcal{E}_f$ at T = 0.

Figure 2: Plots of the experimental and calculated values of the spin-orbit parameter η for $Hg_{0.8}Cd_{0.2}Te$ at 4.2K after De Vos and Herlach (reference 18). Experimental values for σ_{xy} σ_{xx} and $\omega_c\tau$ as a function of magnetic field also are given. The analysis is described in the text.

Energy Level Diagram
Two-Band Model

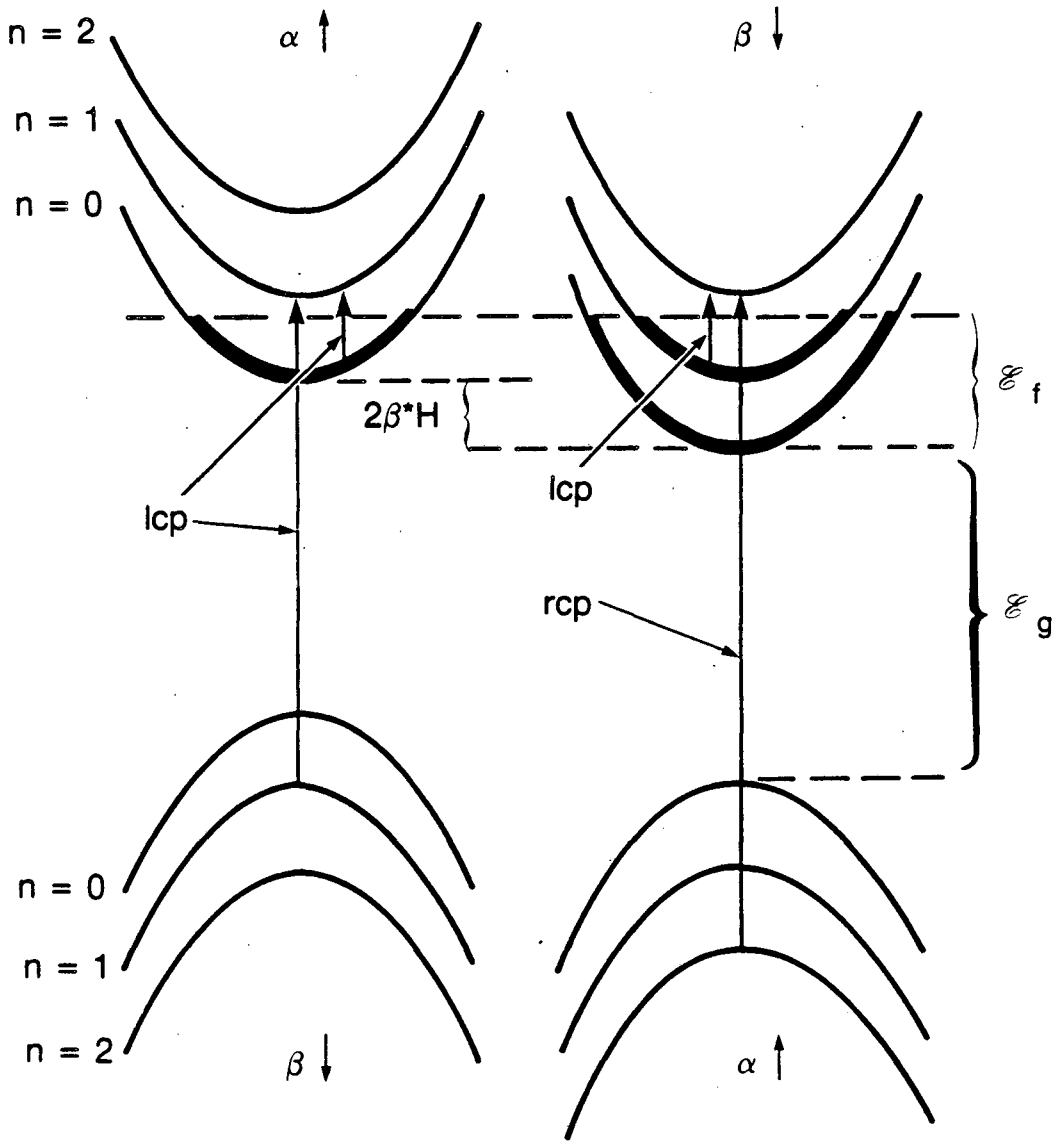

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Fig. 1

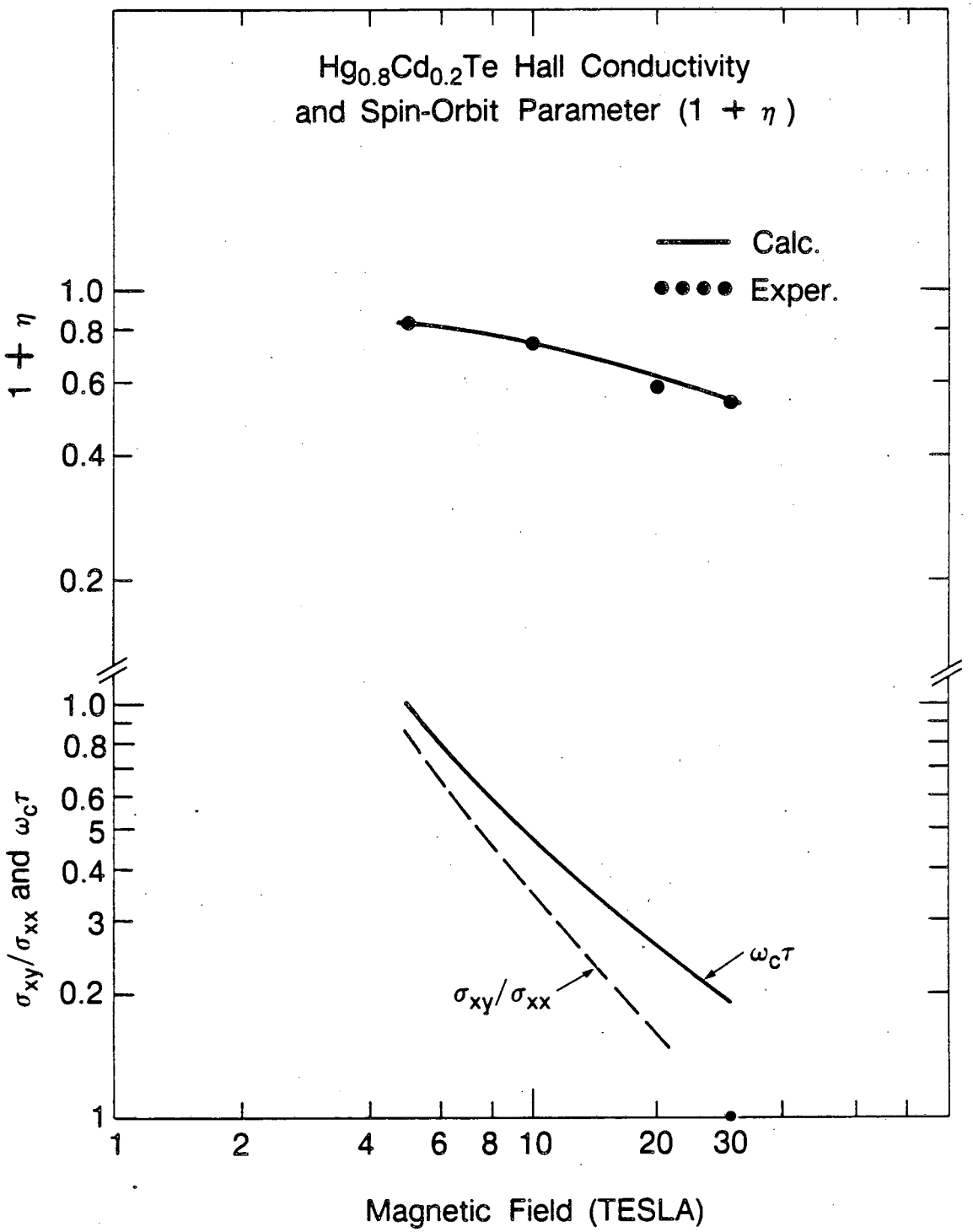

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Fig. 2

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