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CAN AND DOES THE POMERON OCCUR MORE THAN ONCE IN A SINGLE PROCESS?*

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ABSTRACT

A study of high energy diffractive amplitudes (the elastic amplitude being a special case), has revealed the following regularities at small momentum transfers: (a) They all tend to be almost purely imaginary, and (b) They all have the same energy dependence, leading to universal, constant (modulo logarithms) cross sections at high energies. In this paper, it is assumed that these regularities are produced by an underlying, common mechanism, which is defined as the pomeron. The question then addressed is whether the pomeron, so defined, can and does occur more than once in a single process.

It is demonstrated that various models for the pomeron (involving Regge poles, Regge cuts, geometric ideas like diffraction, etc.) lead to different answers to this question, none of them quantitative. By contrast, the introduction of the pion-pole dominance (PPD) hypothesis is shown to lead to a model-independent quantitative answer. Assuming just the above definition of the pomeron, the PPD hypothesis predicts certain processes that must be termed multi-pomeron by the advocates of all models, and provides estimates for their cross sections. The predictions of this hypothesis are compared with experiment.

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It is shown that PPD leads to, and sets lower bounds for, inclusive triple-pomeron cross sections assuming no more than our general definition of the pomeron. It is pointed out that the repetition of the pomeron--guaranteed by PPD--may be used to set upper bounds on asymptotic total cross sections. The crucial property of the result --that total cross sections must eventually die away--is that it does not rely on any model-dependent property of the pomeron, such as factorization.

1. INTRODUCTION

Consider the collision of two particles a and b. We shall call this process a diffractive process if:

(i) The final particles fall into two clusters A and B (in rapidity) centered around particles a and b respectively, and

(ii) The quantum numbers of A and B are those of a and b respectively.

It should be emphasized that we use the word diffraction to refer only to these two properties of an event and do not imply any underlying optical model mechanism. Clearly elastic events fall under the class of diffractive events as defined above.

Imagine a rapidity plot of an event in which a and the cluster A occupy one end, say the left end, while b and B occupy the right end. If there is a large rapidity gap between the rightmost member of A and the leftmost member of B, we shall term it a high energy diffractive event. The following regularities have been detected empirically in the study of the amplitudes for such events:

(i) They all tend to be purely imaginary in the "forward" directions, that is, in regions of small momentum transfer t across the large rapidity gap.

(ii) They all have the same energy dependence in the small t region, leading to the universal, constant† cross section.

In the elastic case, these two properties, together with the optical theorem, imply that total cross sections at high energies are constant.

The universality of these two properties of diffractive amplitudes at high energies suggests a common underlying mechanism. It is assumed here that such a mechanism exists, and is called the pomeron. No specific models such as Regge poles, cuts or optical descriptions are assumed for the pomeron. It is simply defined by the context in which it occurs--as the controlling mechanism behind all high energy diffractive processes.

It should be pointed out that the word pomeron was originally coined by the Reggeists to stand for a factorizable Regge pole, which was their model for this mechanism. To avoid confusion, I shall use the word pomeron when referring to the mechanism in a model independent way and the word pomeron pole, when referring to the Reggeist's model for it.

The question before us is this: "Can the pomeron, as defined above, occur more than once in a single process?" In what follows, we shall try to answer this question restricting ourselves to a subset of diffractive events--the elastic ones. This is done only in the

† Unless otherwise stated, the word constant should be taken modulo logarithms.

interest of simplicity and brevity. In other words, whereas we shall consider from now on, only those situations in which the pomeron controls high energy elastic amplitudes, the conclusions we reach about its multiple occurrence are valid for the pomeron defined in the broad sense, as the mechanism behind all high energy diffractive events.

Consider the total cross section for two particles a and b. It typically has an energy dependence shown in fig. 1. There is a low energy resonance region characterized by sharp bumps which gives way to a smooth Regge region around S_{ab}^{R}. At higher energies, around S_{ab}^{*}, the Regge region turns into a flat region. The interesting fact is that while the two lower energy regions differ in their shapes as we change the particles a and b, the region above S_{ab}^{*} has a universal form. From the optical theorem, this implies that the corresponding elastic amplitudes must have a universal energy dependence in the forward direction. It is also found in the region above S_{ab}^{*}, that the elastic amplitudes are almost purely imaginary at small t. This then is the high energy diffractive region referred to earlier and according to our definition, the pomeron controls the elastic amplitude above the "pomeron threshold", S_{ab}^{*}. Can the pomeron, so defined, occur more than once in a single process?

There is no unanimity among the theorists in their answers, since different factions of theorists believe in different models and different models give different answers. This is not surprising, considering the diversity in the models. While the reggeists argue among themselves on whether to represent the pomeron by a factorizable Regge pole or by some nonfactorizable object like a cut in the J-plane; the advocates of the geometric models speak in terms of absorbing target

discs and projectiles diffracting around them. This state of affairs is elaborated in section 2.

Is there a model independent way of answering the question? Can one, assuming no more than a definition of the pomeron as the mechanism controlling all high energy elastic amplitudes, decide the question of its multiple occurrence? One can, if one steps outside current high energy ideas and invokes the old notion of pion pole dominance (PPD). It is shown in section 3 that armed with this hypothesis, one can define multi-pomeron processes, and estimate their cross sections, assuming no more than our general definition of the pomeron. In short, PPD provides a model-independent† and quantitative answer to the question of multi-pomeron processes. The predictions of this hypothesis are compared with experiment in section 4.

One can also use PPD to define and set lower bounds for inclusive triple-pomeron cross sections; as well as to set upper bounds on asymptotic, pomeron dominated cross sections; all in a model independent fashion. These ideas are discussed in section 5.

2. WHAT DO THE DIFFERENT MODELS OF THE POMERON SAY ABOUT THE QUESTION OF MULTI-POMERON PROCESSES?

I will consider just three models. They will suffice to show that the question of multi-pomeron processes, if analyzed within the language of the existing theories of the pomeron, becomes highly model dependent.

† The PPD hypothesis is, itself, a model. The words "model independent," as used in this paper, should be taken to mean "independent of any models for the pomeron."

A. The pomeron pole model

In the Regge language, a pomeron pole at† $J = 1$, with even signature, is the most economical way to explain the regularities mentioned earlier. The unit intercept provides the s^1 behavior, while the signature factor, $i - \cot\left(\frac{1}{2}\pi\alpha_P(t)\right)$, provides the correct phase at $t = 0$.

The elastic amplitude, for a typical $ab \to ab$ process, has the following form, when dominated by the pomeron pole:

$$M_{ab\to ab}(s,t) = \beta_{aaP}(t)\,(s)^{\alpha_P(t)}\,\beta_{bbP}(t) \quad . \tag{2.1}$$

This amplitude is represented pictorially in fig. 2. The factorized form allows us to abstract the pole, with a trajectory $\alpha_P(t)$, and speak of it in other reactions. Consider, for example, the process $ab \to ab\pi^+\pi^-$; in a part of phase space where the rapidity ordering of the particles is as shown in fig. 3.

If the subenergy $s_{a+} = (P_a^f + P_+)^2 > s_{a+}^*$, and the subenergy $s_{b-} = (P_b^f + P_-)^2 > s_{b-}^*$, Regge theory gives for the amplitude,

$$M_{ab\to ab\pi^+\pi^-} = \beta_{aaP}(t_1)(s_{a+})^{\alpha_P(t_1)}\,\gamma_{\pi\pi PP}(s_{\pi\pi}^c,t,t_1,t_2)$$

$$\times\,(s_{b-})^{\alpha_P(t_2)}\,\beta_{bbP}(t_2) \quad , \tag{2.2}$$

as depicted in fig. 4.

Let us summarize what Regge theory tells us, granted that the pomeron is indeed a factorizable pole.

† When I speak of a moving singularity, such as the pomeron pole, $\alpha_P(t)$, being <u>at</u> $J = 1$, I mean $\alpha_P(0) = 1$.

(a) It clearly defines a double-pomeron process as one in which the pomeron pole occurs twice in the amplitude, as in eq. (2.2) or fig. 4. The pomeron pole encountered here is the one from the elastic reaction that originally defined it [eq. (2.1) or fig. 2].

(b) While Regge theory says that the external couplings, $\beta(t)$, are the same ones encountered in the elastic case, all it says of γ, the central coupling, is that γ it independent of a and b. It gives neither the scale of γ, nor the dependence on the variables, $S_{\pi\pi}{}^{c}$, t, t_1, and t_2.

(c) Regge theory does, however, give the dependence of the amplitude on the subenergies, S_{a+} and S_{b-}. This dependence may be used by the experimentalist to identify double-pomeron processes.

In short, granted a pomeron pole, Regge theory admits and defines a double-pomeron process, but leaves it to experiment to set the scale or rate. This conclusion is true for a general multi-pomeron process.

B. <u>The Regge cut model of the pomeron</u>

Theoretical analysis following the introduction of the pomeron pole has indicated that such a simple description of the pomeron leads to inconsistencies, For one thing, if there is a pomeron pole at $J = 1$, as suggested by the observed constancy of total cross sections, the multi-pomeron branch points accumulate at $J = 1$ [1]. For another, starting with a factorizable pomeron pole at $J = 1$, one can get into situations where some partial cross sections exceed the total, unless the triple-pomeron coupling, $g_{PPP}(t)$, vanishes at $t = 0$ [2,12]. At present, when neither $g_{PPP}(0)$ nor the importance of multi-pomeron branch points is known, the J-plane singularity associated with the pomeron is obscure. What does Regge theory say about the possibility

of multi-pomeron processes, if the pomeron is represented by a non-factorizable J-plane singularity, such as a cut? Strictly speaking, it is incorrect to speak of the recurrence (single or multiple), of a nonfactorizable singularity. The reason is that such singularities, unlike factorizable poles, do not have an identity independent of the specific reaction they occur in. For example, if the leading J-plane singularity in the high-energy elastic ab amplitude were a cut, we could not dissociate the cut from the particles a and b. The only time we can be sure that this same cut occurs in a different process, is when the amplitude involves explicitly the high energy a-b amplitude as a factor.

There is, however, a slightly nonrigorous way of speaking of a nonfactorizable singularity without associating it with a specific reaction, and that is by its location in the J-plane or, alternatively, by the energy dependence it produces. Motivated by the universal high energy dependence of all total cross sections, one may assume that their J-planes are universal, <u>in that their leading singularities will have the same location</u>. If, therefore, one defines the salient feature of the pomeron to be this energy dependence, one may define a multi-pomeron process as one in which this dependence is repeated. For example, in the reaction $ab \to ab\pi^+\pi^-$ discussed earlier (fig. 3), with $S_{a+} > S_{a+}{}^{*}$ and $S_{b-} > S_{b-}{}^{*}$, if one finds the same dependence of the amplitude on these variables as in the elastic a-π and b-π reactions, respectively, one may refer to this reaction as a double-pomeron process. While such a definition tells the experimentalist what to look for, Regge-cut theory does not provide an explicit form such as eq. (2.2), for the amplitude of this process.

C. The geometric or diffraction model [3]

In this model, the collision of particles a and b is viewed in geometric terms. The projectile a sees the target b as a disc. At high energies, the disc becomes highly absorbing, due to the preponderance of inelastic channels. In a naive sense, the constancy of total cross sections may be understood in terms of a constant radius, R, of the disc. The phase is largely controlled by the absorptivity. To see this connection, consider the following rather artificial, but illustrative example. For a collision of spinless particles, the partial wave series for the amplitude is given by

$$M(S,\theta) = \sum_{\ell=0}^{\infty} (2\ell + 1) \frac{\left(\eta_\ell e^{2i\delta_\ell} - 1\right)}{2ik} P_\ell(\cos\theta) .$$

Let us resort to the following simple minded description of the scattering:

(a) The target disc absorbs $(\eta_\ell = 0)$ all partial waves that impinge on it, i.e., till $\ell = \ell_{max} = kR$; where k is the momentum of the projectile in the target rest frame,

(b) All higher partial waves go unaffected, $(\eta_\ell = 1, \ \delta_\ell = 0)$.

The phase of the amplitude is then clearly imaginary. In practice, of course, the description is more complicated [4].

It is curious that the geometric diffraction model, which, despite its vastly different logic, concurs with the Regge pole model regarding many of the features of the pomeron in high energy elastic amplitudes, takes a very different stand on the question of multi-pomeron processes. Where are the two absorbing discs in the reaction $ab \to ab\pi^+\pi^-$ that might justify calling this reaction a double-

pomeron process? Advocates of the geometric model see no reason for--indeed no meaning for--the repetition of the pomeron.

Having discussed at some length the various models of the pomeron and the varying answers they give to the question of multi-pomeron processes, we are now ready to embark on a study of the PPD hypothesis and the model-independent answer it provides.

3. THE PION POLE DOMINANCE HYPOTHESIS

In this section, the question of multi-pomeron processes will be discussed, assuming no more than our general definition of the pomeron. For simplicity, let us consider a specific reaction, $\pi^- p \to \pi^- p \pi^+ \pi^-$. Let us go to the part of phase space shown in the rapidity plot of fig. 5. It is a general property of the amplitude that, when $t = (P_a^f + P_+ - P_a^i)^2 = \mu^2$, it is given by a pion pole, with a factorizable residue:

$$M_{\pi^- p \to \pi^- p \pi^+ \pi^-} \xrightarrow[t \to \mu^2]{} \frac{A_{\pi^+\pi^-}(V_L)\, A_{\pi^- p}(V_R)}{t - \mu^2} , \qquad (3.1)$$

as pictorially represented in fig. 6.

In eq. (3.1), the factor $A_{\pi^+\pi^-}(V_L)$ is the elastic, $\pi^+\pi^-$ scattering amplitude, as a function of the variables, V_L, associated with the left blob. A similar definition holds for $A_{\pi^- p}(V_R)$ at the right blob.

The crucial point is that if the two subenergies, S_{a+} and S_{a-}, exceed the pomeron thresholds, S_{a+}^* and S_{b-}^*, the pomeron will occur in each blob by definition, and produce the characteristic subenergy dependence and phase in the two elastic amplitudes.

This process must be termed double-pomeron by any standards, since the precise situations that contain the pomeron by definition, occur twice. The form of the amplitude in eq. (3.1) allows us another way of seeing this. Let us use, in eq. (3.1), the principle of CPT invariance to replace the amplitude $A_{\pi^- p}(V_R)$ by the amplitude for the CPT-transformed process, $A_{\pi^+ \bar{p}}(V_R)$. We may now see the amplitude M as describing a two stage process--the reaction $\pi^+ \bar{p} \to \pi^+ \bar{p}$ followed by the reaction $\pi^+ \pi^- \to \pi^+ \pi^-$--in which the π^+ going into the second collision is the one that came out of the first. At $t = \mu^2$, this π^+ is a real pion, and the two collisions are real collisions and can be separated in space-time. Clearly these two elastic events are independent, and the pomeron, whatever be the model for it, will occur in each, if the subenergies are above the pomeron thresholds. We thus see that there are really two discs in this process--one in each elastic collision. By the same token, there are two pomeron poles or two Regge cuts or two of whatever-you-think-the-pomeron-is. There is, however, a catch to this argument. The point $t = \mu^2$ where these considerations apply, is outside the physical region which is confined to negative t. The redeeming factor is the smallness of the quantity μ^2 ($\simeq 0.02$ GeV^2) which prompts the following hypothesis of PPD: The amplitude in the physical region is given by the factorizable function [eq. (3.1)] defined at the pole, multiplied by a t-dependent form factor, $f(t)$. Although the entire physical region is not close to the pion pole, the region where the amplitude is significant is close to it, since the $(\text{amplitude})^2$ contains the factors $(t - \mu^2)^{-2}$ and $f(t)$, both of which are rapidly falling functions

of $t' = -t$ †. Support for the PPD hypothesis and the specific choice of the form factor appropriate to this problem are discussed at length in the Appendix. For the present let us accept a simple-minded form factor given by $f(t') = 1$ for $0 < t' < T$, and zero beyond. The Appendix will justify this choice and provide the value for T.

Starting with the matrix element M of eq. (3.1), we can integrate $|M|^2$ over t' up to T, over the blob subenergies from the pomeron thresholds up to the kinematically allowed maxima, to get $\sigma^{DP}_{+-}(S,T)$, the double-pomeron cross section for this ordering (fig. 5) of the central pions. The following is the result:

$$\sigma^{DP}_{+-}(S,T) = \frac{2.5}{16\pi^3 S^2} \int_{S_{a+}^*}^{ST/(S_{b-}^* - m^2)} dS_{a+}\, S_{a+}\, \sigma^{e\ell}_{\pi^+\pi^-}(S_{a+})$$

$$\times \int_{S_{b-}^*}^{[(ST/S_{a+})+m^2]} dS_{b-}(S_{b-} - m^2)\sigma^{e\ell}_{\pi^- p}(S_{b-})$$

$$\times \int_{t'_{min}=[S_{a+}(S_{b-}-m^2)/S]}^{T} \frac{dt'}{t'^2}\ \text{mb} \quad . \qquad (3.2)$$

This formula is from ref. [5], adapted to a situation where the energies, S, S_{a+}, and S_{b-} are large and the pion mass is ignored. (The last approximation leads to little error, due to the t'_{min} limit.) The

† We shall be using both variables t' and t in the future.

kinematical upper limit on the subenergies guarantees that t'_{min} never exceeds T. The proton mass is m.

To estimate $\sigma^{DP}_{+-}(S,T)$ from eq. (3.2), one can feed in the empirical π-π and π-p cross sections and perform a numerical integration. Since these calculations are anyway quite approximate, let us resort to a simplification that gives a quick estimate. Let us replace the elastic cross sections, which vary slightly with the subenergies, by constants $\sigma^{e\ell}(\infty)$, that represent their average behavior in the region between the pomeron thresholds and the kinematically allowed maxima. With this simplification the integral can be easily performed to give

$$\sigma^{DP}_{+-}(S,T) = \frac{T \times 2.5 \times \sigma^{e\ell}_{\pi\pi}(\infty) \times \sigma^{e\ell}_{\pi p}(\infty)}{16\pi^3}\left[\frac{1}{2}\log Z - \frac{3}{4} + \frac{1}{Z} - \frac{1}{4Z^2}\right] \text{ mb} \tag{3.3}$$

where

$$Z = \frac{ST}{S_{a+}^{*}(S_{b-}^{*} - m^2)} .$$

Choosing $\sigma^{e\ell}_{\pi\pi}(\infty) = 3$ mb, $\sigma^{e\ell}_{\pi p}(\infty) = 5$ mb, $S_{a+}^{*} = 2$ GeV^2, $S_{b-}^{*} = 4$ GeV^2, $T = 0.25$ GeV^2, leads to

$$\sigma^{DP}_{+-} = 13.4\ \mu\text{b} \qquad \text{at } S = 386 \text{ GeV}^2.$$

The value of S is chosen to facilitate comparison with recent experiments performed at NAL at 205 GeV/c. While the choice of T is discussed in the Appendix, it must be mentioned here that it could be lower in principle, but not likely to be lower than 0.125 GeV^2.

The pomeron thresholds are chosen to eliminate all prominent resonances. Some contamination from lower trajectories is inevitable. This question is taken up later.

A similar calculation for σ^{DP}_{-+}, the cross section for events with the other ordering of the two central pions in rapidity, yields a formula similar to eq. (3.1), with

$$Z = \frac{ST}{S_{a-}^{*}(S_{b+}^{*} - m^2)} .$$

In this formula, S_{a-}^{*} is the pomeron threshold for a $\pi^-\pi^-$ system. Due to the lack of any structure in the cross section in this channel, it is hard to select a value for S_{a-}^{*}. The following alternative criterion for pomeron dominance is suggested and is to be observed both in the calculation of the theoretical estimate and in the experimental selection of double-pomeron events: The $\pi^-\pi^-$ subsystem is pomeron dominated when the rapidity gap Δy, between the two pions is two units or more. For a phenomenological connection between $\Delta y \geq 2$ and pomeron dominance, see ref. [7]. The theoretical estimate, which deals with subenergies rather than rapidities, requires us to convert a minimum rapidity gap $\Delta y = 2$ into a minimum subenergy S_{a-}^{*}. If the two pions have transverse momenta $\vec{P}_a$ and $\vec{P}_-$, and have $\Delta y = 2$ their subenergy is

$$S_{a-}^{*}(\vec{P}_a,\vec{P}_-) = 2\mu^2 + \left(|\vec{P}_a|^2 + \mu^2\right)^{\frac{1}{2}}\left(|\vec{P}_b|^2 + \mu^2\right)^{\frac{1}{2}} \cosh 2 - 2\vec{P}_a\cdot\vec{P}_b .$$

Assuming that on the average,

(i) $|\vec{P}_a| = |\vec{P}_b| \simeq 0.3$ GeV/c

(ii) $\vec{P}_a \cdot \vec{P}_b \simeq 0$ we get $S_{a-}^{*} = 0.8$ GeV^2. With the other parameters same as before, we get

$$\sigma^{DP}_{-+} = 20.4\ \mu b \quad \text{at} \quad S = 386\ \text{GeV}^2.$$

The total double-pomeron cross section is given by

$$\sigma^{DP}(S = 386\ \text{GeV}^2) = \sigma^{DP}_{+-} + \sigma^{DP}_{-+} = 33.8\ \mu B \ .$$

The same set of parameters yields for the reaction $pp \to pp\pi^+\pi^-$ a total double-pomeron cross section of 31.3 μb at 205 GeV/c ($S = 387$ GeV^2).

Comparison with experiment: Recently two groups have measured double pomeron cross sections as defined in this paper. The reaction $\pi^- p \to \pi^- p\pi^+\pi^-$ at 205 GeV/c was studied by an NAL-LBL-UC Berkeley collaboration [7], while the reaction $pp \to pp\pi^+\pi^-$ at 205 GeV/c was studied by the Argonne group [8]. Omitting details of the experiments since they may be found in the references quoted, I present below the comparison between the theoretical estimates of the cross sections and the empirically measured values.

Reaction	P_{lab}	σ (experiment)	σ (theory)
$\pi^- p \to \pi^- p\pi^+\pi^-$	205 GeV/c	30 ± 10 μb	33.8 μb
$pp \to pp\pi^+\pi^-$	205 GeV/c	44 ± 15 μb	31.3 μb .

We find that the measured cross sections are compatible with the theoretical estimates. As a result of the rather low values of the S^* used here, there is surely some contamination from lower trajectories. Raising these minimum subenergies (in the theoretical estimate and in the experimental selection of events) will lead to "purer" double-pomeron cross sections. At the present energies and statistics, such a move will lead to prohibitively low cross sections. In future experiments with higher energies or statistics or both, this will be a desirable as well as feasible modification.

In addition to providing an estimate of the integrated cross section, the PPD hypothesis also makes two predictions on differential cross sections. These could not be meaningfully tested with the present statistics.

(i) The t' distribution: Consider the general reaction $ab \to ab\pi^+\pi^-$. By integrating eq. (3.2) over the subenergies we obtain

$$\frac{d\sigma}{dt'} = \frac{2.5 \times \sigma^{e\ell}_{ai}(\infty)\ \sigma^{e\ell}_{bj}(\infty)}{16\pi^3} \times f(Z)\left[\frac{1}{2}\log Z - \frac{1}{4} + \frac{1}{4Z^2}\right] \text{mb/GeV}^2 \ . \tag{3.3'}$$

This formula refers to a specific rapidity ordering of the central pions--pion i nearest to a and pion j is nearest to b. In the formula, $Z = t'/t_0$, where $t_0 = (S_{ai}^{*} - m_a^2)(S_{bj}^{*} - m_b^2)/S$ and $f(Z)$ is the form factor. As mentioned in the Appendix, [eq. (A.3)], the form factor appropriate to these calculations is $f(t') = e^{-4t'}$. In eqns. (3.2 and 3.3), where the aim was to integrate over t', this form was replaced for convenience by $f(t') = 1$ for $0 < t' < T = 1/4$ and zero beyond. For the differential cross section of course, we

must use the exponential form. Typically $d\sigma/dt'$ rises from t_0 up to $10\ t_0$ and falls monotonically thereafter. For example, in the process depicted in fig. 6, $t_0 = 0.016\ \mathrm{GeV}^2$ and the peak is around $t' = 0.16\ \mathrm{GeV}^2$. Unlike quasi-two-body reactions, which typically fall monotonically in t', these cross sections are predicted to first rise and then fall. They owe this property to the fact that here the two blob masses do not have to lie in some resonance band but are allowed to vary. As t' increase from t_0, the allowed range of mass variation increases, while the factors $f(t')$ and $(t' + \mu^2)^2$ decrease.

(ii) Distribution in $S_{\pi\pi}{}^c$: According to the Steinman relations [9], the amplitude cannot have simultaneous poles in t and in $S_{\pi\pi}{}^c$, the $(\mathrm{mass})^2$ of the two central pions. Thus the residue, R, of the pole at $t = \mu^2$, will not have pole in $S_{\pi\pi}{}^c$, say due to the f meson. According to the PPD hypothesis, there exists a (physical) region of small t' $(= -t)$ in which the amplitude is essentially what is found at the pole (except for a t-dependent form factor which introduces no singularity in $S_{\pi\pi}{}^c$). In this region of "small" t', if one divides the events into bins (of width $0.05\ \mathrm{GeV}^2$ for example) and plots within each bin the distribution of events versus $S_{\pi\pi}{}^c$, one should see none of the resonances of the dipion system. Conversely, the t' above which these resonances show up would mark the breakdown of the PPD hypothesis, telling us what "small" t' means. Such a test, which can be done in quasi-two-body reactions as well, will tell us in one stroke the validity of the Steinman relations as stated above, and the range of validity of the PPD hypothesis.

We thus infer from the Steinman relations that PPD is challenged not only by the neglected singularities in t but also by the

singularities in $S_{\pi\pi}{}^c$. At a fixed value of $S_{\pi\pi}{}^c$, if we increase $-t$, the neglected singularities in t compete with the pion pole. At some fixed t, if we vary $S_{\pi\pi}{}^c$, a pole in $S_{\pi\pi}{}^c$ can dominate the amplitude if we get sufficiently close to it. Should this happen, the pion pole will be absent in the amplitude according to the Steinman relations. In our example, if we focus on the f-meson pole in $S_{\pi\pi}{}^c$, the closest we can get to it, by varying $S_{\pi\pi}{}^c$ along the real axis, is given by the imaginary part of the pole position, which is equal to the product of its mass and width, with a value of about $0.2\ \mathrm{GeV}^2$. At this point of closest approach, we can say roughly that PPD will be challenged by the f-meson pole for $-t$ around $0.2\ \mathrm{GeV}^2$, assuming equal residues for the two poles. Thus the breakdown of the PPD hypothesis can be brought about by either the neglected singularities in t or the neglected singularities in $S_{\pi\pi}{}^c$. The former could be detected by a study of density matrix elements in quasi-two-body reactions and the latter by a search for resonances in $S_{\pi\pi}{}^c$.

It is interesting to study two earlier attempts at detecting double-pomeron processes in the light of the PPD hypothesis. Leipes, Zweig, and Robertson (LZR) [10] studied $\pi^- p \to \pi^- p \pi^+ \pi^-$ at 25 GeV/c while Rushbrooke and Webber (RW) [11] studied $pp \to pp\pi^+\pi^-$ at 6-25 GeV/c. Both assumed the double-Regge pole form of the amplitude, eq. (2.2), for the double-pomeron process and found that such an amplitude had negligible weight in their fit to the double-Regge region. This means either that the central coupling γ is very small, or that the pomeron is not a factorizable pole and the amplitude doesn't contain a factored component like eq. (2.2).

Does the failure of these two analyses to detect double-pomeron processes conflict with the estimates of PPD? No! The reason is that the PPD formula gives a miniscule $4\ \mu b$ for the experiment of LZR ($S = 50\ GeV^2$) and a similar result for that of RW. Instead of using the formula we can see the smallness of the PPD estimate in the following way. For the double-pomeron process to occur via PPD, we require not only that the two end blobs be massive, but that the central link be kinematically allowed to have small t's. In the $\pi^- p \to \pi^- p \pi^+ \pi^-$ reaction that we just discussed, we saw that

$$t'_{min} = \frac{S_{a+}(S_{b-} - m^2)}{S} .$$

Assuming that the only sizeable cross sections are for those reactions in which a t', of say $0.1\ GeV^2$, is accessible, we need an S given by

$$\frac{S_{a+}^*(S_{b-}^* - m^2)}{S} < \frac{1}{10} ,$$

using the smallest values of S_{a+} and S_{b-} compatible with the double-pomeron region. Using the S^* values quoted earlier, this condition requires $S > 45\ GeV^2$, a requirement barely met in the LZR experiment. A similar consideration applies to the RW experiment.

In the language of these two analyses, involving pomeron poles, the PPD hypothesis sets a lower bound on the central coupling, $\gamma(S_{\pi\pi}^C, t, t_1, t_2)$, by focussing on the pion pole at $t = \mu^2$, with a residue known from elastic experiments (fig. 7).

In their analysis, LZR conclude that the absence of an f resonance in $S_{\pi\pi}^C$ further corroborates the absence of the double-

pomeron events. This conclusion is true only as long as γ is controlled by a pole in $S_{\pi\pi}^C$ (fig. 8). If such a pole were present with a substantial residue, it would lead to double-pomeron events at lower energies, since t' need not be small. Their analysis essentially indicates the absence of such a pole.

In the PPD induced, double-pomeron processes, the situation is just the opposite--namely, the absence of resonant structure in $S_{\pi\pi}^C$ accompanies the controlling mechanism, the pion pole.

5. FURTHER APPLICATIONS OF PPD

A. Triple-pomeron cross sections

The PPD hypothesis, together with the definition of the pomeron, may be used to define and set lower bounds for inclusive triple-pomeron cross sections. Consider, for example, the reaction $p(P_a) + p(P_b) \to p(P_c) + X$, the parentheses containing the momenta of the protons. Let us restrict ourselves to events in which P_c is very close to P_a. Let M_x be the mass of the undetected particles, X. We are interested in the inclusive cross section,

$$\frac{d\sigma}{dt\, d(M_x^2/S)} , \text{ where } t = (P_c - P_a)^2 \text{ and } M_x^2 = (P_a + P_b - P_c)^2 .$$

Consider all exclusive events in this region with the property that of all the particles in the cluster X, the one nearest to the proton, in rapidity, is a pion of momentum P_d (fig. 9). The contribution of these exclusive events to the inclusive cross section involves, among other integrations, one over $u = (P_d + P_c - P_a)^2$, from zero to the kinematical limit in the negative u region. At $u = \mu^2$, the amplitude factorizes:

$$M \xrightarrow[u \to \mu^2]{} \frac{A_{\pi p}(S_1,t)\, A_{\pi p \to x'}(V_{x'})}{u - \mu^2} \quad . \tag{5.1}$$

Using PPD, we may integrate this $|M|^2$, over a region of small negative u, up to -U. The integral over $V_{x'}$ is done using the optical theorem. These operations are best represented pictorially (fig. 10). The result is $d\sigma^{\pi}/dt\, d(M_x^2/S)$, the contribution to $d\sigma/dt(d\, M_x^2/S)$ from the pion pole in u.

If the three blobs of fig. 10 have subenergies above the pomeron thresholds, the pomeron will occur in each of them doing its job. This then is a triple-pomeron process in a model-independent sense. One can estimate the magnitude of this pion-pole contribution using πp elastic and total cross section data.

It is only when one speaks of a triple-pomeron pole coupling, $g_{PPP}(t)$; that one needs to put pomeron poles in the blobs. Such a calculation has been done by Sorensen [6] who estimated $g_{PPP}(t)$. His paper also contains the phase space details omitted here.

B. Asymptotic bounds on total cross sections

Theorists have repeatedly been driven [2,12] to the conclusion that if the pomeron were to be a factorizable Regge pole, it couldn't be at $J = 1$ (i.e., all total cross sections must eventually die away), unless the triple-pomeron coupling $g_{PPP}(t)$ vanished at $t = 0$. This result is arrived at by repeating the pomeron in certain judiciously chosen circumstances, either exclusively or inclusively. To ensure its repetition these authors assumed its factorization, and their results seem to rely on this assumption.

On the other hand, we have seen that using PPD, the pomeron may be kept inside blobs and repeated using just the factorizablitiy

of the pion pole. It follows that the ailments accompanying an asymptotically constant,† pomeron-dominated cross section will ensue, forcing total cross sections to eventually die away. The crucial feature of the result is that it is independent of the J-plane singularity associated with the pomeron.

To get the bounds, one needs to find appropriate situations with repeated pomerons, to avoid pitfalls of multiple counting, and to do the phase space. These details will be discussed and the bounds derived in a subsequent paper.

6. CONCLUSIONS

The universal energy dependence and phase of high-energy diffractive amplitudes (of which the elastic is a special case), suggests an underlying mechanism. In this paper, such a mechanism was assumed to exist, and defined to be the pomeron. The question taken up was "Can and does the pomeron, so defined, occur more than once in a single process?" An analysis of various models of the pomeron indicated that different models gave different results, none of them quantitative. The introduction of the PPD hypothesis provided a model independent, quantitative answer whose utility was demonstrated in the specific reaction, $\pi^- p \to \pi^- p \pi^+ \pi^-$. At the pion pole (fig. 6), the production amplitude factored into a product of two elastic amplitudes, $A_{\pi^+\pi^-}$ and $A_{\pi^- p}$. Since these elastic amplitudes contain the pomeron by definition at high energies; the situation at the pion pole is a double-pomeron process in a model independent sense. In

† In this context the word "constant" has the usual meaning and not modulo logs.

terms of the space-time description outlined earlier, possible at $t = \mu^2$, one may see the process as one in which a real π^+ of mass μ, first suffers an elastic collision with a $\bar{p}$, and then proceeds to collide elastically with the π^-. Since there are two collisions, there are two pomerons, granted large enough subenergies. The PPD hypothesis allows a continuation of these ideas, valid at $t = \mu^2$, to the nearby physical region.

A comparison with two recent experiments, $\pi^- p \to \pi^- p \pi^+ \pi^-$ and $pp \to pp\pi^+\pi^-$, both at 205 GeV/c, shows that the observed cross sections are compatible with the theoretical estimates. Further tests of the PPD hypothesis, which must await experiments with greater energies, statistics or both, are suggested. A study of two earlier attempts at detecting double-pomeron cross sections shows that their negative results are compatible with the PPD model.

It was shown that PPD, together with no more than our general definition of the pomeron, leads to lower bounds on triple-pomeron processes. It was pointed out that using PPD to repeat the pomeron, one could derive upper bounds on the asymptotic pomeron dominated cross sections, without making model-dependent assumptions about the pomeron, such as its factorizability. If the present degree of validity of PPD persists asymptotically, the result that total cross sections must eventually die away seems inescapable, no matter what the nature of the singularity associated with the pomeron.

The philosophy throughout this paper has been to use the pion to analyze the pomeron, rather than to use the pomeron to analyze itself. The pomeron, whose nature is enigmatic, is kept within blobs, and only the pion, whose properties (particularly its factorizability) are certain, is explicitly shown. The catch is that the

factorizability of the amplitude is guaranteed only at the pion pole and not in the physical region. One has to assume, via PPD, that this crucial property is not lost in the transit from the pole, to the physical region, 0.02 GeV^2 away. This seems plausible (due to the smallness of μ^2), has worked in the past, works at $S \simeq 400$ GeV^2, but can never be proved.

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APPENDIX

The arguments supporting the existence of double-pomeron cross sections rested on two assumptions:

(1) At the pion pole the amplitude factorizes--the residue R is a product of two elastic amplitudes, each of which will contain the pomeron if the subenergies are large enough.

(2) This behavior will persist in the small t' region. The only difference from the situation at the pole will be the inclusion of a form factor, $f(t)$, (the PPD hypothesis).

The first assumption will not be discussed here since it is a widely accepted and basic property of the amplitude. The second notion involves a guess as to how the amplitude behaves in the physical region of small t', knowing its behavior at the pole. These are essentially two schools of thought that make two different guesses.

A. The S-matrix approach. Here the problem is viewed as that of guessing the behavior of an analytic function near a pole with a known residue. There is no systematic way to do this. The PPD hypothesis is a guess prompted by the notion that since the physical region is close to the pole, the amplitude should not vary too much in going from the pole to the physical region. Only experiment can decide the validity of such a guess and if it proves a valid guess, to decide its range (in t') of validity. We shall return to this question later.

B. The absorption model [13-15]. This model has proved very useful in the study of quasi-two-body reactions. Here one essentially identifies the amplitude at $t = \mu^2$ with a single Feynman diagram (the "pion pole" diagram), since at this point it dominates over the other diagrams (fig. 11a). Away from $t = \mu^2$ the neglected diagrams

have to be considered. The crux of the absorption model is that the effect of the neglected diagrams may be incorporated by the inclusion of initial and final state interactions (fig. 11b) [13,15]. Once again only experiment can decide the validity of this guess.

Over the last few years, numerous quasi-two-body reactions have been studied to test and compare the two guesses or models. Both models are required to explain the empirical fact that often the fall in t of the differential cross section is sharper than what the pion pole factor $(t - \mu^2)^{-2}$ would indicate. In the S-matrix approach, this is achieved by the incorporation of form factor [16,17]. While these form factors, suitable for describing final states containing resonances or stable particles of definite spin, have kinematical and dynamical notions behind them, they are not free of arbitrary parameters that must be deduced from experiment [18].

In the absorption model, the sharp collimation in t is a result of the initial and final state interactions. To the extent that the initial state interaction is given by the elastic scattering data (see fig. 11b), it is free of parameters. The final state interactions, since they are not subject to direct measurement, must be handled either via additional assumptions or additional parameters that may be empirically determined [13].

In short, both models can usually describe any differential cross section $d\sigma/dt'$ with the help of judiciously chosen parameters. By contrast, the study of the density matrix elements, ρ_{ij}, of the decaying final state resonances, such as the ρ^0 in the reaction $\pi^- p \to \rho^0 \Delta^0$, can distinguish the two models. The PPD model, with a factorizable amplitude, predicts that in the decay of the ρ-meson, all

the density matrix elements will vanish for all values of t' in the Gottfried-Jackson frame, with the exception of ρ_{00}, which will be unity [13]. The absorption model, with a nonfactorizable amplitude can admit a nonzero value for all ρ_{ij}. However, for small t' the predictions of this model approach the values given by the PPD model.

The empirical situation is as follows. One finds that for small t' (usually up to 0.15-0.2 GeV^2) ρ_{00} is between 0.8 and 1, while the others are very small, usually around 0.05 [19-21]. For larger values of t', the results differ substantially from the PPD predictions. The absorption model, although parameter dependent, is able to accommodate and describe these matrix elements in this region.

We have seen that in our problem, the bulk of the t' integration comes from small t' (around 0.15 GeV^2 for the specific process depicted in fig. 6). Based on the study of the density matrix elements in this region, we may say that in this range of t', the PPD and absorption models are indistinguishable and in agreement with experiment. After all both of them have to agree at the pole, and if the process is a smooth one the merger could be expected around small t'.

Further evidence for factorization at small t' comes from a study of $\pi^- p \to \rho^0 \pi^- p$ at 6 and 8 GeV/c, described in ref. [22]. Here the PPD model is assumed for $t' - t'_{min} < 0.3$ GeV^2 and the off-shell π-p cross section (lower vertex in fig. 12a) $d\sigma^{off}/d\Omega$, is extracted and found to have the same angular dependence as its on-shell counterpart, except for an overall scale. It is also found that if the lower vertex is allowed to be inelastic (lower vertex, fig. 12b), and the off-shell cross section for the process $\pi^- p \to \pi^- \pi^0 p$ is derived, then the ratio $\sigma^{off}(\pi^- p \to \pi^- \pi^0 p)/\sigma^{off}(\pi^- p \to \pi^- p)$ agrees with the on-shell ratio.

Another factor that controls the success of PPD, in addition to the smallness of t', is the absence of competing mechanisms. In the process $\pi^- p \to \pi^- p \pi^+ \pi^-$ it is clear that the link carrying the momentum transfer t must have $I^G = 1^-$. The same conclusion may be reached for the process $pp \to pp\pi^+\pi^-$ if one makes the additional assumption that the two protons at the two ends do not send any quantum numbers to the central pions (which is tantamount to assuming a factorizable pomeron controlling the two end blobs). This means that the π and the A_2 are the only possible objects that can be exchanged across that link. A study of the reaction $\pi^- p \to \rho^0 n$ [23], shows that when the π and the A_2 are present, the A_2 begins to stand out for $t' - t'_{min}$ greater than 0.3 GeV^2. This conclusion is based on a study of the density matrix elements of the decaying ρ meson and seeing at what t' the PPD predictions break down, forcing the inclusion of the A_2 in the description. While this state of affairs is not expected to be universal, it does lend some support to our ignoring the A_2 at smaller values of t'.

If one is persuaded by the above-mentioned arguments that the PPD model will provide a good description of a process at small t's, there still remains the problem of what form factor is to be employed in the double-pomeron process. The standard form factors of the quasi-two-body reactions are not applicable here since they pertain to final states of definite spins--resonances or stable particles; while in the double-pomeron case these resonances have been specifically excluded. There is, however, a theoretical model of form factors that is valid in precisely this context. By solving the multiperipheral integral equation with variable masses for the external pions it is possible to derive the dependence of the high-energy elastic amplitudes on the

external masses, i.e.; the form factors [24]. Omitting details of the calculation, as they may be found in the reference quoted, I present here the final formula that is applicable to the present process: If two on-shell pions couple to a reggeon of spin $\alpha(u)$ and mass $(u)^{\frac{1}{2}}$, with a coupling $\beta(u)$, then the effect of taking one of the pions off-shell to a mass $(t)^{\frac{1}{2}}$, changes the coupling to

$$\beta(u,t) = \beta(u)\left[\frac{u_0 - \frac{\mu^2}{2} + \frac{u}{4}}{u_0 - \frac{1}{2}\left(\mu^2 - \frac{u}{2} + t\right)}\right]^{1+\alpha(u)} . \qquad (A.1)$$

In this formula u_0 is the scale factor and represents the $(\text{mass})^2$ of the π-π resonance that goes into the kernel of the integral equation. Since there are at least two prominent resonances to be considered, namely the ρ- and the f-mesons, the authors of ref. [24] recommend a value of 1 GeV^2 for u_0, which in addition to representing the mean of the two resonance masses also gives a good result in the numerical solution of the integral equation [24]. In principle the value of u_0 could be smaller, but not smaller than 0.5 GeV^2, the mass squared of the meson.

In incorporating these form factors into our calculation the following considerations are relevant:†

(i) Since u represents the momentum transfer in the two elastic processes at the two ends of the pion link (see, for example, fig. 6), it is usually very small, since at high energies, the bulk of the elastic cross section comes from $u \lesssim 0.1\ \text{GeV}^2$. We may therefore drop factors like $u/2$ and $u/4$, as well as μ^2 in eq. (A.1), in comparison with u_0 and t. While t can be very small, it is only

† We are forced here to associate pomeron Regge poles with the two pomerons in the blobs (see fig. 6). This is a necessary evil for getting the form factors.

at larger t (around 0.15 or 0.2 GeV^2) that the form factor plays a significant role, providing the cut off. We shall ignore the slope of the pomeron and set $\alpha = 1$. The form factor then simplifies to

$$\beta(u,t) = \beta(u)\left(\frac{u_0}{u_0 - \frac{1}{2}t}\right)^{1+1} . \qquad (A.2)$$

(ii) This factor occurs to the fourth power in the double-pomeron cross section and leads to an overall form factor

$$f(t) = \left[\frac{u_0}{u_0 - \frac{1}{2}t}\right]^8 \simeq e^{4t/u_0} = e^{-4t'/u_0} \qquad (A.3)$$

for small t'. Choosing $u_0 = 1\ \text{GeV}^2$ leads to a form factor $f(t') = e^{-4t'}$ [while $u_0 = 0.5\ \text{GeV}^2$ would lead to $f(t') = e^{-8t'}$]. For simplicity this form factor was replaced by a flat one that simply cut off the integral [eq. (3.2)] at $t' = 1/4\ \text{GeV}^2$, leading to a result like eq. (3.3). If instead, one performs the integrations using the exponential form factors, one gets an answer in terms of exponential functions. Numerically, the result of such a calculation is about 20-25% larger than that coming from a simple formula like eq. (3.3). Considering the other approximations and uncertainties in this calculation, such as the value of u_0, this difference is considered not important enough to justify abandoning the simple formula, eq. (3.3).

The following important point is worth underscoring. For the purposes of deriving the model independent bounds on asymptotic total cross sections that were mentioned in section 5B, it is sufficient to

know that there exists a physical region of nonzero measure in t' in which the production amplitude factorizes, as it does at the pole. On the other hand, to make a realistic estimate of the double-pomeron cross section, one must estimate the range in t' over which this factorization will persist. While the range of validity of the PPD hypothesis may be controversial, it seems very clear from a study of quasi-two-body reactions that there definitely exists a range of small t' over which the amplitude factorizes to a very good approximation and is dominated by the pion pole. For example, at very small t', all density matrix elements approach the PPD values [19-21]. To extend this result from the quasi-two-body reactions to the double-pomeron process, one simply needs to increase the masses of the end blobs from the resonance region to the pomeron region. Is this increase likely to produce any significant changes? It appears not, from the following consideration. In ref. [20] we find that if in the reaction $\pi^- p \to \rho^0 \Delta^0$, we increase the mass of the π-π system till we reach the reaction $\pi^- p \to f^0 \Delta^0$, the density matrix elements in the very small t' region remain the same. One finds, for example that $\rho_{00} = 0.91 \pm 0.07$ for $0 < t' - t'_{min} < 2\mu^2$, in ρ-decay, while $\rho_{00} = 0.88 \pm 0.11$ for $0 < t' - t'_{min} < 5\mu^2$ in f-decay. The slight decrease in ρ_{00} in going from the ρ to the f-meson may be understood in terms of the increase in t'_{min} and the increase in the range of t'.

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FIGURE CAPTIONS

Fig. 1. A typical total cross section as a function of energy.

Fig. 2. The elastic amplitude $M_{ab\to ab}(S,t)$ in the pomeron dominated region.

Fig. 3. The rapidity plot for the process $ab \to ab\pi^+\pi^-$ in the region of interest.

Fig. 4. The multi-Regge production amplitude $M_{ab\to ab\pi^+\pi^-}$ in the double-pomeron region.

Fig. 5. The rapidity plot for the reaction $\pi^- p \to \pi^- p\pi^+\pi^-$ in the region of interest.

Fig. 6. The production amplitude $M_{\pi^- p\to \pi^- p\pi^+\pi^-}$ at the pion pole.

Fig. 7. The PPD model for $\gamma_{\pi\pi pp}(S_{\pi\pi}{}^C, t, t_1, t_2)$. The β's are known by factorization from the elastic experiments.

Fig. 8. The LZR model for $\gamma_{\pi\pi pp}(S_{\pi\pi}{}^C, t, t_1, t_2)$.

Fig. 9. Rapidity plot for $pp \to pX$, with the "left-most" particle in X being a pion.

Fig. 10. Calculating $d\sigma^\pi/dt\, d(M_x{}^2/S)$, the pion's contribution to the inclusion cross section. The prime on Σ' tells us to keep $M_x{}^2$ fixed when summing over P_d.

Fig. 11. (a) The amplitude for the reaction $\pi^- p \to \rho^0\Delta^0$ at the pion pole.

(b) The amplitude for the same process, away from the pion pole, in the absorption model. The blobs denote initial and final state interactions.

Fig. 12. (a) The reaction $\pi^- p \to \rho^0\pi^- p$ in the PPD model.

(b) The reaction $\pi^- p \to \rho^0\pi^-\pi^0 p$ in the PPD model.

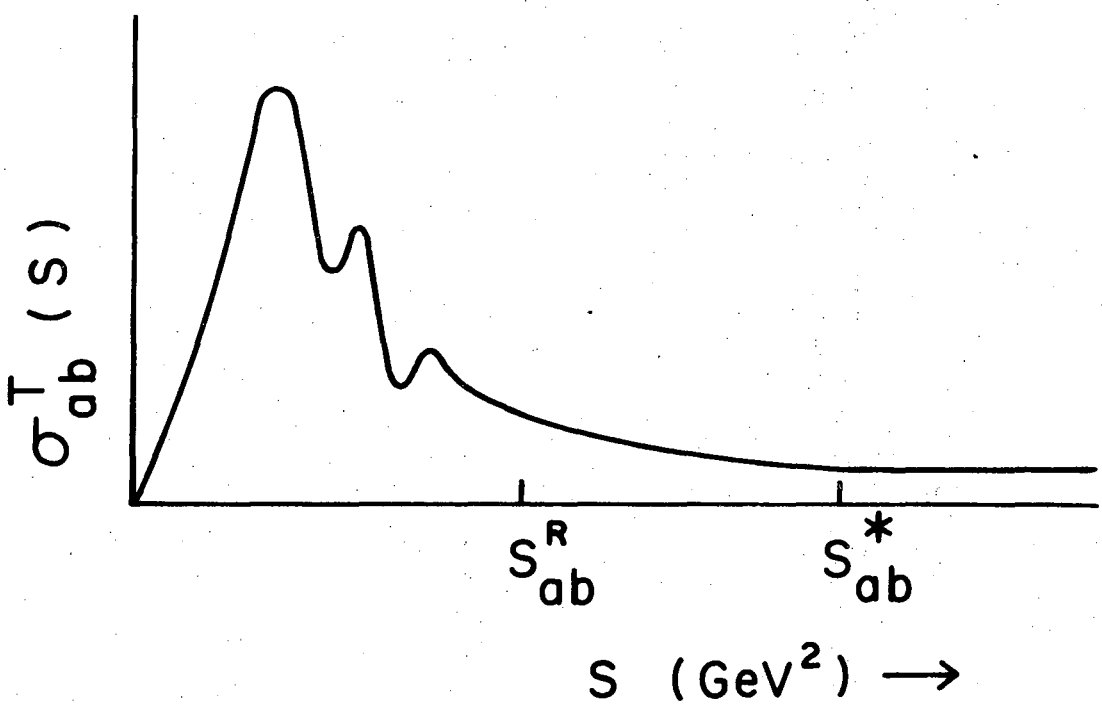

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Fig. 1

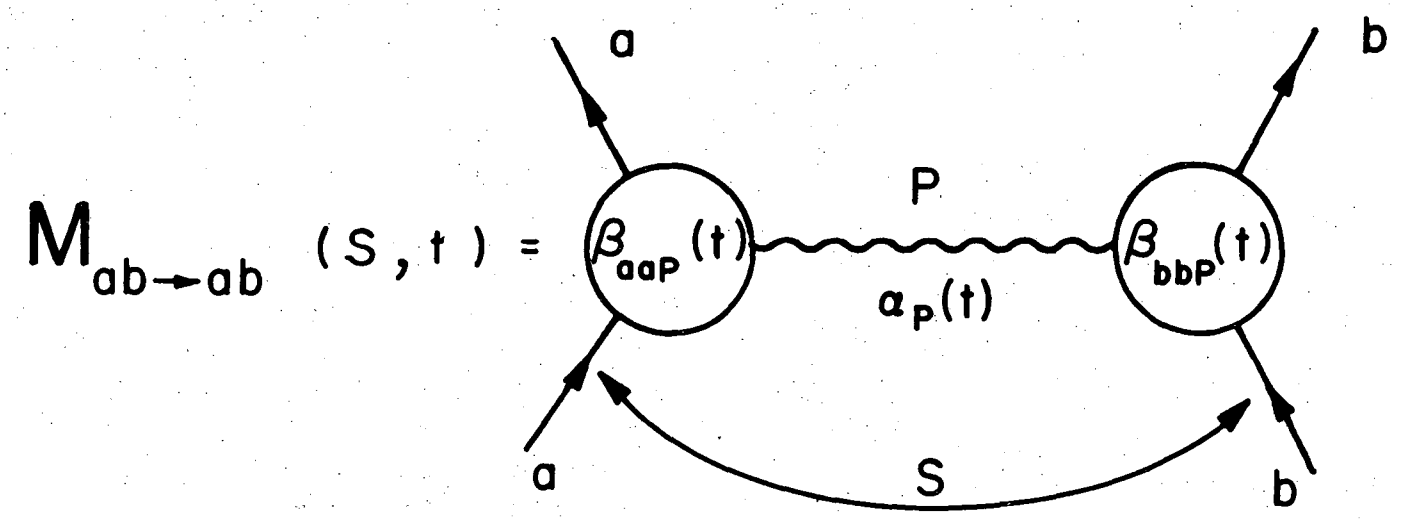

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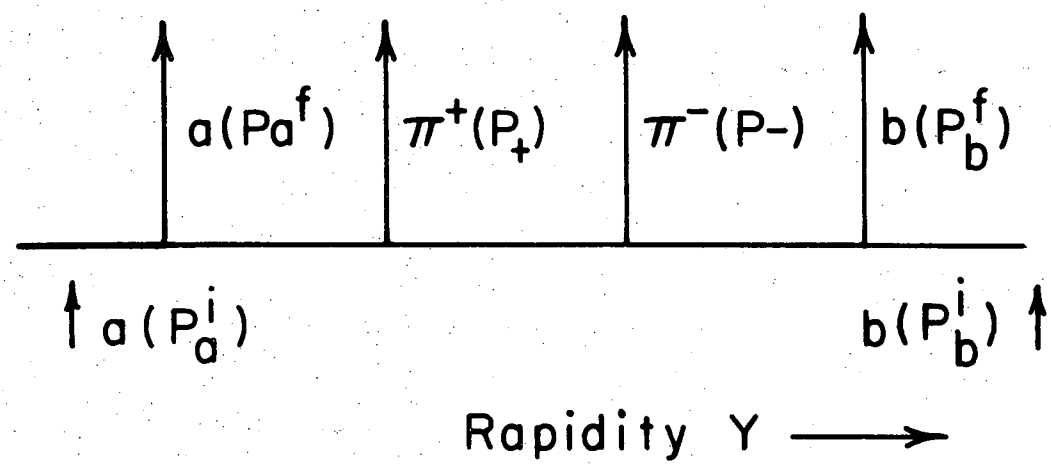
Fig. 2



XBL737-3409

Fig. 3

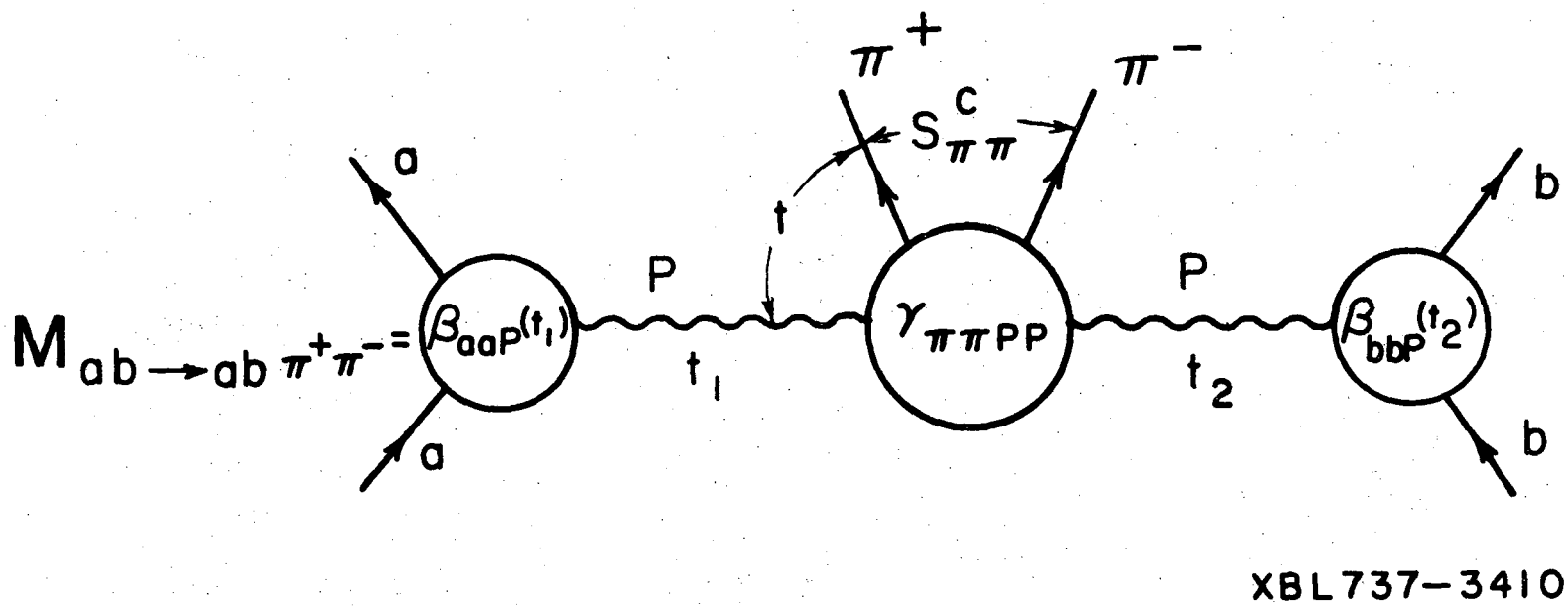
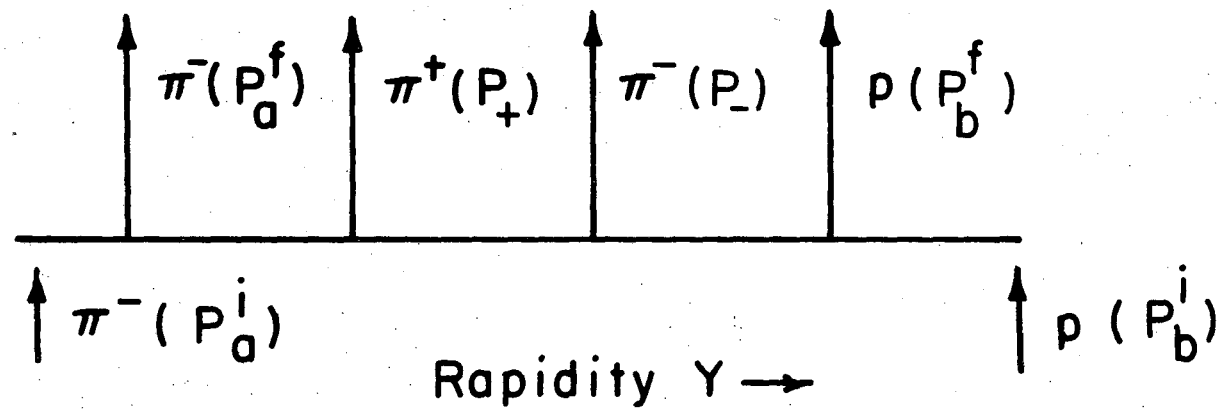
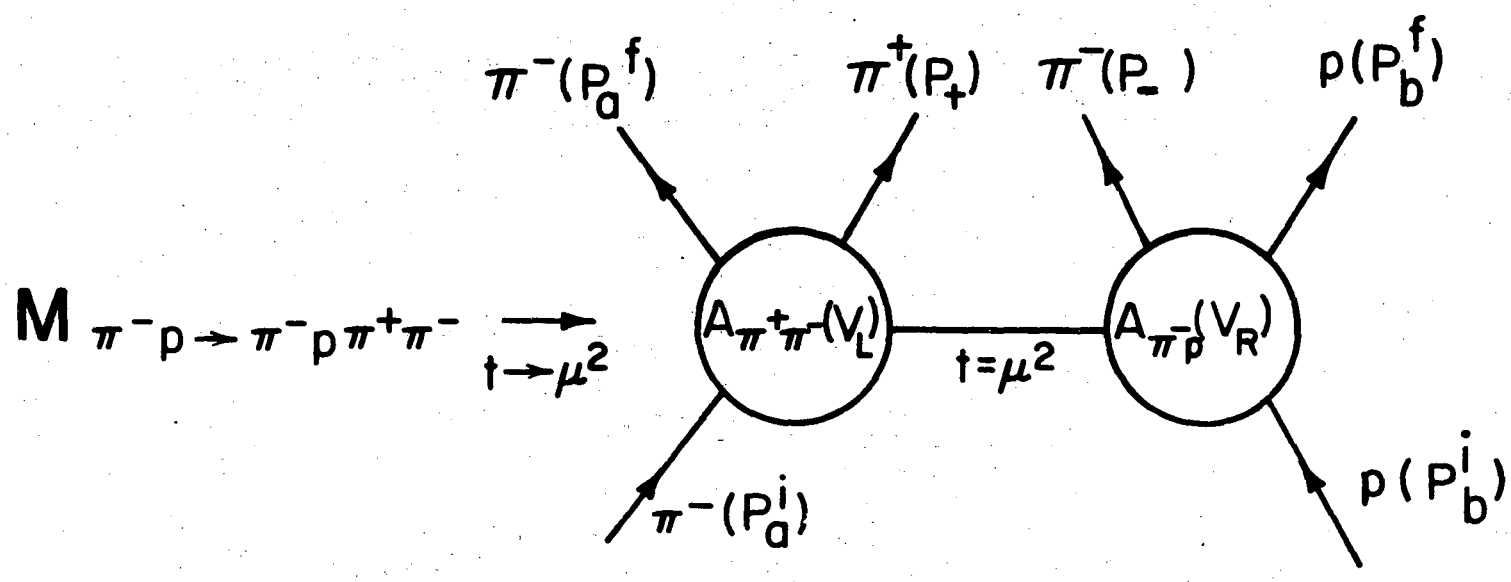

$M_{ab \to ab\pi^+\pi^-} =$
a
a
$\beta_{aaP}(t_1)$
P
t_1
t
π^+
π^-
$S^c_{\pi\pi}$
$\gamma_{\pi\pi PP}$
P
t_2
$\beta_{bbP}(t_2)$
b
b
XBL737-3410

Fig. 4



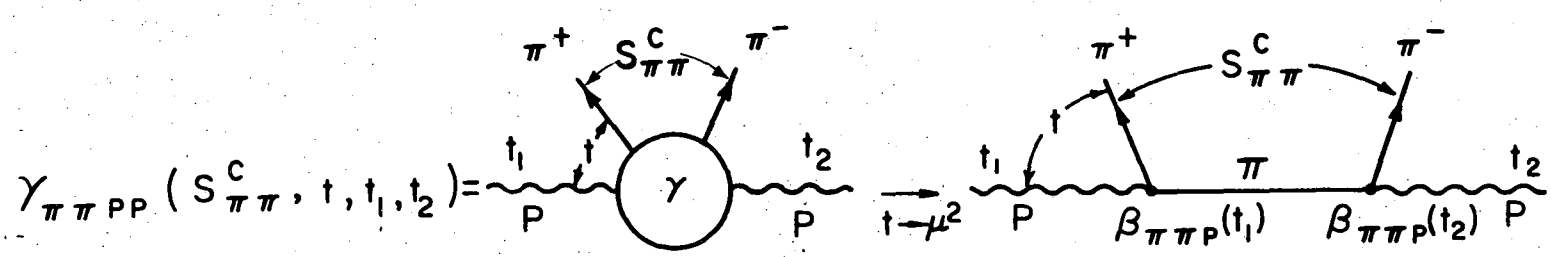
XBL737-3411

Fig. 5



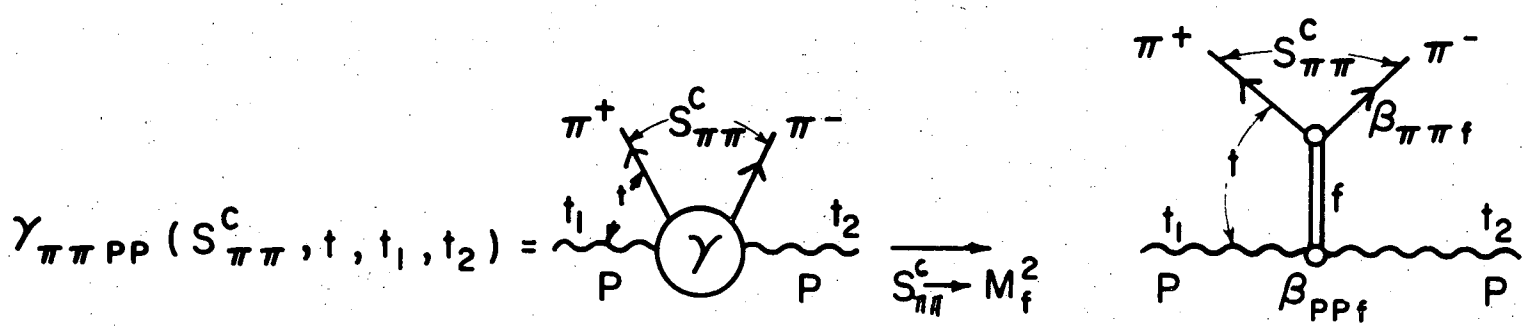
XBL737-3412

Fig. 6



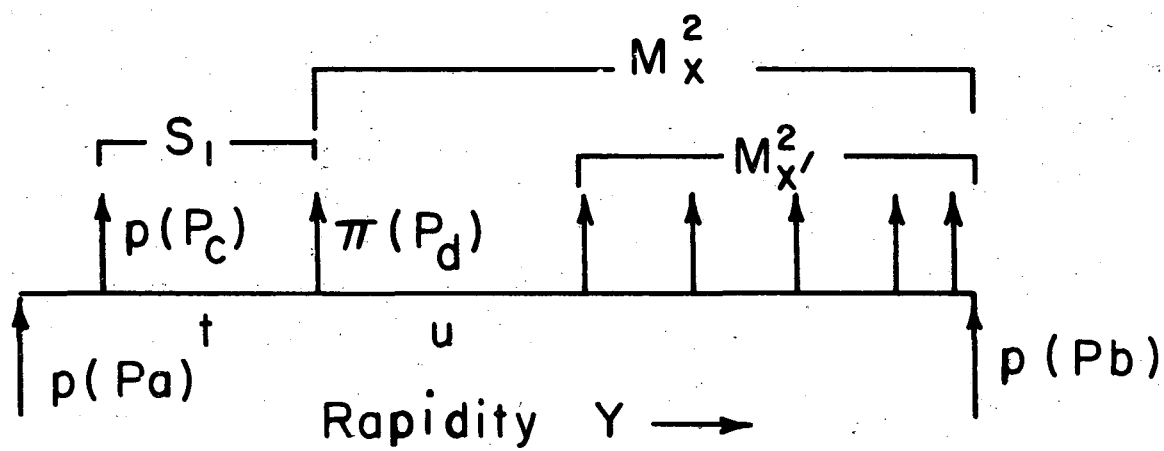
XBL737-3415

Fig. 7



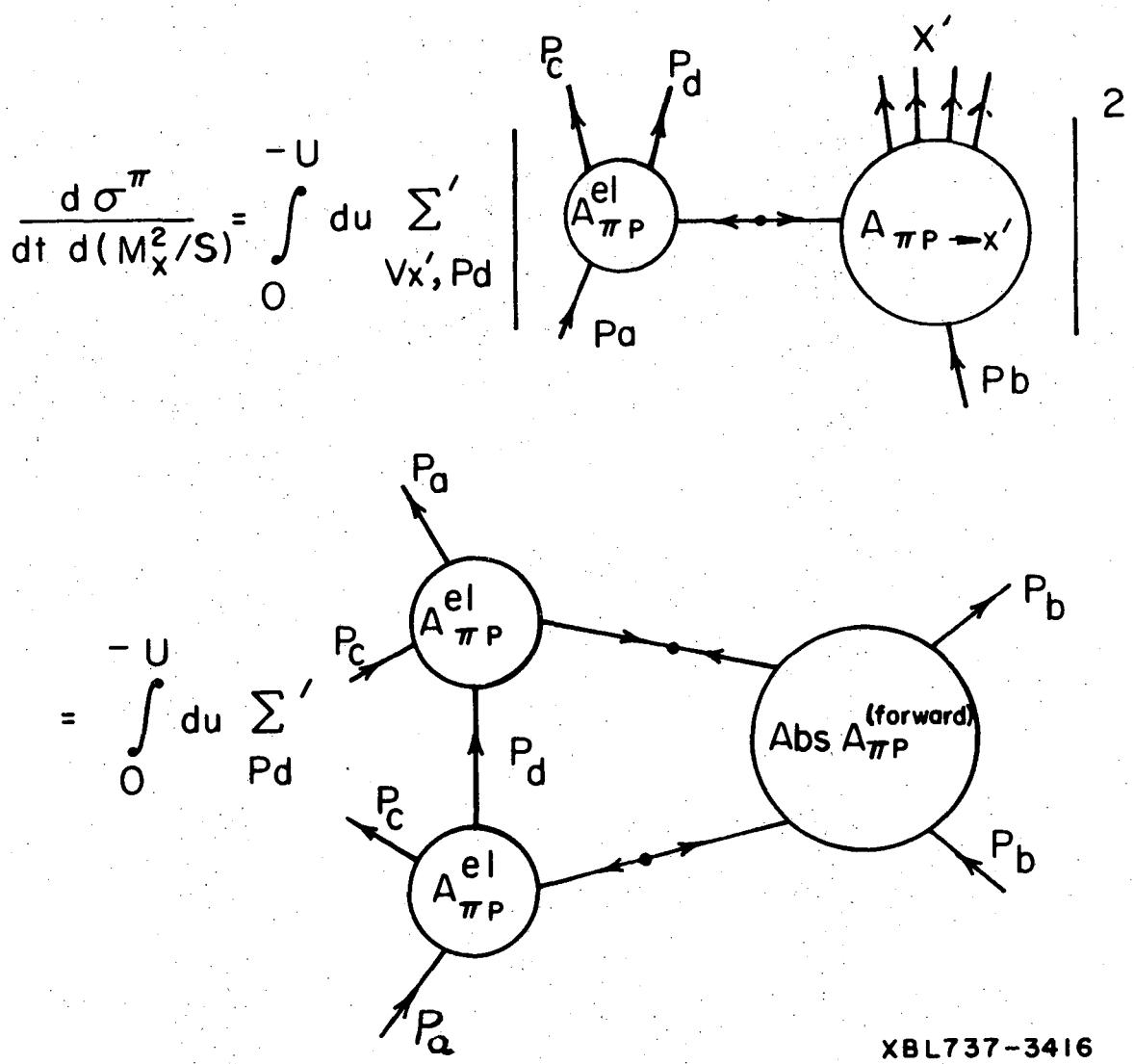
XBL737-3413

Fig. 8



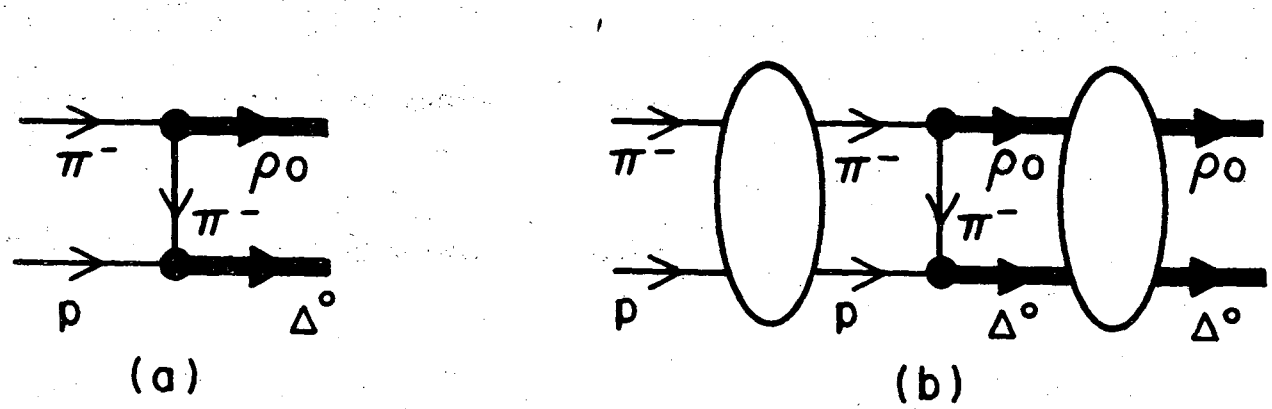
XBL737-3414

Fig. 9



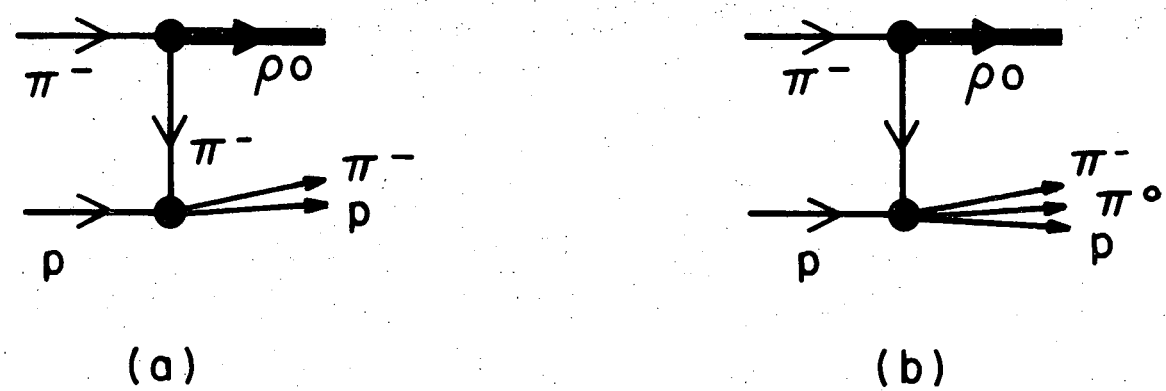
$\frac{d\sigma^{\pi}}{dt\, d(M_x^2/S)} = \int_0^{-U} du \sum'_{Vx', Pd}$
$A^{el}_{\pi P}$
$A_{\pi P \to x'}$
P_c
P_d
P_a
P_b
X'
2
$= \int_0^{-U} du \sum'_{Pd}$
$A^{el}_{\pi P}$
$A^{el}_{\pi P}$
Abs $A^{(forward)}_{\pi P}$
P_a
P_c
P_d
P_c
P_a
P_b
P_b
XBL737-3416

Fig. 10



XBL7310-4176

Fig. 11



XBL7310 -4175

Fig. 12

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