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Light Guide Design Principles

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Light Guide Design Principles

Kenneth Johnson and Stephen Selkowitz

ABSTRACT

A general theory of optical transport systems has been developed that can be used to determine preliminary design specifications for light guide systems. Several generic light guide types are analyzed, including hollow reflective light guides, prism light guides, solid dielectric and fluid-filled light guides, lens guides, and open light wells. Minimum theoretical aperture requirements are determined for each type as a function of the specified optical transport efficiency and design parameters (light guide length, transmitted luminous flux, etc.). Generally, a system's aperture requirement would be inversely related to its cost. Solid dielectric (e.g., optical fiber) light guides would be very compact and practical for retrofit applications, but their high cost would preclude their use for long-distance optical transport. Open light wells would be the simplest and least costly option, but would require the greatest aperture area. Hollow reflective light guides, prism light guides, or lens guides may offer the best compromise between cost and space requirements. But in order to achieve optical concentrations and efficiencies near the theoretical limit, the collector system would need to maintain optical and tracking tolerances exceeding the capabilities of existing systems, so further advances in core daylighting will require improvements in collector technology

INTRODUCTION

The amount of sunlight available under typical clear sky conditions would be more than sufficient to supply a building's entire lighting needs, provided that the light can be conducted efficiently from the collector to the building's core zones. Several types of optical transport mechanisms have been proposed for core daylighting systems, including hollow reflective light guides (LBL 1985; Spear 1986); prism light guides (Whitehead et al. 1982, 1984); dielectric light guides, i.e., solid or fluid-filled lightguides (Cariou et al. 1982; Fraas et al. 1983); and lens guides (Duguay and Edgar 1977; Duguay and Aumiller 1979). Recent advances in light guide materials have greatly enhanced the technical and economic feasibility of a couple of these systems. LBL is currently conducting experimental model studies of light guide systems using a new high-reflectance, silver-backed film, which exhibits 95 % reflectance and has good environmental characteristics (Spear 1986). Prism light guides do not currently exhibit this level of efficiency (Whitehead et al. 1982); but it is likely that refinements of the production processes could improve their optical performance very significantly. A solid light guide composed of a recently developed experimental plastic material (Kaino et al. 1983); could transport highly concentrated sunlight over a 50 ft distance with around 80 % transmission efficiency.

Although good progress is being made in the development of light guide materials, their effective use will require general optical design principles and techniques that are currently lacking. Published descriptions of core daylighting systems generally only discuss the systems from a conceptual viewpoint and do not provide the kinds of design and performance specifications and optimization criteria that would be required to implement any of these concepts. However, an optical theory of light guide systems has recently been developed under LBL sponsorship (Johnson 1986); which can be used to determine rough preliminary design and performance specifications for a light guide system, assuming that the collector has certain optimal characteristics. We will breifly outline this theory and will then examine several examples illustrating its practical application.

GENERAL DESIGN APPROACH

The following discussion primarily concerns systems that employ two-axis tracking collectors, although one example of a nontracking system will also be considered.

In designing a light guide system, we begin with an initial specification that defines certain fixed material, structural, and optical parameters of the system. The material parameters include data on the optical properties of the light guide materials (e.g., surface reflectances, refractive indices, etc.). The primary structural specification is the light guide length. The optical specifications would include the collector's tracking accuracy and its transmittance. Also, we would need information on the optical characteristics of the distribution system. For our purposes, we will simply characterize the distribution system in terms of a coefficient of utilization (CU), which is defined as the fraction of the light guide's output flux that reaches the illuminated area.

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Aside from the fixed design parameters, we also have two free design parameters, the light guide aperture area, $A_{light\ guide}$, and the collector aperture area, $A_{collector}$, which must be determined to satisfy certain design constraints. The primary design constraint defines the level of flux the light guide should emit under specified solar insolation conditions. The required flux output, Φ_{output} (in lumens), can be calculated from the floor area, $A_{distributed}$ (in square feet), over which the flux is distributed, the desired flux density level, $E_{distributed}$ (in footcandles), in the distribution space, and the coefficient of utilization, CU* :

$$\Phi_{output} = A_{distributed}\ E_{distributed}\ /\ CU\ . \qquad (1)$$

The solar insolation may be specified in terms of the flux density, E_{sun}, at the collector aperture or it may alternatively be specified in terms of the sun's luminance, L_{sun}. The direct normal solar flux density, E_{sun} (in footcandles), can be calculated from L_{sun} (in candelas per square foot) and the sun's angular radius, $\theta_{sun} = 0.27^{\circ}$, from the relation

$$E_{sun} = L_{sun}\ \pi \sin^2 \theta_{sun} \qquad (\theta_{sun} = 0.27^{\circ})\ . \qquad (2)$$

The specification of Φ_{output} reduces the number of degrees of freedom in the system design from two to one, and the remaining degree of freedom would be used to minimize the collector's and light guide's combined cost. (The cost would include the value of occupancy space displaced by the light guide, as well as material costs.) There would generally be a trade-off between the light guide cost and the collector cost, since decreasing the aperture of the light guide would generally require a compensating increase in the collector aperture size, and vice versa. This trade-off is illustrated in Figure 1. The left-hand diagram schematically illustrates a collector system as a lens, which focuses the sun disk onto a hollow reflective light guide's entrance aperture. The light guide aperture can be made smaller by shortening the collector's focal length, as illustrated on the right. However, this modification increases the number of surface reflections that rays in the light guide must undergo, because the light guide walls are closer together and also because rays from the edge of the collector enter the light guide at a larger axial divergence angle in the short focal length system. Thus, the surface reflection losses are greater, resulting in reduced flux transmittance through the light guide. In order to compensate for reduced light guide transmittance and maintain the flux output, Φ_{output}, at the specified level, more light would have to be collected at the system's input end; consequently, the collector aperture would need to be enlarged.

(If the light guide in Figure 1 is a dielectric light guide, its flux transmittance would be somewhat less sensitive to the light guide size since the transmittance of a ray would depend on the ray's path length through the light guide, rather than the number of reflections it makes.)

The above considerations suggest an overall design strategy that we will discuss qualitatively and will then formulate in mathematical terms. First, in order to keep the collector size and cost within reasonable bounds, we should stipulate some criterion to ensure that the light guide's flux transmittance, $T_{light\ guide}$, is not unreasonably low. The criterion we will use is defined as follows: For each ray that transmits through the light guide, we can define a ray transmittance factor, τ, which represents the ratio of the ray's output luminance to its input luminance. (In other words, τ represents the light guide's flux transmittance for a very narrow, well-collimated beam enclosing the ray.) We will stipulate that every ray in the beam should transmit through the light guide with a ray transmittance abouve a specified minimum value, τ_{min}. This "ray transmittance constraint" guarantees that the light guide's total flux transmittance, $T_{light\ guide}$, will exceed τ_{min}.

We will use τ_{min} as one of our two free design parameters, in lieu of $A_{collector}$. (A high τ_{min} value would be favored if the collector is the major expense item, whereas a low τ_{min} value would be more optimum if the light guide-related costs are dominant.) For any specified values of the two free parameters, τ_{min} and $A_{light\ guide}$, the collector design will be optimized (subject to the ray transmittance constraint) to maximize the light guide's flux output, Φ_{output}. The collector design includes its aperture area, $A_{collector}$, so this optimization condition defines $A_{collector}$, as well as Φ_{output}, as a function of τ_{min} and $A_{light\ guide}$. The system's aperture-related cost would be defined as a function of $A_{collector}$ and $A_{light\ guide}$, so these costs may also be determined from τ_{min} and $A_{light\ guide}$. The two free parameters would be chosen to satisfy two conditions: The light guide's output flux, Φ_{output}, should be at the specified level, and, subject to this constraint, the system cost should be minimized.†

In order to implement the above design procedure, we need to know how to determine, for any specified light guide configuration and any specified minimum ray transmittance, τ_{min}, the maximum flux Φ_{output} that can be channeled out of the light guide (under the specified solar insolation conditions); and we also need to know what kind of collector design will achieve this level of flux output. The flux output level depends on the characteristics of the input radiation, but this dependence can be factored out if we make a couple lf simple assumptions about the solar insolation and the collector. First, we assume that the input radiation is direct sunlight (of uniform luminance, L_{sun}, with a divergence half-

* See ANSI / IES RP-16 (1980) for a summary of photometric definitions and nomenclature.

† A more sophisticated design optimization procedure might instead optimize the system's projected life-cycle savings, taking into account the system's influence on HVAC loads.

angle of $\theta_{sun} = 0.27^{\circ}$) that impinges on the collector aperture at normal incidence. We also assume that the collector's ray transmittance is uniform (i.e., identical for all incident rays) and equal to a fixed value, $T_{collector}$, which is independent of the collector design. In addition, we assume that the collector's tracking error is negligible; so the collector's output beam is perfectly stationary. Thus, the beam that is fed into the light guide is stationary and has a uniform luminance of

$$L_{input} = T_{collector} L_{sun} . \tag{3}$$

If we know the geometry (i.e., size and shape) of the beam that is fed into the collector and also its luminance, L_{input}, we can calculate the output flux, Φ_{output}. Since L_{input} is specified, Φ_{output} depends only on the beam geometry, which is determined by the collector design. Thus, the collector optimization problem reduces to one of determining what light guide input beam geometry will maximize Φ_{output}, under the constraint that all rays in the beam traverse the light guide with ray transmittances above τ_{min}; and the collector's output beam should optimally comprise this particular set of rays.

We will not get involved in the specifics of the collector design in the following examples, but will simply assume that the collector produces the optimum beam geometry as described above. We will, however, calculate the collector's aperture area requirement, $A_{collector}$, by using an energy balance criterion: First, we will calculate the light guide's flux transmittance, $T_{light\ guide}$ (which depends on the light guide design and τ_{min}). From this, we can calculate the amount of flux, Φ_{input}, that enters the light guide:

$$\Phi_{input} = \Phi_{output} / T_{light\ guide} . \tag{4}$$

Taking into account the collector transmittance, we then determine how much solar flux, $\Phi_{collected}$, must be intercepted by the collector aperture:

$$\Phi_{collected} = \Phi_{input} / T_{collector} . \tag{5}$$

Knowing how much flux must be collected, we can determine the required collector area:

$$A_{collector} = \Phi_{collected} / E_{sun} , \tag{6}$$

where E_{sun} is the incident solar flux density, which may be specified or may be determined from the sun luminance L_{sun} by using Equation 2.

In the above design outline, the collector is assumed to meet several idealistic conditions that may not apply in practice. First, we have implicitly assumed in calculating $A_{collector}$ that all of the direct sunlight that enters the collector gets channeled into the light guide, whereas in practice some rays could be blocked within the collector system. Also, we have assumed that the collector's output beam has a specific optimum shape that maximizes the light guide's flux transmittance, but in practice we would not have enough control over the beam shape to satisfy this condition exactly. Due to these limitations, the required collector and/or light guide aperture areas would actually be somewhat larger than the theoretical values predicted by our design model. However, the aperture areas of a well-designed system could in practice be expected to be well within a factor of two of their theoretical values.

The collector's ray transmittance was assumed to be uniform and independent of the collector design. This assumption may not be exactly valid, but it is a reasonably good approximation.

We have also assumed that the collector tracking error is negligible. Specifically, the pointing error should be small in comparison to the sun's angular radius (0.27°) in order for the results of our analysis to be valid. A significant tracking error could be taken into account, however, by making the following modifications in the analysis: We redefine θ_{sun} as the angular radius of the collector's field of view (which would be large enough to accommodate the sun's angular radius, 0.27°, plus the tracking error); and we redefine L_{sun} as the average luminance over the collector's field of view. L_{sun} is implicitly defined in terms of the incident solar illuminance E_{sun} and θ_{sun} by Equation 2.

DESIGN PROCEDURE

In implementing the design approach outlined above, the primary design tool we will be using is a characteristic function, $\mathcal{A}$, which determines how much flux, Φ_{input}, is in the light guide's input beam. Φ_{input} is proportional to the input beam's luminance, L_{input}. The proportionality factor, which is a function of τ_{min}, will be denoted $\pi\, \mathcal{A}(\tau_{min})$:

$$\Phi_{input} = L_{input}\, \pi\, \mathcal{A}(\tau_{min}) . \tag{7}$$

(The constant π is factored out of the proportionality term to preserve notational consistency with Johnson, 1986.) The form of function $\mathcal{A}$ depends on the specific type of light guide system that is being analyzed (e.g., square-section hollow reflective, circular-section dielectric, etc.), and $\mathcal{A}$ is implicitly a function of the light guide's design parameters (aperture

dimension, wall reflectance, etc.). $\mathcal{A}$ has units of area. (If Φ_{input} is specified in lumens and L_{input} is specified in candelas per square foot, then $\mathcal{A}$ has units of square feet.) $\mathcal{A}(\tau)$ is defined for any ray transmittance value, τ, from zero to τ_{max}, where τ_{max} is the light guide's maximum ray transmittance. (Generally, τ_{max} is the ray transmittance of rays that are directed parallel to the light guide axis. For hollow light guides, $\tau_{max} = 1$.)

The light guide's input flux, Φ_{input}, and flux transmittance, $T_{light\ guide}$, determine its output flux:

$$\Phi_{output} = T_{light\ guide}\ \Phi_{input} \tag{8}$$

so Equation 7 can be restated in terms of the specified output flux:

$$\Phi_{output} = T_{light\ guide}\ L_{input}\ \pi\ \mathcal{A}(\tau_{min}) . \tag{9}$$

The light guide's characteristic function, $\mathcal{A}$, also determines its flux transmittance, $T_{light\ guide}$. The following formula is derived by Johnson (1986):

$$T_{light\ guide} = \tau_{min} + (1/\mathcal{A}(\tau_{min})) \int_{\tau_{max}}^{\tau_{min}} \mathcal{A}(\tau)\, d\tau . \tag{10}$$

Equations 9 and 10 form the basis of our design procedure. $\mathcal{A}(\tau)$ is implicitly a function of $A_{light\ guide}$, so for a fixed value of τ_{min}, these two equations can be solved for the two unknowns, $A_{light\ guide}$ and $T_{light\ guide}$. We can also determine $A_{collector}$ from Equations 4, 5, and 6. $A_{light\ guide}$ and $A_{collector}$ determine the system's aperture-related cost, and τ_{min} may be chosen to minimize this cost. (Since we do not have a cost model, however, we will simply specify a reasonable value for τ_{min} as a fixed design parameter in each of the design examples.)

Equations 9 and 10 can be solved graphically by plotting the two $A_{light\ guide}$ vs. $T_{light\ guide}$ relations represented in Equations 9 and 10 and finding the point where the two curves intersect.* The Appendixes present graphical data representing Equation 10 for several generic light guide types. These data save the trouble of numerically evaluating the integral in Equation 10 - a curve representing Equation 9 can be overlaid on the appropriate graph in the Appendixes to locate the design values of $A_{light\ guide}$ and $T_{light\ guide}$.

In some cases, Equations 9 and 10 will have a form that considerably simplifies the above design procedure. For the case of a dielectric light guide (which we will discuss below), Equation 10 has no dependence on $A_{light\ guide}$, so we can calculate $T_{light\ guide}$ directly from Equation 10. This $T_{light\ guide}$ value can then be used in Equation 9 to determine $A_{light\ guide}$. (Equation 9 will be in a form that can be algebraically solved for $A_{light\ guide}$.) In the case of a hollow light guide, we can, under certain circumstances, make reasonable approximations which will also convert 9 and 10 into the same kind of form.

DESIGN EXAMPLES

For the following examples, we will consider a system that illuminates a core zone of area $A_{delivered} = 10{,}000\ ft^2$ at a nominal flux density of $E_{distributed} = 50$ fc. We assume a coefficient of utilization, CU, of 0.5 for the distribution system. From Equation 1, the light guide's nominal flux output is

$$\Phi_{output} = 1{,}000{,}000\ \text{lm} . \tag{11}$$

We assume a light guide length, Z, of 50 ft:

$$Z = 50\ \text{ft} . \tag{12}$$

The assumptions that we will make about the available solar insolation will be defined separately for the nontracking system and the tracking systems.

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* We can alternatively solve Equations 9 and 10 numerically by a simple iterative procedure: First, we make an initial estimate of $T_{light\ guide}$ (e.g., $T_{light\ guide} = 1$). We substitute this value in Equation 9, which we use to numerically calculate $A_{light\ guide}$. This $A_{light\ guide}$ value is used in Equation 10 to calculate an improved estimate of $T_{light\ guide}$. $T_{light\ guide}$ is then substituted back into Equation 9 to recalculate $A_{light\ guide}$, and we repeat the procedure iteratively until the calculation converges to a stable solution. (The algorithm will normally converge if $T_{light\ guide}$ is reasonably high.)

Since we do not have a system cost model on which to base our designs, we will not treat τ_{min} as a design variable but will instead simply pick a reasonable value as a design constant for each example.

NONTRACKING SYSTEM (SQUARE-SECTION HOLLOW REFLECTIVE LIGHT GUIDE)

The first system that we will analyze will be a square-section hollow reflective light guide that is coupled to a nontracking collector. The light guide is, in essence, a hollow duct with mirrored walls. The collector could comprise a large Fresnel lens that feeds into a light funnel, as illustrated in Figure 2 . [The collector's field of view would be limited to the region of the sky where the sun is located during periods of peak energy demand. The diurnal variability of the transmitted luminance could be mitigated by using a diffuser in the light guide's entrance aperture.] We will denote the light guide's width dimension as W (W will be chosen to meet the output flux specification), and we will assume a wall reflectance ρ of 0.95:

$$\rho = 0.95 . \tag{13}$$

Although the design procedure developed above does not apply to nontracking systems, it can be adapted to this case by the following means: First, we will model the performance of the system under conditions in which the input beam's luminance distribution is equivalent to its time-averaged input luminance under actual operating conditions. (The luminance is averaged over the period of time when the sun is unobstructed and is within the collector's field of view). Second, we will assume that the (time-averaged) beam at the light guide's entrance aperture is completely diffuse (i.e., uniform and isotropic), with an input flux density, E_{input}, of 10,000 fc:

$$E_{input} = 10{,}000 \text{ fc} . \tag{14}$$

The flux density, E_{input}, of a completely diffuse beam is related to its luminance, L_{input}, by a relation analogous to Equation 2 but with 90° substituted for the beam's divergence half-angle:

$$E_{input} = \pi L_{input} . \tag{15}$$

so Equation 14 determines the input luminance.

The input beam's minimum ray transmittance is zero, since the rays in the beam that are nearly perpendicular to the light guide axis are completely absorbed by multiple surface reflections:

$$\tau_{min} = 0 . \tag{16}$$

(We could design the system for a higher τ_{min} value, but a low value is favorable for nontracking systems because the collector cost is not very significant in comparison to the cost of occupancy space displaced by the light guide.) For this value of τ_{min}, $\mathcal{A}(\tau_{min})$ is simply equal to the light guide's aperture area:

$$\mathcal{A}(0) = A_{light\ guide} = W^2 . \tag{17}$$

From Equations 15 and 17, Equation 9 reduces to

$$\Phi_{ouput} = T_{light\ guide} E_{input} W^2 . \tag{18}$$

This gives us one of the two relations that we need to calculate $A_{light\ guide}$ and $T_{light\ guide}$. The other relation, Equation 10, will not be quite so simple.

In order to evaluate Equation 10, we need to know the explicit form of the function $\mathcal{A}(\tau)$ for any value of τ. For the case of a square-section hollow reflective light guide of length Z, width W, and reflectance ρ, this function has the form

$$\mathcal{A}(\tau) = \frac{4}{\pi} W^2 b \tan^{-1}(b) \tag{19}$$

where

$$b = 1/\sqrt{(1+2(\frac{Z \ln \rho}{W \ln \tau})^2)} . \tag{20}$$

We also need to know the light guide's maximum ray transmittance τ_{max}, which is 1:

$$\tau_{max} = 1 . \tag{21}$$

Substituting Equations 16, 19, and 21 into Equation 10, we can solve for $T_{light\ guide}$ as a function of the parameter -(Z/W) ln ρ. This function is illustrated in Figure 3. The dashed line in Figure 3 represents Equation 18 (with Φ_{output} = 1,000,000 lm, Z = 50 ft., and ρ = 0.95). The point where the two curves intersect defines the design values for the parameter -(Z/W) ln ρ and $T_{light\ guide}$:

$$-(Z/W)\ \ln \rho = 0.22 \tag{22}$$

$$T_{light\ guide} = 0.71 \tag{23}$$

From Equations 12, 13, and 22, we obtain the light guide aperture area:

$$A_{light\ guide} = W^2 = 140\ ft^2 \tag{24}$$

A light guide system that displaces 140 ft^2 on each floor it traverses would constitute a major architectural element, so this type of system would not be practical for retrofit applications. An additional disadvantage of this system is that it would be difficult to efficiently distribute highly diffuse light from such a large aperture over 10,000 ft^2 of illuminated space. One approach that might be considered to facilitate light distribution would be to use several smaller, evenly distributed light guides instead of a single large central light guide. The problem with this approach is that the smaller light guides' flux transmittance would be lower than that of a large light guide, so their apertures would have to be enlarged somewhat to compensate. For example, if we were to divide the 140 ft^2 aperture area equally amoung four separate light guides, their flux transmittance would be only 0.54 (compared to 0.71 for the single light guide), so their combined flux output would only be around 770,000 lm. In order to achieve the specified 1,000,000 lm output level, their cumulative aperture area would need to be increased to 174 ft^2. (The larger aperture area would increase the flux transmittance slightly to 0.57 .)

In weighing the trade-off between a single vs. multiple light guide system, we should take into account the improved light distribution that would be achieved by using more than one light guide. If the coefficient of utilization can be increased from our assumed value of 0.5 to 0.65 by using four light guides, the improved distribution efficiency would just balance the reduced light guide transmittance, so a 140 ft^2 total light guide cross section would still suffice. Thus, a four - light guide system could perhaps be practical; but if many more light guides are used, the decrease in flux transmittance would outweigh any gain in distribution efficiency.

TRACKING SYSTEMS

For the following exampes, we will assume that the light guide is coupled to a two-axis tracking collector with a flux transmittance, $T_{collector}$, of 0.75:

$$T_{collector} = 0.75 . \tag{25}$$

We will require that the system deliver illumination at the specified 50 fc level under conditions of 5000 fc direct normal solar insolation, E_{sun}:

$$E_{sun} = 5000\ fc . \tag{26}$$

From Equations 2, 3, and 26, we determine the luminance, L_{input}, at the collector's input aperture:

$$L_{input} = T_{collector} \frac{E_{sun}}{\pi \sin^2\theta_{sun}} = 5.4\ (10^7)\ cd/ft^2 . \tag{27}$$

SQUARE-SECTION HOLLOW REFLECTIVE LIGHT GUIDE

We will now investigate how the size of the light guide system discussed above changes when we use a tracking collector. We again consider a square-section hollow reflective light guide of width W, length Z = 50 ft, and wall reflectance ρ = 0.95 .

Appendix 1 presents a graphical design tool that applies to a square-section hollow reflective light guide of any design configuration. (Appendix 2 presents the same data for a circular-section hollow reflective light guide.) The curves

represent the light guide's flux transmittance, $T_{light\ guide}$ (as defined by Equation 10), as a function of the parameter $-(Z/W)\ \ln \rho$ for several values of τ_{min}. (Figure 3 illustrates the particular case $\tau_{min} = 0$.) Notice that the curves flatten out beyond the point where $-(Z/W)\ \ln \rho$ becomes large in comparison to 1. This implies that if this asymptotic condition holds:

$$-(Z/W)\ \ln \rho \gg 1 \qquad \text{(asymptotic condition)} \tag{28}$$

then $T_{light\ guide}$ does not depend significantly on the light guide design and is essentially a function only of τ_{min}. The asymmptotic value of $T_{light\ guide}$ can be determined using the following asymptotic approximation of Equation 19:

$$\mathcal{A}(\tau) \cong \frac{2}{\pi}\left(\frac{W^2 \ln \tau}{Z \ln \rho}\right)^2 . \quad \text{(asymptotic approximation)} \tag{29}$$

Using this approximation, Equation 10 reduces to*

$$T_{light\ guide} \cong 2(1-\tau_{min}(1-\ln\tau_{min}))\ /\ (\ln\tau_{min})^2 . \quad \text{(asymptotic approximation)} \tag{30}$$

We can also use Equation 29 to solve Equation 9 for $A_{lightguide}$:

$$A_{light\ guide} = W^2 \cong \sqrt{\left(\frac{\Phi_{output}}{2\ T_{light\ guide}\ L_{input}}\right)}\ \frac{Z \ln \rho}{\ln \tau_{min}} .$$

$$\text{(asymptotic approximation)} \tag{31}$$

We will assume for our example that the light guide width, W, is sufficiently small that Relation 28 holds, and we will use Equations 30 and 31 to calculate W. The τ_{min} value that we will specify will be determined so that the light guide's flux transmittance is acceptable. If we pick a value of 0.34 for τ_{min}, Equation 30 predicts a reasonable light guide transmittance, $T_{light\ guide}$, of 0.5, so we will specify this value for our design:

$$\tau_{min} = 0.34 , \tag{32}$$

$$T_{light\ guide} = 0.50 . \tag{33}$$

We now have all the values that we need to calculate $A_{light\ guide}$. Substituting Equations 11, 12, 13, 27, 32, and 33 in Equation 31, we obtain

$$A_{light\ guide} = W^2 = 0.32\ ft^2 . \tag{34}$$

(Note that $-(Z/W)\ \ln \rho = 4.5 \gg 1$, so the asymptotic condition, Relation 28, holds as we had assumed.) Using Equations 4, 5, and 6, we can also determine the collector's aperture area:

$$A_{collector} = 533\ ft^2 . \tag{35}$$

The geometric concentration in the light guide ($A_{collector}/A_{light\ guide}$) is over 1600. Taking into account collector losses and the transmitted beam's obliquity relative to the light guide wall, the luminous flux incident on the wall would only be about 50 suns; and since the wall is 95% reflective only about 2.5 suns of luminous flux would actually be absorbed. (The absorbed radiant flux would be somewhat less than 2.5 suns since the collector would filter out much of the infrared and ultraviolet radiation.) Thus, this level of flux concentration would not necessarily exceed the light guide's thermal tolerance, although it would pose a fire safety concern.

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* If τ_{min} is close to 1, Equation 30 reduces approximately to

$$T_{light\ guide} \cong (1+2\tau_{min})\ /\ 3 \qquad \text{(asymptotic approximation; } \tau_{min} \cong 1) .$$

SQUARE-SECTION PRISM LIGHT GUIDE

A prism light guide (Whitehead et al. 1982) is basically a hollow rectangular-section duct with transparent walls, with the outer surfaces of the walls bearing a corrugated pattern of prism-shaped ridges running parallel to the light guide axis. Light is confined in the light guide by means of total internal reflection at the walls' corrugated surfaces. Heat gain in the light guide would not be likely to pose a problem, because the light guide material (acrylic) has little absorption in the visible spectrum. (The wavelengths it does absorb strongly could be filtered out in the collector.) The prism light guide does, however, exhibit significant transmittance loss due to light scattering from imperfections in the prism geometry that are unavoidable in the manufacturing process. In our optical model, we will describe these scattering losses in terms of a wall reflectance coefficient, ρ , that is analogous to the surface reflectance of a hollow reflective light guide.

We will consider here the case of a square-section prism light guide, which is characterized by its length Z, width W, and reflectance coefficient ρ . The values we will assume for Z and ρ will be the same ones used in the last section ($Z = 50$ ft, $\rho = 0.95$), and we will also define $\tau_{min} = 0.34$. We also need to know the refractive index, n_w, of the light guide's wall material. The refractive index of acrylic is

$$n_w = 1.491 . \tag{36}$$

The general expression for a prism light guide's characteristic function $\mathcal{A}(\tau)$ is very complicated, but there is a broad range of conditions under which the expression has a form identical to that of a hollow reflective light guide (Equations 19 and 20). This range of conditions is defined by the relation

$$1/\sqrt{1 + \left(\frac{Z \ln \rho}{W \ln \tau_{min}}\right)^2} \leq (\sqrt{2}-1)\sqrt{(n_w^2-1)} . \tag{37}$$

If this "equivalence condition" holds, the prism light guide would be optically equivalent to a hollow reflective light guide, and the previously calculated light guide aperture area requirement ($W^2 = 0.32$ ft^2) would also apply to the prism light guide. With the previous design specifications, the expression on the left of Relation (37) evaluates to 0.23 and the expression on the right is 0.46, so this design is well within the range of the equivalence condition.

CIRCULAR-SECTION SOLID DIELECTRIC LIGHT GUIDE

The light guide system that we will now investigate consists of a solid circular-section rod of high-transmittance optical material that is coated with a thin cladding of a different material whose refractive index is slightly lower than that of the core material. Light is confined inside the core via total internal reflection at the core-cladding interface. The light guide parameters are its length, Z; aperture radius, R; the core material's refractive index, n_i; the refractive index, n_o, of the cladding; and the core material's optical attenuation coefficient, α (i.e., its optical loss per unit length due to absorption and scattering) . We will assume the following material parameters:

$$n_i = 1.485, \tag{38}$$

$$n_o = 1.385, \tag{39}$$

$$\alpha = 0.0035 \text{ ft}^{-1} . \tag{40}$$

These values are characteristic of plastic optical fibers that have been fabricated under laboratory conditions using the material described in Kaino et al. (1983). (The attenuation coefficient of 0.0035 ft^{-1} corresponds to a 50 dB/km loss.)

In contrast to the case of hollow reflective light guides, a dielectric light guide's flux transmittance, $T_{light\ guide}$, is not highly sensitive to τ_{min}. Therefore, only a marginal gain in flux transmittance (and consequent collector area reduction) can be achieved by specifying a high value of τ_{min}. On the other hand, the light guide's aperture area is strongly sensitive to τ_{min} , so a low value of τ_{min} would be favored for this type of light guide. Any penalty that is incurred in collector cost by choosing a low τ_{min} value would likely be more than offset by light guide cost minimization, so we will take τ_{min} to be zero:

$$\tau_{min} = 0 . \tag{41}$$

A dielectric light guide's characteristic function, $\mathcal{A}(\tau)$, is defined as follows: We calculate two auxiliary parameters,

$$b = \sqrt{n_i^2 - n_o^2} , \tag{42}$$

$$c = n_i \sqrt{1 - (\alpha Z / \ln \tau)^2} \; ; \tag{43}$$

and we define $\mathcal{A}(\tau)$ by one of two expressions, depending on which of b or c is larger:

If $b \geq c$, then

$$\mathcal{A}(\tau) = \pi R^2 c^2;$$

if $b \leq c$, then

$$\mathcal{A}(\tau) = 2R^2[\pi b^2 + (c^2 - 2b^2)\sin^{-1}(b/c) - b\sqrt{(c^2 - b^2)}] \, . \tag{44}$$

An essential feature of the above expression is that the dependence of $\mathcal{A}(\tau)$ on the light guide's aperture area, $A_{\text{light guide}} = \pi R^2$, is one of a simple proportionality relationship. Thus, if we substitute Equation 44 in Equation 9 we get a relation that can be easily solved for $A_{\text{light guide}}$. With $\tau_{min} = 0$, this relationship yields

$$A_{\text{light guide}} = \pi R^2 = \frac{\Phi_{\text{output}}}{2\, T_{\text{light guide}}\, L_{\text{input}}\, [\pi(n_i^2 - n_o^2) + (2n_o^2 - n_i^2)\cos^{-1}(n_o/n_i) - n_o\sqrt{n_i^2 - n_o^2}\,]} \tag{45}$$

$$(\text{for } \tau_{min} = 0) \, .$$

Substituting the values from our example (equations 11, 27, 38, and 39), Equation 45 reduces to

$$A_{\text{light guide}} = \pi R^2 = (0.012 \text{ ft}^2) / T_{\text{light guide}} \, . \tag{46}$$

To evaluate Equation 10, we need to know the light guide's maximum ray transmittance, τ_{max}, which is

$$\tau_{max} = e^{-\alpha Z} . \tag{47}$$

For our example (Equations 12 and 40), this value is

$$\tau_{max} = 0.84 \, . \tag{48}$$

Appendix 4 presents a graphical design tool that represents Equation 10 for a circular-section dielectric light guide whose index ratio, n_i / n_o, is equal to our assumed value ($n_i / n_o = 1.485 / 1.385$).* The curves illustrate the ratio $T_{\text{light guide}} / \tau_{max}$ (as defined by Equations 10 and 47) as a function of τ_{max} for several values of the ratio τ_{min} / τ_{max}. Note that $T_{\text{light guide}}$ has no dependence on R, so we can obtain $T_{\text{light guide}}$ directly from the bottom curve:

$$T_{\text{light guide}} = 0.82 \, . \tag{49}$$

From Equations 46 and 49, we get

$$A_{\text{light guide}} = \pi R^2 = 0.015 \text{ ft}^2 \, . \tag{50}$$

We can also calculate the collector area from Equations 4, 5, and 6:

$$A_{\text{collector}} = 327 \text{ ft}^2 \, . \tag{51}$$

The geometric concentration in the light guide ($A_{\text{collector}} / A_{\text{light guide}}$) is close to 22,000, which is greater by an order of magnitude than the practical concentration limits of hollow light guides. The system would use a light guide aperture that is only about 1.7 inches in diameter to illuminate 10,000 ft^2 of floor space. Thus, dielectric light guides could be very practical for retrofit applications. One significant advantage of dielectric light guides is that the light guide transmittance does not depend on the light guide's aperture dimension, so there is no reason why the 0.015 ft^2 light guide aperture area would need to be concentrated in a single central light guide - it could be divided up among several smaller light guides, or a large number of flexible optical fibers could be used. Smaller light guides would be preferable to a large central light guide for a number of reasons: They would have better heat dissipation; they could be fit into small conduits and bent around tight corners more easily; the output flux could be delivered at several widely separated distribution

* Appendix 3 presents the same data for a rectangular-section dielectric light guide.

points; and the 327 ft^2 collector area could be divided up among several collectors of reasonable size and cost rather than consolidating it all in a single 20 ft diameter collector unit.

Dielectric light guides do, however, have one particular disadvantage in relation to hollow light guides: Whereas a hollow light guide's flux transmittance can always be increased by enlarging its aperture, a dielectric light guide's transmittance can never exceed an upper limit, τ_{max}, which is determined by the material's optical attenuation coefficient, α, and the ligtht guide length, Z. This limitation does not seriously constrain the performance of the system discussed above because we have assumed a very low attenuation coefficient; but if we were to use materials that are currently commercially available the attenuation losses might limit the practical light guide length to distances much shorter than 50 ft.

FLUID-FILLED DIELECTRIC LIGHT GUIDE

A fluid-filled dielectric light guide would consist of a transparent, fluid-filled pipe that confines light by means of total internal reflection at the pipe's outer surface. If the pipe wall is fairly thin, it would have no significant optical influence on the transmitted beam, and the device would be optically equivalent to a solid dielectric light guide that has a refractive index equivalent to that of the fluid material and has no cladding (i.e., the cladding material is air). The pipe material's optical attenuation would not need to be nearly as low as that of a solid dielectric light guide, since a typical ray's path would pass through the pipe wall over only a small fraction of its length. In addition, the volume of optical material in the pipe is small, so the cost of the pipe itself could be quite low. The fluid might be an inexpensive material like water, so this system could have a considerable cost advantage over solid dielectric light guides. An additional possible benefit of fluid-filled light guides would be that absorbed heat could be efficiently dissipated (or utilized) by cycling the fluid through a heat exchanger.

We will consider here a design example in which the fluid material is water, and we will assume that optical losses in the pipe wall are negligible. The light guide is optically equivalent to a solid dielectric light guide whose core index, n_i, is equal to that of water (n_i = 1.333) and whose cladding index is 1 (n_o = 1). The analysis is complicated by the strong spectral selectivity of water. Generally, all wavelengths longer than 600 nm would be absorbed by the light guide and only the blue-green portion of the spectrum would be transmitted. (The chromatic quality of the illumination should be taken into account in assessing the light guide's performance.) In order to accurately account for the light guide's spectral selectivity, its flux transmittance must be calculated (by the procedure outlined above) for a range of wavelengths, taking into account the wavelength dependence of water's attenuation coefficient, α. The light guide's actual operating flux transmittance, $T_{light\ guide}$, would be calculated as a weighted average of its wavelength-dependent transmittance, using the sun's spectral luminance distribution as a weighting function.*

For the example we are considering, a water-filled light guide would have a flux transmittance of $T_{light\ guide}$ = 0.47, an aperture area of $A_{light\ guide}$ = 0.011 ft^2 (i.e., about a 1.5 inch diameter aperture), and it would require a collector area of 561 ft^2. Due to the large collector area required and the chromatic characteristics of the output flux, this system may not be very practical. However, a shorter light guide would show more favorable performance. For example, if we change the length specification to Z = 15 ft, we obtain the following results: $T_{light\ guide}$ = 0.78, $A_{light\ guide}$ = 0.0069 ft^2 (i.e., the diameter is less than 1.2 inch), and $A_{collector}$ = 341 ft^2. These values assume that $\tau_{min} = 0$. If we specify a higher τ_{min} value, some marginal reduction in collector size could be gained at the expense of increased light guide area. But the light guide material is cheap and, moreover, a larger light guide aperture might be desired to facilitate heat dissipation; so a higher τ_{min} value might be more optimum.

LENS GUIDE

Figure 4 illustrates a design concept for an optical transport mechanism that uses lenses rather than reflective means to confine the transmitted beam. The collector system is illustrated schematically as a large objective lens that focuses the sun disk onto the aperture of a small field lens in the objective element's focal plane. A movable mirror between the two elements keeps the focused beam stationary as the objective element moves to track the sun. The field lens projects a focused image of the objective aperture onto the lens guide's entrance aperture. This image fills the aperture of the lens guide's first lens, which images the field lens aperture onto the second lens element's aperture. Each succeeding lens in the lens guide images the preceeding lens aperture onto the next lens aperture (e.g., lens 2 images lens 1 onto lens 3, etc.). The last lens focuses the aperture of the preceeding lens onto the lens guide's output aperture.

The analysis of this system does not quite fit into the framework that was developed earlier, but the general design approach is very similar. We will specify the design in terms of the following parameters: the length, Z, of the lens guide; its aperture radius, R; the number, N, of lenses (the lenses are equally spaced at a separation distance of Z/N); and the

---

* The following data were used in this analysis: The wavelength-dependent attenuation coefficient, α, was derived from data in Hale and Querry (1973) ($\alpha = 4\pi k / \lambda$, where k is the imaginary part of the complex refractivce index and λ is the wavelength); and the spectral luminance of sunlight was inferred from the radiant flux density data in Boer (1977) (tabulation on the bottom of p. 534) and the photopic eye response data in the IES Lighting Handbook (p. 3-5) .

lens flux transmittance, T_{lens}. We assume that the lenses have a large focal ratio:

$$Z/N >> R \tag{52}$$

The design procedure will be based on two fundamental relations that are analogous to Equations 9 and 10. In lieu of Equation 9, the lens guide's output flux is given by

$$\Phi_{output} = T_{light\ guide} L_{input} (N\pi R^2/Z)^2; \tag{53}$$

and in lieu of Equation 10, its transmittance is

$$T_{light\ guide} = T_{lens}^{N} . \tag{54}$$

The parameters T_{lens}, Z, L_{input}, and Φ_{output} would generally be specified, and the design problem would be to calculate N, R, and $T_{light\ guide}$ from Equations 53 and 54. Since we have three unknowns but only two constraints, the extra degree of freedom could be used to optimize the cost trade-off between the collector and the light guide system. But since we have no cost data, we will simply specify a reasonable minimum light guide transmittance, $T_{light\ guide}$, and will use this value to solve for N and R.

The lenses might , in practice, be cast acrylic elements with a single-layer, MgF_2 antireflective coating on both surfaces. The coating would limit reflection losses to about 1.6% per surface (Handbook of Plastic Optics 1983), so the lens transmittance would be

$$T_{lens} = 0.968 . \tag{55}$$

We will stipulate that the lens guide's transmittance is at least 75%:

$$T_{light\ guide} \geq 0.75 . \tag{56}$$

From Equations 54, 55, and 56, we can determine an upper bound on the number of lenses that can be used:

$$N = \ln (T_{light\ guide}) / \ln (T_{lens}) \leq \ln (0.75) / \ln (0.968) = 8.85 \tag{57}$$

In order to minimize the lens guide's aperture size, we should use as many lenses as are allowed by Relation 57; so we will use 8 lenses:

$$N = 8 . \tag{58}$$

With 8 lenses, the lens guide transmittance is

$$T_{light\ guide} = 0.77 . \tag{59}$$

Having determined N and $T_{light\ guide}$, we can solve Equation 53 for $A_{light\ guide}$:

$$A_{light\ guide} = \pi R^2 = \sqrt{\frac{\Phi_{output}}{T_{light\ guide}\ L_{input}}}\ \frac{Z}{N} . \tag{60}$$

Substituting the numbers from our example (Equations 11, 12, 27, 58, and 59), we obtain

$$A_{light\ guide} = \pi R^2 = 0.97\ ft^2. \tag{61}$$

The lenses have an aperture diameter of 2R = 13.4 inches. Except for the first element, each lens has a focal length of Z / (2N) = 37.5 inches. The geometric concentration ($A_{collector}$ / $A_{light\ guide}$) in the lens guide is 356. Heat gain could possibly be a problem at this concentration level, but the field lens (Figure 4) could function as a spectral filter, eliminating most of the nonvisible radiation that is absorbed by the lens material. In addition, the lenses would dissipate heat efficiently if they were formed as thin, Fresnel-type elements.

OPEN LIGHT WELL

The simplest light transport mechanism would be an open light well through which an unconfined beam of light is projected. This type of system is, in essence, a lens guide that uses just one lens, a projection lens at the top of the guide. If we again assume a lens transmittance of T_{lens} = 0.968, we obtain for N = 1 a lens guide transmittance of $T_{light\ guide}$ = 0.968, a lens apperture area of $A_{light\ guide}$ = 6.93 ft^2, and a collector area of $A_{collector}$ = 276 ft^2. The light well's

diameter is 36 inches, and the geometric concentration in the light well ($A_{collector}$ / $A_{light\ guide}$) is 40.

Except for the large size of the light well, this system is superior to all of the systems considered above in nearly every respect. It is extremely simple and uses very little optical material. (The projection lens could be a thin, light weight Fresnel element.) Its flux transmittance is very high; so the required collector area is the smallest of any system we have considered. (One drawback of the system in comparison to dielectric light guides is that the entire 276 ft^2 collector area may need to be consolidated in a single 19 ft diameter aperture. However, this limitation could perhaps be circumvented by using solid light guides such as optical fibers in the collector system to combine the output of a number of small collectors into a single light well.) The geometric concentration in the light well is lower than any of the other systems', which is favorable from a safety standpoint. (The field lens in the collector system would act as a spatial filter, ensuring that no light strays outside of the light well boundaries.) There are no optical surfaces in the light well that need to be protected, so the light well could perhaps also serve as an HVAC duct, offsetting its size disadvantage. Since the light well does not need an enclosure, it could be located outside the building or in an atrium space. (One might even consider configurations in which the beam is directed from a ground-based collector to the top of a building, or between buildings, etc.)

CONCLUSIONS

A core daylighting system, which uses sun-tracking solar collectors and light guides to channel sunlight to deep interior building spaces, could potentially offset a building's entire electric lighting load during periods of sunshine availability. In conjunction with efficient dimmable electric luminaires, such a system could provide a comfortable lighting environment while accruing considerable energy savings.

This paper summarizes results of a recent theoretical investigation pertaining to the light guide components of core daylighting systems. Several generic light guide types are analyzed, including hollow reflective light guides, prism light guides, solid dielectric and fluid-filled light guides, lens guides, and open light wells. Minimum theoretical aperture requirements are determined for each type as a function of the specified optical transport efficiency and design parameters (light guide length, transmitted luminous flux, etc.). Generally, a system's aperture requirement would be inversely related to its cost. Solid dielectric (e.g., optical fiber) light guides would be very compact and practical for retrofit applications, but their high cost would preclude their use for long-distance optical transport. Open light wells would be the simplest and least costly option, but would require the greatest aperture area. Hollow reflective light guides, prism light guides, or lens guides may offer the best compromise between cost and space requirements.

A general conclusion of this study is that the aperture requirements for practical light guide components could, in theory, be small enough for retrofit applications (e.g., a 95% reflective, 50-ft hollow light guide supplying 10,000 illuminated-ft^2 would have a minimum aperture requirement of only about 0.32 ft^2). But in order to achieve optical concentrations and efficiencies near the theoretical limit, the collector system would need to maintain optical and tracking tolerances exceeding the capabilities of existing systems, so further advances in core daylighting will require improvements in collector technology.

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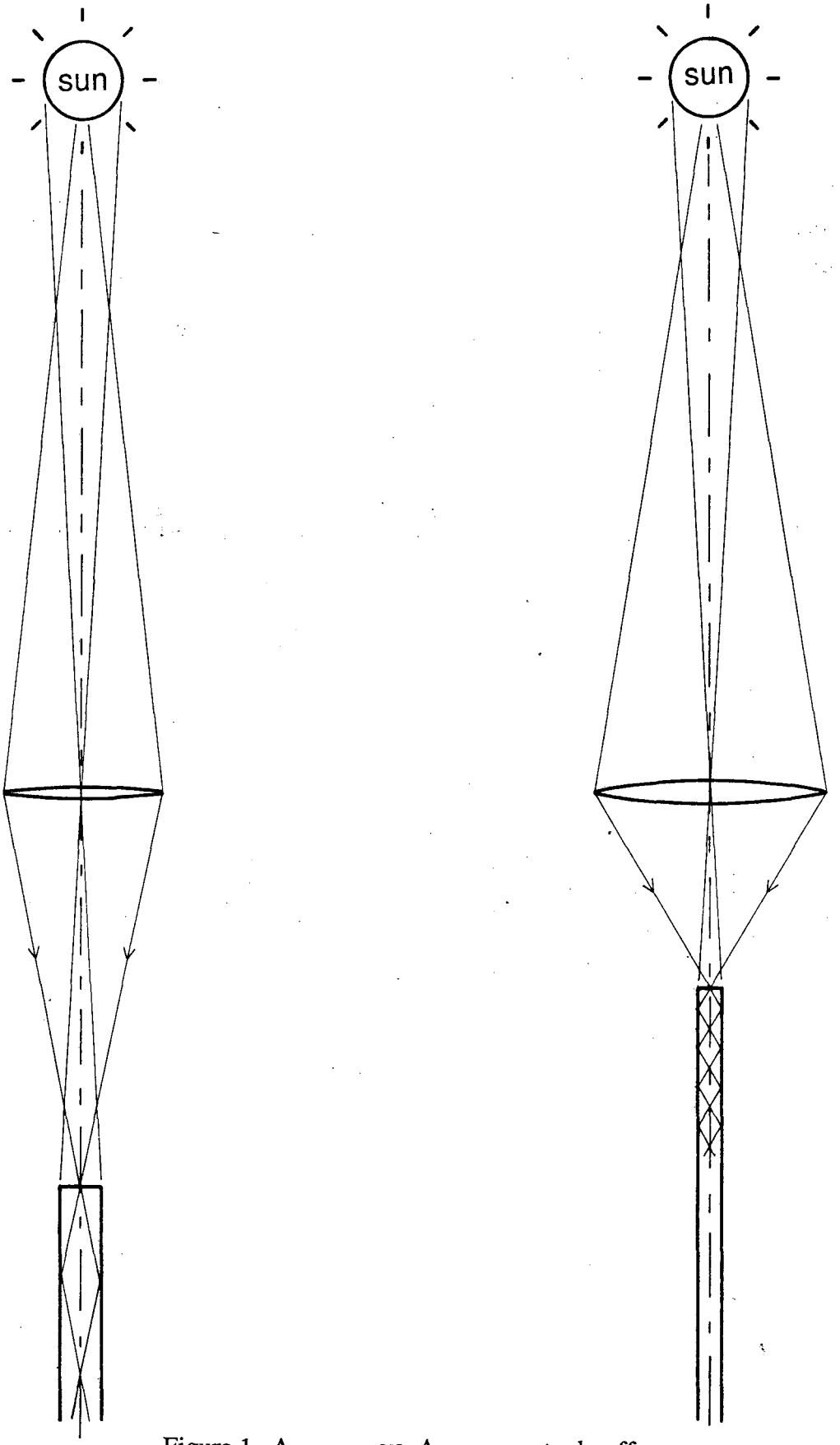


Figure 1. $A_{collector}$ vs. $A_{light\ guide}$ trade-off.

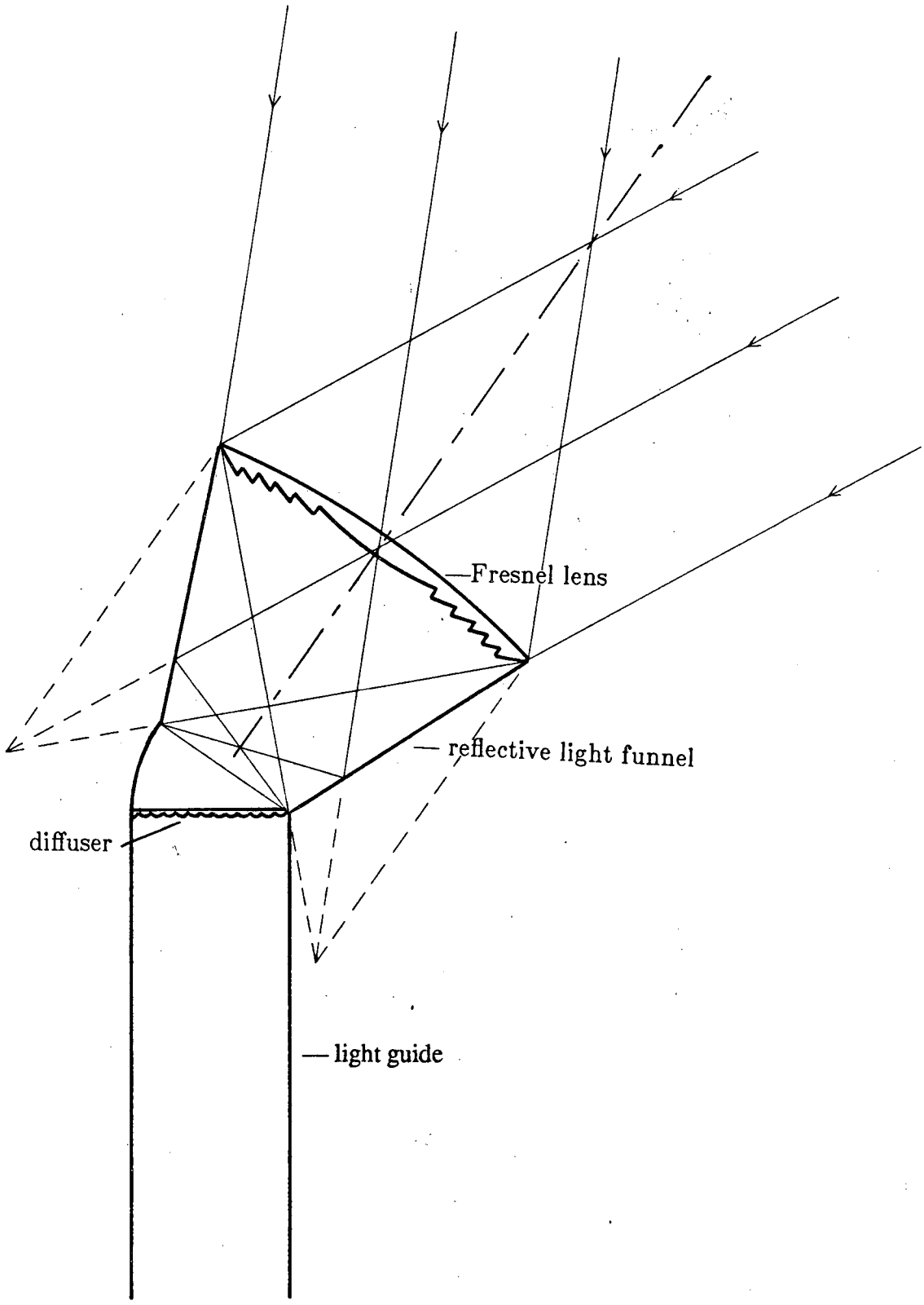


Figure 2. Nontracking collector.

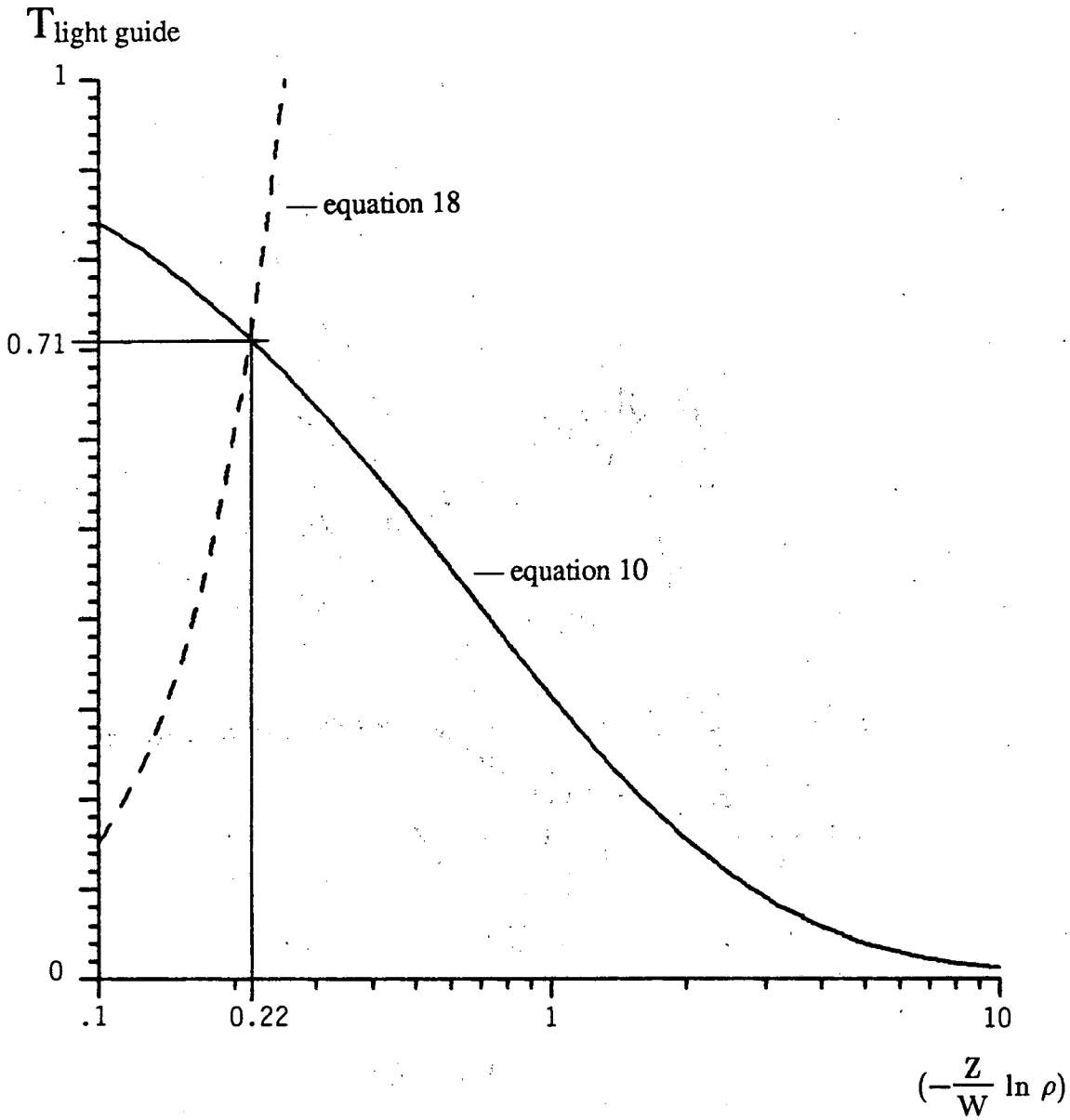


Figure 3. Graphical determination of light guide width, W.

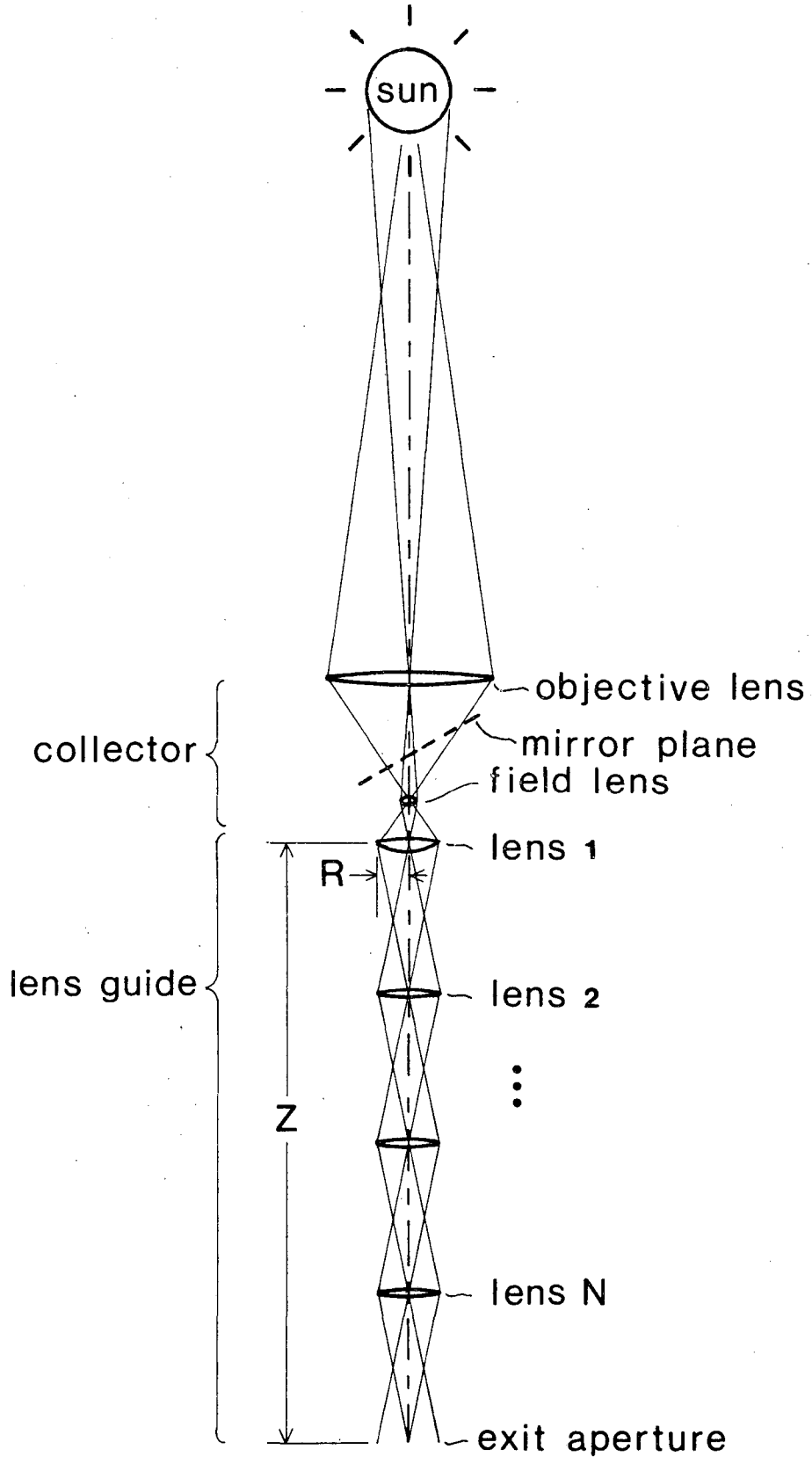
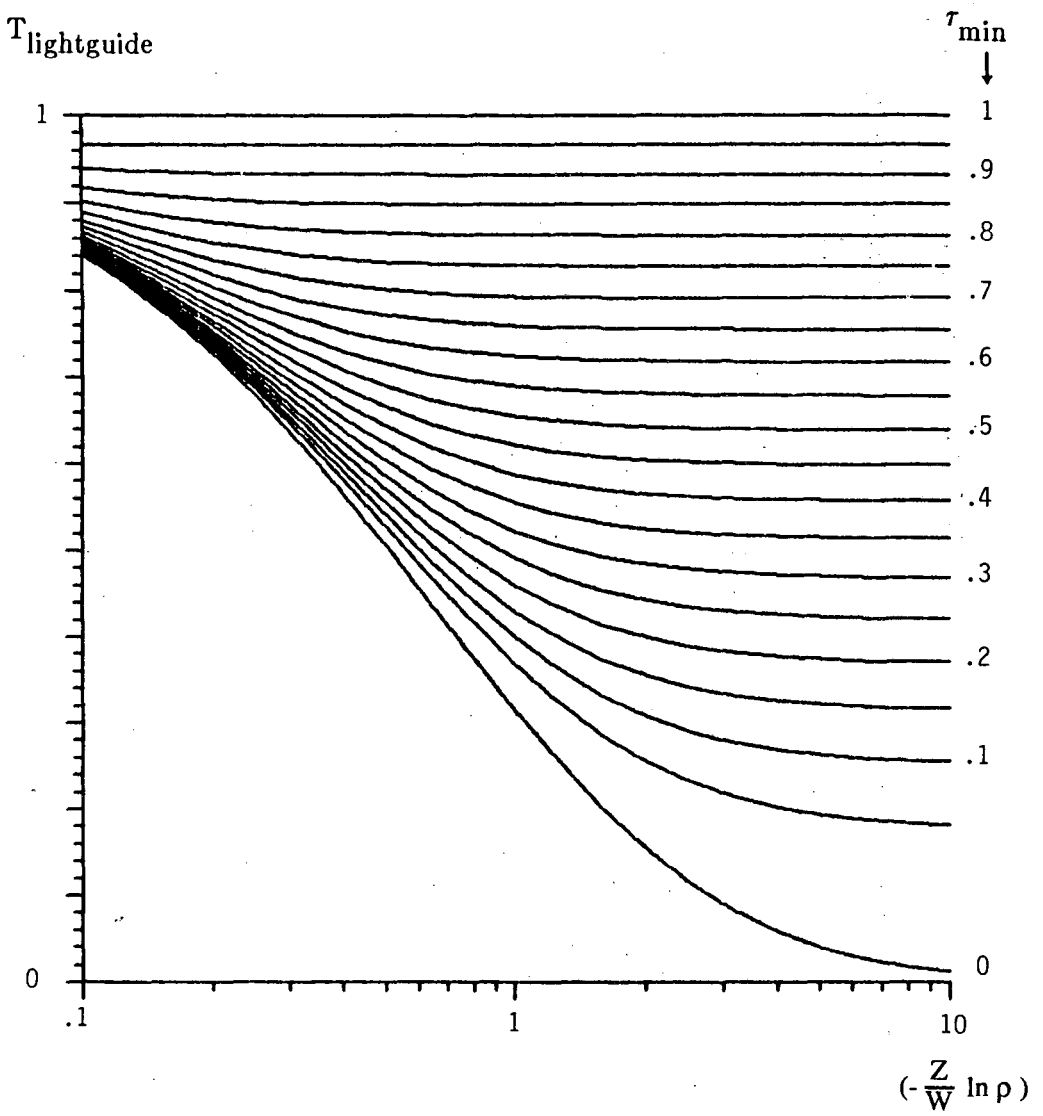


Figure 4. Lens guide.

Appendix 1. Transmission Characteristics of a Square-Section Hollow Reflective Light Guide



Design Parameters:

Z = light guide length
W = aperture width
ρ = wall reflectance

Characteristic Function:

$$\mathcal{A}(\tau) = \frac{4}{\pi} W^2 b \tan^{-1}(b)$$

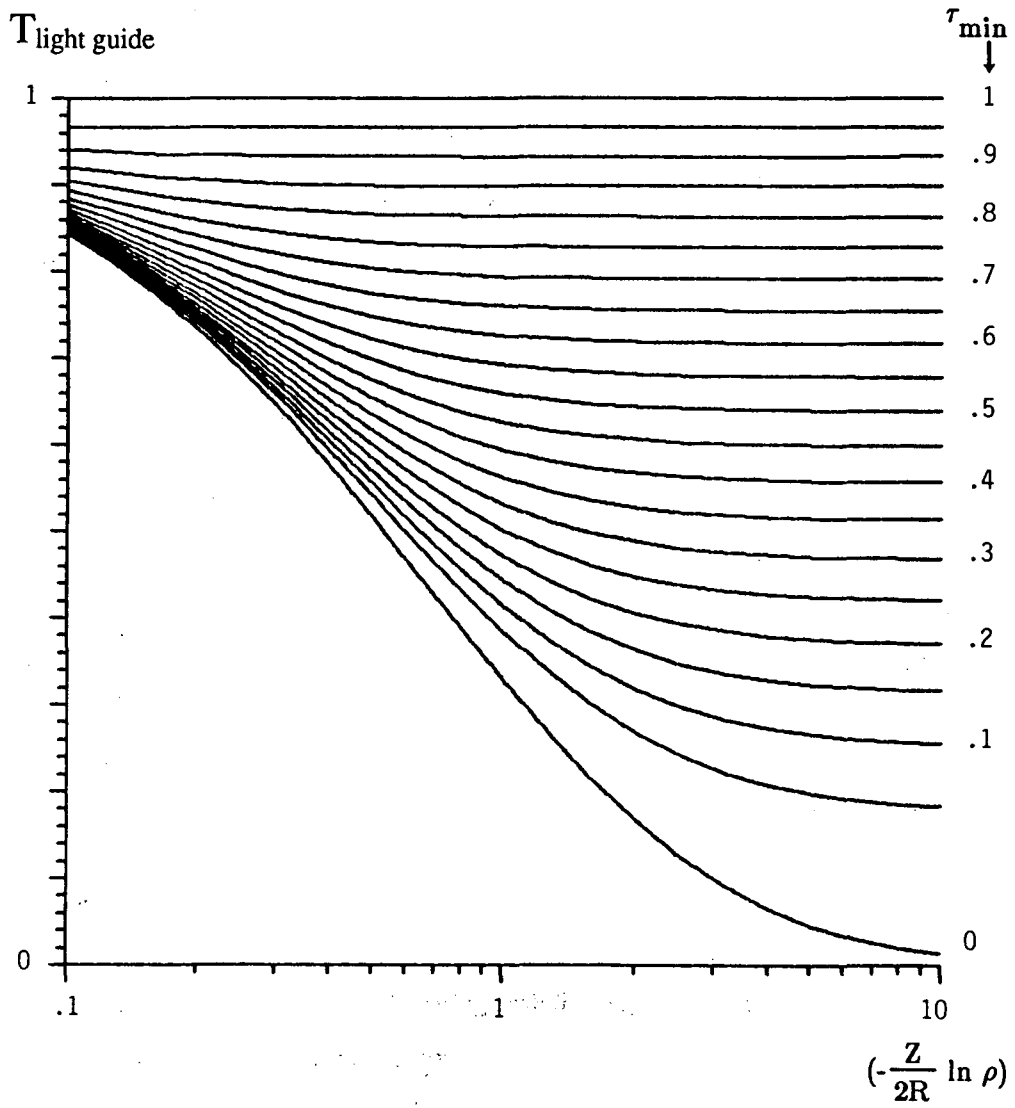
$$\text{where } b = 1/\sqrt{1+2\left(\frac{Z \ln \rho}{W \ln \tau}\right)^2}$$

Asymptotic Approximations $(-\frac{Z}{W} \ln \rho >> 1)$:

$$\mathcal{A}(\tau) \cong \frac{2}{\pi} \left(\frac{W^2 \ln \tau}{Z \ln \rho}\right)^2$$

$$T_{light\ guide} \cong 2(1-\tau_{min}(1-\ln\tau_{min})) / (\ln\tau_{min})^2$$

Appendix 2. Transmission Characteristics of a Circular-Section Hollow Reflective Light Guide



Design Parameters:

Z = light guide length
R = aperture radius
ρ = wall reflectance

Characteristic Function:

$$\mathcal{A}(\tau) = \pi R^2 (1 - 2(\frac{1}{b^2} - 1)(1 - \sqrt{1 - b^2}))$$

$$\text{where } b = 1/\sqrt{1 + (\frac{Z \ln \rho}{2R \ln \tau})^2}$$

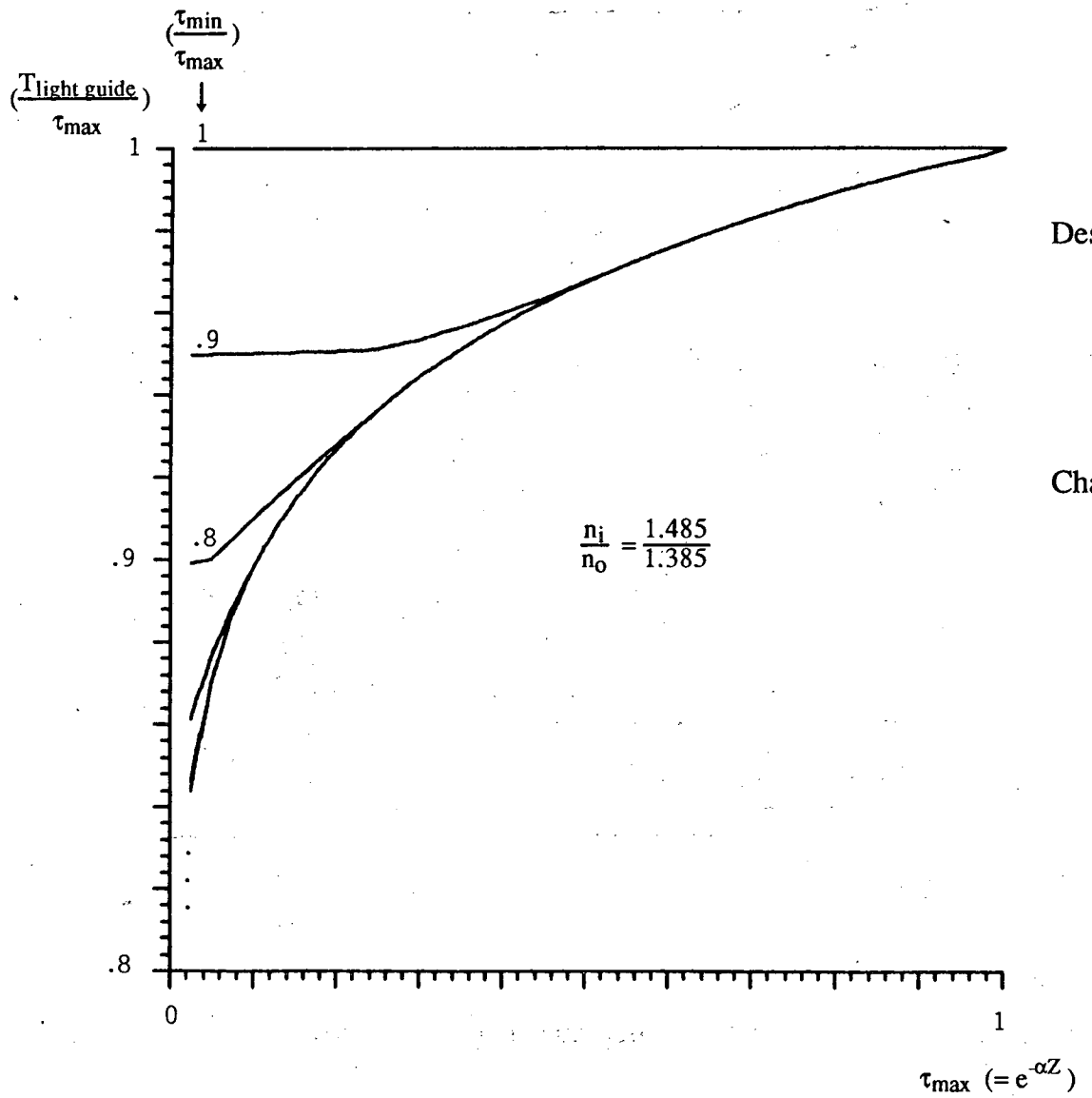
Asymptotic Approximations $(-\frac{Z}{2R} \ln \rho >> 1)$:

$$\mathcal{A}(\tau) \cong 3\pi (\frac{R^2 \ln \tau}{Z \ln \rho})^2$$

$$T_{light\ guide} \cong 2(1 - \tau_{min}(1 - \ln \tau_{min})) / (\ln \tau_{min})^2$$

Appendix 3.

Transmission Characteristics of a Rectangular-Section Dielectric Light Guide $\left(\frac{n_i}{n_o} = \frac{1.485}{1.385}\right)$



Design Parameters:

Z = light guide length
W1,W2: aperture dimensions
n_i = core refractive index
n_o = cladding refractive index
α = attenuation coefficient

Characteristic Function:

$$b = \sqrt{n_i^2 - n_o^2}$$

$$c = n_i \sqrt{1 - (\alpha Z / \ln \tau)^2}$$

if $b \geq c$, then

$$\mathcal{A}(\tau) = W_1 W_2 c^2$$

if $b \leq c \leq \sqrt{2}\, b$, then

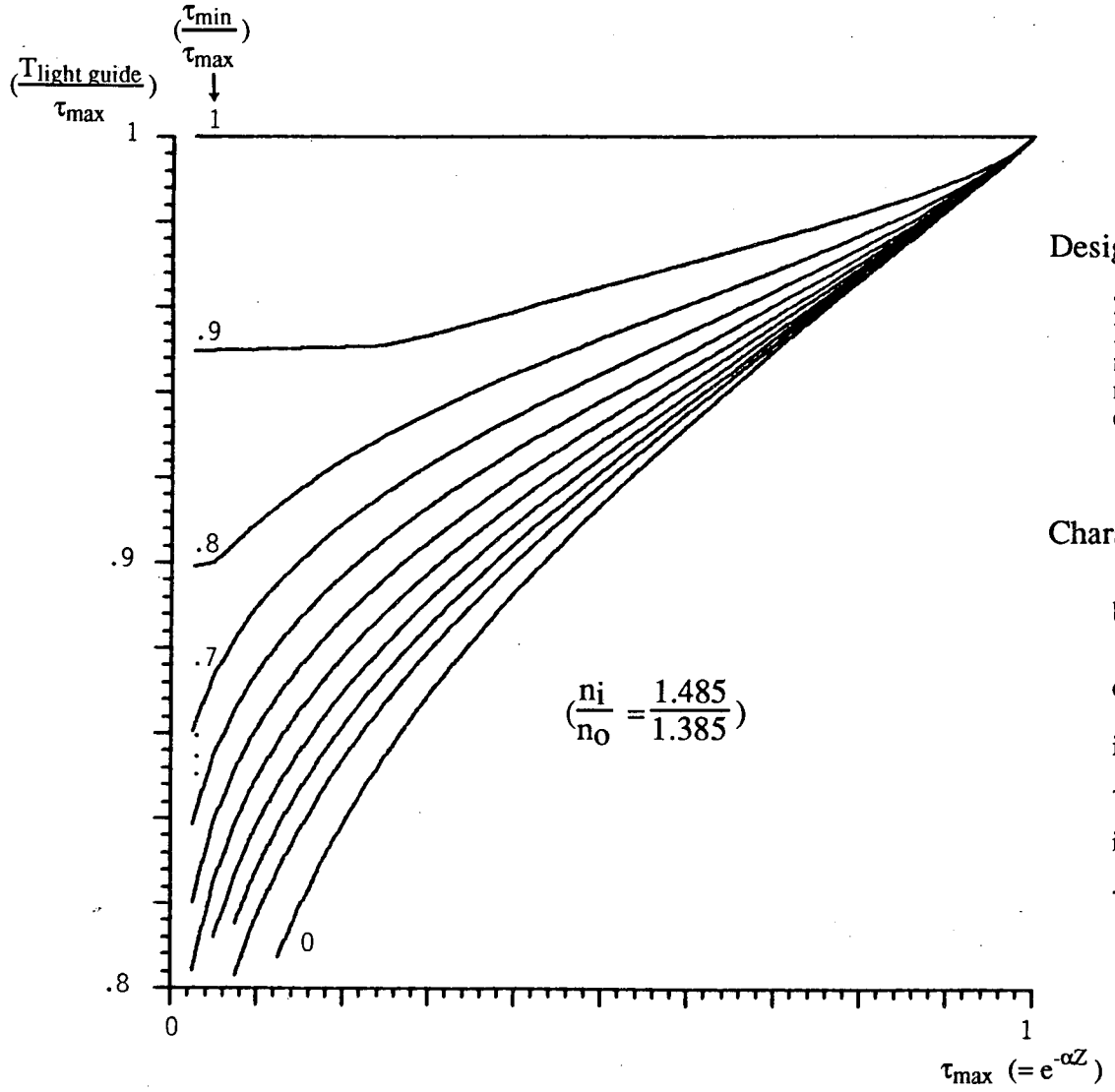
$$\mathcal{A}(\tau) = \frac{4}{\pi} W_1 W_2 \left[b \sqrt{c^2 - b^2} + c^2 \left(\frac{\pi}{4} \cos^{-1}\left(\frac{b}{c}\right)\right)\right]$$

if $c \geq \sqrt{2}\, b$, then

$$\mathcal{A}(\tau) = \frac{4}{\pi} W_1 W_2 b^2$$

Appendix 4.

Transmission Characteristics of a Circular-Section Dielectric Light Guide $\left(\frac{n_i}{n_o} = \frac{1.485}{1.385}\right)$



Design Parameters:

Z = light guide length
R = aperture radius
n_i = core refractive index
n_o = cladding refractive index
α = attenuation coefficient

Characteristic Function:

$$b = \sqrt{n_i^2 - n_o^2}$$

$$c = n_i \sqrt{1 - (\alpha Z / \ln \tau)^2}$$

if $b \geq c$, then

$$\mathcal{A}(\tau) = \pi R^2 c^2$$

if $b \leq c$, then

$$\mathcal{A}(\tau) = 2R^2[\pi b^2 + (c^2 - 2b^2)\sin^{-1}(b/c) - b\sqrt{(c^2 - b^2)}]$$

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