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GAUGE-FIELDS AND INTEGRATED QUANTUM-CLASSICAL THEORY

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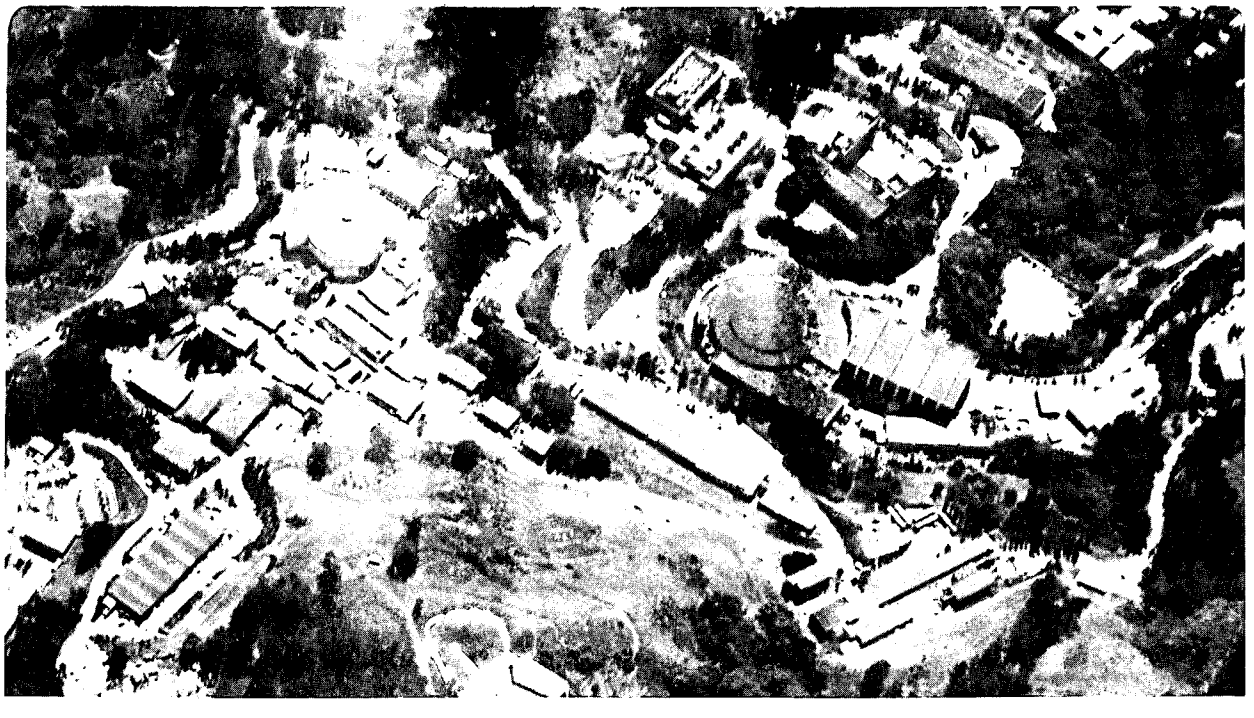

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Gauge-Fields and Integrated Quantum-Classical Theory

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Abstract

Physical situations in which quantum systems communicate continuously to their classically described environment are not covered by contemporary quantum theory, which requires a temporary separation of quantum degrees of freedom from classical ones. A generalization would be needed to cover these situations. An incomplete proposal is advanced for combining the quantum and classical degrees of freedom into a unified objective description. It is based on the use of certain quantum-classical structures of light that arise from gauge invariance to coordinate the quantum and classical degrees of freedom. Also discussed is the question of where experimenters should look to find phenomena pertaining to the quantum-classical connection.

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1. INTRODUCTION

This paper responds to a number of issues raised by previous speakers. It responds in particular to the challenge by Professor Wheeler to use quantum theory to construct not only devices and machines, but also the universe itself 1/; to the reminder by Wigner 2/ of the limited scope of contemporary quantum theory; and to Wigner's concerns pertaining to spacetime points. It adds depth to the remarks of Wigner 2/, Zurek 3/, and Joos 4/ about the importance of the interplay between the whole and its parts. And it addresses also the crucial question of where experimentalists should look to find phenomena that may shed light on the nature of the connection between the classical and quantum aspects of Nature.

To set the problem it is useful to distinguish between the "strict" and "informal" levels of the Copenhagen interpretation. Both levels adhere to the basic precept of the Copenhagen interpretation, which is that the quantum formalism "merely offers rules of calculation for the deduction of expectations pertaining to phenomena obtained under well defined experimental conditions specified by classical physical concepts" 5/. Thus both levels recognize and insist that the quantum formalism should be regarded merely as a "tool" for making predictions about observations, and that the "realities" with which the

formalism deals are the observations and knowledge of the community of communicating observers 6/.

However, this pragmatic character of the Copenhagen interpretation does not mean Copenhagenists deny the existence of external reality. There is certainly a tacit acceptance throughout the writings of Bohr that "our observations" are observations of (macroscopic) qualities that exist independently of their being observed by anybody. Heisenberg goes still further. In his 1958 book, "Physics and Philosophy," in the chapter on the Copenhagen interpretation, he speaks of "possibilities" and "actualities", and says:

> If we want to describe what happens in an atomic event we have to realize that the word "happens" can apply only to the observations, not to the state of affairs between two observations. It applies to the physical not the psychical act of observation, and we may say that the transition from the "possible" to the "actual" takes place as soon as the interaction of the object with the measuring device, and thereby the rest of the world, has come into play; it is not connected with the act of registration of the result in the mind of the observer. The discontinuous change of the probability function, however, takes place with the act of registration, because it is the discontinuous change in our knowledge of the instant of recognition that has its image in the discontinuous change of the probability function 7/.

The passage just quoted distinguishes nicely between the two levels of description. At the "strict" level one deals only with observations, knowledge, and the

formalism, as indicated in the final sentence. But at the "informal" level one speaks of what actually happens.

The description provided at the informal level is imprecise: it is not spelled out in detail. This imprecision is acceptable because the informal description does not enter into the calculations, which are the core of the strict interpretation. This gratuitous character of the informal level of description is acknowledged in the opening phrase: "If one wants to describe what happens...".

The tension between the strict and informal levels of the Copenhagen interpretation can be viewed as the motive force behind this conference: If there is some as yet imprecisely spelled out transition from possible to actual, or from quantum to classical, occurring at the level of the macroscopic devices, then not every physicist can suppress his curiosity about the nature of this basic mechanism.

2. MODEL UNIVERSE

A model universe that accords generally with Heisenberg's conception of external reality consists of an alternating sequence of density matrices and projection operators,

$$\{\ldots \rho_{i-1}, P_i, \rho_i, P_{i+1}, \ldots ,$$

where the ρ's are density matrices corresponding to Heisenberg states, and

$$\rho_i = P_i \rho_{i-1} P_i / \mathrm{Tr}\, P_i \rho_{i-1}.$$

Here P_i is an element of some specified set $\{P_z\}_i$ of

projection operators, z labels the elements of this set, and

$$\sum P_z = I.$$

This sum is over the elements of $\{P_z\}_i$. The probability that P_z will be selected by Nature from the set $\{P_z\}_i$ is Prob $(P_z) = \mathrm{Tr}\, P_z \rho_{i-1}$.

As an example consider a measurement in which a pointer with center of mass x swings perceptibly either to the right, $x>a>0$, or to the left, $x<b<0$. In this case $\{P_z\}_i$ can be the set consisting of P_1 and P_2, where (at any one of an appropriate continuum of times -- after the measurement) $P_1|x\rangle = \Theta(x)|x\rangle$, $P_2 = I-P_1$, and $\Theta(x)$ is the Heaviside function.

More generally, we can define a "measurement" to be a situation in which some quantity well-described by classical physics (e.g., the center of mass of the pointer) ends up in some one of a set of well-separated "bins". Then $\{P_z\}_i$ is the set of projection operators onto the set of distinct "bins".

3. CONTINUUM PROBLEM

This simple model encounters difficulties if one considers the continuum problem raised by Einstein 8/. The situation involves a radioactive source, a detector, a mechanism, and a macroscopic object. The object is given a kick by the mechanism when a decay is detected. After a while the position x of the center of mass of the macroscopic object will be spread continuously over a

macroscopic interval: there is no natural evolution into well-separated "bins". Hence we can say that no "measurement" has taken place. But in an objective description of Nature itself, one must specify whether, in this continuum case, some "reduction of the wave packet" occurs, and if so what the corresponding set $\{P_z\}_i$ is.

4. THE PROBLEM

Fundamentally, our problem is that, strictly speaking, contemporary quantum theory applies only under special conditions of "separation" 9/: some part of the universe separates from the rest for an interlude, and later rejoins the whole. The process of separation is the preparation, and the process of rejoining is the detection. During the interlude between the preparation and detection the "quantum system" must be separated, or isolated, from the rest of the universe, in the sense that it does not act significantly upon the rest. For if the quantum system acts significantly upon some other part of the universe, and hence phase correlations are established, then that other part must be included in the quantum system, in order for the quantum laws to hold. The observer and his devices must remain outside the quantum system, in the rest of the universe 9/ 10/.

This separation condition is well-satisfied the domain of atomic physics. However, both Bohr 11/ and Heisenberg 12/ have emphasized the problematic character of

the applicability quantum theory, conceived in this way, to, for example, biological systems. If we want to provide a foundation for all of the sciences then we must consider not only isolated systems but also systems that exist in close liaison with their environments, and act significantly upon their environment. Measuring devices are special cases of such systems.

The specific problem posed by these considerations is to construct a theory for the emergence of distinct classically describable form from amorphous quantum potentiality in systems that exist in close liaison with their environment. Contemporary quantum theory should emerge in the limiting case in which the system does not act significantly upon its environment. The theory would subsume measurement theory, which deals with the special case in which certain "classical" variables become effectively confined in well-separated "bins" by the natural evolution of the system.

5. TWO PRINCIPLES

The problem just posed involves passing from a limiting case to a more general case. To pass from a special case to a general case some principles are needed. I shall accept here two principles:

1. The first principle is that the solution should be tight-knit. There should be no arbitrariness: no accidents. The entire

scheme of things should fit together into a tight-knit unity 13/. This nonarbitrariness includes even the Lagranian. Normally the theory of measurement is approached from a very general point of view in which the specific nature of the interactions occurring in Nature is regarded as irrelevant. I shall assume just the opposite; that the specific nature of the interactions are of crucial importance.

2. The second principle is that the form that emerges from the actualization of quantum potentialities should be strictly describable in terms of the concepts of classical physics. Bohr's phrase "specified by classical physical concepts" is often interpreted as entailing merely that the classical concepts are appropriate in some fuzzy, imprecise kind of way. To eliminate fuzziness from our conception of the quantum-classical connection, and thereby introduce a very restrictive condition, I interpret Bohr's words in a strict way, and demand that the qualities actualized by Heisenberg's measurements be strictly describable in terms of the concepts of classical physics. In this case the collection of Heisenberg's

transitions from "possible" to "actual" will actualize a world that is strictly describable in terms of the variables and concepts of classical physics.

6. TWO NON-ACCIDENTS

I shall take the following two features of the world in which we live to be nonaccidential:

1. Light (electromagnetism) produces the form of the macroscopic world in which we live. It provides the forms of objects and devices, of our bodies and brains, and of our principal means of communication. [Gravity should perhaps ultimately be included, but for the present I consider only light.]
2. Gauge-fields play a central role in Nature. Hence structures arising from gauge invariance are likely to be important in the structure of Nature.

7. THE TECHNICAL BASIS

The gauge-invariance property of electro-magnetism plays an essential role in preserving the correspondence between quantum theory and classical physics in macroscopic limits. The infrared divergences associated with the masslessness of the photon appear to disrupt this correspondence, by altering the character of physical-region

singularities. But the correspondence is rescued by gauge invariance, which leads to the following expression for the S matrix 14/:

$$S = \sum_P U(P_h)\, \tilde{S}(P_{hq}) \tag{1}$$

$$\simeq \sum_P U(P_h)\, \tilde{S}(P_h)\ . \tag{2}$$

In Eq. (1) there is a sum over all Feynman paths P. There is an arbitrary separation of the photon interaction into hard and soft parts, and then a further separation of the soft part into its "classical" and "quantum" parts. The "hard path" P_h arises from P by omitting all soft vertices, and P_{hq} arises from P by omitting all "classical vertices", at which the classical part of the photon interaction couples.

Equation (1) has a rather amazing form, which is very useful for the consideration of infrared problems. The expression $\tilde{S}(P_{hq})$ involves only the hard and quantum parts of the interaction, and is expressible in terms of Feynman rules with a modified photon interaction term. It gives no infrared problems. The quantum coupling vanishes in the limit $|\mathbf{k}|_{\text{Eucl.}} \to 0$. This leads to Eq. (2) if the separation between hard and soft photons is made at small $|\mathbf{k}|_{\text{Eucl.}}$

All of the contributions from the "classical" interactions are collected into the factor $U(P_h)$, which, however, involves only the path P_h, which contains none of the soft photon vertices of P. In fact, all of the

"classical" interactions are shifted to the hard-interaction vertices. An arbitrary number of identical classically-coupled photons are emitted from each hard-interaction vertex. They constitute the bremsstrahlung radiation from that vertex.

This occurrence of arbitrary numbers of identical photons that do not depend upon the presence or absence of other soft photons allows the sum over all numbers of classically coupled photons to be expressed in closed form. The result $U(P_h)$ is a (pseudo) unitary operator which, acting upon the photon vacuum state, gives precisely the coherent photon state that corresponds to the (soft-mode part of the) classical electromagnetic field radiated by a classical particle of charge e moving on the spacetime path P_h.

The close connection between coherent states and classical physics is well-known 15/ 16/. For each single mode i the radiation field from the classical charge moving on P_h can be described asymptotically by an amplitude $a_i(P_h)$, and its time derivative $\dot{a}_i(P_h)$. This pair of numbers is equivalent to a complex number $z_i(P_h)$. The properties of the coherent state $|z_i(P_h)\rangle$ that correspond to this classical radiation field are defined by the single complex number $z_i(P_h)$, together with the identification of the mode function $\emptyset_i(x,t)$.

This coherent state of the quantum formalism has many properties that are similar to the properties of the

corresponding classical state 15/ 16/. Thus we have in Eq. (1) the automatic emergence from gauge invariance of quantum mechanical structures that correspond closely to classical physics. They are naturally described in terms of the variables of classical physics, and have only the classical degrees of freedom.

The central idea of the following proposal is to give special ontological status of these naturally occurring quantum-classical structures of light.

8. THE PROPOSAL

To give definite form the objective model described in section 2 we must specify the sets $\{P_z\}_i$. The proposal is to give special status to the coherent states of light by taking each set $\{P_z\}_i$ to be the set of projection operators $P_z=|z\rangle\langle z|$, [where $z \in \mathbb{C}^n$ and $|z\rangle=|z_1\rangle_1 \otimes \ldots |z_n\rangle_n$] onto the coherent states built on a set of modes $\{\emptyset_1, \ldots, \emptyset_n\}_i$ of the electromagnetic field. The specified set $\{\emptyset_1, \ldots, \emptyset_n\}_i$, which depends on i, consists of modes with "large" numbers of photons. The definition of "large" will be discussed later.

9. APPLICATION TO THE CONTINUUM PROBLEM

Consider again the Einstein continuum problem. Focus attention on the detection device, and on the soft photons emitted during the transition from the initial to the final state of this device. Suppose the set of modes

$\{\emptyset_1, \ldots, \emptyset_n\}_i$ with "large" numbers of photons is given. And suppose for simplicity that there are initially no photons in these modes, and that the photons are sufficiently soft so that Eq. (2) can be used. Then the probability density for Nature to choose the projector P_z from $\{P_z\}_i$ is

$$\text{Prob}\,(P_z) = \text{Tr}\, P_z\, S\, \rho_{i-1}\, S^\dagger$$

$$= \sum_{P_h} \sum_{P_h'} \langle z|z(P_h)\rangle\langle z(P_h')|z\rangle$$

$$\times\ \text{Tr}\, \tilde{S}(P_h)\, \rho_{i-1}\, \tilde{S}^\dagger(P_h').$$

The coherent-state matrix element is 15/

$$\langle z|z(P_n)\rangle =$$

$$\exp\sum_i [-\tfrac{1}{2}|z_i - z_i(P_h)|^2 + i\ \text{Im}\ z_i^*\ z_i(P_h)],$$

where the scale of the exponential fall-off factor is a change of photon number by one unit. Thus the selected z will be close to the $z(P_h)$ for some set of important contributions P_h to the transition under consideration. Conversely, once the $P_z = P_i$ is selected (by Nature) the same matrix element in

$$\rho_i = P_i\, \rho_{i-1}\, P_i / \text{Tr}\, P_i\, \rho_{i-1}$$

will damp out the contributions from the Feynman paths P_h for which

$$|z(P_h) - z| > \sim 1.$$

The well-known relationship 15/

$$\pi^{-n} \int dz\ P_z = 1$$

where

$$dz = dz_1 \ldots dz_n$$

and

$$dz_i = dx_i dy_i,$$

with

$$z_i = x_i + iy_i,$$

replaces

$$\sum P_z = I.$$

It guarantees that the results will conform to the demands of probability theory, and also those of ordinary measurement theory in the measurement-theory cases where z space can be divided into well-separated "bins", with the contributing domain of the variable $z(p_h)$ effectively confined to the interiors of these "bins" 16/.

10. SELECTIONS OF MODES

To complete the theory the rule must be given for selecting, for each i, that set of modes $\{\emptyset_1, \ldots, \emptyset_n\}_i$ having "large" numbers of photons. Certain self-consistency conditions, which have not previously been mentioned, will, it is hoped, fix these sets of modes. These conditions should also shape the detailed form of the theory in various other ways.

Though I had expected these aspects of the theory to be worked out in time for this conference, the theory is, at present, still in a developmental stage: the self-consistency conditions are, fortunately, rather severe.

11. BASIC FEATURES OF MODEL

In spite of its incomplete character the model described above has some features that are, I believe, worth mentioning. It would account for the emergence of distinct form from the amorphous quantum potentialities even in continuum situations, which occurring ubiquitously in Nature. I believe that most of the completing models, including the many-worlds interpretation, fail to account for the emergence of distinct form in continuum situations, and they fail to specify in continuum situations the precise character of the distinct forms that can emerge.

The forms that emerge, according to this model, are completely quantum mechanical in character, yet describable in terms of the concepts of classical electromagnetic theory. These special quantum-classical forms of light play a role similar to that ascribed by the Copenhagen interpretation to the human observer, namely the role of specifying the special aspects of Nature that characterize the distinctive features of actual situations. Light, rather than knowledge, becomes, in this theory, the substance of actual being.

Notice that the classical electromagnetic field is described in ordinary three-dimensional space, not in 3n-dimensional configuration space. On the other hand, only certain components of this field associated with soft photons are actualized. Thus spacetime points do not occur at the level of the actual.

12. WHERE SHOULD EXPERIMENTALISTS LOOK?

The distinction between quantum systems and classical systems is often confused with the distinction between microscopic and macroscopic. Thus we have seen at this conference many experiemental contributions, ostensibly pertaining to the problem of the quantum-classical connection, showing the high degree of conformity of phenomena to the predictions of quantum theory both for large systems, and for systems involving large numbers of elementary excitations (e.g., Cooper pairs). It must be stressed, however, that the condition for a system to be a quantum system, according to the Copenhagen view, is not a condition of smallness, or of smallness of the number degrees of freedom. The condition is rather that the quantum system not be communicating to the rest of the world, during the interlude between its preparation and detection.

In this connection, it is of course understood that the "system" involved here refers to a set of degrees of freedom. The degrees of freedom of a macroscopic object can be parameterized in an infinitude of ways. Any set of degrees of freedom that do not act significantly upon the remaining ones, or upon the rest of the world, is a "quantum system", to which quantum theory should apply. Phenomena related to the quantum-classical connection should occur only in systems that are communicating, but not too strongly, to the rest of the world.

The interplay between quantum systems and their environments has been the focal point of many contributions to this conference (Zurek, Joos, Glauber, Namiki, Session IB), and is the subject of a host of recent theoretical papers 17/. In these studies the environment is generally regarded as part of the whole quantum mechanical system. This is justified only to the extent that the environment is not communicating information about itself to the rest of the world.

This raises the question of what can be said in situations where the system is interacting significantly upon its environment, and the environment is acting significantly on the rest of the world; i.e., in situations where there is no separation of the kind mandated by contemporary quantum theory.

One conceivable possiblity (diametrically opposed to the Copenhagen interpretation) is to consider the entire universe as a quantum system. This immediately pushes the problem of the distinctness of our perceptions (and their describability in terms of the concepts of classical physics) onto the mind-body problem. Quantum theory becomes useless in a pracical sense, both because of the need to bring the whole universe, including human brains, into our calculations, and because of the fundamental problem of understanding how distinct perceptions can ever emerge from amorphous potentialities without introducing some fundamental element of distinctness.

The present model shifts the quantum-classical transition away from the level of the mind-body interaction down to the level where a limited system is sending signals to its environment via soft-photon coherent-state radiation. This shifts the calculations from a realm where they are essentially intractible, to the realm of soft-photon interactions with limited systems. These interactions are perhaps the best understood and most tractible in all of physics.

Space does not permit sample calculations, which will be given elsewhere. Roughly speaking, systems copiously emitting coherent-state soft photons should behave more classically than would be predicted by naive quantum mechanical calculations that do not fully account for the coupling of the quantum system to the classically described part of Nature.

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APPENDIX: The Preferred Basis

The problem of the preferred basis has been discussed at this conference, but not resolved. Suppose, for example, that the interaction between the system and its environment is governed by

$$H_{SE} = a\left[\frac{1}{2}(1+\sigma_z)(a_1+a_1^*) + \frac{1}{2}(1-\sigma_z)(a_2+a_2^*) + \frac{1}{2}(1+\sigma_z)(a_3+a_3^*)\right],$$

where the Pauli matrices σ_i act in the system space and a_i^* creates a quantum in mode i of the environment. No nontrivial operator in the system space commutes with H_{SE}. This is the normal situation: different oscillators of the environment generally couple to different projection operators in the system space. Thus if one asks whether photons couple to position or momentum the answer is neither: each photon mode is coupled in a different way.

In the theory discussed here the preferred basis vectors are the coherent states. The expectation value of an operator P in a state with initial density matrix ρ is typically

$$\langle P\rangle_\rho = \int \frac{dz_1}{\pi}\cdots\frac{dz_m}{\pi}\,\mathrm{tr}\,PP_{z_m}P_{z_{m-1}}\cdots P_{z_1}\rho P_{z_1}\cdots P_{z_m}$$

where, for each i, the set

$$z_i = (z_{i1}, z_{i2}, \cdots, z_{in(i)})$$

specifies some set of modes, which generally depend on i. Taking for simplicity the case $m = 1$ we have

$$\begin{aligned}\langle P\rangle_\rho &= \int \frac{dz}{\pi}\,\mathrm{tr}\,PP_z\rho P_z \\ &= \int \frac{dz}{\pi}\,\mathrm{tr}\,P_zP\rho P_z \\ &+ \int \frac{dz}{\pi}\,\mathrm{tr}[P,P_z]\rho P_z \\ &= \mathrm{tr}\,P\rho + \int \frac{dz}{\pi}[P,P_z]\rho P_z\end{aligned}$$

The second term is the effect of the collapses of the wave function.

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