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THE ELECTRIC DIPOLE OF A GUIDING CENTER
AND MOMENTUM CONSERVATION

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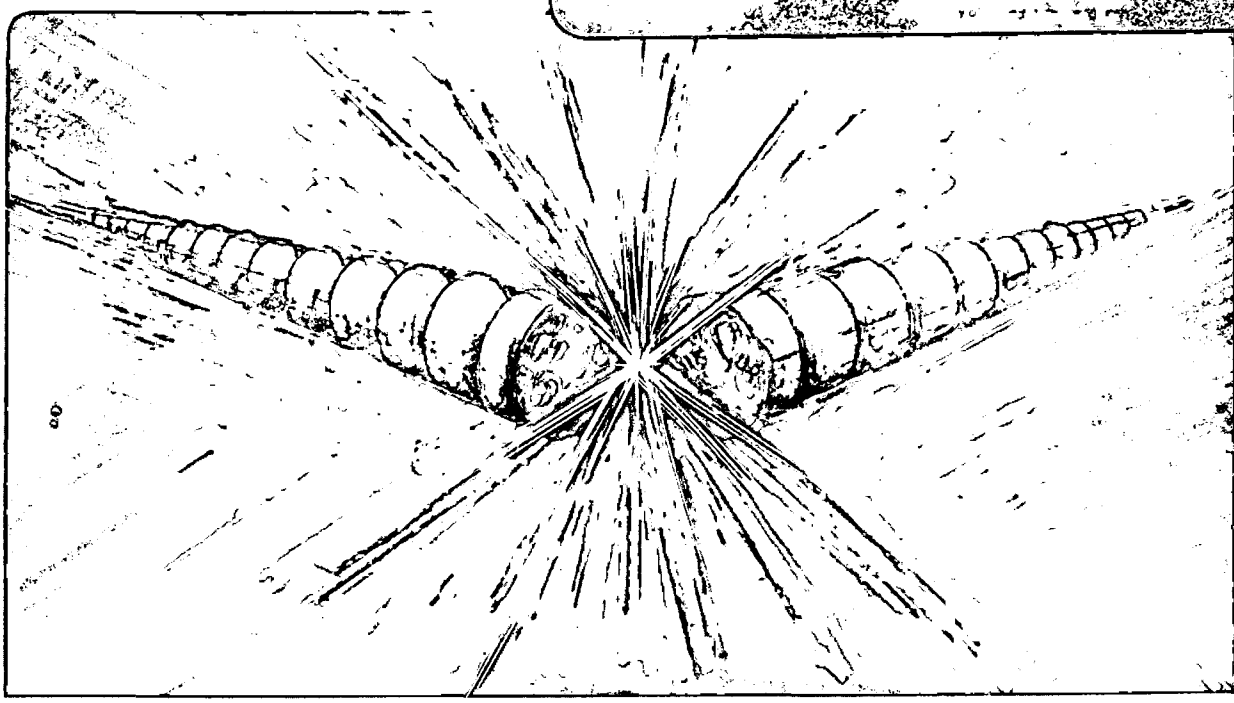

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# The Electric Dipole of a Guiding Center and Momentum Conservation*

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## Abstract

Pfirsch has shown that the charge and current density in the guiding-center representation have contributions from electric dipole moments and magnetic dipole corrections. We explain these moments in terms of elementary guiding-center theory, and point out their role in the law of momentum conservation.

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In developing a self-consistent plasma dynamics in the guiding-center (g.c.) representation, it is necessary to distinguish between the particle density and the g.c. density, as the source of electric field. This distinction is responsible for "finite gyroradius" effects, as is well known.

An additional effect, heretofore overlooked, has recently been discovered by Pfirsch.[1] In a careful analysis, based on Wimmel's modification[2] of Littlejohn's guiding-center Lagrangian,[3] he obtained the following expressions for the electric dipole moment $\underline{p}$ of a g.c.:

$$\underline{p} = (mc/B)\,\hat{b} \times \underline{v}_D, \tag{1}$$

and for the correction $\underline{\mu}'$ to its magnetic moment:

$$\underline{\mu}' = (m/B)(v_{\parallel}\,\underline{v}_D - \hat{b}\,\underline{v}_D \cdot \underline{v}_E) \tag{2}$$

Here the g.c. velocity is $v_{\parallel}\hat{b} + \underline{v}_E + \underline{v}_D$, the sum of parallel velocity, (zero order) electric drift $\underline{v}_E = (c/B)\,\underline{E} \times \hat{b}$, and the classical (first-order) drift $\underline{v}_D$, due to $\nabla_{\perp}B$, to field-line curvature, and to the acceleration $d\underline{v}_E/dt$ (so-called "polarization" drift).

We have independently discovered this same result, from a covariant Lagrangian analysis[4] motivated by the search for a momentum conservation law[5] in the g.c. representation. We found that the total momentum density $\underline{g}$ is (in the non-relativistic limit) the sum of two terms:

$$\underline{g} = \underline{g}_{Mink} + \underline{g}_{kin}, \tag{3}$$

where the Minkowski momentum density is

$$\underline{g}_{Mink} = \underline{D} \times \underline{B}/4\pi c \tag{4}$$

and the kinetic momentum density is

$$\underline{g}_{kin} = \Sigma\, n_{gc} m \langle v_{\parallel}\hat{b} + \underline{v}_E \rangle, \tag{5}$$

the sum of parallel and electric-drift contributions. The associated "macrofields" $\underline{D}$ and $\underline{H}$, defined by

$$\underline{D} = \underline{E} + 4\pi\underline{P},$$

$$\underline{H} = \underline{B} - 4\pi\underline{M}, \tag{6}$$

satisfy the usual macro-equations:

$$\nabla\cdot\underline{D} = 4\pi\rho_{gc},$$

$$c\ \nabla\times\underline{H} - \partial\underline{D}/\partial t = 4\pi\underline{j}_{gc}, \tag{7}$$

where the total charge and current density $(\rho,\underline{j})$ are related to their g.c. counterparts in the standard way:

$$\rho = \rho_{gc} - \nabla\cdot\underline{P},$$

$$\underline{j} = \underline{j}_{gc} + c\ \nabla\times\underline{M}. \tag{8}$$

The polarization $\underline{P}$ and magnetization $\underline{M}$ are the corresponding dipole densities:

$$\underline{P} = \Sigma\ n\langle\underline{p}\rangle,$$

$$\underline{M} = \Sigma\ n\langle-\hat{b}\mu_0 + \underline{\mu}'\rangle, \tag{9}$$

with $\mu_0 = 1/2\ m\ v_\perp^2/B$ the zero-order magnetic moment. Identical results have been obtained for the non-relativistic problem by Pfirsch and Morrison.[6]

It is the purpose of this paper to explain the origin of the dipoles, $\underline{p}$ and $\underline{\mu}'$, and to interpret the Minkowski momentum. The electric dipole $\underline{p}$ represents a displacement $\delta\underline{r}$ between the gyro-average position of a particle and the position of its g.c.:

$$\underline{p} = e\ \delta\underline{r}; \tag{10}$$

thus the displacement is, by (1),

$$\delta\underline{r} = \Omega^{-1}\ \hat{b} \times \underline{v}_D, \tag{11}$$

where $\Omega = eB/mc$ is the signed gyrofrequency. Taking the time-derivative of (11), we have

$$d(\delta\underline{r})/dt = \Omega^{-1}\ \hat{b} \times d\underline{v}_D/dt \tag{12}$$

(treating $\Omega$ and $\hat{b}$ as constant here), which we recognize as the second-order inertial drift of the gyro-mean particle position due to the acceleration of

the first-order drift $\underline{v}_D$. Since (12) is a standard result, obtained by iteration of the Lorentz force equation, we can now work backwards to deduce (1).

The magnetic moment correction $\underline{\mu}'$ is interpreted as the standard contribution[7] of a moving electric dipole:

$$\underline{\mu}' = \underline{p} \times \underline{v}/c. \tag{13}$$

Upon substituting the zero-order $\underline{v} = v_{\parallel}\hat{b} + \underline{v}_E$ into (13), we see that (2) is simply a direct consequence of (1).

Finally, we examine the Minkowski momentum. By (4), (6), (9), and (1), we see that

$$\underline{g}_{Mink} = \underline{E} \times \underline{B}/4\pi c + \Sigma\, n_{gc} m \langle \underline{v}_D \rangle. \tag{14}$$

Thus the momentum density $\underline{g}$ is, by (3) and (5),

$$\underline{g} = \underline{E} \times \underline{B}/4\pi c + \Sigma\, n_{gc} m \langle v_{\parallel}\, \hat{b} + \underline{v}_E + \underline{v}_D \rangle, \tag{15}$$

just as one would expect intuitively. (The two alternative expressions, (3) and (15), resolve the ancient Abraham-Minkowski controversy for the g.c. problem).

In closing, it must be stressed that, although the dipoles (1) and (2) are second order effects, they cannot be discarded. For they appear to first order in (15) and (9), and result from an exact Noether-symmetry analysis[5,6] of the system Lagrangian action principle.

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