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(AND SUCH VECTOR MESONS WITH  $CP = -1$ )

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July 9, 1973

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Prepared for the U. S. Atomic Energy Commission  
under Contract W-7405-ENG-48.

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## GAUGE THEORIES INCLUDING THE BUDDHA (AND SUCH

VECTOR MESONS WITH  $CP = -1$ )\*

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July 9, 1973

## ABSTRACT

We construct natural generalizations of the gauge theory of hadrons (and leptons), now to include the  $B(1235)$  and possibly other mesons with  $CP = -1$ . We are led to a concept of "pseudospin" multiplets including  $\rho$ ,  $\rho'$ ,  $A_1$ , and B-type particles; this provides then a renormalizable approach to all known particle types with  $J \leq 1$ . A number of interesting mass relations and intriguing structure is found.

\* This work was supported in part by the U. S. Atomic Energy Commission.

## 1. Introduction

At least one vector meson with  $CP = -1$ , namely the Buddha particle  $[B(1235)]$ , is known to exist in nature <sup>1)</sup>. Others, such as  $h$  (the singlet  $B$ ), have popped up from time to time on (at least) theoretical grounds <sup>2-4)</sup>. While gauge theories of  $CP = +$  vector mesons have been successful over the years <sup>5)</sup>, I am not aware of any theories including such B-type particles. Especially with the advent of renormalizable gauge theories <sup>6-8)</sup>, in principle capable of describing all particles with  $J \leq 1$ , we feel "B-particles" can and should be studied. It is our purpose here to discuss the principles of including B-type particles in gauge theories, and we will construct a number of models.

Most of our model-building will be hadronic, because of the known  $B$  itself. However, we do not claim to have explicitly discussed all hadronic  $B$  models that follow from our general ideas, and we also note that it may be useful to consider such particles in the weak interactions as well.

In our way of doing things, a number of general features emerge. (1) We include the B-type particles in "pseudospin" <sup>9)</sup> multiplets along with  $\rho$ ,  $A_1$ , and  $\rho'$  type particles. This is a very natural extension of the gauge theory of hadrons <sup>7)</sup>, and leads to the idea that the underlying (algebraic) symmetry group of strong interactions is larger than say  $SU(3) \otimes SU(3)$ : Where  $SU(3) \times SU(3)$  is generated say by  $\lambda^\alpha (t_0 \pm t_3)$  [ $t$ 's being Pauli matrices], our symmetries are generated by the completion of this group to  $\lambda^\alpha t_\beta$  ( $\beta = 0, 1, 2, 3$ ). [ $t_\beta$ , we say, generates the pseudospin.] These algebras are isomorphic to those used by Gilman and Kugler <sup>10)</sup>, but their role is different here, because in our notation,  $t_2$  will have

CP = -1. In particular, if the internal symmetry is SU(3), then, with pseudospin, we have in all an SU(6). Of course, these larger symmetries are in general badly broken. (2) These symmetry groups are intrinsically parity conserving in the sense that the entire multiplet is described by only one gauge coupling constant ("left" and "right" are locked). More will be said about this later. (3) As far as vector meson spectra are concerned, we find one SU(2) model in which  $m_{\rho}^2 + m_{\rho'}^2 = 2m_B^2$ ; this is remarkably accurate. In the same model, an isoscalar  $A_1$  [D(1285)?] is found degenerate with B. In another SU(2) model, assuming the existence of h, we find it most naturally degenerate with  $A_1$ . The SU(3) model may be unsatisfactory, in that it indicates that the  $\rho'$  is the first  $\rho$ -recurrence at  $m_{\rho'}^2 = \frac{3}{2}$ , rather than the observed  $\rho'$  at  $m_{\rho'}^2 = \frac{5}{2}$ . (4) The gauge theory of hadrons' problem ( $G_A = \frac{1}{2}$ ) is modified here, and in general improved. (5) In some of the models, the intriguing possibility of calculable pseudoscalar-baryon coupling constant emerges. This is directly connected with the existence of the CP = -1 vector mesons <sup>11</sup>).

## 2. Fermions and Pseudospin

As far as I can tell, the stumbling block in constructing gauge theories with B-type particles is the observation that such particles must have a derivative coupling into say baryon-antibaryon (or any diagonal coupling to a fermion). Thus, at first sight, B particles are simply nonrenormalizable. This reasoning is however specious, as we shall see. The point is that B can couple off-diagonally between two fermions in the gauge Lagrangian. One test

that its CP is fixed negative would be that higher order corrections do indeed generate the appropriate derivative diagonal couplings.

We intend realizing just such a situation; and we shall do so as a very natural extension of the gauge theory of hadrons <sup>7)</sup>. In that model, the fermion vector meson system can be phrased [in a U(4) notation]

$$Q = \begin{pmatrix} q_L \\ q'_L \\ q_R \\ q'_R \end{pmatrix} \quad \begin{aligned} q_L &= (1 - \gamma_5)q \\ q'_L &= (1 + \gamma_5)q' \\ &\vdots \end{aligned} \quad (2.1)$$

with V and A, the vector and axial-vector gauge particles, coupling as

$$t_0 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix}, \quad t_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \quad q \rightarrow Sq$$

$$S = \exp(it \cdot \alpha) \quad (2.2)$$

respectively. Here  $\sigma_\alpha$  are ordinary Pauli-matrices (we are suppressing internal SU(2), SU(3), etc. for this discussion). As written, the primed quark is transforming with the opposite sign of  $\gamma_5$ , to remove axial-vector anomalies.

Once we have removed the anomalies with the extra quark, it turns out that the same space supports two more gauge transformations, and anomalies are still absent. We take these to be

$$t_1 = \left( \begin{array}{c|c} \sigma_1 & 0 \\ \hline 0 & \sigma_1 \end{array} \right), \quad t_2 = \left( \begin{array}{c|c} \sigma_2 & 0 \\ \hline 0 & -\sigma_2 \end{array} \right) \quad (2.3)$$

Assuming that  $q, q'$  have the same parity and charge conjugation, we see immediately that  $t_1$  corresponds to  $\rho'$ ,  $t_2$  to a B-type particle. The details of C and P for these models are found in Appendix A. Together, all four particles are a representation of the  $U(2)$  (pseudospin) group

$$t_\alpha = \left( \begin{array}{c|c} \sigma^\alpha & 0 \\ \hline 0 & \sigma^\alpha \end{array} \right), \quad \hat{\sigma}^\alpha = \sigma_1 \sigma^\alpha \sigma_1 \quad (2.4)$$

It will prove useful also to note the  $SU(2)$  subgroup which commutes with pseudospin,

$$\begin{aligned} \mathcal{U}: \mathcal{U}_1 &= \left( \begin{array}{c|c} 0 & \sigma_1 \\ \hline \sigma_1 & 0 \end{array} \right) & \mathcal{U}_2 &= \left( \begin{array}{c|c} 0 & -i\sigma_1 \\ \hline i\sigma_1 & 0 \end{array} \right) \\ \mathcal{U}_3 &= \left( \begin{array}{c|c} 1 & 0 \\ \hline 0 & -1 \end{array} \right) \quad (2.5) \\ (t^\alpha, \mathcal{U}) &= 0 \end{aligned}$$

It does not appear that gauge particles can be attached to  $\mathcal{U}_1$  or  $\mathcal{U}_2$  because these mix left and right quarks.  $\mathcal{U}_3$  could be used alone however, and would evidently provide an  $A'_1$ . This would provide a larger, more chirally symmetric pseudospin group [with members  $\rho A_1, B, \rho'_1 A'_1$ ] but we will not go into these interesting models in this paper.

As an immediate application of  $\mathcal{U}$ , however, we notice that we fix the C and P of  $q, q'$  to be the same by writing the gauge-

invariant mass term

$$M' \bar{q} \mathcal{U}_1 q \quad (2.6)$$

Thus the C's and P's of all particles are now as assumed above<sup>12)</sup>. It is curious that within the meson Lagrangian itself (without fermions) there is no way of ascertaining more than the relative C, P of  $\rho', B$ . As we shall see below, however, spontaneous breakdown will establish their absolute C, P, even within the mesons. Such mechanisms will also give further (diagonal) quark masses.

### Algebras

The desired internal symmetry can now be juxtaposed with the pseudospin. For the simplest case of  $SU(2)$ , we distinguish three models:

$$(a) \quad t_{0\tau}, t_{3\tau}, t_{1\tau}, t_2 : (\rho, A_1, \rho', h) \quad (2.7a)$$

$$(b) \quad t_{0\tau}, t_{3\tau}, t_1, t_{2\tau} : (\rho, A_1, \omega', B) \quad (2.7b)$$

$$(c) \quad t_{0\tau}, t_3, t_{1\tau}, t_{2\tau} : (\rho, \omega_5, \rho', B) \quad (2.7b)$$

where  $\tau$  are another set of Pauli matrices ( $\tau^\alpha, \sigma^\beta = 0$ , this time representing isospin. Here  $B$  and  $h$  have the quantum numbers of  $B(1235)$  and the oft-conjectured isoscalar Buddha. To each of these models, an  $\omega$ , transforming as  $t_0$ , may be added trivially if desired.

Each of these models has an  $O(5)$ -like algebra. For example, in the case of model (a):

$$\begin{aligned}
(Q_\alpha, Q_\beta^5) &= i \epsilon_{\alpha\beta\gamma} Q_\gamma^5, & (Q_\alpha, Q_\beta) &= i \epsilon_{\alpha\beta\gamma} Q_\gamma \\
(Q_\alpha^5, Q_\beta^5) &= i \epsilon_{\alpha\beta\gamma} Q_\gamma, & (Q_\alpha, Q'_\beta) &= i \epsilon_{\alpha\beta\gamma} Q'_\gamma & (2.8) \\
(Q_\alpha, Q_h) &= 0, & (Q_\alpha^5, Q'_\beta) &= i \delta_{\alpha\beta} Q_h, & (Q_\alpha^5, Q_h) &= -i Q'_\alpha \\
(Q'_\alpha, Q_h) &= i Q_\alpha^5, & (Q'_\alpha, Q'_\beta) &= i \epsilon_{\alpha\beta\gamma} Q_\gamma
\end{aligned}$$

where  $Q_\alpha, Q_\alpha^5$  form the usual  $SU(2) \otimes SU(2)$  (of vector and axial vector currents associated with  $t_0$  and  $t_3$ ), while  $Q'_\alpha$  are an additional set of ( $Q'_\alpha \sim t_1$ ) vector charges, and  $Q_h$  is the second-class isoscalar charge ( $t_2$ ). Similar algebras for models (b) and (c) can be read off, remarking that, in (c), there is no local realization of  $SU(2)$  axial vector transformations.

Similarly, one can write models for arbitrary internal symmetry. Most interesting presumably would be the models for  $U(2)$ , generated by  $t^{\alpha\beta}$  ( $\beta = 0, 1, 2, 3$ ), and  $SU(3)$ , generated by  $t^{\alpha\beta}$  ( $\beta = 0, \dots, 8$ ). These models essentially combine the features of the prototype models (a), (b), and (c), and, except for a remark about possible trouble with  $SU(3)$  in Sec. 3, will not be discussed here in detail.

A final relevant remark in this section is that we have also studied introducing B-type particles on  $qq'$  where both quarks transform with the same sign of  $\gamma_5$ . Then, e.g., one can take [in a similar  $U(4)$  notation]

$$\begin{aligned}
L &= \begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix} \mathcal{I}, & R &= \begin{pmatrix} 0 & & \\ & 1 & \\ & & 1 \end{pmatrix} \mathcal{I}, \\
L' &= \begin{pmatrix} \sigma_2 & & \\ & 1 & \\ & & 0 \end{pmatrix} \mathcal{I}, & R' &= \begin{pmatrix} 0 & & \\ & 1 & \\ & & \sigma_2 \end{pmatrix} \mathcal{I} & (2.9)
\end{aligned}$$

where  $L, R$  form the usual  $SU(2) \otimes SU(2)$ , and where  $L', R'$  are of abnormal content. Particle content in such models always seem to involve (with B) an abnormal  $\rho$  particle (with  $C = +$ ); further, these models have anomalies, and are not "parity-conserving". For all these reasons, our  $\gamma_5$ -doubled quark system appears natural. We emphasize, however, that our pseudospin-vector-meson systems may be taken on their own, and other (baryonic?) representations may be sought instead of the quarks.

### 3. Spontaneous Breakdown and Vector Systems

We will catalogue here a number of the simpler scalar multiplets possible for the Abelian theory. The analogous multiplets for the models with isospin are given in Appendix A. The smallest representations are of course vector, say  $\Sigma \rightarrow S \Sigma S^{-1}$ . These are representations which potentially can couple directly to the fermions, and which are the analogue of the usual  $(\pi, \sigma)$  multiplet [say  $(3, \bar{3})$  when using  $SU(3) \otimes SU(3)$ ]. It will be convenient to distinguish three of these, depending on their C and P content:

$$\begin{aligned}
\Sigma_1 &= \sigma t_1 + \pi t_2 + P t_3 \\
\Sigma_2 &= \pi' t_1 + \sigma' t_2 + S t_3 \\
\Sigma_3 &= P' t_1 + S' t_2 + \sigma'' t_3
\end{aligned} \tag{3.1}$$

where  $\sigma$ ,  $\pi$ ,  $P$ ,  $S$  type fields are characterized by  $\sigma(C = P = +)$ ,  $\pi(C = -P = +)$ ,  $P(C = P = -)$ , and  $S(P = -C = +)$  particles. The couplings, if desired, to the fermions are

$$\bar{q}(g_1 \mathcal{U}_1 \Sigma_1 + g_2 \mathcal{U}_2 \Sigma_2 + g_3 \mathcal{U}_3 \Sigma_3)q \quad (3.2)$$

but the  $\Sigma_3$  coupling vanishes explicitly. The couplings to the vector mesons are as usual for vector representations. We give all three representations for reference; in fact we do not necessarily intend using all of them at once in a given model.

Another, even more useful representation is the M-type field of Ref. 7, which, as in Ref. 8, is necessary to provide unification with the nonstrong interactions. M transforms  $M \rightarrow SM(S')^{-1}$ , where the primed group is that of nonstrong interactions; M has no direct coupling to our fermions. Its representation content is

$$M = t_0(\sigma + iS) + t_1(\sigma' + iS') + t_2(\pi + iP) + t_3(P + i\pi). \quad (3.3)$$

Other representations such as complex vectors may also be useful, but, for this paper, we will confine ourselves to the foregoing.

The C and P content of these representations are fixed for all but M by the fermions. (See Appendix A) The content of each multiplet is, however, fixed anyway by the assumed pattern of spontaneous breakdown

$$\langle \Sigma_i \rangle = t_i v_i \quad (3.4)$$

$$\langle M \rangle = \kappa_0 t_0 + \kappa_1 t_1$$

This leads to the general vector meson masses

$$m_{\rho\rho'}^2 = f^2 \begin{bmatrix} \kappa_0^2 + \kappa_1^2 & 2\kappa_0\kappa_1 \\ 2\kappa_0\kappa_1 & \kappa_0^2 + \kappa_1^2 + v_2^2 + v_3^2 \end{bmatrix}$$

$$m_{A_1}^2 = f^2(\kappa_0^2 + \kappa_1^2 + v_1^2 + v_2^2) \quad (3.5)$$

$$m_B^2 = f^2(\kappa_0^2 + \kappa_1^2 + v_1^2 + v_3^2)$$

Here  $m_{\rho\rho'}^2$  is appropriate when the representation content allows mixing between the  $t_0(\rho)$  and  $t_1(\rho')$  type particles [e.g.,  $\rho\rho'$  mixing in models (a) and (c)]--otherwise the masses in the  $t_0 t_1$  system are just the diagonal entries [model (b)].  $m_{A_1}^2, m_B^2$  are generic here for normal and abnormal axial vectors, whatever isospin we choose.  $f$  is of course the strong gauge coupling constant.

This  $\rho\rho'$  mixing determines, as promised, the absolute C and P of  $\rho'$  and B type particles even in the pure meson system (given C and P of  $\rho$ ). Using Eq. (3.5), we will now distinguish appropriate scalar systems for the models (a), (b), and (c).

Model (a) ( $\rho, A_1, \rho', h$ ). The simplest scalar system here is just M (no  $\Sigma$ 's). This has a number of interesting features worth mentioning. In the first place, the mass spectrum is  $m_\rho^2 = f^2(\kappa_0 - \kappa_1)^2$ ,  $m_{\rho'}^2 = f^2(\kappa_0 + \kappa_1)^2$ ,  $m_{A_1}^2 = f^2(\kappa_0^2 + \kappa_1^2) = m_h^2$ . The model predicts h degenerate with  $A_1$ . We also get the sum rule  $m_\rho^2 + m_{\rho'}^2 = m_{A_1}^2 + m_h^2$ , together with an equal spacing rule:  $m_{\rho'}^2$  is as far above  $m_{A_1}^2 = m_h^2$  as  $m_\rho^2$  is below it. Thus this  $\rho'$  is

not the  $\rho'$  of experiment at  $m_{\rho'}^2 = 5/2$ . Instead, taking known masses of  $\rho$ ,  $A_1$ , we find  $m_{\rho'}^2 = 3/2$ , the position of the as-yet-undiscovered first dual  $\rho'$ . Later we will discuss (with  $v_3$ ) raising  $\rho'$  to  $5/2$ , with a corresponding raise of  $h$ .

Another interesting feature here is a calculable pion-quark coupling:  $M$  does not couple directly to fermions, but we have determined that, say, the remaining pion in  $M$  does couple diagonally via loops. It is easy to check that the  $\rho, h$  intermediate state induces a direct coupling of the form  $\sim M' f^3 \bar{q} t_2 \mathcal{U}_1 q$  which is just like the pion in  $\Sigma_1$  would couple. Similarly, as must occur,  $h$  develops diagonal quark couplings through a  $\rho, A_1$  intermediate state.

Another feature is that the  $G_A = 1/2$  (for "bare" quarks)<sup>13)</sup> result of the original gauge models is in general modified in these theories. Letting the weak interactions transform  $M$  as  $(t'_0 \pm t'_3)$  from the right, we can calculate

$$(G_A)_{q\pm} = \pm f^2 \frac{(\kappa_0^2 - \kappa_1^2)}{m_{A_1}^2} \quad (3.6)$$

The  $\pm$  is for the quarks which transform as  $\pm \gamma_5$ . In fact, this result is perfectly general (as long as  $A_1$  exists, and is not restricted to just  $M$ ); in this simple case, however, Eq. (3.6) reduces to

$$(G_A)_q = \frac{m_{\rho'}}{m_{A_1}^2} \quad (3.7)$$

where we've assumed the (+) quark is dominating the low-lying fermion spectrum. Using  $m_{\rho'}^2 = 3/2$ , we get  $G_A \sim 0.85$  which is an improvement over the old unified models. This modification (and

improvement) of  $G_A$  is a general feature of all our type of models, and traceable directly to the intrinsic  $\rho\rho'$  mixing. Of course, for real baryons,  $G_A$  depends on Eq. (3.6) times a factor reflecting the composition of the baryons: If the baryons are taken in a symmetric SU(6) multiplet<sup>14)</sup>, e.g., we get to multiply by  $5/3$ :

$$(G_A)_{\text{BARYON}} \sim 1.3.$$

Model (c) ( $\rho, \omega_5, B, \rho'$ ). This is a model without  $A_1$ ; although the effects of an  $A_1$  may be present as a kinematical enhancement, it is not likely this model can be successfully unified with the nonstrong interactions ( $G_{A, \text{trees}} = 0$  etc.). Still, the model is very interesting for hadrons. If we proceed with just  $M$ , we find  $\omega_5$  [D(1285)?] degenerate with  $B$ . In fact,

$$m_{\rho}^2 + m_{\rho'}^2 = 2m_B^2 \quad (3.8)$$

fixing  $m_{\rho}^2 = \frac{1}{2}$ ,  $m_B^2 = \frac{3}{2}$ , we find indeed the experimentally known  $\rho'$  at  $m_{\rho'}^2 = \frac{5}{2}$ ! In this model we predict the  $\rho'$  mass in terms of  $\rho$  and  $B$ .

Model (b) ( $\rho, A_1, B, \omega'$ ). This is a model without  $\rho\rho'$  mixing. We must split  $A_1$  from  $\rho$ , and raise  $B$  even more than  $A_1$ . Thus, in addition to  $M$ , we must introduce some  $v_3$  and (at least) either  $v_1$  or  $v_2$ . With  $v_1$ , we find  $m_{\omega'}^2 = m_{\rho}^2 + m_B^2 - m_{A_1}^2 \simeq m_{A_1}^2$ , so  $\omega'$  is a candidate for  $\phi$ [1020]; with  $v_2$ ,  $m_{\omega'}^2 = m_{A_1}^2 + m_B^2 - m_{\rho}^2 \simeq 2m_{A_1}^2$ , so here  $\omega'$  is closer to  $\phi'$ (1675). This model seems less attractive, both on grounds of vector masses, and because of the large number of scalars.

A necessary remark in this section is that the models we've discussed so far have the minimal number of scalars to achieve interesting vector spectra. Other  $\Sigma$ 's may be added with corresponding complications.

The case of  $SU(3)$  is not totally satisfactory. Here we must have  $\rho, \rho', A_1, B$  all together (with strange mesons) in the same multiplet. Then we see immediately from (3.5) that (fixing  $m_\rho^2 = \frac{1}{2}, m_{A_1}^2 = 1, m_B^2 = \frac{3}{2}$ ) one cannot get  $m_{\rho'}^2$  greater than 2. (It is easy, of course, to put  $m_{\rho'}^2 = \frac{3}{2}$ ). So, at the  $SU(3)$  level, our models, as they stand, are giving the "first" dual  $\rho'$ ;  $SU(3)$  appears unsatisfactory here unless a lower  $\rho'$  is discovered<sup>15)</sup>. It also remains possible that some other scalar representation can split the multiplet differently.

#### 4. Structure of the Scalar Systems

We begin, for simplicity with a discussion of the "Abelian" case, which will illustrate most of the principles. Further, we will assume, at first, that there are no "insertions" in the "primed" side of  $M$ . Either  $\kappa_0$  or  $\kappa_1$  vacuum expectation value would break the system down from eight symmetries (four local, four "primed" global) to four final symmetries, being the product  $U(2)$  group of primed and unprimed. The four Goldstone bosons are eaten as four vectors are raised. However, we need in general  $\kappa_0$  and  $\kappa_1$ . Together, the final number of symmetries is reduced to two, being the product group  $t_0$  and  $t_1$ . Now there are two remaining real Goldstone bosons-- corresponding to a  $\pi$  and a  $P$ . In the case of model (a), e.g., this translates into a zero mass  $\pi$  and  $P$  (isosinglet). These are

consequences of spontaneous breakdown of  $t_3$  and  $t_2$  respectively. Their degeneracy is a consequence of the persisting  $t_1$  global invariance. At this stage, we remark that the (a) model appears completely analogous to the low spectrum of Brower's model<sup>4)</sup>. (He found  $\rho\rho'$  trajectories split equally around a degenerate  $P_\pi$  trajectory.) The possibility of a very low mass  $P$  is discussed further in Appendix B.

Here, however, we can change this situation if desired: e.g., a  $t_1$  insertion such as  $\text{Tr}(M^\dagger M(\alpha_0 t_0 + \alpha_1 t_1))$  will raise  $\pi$  and  $P$  away from zero mass, but they would remain degenerate. A  $t_3$  insertion, which would raise  $P$  above  $\pi$  is unfortunately parity violating and cannot be introduced directly. A  $t_3$  insertion can be achieved, however, via the introduction of an extra  $\Sigma_3$ . Then, we can have the term  $\text{Tr}(t_3 M^\dagger \Sigma_3 M)$ . In such a model (just  $t_3$  insertion), only one  $\pi$  remains at zero mass<sup>16)</sup>, while  $P$  is raised. Concurrently, the presence of  $\Sigma_3$  now further raises  $\rho'$  and  $h$ ; the  $\pi$ - $P$  and  $A_1$ - $h$  splitting is not necessarily correlated in size. The possibility of  $\rho'$  being the observed  $\rho'$  at  $\frac{5}{2}$  is now reopened in this model. If we set  $\rho'$  at that mass, we find  $m_h^2 = 2$ . Further, Eq. (3.6) for  $(G_A)_{\text{quark}}$  can be reexpressed,

$$m_\rho^2 m_{\rho'}^2 = m_A^4 G_A^2 + m_A^2 (m_h^2 - m_{A_1}^2) \quad (4.1)$$

so we would, unfortunately, be back at  $(G_A)_{\text{quark}} \sim \frac{1}{2}$ .

Further,  $t_3$  and/or  $t_1$  insertions can be reconciled with the usual nonstrong interactions [ $(t_0' + t_1')$  local gauge group]: We must, of course, make such insertions indirectly through the spontaneous breakdown of "weak" Higgs' fields  $\phi$ . For example, to get

a  $t_1$  insertion we would use

$$\phi_1 \equiv t_1 \sigma' + t_2 \pi \cdot \tau, \quad \phi \rightarrow S' \phi S'^{-1} \quad (4.2)$$

This field, being like the  $t_1, t_2$  part of  $\Sigma_1$ , is essentially Weinberg's <sup>6)</sup> scalar  $\phi$ , and is enough to provide adequate spontaneous breakdown in the usual weak interactions, while introducing a  $t_1$  insertion via  $\text{Tr}(\phi_1 M^\dagger M)$  (or  $\text{Tr}(\phi_1 M^\dagger \Sigma_1 M)$ ). Alternately, a  $t_3$  insertion can be achieved via a weak scalar  $\phi_3$  (like  $\Sigma_3$ ), taken in the combination  $\text{Tr}(\phi_3 M^\dagger \Sigma_3 M)$ . Parity-violating terms like  $\text{Tr}(\phi_1 \phi_3)$  can be included to avoid extra Goldstone bosons. All this can be done without expanding the weak interactions to include  $t_1', t_2'$  currents in analogy with the strong interactions. We shall, however, return to such subjects below.

Although the scalar systems in model (b) and SU(3) are easily discussed in the terms we have just employed for the Abelian case and case (a), our model (c) (no  $A_1$ ) is very unusual, and deserves separate comment. As one can see from Appendix A, there are no pions in the appropriate M. Presumably, this is related to not needing any to be eaten by an  $A_1$ . Pions can be included via  $\Sigma$ -type fields, as in the Appendix (or a more complicated M), but in all the multiplets we have found, the pion occurs together with fields that can have no parity and isospin conserving vacuum expectation value (see Appendix A). Thus, in these models, we find no reason for the pion to be zero mass (i.e., Goldstone). The point is of course that  $A_1$  type (global) transformations in such a model correspond to an intrinsically broken symmetry (by vector representation content). If a reason could be found for getting the  $\pi$  to

zero mass in the trees, such a representation would quickly generate  $m_\pi^2$  via loops (pseudo-Goldstone <sup>17)</sup>). In the absence of such a mechanism, we must conclude that a gauge  $A_1$  should always be incorporated from the start. This is distressing, however, in relation to the sum rule (3.8), which is so good.

## 5. Parity-Conserving Strong Interactions and Miscellaneous Topics

As mentioned in the Introduction, the basic pseudospin-symmetry group of these models, taken with say SU(2), SU(3) ..., is intrinsically parity conserving. Both "left" and "right" vector mesons are locked together with a single coupling  $f$ . On the other hand, the scalar representations that we have introduced have no such nice property. For example, one can (a priori) introduce parity-violating insertions of the form  $\text{Tr}(M^\dagger M(\alpha_0 t_0 + \alpha_3 t_3))$  in the potential. After that, of course, one can have  $\langle P \rangle \neq 0$  as a complementary source of spontaneous parity violation.

It is an interesting question whether or not we can stop such insertions in one way or another. If we have the ordinary  $(t_0' \pm t_3')$  weak interactions, acting on the M we have introduced, we certainly cannot stop them in a gauge invariant manner. (We can of course hold them small by hand as usual <sup>18)</sup>). One path toward stopping the insertions is through expanding the weak interactions to include the pseudospin group (and second class currents--to be suppressed). In such a situation, the group structure is so tight that it is no longer possible to violate parity by representation content, as is now common. Rather, we must take the whole parity-conserving multiplet. Then parity would have to be broken

spontaneously, presumably through a  $\langle P_{\text{weak}} \rangle \neq 0$ . This is an intriguing possibility that we will explore elsewhere.

Another possibility is that other scalar representations be used that brook no insertions. We have found a number of simple models of this, but they are physically unsatisfactory. As an example, consider in the "Abelian" case, the "SU(2)-M" representation

$$M_{\text{SU}(2)} = t_0 M_0 + it_1 M \quad (5.1)$$

This 4-component representation supports no  $t_0$  transformation. If we couple  $t_1$  in the manner

$$M_{\text{SU}(2)} \rightarrow S(t_1) M_{\text{SU}(2)} \quad (5.2)$$

$$q \rightarrow S(t_1) q$$

and a U(1) as

$$M_{\text{SU}(2)} \rightarrow M_{\text{SU}(2)} e^{-i\alpha t_1} \quad (5.3)$$

$$q \rightarrow e^{i\alpha t_0} q$$

then we have a model with all but one vector mesons raised and only one  $\sigma$ -like scalar left. This vector meson transforms like the product group  $t_1$ , and we can raise this with the introduction of

$$\Sigma_2 \rightarrow S(t_1) \Sigma_2 S^{-1}(t_1) \quad (5.4)$$

The resulting model has all four vector mesons raised, and one  $\pi$  and two  $\sigma$ 's left in the scalar sector. In this model, we have used our B-type particle to "eat the P", and the resulting system is

completely parity-invariant. A weak left-handed W can be attached as  $t_1'$  on the right of M, and  $u_3$  to the quarks. No parity-violating insertions are possible here, but, unfortunately, this model does not appear readily extendable to internal symmetry. Still, the idea of parity-conserving strong interactions, say by eating all P's<sup>19)</sup>, is very interesting, and we are not yet convinced such models cannot be made more physical.

We have some remarks about the issue of  $\pi_0 \rightarrow 2\gamma$ , (with respect to the quark doubling). In the first place, it is worth re-emphasizing that the vector (and scalar) systems of this paper can be viewed on their own right, to be attached later to appropriate baryons. From this point of view, we need classify various fermions under the pseudospin group. On the other hand, staying with our original (motivating) fermions, it looks that  $\pi_0 \rightarrow 2\gamma$  can be taken correct<sup>20)</sup>: In the case of the calculable pion-quark couplings, the question is open at the moment, but the pion in the  $\Sigma_2$  multiplet works quite well on its own. In fact, this pion, while coupling (say) with  $g_2, -g_2$  to  $q, q'$  respectively, gives masses  $m_q = g_2 v_2$ ,  $m_{q'} = -g_2 v_2$ . For simplicity then, think of zero quark mixing ( $M' = 0$ ). Writing  $q' \equiv \gamma_5 q''$ , we have two degenerate quarks  $q, q''$  at the same mass, and with opposite parity. Their pion couplings are both  $+g_2$ , so the  $q, q''$  contributions add and  $\pi_0 \rightarrow 2\gamma$  receives in fact an extra (good) factor of 2. At the same time, the B-quark coupling is  $i(\bar{q} \gamma^\mu q'' - \bar{q}'' \gamma^\mu q) B_\mu$  which is quite correct, because  $q''$  has opposite parity.

We have a remark to make about M-type particles in general. We have seen here that such are always best thought of as bound states in just such a  $\gamma_5$ -doubled quark system. Indeed, in the original

model 7,8) if one introduces a  $q'$  transforming with the opposite sign of  $\gamma_5$  under the primed group, we have the coupling  $\bar{q}'_R M_L^\dagger q_L$  etc. 21) In fact such an observation leads to the idea of an M-free theory based on  $q, q'$  with a gluon coupling between them. M's can be called into existence as bound states by searching for a solution with a mixing mass  $\bar{q}'q$ . All this is in direct analogy to the way pions are found with just one quark. Such theories would also be, in principle, parity conserving. Analogously, this idea can be extended to the gauge groups of this paper.

As a final remark, we note that these theories might be expected to Reggeize 22) better than the original M models. For example, here the  $h$  meson, being the next member on the  $\pi, h$  trajectory, may help  $\pi$  Reggeize. By the same reasoning P might be helped to reggeize (even in the original models) by  $A_1$ --especially if P is set to zero mass.

I would like to thank Professor K. Bardakci, Dr. K. Lane, Professor H. Bingham, Professor A. Goldhaber, and especially Professor I. Bars for helpful conversations.

### Appendix A. C, P, and Scalar Representations

In the text, we fixed the C and P content of  $q, q'$  to be the same. As we shall note below, the whole theory can be redone with different assumptions; first, however, we want to give the matrix C and P transformation properties of all relevant fields with this convention. C and P transformations on the quarks [in the U(4) notation] are

$$U_C q U_C^{-1} = C C' \bar{q} \quad (A.1)$$

$$U_P q U_P^{-1} = C' q \quad (A.2)$$

where  $C = i\gamma^2\gamma^0$  is the fermionic charge conjugation operation, while  $C' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  operates in the pseudospin space. The transformation properties of the vector mesons are then

$$U_C v U_C^{-1} = -C' v^T C' \quad (A.3)$$

$$U_P v U_P^{-1} = -C' v C' \quad (A.4)$$

The transformation properties of the  $\Sigma$ 's can be read off from the manner in which they couple to fermions [see Eq. (3.2)]; note the C and P properties of  $U$ ,

$$U_C \Sigma_1 U_C^{-1} = C' \Sigma_1^T C' \quad (A.5)$$

$$U_P \Sigma_1 U_P^{-1} = C' \Sigma_1 C' \quad (A.6)$$

For  $\Sigma_2$ , the parity is reversed and, for  $\Sigma_3$ , both parity and charge conjugation are reversed. In fact, this argument is formal for  $\Sigma_3$ , as it does not couple to these fermions. However, it is a useful field through its couplings to the vector mesons, and its C,P is fixed through its assumed spontaneous breakdown ( $\langle\sigma\rangle \neq 0$ ). We will assume the complex field M to transform as

$$u_c M u_c^{-1} = C' M^\dagger C' \quad (A.7)$$

$$u_p M u_p^{-1} = C' M C' \quad (A.8)$$

and use allowed vacuum expectation values accordingly. Other M's are possible, but do not appear useful.

This completes a description of the  $\rho, \rho', A_1, B$  type models. We want to note that a different kind of model involving  $\rho, A_1, A_1', \rho_{AB}$  (abnormal  $\rho$  with  $C = +$ ) is also possible. The way to obtain such models is, say, to leave the quarks alone, but use a  $\tau_2$  conjugation (instead of  $\tau_1$ ) for the vectors. That is

$$t^\alpha = \left( \begin{array}{c|c} \sigma^\alpha & \\ \hline & \hat{\sigma}^\alpha \end{array} \right), \quad \hat{\sigma}^\alpha = \sigma_2 \sigma^\alpha \sigma_2. \quad (A.9)$$

Then  $t_2 \sim \rho_{AB}$ . We have avoided explicit discussion of these models on physical grounds. With this remark, one can see how to construct models of  $\rho \rho' A_1 B$  even when  $q, q'$  have opposite C and P. One needs only use the  $\tau_2$  conjugated gauge particles.

Using these transformation properties, one can check the C and P of the following assortment of scalar multiplets for each of the SU(2) models of Sec. 2.

Theory (a) ( $\rho, \rho', A_1, h$ ):

$$M = t_0(\sigma + i\tau \cdot \underline{S}) + t_1(\sigma' + i\tau \cdot \underline{S}') + t_2(iP + \tau \cdot \underline{\pi}) + t_3(P + i\tau \cdot \underline{\pi}')$$

$$\Sigma_1 = \sigma t_1 + t_2 \tau \cdot \underline{\pi} + P t_3 \quad (A.10)$$

$$\Sigma_2 = t_0 \tau \cdot \underline{\pi} + t_1 \tau \cdot \underline{\pi}' + t_2 \sigma + t_3 \tau \cdot \underline{S}$$

$$\Sigma_3 = t_1 P + t_2 \tau \cdot \underline{S} + t_3 \sigma$$

Notice that the fields in different multiplets are in fact distinct.

Theory (b) ( $\rho, A_1, B, \omega'$ ):

$$M = t_0(\sigma + i\tau \cdot \underline{S}) + t_1(i\sigma' + \tau \cdot \underline{\sigma}) + t_2(\eta + i\tau \cdot \underline{P}) + t_3(P + i\tau \cdot \underline{\pi})$$

$$\Sigma_1 = t_0 \tau \cdot \underline{\sigma} + t_1 \sigma + t_2 \tau \cdot \underline{\pi} + t_3 \tau \cdot \underline{P} \quad (A.11)$$

$$\Sigma_2 = t_1 \tau \cdot \underline{\pi} + t_2 \sigma + t_3 S$$

$$\Sigma_3 = t_1 \tau \cdot \underline{P} + t_2 S + t_3 \sigma$$

where  $\eta$  has  $J^{PCG} = 0^{-++}$ .

Theory (c) ( $\rho, D, \rho', B$ ):

$$M = t_0(\sigma + i\tau \cdot \underline{S}) + t_1(\sigma' + i\tau \cdot \underline{S}') + t_2(\eta + i\tau \cdot \underline{P}) + t_3(i\eta + \tau \cdot \underline{P})$$

$$\Sigma_1' = t_1\sigma + t_2\sigma' + t_3\tau \cdot \underline{S}$$

$$\Sigma_1'' = t_0\tau \cdot \underline{g} + t_1\tau \cdot \underline{g}' + t_2\tau \cdot \underline{\pi} + t_3P \quad (\text{A.12})$$

$$\Sigma_2 = t_3\tau \cdot \underline{S} + t_2\sigma + t_1\eta$$

$$\Sigma_2' = t_1\tau \cdot \underline{\pi} + t_0\tau \cdot \underline{\pi}' + t_3S + t_2\sigma \cdot \underline{\tau}$$

Analogous multiplets for  $SU(3)$  are even easier to construct than these, and are left as an exercise for the reader.

#### Appendix B. Possible Low Mass P and Exchange Degeneracy

We have seen in the text that it appears possible to raise P above  $\pi$  in these models, but it is somewhat cumbersome, requiring a considerable number of new scalar fields; indeed, the simplest (a) model, with just the M scalar, keeps them degenerate (this is the extra symmetry  $t_1\tau$ , corresponding to  $\rho'$  transformations). This is also the situation found by Brower<sup>4)</sup>.

In fact, for a considerable time prior to the construction of the Buddha-type models, I. Bars and the author have been discussing together the possibility of a very low mass P particle, say degenerate with the pion, or very close. The following ideas of this Appendix arose in collaboration with I. Bars.

We realized that the very existence of a P particle in the M-models<sup>7-8)</sup> is a sign that the Lagrangians are giving a simple realization of exchange degeneracy. For some time physicists have been familiar with the  $(\rho, f)$ ,  $(\omega, A_2)$ ,  $(\pi, h)$ ,  $(\sigma, \rho')$ ,  $(\eta, B)$  exchange degeneracies, and a so-called "pseudotensor" trajectory<sup>3)</sup> with intercept near zero and approximately degenerate with  $A_1$ . This pseudotensor trajectory, usually assumed to begin particle content at  $J = 2$ , is "smoothing" the  $A_1$  behavior in the cross channel (say  $\pi\rho$  scattering). Our Lagrangians are clearly giving "smoothed" (no  $\Delta I = 2$ , unitarity bounded) physics, but realized on systems with  $J \leq 1$ . In this sense, it is not surprising that P is found in general in these models: It has the quantum numbers of a particle at  $J = 0$  on the "pseudotensor" trajectory.

This is, of course, all the more striking in the B-models here, where P most naturally occurs degenerate with  $\pi$ . On the

other hand, it may be the case that  $P$  should always be raised by hand, and thought of as a low spin representation of the higher  $J$  part of the pseudotensor trajectory.

In any case, Bars and I have also found that a very low mass  $P$  is a very peculiar particle indeed, and may possibly have eluded detection thus far. We are presently pursuing this investigation <sup>22)</sup>.

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9. R. Brower, in Ref. 4, has used this term previously. In fact, our usage corresponds rather closely with his.
10. F. Gilman and M. Kugler, Phys. Rev. Letters 30, 518 (1973).
11. The idea of calculable  $\pi$ -N couplings arose some time ago in discussions with I. Bars. At that time, however, with only the  $CP = +$  mesons of the original models, the idea did not work.
12. One can check at this point that, in fact, loops do induce diagonal couplings of  $B$  with quark.
13. Here, as well as in the original models,  $G_A$  will change calculably in higher order.
14. We wish to thank K. Bardakci for informing us of this.

15. The  $\rho'$  at  $m_{\rho'}^2 = \frac{5}{2}$  can always be added via the methods of I. Bars and K. Lane, "Gauge Model for the Pion Mass and the  $\rho'$  Vector Meson;" I. Bars and K. Lane, "Current Algebra and the Pion Mass in a Gauge Model;" I. Bars, M. B. Halpern, and K. Lane, "Hadronic Origin of the Pion Mass;" University of California, Berkeley, preprints. In this last reference,  $\rho'$  data and pion mass are in excellent agreement.
16. If, by choosing no  $t_1$  insertions, we leave the pion at zero mass, then it can pick up a calculable mass either through the weak interactions, or through the hadronic mechanism of Ref. 15.
17. S. Weinberg, Phys. Rev. Letters 29, 1698 (1972).
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19. This conjecture was arrived at in collaboration with I. Bars.
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21. It is intriguing to speculate that these  $q'$  fields are the  $\gamma_5$ -doubled leptons, with a very weak coupling strength for  $\bar{q}'_R M_L^\dagger q_L$  etc. Then the M's are quark-lepton "bound states."
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23. I. Bars and M. B. Halpern, to be published. Peculiarities of a very low mass P include the following. It is stable under the strong interactions, and does not couple to baryon-antibaryon. It has only very weak (order  $G_F^2, \alpha^2 G_F^2$ ) leptonic weak decays, and a primary electromagnetic  $2\gamma e^+ e^-$  decay, which may even be suppressed. Its most salient feature appears to be that  $\rho$  mesons can have strong P-decay modes. We have calculated,

e.g., that  $\Gamma(\rho^+ \rightarrow \pi^+ + X)/\Gamma(\rho^0 \rightarrow \pi^+ + X) \approx \frac{11}{9}$ , where X is a missing pseudoscalar with small mass. The asymmetry arises because of  $\rho^+ \rightarrow \pi^+ + P$  can occur, but there is no corresponding  $\rho_0$  mode.

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