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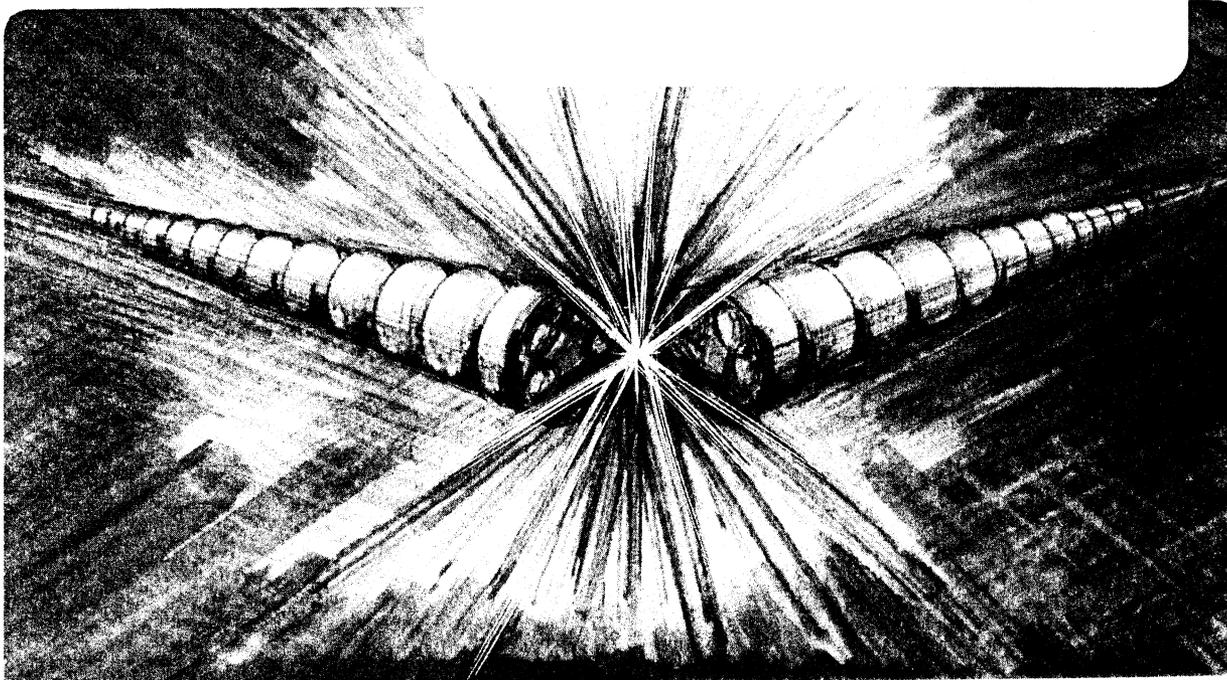
**ZAP USER'S MANUAL**

M.S. Zisman, S. Chattopadhyay, and J.J. Bisognano

December 1986

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ZAP USER'S MANUAL\*

Michael S. Zisman, Swapan Chattopadhyay and Joseph J. Bisognano<sup>†</sup>

Lawrence Berkeley Laboratory  
University of California  
Berkeley, CA 94720

December 1986

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<sup>†</sup>Now at CEBAF, 12070 Jefferson Ave., Newport News, VA 23606.



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## I. INTRODUCTION

In this manual, we describe the use and content of Phase I of the accelerator physics code ZAP [Version Date 1-DEC-86], which calculates the performance of a storage ring in terms of the effects of beam-intensity-dependent phenomena and the limitations they impose. Basically, the code is intended to be an assemblage of "tools" by which an accelerator physicist can explore the properties of a particular storage ring design or can perform parameter studies that will elucidate the trade-offs involved in particular choices of parameters. For this reason, the coupling among the various routines is relatively weak, i.e., the outputs (or inputs) of a particular routine are not, in general, transmitted to subsequent routines. The designer thus has the freedom to explore parameter variations without too many constraints placed upon him by the structure of the code itself. Of course, this philosophy makes the user ultimately responsible for the quality of the results, as it must be for a code of this type.

The code has been designed primarily for the study of electron storage rings. However, appropriate modifications for protons and heavy ions have been added for most options, so that essentially any storage ring can be examined.

Our aim in designing ZAP has been to make the code interactive and as user friendly as possible. For certain routines, where the calculations can be time-consuming or where it is useful to run a whole series of calculations at once, a batch input option has been provided. The desired running mode in these cases is the choice of the user. We have attempted to make the code forgiving of at least the errors that can easily be anticipated, and we have tried to make the inputs self-prompting and (relatively) unambiguous.

The code has been written for a VAX computer and should run without modification on any model VAX using the VMS operating system, from a MicroVAX to an 8800. An effort has been made to keep the Fortran-77 coding reasonably "vanilla" so

that the modifications required to run the code on non-VAX hardware should be minimal. [One notable exception to this involves the date and time routines, which are machine dependent and must be replaced to run on another computer. Another area where local modification may be required is in the file handling, i.e., the "OPEN" statements.]

In order to allow us to locate and correct any errors or obscurities that might arise in a complicated code such as ZAP, we urge you to contact us regarding any unusual results, etc., that are encountered when using the code. It would be helpful in such cases to have as much documentation as possible regarding the inputs being used and the resultant output, so that problems can be reproduced at LBL. In general, comments, questions, suggestions, or praise may be directed to LBL via electronic mail addressed to:

CSA2::ZISMAN [node 41.197]      via Decnet

or

ZISMAN@LBL.BITNET      via Bitnet

or

ZISMAN@LBL.ARPA      via ARPAnet.

Alternatively, information can be sent via normal mail to:

Michael S. Zisman  
Mail Stop 47/112  
Lawrence Berkeley Laboratory  
1 Cyclotron Road  
Berkeley, CA 94720  
U.S.A.

We clearly cannot be responsible for changes to the code made by others. Thus, if any non-trivial changes or improvements are made to ZAP, we ask that you change the name of your version and refer to it in print as "based on the code ZAP" or "a modified

version of the code ZAP". A preferable alternative would be to send us the suggested changes so that we can incorporate them -- at our discretion -- into future ZAP releases. This latter method should minimize the possibility of inadvertently introducing an error into the code, and will also make any improvements available to the widest group of ZAP users. Although initial dissemination of the code will be directly from LBL, it is likely that there will be secondary distribution as well. To provide information for users on improvements, bug fixes, etc., we would appreciate having at least one person from each laboratory using the code notify us. In this way we can maintain an up-to-date distribution list. We expect to continue work on ZAP, and a "Phase-II" version will be released at some future date. In this manual we will first give an overview of ZAP and indicate the various types of calculations that can be done with the code. Next, we will present examples of using the code, indicating what the input parameters are and providing some sample outputs. Finally, we will describe the physics behind the code, indicating the formulae employed in the various routines.

Michael S. Zisman  
Lawrence Berkeley Laboratory

Swapan Chattopadhyay  
Lawrence Berkeley Laboratory

Joseph J. Bisognano  
CEBAF

## II. DESCRIPTION

In this section we will provide a brief overview of the code and the types of calculations that can be performed with it. In general, we will assume that the code is being run interactively. The changes required to run selected routines in a "batch" mode, that is, with inputs obtained from a disk file, will be described separately under the heading of Data Preparation later in this section. Examples of using the code and sample outputs will be presented in Section III; it is suggested that new users of ZAP read Section II in its entirety before proceeding to the specific examples. The actual formulae utilized by the code, and references to their derivations, can be found in Section IV.

### Compiling ZAP

To compile ZAP under VMS, the user should perform the following sequence of DCL commands:

- (1) SET DEFAULT [USER.SUBDIR]  
where [USER.SUBDIR] is the directory in which the source code ZAP.FOR resides
- (2) FORTRAN ZAP or FORTRAN/LIST ZAP  
depending on whether or not a listing is desired
- (3) LINK ZAP  
to create an executable image, ZAP.EXE
- (4) DELETE ZAP.OBJ;\*  
to get rid of unnecessary object file

- (5) To run the code from a terminal, invoke it with RUN ZAP.
- (a) To run the code from a batch queue, create the following command procedure called ZAP.COM:

```
$ SET DEF [USER.SUBDIR]
$ RUN ZAP
1
```

(The last line, with the 1 in it, is required to set the code to batch mode.)

Having created the ZAP.COM file, the batch job is invoked by SUBMIT ZAP.

For some cases, it may be necessary or desirable to compile the code with the G\_FLOAT option turned on. In this case, several changes are required to the source code itself. The variable EXPMAX (in the main program only, not in any single-precision subroutines) must be changed from 87.0 to 700.0, and the value of ARGST should be reduced from 2.9 to 2.7 in routines CBML and CBMT. Having done so, recompile the code as in step (2) above, but with FORTRAN/G\_FLOAT ZAP or FORTRAN/G\_FLOAT/LIST ZAP. The other steps remain unchanged. If the computer does not have G\_FLOAT hardware available, these steps will still work, but the performance of the code will degrade by a large amount. Our own (inadvertent) experience indicates a slowdown of nearly a factor of 100 without the hardware G\_FLOAT; with the hardware, the slowdown should be of no concern.

#### File Assignments

A list of the various files utilized by ZAP is given in Table 1. These files, with the names indicated, are assumed to be present in whatever default directory is being used when the code is executed. Note that INPUT files (ZAP.DAT, ZAPLAT.DAT, CBMLRFI.DAT, CBMTRFI.DAT) must be prepared by the user prior to executing the code. However, they need not be present unless they are actually required for a

---

Table 1  
ZAP Input/Output File Assignments

<u>FILE</u>	<u>NAME</u>	<u>USE</u>
1	ZAP.DAT	Batch inputs for Main Menu options 3,4,7,8,9
2	ZAPOUT.DAT	Output for all routines (batch and interactive)
5	(TERMINAL.)	Interactive input file (VMS default is terminal)
6	(TERMINAL.)	Interactive prompt file (VMS default is terminal)
11	ZAPLAT.DAT	Lattice table and dynamic aperture data for Main Menu options 7,8,9
12	CBMLRFi.DAT*	RF longitudinal resonator input data for Main Menu options 1,3
12	CBMTRFi.DAT*	RF transverse resonator input data for Main Menu option 4

---

\* [i = 1 to 3]

particular calculation. [For example, the batch input file ZAP.DAT is not needed unless the batch program mode (see Startup, below) has been selected.]

It is advised that the various data files be prepared under their own specific names and subsequently copied for use into the "generic" files used by ZAP. In this way, the generic files can routinely be purged, but the originals are easily identified and can be recopied as needed. [The alternative of having 15 files, all called ZAPLAT.DAT, has been found empirically to work badly!]

### Startup

When ZAP is first loaded (using the DCL command "RUN ZAP"), the choice of interactive or batch mode must be made; this cannot be subsequently changed without ending and restarting the code. Upon startup, the code prompts:

ENTER PROGRAM MODE  
[-1 = END PROGRAM, 0 = INTERACTIVE, 1 = BATCH]

An entry of 0 (zero) here produces a "Main Menu" from which all subsequent use of the code is controlled.

### Program Control

Because of our design philosophy, there is not a unique "flow chart" for ZAP. The Main Menu appears on the terminal as follows:

```
ENTER CALCULATION TYPE
0 = END
1 = SINGLE BUNCH THRESHOLDS
2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING
  TABLES
3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES
4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES
5 = GAS SCATTERING LIFETIME
6 = FREE ELECTRON LASER FORMULAE
7 = INTRABEAM SCATTERING
8 = TOUSCHEK SCATTERING
9 = ION TRAPPING FORMULAE
```

The user is free to utilize any, or all, of these options as he sees fit. There is, however, one distinction among the above Main Menu options that should be noted: options 1-6 are stand-alone options, in the sense that their inputs can all be provided interactively from the terminal; options 7-9, however, require a table of lattice functions that has previously been written to a disk file called ZAPLAT.DAT. Upon completion of any option, the user can return to the Main Menu to select further options or end the code. The individual menu options will be described briefly below.

### Main Menu Options

1 = SINGLE BUNCH THRESHOLDS

This option calculates, as a function of RF voltage, a table of single-bunch parameters based on the longitudinal microwave and transverse fast-blowup (or, if it is lower, transverse mode-coupling)

thresholds. Calculated are bunch lengths, synchronous phase angle, synchrotron tunes, (combined) resistive wall and parasitic mode energy loss estimates, momentum half-height of the RF bucket, threshold currents for both longitudinal and transverse instabilities, and the bunch current corresponding to the more severe of the instability thresholds. These values can be based on the phenomenological "SPEAR Scaling" law for short bunches at the user's discretion. To provide an estimate of the RF cavity contribution to the broadband impedance, it is possible to utilize the same data on higher-order cavity modes that are required for longitudinal coupled-bunch calculations. The requisite files are indicated in the description of option 3 below, and the inputs needed are discussed in the section on Data Preparation.

Because it provides, at a glance, much of the information required to select the appropriate RF parameters for a storage ring, this option will usually be the first one used in a parameter study.

## 2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES

This option comprises three different "utility" routines that can be used in parameter studies.

The first routine produces a table, as a function of beam current, of longitudinal bunch parameters, based on the longitudinal microwave instability ("turbulent bunch lengthening"), the effect of potential well distortion, or the combined effect of both phenomena. Calculations can be done for either a Gaussian or parabolic bunch shape. For a given set of RF parameters and effective impedance, the code calculates the number of particles per bunch, rms bunch length, rms momentum spread and peak current. These values can be based on the phenomenological "SPEAR Scaling" law for short bunches at the user's discretion. The bunch length and momentum spread values from this routine should typically be used as starting values for intrabeam scattering or Touschek scattering calculations, so that the latter calculations are performed in a manner that takes bunch lengthening into account in a consistent fashion.

The second routine produces a table of various parameters needed for calculations of electron storage rings, as a function of energy. Values of the synchrotron radiation energy loss, natural emittance, radiation damping rates (transverse and longitudinal), and natural momentum spread are provided. All values are scaled from a set of starting values at a specified energy. These values are required for many ZAP routines when looking at a ring design as a function of energy.

The third utility routine provides, as a function of energy, values of the unnormalized emittance (for protons or ions) based on an input normalized emittance value. This is useful because ZAP utilizes unnormalized emittance values for all its calculations.

### 3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES

This option performs longitudinal coupled-bunch calculations for equally spaced Gaussian or parabolic bunches. For a given set of input parameters, the code indicates the modes having the fastest growth rates and those having the largest frequency shifts. For Gaussian bunches, the calculations can be performed according to the formalism of Wang or that of Zotter (see Section IV for details); for parabolic bunches, the Zotter formalism is always used. The summations required for these calculations can be evaluated explicitly or via analytic expressions at the user's option. Landau damping is also considered, and the selected modes are marked as stable [S], unstable [U], or Landau damped [D] as appropriate. This routine requires data on the higher-order modes of the RF cavity, which must either be entered manually from the terminal or provided in up to three separate files [CBMLRF1.DAT, CBMLRF2.DAT, or CBMLRF3.DAT], referred to in the input list as sets 1, 2, and 3, respectively. The required inputs are described below under the heading of Data Preparation. Note that none of these files are required if set 0 is selected, and that, for example, set 1 is not required if only set 2 is actually selected by the user.

For a single cavity mode, the code can optionally adjust the frequency to fall exactly on a beam spectral line, thus giving the maximum effect that can be attributed to that particular mode.

### 4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES

This option provides the same information as Main Menu option 3, but for the transverse case. The higher modes must be provided as detailed in option 3, in files called CBMTRF1.DAT, CBMTRF2.DAT, or CBMTRF3.DAT. Landau damping calculations for the higher-order non-rigid transverse modes ( $a > 0$ ) are based solely on the average synchrotron tune spread of the bunch. Landau damping of the rigid dipole transverse mode ( $a = 0$ ) is absent if there is only a synchrotron tune spread, but can still be calculated (in the Wang formalism only) by entering a finite betatron tune spread value. This option also calculates the Laslett tune shifts of the transverse betatron tunes.

### 5 = GAS SCATTERING LIFETIME

This option calculates e-folding electron beam lifetimes for gas scattering. Both elastic and inelastic (Bremsstrahlung) processes are considered. At the user's option, the lifetime calculation can be based on the assumption of equal limitations in both transverse planes (as is normally assumed) or a limitation in only one plane (as might happen, for example, because of a narrow-gap undulator).

### 6 = FREE ELECTRON LASER FORMULAE

This option evaluates the Free Electron Laser performance of a ring, based on peak current and emittance values as determined, for

example, from other ZAP options. For a specified wavelength and undulator gap, the parameters for the required undulator are calculated. Along with the beam parameters, a value for the FEL gain parameter and e-folding length are calculated. The degradation in performance due to the finite beam momentum spread is then evaluated assuming, at the user's choice, a Gaussian, Lorentzian, or rectangular momentum distribution.

## 7 = INTRABEAM SCATTERING

This option calculates beam growth rates (in all three dimensions) due to the effects of intrabeam scattering. In general, the starting values for bunch length and momentum spread should be based on results from Main Menu option 2, but the user is not constrained to do so. If non-zero synchrotron radiation damping rates are entered (for electrons), the code iterates to obtain the equilibrium emittance based on the balance among the competing effects of quantum fluctuations, intrabeam scattering, and radiation damping. Otherwise, it simply evaluates the rates at the specified value of (unnormalized) beam emittance. This option requires a table of lattice betatron functions at various points along the ring; the data must be provided in a file called ZAPLAT.DAT. The information needed will be described below in the section on Data Preparation. The overall rates calculated by the code are weighted averages of those calculated point-by-point throughout the lattice.

## 8 = TOUSCHEK SCATTERING

This option evaluates the Touschek scattering half-life for the ring as a weighted average over the lifetimes calculated point-by-point throughout the lattice. The momentum acceptance at any given lattice point is based on the minimum value of the RF acceptance, the physical or the dynamic aperture. Alternatively, the momentum limits can be specified explicitly (for the dispersive and non-dispersive regions of the ring) if they are already known (e.g., from a tracking calculation) or can be estimated.

If the ring aperture has been given a non-zero value in the lattice file, or if dynamic aperture data are provided, the code estimates the transverse momentum limitations of the lattice due to the effects of dispersion. This information is (optionally) made available to the Touschek scattering routine. These transverse limits arise because a particle that undergoes a momentum-changing collision has its orbit subsequently displaced in the dispersive regions of the lattice. For a sufficiently large momentum change, the orbit will exceed the available physical aperture (given in the last column of the ZAPLAT.DAT lattice table) and the particle will be lost. Moreover, in the dispersive regions of the lattice, a particle that changes momentum will also excite a betatron oscillation. This oscillation, coupled with the closed-orbit distortion, further shrinks the maximum acceptable momentum excursion. Thus, in the dispersive region, the momentum acceptance is potentially constrained by either the physical or the dynamic aperture

limitation (i.e., the maximum acceptable betatron oscillation amplitude). The limiting momentum change in the dispersive region based on each constraint is tabulated, along with the lattice location where the particle was predicted to be lost, in the aperture printout that immediately follows the lattice summary. In addition, the maximum acceptable momentum change in the non-dispersive region is indicated, along with the locations of maximum betatron amplitude and maximum dispersion.

If the equilibrium emittance has been calculated (for electrons) in option 7, this new emittance value is automatically utilized in the Touschek calculation. Thus the beam blowup from IBS is taken into account in a self-consistent manner.

#### 9 = ION TRAPPING FORMULAE

This option evaluates a few parameters relevant to the effects of ion trapping (for electrons). The critical masses for trapping are calculated, along with the limiting ion density, the neutralization factor, the equivalent ion "pressure" (useful to estimate the effects on beam lifetime), and the ion-induced tune shifts, all assuming full neutralization.

#### Data Preparation

Here we give details of the input files required for running ZAP in both interactive and batch modes. All numerical inputs are free format, and can be separated with either spaces or commas.

## Interactive Mode

In the interactive mode, there are three types of data files that may be needed for running ZAP (in addition to the inputs prompted from the user's terminal). These are:

- (A) Longitudinal RF cavity modes  
[used for Main Menu options 1 and 3]
- (B) Transverse RF cavity modes  
[used for Main Menu option 4]
- (C) Lattice and aperture data  
[used for Main Menu options 7, 8, and 9].

Instructions for preparing each are given below. Following the description of the data files will be a general discussion of input from the user's terminal.

- (A) Longitudinal RF cavity modes [CBMLRFi.DAT,  $i = 1$  to 3]

Required inputs:

- (1) RF frequency (MHz); number of modes to be input (20 maximum).
- (2) Header (alphanumeric, <80 characters) describing the data set.
- (3) Resonator parameters for one mode:  
angular frequency, i.e.,  $\omega$  (MHz); shunt impedance (Mohms);  $Q$ .
- (3a) etc. (for additional modes).

A sample data set is given in Table 2. Note that the code will add its own  $Q = 1$  broadband resonator, so the user should not, in general, add one of his own here. Although the coupled-bunch calculation will function properly, the estimated RF broadband contribution will be badly in error if a broadband resonator is appended to

the list by the user. If it is necessary to add a broadband resonator here, it is suggested that the code be run once without the additional resonator to get a proper estimate of the RF broadband contribution. This value can subsequently be entered manually in the single-bunch cases by selecting RF Set 0.

---

Table 2

Sample CBMLRFi.DAT Data Set

500,11		
STANDARD TEST PARAMETERS (500 MHz) - KEK CAVITY - LONGITUDINAL		
3142	9.860	44000
4764	2.910	37000
6585	0.100	34000
8181	1.340	112000
8344	0.418	34000
10420	0.328	34000
10730	0.828	34000
11690	0.100	34000
12330	0.106	34000
13330	0.587	34000
13620	0.100	34000

---

(B) Transverse RF cavity modes [CBMTRFi.DAT, i = 1 to 3]

Required inputs:

(1) RF frequency (MHz); number of modes to be input (20 maximum).

(2) Header (alphanumeric, <80 characters) describing the data set.

(3) Resonator parameters for one mode:

angular frequency, i.e.,  $\omega$  (MHz); transverse impedance  
(Mohms/m); Q.

(3a) etc. (for additional modes).

A sample data set is given in Table 3. Note that these inputs contain essentially the same information as those for the longitudinal case, with the exception that the transverse impedance is in units of  $M\Omega/m$  rather than  $M\Omega$ .

Table 3.

Sample CBMTRFi.DAT Data Set

---

500,12

STANDARD	RF TEST	PARAMETERS (500 MHz) - KEK CAVITY - TRANSVERSE
4329.1	0.990	45000
5210.6	11.590	56000
6728.0	26.640	40000
7149.6	0.270	45000
7822.6	3.230	95000
8461.6	1.000	40000
9911.7	1.000	40000
10798.3	1.000	40000
12256.6	1.000	40000
12551.3	1.000	40000
12704.6	1.000	40000
13396.4	1.000	40000

---

(C) Table of lattice and aperture data [ZAPLAT.DAT]

Required inputs:

- (1) No. of data points (400 maximum); ring circumference (m).
- (2) Title [header record, <80 characters, in alphanumeric format].
- (3)  $s$  (m);  $\beta_x$  (m);  $\alpha_x$ ;  $\beta_y$  (m);  $\alpha_y$ ;  $D$  (m);  $D'$ ; aperture (m).
- (3a) etc. [one line for each lattice point].
- (4) No. of dynamic aperture data points for positive momentum deviation.

- (4a) etc. If input (4) is non-zero, [positive] momentum deviation ( $dp/p$ ); dynamic aperture (m).
- (5) [If input (4) was negative] no. of dynamic aperture data points for negative momentum deviation.
- (5a) etc. If input (5) is non-zero, [negative] momentum deviation ( $dp/p$ ); dynamic aperture (m).

A few comments are in order here. The aperture value for one lattice point is retained for subsequent points if a zero value is encountered. Thus, if the aperture column is automatically provided by your lattice code, or a ZAP translation routine, as 0.0 (zero) for each point, it is only necessary to enter the aperture value for the very first lattice point and it will be used everywhere. (Similarly, to change the aperture value in a local region, enter the new value at the appropriate lattice point, continue with subsequent lattice point aperture values set to zero, and then "reset" the aperture to its original value where appropriate.) Having subsequent aperture values of zero, however, is NOT the same as having no entry at all, since the program will simply move to the next record to find a value. The zero value MUST be there explicitly for each lattice point. With regard to the dynamic aperture calculation, the input data must be derived from tracking the lattice. The required values correspond to the maximum stable betatron amplitude for a given momentum deviation, taken (arbitrarily) at the lattice location where the dispersion is maximum. If synchrotron oscillations are considered, the results should be symmetric about  $dp/p = 0$ ; this is assumed if the number of data points (input 4 above) is a positive number. If the number of points is negative then, after the positive  $dp/p$  values are read, the number of points for negative  $dp/p$  is read and a second group of values is input. (The  $dp/p = 0$  point, which must be the first "positive" point read in, should not be repeated in the "negative" table.) A sample data set (abridged) is shown in Table 4.

---

Table 4

Sample ZAPLAT.DAT Data Set

```

55,143.16
CF143 ZAP TEST LATTICE [see Particle Accel. 18, 223 (1986)]
0.0000 15.07971 0.00000 5.22260 0.00000 0.00000 0.00000 0.03000
0.2500 15.08386 -0.01658 5.23457 -0.04787 0.00000 0.00000 0.00000
0.5000 15.09629 -0.03316 5.27048 -0.09574 0.00000 0.00000 0.00000
.
.
. [you get the idea]
.
.
11.5233 1.22352 0.55953 2.75103 -0.32437 0.19808 -0.14021 0.00000
11.7267 1.05513 0.27236 2.85168 -0.16783 0.17677 -0.06963 0.00000
11.9300 1.00000 0.00000 2.88600 0.00000 0.16970 0.00000 0.00000
5 / DYNAMIC APERTURE CALCULATION
0.000 0.030
0.010 0.027
0.023 0.021
0.035 0.011
0.036 0.000

```

---

Input from the Terminal

Input from the terminal depends on the particular routine being run. However, the philosophy of the user interaction is similar in all cases. Inputs are always prompted for, including the units of the input being requested. Numerical inputs are all free format; by (Fortran) convention an entry of a slash (/) will preserve the current value of a variable (provided it has already been suitably entered). This feature is convenient for those routines where the inputs are sequential, rather than vectorized, since it is still possible to cycle relatively quickly through the required inputs to change a single entry. At present, the sequential (non-vectorized) inputs are found only in Main Menu options 5, 6 and parts of option 2. Alphanumeric inputs, on the other hand, are formatted and must be entered as requested. These values cannot be retained by using a slash; the input (always a single character) must be re-entered when cycling through an input routine. Although the code always prompts for alphanumeric values in upper

case, responses in lower case are also valid. Thus, entering either "Y" or "y" will be correctly interpreted as YES by the code. In general, if a recognizable error is made during input, e.g., typing an alphanumeric character in response to a request for numerical input, the code will simply ask the question again. On the other hand, more subtle errors, such as entering the synchrotron radiation energy loss in keV/turn, rather than MeV/turn as requested by the code, will not be caught. In most routines, the use of the list option may allow the user to notice and correct the error before actually attempting to perform the calculation with incorrect data. Beyond that, either the code will abort with an error or the user will rediscover the familiar Law of Computing referred to as GIGO (garbage in, garbage out).

In the cases where a given input subroutine is shared by more than one calculation routine, the common inputs are available to all of the latter. That is, the inputs for Main Menu options 1, 3, and 4 are shared, as are (separately) the inputs for options 7, 8, and 9. On the other hand, the inputs for option 2 (which look rather similar to those for options 1, 3, and 4) are NOT available elsewhere and must be entered the first time option 2 is selected, regardless of whether or not the questions for options 1, 3, or 4 have previously been answered. While this may seem confusing for new ZAP users, the following discussion should prove helpful. The code knows whether or not it has data available for use when a particular option is selected. If, upon selecting a given option, the code begins immediately to prompt for inputs, you must provide them. On the other hand, if the code says:

```
KEEP SAME PARAMETERS? [Y = YES, N = NO, ? = LIST VALUES, R = RETURN  
                        TO MAIN MENU]
```

then it DOES have a complete set of inputs available to perform the requested calculation. To inspect the values it has, type a question mark (?) and they will be displayed on the terminal for you. If any of the values are incorrect, either because

they were mis-entered or because they were left over from a previous calculation, the incorrect value(s) can be changed (one at a time) by typing "N" and selecting the item to be changed from a subsequent menu list. This feature will be illustrated in Section III, where actual examples of using the code are presented.

If the majority of the parameters need changing, it is usually faster to enter a menu code of 0 (zero), in which case all required inputs are requested. Remember that, because the numerical inputs are free format, any individual entry can be retained by entering a slash (/) followed by a carriage return. As mentioned, however, this does not apply to alphanumeric inputs, e.g., Y for yes, which must be explicitly reentered.

The requested calculation is only performed after responding "Y" to the KEEP SAME PARAMETERS? question. Upon completion of a calculation, the terminal will beep at you (to wake you up) and request further input or provide a path for getting back to the Main Menu, from which the code may be ended.

Another point worth making is that the code does not prompt for information it doesn't need for a particular case. For example, the radiation damping rates are requested in Main Menu option 7 if an electron case is being run, but not if protons have been specified. Thus, the questions may change slightly depending on what case is being considered by the user. The convention of the code is to determine the type of particle from the energy input value. A positive value for energy is interpreted as the total (as opposed to kinetic) energy of an electron. A negative energy value is interpreted as a kinetic energy per amu; in addition, the charge state and mass of the particle are then requested.

#### Batch Mode

As mentioned earlier, only a subset of ZAP options can be run in a "batch" mode, i.e., with inputs taken from a disk file ZAP.DAT. The options that may be run this way are the coupled-bunch calculations (Main Menu options 3 and 4) and the intrabeam scattering, Touschek scattering, and ion trapping calculations (Main Menu options 7, 8,

and 9). The required inputs are given below; a sample data set is given in Table 5. The data in Table 5 would reproduce the examples given in Section III for Main Menu options 3, 7, 8, and 9.

Note that, in the notation used in this manual, a batch run does not necessarily mean that the job has been run in a batch queue (i.e., with the DCL command "SUBMIT") but only that all program control after the startup question will be from the disk file, rather than from the user's terminal, even if the job is being run interactively as far as the operating system is concerned (i.e., with a DCL "RUN" command).

---

Table 5.

Sample ZAP.DAT Data Set\*

```

1/COUPLED BUNCH CALCULATION
1/LONGITUDINAL COUPLED BUNCH CALCULATION
143.16,9.4416,9.7969,3,4.923E-3
750,1,0,0,0
2,1,2,1,3
1.0016E-3,0.582,8.099E-9,8.099E-10,1
1.7883E10,119,3,1
0/**ENDS COUPLED BUNCH CALCULATION
2/IBS ETC.
750,4.1844E-9,4.1844E-10,1.0016E-3,0.582,1.7883E10,-28.818
38.262,3.7151E-4,0.216
4.923E-3,500,1.5,8.008E-3,0,0
0/ENDS TOUSCHEK CALCULATION
1/ION TRAPPING CALCULATION
-1/ENDS ION TRAPPING CALCULATION
0/**ENDS IBS ETC. CALCULATION
0/*****ENDS PROGRAM*****

```

---

\*Entries after a slash (/) will not be read by ZAP, and serve as a useful way of adding comments to make the input data set (slightly) more readable.

Required ZAP Batch Inputs:

(A) IBATCH [0 = end program; 1 = coupled bunch; 2 = IBS etc.]

For IBATCH = 1 the remaining inputs are:

(B) ITYPE [0 = read new IBATCH; 1 = long. calc.; 2 = transv. calc.]

For ITYPE = 1 or 2:

- (1) circ. (m); avg.  $\beta_x$  (m); avg.  $\beta_y$  (m); beam pipe radius (cm); momentum compaction.
- (2) energy (MeV); shape [1 = Gaussian, 2 = parabolic]; formalism [0 = Wang, 1 = Zotter]; debug flag [0 = off, 1 = on]; convergence test [0 gives 1.E-10].
  - (2a) If energy is negative, then Q, A (amu) of ion.
- (3) broadband impedance [ $Z/n$ , ( $\Omega$ )]; RF set [0 = input below, 1-3 = disk files]; no. of bunch modes [max. value is 4]; list of bunch modes to calculate [allowed values are 1-4 longitudinal or 0-3 transverse]  
  
If RF set 0 is selected, then
  - (3a) RF frequency (MHz); no. of resonators [max. value is 20].
  - (3b) (Up to 20 lines)  
resonator angular frequency,  $\omega$  (MHz); shunt impedance ( $M\Omega$  for long.,  $M\Omega/m$  for transv.); Q.
- (4)  $\sigma_p$ ;  $\sigma_\lambda$  (cm); horiz. emittance ( $\pi$  m-rad); vert. emittance ( $\pi$  m-rad); wall material [0 = none, 1 = Al, 2 = stainless steel, 3 = Cu, 4 = cold copper].
- (5) no. of particles per bunch; no. of bunches [0 gives all buckets full]; no. of frequencies to be printed [0 gives 5 values]; summation mode [0 = explicit, 1 = analytic (usually faster)].

For ITYPE = 2 only:

- (6)  $\nu_x$ ;  $\nu_y$ ; horiz. betatron tune spread (rms); chromaticity [ $\xi = d\nu_x/(dp/p)$ ].

Then, back to (B) above.

Inputs for IBS etc. for IBATCH = 2 are:

- (1) Energy (MeV); horiz. emittance ( $\pi$  m-rad); vert. emittance ( $\pi$  m-rad);  $\sigma_p$ ;  $\sigma_\ell$  (cm); no. of particles per bunch; horiz. emittance damping rate [ $= 2/\tau_x$ ] ( $\text{sec}^{-1}$ ).
- (1a) If energy is negative, Q; A (amu) of ion.
- (1b) If damping rate is negative, long. emittance damping rate [ $= 2/\tau_E$ ] ( $\text{sec}^{-1}$ ); natural  $\sigma_p$ ; natural  $\sigma_\ell$  (cm).
- (2) momentum compaction; RF frequency (MHz); RF voltage (MV); synchrotron radiation energy loss (MeV/turn); non-dispersive region momentum acceptance [0 means use RF or aperture limits]; dispersive region momentum acceptance. These data are used for calculating the Touschek lifetime (i.e., for Main Menu option 8).
- (2a) Other "(2)" inputs. Sequence is terminated by a zero momentum compaction factor, followed by a slash (/).
- (3) No. of filled bunches. This value is used for evaluating ion trapping (i.e., for Main Menu option 9).
- (3a) Other "(3)" inputs. Sequence is terminated by a negative input value.

The code then returns to input (1). A zero energy value (that is, a zero followed by a slash) returns to the (A) input for a new IBATCH value.

### Ending the Program

When the series of calculations has been completed, a Main Menu entry of 0 (zero) will end the code. The results of the calculations are then available in the disk file ZAPOUT.DAT for perusal (with the source editor) or printing. A new ZAPOUT.DAT file is created each time the code is started; the user is warned that these files should periodically be purged or deleted or they will eventually consume your entire disk quota!

### Summary of ZAP Routines

For completeness, a summary of the routines used in ZAP is given in Table 6. Although the organization of the code is along the lines of the Main Menu (described earlier), the list in Table 6 should be helpful to any user brave enough to try finding his

way through the source code itself. In general, arguments are passed from the input driver routines to the actual calculation routines through common blocks. To minimize confusion, the variable names are almost always kept the same in all the routines sharing a given parameter. In those cases where it was necessary to use different names in a particular routine, the translation is typically done in a fairly visible manner near the beginning of the particular routine.

---

Table 6.

ROUTINES CONTAINED IN ZAP

<u>Name</u>	<u>Type</u>	<u>Precision</u>	<u>Input Driver</u>	<u>Purpose</u>
APRTUR	SUBROUTINE	R*8	ZAPIN	MOMENTUM APERTURE
BPARAM	SUBROUTINE	R*8	--	LONG. BUNCH PARAM.
CBML	SUBROUTINE	R*8	INPUT	LONG. COUPLED BUNCH
CBMT	SUBROUTINE	R*8	INPUT	TRANS. COUPLED BUNCH
COULOM	SUBROUTINE	R*4	--	COULOMB LOG (FOR ZIBS)
FEL	SUBROUTINE	R*8	--	FREE ELECTRON LASER CALC.
GASCAT	SUBROUTINE	R*8	--	GAS SCATTERING
INPUT	SUBROUTINE	R*8	--	THRESH/CBML/CBMT INPUT
THRESH	SUBROUTINE	R*8	INPUT	SINGLE-BUNCH THRESHOLDS
TRPION	SUBROUTINE	R*4	ZAPIN	ION TRAPPING ESTIMATE
ZAPIN	SUBROUTINE	R*4	--	ZIBS/ZTOUSH/TRPION INPUT
ZCERF	SUBROUTINE	R*8	--	COMPLEX ERROR FUNCTION
ZIBS	SUBROUTINE	R*4	ZAPIN	INTRABEAM SCATTERING
ZJ	FUNCTION	R*8	--	BESSEL FUNCTIONS (FOR FEL)
ZTOUSH	SUBROUTINE	R*4	ZAPIN	TOUSCHEK SCATTERING

---

### III. EXAMPLES

In this section, we give examples of both the required inputs and resultant outputs for each Main Menu option in ZAP. Obviously it is impractical to provide a sample of each and every possible parameter choice in the code, so we will comment at various places about alternative answers and what they would imply. The examples will follow the organization of the Main Menu. For each Main Menu entry we show a copy of the inputs and prompts for a typical terminal session. These are annotated to correspond to the "input notes" that follow. The resultant outputs (also annotated) are shown next, followed by the "output notes." Notes on the inputs are labeled 1A, 1B, etc. (for option 1); notes on the outputs are labeled 1a, 1b, etc. In general, the parameters selected for the examples have all been taken for the same machine, denoted CF143.

#### Typical Analysis

Before turning to the examples of the individual Main Menu items, it is worthwhile to outline here a "typical" analysis sequence. Because of the manner in which the code was designed, the user is, of course, free to approach his problem in the way that seems best to him, but the following sequence has proved itself fairly efficient in doing performance studies of numerous electron storage rings at LBL. In what follows, we will assume that the user wishes to evaluate the performance of an electron storage ring lattice that has been made available from a lattice design code, such as SYNCH or MAD. An efficient use of ZAP might involve the steps outlined below:

- 1) If various energies are to be considered, take the requisite data from the lattice code and run the energy scaling table, Main Menu option 2-2. Assuming you don't misplace the output, these values will then be quickly available when you need to provide input values for any of the other ZAP routines.
- 2) Run Main Menu option 1 for a plausible range of RF voltages (and RF frequencies) to get the "lay of the land." Use these results to pick one

(or more) interesting RF scenarios to pursue. This choice will generally be based upon such issues as bunch length, RF momentum acceptance (which affects lifetime), or obtainable beam current. In practice, it is usually the case that the desired features will involve several of the above, so that trade-offs will be necessary. Thus, it is often true that a number of alternatives (emphasizing different features) should be followed at this stage of the analysis.

- 3) Having settled on one (or more) RF scenarios, run Main Menu option 2-1 to look at bunch lengthening as a function of beam current. These tables will provide the bunch length and momentum spread values to be used in subsequent ZAP routines. These tables should be generated for different beam energies and/or different RF parameters as appropriate.
- 4) Run Main Menu options 7,8,9 (for all scenarios being considered) to obtain information on emittance growth and Touschek lifetime. In the latter case, it may be informative to look at different assumptions about the momentum acceptance, e.g., RF only, physical or dynamic aperture, or tracking limits. This is easy to do in either the batch or interactive mode, once the IBS equilibrium emittance has been obtained. If there are many cases to run here, it can be quite convenient to set up a batch file for the various RF parameters, energies, etc., and run them all at once. Once the basic file is set up, making changes and rerunning cases is very rapid.
- 5) Run Main Menu option 5 to evaluate gas scattering lifetimes. Combining these results with the Touschek lifetime calculations gives the overall beam lifetime.
- 6) If desired, and if RF cavity higher-order mode data are available, Main Menu options 3 and 4 can be run to look at coupled-bunch instability growth times. These results can be used to see if the lattice is likely to give problems from coupled-bunch instabilities, and to assess feedback requirements.
- 7) If FEL use is of interest, Main Menu option 6 can be used to give predicted performance.
- 8) By now your office has been inundated with ZAP outputs, which you should spend some time trying to understand. Having done so, it is time to...
- 9) PUBLISH!

1 = SINGLE BUNCH THRESHOLDS

---

ENTER PROGRAM MODE <---[1A]  
[-1=END PROGRAM, 0=INTERACTIVE, 1=BATCH]  
0

ENTER CALCULATION TYPE <---[1B]  
0 = END  
1 = SINGLE BUNCH THRESHOLDS  
2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES  
3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES  
4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES  
5 = GAS SCATTERING LIFETIME  
6 = FREE ELECTRON LASER FORMULAE  
7 = INTRABEAM SCATTERING  
8 = TOUSCHEK SCATTERING  
9 = ION TRAPPING FORMULAE

1

ENTER RING CIRCUMFERENCE (m)  
143.16

ENTER AVERAGE BETA-X and BETA-Y (m) <---[1C]  
9.4416,9.7969

ENTER BEAM PIPE RADIUS (cm)  
3

ENTER RF CAVITY EQUIV. RADIUS (cm) <---[1D]  
[ZERO MEANS USE BEAM PIPE RADIUS]  
0

SELECT RF INPUT [0=TERMINAL, 1-3=DISK FILES] <---[1E]  
1

RF MODES HEADER IS:  
STANDARD TEST PARAMETERS (500 MHz) - KEK CAVITY - LONGITUDINAL

IS THIS CORRECT? (Y/N) <---[1F]  
Y

ENTER BEAM ENERGY (MeV) <---[1G]  
[NEGATIVE VALUE MEANS KINETIC ENERGY (MeV/amu)]  
750

ENTER SYNCHROTRON RAD. ENERGY LOSS (MeV/turn) <---[1H]  
8.0078e-03

ARE WIGGLERS USED? (Y/N) <---[1I]  
n

ENTER NO. OF BUNCHES (ZERO FILLS ALL RF BUCKETS)  
1

ENTER RMS MOMENTUM SPREAD  
.001

ENTER MOMENTUM COMPACTION (ALPHA)

4.923e-03

DO YOU WISH TO USE GAUSSIAN BUNCH SHAPE? (Y/N) <---[LJ]  
Y

ENTER RING BROADBAND IMPEDANCE (ohms) <---[LK]  
[NEGATIVE VALUE TURNS OFF "SPEAR SCALING"]  
2

ENTER WALL MATERIAL <---[LL]  
[0=no resistive wall,1=aluminum,2=stainless steel,3=copper,4=cold copper]  
1

CALCULATE POTENTIAL WELL DISTORTION? (Y/N)  
n

ENTER RF VOLTAGE RANGE (MV): MIN,MAX,STEP  
0.5,3,0.5

ENTER VOLTAGE per CELL (kV)  
[ZERO GIVES 500 kV/cell]  
0

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[LM]  
?

CIRCUMFERENCE= 143.160 m  
BEAM PIPE RADIUS= 3.000 cm  
RF CAVITY RADIUS= 3.000 cm  
AVG. BETA-X, BETA-Y= 9.442 m 9.797 m  
WIGGLER= N  
RF SET 1: FRF= 500.00 MHz, Z/n= 0.3303 ohms/cell  
ENERGY= 750.000 MeV  
SYNCH. RAD. E LOSS= 8.008E-03 MeV/turn  
NO. OF BUNCHES= 1  
MOMENTUM SPREAD= 1.000E-03  
MOMENTUM COMPACTION= 4.923E-03  
BUNCH SHAPE= GAUSSIAN  
RING BB IMPEDANCE= 2.000 ohms, SPEAR SCALING ON  
WALL MATERIAL= Al  
POTL. WELL DISTORTION= N  
VOLTAGE RANGE (MeV)= 0.500 to 3.000 in steps of 0.500  
VOLTAGE per CELL= 500.000 kV

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[LN]  
Y

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[LO]  
n

ENTER MENU CODE (? FOR LIST)  
?

MENU CODES ARE:  
O = CHANGE ALL PARAMETERS  
C = CIRCUMFERENCE  
B = BETA AVERAGE  
R = BEAM PIPE RADIUS  
D = RF CAVITY DIMENSION (RADIUS)  
W = WIGGLER PARAMETERS

F = RF PARAMETERS  
G = BEAM ENERGY  
U = SYNCHROTRON RAD. LOSS  
K = NO. OF BEAM BUNCHES  
P = MOMENTUM SPREAD  
A = MOMENTUM COMPACTION (ALPHA)  
S = BUNCH SHAPE  
Z = RING BROADBAND IMPEDANCE  
M = WALL MATERIAL  
L = POTENTIAL WELL DISTORTION  
V = RF VOLTAGE RANGE

(NOTE: For NUMERICAL inputs, a slash (/) preserves the current value.)

ENTER MENU CODE (? FOR LIST) <---[1P]  
z

ENTER RING BROADBAND IMPEDANCE (ohms)  
[NEGATIVE VALUE TURNS OFF "SPEAR SCALING"]  
-2

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[1Q]  
n

ENTER MENU CODE (? FOR LIST)  
l

CALCULATE POTENTIAL WELL DISTORTION? (Y/N)  
y

ENTER X and Y EMITTANCE VALUES (pi-m)  
[ZERO OMITTS SPACE CHARGE TERM]  
8.099E-09,8.099E-10

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[1R]  
y

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[1S]  
r

ENTER CALCULATION TYPE <---[1T]  
0 = END  
1 = SINGLE BUNCH THRESHOLDS  
2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES  
3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES  
4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES  
5 = GAS SCATTERING LIFETIME  
6 = FREE ELECTRON LASER FORMULAE  
7 = INTRABEAM SCATTERING  
8 = TOUSCHEK SCATTERING  
9 = ION TRAPPING FORMULAE

0

## Input Notes

- 1A) This question appears only once, when the code is first loaded. The "batch" mode here refers to the option of running the code with inputs taken from a (previously prepared) disk file ZAP.DAT, whether or not the code is being run in a batch mode (i.e., via a SUBMIT command). See discussion in Section II. Only Main Menu options 3, 4, 7, 8, and 9 can be run in the batch mode.
- 1B) Listing of the Main Menu, which directs all subsequent interactive running of the code. In this example, we have selected option 1.
- 1C) These values can be obtained from the ZAP printout that summarizes the lattice, which is described under Main Menu option 7. To be conservative, the code uses the larger of the two input values to calculate the transverse thresholds. If desired, different values may be used here, e.g., the beta values at the RF cavity location (if known).
- 1D) This value is used for scaling the RF broadband contribution to the transverse impedance (see Section IV). The implied difference in cutoff frequencies between the beam pipe and the RF cavity is not considered elsewhere in the code.
- 1E) A previously prepared set of cavity modes (that given in Table 2 of Section II) has been selected. If set 0 (zero) is selected, the code will subsequently request the RF frequency and the equivalent RF broadband contribution per cell (see Section IV). The latter quantity can be obtained from the informational printout in Main Menu option 3, if the higher-order cavity modes are known (see note 3f).
- 1F) Saying "no" here would return the program to the SELECT RF INPUT question (for example, if the header indicated that the wrong set had been selected).
- 1G) The flag to choose a proton or heavy ion is here. The code will subsequently request values for the particle charge state and mass number if a negative energy value is input. For the case shown, the positive value is interpreted as a total energy of an electron (as opposed to kinetic energy); a negative energy is taken to be a kinetic energy.
- 1H) This question will not appear unless the beam energy has been entered as a positive value. Note that the dimensions are MeV/turn; an incorrect value here (such as typing a value in keV/turn) will generally cause the program to abort; your punishment will thus consist of having to retype all of the inputs!
- 1I) The use of damping wigglers in the ring increases the synchrotron radiation, and hence the free-space impedance value. The wiggler gap and (absolute) bending angle would be requested here if this response were in the affirmative. This question will not appear unless the beam energy has been entered as a positive value.
- 1J) The Gaussian shape is the default for electrons, whereas the default shape for protons is parabolic. However, in either case the "other" bunch shape can be selected by saying "no" here.

- 1K) The choice of using SPEAR scaling is made here. A positive input, as shown here, uses SPEAR scaling; a negative input does not.
- 1L) This input is used to select the resistivity for the resistive wall energy loss (see Section IV). The values for the first three materials are taken from the Particle Properties Data Booklet; the fourth value corresponds to reducing the room temperature resistivity of copper (somewhat arbitrarily) by a factor of 30.
- 1M) This is the "action" question. To see what case the program thinks you've asked it to do, enter a question mark (?); the indicated list of parameters will magically appear on your terminal.
- 1N) Saying "yes" here causes the requested calculation to be performed, giving the first of the sample outputs shown below.
- 1O) Upon completion of the calculation, the terminal will beep at you and ask this question again. At this point, sensible answers are "no" (as shown) to change parameters for a subsequent calculation, or "return" to get back to the main menu (see note 1S). Answering "yes" again here would repeat the calculation just performed, giving rise to a second (identical) page of output; this choice is probably useful only if xeroxing costs more than computer time at your facility. After answering "no," there is a secondary menu that allows the user to change the value of any of the various input parameters, leaving the other parameters fixed. Because only one parameter can be changed at a time, this technique can be slow if many values must be modified before the next calculation. Therefore, entering 0 (zero) for the menu item here tells the code to cycle through all parameter inputs again. Remember that any numerical input can be retained by typing a slash (/) for that input; alphanumeric inputs, such as "y" must be explicitly reentered.
- 1P) Here we change the impedance value to turn off the SPEAR scaling option. The same numerical value of 2 ohms will be used.
- 1Q) To change a second parameter, we say "no" again. This time we turn on the approximate potential well calculation.
- 1R) Now we're ready for action; go for it! This response produces the second sample output below.
- 1S) Enough of this Main Menu option, let's go back to the Main Menu to try something else.
- 1T) At this point we can select another type of calculation or end the code to print (or examine on the screen) our outputs, which reside in the newly created ZAPOUT.DAT file in the current default directory. In the example shown, we quit here and printed the outputs shown below.

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\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

THRESHOLD CURRENT ESTIMATES FOR ELECTRONS IN 1 GAUSSIAN BUNCHES  
IN A PIPE OF A1

INPUT PARAMETERS

RING CIRCUMFERENCE 143.160 m RING RADIUS 22.785 m  
 RF FREQUENCY 500.000 MHz HARMONIC NO. 238  
 BEAM ENERGY 750.000 MeV MOMENTUM SPREAD 1.000E-03  
 AVERAGE BETA 9.797 m U0 8.008E-03 MeV/turn  
 MOMENTUM COMPACTION 0.004923 ETA 0.004923  
 BROADBAND IMPEDANCE 2.000 ohms RF B.B. IMPEDANCE 0.330 ohms/cell <---[1a]  
 BEAM PIPE RADIUS 3.000 cm RF EQUIV. RADIUS 3.000 cm

OUTPUT PARAMETERS <---[1b]

SIGTAU (psec)	SIGMA-L (mm)	VRP (MV)	PHI-S (deg)	NU-S	DE(RW+BB) ([MeV/Q]/turn)	BUCKET HH (DP/P)	NCELL	Zp/n-eff (ohms)	Zt (kohms/m)	IPL (A)	IPT (A)	Ib (mA)	Nb	Itot (A)
34.	10.1	0.500	178.0	0.011146	9.392E-03	1.851E-02	1	0.395	118.0	58.7	216.2	10.3	3.084E+10	10.3*
24.	7.1	1.000	179.1	0.015766	8.243E-03	2.657E-02	2	0.395	134.7	58.7	378.9	7.3	2.180E+10	7.3*
19.	5.8	1.500	179.4	0.019311	7.873E-03	3.269E-02	3	0.395	151.4	58.7	505.6	6.0	1.780E+10	6.0*
17.	5.0	2.000	179.5	0.022298	7.758E-03	3.783E-02	4	0.395	168.2	58.7	607.1	5.2	1.542E+10	5.2*
15.	4.5	2.500	179.6	0.024930	7.757E-03	4.235E-02	5	0.395	184.9	58.7	690.2	4.6	1.379E+10	4.6*
14.	4.1	3.000	179.7	0.027310	7.815E-03	4.643E-02	6	0.395	201.6	58.7	759.5	4.2	1.259E+10	4.2*

\* Printed value has been multiplied by 1000. <---[1c]



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\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

POTENTIAL WELL DISTORTION ESTIMATES FOR ELECTRONS IN 1 GAUSSIAN BUNCHES <---[19]  
IN A PIPE OF A1

INPUT PARAMETERS

-----  
RING CIRCUMFERENCE 143.160 m RING RADIUS 22.785 m  
RF FREQUENCY 500.000 MHz HARMONIC NO. 238  
BEAM ENERGY 750.000 MeV MOMENTUM SPREAD 1.000E-03  
AVERAGE BETA 9.797 m U0 8.008E-03 MeV/turn  
MOMENTUM COMPACTION 0.004923 ETA 0.004923  
BROADBAND IMPEDANCE 2.000 ohms RF B.B. IMPEDANCE 0.330 ohms/cell  
BEAM PIPE RADIUS 3.000 cm RF EQUIV. RADIUS 3.000 cm  
X-EMITTANCE 8.099E-09 pi-m Y-EMITTANCE 8.099E-10 pi-m <---[1h]

OUTPUT PARAMETERS <---[11]

-----  
VRF (MV) CBUNCH (A) SIGL0 (cm) NUS0 (Z/n)eff (ohms) SIGL (cm) SIGP NUS  
0.500 1.754E-03 1.006 0.011148 2.709E-02 1.000 1.000E-03 0.011214  
1.000 1.086E-03 0.711 0.015767 4.533E-02 0.705 1.000E-03 0.015904  
1.500 7.887E-04 0.581 0.019311 5.040E-02 0.576 1.000E-03 0.019477  
2.000 6.151E-04 0.503 0.022298 5.352E-02 0.499 1.000E-03 0.022482  
2.500 5.004E-04 0.450 0.024931 5.569E-02 0.446 1.000E-03 0.025124  
3.000 4.189E-04 0.411 0.027310 5.730E-02 0.408 1.000E-03 0.027510

## Output Notes

- 1a) The RF contribution to the broadband impedance for a single RF cell. The estimate is based on adding the slopes of the higher-order modes and then finding the strength of the equivalent  $Q=1$  broadband resonator (see Section IV).
- 1b) Output parameters are:
- SIGTAU: rms bunch length (in psec).
- SIGMA-L: rms bunch length (in mm).
- VRF: RF voltage, specified as a range in the input.
- PHI-S: synchronous phase angle, based on the sum of the synchrotron radiation loss (for electrons), the resistive wall loss, and the broadband energy loss.
- NU-S: synchrotron tune value, i.e., ratio of synchrotron oscillation frequency to beam rotation frequency.
- DE(RW+BB): sum of resistive wall energy loss (if a wall material is specified) and broadband energy loss. The chosen wall material, if any, is printed in the calculation title, e.g., "IN A PIPE OF Al" in this example.
- BUCKET HH: momentum half-height of RF bucket. This value is important in determining the Touschek lifetime of a lattice.
- NCELL: number of RF cells required to achieve specified voltage, based on the maximum voltage per cell specified in the input. The total RF contribution to the broadband impedance is given by  $NCELL * Z_{NBRRF}$ , where the latter quantity is that discussed in note 1a) above.
- Zp/n-eff: value for the effective longitudinal impedance used to evaluate the longitudinal microwave threshold current. If the SPEAR scaling option is on, and the rms bunch length is below the (input) value for the beam pipe radius, this value may be reduced considerably compared with the input impedance value (see Sec. IV). The value used by the code is not allowed to drop below the "free space" impedance of  $300b/R$ ; this limit explains the constant impedance values shown in the example.
- Zt: transverse impedance value; obtained from the longitudinal impedance by scaling with  $2R/b^2$ . SPEAR scaling is not used for the transverse value, i.e., the transverse impedance is based upon the unscaled longitudinal impedance. If the RF cavity equivalent radius (see note 1D) has been given a non-zero value, the RF broadband contribution is scaled separately from that for the vacuum chamber.

- IPL: peak current from the longitudinal threshold.
- IPT: peak current from the transverse threshold; the value printed is based on the lower of the transverse fast blowup or transverse mode-coupling thresholds.
- Ib: single-bunch current corresponding to the lower of the longitudinal and transverse threshold values.
- Nb: number of particles per bunch corresponding to the lower of the longitudinal and transverse threshold values.
- Itot: total ring current, obtained as the product of the single-bunch current (Ib) and the total number of bunches specified in the input.
- lc) To increase the "dynamic range" of various output columns, the values can be shifted (under program control) by a factor of 1000 up or down. Values increased by a factor of 1000 are marked with "\*"; those decreased are marked with "#". The definitions of the two flags are indicated at the bottom of the printout if they are used. In the example shown, the total current has been multiplied by 1000, so the printed values are in milliamperes rather than the amperes listed in the column header.
- ld) This message is printed if the (default) SPEAR scaling option is not used.
- le) Note that in this case the impedance increases with increasing RF voltage due to the extra contribution from the additional RF cells.
- lf) The peak current values in this case are much lower than with SPEAR scaling on, because the impedance values are much higher. For long bunches, the results with and without SPEAR scaling would be identical.
- lg) The results of an approximate estimate of potential well distortion effects are shown in this output.
- lh) Emittance values are used to calculate the space-charge impedance. They can be input as zero to skip this contribution.
- li) Output parameters are:
- VRF: RF voltage.
- CBUNCH: single-bunch current. This value is identical to the column labeled Ib in the threshold output (see note lb above).
- SIGL0: original bunch length without potential well distortion. This value is identical to that labeled SIGMA-L in the threshold output (see note lb above).
- NU0: original synchrotron tune value without potential well distortion. This value is identical to that labeled NU-S in the threshold output (see note lb above).

(Z/n)<sub>eff</sub>: value of  $\text{Im}(Z/n)$  [see Sec. IV]. A more rigorous combined potential well distortion and longitudinal microwave calculation can be obtained from Main Menu option 2, an example of which is given below.

SIGL: rms bunch length based on potential well distortion estimate.

SIGP: rms momentum spread ( $dp/p$ ) based on potential well distortion estimate.

NU<sub>s</sub>: synchrotron tune value based on potential well distortion estimate.

2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES

---

ENTER PROGRAM MODE <---[2A]  
[-1=END PROGRAM, 0=INTERACTIVE, 1=BATCH]  
0

ENTER CALCULATION TYPE <---[2B]  
0 = END  
1 = SINGLE BUNCH THRESHOLDS  
2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES  
3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES  
4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES  
5 = GAS SCATTERING LIFETIME  
6 = FREE ELECTRON LASER FORMULAE  
7 = INTRABEAM SCATTERING  
8 = TOUSCHEK SCATTERING  
9 = ION TRAPPING FORMULAE

2

CALCULATION TYPE? <---[2C]  
0=RETURN TO MAIN MENU  
1=BUNCH PARAMETERS (LONG. MICROWAVE and/or POTENTIAL WELL DISTORTION)  
2=ENERGY SCALING (electrons)  
3=ENERGY SCALING (protons or ions)

(NOTE: For NUMERICAL inputs, a slash (/) preserves the current value.)

1

ENTER RING CIRCUMFERENCE (m)  
143.16

ENTER BEAM PIPE RADIUS (cm)  
3

ENTER RF FREQUENCY (MHz) <---[2-1A]  
500

ENTER RF VOLTAGE (MV) <---[2-1B]  
(ZERO ALLOWS SELECTION OF BUCKET HEIGHT)  
1.5

ENTER RF CAVITY BROADBAND IMPEDANCE [Z/n] (ohms/cell) <---[2-1C]  
0.33

ENTER RF VOLTAGE per CELL (kV)  
[ZERO GIVES 500 kV/cell]  
0

ENTER EFFECTIVE BROADBAND IMPEDANCE [Z/n] (ohms) <---[2-1D]  
[NEGATIVE VALUE TURNS OFF "SPEAR SCALING"]  
2

ENTER BEAM ENERGY (MeV) <---[2-1E]  
[NEGATIVE VALUE MEANS KINETIC ENERGY (MeV/amu)]  
750

ENTER SYNCHROTRON RAD. ENERGY LOSS (MeV/turn) <---[2-1F]

8.0078e-03

ARE WIGGLERS USED? (Y/N) <---[2-1G]

n

DO YOU WISH TO USE GAUSSIAN BUNCH SHAPE? (Y/N) <---[2-1H]

y

ENTER NO. OF BUNCHES (ZERO FILLS ALL RF BUCKETS)

1

ENTER NATURAL MOMENTUM SPREAD <---[2-1I]

3.7151e-04

ENTER MOMENTUM COMPACTION (ALPHA)

4.923e-03

CALCULATE POTENTIAL WELL DISTORTION? (Y/N) <---[2-1J]

[ENTER O(nly) TO OMIT MICROWAVE CALCULATION]

n

ENTER AVG. CURRENT [INITIAL,FINAL,STEP] (mA)

1,20,1

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO SUBMENU]<-[2-1K]

?

CIRCUMFERENCE= 143.160 m  
BEAM PIPE RADIUS= 3.000 cm  
WIGGLER= N  
FRF= 500.00 MHz VRF= 1.5000 MV  
VOLTAGE per CELL= 500.000 kV Z/n= 0.3300 ohms/cell  
ENERGY= 750.000 MeV  
SYNCH. RAD. E LOSS= 8.008E-03 MeV/turn  
NO. OF BUNCHES= 1  
NATURAL MOMENTUM SPREAD= 3.715E-04  
MOMENTUM COMPACTION= 4.923E-03  
BUNCH SHAPE= GAUSSIAN  
RING BB IMPEDANCE= 2.000 ohms, SPEAR SCALING ON  
POTL. WELL DISTORTION= N  
CURRENT RANGE (mA)= 1.000 to 20.000 in steps of 1.000

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO SUBMENU]<-[2-1L]

y

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO SUBMENU]<-[2-1M]

n

ENTER MENU CODE (? FOR LIST) <---[2-1N]

z

ENTER EFFECTIVE BROADBAND IMPEDANCE [Z/n] (ohms)  
[NEGATIVE VALUE TURNS OFF "SPEAR SCALING"]

-2

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO SUBMENU]

y

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO SUBMENU]

n

```

ENTER MENU CODE (? FOR LIST) <---[2-10]
d
CALCULATE POTENTIAL WELL DISTORTION? (Y/N)
[ENTER 0(nly) TO OMIT MICROWAVE CALCULATION]
y
ENTER X and Y EMITTANCE VALUES (pi m-rad) <---[2-1P]
[ZERO OMITTS SPACE CHARGE TERM]
8.099e-09,8.099e-10
ENTER AVERAGE BETA-X and BETA-Y (m)
9.4416,9.7969
ENTER WALL MATERIAL <---[2-1Q]
[0=no resistive wall,1=aluminum,2=stainless steel,3=copper,4=cold copper]
1
KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO SUBMENU]
y
KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO SUBMENU]<-[2-1R]
r
CALCULATION TYPE? <---[2-2A]
0=RETURN TO MAIN MENU
1=BUNCH PARAMETERS (LONG. MICROWAVE and/or POTENTIAL WELL DISTORTION)
2=ENERGY SCALING (electrons)
3=ENERGY SCALING (protons or ions)
(NOTE: For NUMERICAL inputs, a slash (/) preserves the current value.)
2
ENTER SCALING VARIABLES: <---[2-2B]
ENERGY (MeV)
1500
U0 (MeV/turn)
0.12813
TRANSVERSE SYNCH. RAD. DAMPING RATE [= 2/TAUx] (1/sec)
230.544
LONGITUDINAL SYNCH. RAD. DAMPING RATE [= 2/TAUe] (1/sec)
306.096
EPS0 (pi m-rad)
1.8411e-08
NATURAL MOMENTUM SPREAD
7.4302e-04
ENTER ENERGY RANGE [MIN,MAX,STEP] (MeV) <---[2-2C]
500,2000,50
CALCULATION TYPE? <---[2-3A]
0=RETURN TO MAIN MENU
1=BUNCH PARAMETERS (LONG. MICROWAVE and/or POTENTIAL WELL DISTORTION)
2=ENERGY SCALING (electrons)
3=ENERGY SCALING (protons or ions)
(NOTE: For NUMERICAL inputs, a slash (/) preserves the current value.)

```

3

ENTER CHARGE, MASS (amu) FOR ION  
92,238

ENTER NORMALIZED EMITTANCES [EPSX,EPY] (pi m-rad) <---[2-3B]  
1.e-06,1.e-06

ENTER ENERGY RANGE [MIN,MAX,STEP] (MeV/amu) <---[2-3C]  
1000,10000,1000

CALCULATION TYPE? <---[2-3D]  
0=RETURN TO MAIN MENU  
1=BUNCH PARAMETERS (LONG. MICROWAVE and/or POTENTIAL WELL DISTORTION)  
2=ENERGY SCALING (electrons)  
3=ENERGY SCALING (protons or ions)

(NOTE: For NUMERICAL inputs, a slash (/) preserves the current value.)

0

ENTER CALCULATION TYPE <---[2D]  
0 = END  
1 = SINGLE BUNCH THRESHOLDS  
2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES  
3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES  
4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES  
5 = GAS SCATTERING LIFETIME  
6 = FREE ELECTRON LASER FORMULAE  
7 = INTRABEAM SCATTERING  
8 = TOUSCHEK SCATTERING  
9 = ION TRAPPING FORMULAE

0

## Input Notes

- 2A) This question appears only once, when the code is first loaded. The "batch" mode here refers to the option of running the code with inputs taken from a (previously prepared) disk file ZAP.DAT, whether or not the code is being run in a batch mode (i.e., via a SUBMIT command). See discussion in Section II. Only Main Menu options 3, 4, 7, 8, and 9 can be run in the batch mode.
- 2B) Listing of the Main Menu, which directs all subsequent interactive running of the code. In this example, we have selected option 2.
- 2C) For this option there are three sub-options that can be selected. This list is referred to as a "sub-menu." Initially, we choose sub-option 1, which calculates bunch-lengthening tables.
- 2-1A) The RF frequency is used to obtain the harmonic number of the ring.
- 2-1B) If a certain longitudinal momentum acceptance is desired, the voltage is entered as zero here. The code then requests a value for the RF bucket half-height. This option can be especially useful if a sequence of energies is to be run, since the RF voltage will automatically change in such a way as to maintain the specified acceptance value at different energies.
- 2-1C) The broadband impedance of the RF cavity (per cell) should be entered here. For a given series of longitudinal modes, this value can be obtained from the code either from the header of the Main Menu option 1 printout, or the informational output from Main Menu option 3 (see notes 1a or 3f and Section IV).
- 2-1D) The choice of using SPEAR scaling is made here. A positive input, as shown here, uses SPEAR scaling; a negative input does not (see Section IV).
- 2-1E) The flag to choose a proton or heavy ion is here. The code will subsequently request values for the particle charge state and mass number if a negative energy value is input. For the case shown, the positive value is interpreted as a total energy of an electron (as opposed to kinetic energy); a negative energy is taken to be a kinetic energy.
- 2-1F) This question will not appear unless the beam energy has been entered as a positive value. Note that the dimensions are MeV/turn; an incorrect value here (such as typing a value in keV/turn) will generally cause the program to abort; your punishment will thus consist of having to retype all of the inputs!
- 2-1G) The use of damping wigglers in the ring increases the synchrotron radiation, and hence the free space impedance value. The wiggler gap and (absolute) bending angle would be requested here if this response were in the affirmative. This question will not appear unless the beam energy has been entered as a positive value.
- 2-1H) The Gaussian shape is the default for electrons, whereas the default shape for protons is parabolic. However, in either case the "other" bunch shape can be selected by saying "no" here.

- 2-1I) The rms relative momentum spread determined from synchrotron radiation effects. This is generally available from the output of a lattice design code. The threshold current for turbulent bunch lengthening is based on this value. For protons or heavy ions, the initial rms relative spread is used here.
- 2-1J) This option selects whether the bunch lengthening calculation is based on the longitudinal microwave instability (turbulent bunch lengthening), on potential well distortion, or on the combined influence of both effects (see Section IV). Although the calculations of turbulent bunch lengthening are straightforward, the user is cautioned that those involving potential well distortion sometimes suffer from numerical problems. In the example shown, we select the microwave calculation alone.
- 2-1K) This is the "action" question. To see what case the program thinks you've asked it to do, enter a question mark (?); the indicated list of parameters will magically appear on your terminal.
- 2-1L) Saying "yes" here causes the requested calculation to be performed, giving the first of the sample outputs shown below.
- 2-1M) Upon completion of the calculation, the terminal will beep at you and ask this question again. At this point, sensible answers are "no" (as shown) to change parameters for a subsequent calculation, or "return" to get back to the sub-menu. Answering "yes" again here would repeat the calculation just performed, giving rise to a second (identical) page of output; this choice is probably useful only if xeroxing costs more than computer time at your facility. After answering "no," there is a secondary menu that allows the user to change the value of any of the various input parameters, leaving the other parameters fixed. Because only one parameter can be changed at a time, this technique can be slow if many values must be modified before the next calculation. Therefore, entering a menu item of 0 (zero) here tells the code to cycle through all parameter inputs again. Remember that any numerical input can be retained by typing a slash (/) for that input; alphanumeric inputs, such as "y" must be explicitly reentered.
- 2-1N) Here we change the impedance value to turn off the SPEAR scaling option. The same numerical value of 2 ohms will be used. The subsequent "yes" answer causes the new calculation to be carried out.
- 2-1O) To change a second parameter, we say "no" again. This time we turn on the approximate potential well calculation.
- 2-1P) When including potential well distortion (see Section IV) we need the emittance and average beta values to calculate the space charge impedance. If the emittance values are entered as 0,0 (two zeros, because there are two variables to read) this term is skipped.
- 2-1Q) The resistive wall term is included in the potential well calculation if a wall material is specified. The values for the first three materials are taken from the Particle Properties Data Booklet; the fourth value corresponds to reducing the room temperature resistivity of copper (somewhat arbitrarily) by a factor of 30.

- 2-1R) Enough of this sub-menu option, let's go back to the sub-menu to try something else.
- 2-2A) From here we can either exit to the Main Menu or try another choice from the sub-menu. Being energetic, let's go on to an energy scaling calculation for electrons.
- 2-2B) Here we enter a complete set of the required values at a specified beam energy. The code then produces a table of the same parameters over a specified energy range. In addition to the energy, the required parameters are:
- UD:           synchrotron radiation energy loss per turn.
- 2/TAUx:   horizontal emittance damping rate; calculated from the horizontal betatron damping time that is available from a lattice code. The factor of 2 is used to convert the rate to an emittance damping rate (remember that the emittance is proportional to the square of the beam size).
- 2/TAUe:   longitudinal emittance damping rate; calculated from the energy damping time that is available from a lattice code. The factor of 2 is used to convert the rate to an emittance damping rate (remember that the emittance is proportional to the square of the beam energy spread).
- EPS0:       the (uncoupled) natural emittance, available from a lattice code.
- 2-2C) The energy range does not have to include the "scaling" energy, but it's safer to do so to ensure that you didn't mistype one of the inputs, thus generating an incorrect table. These values are used for a number of ZAP routines, and the output from this utility routine will prove exceedingly useful if you don't lose it on your desk. After this calculation, the code returns automatically to the sub-menu. To generate another electron scaling table, you would answer 2 again. In this case, we'll move on to sub-option 3.
- 2-3A) Here we select the proton/ion energy scaling routine. This routine converts normalized emittance values (which are often used for these particles) to the unnormalized values required by ZAP.
- 2-3B) This is the **ONLY** place in the code where normalized emittance values should be used!
- 2-3C) These energies are automatically assumed to be kinetic energies in MeV/amu. They should not be entered as negative values here (as would be required elsewhere in the code).
- 2-3D) Here too, the code returns automatically to the sub-menu. By now we're tired of this option, so we return to the Main Menu.
- 2D) At this point we can select another type of calculation or end the code to print (or examine on the screen) our outputs, which reside in the newly created ZAPOUT.DAT file in the current default directory. In the example shown, we quit here and printed the outputs shown below.

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LONGITUDINAL BUNCH PARAMETERS FOR ELECTRONS IN GAUSSIAN BUNCHES <--[2-1a]  
(MICROWAVE)

CIRCUMFERENCE 143.1600 m RADIUS 22.7846 m  
 HARMONIC NO. 238 NO. OF BUNCHES 1  
 ALPHA 0.004923 ETA 0.004923  
 U0 8.008 keV/turn PIPE RADIUS 3.000 cm  
 z/n 2.000 ohms z/n for RF 0.330 ohms/cell  
 NATURAL SIGMAP 3.7151E-04 NATURAL SIGMAL 0.216 cm <---[2-1b]  
 ETOT 750.000 MeV GAMMA 1467.701 BETA 1.000000  
 VRF 1500.000 kV PHIS 179.694 deg RF BUCKET H.H. 3.283E-02  
 FRF 500.000 MHz CAVITY VOLTAGE 500.000 kv/cell

Iavg (mA)	Ipeak (A)	Nb	SIGMA l (cm)	SIGMA p <---[2-1c]
1.000	17.841	2.9805E+09	0.320	5.5118E-04
2.000	28.320	5.9610E+09	0.403	6.9444E-04
3.000	37.110	8.9415E+09	0.462	7.9494E-04
4.000	44.956	1.1922E+10	0.508	8.7494E-04
5.000	52.167	1.4902E+10	0.547	9.4250E-04
6.000	58.909	1.7883E+10	0.582	1.0016E-03
7.000	65.285	2.0863E+10	0.612	1.0544E-03
8.000	71.363	2.3844E+10	0.640	1.1024E-03
9.000	77.193	2.6824E+10	0.666	1.1465E-03
10.000	82.810	2.9805E+10	0.690	1.1875E-03
11.000	88.242	3.2785E+10	0.712	1.2258E-03
12.000	93.512	3.5766E+10	0.733	1.2619E-03
13.000	98.638	3.8746E+10	0.753	1.2960E-03
14.000	103.633	4.1727E+10	0.772	1.3284E-03
15.000	108.511	4.4707E+10	0.789	1.3593E-03
16.000	113.282	4.7688E+10	0.807	1.3889E-03
17.000	117.954	5.0668E+10	0.823	1.4172E-03
18.000	122.536	5.3649E+10	0.839	1.4445E-03
19.000	127.033	5.6629E+10	0.854	1.4708E-03
20.000	131.452	5.9610E+10	0.869	1.4961E-03

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LONGITUDINAL BUNCH PARAMETERS FOR ELECTRONS IN GAUSSIAN BUNCHES  
(MICROWAVE) [\*\*\*SPEAR SCALING OFF\*\*\*] <---[2-1g]

CIRCUMFERENCE 143.1600 m RADIUS 22.7846 m  
 HARMONIC NO. 238 NO. OF BUNCHES 1  
 ALPHA 0.004923 ETA 0.004923  
 U0 8.008 keV/turn PIPE RADIUS 3.000 cm  
 Z/n 2.000 ohms Z/n for RF 0.330 ohms/cell  
 NATURAL SIGMAP 3.7151E-04 NATURAL SIGNAL 0.216 cm  
 ETOT 750.000 MeV GAMMA 1467.701 BETA 1.000000  
 VRF 1500.000 kV PHIS 179.694 deg RF BUCKET H.H. 3.283E-02  
 FRF 500.000 MHz CAVITY VOLTAGE 500.000 kV/cell

Iavg (mA)	Ipeak (A)	Nb	SIGMA l (cm)	SIGMA p
1.000	9.086	2.9805E+09	0.629	1.0822E-03
2.000	14.424	5.9610E+09	0.792	1.3635E-03
3.000	18.900	8.9415E+09	0.907	1.5608E-03
4.000	22.896	1.1922E+10	0.998	1.7179E-03
5.000	26.569	1.4902E+10	1.075	1.8506E-03
6.000	30.003	1.7883E+10	1.142	1.9665E-03
7.000	33.250	2.0863E+10	1.202	2.0702E-03
8.000	36.345	2.3844E+10	1.257	2.1644E-03
9.000	39.314	2.6824E+10	1.307	2.2511E-03
10.000	42.175	2.9805E+10	1.354	2.3316E-03
11.000	44.942	3.2785E+10	1.398	2.4068E-03
12.000	47.626	3.5766E+10	1.439	2.4777E-03
13.000	50.236	3.8746E+10	1.478	2.5447E-03
14.000	52.781	4.1727E+10	1.515	2.6083E-03
15.000	55.265	4.4707E+10	1.550	2.6690E-03
16.000	57.695	4.7688E+10	1.584	2.7270E-03
17.000	60.074	5.0668E+10	1.616	2.7827E-03
18.000	62.408	5.3649E+10	1.647	2.8362E-03
19.000	64.698	5.6629E+10	1.677	2.8878E-03
20.000	66.949	5.9610E+10	1.706	2.9376E-03

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LONGITUDINAL BUNCH PARAMETERS FOR ELECTRONS IN GAUSSIAN BUNCHES  
(MICROWAVE + POTENTIAL WELL DISTORTION) [\*\*\*SPEAR SCALING OFF\*\*\*]

CIRCUMFERENCE 143.1600 m RADIUS 22.7846 m  
HARMONIC NO. 238 NO. OF BUNCHES 1  
ALPHA 0.004923 ETA 0.004923  
U0 8.008 keV/turn PIPE RADIUS 3.000 cm  
Z/n 2.000 ohms Z/n for RF 0.330 ohms/cell  
NATURAL SIGMAP 3.7151E-04 NATURAL SIGMAL 0.216 cm  
ETOT 750.000 MeV GAMMA 1467.701 BETA 1.000000  
VRF 1500.000 kV PHIS 179.694 deg RF BUCKET H.H. 3.283E-02  
FRF 500.000 MHz CAVITY VOLTAGE 500.000 kV/cell

POTENTIAL WELL DISTORTION PARAMETERS: <---[2-1e]  
AVG. BETA-X 9.442 m AVG. BETA-Y 9.797 m  
X-EMITTANCE 8.099E-09 pi m-rad Y-EMITTANCE 8.099E-10 pi m-rad  
WALL MATERIAL IS Al

Iavg (mA)	Ipeak (A)	Nb	SIGMA 1 (cm)	SIGMA P
1.000	9.144	2.9805E+09	0.625	1.0788E-03
2.000	14.550	5.9610E+09	0.785	1.3576E-03
3.000	19.095	8.9415E+09	0.897	1.5529E-03
4.000	23.158	1.1922E+10	0.986	1.7082E-03
5.000	26.896	1.4902E+10	1.062	1.8393E-03
6.000	30.392	1.7883E+10	1.128	1.9539E-03
7.000	33.698	2.0863E+10	1.186	2.0564E-03
8.000	36.850	2.3844E+10	1.240	2.1496E-03
9.000	39.872	2.6824E+10	1.289	2.2353E-03
10.000	42.783	2.9805E+10	1.335	2.3150E-03
11.000	45.596	3.2785E+10	1.378	2.3895E-03
12.000	48.325	3.5766E+10	1.418	2.4597E-03
13.000	50.977	3.8746E+10	1.456	2.5261E-03
14.000	53.559	4.1727E+10	1.493	2.5893E-03
15.000	56.021	4.4707E+10	1.529	2.6509E-03
16.000	58.480	4.7688E+10	1.563	2.7087E-03
17.000	60.887	5.0668E+10	1.595	2.7641E-03
18.000	63.245	5.3649E+10	1.625	2.8174E-03
19.000	65.557	5.6629E+10	1.655	2.8688E-03
20.000	67.828	5.9610E+10	1.684	2.9185E-03

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PARAMETER SCALING TABLES FOR ELECTRONS <---[2-2a]

ENERGY (MeV)	U0 (MeV/turn)	EPS0 (pi m-rad)	GSR (1/sec)	GSRL (1/sec)	NAT. SIGP
500.000	1.5819E-03	2.0457E-09	8.5387	11.3369	2.4767E-04
550.000	2.3160E-03	2.4753E-09	11.3650	15.0894	2.7244E-04
600.000	3.2801E-03	2.9458E-09	14.7548	19.5901	2.9721E-04
650.000	4.5179E-03	3.4572E-09	18.7595	24.9071	3.2198E-04
700.000	6.0768E-03	4.0095E-09	23.4301	31.1084	3.4674E-04
750.000	8.0081E-03	4.6027E-09	28.8180	38.2620	3.7151E-04
800.000	1.0367E-02	5.2369E-09	34.9744	46.4359	3.9628E-04
850.000	1.3212E-02	5.9120E-09	41.9505	55.6981	4.2104E-04
900.000	1.6606E-02	6.6280E-09	49.7975	66.1167	4.4581E-04
950.000	2.0615E-02	7.3849E-09	58.5667	77.7597	4.7058E-04
1000.000	2.5310E-02	8.1827E-09	68.3093	90.6951	4.9535E-04
1050.000	3.0764E-02	9.0214E-09	79.0766	104.9909	5.2011E-04
1100.000	3.7056E-02	9.9010E-09	90.9197	120.7152	5.4488E-04
1150.000	4.4267E-02	1.0822E-08	103.8900	137.9359	5.6965E-04
1200.000	5.2482E-02	1.1783E-08	118.0385	156.7212	5.9442E-04
1250.000	6.1791E-02	1.2785E-08	133.4167	177.1389	6.1918E-04
1300.000	7.2287E-02	1.3829E-08	150.0756	199.2572	6.4395E-04
1350.000	8.4066E-02	1.4913E-08	168.0666	223.1440	6.6872E-04
1400.000	9.7229E-02	1.6038E-08	187.4408	248.8674	6.9349E-04
1450.000	1.1188E-01	1.7204E-08	208.2495	276.4954	7.1825E-04
1500.000	1.2813E-01	1.8411E-08	230.5440	306.0960	7.4302E-04
1550.000	1.4609E-01	1.9659E-08	254.3754	337.7373	7.6779E-04
1600.000	1.6587E-01	2.0948E-08	279.7950	371.4872	7.9255E-04
1650.000	1.8760E-01	2.2277E-08	306.8541	407.4138	8.1732E-04
1700.000	2.1139E-01	2.3648E-08	335.6038	445.5851	8.4209E-04
1750.000	2.3738E-01	2.5059E-08	366.0953	486.0691	8.6686E-04
1800.000	2.6569E-01	2.6512E-08	398.3800	528.9339	8.9162E-04
1850.000	2.9646E-01	2.8005E-08	432.5091	574.2474	9.1639E-04
1900.000	3.2984E-01	2.9539E-08	468.5337	622.0778	9.4116E-04
1950.000	3.6595E-01	3.1115E-08	506.5052	672.4929	9.6593E-04
2000.000	4.0495E-01	3.2731E-08	546.4747	725.5609	9.9069E-04

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PARAMETER SCALING FOR Q= 92., A= 237.950 IONS <---[2-3a]

NORMALIZED EMITTANCE VALUES:

HORIZONTAL= 1.0000E-06 VERTICAL= 1.0000E-06

ENERGY (MeV/amu)	EPSX (pi m-rad)	EPSY (pi m-rad)
1000.000	5.5052E-07	5.5052E-07
2000.000	3.3512E-07	3.3512E-07
3000.000	2.4388E-07	2.4388E-07
4000.000	1.9235E-07	1.9235E-07
5000.000	1.5902E-07	1.5902E-07
6000.000	1.3562E-07	1.3562E-07
7000.000	1.1826E-07	1.1826E-07
8000.000	1.0487E-07	1.0487E-07
9000.000	9.4208E-08	9.4208E-08
10000.000	8.5524E-08	8.5524E-08

## Output Notes

- 2-1a) The label indicates the type of calculation, i.e., microwave only, combined microwave and potential well distortion, or potential well distortion only.
- 2-1b) The natural bunch length (based on the input value for the natural momentum spread along with the specified RF parameters) is always given here. Values are rms quantities.
- 2-1c) Output parameters are:
- Iavg: average ring current, specified as an input range.
  - Ipeak: peak current in a single bunch, based on the derived number of particles per bunch (Nb) and bunch length (SIGMA l).
  - Nb: number of particles per bunch corresponding to the average current Iavg and the number of bunches specified in the input.
  - SIGMA l: rms bunch length at the specified average current resulting from the effect(s) listed in the title [see note 2-1a)].
  - SIGMA p: rms momentum spread at the specified average current resulting from the effect(s) listed in the title [see note 2-1a)].
- 2-1d) Additional message printed when SPEAR scaling option is not used.
- 2-1e) Additional echoing of inputs when potential well distortion is selected. The average beta values and the emittances are used to estimate the space-charge impedance; the wall material, if specified, is used for the resistive wall impedance.
- 2-2a) Output from sub-option 2. The values are scaled from a complete set of input values at a specified energy. The columns are:
- UO: synchrotron radiation energy loss per turn.
  - EPSO: (uncoupled) natural emittance. If the coupling ratio will not be changed for a particular study, the coupled emittance value can be used here, but experience has shown that converting an uncoupled emittance value to various coupling ratios is safer than converting a coupled emittance value to that for another coupling.
  - GSR: transverse synchrotron radiation emittance damping rate, defined as  $2/\tau_x$  (where  $\tau_x$  is the damping time for horizontal betatron oscillations). This value is used as the damping term in calculations of the equilibrium transverse emittance [see Main Menu option 7].
  - GSRL: longitudinal synchrotron radiation emittance damping rate, defined as  $2/\tau_E$  (where  $\tau_E$  is the damping time for energy oscillations). This value is used as the damping term in calculations of the equilibrium longitudinal emittance [see Main Menu option 7].

NAT. SIGP: Natural rms relative momentum spread  $(\Delta p/p)_{\text{rms}}$ .

- 2-3a) Output from sub-option 3. The unnormalized emittance values (EPSX and EPSY) are scaled from input values for the normalized emittance. Note that, with the single exception of the inputs for this sub-option, all ZAP routines that require emittance values are expecting unnormalized emittances.

3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES

---

ENTER PROGRAM MODE <---[3A]  
[-1=END PROGRAM, 0=INTERACTIVE, 1=BATCH]  
0

ENTER CALCULATION TYPE <---[3B]  
0 = END  
1 = SINGLE BUNCH THRESHOLDS  
2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES  
3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES  
4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES  
5 = GAS SCATTERING LIFETIME  
6 = FREE ELECTRON LASER FORMULAE  
7 = INTRABEAM SCATTERING  
8 = TOUSCHEK SCATTERING  
9 = ION TRAPPING FORMULAE  
3

ENTER RING CIRCUMFERENCE (m)  
143.16

ENTER AVERAGE BETA-X and BETA-Y (m) <---[3C]  
9.4416,9.7969

ENTER BEAM PIPE RADIUS (cm)  
3

ENTER BEAM ENERGY (MeV) <---[3D]  
[NEGATIVE VALUE MEANS KINETIC ENERGY (MeV/amu)]  
750

ENTER MOMENTUM COMPACTION (ALPHA)  
4.923e-03

DO YOU WISH TO USE GAUSSIAN BUNCH SHAPE? (Y/N) <---[3E]  
Y

ENTER RING BROADBAND IMPEDANCE (ohms) <---[3F]  
[ZERO OMITTS BROADBAND RESONATOR]  
2

DEBUG PRINTOUT REQUIRED? (Y/N) <---[3G]  
n

SELECT FORMALISM [0 = Wang; 1 = Zotter (BBI)] <---[3H]  
0

ENTER NO. OF MODES REQUIRED [.LE. 4] <---[3I]  
2

ENTER 2 MODE VALUES [ONLY 1-4 ALLOWED] <---[3J]  
1,3

SELECT RF INPUT [0=TERMINAL, 1-3=DISK FILES] <---[3K]  
3

\*\*\*ERROR OPENING FILE CBMLRF3.DAT :

TRY ANOTHER INPUT FILE or  
EXIT FROM PROGRAM AND CREATE FILE or  
USE TERMINAL INPUT OPTION (as penance!)

SELECT RF INPUT [0=TERMINAL, 1-3=DISK FILES]

1

RF MODES HEADER IS:

STANDARD TEST PARAMETERS (500 MHz) - KEK CAVITY - LONGITUDINAL

IS THIS CORRECT? (Y/N)

Y

ENTER X and Y EMITTANCE VALUES (pi-m) <---[3L]  
[ZERO OMITTS SPACE CHARGE TERM]

8.099e-09,8.099e-10

ENTER WALL MATERIAL <---[3M]

[0=no resistive wall,1=aluminum,2=stainless steel,3=copper,4=cold copper]

1

ENTER RMS BUNCH LENGTH (cm) <---[3N]

0.582

ENTER RMS MOMENTUM SPREAD

1.0016e-03

ENTER NO. OF PARTICLES PER BUNCH

1.7883e10

ENTER NO. OF BUNCHES (ZERO FILLS ALL RF BUCKETS) <---[3O]

119

ENTER NUMBER OF VALUES TO BE SORTED (0 GIVES 5 VALUES) <---[3P]

3

USE ANALYTIC SUMMATION? (Y/N) <---[3Q]

Y

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[3R]

?

C:	143.160	M:	2 - 1 3
B:	9.442/ 9.797	F:	SET 1
R:	3.000	E:	8.099E-09/ 8.099E-10
G:	750.000	W:	Al
A:	4.923E-03	L:	0.582
S:	GAUSSIAN	P:	1.002E-03
Z:	2.000	N:	1.788E+10
D:	n	K:	119
V:	W	O:	3
T:	1.000E-10	Y:	A

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[3S]

Y

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[3T]

r

ENTER CALCULATION TYPE

- 0 = END
- 1 = SINGLE BUNCH THRESHOLDS
- 2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES
- 3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES
- 4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES
- 5 = GAS SCATTERING LIFETIME
- 6 = FREE ELECTRON LASER FORMULAE
- 7 = INTRABEAM SCATTERING
- 8 = TOUSCHEK SCATTERING
- 9 = ION TRAPPING FORMULAE

## Input Notes

- 3A) This question appears only once, when the code is first loaded. The "batch" mode here refers to the option of running the code with inputs taken from a (previously prepared) disk file ZAP.DAT, whether or not the code is being run in a batch mode (i.e., via a SUBMIT command). See discussion in Section II. Only Main Menu options 3, 4, 7, 8, and 9 can be run in the batch mode.
- 3B) Listing of the Main Menu, which directs all subsequent interactive running of the code. In this example, we have selected option 3.
- 3C) These values can be obtained from the ZAP lattice summary printout, which is described under Main Menu option 7 below.
- 3D) The flag to choose a proton or heavy ion is here. The code will subsequently request values for the particle charge state and mass number if a negative energy value is input. For the case shown, the positive value is interpreted as a total energy of an electron (as opposed to kinetic energy); a negative energy is taken to be a kinetic energy.
- 3E) The Gaussian shape is the default for electrons, whereas the default shape for protons is parabolic. However, in either case the "other" bunch shape can be selected by saying "no" here.
- 3F) In this Main Menu option, we do not consider SPEAR scaling. The chamber broadband component is not used in the coupled-bunch calculation if this value is input as 0 (zero). Note that the RF broadband component should not be explicitly included here, as it is contained implicitly in the RF modes themselves.
- 3G) The debug printout here prints, for each mode, the real and imaginary parts of the effective impedance. Note that this printout includes ALL calculated modes, not just the top few values requested in the sort (see note 3P), so a lot of paper can be generated if you say "yes" here.
- 3H) For a Gaussian bunch shape, either the Wang or Zotter formalism can be selected (see Section IV). If the bunch shape is chosen to be parabolic, only the Zotter formalism is used by the code. The Zotter formalism is that used in the CERN BBI code.
- 3I) Up to 4 synchrotron modes ( $A = 1, 2, 3, \text{ or } 4$ ) may be selected. If you want all 4 modes, enter 4 here and the next input, listing the individual modes, is skipped. If all modes are not needed, enter a number from 1 to 3.
- 3J) If the answer to the previous question was 1, 2, or 3, enter the actual modes desired here. Note that they need not be entered in numerical order here, and that "repeat" entries are permissible (albeit wasteful, but it's your time and your money), e.g., an answer of 3,1 would give the same outputs as in the example, but with the two pages in reverse order. Because only modes 1, 2, 3, 4 may be calculated, any input mode value greater than 4 is treated as a 4. Thus, entering the values 1,4,5 would give calculations for  $A = 1$ , then  $A = 4$ , followed by another  $A = 4$  calculation; xeroxing the first  $A = 4$  page is definitely faster!
- 3K) The list of RF modes can either be entered manually here by selecting set 0, or can be chosen from one of three previously prepared data sets, called

CBMLRF1.DAT, CBMLRF2.DAT, and CBMLRF3.DAT. In case you pick the wrong set, as shown in the example, the code scolds you and gives you another chance. In this example, the code does locate the requested parameters the second time. If the parameters are simply not there, enter set 0, put in whatever comes to mind, and proceed until the KEEP SAME PARAMETERS question, then end the code and create the required files. (Alternatively, CTRL-Y will stop the code if it is running under VMS on a VAX.) If set 0 is chosen, the required input values for omega, RS, and Q for the specified number of resonators are requested here. (These inputs are tedious if more than one or two resonant modes are to be used, and it is recommended that, in general, a data set be prepared in file CBMLRFi.DAT, as described in Section II.) If only a single resonator is specified, it is possible to have the code place the resonator directly on a synchrotron sideband; to use this option, the angular frequency (omega) value for the single resonator must be input as a negative number. If more than one resonator is selected, this option is disabled.

- 3L) The emittance values are used to obtain the space charge impedance term (see Section IV). Entering 0,0 here (remember that two inputs are required) omits this term from the calculation.
- 3M) This input is used to select the resistivity for the resistive wall impedance term (see Section IV). The values for the first three materials are taken from the Particle Properties Data Booklet; the fourth value corresponds to reducing the room temperature resistivity of copper (somewhat arbitrarily) by a factor of 30. An entry of 0 (zero) here omits this term from the calculation. Because there is no analytic expression for this impedance term, its contribution must be summed explicitly, even if the analytic summation option is selected (see note 3Q). If many bunches are needed, the calculations (in the analytic mode) will be greatly speeded up by not using this term. In such cases, it is worthwhile to try a few sample runs with and without the resistive wall term. If, as is often true, the contribution from the resistive wall term is negligible, you'll do a lot better to skip it. In the case of the explicit summation option, the incremental time is less of an issue.
- 3N) The rms bunch length for a given beam current can be obtained from Main Menu option 1 or 2-1. The issue of SPEAR scaling is thus implicitly dealt with here. In this case, the value corresponds to the 6 mA (per bunch) value from option 2-1 (see above). The values for momentum spread and number of particles per bunch in the next two input questions come from the same place. Thus, these calculations take the bunch lengthening into account in a self-consistent fashion.
- 3O) Because the theory used in this part of the code applies only to the case of equally-spaced bunches, this value must correspond to either the harmonic number (h), an integral submultiple of h, or 1. If you don't know what the harmonic number is, enter a 0 (zero) here initially. When you reach the KEEP SAME PARAMETERS? question, enter ? (question mark) to see the parameter values. The number of bunches (which you have set equal to the harmonic number) appears under the menu item K. You can then, if desired, get out your trusty calculator and calculate a different value for the number of bunches; change the value by answering "no" to the KEEP SAME PARAMETERS? question and then change menu item K as appropriate. Note that the code does not check your input value here, but it does do so if you type either "yes" or "?" for the KEEP... question. At this time an error message is generated and you will be forced to change some parameter value. It is worth pointing out that problems

in this section of input code are often related to being somewhat sloppy about entering either the ring circumference or the RF frequency. If these values are rounded inappropriately, it is quite possible to have the code calculate a harmonic number of 299 instead of the actual value of 300; in this case, the bunch number you pick may no longer be an integral submultiple. Be careful!

- 3P) Because the sorting of many modes is time consuming, it is recommended that only a few (say 5 or fewer) modes be chosen here. However, if you wish, all modes (up to 1000) can be sorted. Note that the limit of 1000 modes here refers only to the sorting itself. The number of modes that the program can calculate is limited only by your patience and computer budget, but only the fastest 1000 modes (and largest 1000 frequency shifts) can be printed. Up to now, there have been no complaints about this limitation.
- 3Q) The required summation to obtain the effective impedance (see Section IV) can be carried out either explicitly or by means of analytic expressions due to Zotter. The latter technique is generally much faster, but suffers from numerical problems; the former technique has rather unusual convergence properties. When performing calculations in a new parameter regime, it is strongly recommended that the user perform duplicate calculations using BOTH summation options. If the results agree, one can be quite confident that there are no unobserved numerical problems. In the analytic mode the code does attempt to flag cases where numerical problems may occur. However, the problem is a "soft" error, i.e., it is difficult to know exactly when you're in trouble. Similarly, the only way to be sure of convergence in the explicit summation is to make the value of the convergence parameter ZTEST much smaller to see if the results change.
- 3R) At last, we've entered everything. Because the list is so long, the list output is coded in terms of the parameter menu (which can be listed by answering "no" here followed by a ? (question mark) for the menu code). Within the limits imposed by a one-character code, most of the menu codes are (at least intended to be) mnemonic.
- 3S) Now perform the requested calculation.
- 3T) After completing this result, we will take advantage of the fact that these inputs are shared with the transverse coupled-bunch calculation, Main Menu option 4. To do so, we return to the Main Menu, but do not end the code. The subsequent inputs will be found in the description of Main Menu option 4 below. For clarity, the outputs from the calculation just performed are shown separately from those from the Main Menu option 4 results. In reality, however, if the calculations are continued as indicated, the outputs for both calculations will follow one another in a single ZAPOUT.DAT file.

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\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

[3a]--> 500.000 MHz RF CAVITY AND BROADBAND RESONATOR PARAMETERS (LONGITUDINAL)

STANDARD TEST PARAMETERS (500 MHz) - KEK CAVITY - LONGITUDINAL <---[3b]

J	OMEGAR (MHz)	RS (Mohm)	Q <---[3c]
1	3142.000	9.860	44000.
2	4764.000	2.910	37000.
3	6585.000	0.100	34000.
4	8181.000	1.340	112000.
5	8344.000	0.418	34000.
6	10420.000	0.328	34000.
7	10730.000	0.828	34000.
8	11690.000	0.100	34000.
9	12330.000	0.106	34000.
10	13330.000	0.587	34000.
11	13620.000	0.100	34000.
12	9993.082	0.002	1. <---[3d]

\*\*\*ZTEST= 1.0000E-10 <---[3e]

EQUIV. BROADBAND RESONATOR CONTRIBUTION FROM RF IS: <---[3f]  
Z/n= 0.3303 ohms/cell

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\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

LONGITUDINAL COUPLED BUNCH INSTABILITY (ANALYTIC) ESTIMATES <---[3g]  
[ WANG FORMALISM] FOR ELECTRONS IN GAUSSIAN BUNCHES IN A PIPE OF Al

-----  
INPUT PARAMETERS

RING CIRCUMFERENCE 143.160 m RING RADIUS 22.785 m  
RF FREQUENCY 500.000 MHz HARMONIC NO. 238  
NO. OF BUNCHES 119 Z/n BROADBAND 2.000 ohms  
BEAM ENERGY 750.000 MeV MOMENTUM SPREAD 1.002E-03  
MOMENTUM COMPACTION 0.004923 ETA 0.004923  
X-EMITTANCE 8.099E-09 pi-m Y-EMITTANCE 8.099E-10 pi-m  
AVERAGE BETA-X 9.442 m AVERAGE BETA-Y 9.797 m  
NO. OF PARTICLES 1.788E+10 BUNCH CURRENT 6.000E-03 A  
SYNCHROTRON TUNE 0.019302 SIGMA-L 0.006 m  
BEAM PIPE RADIUS 3.000 cm

SORT ON GROWTH RATES FOR MODE A= 1 <---[3h]

S	NMAX	FREQ. SHIFT (1/sec)	NU-S SHIFT	GROWTH RATE (1/sec)	GROWTH TIME (sec)
61	146	1.3498E+04	1.0259E-03	4.9121E+03	2.0358E-04 U
5	146	1.2000E+04	9.1201E-04	2.4155E+03	4.1400E-04 U
78	149	-2.1718E+04	-1.6506E-03	1.1559E+03	8.6511E-04 U

SORT ON FREQUENCY SHIFTS FOR MODE A= 1 <---[3j]

S	NMAX	FREQ. SHIFT (1/sec)	NU-S SHIFT	GROWTH RATE (1/sec)	GROWTH TIME (sec)
41	143	-2.8082E+04	-2.1343E-03	-3.6971E+03	-2.7049E-04 S
78	149	-2.1718E+04	-1.6506E-03	1.1559E+03	8.6511E-04 U
118	159	-2.0543E+04	-1.5613E-03	-1.2820E+02	-7.8004E-03 S

\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

LONGITUDINAL COUPLED BUNCH INSTABILITY (ANALYTIC) ESTIMATES  
 [ WANG FORMALISM] FOR ELECTRONS IN GAUSSIAN BUNCHES IN A PIPE OF Al

INPUT PARAMETERS  
 -----

RING CIRCUMFERENCE 143.160 m RING RADIUS 22.785 m  
 RF FREQUENCY 500.000 MHz HARMONIC NO. 238  
 NO. OF BUNCHES 119 Z/n BROADBAND 2.000 ohms  
 BEAM ENERGY 750.000 MeV MOMENTUM SPREAD 1.002E-03  
 MOMENTUM COMPACTION 0.004923 ETA 0.004923  
 X-EMITTANCE 8.099E-09 pi-m Y-EMITTANCE 8.099E-10 pi-m  
 AVERAGE BETA-X 9.442 m AVERAGE BETA-Y 9.797 m  
 NO. OF PARTICLES 1.788E+10 BUNCH CURRENT 6.000E-03 A  
 SYNCHROTRON TUNE 0.019302 SIGMA-L 0.006 m  
 BEAM PIPE RADIUS 3.000 cm

SORT ON GROWTH RATES FOR MODE A= 3

S	NMAX	FREQ. SHIFT (1/sec)	NU-S SHIFT	GROWTH RATE (1/sec)	GROWTH TIME (sec)
61	181	-1.5986E+03	-1.2150E-04	9.6861E+00	1.0324E-01 U
104	190	-1.6221E+03	-1.2328E-04	1.0979E+00	9.1081E-01 U
83	191	-1.6222E+03	-1.2329E-04	5.0906E-01	1.9644E+00 U

SORT ON FREQUENCY SHIFTS FOR MODE A= 3

S	NMAX	FREQ. SHIFT (1/sec)	NU-S SHIFT	GROWTH RATE (1/sec)	GROWTH TIME (sec)
41	180	-1.6312E+03	-1.2397E-04	-8.8949E+00	-1.1242E-01 S
78	196	-1.6262E+03	-1.2359E-04	1.0506E-01	9.5186E+00 U
57	198	-1.6261E+03	-1.2359E-04	-3.4317E-02	-2.9140E+01 S

\*\*\* CPU TIME FOR CALCULATION WAS 50.000 sec  
 \*\*\*ELAPSED TIME FOR CALCULATION WAS 50.418 sec

## Output Notes

- 3a) Summary of longitudinal resonator parameters used in the coupled-bunch calculation. This page is always the first one printed for a longitudinal coupled-bunch case. The input RF frequency is given here; this value is used to obtain the harmonic number for the ring and does not need to correspond to any of the cavity modes listed [see note 3c)].
- 3b) Header record from the CBMLRFi.DAT file. This line is blank if "Set 0", i.e., input from the terminal, was selected.
- 3c) The various output columns are:
- J: Resonator index (supplied by the code).
  - OMEGAR: Resonant (angular) frequency of the cavity mode. Note that the input for these modes (but not for the RF frequency) is not frequency but  $\omega = 2\pi f$ .
  - RS: Shunt impedance of the cavity mode.
  - Q: Quality factor for the cavity mode.
- 3d) Values for the broadband component of the ring are given here. These quantities, based on the input values for the beam pipe radius (to determine the resonance angular frequency,  $c/b$ ) and broadband  $Z/n$  (which is converted to  $RS$  by the code), are automatically appended to the cavity mode list, and should not be input by the user. A  $Q = 1$  resonator is assumed.
- 3e) Value of the convergence test parameter that is used to terminate the summation if the explicit (as opposed to analytic) summation option was selected or if the resistive wall term is active (that is, a wall material has been specified). The default value is  $1E-10$ ; this value is never asked for automatically, but it can be "changed" interactively via the menu. It is occasionally useful to try a smaller value to ensure proper convergence of the summation; alternatively, a larger value will speed up the convergence, albeit sometimes at the expense of accuracy. [Because of the rather unusual convergence properties of the summation, the default value must usually be set to a low value. This does not imply that the answer is being sought to this level of precision. A poorly chosen value for  $ZTEST$  can lead to substantial errors in the resulting growth times.] A good test of the correctness of a given calculation is to compare the results of using both the analytic and explicit summation options. Although both techniques have potential drawbacks, agreement between their results gives an unambiguous indication that the calculations do not suffer from numerical problems.
- 3f) RF contribution to the ring broadband impedance, calculated from the higher-order modes listed for the cavity. The value printed here is only informational. It can be either separately calculated (if the set of cavity modes is available) or manually input (based on this printout value) in Main Menu option 1. If the user has included his own low- $Q$  broadband resonator in the list, the value printed here will be badly in error. Thus, the modes used in Main Menu option 1 must not include a user-supplied low- $Q$  component. The coupled-bunch calculations (Main Menu option 3), however, would still be performed properly if a user-supplied low- $Q$  resonator mode were included.

- 3g) Header that indicates the type of calculation being done. In this example, the analytic (as opposed to the explicit) summation option is used, the Wang (as opposed to Zotter) formalism has been selected, an electron (as opposed to proton or heavy ion) beam in Gaussian (as opposed to parabolic) bunches is taken, and a resistive wall of aluminum (as opposed to stainless steel, copper, or cryogenic copper) is utilized. (The numerous variants of this header are presumably obvious from the above description; more details can be found in Section IV.) A new page is generated for each value of the synchrotron mode number, A, requested in the input.
- 3h) List of the fastest growing modes, i.e., a sort on the GROWTH RATE column, for synchrotron mode A. Note that all positive (growing) values will appear before any negative (damped) values are listed.
- 3i) Outputs are:
- S: coupled-bunch mode number (see Section IV).
- NMAX: number of terms summed to reach convergence. Note that, in the case of the analytic option this value will still be non-zero if the resistive-wall impedance is included, since these terms cannot be summed analytically and must be explicitly evaluated. Changes in the convergence parameter ZTEST (see note 3e) will manifest themselves here.
- FREQ. SHIF T: coherent mode (angular) frequency shift (see Section IV).
- NU-S SHIF T: frequency shift in tune, i.e., the frequency shift in the previous column divided by the beam revolution frequency.
- GROWTH RATE: e-folding growth rate for the indicated mode.
- GROWTH TIME: reciprocal of the growth rate.
- : the letter (S, U, or D) to the right of the output line indicates whether the mode is stable, Landau unstable, or Landau damped (see Section IV).
- 3j) List of the modes with the largest frequency shifts, i.e., a sort on the FREQ. SHIF T column, for synchrotron mode A. Note that this sort is based only on the magnitude of the shifts, without regard to sign.

4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES

---

ENTER CALCULATION TYPE <---[4A]

- 0 = END
- 1 = SINGLE BUNCH THRESHOLDS
- 2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES
- 3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES
- 4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES
- 5 = GAS SCATTERING LIFETIME
- 6 = FREE ELECTRON LASER FORMULAE
- 7 = INTRABEAM SCATTERING
- 8 = TOUSCHEK SCATTERING
- 9 = ION TRAPPING FORMULAE

4

ENTER NO. OF MODES REQUIRED [.LE. 4] <---[4B]

1

ENTER 1 MODE VALUES [ONLY 0-3 ALLOWED] <---[4C]

0

SELECT RF INPUT [0=TERMINAL, 1-3=DISK FILES] <---[4D]

1

RF MODES HEADER IS:

STANDARD RF TEST PARAMETERS (500 MHz) - KEK CAVITY - TRANSVERSE

IS THIS CORRECT? (Y/N)

Y

ENTER NU-X, NU-Y <---[4E]

7.85,4.35

ENTER RMS HORIZONTAL TUNE SPREAD <---[4F]

[ZERO SKIPS LANDAU DAMPING CALC. FOR MODE 0]

3.e-4

ENTER CHROMATICITY ("SYNCH" DEFINITION) <---[4G]

0.1

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[4H]  
?

C:	143.160	M:	1 - 0
F:	SET 1	E:	8.099E-09/ 8.099E-10
R:	3.000	W:	A1
G:	750.000	L:	0.582
A:	4.923E-03	P:	1.002E-03
S:	GAUSSIAN	N:	1.788E+10
Z:	2.000	K:	119
D:	n	O:	3
V:	w	X:	0.100
Q:	7.850/ 4.350 [DNUX= 3.00E-04]	Y:	A
T:	1.000E-10		

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[4I]  
Y

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[4J]  
r

ENTER CALCULATION TYPE <---[4K]

- 0 = END
- 1 = SINGLE BUNCH THRESHOLDS
- 2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES
- 3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES
- 4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES
- 5 = GAS SCATTERING LIFETIME
- 6 = FREE ELECTRON LASER FORMULAE
- 7 = INTRABEAM SCATTERING
- 8 = TOUSCHEK SCATTERING
- 9 = ION TRAPPING FORMULAE

0

## Input Notes

- 4A) We pick up here at the end of the Main Menu option 3 calculation. If this option is selected first, the other requested inputs will be essentially the same as those shown in the example for Main Menu option 3, along with the additions/changes shown in the Main Menu option 4 sample input.
- 4B) Up to 4 synchrotron modes ( $A = 0, 1, 2, \text{ or } 3$ ) may be selected. If you want all 4 modes, enter 4 here and the next input, listing the individual modes, is skipped. If all modes are not needed, enter a number from 1 to 3.
- 4C) If the answer to the previous question was 1, 2, or 3, enter the actual modes desired here. Note that they need not be entered in numerical order here, and that "repeat" entries are permissible (albeit wasteful, but it's your time and your money). Because only modes 0, 1, 2, 3 may be calculated, any input mode value greater than 3 is treated as a 3. Thus, entering the values 1,3,5 would give calculations for  $A = 1$ , then  $A = 3$ , followed by another  $A = 3$  calculation; xeroxing the first  $A = 3$  page is definitely faster!
- 4D) The list of RF modes can either be entered manually here by selecting set 0, or can be chosen from one of three previously prepared data sets, called CBMTRF1.DAT, CBMTRF2.DAT, and CBMTRF3.DAT. See note 3K in the previous example.
- 4E) The tune values, available from your lattice code, are used to calculate the betatron frequency  $\omega$  and for the Laslett tune shifts (see Section IV).
- 4F) The Landau damping for mode zero comes only from a betatron tune spread. If a value is entered here, the code calculates whether the spread is sufficient to Landau damp a given  $A = 0$  bunch mode. For higher modes ( $A > 0$ ), this parameter is not used.
- 4G) Chromaticity here is defined as in the LBL lattice code SYNCH, i.e.,  $d\nu/(dp/p)$ . See Section IV.
- 4H) Here we list the parameters. As promised, the previously entered inputs from Main Menu option 3 are available here. Because of the large number of parameters, they are coded as in the list of menu codes, which can be obtained by answering "no" here, followed by a menu code of "?".
- 4I) Perform the calculation.
- 4J) Go back to the Main Menu.
- 4K) End the code and print the outputs. As mentioned in note 3T, the output below has been separated from the output for Main Menu option 3 for clarity. In reality, both outputs would appear sequentially in the same ZAPOUT.DAT file.

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\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

[4a]----> 500.000 MHz RF CAVITY AND BROADBAND RESONATOR PARAMETERS (TRANSVERSE)

STANDARD RF TEST PARAMETERS (500 MHz) - KEK CAVITY - TRANSVERSE <---[4b]

J	OMEGAR (MHz)	RT (Mohm/m)	Q <---[4c]
1	4329.100	0.990	45000.
2	5210.600	11.590	56000.
3	6728.000	26.640	40000.
4	7149.600	0.270	45000.
5	7822.600	3.230	95000.
6	8461.600	1.000	40000.
7	9911.700	1.000	40000.
8	10798.300	1.000	40000.
9	12256.600	1.000	40000.
10	12551.300	1.000	40000.
11	12704.600	1.000	40000.
12	13396.400	1.000	40000.
13	9993.082	0.101	1. <---[4d]

\*\*\*ZTEST= 1.0000E-10 <---[4e]

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\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

TRANSVERSE COUPLED BUNCH INSTABILITY (ANALYTIC) ESTIMATES <---[4f]  
[ WANG FORMALISM] FOR ELECTRONS IN GAUSSIAN BUNCHES IN A PIPE OF A1

INPUT PARAMETERS  
-----

RING CIRCUMFERENCE 143.160 m RING RADIUS 22.785 m  
 RF FREQUENCY 500.000 MHz HARMONIC NO. 238  
 NO. OF BUNCHES 119 z/n BROADBAND 2.000 ohms  
 BEAM ENERGY 750.000 MeV MOMENTUM SPREAD 1.002E-03  
 MOMENTUM COMPACTION 0.004923 ETA 0.004923  
 X-EMITTANCE 8.099E-09 pi-m Y-EMITTANCE 8.099E-10 pi-m  
 1/AVG. BETA-X INV. 2.902 m 1/AVG. BETA-Y INV. 5.238 m  
 NO. OF PARTICLES 1.788E+10 BUNCH CURRENT 6.000E-03 A  
 SYNCHROTRON TUNE 0.019302 SIGMA-L 0.006 m  
 X BETATRON TUNE 7.850000 Y BETATRON TUNE 4.350000  
 CHROMATICITY 0.100000 BEAM PIPE RADIUS 3.000 cm  
 HORIZ. TUNE SPREAD 3.000E-04  
 LASLETT TUNE SHIFTS (NO NEUTRALIZATION): <---[4g]  
 COHERENT -2.089E-03 INCOHERENT -8.569E-03  
 LASLETT TUNE SHIFTS (FULL NEUTRALIZATION): <---[4g]  
 COHERENT -2.071E-03 INCOHERENT 2.394E+02

SORT ON GROWTH RATES FOR MODE A= 0 <---[4h]

S	NMAX	FREQ. SHIFT (1/sec)	NU SHIFT	GROWTH RATE (1/sec)	GROWTH TIME<-[4i] (sec)
45	116	4.7132E+04	3.5821E-03	2.2325E+03	4.4793E-04 U
111	134	4.6025E+04	3.4979E-03	1.2795E+02	7.8157E-03 U
76	134	4.3455E+04	3.3026E-03	8.3851E+01	1.1926E-02 U

SORT ON FREQUENCY SHIFTS FOR MODE A= 0 <---[4j]

S	NMAX	FREQ. SHIFT (1/sec)	NU SHIFT	GROWTH RATE (1/sec)	GROWTH TIME (sec)
45	116	4.7132E+04	3.5821E-03	2.2325E+03	4.4793E-04 U
28	133	4.6976E+04	3.5702E-03	-2.6464E+01	-3.7787E-02 S
72	138	4.6886E+04	3.5634E-03	2.7266E+01	3.6675E-02 U

\*\*\* CPU TIME FOR CALCULATION WAS 19.540 sec  
 \*\*\*ELAPSED TIME FOR CALCULATION WAS 19.641 sec

## Output Notes

- 4a) Summary of transverse resonator parameters used in the coupled-bunch calculation. This page is always the first one printed for a transverse coupled-bunch case. The input RF frequency is given here; this value is used to obtain the harmonic number for the ring and does not correspond to any of the cavity modes listed [see note 4c)].
- 4b) Header record from the CBMTRFi.DAT file. This line is blank if "Set 0", i.e., input from the terminal, was selected.
- 4c) The various output columns are:
- J: Resonator index (supplied by the code).
  - OMEGAR: Resonant (angular) frequency of the cavity mode. Note that the input for these modes (but not for the RF frequency) is not frequency but  $2\pi f$ .
  - RT: Transverse impedance (Mohms/m) of the cavity mode.
  - Q: Quality factor for the cavity mode.
- 4d) Values for the broadband component of the ring are given here. These quantities, based on the input values for the beam pipe radius (to determine the resonance angular frequency,  $c/b$ ) and broadband  $Z/n$  (which is converted to RT by the code), are automatically appended to the cavity mode list, and should NOT be input by the user. A  $Q = 1$  resonator is assumed.
- 4e) Value of the convergence test parameter; it is used to terminate the summation if the explicit (as opposed to analytic) summation option was selected or if the resistive wall term is active (that is, a wall material has been specified). The default value is  $1E-10$ ; this value is never asked for automatically, but it can be "changed" interactively via the menu. It is useful to occasionally try a smaller value to ensure proper convergence of the summation; alternatively, a larger value will speed up the convergence, albeit sometimes at the expense of accuracy. [Because of the rather unusual convergence properties of the summation, the default value must usually be set to a low value. This does not imply that the answer is being sought to this level of precision. A poorly chosen value for ZTEST can lead to substantial errors in the resulting growth times.] A good test of the correctness of a given calculation is to compare the results of using both the analytic and explicit summation options. Although both techniques have potential drawbacks, agreement between their results gives an unambiguous indication that the calculations do not suffer from numerical problems.
- 4f) Header that indicates the type of calculation being done. In this example, the analytic (as opposed to the explicit) summation option is used, the Wang (as opposed to Zotter) formalism has been selected, an electron (as opposed to proton or heavy ion) beam in Gaussian (as opposed to parabolic) bunches is taken, and a resistive wall of aluminum (as opposed to stainless steel, copper, or cryogenic copper) is utilized. (The numerous variants of this header are presumably obvious from the above description; more details can be found in Section IV.) A new page

is generated for each value of the synchrotron mode number, A, requested in the input.

- 4g) Laslett tune shifts (coherent and incoherent) assuming no neutralization and full neutralization (see Section IV).
- 4h) List of the fastest growing modes, i.e., a sort on the GROWTH RATE column, for synchrotron mode A. Note that all positive (growing) values will appear before any negative (damped) values are listed.
- 4i) Outputs are:
  - S: coupled-bunch mode number (see Section IV).
  - NMAX: number of terms summed to reach convergence. Note that in the case of the analytic option this value will still be non-zero if the resistive-wall impedance is included, since these terms cannot be summed analytically and must be explicitly evaluated. Changes in the convergence parameter ZTEST (see note 4e) will manifest themselves here.
  - FREQ. SHIFT: coherent mode (angular) frequency shift (see Section IV).
  - NU SHIFT: transverse frequency shift in tune, i.e., the frequency shift in the previous column divided by the beam revolution frequency.
  - GROWTH RATE: e-folding growth rate for the indicated mode.
  - GROWTH TIME: reciprocal of the growth rate.
  - : the letter (S, U, or D) to the right of the output line indicates whether the mode is stable, Landau unstable, or Landau damped (see Section IV).
- 4j) List of the modes with the largest frequency shifts, i.e., a sort on the FREQ. SHIFT column, for synchrotron mode A. Note that this sort is based only on the magnitude of the shifts, without regard to sign.

5 = GAS SCATTERING LIFETIME

---

ENTER PROGRAM MODE <---[5A]  
[-1=END PROGRAM, 0=INTERACTIVE, 1=BATCH]  
0

ENTER CALCULATION TYPE <---[5B]  
0 = END  
1 = SINGLE BUNCH THRESHOLDS  
2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES  
3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES  
4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES  
5 = GAS SCATTERING LIFETIME  
6 = FREE ELECTRON LASER FORMULAE  
7 = INTRABEAM SCATTERING  
8 = TOUSCHEK SCATTERING  
9 = ION TRAPPING FORMULAE  
5

\*\*\*ZAP\*\*\* GAS SCATTERING CALCULATION FOR ELECTRONS

(NOTE: For NUMERICAL inputs, a slash (/) preserves the current value.)

ENTER ETOTAL (MeV) <---[5C]  
[ZERO RETURNS TO MAIN MENU]  
750

ENTER AVG. BETA OF RING [m] <---[5D]  
9.7969

ENTER LIMITING RING BETATRON ACCEPTANCE [m] <---[5E]  
[NEGATIVE VALUE MEANS LIMIT IS ONLY IN ONE PLANE]  
-4.e-06

ENTER MOMENTUM HALF-APERTURE [%] <---[5F]  
4

ENTER PRESSURE [nTorr] <---[5G]  
(NEGATIVE TO CHANGE TEMPERATURE, ZERO FOR NEW RING VALUES)  
1

ENTER Z OF GAS  
7

ENTER NO. OF Z= 7. ATOMS PER MOLECULE  
2

\*\*\*TOTAL GAS SCATTERING LIFETIME IS 4.864E+00 hrs

ENTER PRESSURE [nTorr] <---[5H]  
(NEGATIVE TO CHANGE TEMPERATURE, ZERO FOR NEW RING VALUES)  
0

ENTER ETOTAL (MeV) <---[5I]  
[ZERO RETURNS TO MAIN MENU]  
/  
/

ENTER AVG. BETA OF RING [m]

/

ENTER LIMITING RING BETATRON ACCEPTANCE [m] <---[5J]

[NEGATIVE VALUE MEANS LIMIT IS ONLY IN ONE PLANE]

4.e-06

ENTER MOMENTUM HALF-APERTURE [%]

/

ENTER PRESSURE [nTorr] <---[5K]

(NEGATIVE TO CHANGE TEMPERATURE, ZERO FOR NEW RING VALUES)

1

ENTER Z OF GAS

/

ENTER NO. OF Z= 7. ATOMS PER MOLECULE

/

\*\*\*TOTAL GAS SCATTERING LIFETIME IS 2.508E+00 hrs

ENTER PRESSURE [nTorr]

(NEGATIVE TO CHANGE TEMPERATURE, ZERO FOR NEW RING VALUES)

0

ENTER ETOTAL (MeV) <---[5L]

[ZERO RETURNS TO MAIN MENU]

0

ENTER CALCULATION TYPE <---[5M]

0 = END

1 = SINGLE BUNCH THRESHOLDS

2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES

3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES

4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES

5 = GAS SCATTERING LIFETIME

6 = FREE ELECTRON LASER FORMULAE

7 = INTRABEAM SCATTERING

8 = TOUSCHEK SCATTERING

9 = ION TRAPPING FORMULAE

0

## Input Notes

- 5A) This question appears only once, when the code is first loaded. The "batch" mode here refers to the option of running the code with inputs taken from a (previously prepared) disk file ZAP.DAT, whether or not the code is being run in a batch mode (i.e., via a SUBMIT command). See discussion in Section II. Only Main Menu options 3, 4, 7, 8, and 9 can be run in the batch mode.
- 5B) Listing of the Main Menu, which directs all subsequent interactive running of the code. In this example, we have selected option 5.
- 5C) Beam (total) energy. An input of 0 (zero) here returns to the Main Menu.
- 5D) If both planes are considered, the value entered here should be the larger of the horizontal and vertical values; this will lead to the more pessimistic value for the calculated lifetime. If the restriction is primarily in a single plane, the corresponding average beta function for that plane should be used.
- 5E) The limiting acceptance is the smallest value anywhere in the lattice of the quantity  $b^2/\beta$ , where  $b$  and  $\beta$  are the aperture and beta function at some particular lattice location. To select a restriction in only one plane, the acceptance is input as a negative number. This would be appropriate, for example, for a ring having a narrow-gap undulator that provides a restricted aperture in only one dimension.
- 5F) This value corresponds to the momentum acceptance of the lattice, which can be either longitudinal or transverse. In general, the same value should be used here as that adopted for Touschek scattering calculations (Main Menu option 8). This parameter is used in evaluating the inelastic scattering (Bremsstrahlung) lifetime.
- 5G) The partial pressure for the gas species is entered here. The density of atoms is calculated based on a temperature of 300 K; if the user desires to use a different value for the temperature, the pressure is input here as a negative value and the code subsequently requests a temperature value.
- 5H) After finishing the calculation (and echoing the result on the user's terminal), the code cycles back to ask for a new partial pressure (for the next gas species). Entering a 0 (zero) here returns the code to the beginning of the gas scattering routine.
- 5I) To continue with additional calculations, we cycle through the required inputs again. In the example shown, we want to use the same energy again, so we need enter only a slash (/) to preserve the current value. In the remaining inputs, we use slashes to preserve all values except for those we wish to change.
- 5J) Now we use a positive value of acceptance, so the restriction is assumed to be equal in both transverse planes.
- 5K) Note that we must explicitly reenter this value, since we changed it to zero in order to return to the top of the code. (If we were merely cycling through a second gas species, a slash would be okay here.) In the present case, entering a slash would have caused the code to branch back to the beginning of the routine without doing a calculation. (One clue that you're going around in circles here

would be the absence of the beep at the end of the calculation. If the terminal doesn't beep at you, then no output has been produced. Also, no output value will be echoed to the scope.)

5L) Now let's quit this routine by entering a zero energy.

5M) Welcome back to the by-now-familiar Main Menu, from which we end the code and print the output shown below.

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\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

GAS SCATTERING CALCULATION FOR ELECTRONS

ETOTAL = 750.000 MeV BETA = 1.000000 GAMMA= 1467.701  
 BETA AVG.= 9.797 m LIMITING ACCEPTANCE= 4.000E-06 m MOMENTUM HALF-APERTURE= 4.000E-02 <---[5b]  
 [\*\*BETATRON ACCEPTANCE LIMIT IN ONE PLANE ONLY\*\*]

[5a]

Z	N	P (Torr)	1/TAUE (1/sec)	TAUE (hrs)	1/TAUB (1/sec)	TAUB (hrs)	1/TAU (1/sec)	TAU (hrs)	TAU <---[5d]
7.	2	1.000E-09	5.365E-05	5.178E+00	3.458E-06	8.032E+01	5.710E-05	4.864E+00	

ETOTAL = 750.000 MeV BETA = 1.000000 GAMMA= 1467.701  
 BETA AVG.= 9.797 m LIMITING ACCEPTANCE= 4.000E-06 m MOMENTUM HALF-APERTURE= 4.000E-02

Z	N	P (Torr)	1/TAUE (1/sec)	TAUE (hrs)	1/TAUB (1/sec)	TAUB (hrs)	1/TAU (1/sec)	TAU (hrs)
7.	2	1.000E-09	1.073E-04	2.589E+00	3.458E-06	8.032E+01	1.108E-04	2.508E+00

## Output Notes

- 5a) Limiting acceptance value  $(b^2/\beta)_{lim}$ , where  $b$  is the aperture radius, and  $\beta$  is the beta function at the limiting point in the lattice.
- 5b) Limiting momentum acceptance for the inelastic scattering (Bremsstrahlung) process. This value should correspond to that used for Touschek scattering calculations, i.e., it should be the lower of the longitudinal (RF bucket) or transverse (physical or dynamic aperture) limits. (See the discussion in Section II under Main Menu Option 8.)
- 5c) As discussed by Le Duff (see Section IV), the lifetime for elastic electron-gas scattering depends on whether the limiting aperture is equal in both transverse planes (as is normally assumed) or is dominated by one plane (as would be the case for a ring with a small undulator gap in one transverse dimension). The latter case -- specified by a negative acceptance value in the input -- is flagged in the output as shown.
- 5d) Output values are:
- Z: atomic number of background gas species.
  - N: number of atoms per molecule for gas Z, i.e., 2 for a diatomic gas like nitrogen.
  - P: gas pressure for gas Z.
  - 1/TAUE: e-folding rate for elastic scattering loss.
  - TAUE: e-folding lifetime for elastic scattering loss.
  - 1/TAUB: e-folding rate for inelastic scattering (Bremsstrahlung) loss.
  - TAUB: e-folding lifetime for inelastic scattering (Bremsstrahlung) loss.
  - 1/TAU: e-folding rate for total (elastic plus inelastic) scattering loss.
  - TAU: e-folding lifetime for total (elastic plus inelastic) scattering loss.

6 = FREE ELECTRON LASER FORMULAE

---

ENTER PROGRAM MODE <---[6A]  
[-1=END PROGRAM, 0=INTERACTIVE, 1=BATCH]  
0

ENTER CALCULATION TYPE <---[6B]  
0 = END  
1 = SINGLE BUNCH THRESHOLDS  
2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES  
3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES  
4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES  
5 = GAS SCATTERING LIFETIME  
6 = FREE ELECTRON LASER FORMULAE  
7 = INTRABEAM SCATTERING  
8 = TOUSCHEK SCATTERING  
9 = ION TRAPPING FORMULAE  
6

\*\*\*ZAP\*\*\* FEL PARAMETER CALCULATION

(NOTE: For NUMERICAL inputs, a slash (/) preserves the current value.)

ENTER RMS MOMENTUM SPREAD <---[6C]  
(NEGATIVE VALUE RETURNS TO MAIN PROGRAM)  
.001

ENTER MOMENTUM DISTRIBUTION <---[6D]  
(1=GAUSSIAN, 2=LORENTZIAN, 3=RECTANGULAR)  
1

ENTER EMITTANCE RATIO (EPSX/EPSY) <---[6E]  
10

ENTER ETOTAL [MeV] <---[6F]  
(ZERO TO CHANGE MOMENTUM SPREAD)  
750

ENTER RADIATION WAVELENGTH [Angstroms] <---[6G]  
(NEGATIVE VALUE ALLOWS CHOICE OF GAP)  
400

ENTER X-EMITTANCE [PI-m] <---[6H]  
3.72e-09

ENTER PEAK CURRENT [amps] <---[6I]  
23.4

FEL Parameters [ 400.0 Angstrom Radiation Wavelength]  
Rho= 9.0540E-04 Rho-eff.= 4.1663E-04  
L = 1.165 m L-eff. = 2.531 m  
Reduction factor= 4.6016E-01 Delta-nu= -1.9000E-03

ENTER ETOTAL [MeV] <---[6J]  
(ZERO TO CHANGE MOMENTUM SPREAD)  
0

ENTER RMS MOMENTUM SPREAD  
(NEGATIVE VALUE RETURNS TO MAIN PROGRAM)

/

ENTER MOMENTUM DISTRIBUTION  
(1=GAUSSIAN, 2=LORENTZIAN, 3=RECTANGULAR)

/

ENTER EMITTANCE RATIO (EPSX/EPSY)

/

ENTER ETOTAL [MeV]  
(ZERO TO CHANGE MOMENTUM SPREAD)  
750

ENTER RADIATION WAVELENGTH [Angstroms]  
(NEGATIVE VALUE ALLOWS CHOICE OF GAP)

/

ENTER X-EMITTANCE [PI-m]  
1.68e-09

ENTER PEAK CURRENT [amps]  
2.7

FEL Parameters [ 400.0 Angstrom Radiation Wavelength]  
Rho= 5.7452E-04 Rho-eff.= 1.4205E-04  
L = 1.835 m L-eff. = 7.423 m  
Reduction factor= 2.4724E-01 Delta-nu= -2.0000E-03

ENTER ETOTAL [MeV]  
(ZERO TO CHANGE MOMENTUM SPREAD)  
0

ENTER RMS MOMENTUM SPREAD <---[6K]  
(NEGATIVE VALUE RETURNS TO MAIN PROGRAM)  
-1

ENTER CALCULATION TYPE <---[6L]  
0 = END  
1 = SINGLE BUNCH THRESHOLDS  
2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES  
3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES  
4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES  
5 = GAS SCATTERING LIFETIME  
6 = FREE ELECTRON LASER FORMULAE  
7 = INTRABEAM SCATTERING  
8 = TOUSCHEK SCATTERING  
9 = ION TRAPPING FORMULAE  
0

## Input Notes

- 6A) This question appears only once, when the code is first loaded. The "batch" mode here refers to the option of running the code with inputs taken from a (previously prepared) disk file ZAP.DAT, whether or not the code is being run in a batch mode (i.e., via a SUBMIT command). See discussion in Section II. Only Main Menu options 3, 4, 7, 8, and 9 can be run in the batch mode.
- 6B) Listing of the Main Menu, which directs all subsequent interactive running of the code. In this example, we have selected option 6.
- 6C) RMS width of momentum distribution. This value is used to obtain the degradation in gain due to the finite beam momentum spread (see Section IV).
- 6D) Shape of momentum distribution (see Section IV).
- 6E) Because this ratio usually remains fixed for electrons, we specify it once here and then ask for only a horizontal emittance value below.
- 6F) This is the "branch point." Entering 0 (zero) here branches back to the beginning of this routine. A positive value continues with the calculation.
- 6G) The required wavelength of the radiation is entered here. Based on Halbach's scaling formula, the code generates a set of undulator parameters to produce these photons. If the code cannot find a valid solution, an error message is generated. In this situation, it will be necessary to modify either the requested wavelength or the default undulator gap (set to 3 mm). To change the gap value, enter the required wavelength as a negative number, and the code will then ask for a gap value.
- 6H) The horizontal emittance value. This should in general be obtained as the equilibrium value from an IBS calculation, i.e., from Main Menu option 7. The natural emittance of a lattice may not be a very good estimate here, because the FEL beam energy is typically rather low, and the bunch density rather high. The sample case in Main Menu option 7 serves to illustrate this point.
- 6I) This value can be obtained from Main Menu option 1. In general, the peak current for an FEL beam will be pushed up to the longitudinal microwave instability limit for the maximum tolerable momentum spread. After this input, the calculation is completed and the FEL parameters are echoed on the terminal. The code then returns to the ENTER ETOTAL prompt.
- 6J) To change parameters, we enter 0 (zero) here and return to the beginning of the routine. Subsequent values that are to be retained can be entered with a slash (/). Note, however, that the energy value must be explicitly reentered because it was changed to zero in order to return to the top of the routine. In the example shown, the emittance and peak current values were changed to correspond to a case without SPEAR scaling (which led to a lower peak current and less emittance growth from IBS).
- 6K) After entering a 0 (zero) for the energy to return to the top of the routine, we enter a negative value here (any negative number will do) to return to the familiar territory of the infamous Main Menu.

6L) Here we end the code and print the output shown below.

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\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

FEL PARAMETER CALCULATION

UNDULATOR PARAMETERS <---[6a]  
Wavelength= 2.29 cm B0= 1.684 T K= 3.608 Gap= 0.300 cm  
BetaXY= 2.971 m Eff. Energy Spread= 2.6021E-04\*  
[\* should be less than actual sigma-p]

BEAM PARAMETERS [ Gaussian Distribution] <---[6b]  
Energy= 750.000 MeV Mom. Spread= 1.0000E-03 Peak Current= 23.400 A  
X-Emittance= 3.7200E-09 pi m-rad Y-Emittance= 3.7200E-10 pi m-rad

FEL Parameters [ 400.0 Angstrom Radiation Wavelength] <---[6c]  
Rho= 9.0540E-04 Rho-eff.= 4.1663E-04  
L = 1.165 m L-eff. = 2.531 m  
Reduction factor= 4.6016E-01 Delta-nu= -1.9000E-03

UNDULATOR PARAMETERS  
Wavelength= 2.29 cm B0= 1.684 T K= 3.608 Gap= 0.300 cm  
BetaXY= 2.971 m Eff. Energy Spread= 1.1751E-04\*  
[\* should be less than actual sigma-p]

BEAM PARAMETERS [ Gaussian Distribution]  
Energy= 750.000 MeV Mom. Spread= 1.0000E-03 Peak Current= 2.700 A  
X-Emittance= 1.6800E-09 pi m-rad Y-Emittance= 1.6800E-10 pi m-rad

FEL Parameters [ 400.0 Angstrom Radiation Wavelength]  
Rho= 5.7452E-04 Rho-eff.= 1.4205E-04  
L = 1.835 m L-eff. = 7.423 m  
Reduction factor= 2.4724E-01 Delta-nu= -2.0000E-03

## Output Notes

6a) Undulator parameters that will provide the requested radiation wavelength. The outputs are:

Wavelength:	undulator period length.
B0:	undulator magnetic field, based on a hybrid (steel/samarium-cobalt) design (see Section IV).
K:	undulator deflection parameter.
Gap:	Guess!
BetaXY:	effective beta function from undulator focusing, assuming equal focusing in both transverse planes (see Section IV).
Eff. Energy Spread:	effective energy spread due to finite beam emittance in undulator.

6b) List of beam parameters; these are an echo of input values. The shape of the momentum distribution (Gaussian, Lorentzian, or rectangular) is shown here. The meanings of the various output values should be unambiguous.

6c) List of resultant FEL performance parameters. The output values (see Section IV) are:

Rho:	FEL gain parameter, in the absence of beam momentum spread.
Rho-eff.:	effective FEL gain parameter, taking account of the finite momentum spread of the beam.
L:	e-folding length for the FEL radiation, in the absence of beam momentum spread.
L-eff.:	effective e-folding length, taking account of the finite momentum spread of the beam.
Reduction factor:	ratio of $\rho_{\text{eff}}/\rho$ , i.e., a measure of the degradation due to the finite beam momentum spread.
Delta-nu:	detuning parameter that leads to maximum reduction factor (i.e., minimum degradation of gain).

7 = INTRABEAM SCATTERING

---

ENTER PROGRAM MODE <---[7A]  
[-1=END PROGRAM, 0=INTERACTIVE, 1=BATCH]  
0

ENTER CALCULATION TYPE <---[7B]  
0 = END  
1 = SINGLE BUNCH THRESHOLDS  
2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES  
3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES  
4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES  
5 = GAS SCATTERING LIFETIME  
6 = FREE ELECTRON LASER FORMULAE  
7 = INTRABEAM SCATTERING  
8 = TOUSCHEK SCATTERING  
9 = ION TRAPPING FORMULAE  
7

LATTICE HEADER IS: <---[7C]  
CF143 ZAP TEST LATTICE [see Particle Accel. 18, 223(1986)]

IS THIS CORRECT? (Y/N)  
Y

ENTER BEAM ENERGY (MeV) <---[7D]  
[NEGATIVE VALUE MEANS KINETIC ENERGY (MeV/amu)]  
750

ENTER X and Y NATURAL EMITTANCE (pi m-rad) <---[7E]  
[NEGATIVE EPSX VALUE TO READ STARTING GUESS]  
4.1844e-09,4.1844e-10

BUNCHED BEAM? (Y/N) <---[7F]  
Y

ENTER NO. OF PARTICLES PER BUNCH  
1.7883e10

ENTER RMS BUNCH LENGTH (cm) <---[7G]  
0.582

ENTER RMS MOMENTUM SPREAD  
1.0016e-3

ENTER TRANS. EMITTANCE DAMPING RATE (sec\*\*-1) <---[7H]  
[NEGATIVE TO CONSIDER LONGITUDINAL GROWTH ALSO]  
-28.818

ENTER LONG. EMITTANCE DAMPING RATE (sec\*\*-1) <---[7I]  
38.262

ENTER NATURAL RMS MOMENTUM SPREAD <---[7J]  
3.7151e-4

ENTER NATURAL RMS BUNCH LENGTH (cm)  
0.216

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[7K]  
?

```
ENERGY= 750.0000 MeV
NAT. EPSX, EPSY= 4.1844E-09, 4.1844E-10 pi m-rad
EPSX, EPSY= 4.1844E-09, 4.1844E-10 pi m-rad
SIGMA-P= 1.0016E-03
SIGMA-L= 0.5820 cm
PARTICLES/BUNCH= 1.7883E+10
S.R. DAMPING RATE (HORIZ.)= 28.8180 sec**-1
S.R. DAMPING RATE (LONG.)= 38.2620 sec**-1
NAT. SIGMA-P= 3.7151E-04
NAT. SIGMA-L= 0.2160 cm
CONVERGENCE CHECK OFF
DEBUG PRINTOUT OFF
```

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[7L]  
Y

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[7M]  
?

```
ENERGY= 750.0000 MeV
NAT. EPSX, EPSY= 4.1844E-09, 4.1844E-10 pi m-rad
EPSX, EPSY= 8.0982E-09, 8.0982E-10 pi m-rad
SIGMA-P= 1.0016E-03
SIGMA-L= 0.5820 cm
PARTICLES/BUNCH= 1.7883E+10
S.R. DAMPING RATE (HORIZ.)= 28.8180 sec**-1
S.R. DAMPING RATE (LONG.)= 38.2620 sec**-1
NAT. SIGMA-P= 3.7151E-04
NAT. SIGMA-L= 0.2160 cm
CONVERGENCE CHECK OFF
DEBUG PRINTOUT OFF
```

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[7N]  
r

ENTER CALCULATION TYPE

- 0 = END
- 1 = SINGLE BUNCH THRESHOLDS
- 2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES
- 3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES
- 4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES
- 5 = GAS SCATTERING LIFETIME
- 6 = FREE ELECTRON LASER FORMULAE
- 7 = INTRABEAM SCATTERING
- 8 = TOUSCHEK SCATTERING
- 9 = ION TRAPPING FORMULAE

## Input Notes

- 7A) This question appears only once, when the code is first loaded. The "batch" mode here refers to the option of running the code with inputs taken from a (previously prepared) disk file ZAP.DAT, whether or not the code is being run in a batch mode (i.e., via a SUBMIT command). See discussion in Section II. Only Main Menu options 3, 4, 7, 8, and 9 can be run in the batch mode.
- 7B) Listing of the Main Menu, which directs all subsequent interactive running of the code. In this example, we have selected option 7.
- 7C) The lattice header from the ZAPLAT.DAT file is shown here to prevent the annoying possibility of running an entire series of calculations on the wrong lattice. A "no" answer here allows the user to exit ZAP and copy the correct lattice into the ZAPLAT.DAT file.
- 7D) In this routine, a negative energy value is again the flag for a proton or heavy ion calculation; the charge state and mass number would be requested in a subsequent input. One "secret" feature of the code is worth noting here. It is often useful to look at only the aperture calculation for the lattice when just starting out. For example, the average beta values needed for Main Menu options 1, 3, or 4 would be printed, and the physical aperture limit would give guidance on sensible values for the RF voltage (since there's no particular benefit to specifying an RF voltage giving a momentum acceptance much greater than the transverse acceptance). If the energy value is entered as 0 (zero) here, the code exits to the Main Menu; ending the program here gives the lattice summary printout and the aperture printout in the ZAPOUT.DAT file.
- 7E) If the code is being used (for an electron case) to find the equilibrium emittance value, the values entered here must be the natural emittance values. If you want to start the search iteration from different values, enter the horizontal (but not the vertical) emittance value as a negative number; the code will then request a second pair of emittance values that serve as a starting guess. Note that the ratio of horizontal to vertical emittance is kept fixed by the code. Thus, if you want to calculate a result for an emittance ratio  $\epsilon_x/\epsilon_y = 10$ , the input natural emittance values must already reflect this coupling. Furthermore, it is important that the ratio of natural emittances be made the same as that for the starting guess. If this is not done, results will be unpredictable!
- 7F) For a bunched beam, the beam intensity is input as the number of particles per bunch (available from outputs of Main Menu options 1 or 2). For an unbunched beam, the intensity is requested in terms of beam current.
- 7G) For a bunched beam, the bunch length is needed to obtain the volume density. This value should, in general, be taken from an output of Main Menu options 1 or 2. In this case, the bunch lengthening phenomenon is taken into account in a self-consistent fashion. Similarly, the momentum spread should come from these routines.
- 7H) To perform a calculation of the equilibrium emittance (for electrons), the emittance damping rate due to synchrotron radiation is needed. (This input is not requested if the energy (see note 7D) is specified as a negative value.) The required value of the horizontal damping time,  $\tau_x$ , is available from a lattice

code. Because we need the emittance damping rate, the value expected by ZAP here is  $2/\tau_x$  (see Section IV). Normally, the code calculates the equilibrium emittance only in the horizontal plane. To optionally calculate the longitudinal growth, a second damping rate is needed. This is requested if the horizontal damping rate has been entered as a negative value. Note that, if the longitudinal calculation is started from values that already include bunch lengthening, the additional longitudinal growth from IBS is usually small; indeed, unless the beam energy is quite low, the bunch lengthened values are already well beyond the equilibrium IBS values and no change occurs in the parameters. The user is warned, however, that the convergence of the longitudinal equilibrium calculation is poor, so the required CPU time for longitudinal calculations can be significant. If the horizontal damping rate is entered as 0 (zero), no iteration is done by the code, and a single output corresponding to the starting emittance values is calculated.

- 7I) The longitudinal emittance damping rate ( $2/\tau_E$ , where  $\tau_E$  is the energy damping time available from a lattice code) is requested if the horizontal emittance damping rate has been input as a negative value. (See also Section IV and note 7H).
- 7J) After the longitudinal damping rate is entered, the natural momentum spread and bunch length are requested. These are available from the header of Main Menu option 2 for a given set of RF parameters. Note that these values should not include any bunch lengthening!
- 7K) Before proceeding, check the values of the input parameters on your terminal. The convergence check switch, which can be activated by saying "no" to the KEEP SAME PARAMETERS question and changing the appropriate menu item, is used to check the behavior of the integration routine. If activated, the default number of steps is doubled (and the calculation takes twice as long); generally, the results will only change by about 0.3% or so. A debug printout can be activated in a similar fashion. This printout is really only for trouble-shooting, i.e., it's rather messy and has little, if any, pedagogical value.
- 7L) If the parameters are okay, perform the calculation by answering "yes" here.
- 7M) Checking the parameter values again here shows the new EPSX and EPSY values; in the example shown, the growth was nearly a factor of two from the natural emittance values. The equilibrium emittance values are automatically available to the Touschek scattering routine (Main Menu option 8) and the ion trapping routine (Main Menu option 9) if they are subsequently called without ending the code. Thus, the effects of both bunch lengthening and emittance growth are taken into account in a self-consistent fashion if we continue the calculation here. This is the recommended manner to use the code.
- 7N) To move on to the Touschek scattering calculation, we return to the Main Menu from here. The output that follows will, for simplicity, include only the aperture and IBS results; the subsequent Touschek scattering and ion trapping outputs will be given separately in the descriptions of these options. In reality, of course, if the code is used as shown in this example, with the calculations done sequentially, all three outputs would appear together in a single ZAPOUT.DAT file.

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\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

CFI43 ZAP TEST LATTICE [see Particle Accel. 18, 223 (1986)] <--[7a]

NUMBER OF LATTICE POINTS= 55 CIRCUMFERENCE= 143.160 m

S	BETAX	ALPHAX	BETAY	ALPHAY	DISPERSION	DISP.PRIME	APERTURE
(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)
1	1.508E+01	0.000E+00	5.223E+00	0.000E+00	0.000E+00	0.000E+00	3.000E-02
2	1.508E+01	-1.658E-02	5.235E+00	-4.787E-02	0.000E+00	0.000E+00	3.000E-02
3	1.510E+01	-3.316E-02	5.270E+00	-9.574E-02	0.000E+00	0.000E+00	3.000E-02
4	1.512E+01	-4.974E-02	5.330E+00	-1.436E-01	0.000E+00	0.000E+00	3.000E-02
5	1.515E+01	-6.631E-02	5.414E+00	-1.915E-01	0.000E+00	0.000E+00	3.000E-02
6	1.518E+01	-8.289E-02	5.522E+00	-2.393E-01	0.000E+00	0.000E+00	3.000E-02
7	1.523E+01	-9.947E-02	5.653E+00	-2.872E-01	0.000E+00	0.000E+00	3.000E-02
8	1.528E+01	-1.160E-01	5.809E+00	-3.351E-01	0.000E+00	0.000E+00	3.000E-02
9	1.534E+01	-1.326E-01	5.989E+00	-3.830E-01	0.000E+00	0.000E+00	3.000E-02
10	1.542E+01	-1.492E-01	6.192E+00	-4.308E-01	0.000E+00	0.000E+00	3.000E-02
11	1.549E+01	-1.658E-01	6.419E+00	-4.787E-01	0.000E+00	0.000E+00	3.000E-02
12	1.558E+01	-1.824E-01	6.671E+00	-5.266E-01	0.000E+00	0.000E+00	3.000E-02
13	1.568E+01	-1.989E-01	6.946E+00	-5.744E-01	0.000E+00	0.000E+00	3.000E-02
14	1.578E+01	-2.155E-01	7.245E+00	-6.223E-01	0.000E+00	0.000E+00	3.000E-02
15	1.589E+01	-2.321E-01	7.568E+00	-6.702E-01	0.000E+00	0.000E+00	3.000E-02
16	1.601E+01	-2.487E-01	7.915E+00	-7.180E-01	0.000E+00	0.000E+00	3.000E-02
17	1.614E+01	-2.653E-01	8.286E+00	-7.659E-01	0.000E+00	0.000E+00	3.000E-02
18	1.628E+01	-2.818E-01	8.681E+00	-8.138E-01	0.000E+00	0.000E+00	3.000E-02
19	1.642E+01	-2.984E-01	9.100E+00	-8.616E-01	0.000E+00	0.000E+00	3.000E-02
20	1.658E+01	-3.150E-01	9.543E+00	-9.095E-01	0.000E+00	0.000E+00	3.000E-02
21	1.674E+01	-3.316E-01	1.001E+01	-9.574E-01	0.000E+00	0.000E+00	3.000E-02
22	1.653E+01	2.666E+00	1.034E+01	-2.832E+00	0.000E+00	0.000E+00	3.000E-02
23	1.581E+01	5.495E+00	1.101E+01	4.888E+00	0.000E+00	0.000E+00	3.000E-02
24	1.463E+01	7.979E+00	1.207E+01	-7.253E+00	0.000E+00	0.000E+00	3.000E-02
25	1.305E+01	9.961E+00	1.358E+01	-1.008E+01	0.000E+00	0.000E+00	3.000E-02
26	9.800E+00	8.617E+00	1.734E+01	-1.140E+01	0.000E+00	0.000E+00	3.000E-02
27	7.019E+00	7.273E+00	2.156E+01	-1.273E+01	0.000E+00	0.000E+00	3.000E-02
28	5.865E+00	5.959E+00	2.362E+01	-1.078E+01	0.000E+00	0.000E+00	3.000E-02
29	4.920E+00	4.876E+00	2.531E+01	-8.414E+00	0.000E+00	0.000E+00	3.000E-02
30	4.148E+00	3.981E+00	2.655E+01	-5.728E+00	0.000E+00	0.000E+00	3.000E-02
31	3.518E+00	3.240E+00	2.730E+01	-2.822E+00	0.000E+00	0.000E+00	3.000E-02
32	2.484E+00	2.668E+00	2.830E+01	-2.879E+00	0.000E+00	0.000E+00	3.000E-02
33	1.650E+00	2.096E+00	2.931E+01	-2.936E+00	0.000E+00	0.000E+00	3.000E-02
34	8.423E-01	1.237E+00	3.003E+01	7.150E-03	8.530E-03	7.004E-02	3.000E-02
35	4.273E-01	4.748E-01	2.931E+01	2.950E+00	3.429E-02	1.415E-01	3.000E-02
36	3.728E-01	-2.500E-01	2.720E+01	5.608E+00	7.778E-02	2.156E-01	3.000E-02
37	6.745E-01	-9.945E-01	2.392E+01	7.723E+00	1.399E-01	2.940E-01	3.000E-02
38	1.356E+00	-1.817E+00	1.979E+01	9.092E+00	2.217E-01	3.781E-01	3.000E-02
39	2.210E+00	-2.451E+00	1.632E+01	8.246E+00	2.973E-01	3.781E-01	3.000E-02
40	3.317E+00	-3.086E+00	1.319E+01	7.400E+00	3.730E-01	3.781E-01	3.000E-02
41	3.317E+00	-3.086E+00	1.319E+01	7.400E+00	3.730E-01	3.781E-01	3.000E-02
42	5.866E+00	-4.196E+00	8.527E+00	5.921E+00	5.053E-01	3.781E-01	3.000E-02
43	9.191E+00	-5.306E+00	4.900E+00	4.441E+00	6.377E-01	3.781E-01	3.000E-02
44	1.030E+01	-3.462E+00	3.970E+00	3.067E+00	6.761E-01	2.359E-01	3.000E-02
45	1.089E+01	-1.248E+00	3.339E+00	2.028E+00	6.964E-01	8.739E-02	3.000E-02
46	1.091E+01	1.101E+00	2.938E+00	1.209E+00	6.979E-01	-6.350E-02	3.000E-02
47	1.035E+01	3.331E+00	2.724E+00	5.223E-01	6.806E-01	-2.127E-01	3.000E-02

48	9.42	9.066E+00	3.097E+00	2.533E+00	4.288E-01	6.380E-01	-2.127E-01	3.000E-02
49	9.62	7.873E+00	2.863E+00	2.381E+00	3.354E-01	5.955E-01	-2.127E-01	3.000E-02
50	9.62	7.873E+00	2.863E+00	2.381E+00	3.354E-01	5.955E-01	-2.127E-01	3.000E-02
51	10.47	3.850E+00	1.870E+00	2.148E+00	-6.185E-02	4.147E-01	-2.127E-01	3.000E-02
52	11.32	1.514E+00	8.771E-01	2.591E+00	-4.591E-01	2.339E-01	-2.127E-01	3.000E-02
53	11.52	1.224E+00	5.595E-01	2.751E+00	-3.244E-01	1.981E-01	-1.402E-01	3.000E-02
54	11.73	1.055E+00	2.724E-01	2.852E+00	-1.678E-01	1.768E-01	-6.963E-02	3.000E-02
55	11.93	1.000E+00	0.000E+00	2.886E+00	0.000E+00	1.697E-01	0.000E+00	3.000E-02

RING AVERAGE VALUES (m) : BETAX= 9.44160E+00 BETAY= 9.79690E+00 DISPERSION= 1.66127E-01 <---[7b]  
ALPHAX= 5.90368E-01 ALPHAY= 9.55567E-02 DISP.PRIME= 1.42406E-02  
BXINV= 3.24930E-01 BYINV= 1.88907E-01

\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

APERTURE CALCULATION <---[7c]

DP/P DYN. APERTURE <---[7d]  
(mm)

0.00000 30.000  
0.01000 27.000  
0.02300 21.000  
0.03500 11.000  
0.03600 0.000

I BETAX DISP. DYN. APERTURE PHYS. APERTURE LOCATION OF <---[7e]  
(m) (m) (DP/P) (DP/P) PHYS. LIMIT

33	1.246E+00	4.265E-03	3.551E-02	4.206E-02	45
34	6.348E-01	2.141E-02	3.352E-02	3.812E-02	45
35	4.001E-01	5.604E-02	3.012E-02	3.497E-02	45
36	5.236E-01	1.088E-01	2.842E-02	3.308E-02	45
37	1.015E+00	1.808E-01	2.694E-02	3.131E-02	45
38	1.783E+00	2.595E-01	2.607E-02	3.021E-02	45
39	2.763E+00	3.352E-01	2.611E-02	3.026E-02	45
40	3.317E+00	3.730E-01	2.617E-02	3.033E-02	45
41	4.591E+00	4.391E-01	2.604E-02	3.017E-02	45
42	7.529E+00	5.715E-01	2.610E-02	3.024E-02	45
43	9.744E+00	6.569E-01	2.611E-02	3.026E-02	45
44	1.059E+01	6.862E-01	2.614E-02	3.030E-02	45
45	1.090E+01	6.971E-01	2.617E-02	3.033E-02	45
46	1.063E+01	6.892E-01	2.619E-02	3.036E-02	45
47	9.708E+00	6.593E-01	2.618E-02	3.035E-02	45
48	8.469E+00	6.168E-01	2.618E-02	3.035E-02	45
49	7.873E+00	5.955E-01	2.617E-02	3.033E-02	45
50	5.862E+00	5.051E-01	2.633E-02	3.053E-02	45
51	2.682E+00	3.243E-01	2.618E-02	3.035E-02	45
52	1.369E+00	2.160E-01	2.696E-02	3.133E-02	45
53	1.139E+00	1.874E-01	2.814E-02	3.276E-02	45
54	1.028E+00	1.732E-01	2.871E-02	3.342E-02	45

\*\*\*LARGEST BETA FUNCTION IS 1.666E+01 m AT LOCATION 20 <---[7f]

\*\*\*LARGEST DISPERSION VALUE IS 6.971E-01 m AT LOCATION 45

\*\*\*MAXIMUM MOMENTUM ACCEPTANCE IS 3.551E-02 (FROM LOCATION 33) IN DISPERSIVE REGION

\*\*\*MINIMUM MOMENTUM ACCEPTANCE IS 2.604E-02 (FROM LOCATION 41) IN DISPERSIVE REGION

\*\*\*MAXIMUM MOMENTUM ACCEPTANCE IS 4.303E-02 ( AT LOCATION 45) IN NON-DISPERSIVE REGION

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\*\*\*ZAP\*\* [VERSION DATE: 01-DEC-86]

INTRABEAM SCATTERING CALCULATION FOR BUNCHED ELECTRONS <---[7g]

ETOTAL= 750.000 MeV BETA= 1.000000 GAMMA= 1467.701 COULOMB LOG= 16.239

TRANSVERSE PARAMETERS (RMS):

X-EMITTANCE= 4.18440E-09 m-rad Y-EMITTANCE= 4.18440E-10 m-rad

NATURAL X-EMITTANCE= 4.18440E-09 m-rad NATURAL Y-EMITTANCE= 4.18440E-10 m-rad

LONGITUDINAL PARAMETERS (RMS):

MOMENTUM SPREAD= 1.0016E-03 BUNCH LENGTH= 0.00582 m

NATURAL MOMENTUM SPREAD= 3.7151E-04 NATURAL BUNCH LENGTH= 0.00216 m

BEAM INTENSITY:

PARTICLES PER BUNCH= 1.7883E+10 BUNCH CURRENT= 0.006000 A

SYNCHROTRON RADIATION (EMITTANCE) DAMPING RATES:

HORIZONTAL= 28.8180 sec\*\*-1 LONGITUDINAL= 38.2620 sec\*\*-1

SPACE CHARGE TUNE SHIFTS:

HORIZONTAL= 1.5362E-03 VERTICAL= 5.6712E-03

\*\*\* (WEIGHTED) AVERAGE RATES (1/sec) :

LONGITUDINAL= 1.934030E+01 HORIZONTAL= 5.647890E+01 VERTICAL= 1.373416E-02

\*\*\* (WEIGHTED) AVERAGE LIFETIMES (sec) :

LONGITUDINAL= 5.170551E-02 HORIZONTAL= 1.770573E-02 VERTICAL= 7.281115E+01

ACUPL= 0.9091

(GSR-ACUPL\*GIBS(EPHAT))\*EPHAT/(GSR\*EPSOX) = -7.816802E-01

(GSRL-GLIBS(EPLHAT))\*EPLHAT/(GSRL\*EPSOIL) = 3.594514E+00

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\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

INTRABEAM SCATTERING CALCULATION FOR BUNCHED ELECTRONS <---[7g]

ETOTAL= 750.000 MeV BETA= 1.000000 GAMMA= 1467.701 COULOMB LOG= 17.230

TRANSVERSE PARAMETERS (RMS):

X-EMITTANCE= 8.09817E-09 m-rad Y-EMITTANCE= 8.09817E-10 m-rad <---[7h]

NATURAL X-EMITTANCE= 4.18440E-09 m-rad NATURAL Y-EMITTANCE= 4.18440E-10 m-rad

LONGITUDINAL PARAMETERS (RMS):

MOMENTUM SPREAD= 1.0016E-03 BUNCH LENGTH= 0.00582 m <---[7i]

NATURAL MOMENTUM SPREAD= 3.7151E-04 NATURAL BUNCH LENGTH= 0.00216 m

BEAM INTENSITY:

PARTICLES PER BUNCH= 1.7883E+10 BUNCH CURRENT= 0.006000 A

SYNCHROTRON RADIATION (EMITTANCE) DAMPING RATES:

HORIZONTAL= 28.8180 sec\*\*-1 LONGITUDINAL= 38.2620 sec\*\*-1

SPACE CHARGE TUNE SHIFTS:

HORIZONTAL= 7.9378E-04 VERTICAL= 2.9304E-03

INDIVIDUAL LATTICE POINT LIFETIMES <---[7j]

NSTEP	BLIM	TLI (1/sec)	TXI (1/sec)	TYI (1/sec)
1	1.0000E+16	1.136219E+01	-7.421898E-03	-8.433942E-03
2	1.0000E+16	1.134425E+01	-7.422353E-03	-8.430985E-03
3	1.0000E+16	1.130867E+01	-7.423239E-03	-8.424856E-03
4	1.0000E+16	1.125599E+01	-7.424513E-03	-8.415115E-03
5	1.0000E+16	1.118703E+01	-7.426112E-03	-8.401134E-03
6	1.0000E+16	1.110280E+01	-7.427967E-03	-8.382122E-03
7	1.0000E+16	1.100449E+01	-7.429996E-03	-8.357158E-03
8	1.0000E+16	1.089340E+01	-7.432135E-03	-8.325238E-03
9	1.0000E+16	1.077091E+01	-7.434312E-03	-8.285288E-03
10	1.0000E+16	1.063844E+01	-7.436484E-03	-8.236244E-03
11	1.0000E+16	1.049739E+01	-7.438611E-03	-8.177035E-03
12	1.0000E+16	1.034913E+01	-7.440678E-03	-8.106646E-03
13	1.0000E+16	1.019497E+01	-7.442692E-03	-8.024128E-03
14	1.0000E+16	1.003613E+01	-7.444675E-03	-7.928620E-03
15	1.0000E+16	9.873718E+00	-7.446670E-03	-7.819368E-03
16	1.0000E+16	9.708774E+00	-7.448723E-03	-7.695708E-03
17	1.0000E+16	9.542211E+00	-7.450905E-03	-7.557094E-03
18	1.0000E+16	9.374833E+00	-7.453283E-03	-7.403085E-03
19	1.0000E+16	9.207359E+00	-7.455937E-03	-7.233343E-03
20	1.0000E+16	9.040403E+00	-7.458949E-03	-7.047632E-03
21	1.0000E+16	8.920583E+00	-7.421110E-03	-6.805310E-03
22	1.0000E+16	8.829101E+00	-7.261622E-03	-6.272296E-03
23	1.0000E+16	8.707702E+00	-6.939719E-03	-5.176122E-03
24	1.0000E+16	8.561476E+00	-6.455726E-03	-3.321578E-03
25	1.0000E+16	8.322424E+00	-5.546608E-03	1.058458E-03

26	160	1.0000E+16	8.100408E+00	-4.316820E-03	9.232792E-03
27	160	1.0000E+16	8.021336E+00	-3.458366E-03	1.706280E-02
28	160	1.0000E+16	8.015452E+00	-2.980919E-03	2.246132E-02
29	160	1.0000E+16	8.061910E+00	-2.583413E-03	2.752450E-02
30	160	1.0000E+16	8.159626E+00	-2.254783E-03	3.204645E-02
31	160	1.0000E+16	8.369529E+00	-1.855094E-03	3.803752E-02
32	160	1.0000E+16	8.721313E+00	-1.374428E-03	4.704752E-02
33	160	1.0000E+16	8.605646E+00	2.218710E+00	5.601457E-02
34	160	1.0000E+16	7.291657E+00	1.104309E+01	5.532271E-02
35	160	1.0000E+16	6.286076E+00	1.787101E+01	4.563908E-02
36	160	1.0000E+16	5.448800E+00	2.083092E+01	3.123897E-02
37	160	1.0000E+16	4.827419E+00	2.371562E+01	1.923901E-02
38	160	1.0000E+16	4.552586E+00	2.588158E+01	1.154111E-02
39	160	1.0000E+16	4.583816E+00	2.589488E+01	6.519102E-03
40	160	1.0000E+16	4.639822E+00	2.595508E+01	4.500863E-03
41	160	1.0000E+16	4.693997E+00	2.680873E+01	1.801741E-03
42	160	1.0000E+16	5.049518E+00	2.857271E+01	-1.775999E-03
43	160	1.0000E+16	5.453205E+00	3.078053E+01	-2.919049E-03
44	160	1.0000E+16	5.682207E+00	3.190876E+01	-3.123072E-03
45	160	1.0000E+16	5.907957E+00	3.304475E+01	-3.157634E-03
46	160	1.0000E+16	6.123917E+00	3.414569E+01	-3.107531E-03
47	160	1.0000E+16	6.382058E+00	3.563470E+01	-2.980598E-03
48	160	1.0000E+16	6.706913E+00	3.744692E+01	-2.814163E-03
49	160	1.0000E+16	6.873359E+00	3.844825E+01	-2.725314E-03
50	160	1.0000E+16	7.531626E+00	4.102447E+01	-2.396534E-03
51	160	1.0000E+16	8.785318E+00	4.901140E+01	-1.044738E-03
52	160	1.0000E+16	1.003737E+01	4.915670E+01	4.780265E-04
53	160	1.0000E+16	1.087370E+01	4.357574E+01	9.144872E-04
54	160	1.0000E+16	1.132361E+01	4.117030E+01	1.215613E-03
56	170	1.0000E+17	7.936172E+00	1.660230E+01	3.901769E-04

\*\*\* (WEIGHTED) AVERAGE RATES (1/sec) : <---[7k]  
 LONGITUDINAL= 8.635381E+00 HORIZONTAL= 1.532022E+01 VERTICAL= 2.720070E-03

\*\*\* (WEIGHTED) AVERAGE LIFETIMES (sec) : <---[7k]  
 LONGITUDINAL= 1.158027E-01 HORIZONTAL= 6.527323E-02 VERTICAL= 3.676375E+02

ACUPL= 0.9091 (GSR-ACUPL\*GIBS(EPSHAT))\*EPSHAT/(GSR\*EPSOX) = 1.000000E+00 <---[71]  
 (GSR-GLIBS(EPLHAT))\*EPLHAT/(GSR\*EPSOL) = 5.628103E+00

## Output Notes

- 7a) The first output from using any of the Main Menu options 7, 8, or 9 is the lattice printout. The list is basically an echo of the input ZAPLAT.DAT file, with the exception that the "zero" aperture values for the second and subsequent lattice points are replaced with the value entered explicitly for the first lattice point.
- 7b) The (s-weighted) values of the indicated lattice parameters averaged throughout the entire ring. The BXINV and BYINV parameters are the averages of 1/BETAX and 1/BETAY, respectively, and are not simply the reciprocals of the average BETAX and BETAY values.
- 7c) If a non-zero physical aperture has been specified, or if dynamic aperture data have been input (see Section II), the aperture routine produces this summary printout.
- 7d) This table is an echo of the input dynamic aperture data, if provided by the user. It is (arbitrarily) assumed by the code that these values correspond to the lattice location where the dispersion has its maximum value. Thus, if tracking results correspond to a different location, they must be scaled appropriately (by the square root of the betatron amplitudes at the two locations). Note that the last entry in the table serves to restrict the maximum dynamic aperture value. This cutoff is not mandatory, and an equivalent (but more gentle) cutoff will be supplied by the code in the absence of a user-supplied zero value.
- 7e) This table gives the available physical and dynamic aperture momentum limits in those portions of the lattice having non-zero dispersion. Note that ZAP averages the input lattice values between adjacent points, so the beta function and dispersion values listed for location 33, for example, actually correspond to the average value of points 33 and 34 in the input list (described under note 7a). (This averaging feature can be disabled, if desired, by entering the number of lattice points as a negative value in the first line of the ZAPLAT.DAT file.) Output values are:
- |                 |   |
|-----------------|---|
| I:              | lattice point index for data averaged between adjacent lattice input points (see above).  |
| BETAX:          | (averaged) horizontal beta function value.  |
| Disp.:          | (averaged) dispersion value.  |
| DYN. APERTURE:  | relative momentum deviation limit corresponding to input dynamic aperture data.   |
| PHYS. APERTURE: | relative momentum deviation limit due to physical aperture, i.e., the momentum value for which the closed-orbit deviation and induced betatron amplitude would cause a particle to hit the beam pipe wall (see Section IV). |

LOCATION OF  
PHYS. LIMIT:

the index of the lattice point where the physical limit is reached. Generally, this will correspond to the location having either maximum betatron amplitude or maximum dispersion.

7f) Below the table, are several summary values. These include:

- the largest beta function value and its lattice location;
- the largest dispersion value and its lattice location;
- the largest (acceptable) momentum deviation in the dispersive region of the lattice, and the location where the deviation occurs;
- the smallest (acceptable) momentum deviation in the dispersive region of the lattice, and the location where the deviation occurs;
- the maximum (acceptable) momentum deviation in the non-dispersive region of the lattice and the location where the limit occurs (which is usually at the location of maximum dispersion).

Note that these summary values, especially that for the physical limit, can be quite informative. In particular, if the physical limitation is severe, there is no benefit in specifying an RF system that has a much larger momentum acceptance (since this will affect the Touschek lifetime adversely).

7g) The output of the IBS calculation for the indicated case. For electrons, the code optionally iterates to find the equilibrium emittance values (see Section IV). In this case, the first page of IBS printout gives an abbreviated output for the starting parameters, followed by a second, fuller, printout of the final equilibrium result. (Only the full printout is produced in cases where no iteration is done.)

7h) The actual values of transverse emittance used by the code to obtain the growth rates. In the electron case, only the values from the full printout correspond to the equilibrium result. These values can be compared with the (user-supplied) natural emittance values, printed in the line below, to assess the magnitude of the IBS effect. The calculated equilibrium emittance values are automatically available to the Touschek scattering (Main Menu option 8) and ion trapping (Main Menu option 9) routines so that the emittance growth is properly taken into account.

7i) The actual values of bunch length and momentum spread used by the code to obtain the growth rates. In the electron case, only the values from the full printout correspond to the equilibrium result (if the user has also elected to consider longitudinal growth). Generally, the longitudinal growth is much lower than the transverse, especially when the bunch has already lengthened due to the influence of the longitudinal microwave instability. Because the "starting values" for the longitudinal parameters are generally obtained from a bunch lengthening calculation (as opposed to the natural values for the ring), the growth from IBS must be obtained by comparing the initial bunch length and momentum spread values (from the abbreviated first-page printout) with the final equilibrium values (from the full printout).

7j) The results for the individual lattice points. Outputs are:

NSTEP: the number of integration steps used.

BLIM: the upper limit for the IBS integrals (see Section IV). This is mainly of interest for the authors of the code, but it's an impressive number to quote at a cocktail party.

TLI: the longitudinal e-folding growth rate.

TXI: the horizontal e-folding growth rate.

TYI: the vertical e-folding growth rate.

7k) Weighted average values of both the growth rates and growth times for all three dimensions. These are the values used in the equilibrium calculations.

7l) Convergence parameters for both the transverse and longitudinal dimensions. The parameters should be roughly 1.0 at equilibrium. (In the case of longitudinal growth, the convergence of the code is rather slow, and the convergence criterion has been relaxed somewhat. Nonetheless, the calculated values for the equilibrium bunch length and momentum spread should have a precision of about 1%. This should be more than sufficient.)

8 = TOUSCHEK SCATTERING

ENTER CALCULATION TYPE <---[8A]

- 0 = END
- 1 = SINGLE BUNCH THRESHOLDS
- 2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES
- 3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES
- 4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES
- 5 = GAS SCATTERING LIFETIME
- 6 = FREE ELECTRON LASER FORMULAE
- 7 = INTRABEAM SCATTERING
- 8 = TOUSCHEK SCATTERING
- 9 = ION TRAPPING FORMULAE

8

ENTER MOMENTUM COMPACTION (ALPHA) <---[8B]

4.923e-3

ENTER RF FREQUENCY (MHz) <---[8C]

500

ENTER RF VOLTAGE (MV)

1.5

ENTER SYNCHROTRON RAD. ENERGY LOSS (MeV/turn) <---[8D]

8.008e-3

ENTER NON-DISP. MOMENTUM ACCEPTANCE <---[8E]

[ZERO MEANS USE APERTURE LIMITS]

0

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[8F]

?

ENERGY=	750.0000 MeV
EPSX, EPSZ=	8.0982E-09, 8.0982E-10 pi m-rad
SIGMA-P=	1.0016E-03
SIGMA-L=	0.5820 cm
PARTICLES/BUNCH=	1.7883E+10
MOMENTUM COMPACTION=	4.9230E-03
RF FREQUENCY=	500.0000 MHz
RF VOLTAGE=	1.5000 MV
SYNC. RAD. ENERGY LOSS=	8.0080E-03 MeV/turn
MOM. ACCEPTANCE (NON-DISP.)=	0.0000E+00
MOM. ACCEPTANCE (DISP.)=	0.0000E+00
CONVERGENCE CHECK	OFF

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[8G]

Y

\*\*\*WEIGHTED TOUSCHEK AVERAGES:

RATE (1/sec)= 4.368457E-05 HALF-LIFE (hrs)= 6.358716E+00

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[8H]  
r

ENTER CALCULATION TYPE

- 0 = END
- 1 = SINGLE BUNCH THRESHOLDS
- 2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES
- 3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES
- 4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES
- 5 = GAS SCATTERING LIFETIME
- 6 = FREE ELECTRON LASER FORMULAE
- 7 = INTRABEAM SCATTERING
- 8 = TOUSCHEK SCATTERING
- 9 = ION TRAPPING FORMULAE

## Input Notes

- 8A) Here we continue with the calculation begun with Main Menu option 7 by selecting option 8. Because some of the required parameters are already available to the code, the input list is somewhat abbreviated compared with starting a fresh calculation. The additional inputs that would be requested are described under Main Menu option 7.
- 8B) The momentum compaction factor is available from a lattice code. It is used to calculate the RF momentum acceptance.
- 8C) The RF parameters, which must correspond to the input values of bunch length and momentum spread used here (or in Main Menu option 7, if they were entered there), are also used to determine the longitudinal momentum acceptance.
- 8D) The synchrotron radiation energy loss is requested (for electrons) to obtain the synchronous phase.
- 8E) If known from tracking (or a well-honed intuition), values for the maximum momentum acceptance in the dispersive and non-dispersive regions may be entered here. If these values are used, the physical and dynamic aperture limits from the aperture routine (see output for Main Menu option 7) are ignored. The expected inputs are fractional values, i.e., a 4% momentum acceptance would be entered here as 0.04. The value in the dispersive region can be the same as (or smaller than) that for the non-dispersive region (see discussion in Section II). If a value of 0 (zero) is entered, as shown in the example, the transverse limits are taken from the aperture routine. To ensure that the RF (longitudinal) limit is used everywhere, enter a large acceptance value (say, 1) for both dispersive and non-dispersive regions. In this case the RF limit must clearly be smaller and will always be selected by the code.
- 8F) Check the various parameters before doing the calculation. As indicated, the bunch parameters, etc., from the previous IBS calculation (Main Menu option 7) are passed to this routine. The convergence test parameter, which can be changed by saying "no" to the KEEP SAME PARAMETERS question and selecting the appropriate menu code, would double the number of integration steps compared with the normal mode.
- 8G) If these parameters are okay, say "yes" to perform the calculation. The resultant lifetime is echoed to the user's terminal so you can see where you are.
- 8H) This routine can be especially informative when repeating calculations. That is, it is easy to change the acceptance limits from 0 (which uses the aperture limits) to 1 (which uses only the RF limits) to some intermediate values that correspond to a guess at the dynamic aperture acceptance. All other parameters are still available, so the multiple outputs are quite easy to generate at this stage. In this case, however, we want to go on to an ion trapping estimate, so that this part of the manual will finally be finished. Therefore, we return to the Main Menu. For clarity, the output from this calculation will be presented separately below. In reality, it would appear in the same ZAPOUT.DAT file as the output from Main Menu option 7 described above.

1-DEC-86 16:20:31

\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

TOUSCHEK SCATTERING CALCULATION FOR BUNCHED ELECTRONS <---[8a]

ETOTAL= 750.000 MeV BETA= 1.000000 GAMMA= 1467.701  
 X-EMITTANCE= 8.0982E-09 m Y-EMITTANCE= 8.0982E-10 m MOMENTUM SPREAD (RMS)= 1.002E-03 <---[8b]  
 RF PARAMETERS:  
 BUCKET HALF-HEIGHT= 3.2828E-02 RF VOLTAGE= 1.5000 MV FREQUENCY= 500.000 MHz  
 BUNCH LENGTH (RMS)= 0.0058 m PARTICLES PER BUNCH= 1.7883E+10 BUNCH CURRENT= 0.006000 A <---[8b]

MOMENTUM COMPACTION= 4.9230E-03 HARMONIC NUMBER= 238

SYNCHRONOUS PHASE ANGLE= 179.694 deg ENERGY LOSS= 8.0080E-03 MeV/turn

INDIVIDUAL LATTICE POINT LIFETIMES <---[8c]

NSTEP	BLIM	ACCEPTANCE (DP/P)	EPSINT	TAUTI (1/SEC)	TAUT (SEC)
1	2.5000E+01	3.2828E-02	9.3172E-01	8.580295E-06	1.165461E+05
2	2.5000E+01	3.2828E-02	9.3223E-01	8.550855E-06	1.169474E+05
3	2.5000E+01	3.2828E-02	9.3325E-01	8.492543E-06	1.177504E+05
4	2.5000E+01	3.2828E-02	9.3479E-01	8.406484E-06	1.189558E+05
5	2.5000E+01	3.2828E-02	9.3684E-01	8.294276E-06	1.205651E+05
6	2.5000E+01	3.2828E-02	9.3940E-01	8.157917E-06	1.225803E+05
7	2.5000E+01	3.2828E-02	9.4247E-01	7.999723E-06	1.250043E+05
8	2.5000E+01	3.2828E-02	9.4606E-01	7.822224E-06	1.278409E+05
9	2.5000E+01	3.2828E-02	9.5015E-01	7.628063E-06	1.310949E+05
10	2.5000E+01	3.2828E-02	9.5476E-01	7.419901E-06	1.347727E+05
11	2.5000E+01	3.2828E-02	9.5988E-01	7.200353E-06	1.388821E+05
12	2.5000E+01	3.2828E-02	9.6552E-01	6.971922E-06	1.434325E+05
13	2.5000E+01	3.2828E-02	9.7166E-01	6.736946E-06	1.484352E+05
14	2.5000E+01	3.2828E-02	9.7832E-01	6.497573E-06	1.539036E+05
15	2.5000E+01	3.2828E-02	9.8549E-01	6.255729E-06	1.598535E+05
16	2.5000E+01	3.2828E-02	9.9317E-01	6.013150E-06	1.663022E+05
17	2.5000E+01	3.2828E-02	1.0014E+00	5.771329E-06	1.732703E+05
18	2.5000E+01	3.2828E-02	1.0101E+00	5.531563E-06	1.807807E+05
19	2.5000E+01	3.2828E-02	1.0193E+00	5.294965E-06	1.888586E+05
20	2.5000E+01	3.2828E-02	1.0290E+00	5.062451E-06	1.975328E+05
21	2.5000E+01	3.2828E-02	1.0277E+00	4.976876E-06	2.009293E+05
22	2.5000E+01	3.2828E-02	9.9915E-01	5.169846E-06	1.934294E+05
23	2.5000E+01	3.2828E-02	9.4037E-01	5.664341E-06	1.765430E+05
24	2.5000E+01	3.2828E-02	8.5502E-01	6.532675E-06	1.530767E+05
25	3.5000E+01	3.2828E-02	7.0585E-01	8.550799E-06	1.169481E+05
26	3.5000E+01	3.2828E-02	5.1953E-01	1.263882E-05	7.912128E+04
27	4.5000E+01	3.2828E-02	3.9799E-01	1.711506E-05	5.842806E+04
28	5.5000E+01	3.2828E-02	3.3315E-01	2.060877E-05	4.852304E+04
29	5.5000E+01	3.2828E-02	2.8010E-01	2.450387E-05	4.080988E+04
30	5.5000E+01	3.2828E-02	2.3679E-01	2.881099E-05	3.470897E+04
31	1.0500E+02	3.2828E-02	1.8540E-01	3.604303E-05	2.774461E+04

32	1.5500E+02	3.2828E-02	1.2771E-01	4.876813E-05	2.050520E+04
33	2.0500E+02	3.2828E-02	6.1322E-02	7.108579E-05	1.406751E+04
34	5.5500E+02	3.2828E-02	1.6992E-02	9.390617E-05	1.064893E+04
35	8.0500E+02	3.0121E-02	1.0679E-02	1.097970E-04	9.107720E+03
36	5.0500E+02	2.8420E-02	1.9108E-02	9.297884E-05	1.075513E+04
37	3.0500E+02	2.6943E-02	3.5639E-02	7.628628E-05	1.310852E+04
38	2.0500E+02	2.6072E-02	5.9027E-02	6.411898E-05	1.559600E+04
39	1.5500E+02	2.6109E-02	9.8022E-02	5.098954E-05	1.961186E+04
40	1.5500E+02	2.6168E-02	1.2233E-01	4.569135E-05	2.188598E+04
41	1.5500E+02	2.6044E-02	1.6500E-01	3.930636E-05	2.544118E+04
42	5.5000E+01	2.6099E-02	2.8140E-01	2.937142E-05	3.404670E+04
43	4.5000E+01	2.6114E-02	3.7180E-01	2.587582E-05	3.864612E+04
44	4.5000E+01	2.6145E-02	4.0827E-01	2.520983E-05	3.966707E+04
45	3.5000E+01	2.6169E-02	4.2332E-01	2.589912E-05	3.861135E+04
46	4.5000E+01	2.6187E-02	4.1532E-01	2.798400E-05	3.573470E+04
47	4.5000E+01	2.6179E-02	3.7924E-01	3.275412E-05	3.053051E+04
48	4.5000E+01	2.6179E-02	3.3175E-01	4.005796E-05	2.496383E+04
49	4.5000E+01	2.6168E-02	3.0837E-01	4.439811E-05	2.252348E+04
50	5.5000E+01	2.6327E-02	2.3090E-01	6.243360E-05	1.601702E+04
51	1.5500E+02	2.6185E-02	9.3862E-02	1.311522E-04	7.624731E+03
52	2.5500E+02	2.6961E-02	5.0510E-02	1.815273E-04	5.508813E+03
53	2.5500E+02	2.8144E-02	4.9791E-02	1.779432E-04	5.619771E+03
54	2.5500E+02	2.8712E-02	4.8371E-02	1.784357E-04	5.604258E+03

\*\*\*WEIGHTED TOUSCHEK AVERAGES: <---[8d]  
RATE (1/sec)= 4.368457E-05 HALF-LIFE (hrs)= 6.358716E+00

## Output Notes

- 8a) Results of Touschek scattering calculation for the indicated case (here, a bunched electron beam).
- 8b) The emittance, bunch length, and momentum spread values are (automatically) obtained from the output of the preceding IBS calculation (Main Menu option 7) if available.
- 8c) Output values are:
- NSTEP: the number of integration steps used.
  - BLIM: the upper limit for the Touschek integral (see Section IV). This is mainly of interest to the authors of the code, or possibly to trivia buffs.
  - ACCEPTANCE: value of the momentum acceptance used at each lattice point. The code uses the smaller of the longitudinal (RF) or transverse acceptance.
  - EPSINT: value of the epsilon parameter from Bruck (see Section IV).
  - TAUTI: the Touschek scattering loss rate.
  - TAUT: the Touschek scattering half-life.
- 8d) Touschek loss rate and half-life, averaged over the ring.

9 = ION TRAPPING FORMULAE

---

ENTER CALCULATION TYPE <---[9A]

- 0 = END
- 1 = SINGLE BUNCH THRESHOLDS
- 2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES
- 3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES
- 4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES
- 5 = GAS SCATTERING LIFETIME
- 6 = FREE ELECTRON LASER FORMULAE
- 7 = INTRABEAM SCATTERING
- 8 = TOUSCHEK SCATTERING
- 9 = ION TRAPPING FORMULAE

9

ENTER NO. OF BUNCHES (ZERO FILLS ALL RF BUCKETS) <---[9B]

1

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[9C]  
?

ENERGY=	750.0000 MeV
EPSX, EPSZ=	8.0982E-09, 8.0982E-10 pi m-rad
SIGMA-L=	0.5820 cm
PARTICLES/BUNCH=	1.7883E+10
NO. OF BUNCHES=	1

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[9D]  
Y

KEEP SAME PARAMETERS? [Y=YES, N=NO, ?=LIST VALUES, R=RETURN TO MAIN MENU]<-[9E]  
r

ENTER CALCULATION TYPE <---[9F]

- 0 = END
- 1 = SINGLE BUNCH THRESHOLDS
- 2 = SINGLE BUNCH LONGITUDINAL PARAMETERS and ENERGY SCALING TABLES
- 3 = LONGITUDINAL COUPLED-BUNCH INSTABILITIES
- 4 = TRANSVERSE COUPLED-BUNCH INSTABILITIES
- 5 = GAS SCATTERING LIFETIME
- 6 = FREE ELECTRON LASER FORMULAE
- 7 = INTRABEAM SCATTERING
- 8 = TOUSCHEK SCATTERING
- 9 = ION TRAPPING FORMULAE

0

## Input Notes

- 9A) Here we continue with the calculation begun with Main Menu options 7 and 8 by selecting option 9. Because most of the required parameters are already available to the code, the input list is very abbreviated compared with starting a fresh calculation. The additional inputs that would be requested for a "new" calculation are described under Main Menu option 7.
- 9B) The only parameter to enter at this stage is the number of filled buckets. If 0 (zero) is entered, all RF buckets are presumed to be filled, that is, the number of bunches is set equal to the harmonic number. This routine merely evaluates a simple formula, and contains (at present) none of the complexities associated with uneven filling of the ring (i.e., leaving a gap in the train of filled bunches).
- 9C) Before doing the calculation, check the parameters. As advertised, the appropriate values from Main Menu option 7 are available here.
- 9D) If the parameters are okay, do the calculation. Remember, after it's done you can stop reading and I can stop writing!
- 9E) Here too, it is trivial to change the number of bunches to see the effect on the critical mass values. In this case, we're getting tired, so we'll return to the Main Menu one last time.
- 9F) Having safely reached the Main Menu for the ninth time, we will reward ourselves by ending the code and printing the results of the calculation. For clarity, the output from this last calculation will be presented separately below. In reality, it would appear in the same ZAPOUT.DAT file as the outputs from Main Menu options 7 and 8 described above.

1-DEC-86 16:20:31

\*\*\*ZAP\*\*\* [VERSION DATE: 01-DEC-86]

ION TRAPPING CALCULATION FOR BUNCHED ELECTRONS <---[9a]

ETOTAL= 750.000 MeV BETA= 1.000000 GAMMA= 1467.701  
X-EMITTANCE= 8.0982E-09 m Y-EMITTANCE= 8.0982E-10 m BUNCH LENGTH (RMS)= 0.0058 m <---[9b]  
NO. OF BUNCHES= 1 PARTICLES PER BUNCH= 1.7883E+10 BEAM CURRENT= 0.006000 A

OUTPUT VALUES  
-----

CRITICAL MASSES: <---[9c]  
HORIZONTAL= 19. VERTICAL= 60.

[9d]->LIMITING ION DENSITY= 8.0721E+14 m\*\*3 NEUTRALIZATION FACTOR= 1.4411E-04

NEUTRAL GAS PARAMETERS: <---[9e]  
PRESSURE= 1.0000E-09 torr TEMPERATURE= 300.0000 K DENSITY= 3.2188E+13 m\*\*3

IONIZATION PARAMETERS:  
SPECIES IONIZATION CROSS SECTION IONIZATION TIME NEUTRALIZATION TIME  
(m\*\*2) (sec) (sec)

HYDROGEN 3.3000E-23 3.1403E+00 3.1403E+00  
NITROGEN 1.6500E-22 6.2806E-01 6.2806E-01

IONIZED GAS PARAMETERS (LIMITING VALUES): <---[9f]  
PRESSURE= 2.5078E-08 torr TEMPERATURE= 300.0000 K DENSITY= 8.0721E+14 m\*\*3

TUNE SHIFTS:  
HORIZONTAL= 4.0602E-01 VERTICAL= 1.3219E+00 ROUND BEAM= 0.0000E+00 <---[9g]

## Output Notes

- 9a) Results of ion trapping calculation. Only the case of an electron beam can be calculated.
- 9b) The emittance and bunch length values are (automatically) obtained from the output of the preceding IBS calculation (Main Menu option 7) if available.
- 9c) Critical masses for ion trapping (see Section IV).
- 9d) Ion density at full neutralization.
- 9e) Neutral gas parameters are fixed at 1 nTorr to simplify scaling of results.
- 9f) The limiting ionized "gas" parameters involve simply converting the limiting ion density at full neutralization into units of pressure. In these terms, the effect of the ions on beam lifetime is easily obtained from a gas scattering calculation (Main Menu option 5). Remember that these are values that correspond to full neutralization, which is a worst-case situation.
- 9g) The round beam (ion-induced) tune shift value is only calculated when the emittance values in the horizontal and vertical planes are equal; otherwise it is printed as 0.0000E+00.

## IV. THEORETICAL FOUNDATIONS

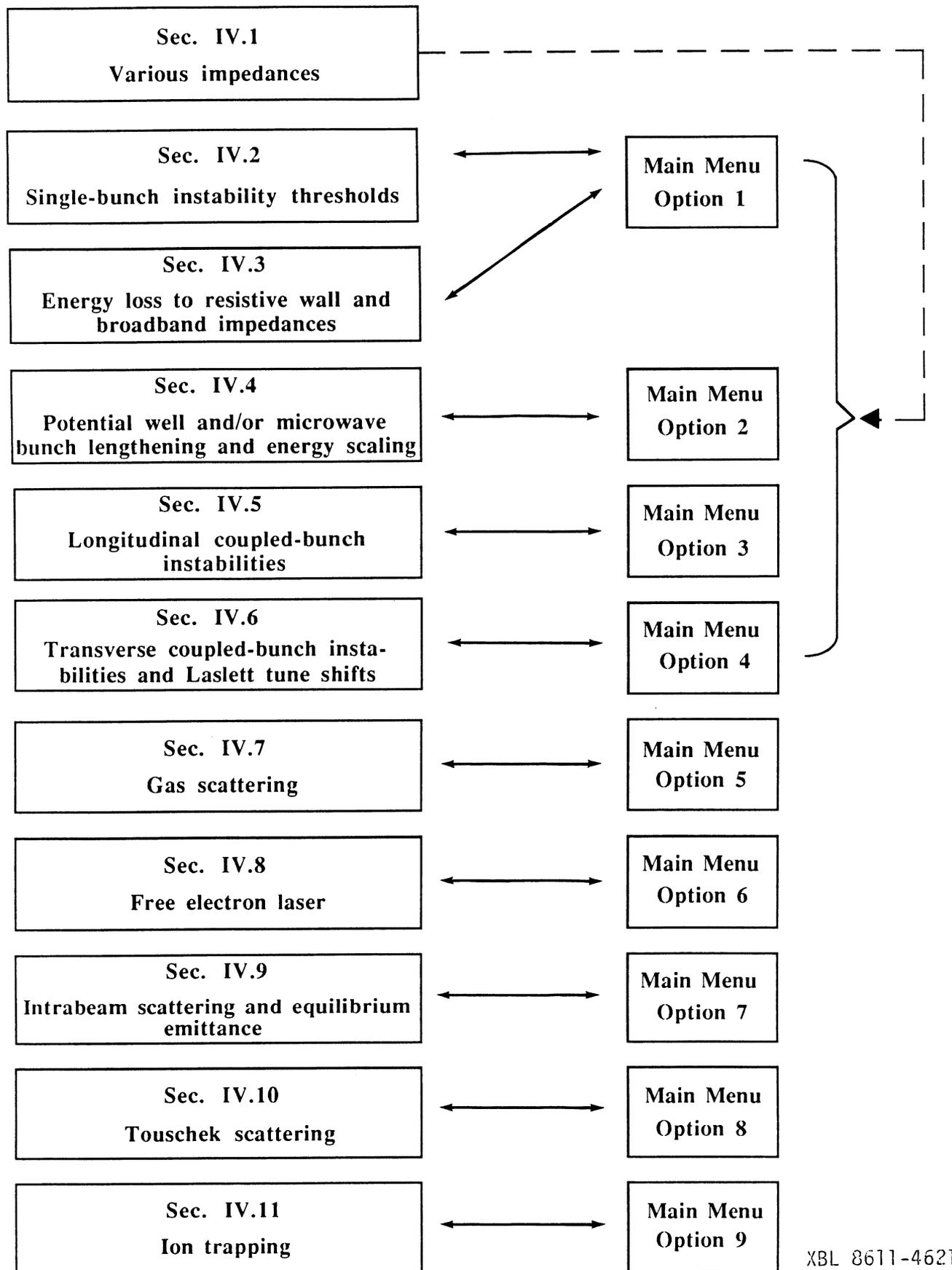
In this section, we summarize the theoretical foundations behind ZAP, providing the relevant formulations, physical models and particularly the equations used in the code in evaluating the various effects listed in the Main Menu as options 1 through 9. The correspondence between the sections in this chapter and the options in the Main Menu is shown in Fig. 1.

Main Menu options 1 through 4 calculate coherent phenomena involving various single-bunch and multibunch collective instabilities. The strengths (growth rates, real frequency shifts) of these instabilities and the limits imposed by them (beam current thresholds) depend critically on the assumed impedance models (including their appropriate magnitudes). We start, therefore, with a discussion of the various impedance models used by ZAP for the purpose of calculating coherent collective beam effects.

### IV.1 Impedances

The interaction of the beam with its environment in the storage ring is described by two frequency-dependent quantities, the longitudinal impedance<sup>1</sup>  $Z_{\parallel}(\omega)$  and the transverse impedance<sup>2</sup>  $Z_{\perp}(\omega)$ . A relativistic particle moving in a storage ring structure trails behind it an electromagnetic wake field that influences the motion of trailing particles. The longitudinal and transverse impedances are appropriately scaled Fourier transforms of the longitudinal and transverse components of the time-varying wake force, in the frequency domain.<sup>3</sup>

For a beam of particles, the longitudinal impedance  $Z_{\parallel}(\omega)$ , expressed in ohms, is the negative (longitudinal) voltage  $V_{\parallel}(\omega)$  per turn induced in the beam per unit current modulation  $I_B(\omega)$  at a frequency  $\omega$ :



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Fig. 1. Correspondence between sections in Chapter IV and options in the Main Menu.

$$Z_{\parallel}(\omega) = - \frac{V_{\parallel}(\omega)}{I_B(\omega)} \cdot \quad (\text{IV.1.1})$$

For transverse effects, it is the transverse dipole wake field (which increases linearly with transverse distance away from the electromagnetic center of the ring and is antisymmetric in sign about that center) that is dominant. The transverse impedance  $Z_{\perp}(\omega)$ , then, is taken to be the transverse force  $F_{\perp}(\omega, s)$  experienced by a unit charge, integrated over one revolution period along the circumferential length, induced by unit beam current with unit transverse displacement,  $x$ :

$$Z_{\perp}(\omega) = \frac{-i \int_0^{2\pi R} F_{\perp}(\omega, s) ds}{e x I_B(\omega)} \cdot \quad (\text{IV.1.2})$$

It is expressed in ohms/meter.

These impedances are generally complex quantities, with their real parts characterizing the dissipative effects of the beam-storage-ring system (growth or damping rates of instabilities, parasitic-mode energy loss, etc.) and their imaginary parts characterizing the effects on the reactive, oscillatory part of the system (coherent oscillation frequency shifts, longitudinally focusing potential well distortion, etc.).

Because of the presence of RF cavities, discontinuities in the vacuum chamber, resistance of the chamber walls, beam self-field or space charge, etc., there are very many different wake fields, and hence impedances, seen by the beam as it circulates. For the coherent motion of the beam, it is the sum of the impedances around the ring that governs beam behavior and instabilities.

### Parasitic Modes of RF Cavities

The impedance due to the RF cavities consists mainly of sharp peaks at frequencies corresponding to the cavity modes. Besides being driven at the fundamental longitudinal accelerating mode, the cavities will also be driven by the beam at their higher-order longitudinal and transverse parasitic modes, which lie at higher frequencies (up to the cutoff frequency determined by the beam pipe radius at the RF cavity noses). These narrow band, high-Q resonance peaks correspond to wake fields that ring in the cavity for a large number of RF oscillations, thus coupling each bunch to the others in the string of bunches in a storage ring. The resulting coupled-bunch motion may be linearly unstable if the beam intensity is high enough. Impedances presented to the beam by these parasitic modes of the RF cavity can be suitably represented by the frequency response of slightly damped, high quality factor (Q) resonators.<sup>4</sup> The impedance of each longitudinal parasitic mode is represented by

$$Z_{\parallel}(\omega) = \frac{R_S}{\left[ 1 + iQ \left( \frac{\omega_R}{\omega} - \frac{\omega}{\omega_R} \right) \right]} \quad (\text{IV.1.3})$$

and that of each transverse parasitic mode by

$$Z_{\perp}(\omega) = \left( \frac{\omega_R}{\omega} \right) \cdot \frac{R_T}{\left[ 1 + iQ \left( \frac{\omega_R}{\omega} - \frac{\omega}{\omega_R} \right) \right]} \quad (\text{IV.1.4})$$

where  $\omega_R = 2\pi f_R$  is the resonance angular frequency, Q is the quality factor and  $R_S(R_T)$  is the strength, or shunt impedance, of the longitudinal (transverse) mode, expressed in ohms (ohms/meter).

Each mode 'j' is described by its own set of values  $\{R_{S(T)}^j, Q^j, \omega_R^j\}$  and each RF cavity is described by its own set of modes  $j = 1, \dots, N$ ; longitudinal and transverse modes are treated separately in ZAP. The resonance parameters for the modes of an RF cavity could be either measured experimentally or calculated by an electromagnetic field code (e.g., URMEI<sup>5</sup>) for a given cavity geometry. In either case, these parameters must be supplied as input to ZAP, either by entering them manually at the terminal in the interactive mode, or by creating disk files, called CBMLRFi.DAT (longitudinal) or CBMTRFi.DAT (transverse), where they can be stored. ZAP is presently limited to using sets of resonance parameters having up to twenty such modes ( $N \leq 20$ ); as many as three different (longitudinal or transverse) resonator sets ( $i = 1, 2, 3$ ) can be accessed by the code at one time.

### Multiple RF Cells

Some amount of care should be exercised in selecting these cavity mode inputs when using ZAP to evaluate a storage ring with more than one identical RF cell. In reality, RF cells are never identical due to various errors and fluctuations. A conservative evaluation can be made simply by enhancing the single RF cell shunt impedances by a multiplicative factor equal to the number of ideally identical cells and using these enhanced shunt impedances as input parameters (with  $\omega_R$  and  $Q$  unchanged). Such an evaluation is, however, pessimistic since--due to cavity fabrication errors and temperature variations--the resonance frequencies will generally differ slightly among the nominally identical cavity cells, effectively broadening each resonance peak. When calculating the instability effects caused by the RF cavity impedance (Main Menu options 3 and 4), it is necessary either to know the exact values of the mode frequencies of individual cells, or to perform a statistical calculation by taking several ensembles of frequency distributions.<sup>6</sup> Such a calculation is beyond the present scope of our code.

An easy, and yet realistic and physical, estimate of the average value of the instability growth rates can be obtained by adopting a suitable de-Qing criterion, where each of the resonances is appropriately de-Qed, i.e., broadened, by lowering its Q-value to represent the spread in resonance frequencies.<sup>7</sup> In order to maintain the overall strength of the resonance, it is also necessary to lower the shunt impedance value of a single resonator by the same factor, thus preserving the area under the real part of the resonance curve (proportional to  $R_S/Q$ ). Physical de-Qing of an RF cavity in the laboratory, by almost any means, also leads to an approximate conservation of  $(R_S/Q)$ . If  $\delta$  is the de-Qing factor, the resonance parameter input to ZAP would be  $(\omega_R, Q/\delta, pR_S/\delta)$ , where  $p$  is the effective number of uncorrelated RF cells. The de-Qing factor can be typically chosen as  $\delta = 10$  for more than 10 RF cells ( $p \geq 10$ ). Because ZAP explicitly considers the number of RF cells in single-bunch calculations (see, for example, Eq. (IV.1.9) below), it is not advisable to put the factor "p" into the higher-order mode list contained in the file(s) CBMLRFi.DAT. To do so would lead to "double counting" of the RF modes in single-bunch estimates. [The inclusion of the de-Qing factor,  $\delta$ , is not a problem, as it does not change the strength of the mode (which scales as  $R_S/Q$ ).] That is, the code is expecting a list of modes for a single RF cell in the CBMLRFi.DAT file. Since the coupled-bunch rates scale linearly with "p", it is a simple matter to scale the longitudinal coupled-bunch rates (growth times) with  $p$  ( $1/p$ ) as appropriate. If the factor  $p$  is actually included in a CBMLRFi.DAT file, then this RF set should not be used in Option 1 to obtain single-bunch thresholds unless the "voltage per cell" parameter is set large enough to ensure that the code will take  $N_{\text{cell}} = 1$  for all its calculations.

### Ring Broadband Impedance

Whereas the localized RF cavities present narrow band resonator impedances that affect predominantly coupled-bunch coherent motion in a storage ring, the overall

global structure of the storage ring along its circumference presents a typically broadband impedance to the beam. This broadband impedance gives rise to an azimuthally short-range wake field that induces coherent motion mainly within a single bunch.<sup>3</sup> Aside from the contributions from the resistive wall of a smooth vacuum chamber and the beam-wall self-fields (space charge), it is very difficult to quantify this so-called ring broadband impedance, which varies from ring to ring and is caused by a large number of electromagnetic elements in the storage ring. Discontinuities in the vacuum chamber, such as bellows and beam collimators, other vacuum chamber cross section variations, and beam instrumentation, such as beam position monitor pickups, feedback loop pickups and kickers, and beam injection and abort kickers, all contribute to this broadband impedance.

A detailed frequency-dependent description of this broadband impedance is useful only if one wishes to calculate the effect of such an impedance on a single bunch in far more detail than is provided by our code. In ZAP, we are merely interested in providing a reasonable estimate of the single-bunch instability thresholds and the effect of the broadband impedance on coupled-bunch coherent motion, based on an overall strength parameter and a simplified (but somewhat universal) shape for this impedance. We have adopted the model<sup>7,8</sup> of a  $Q = 1$  resonator of the type described by Eq. (IV.1.3), centered at the characteristic resonance frequency  $\omega_c = c/b$ , to represent this broadband impedance in the longitudinal case. It is thus given by:

$$[Z_{\parallel}(\omega)]_{\text{Ring}}^{\text{BB}} = \frac{R_S}{\left[ 1 + i \left( \frac{\omega_c}{\omega} - \frac{\omega}{\omega_c} \right) \right]} . \quad (\text{IV.1.5})$$

At low frequencies this impedance is inductive and is proportional to the angular

frequency  $\omega$ , being given by  $-i\omega(R_S/\omega_c)$ . The frequency dependence of this broadband resonator with  $Q = 1$  is depicted in Fig. 2.

It is convenient to characterize the longitudinal broadband impedance in terms of the value of  $|Z_{\parallel}/n|^{BB}$ , where  $n$  is the harmonic of the fundamental angular revolution frequency, i.e.,  $n = \omega/\omega_0$ . For a  $Q = 1$  broadband resonator,  $|Z_{\parallel}/n|^{BB} = R_S(\omega_0/\omega_c)$  at low frequencies; at the resonance frequency  $\omega = \omega_c$ , the value of  $|Z_{\parallel}/n|^{BB}$  is again given by  $R_S(\omega_0/\omega_c)$ . Thus, the value of  $|Z_{\parallel}/n|^{BB}$  remains essentially constant from low frequencies up to the resonance frequency (and then decreases for  $\omega > \omega_c$ ). Accordingly, the longitudinal ring broadband impedance input to ZAP is in the form of a value for  $|Z_{\parallel}/n|^{BB}$  below resonance frequency,  $\omega_c$ . The code then converts this input to a  $Q = 1$  resonator impedance as in Eq. (IV.1.5) with shunt impedance  $R_S = (\omega_c/\omega_0) \cdot |Z_{\parallel}/n|^{BB}$ , centered at  $\omega_c$ . The frequency dependence of  $[Z_{\parallel}/n]^{BB}$ , suitably normalized, is shown in Fig. 3.

The code also converts the longitudinal broadband impedance input to a transverse broadband impedance via the standard prescription<sup>2</sup>

$$\left| Z_{\perp} \right|_{\text{Ring}}^{BB} = \frac{2c}{b^2 \omega} \left| Z_{\parallel} \right|_{\text{Ring}}^{BB} = \frac{2R}{\beta b^2} \left| \frac{Z_{\parallel}}{n} \right|_{\text{Ring}}^{BB} \quad (\text{IV.1.6})$$

Strictly speaking, this relationship between  $Z_{\perp}$  and  $Z_{\parallel}$  is valid only for a uniform beam pipe radius,  $b$ , throughout the ring circumference. However, for the sake of simplicity, this is the model adopted in ZAP. Figure 3 then also depicts the frequency dependence of  $[Z_{\perp}]^{BB}$ , in normalized units.

#### RF-Equivalent Broadband Impedance

The very many sharp resonances of the RF cavity, when acting collectively on a beam, will also have a small broadband impedance component with short wake fields

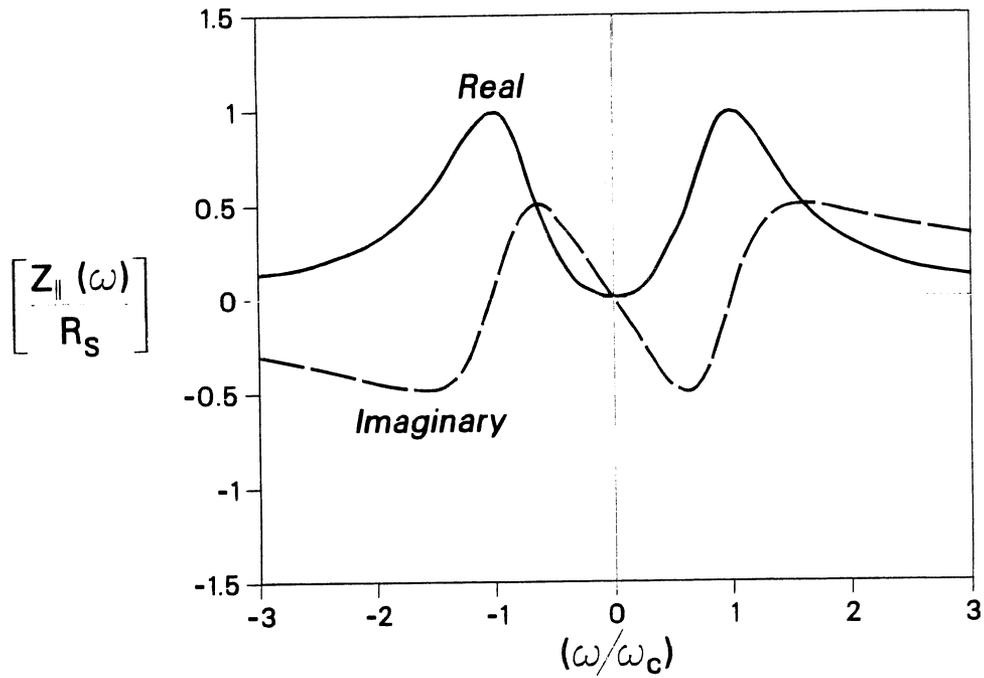
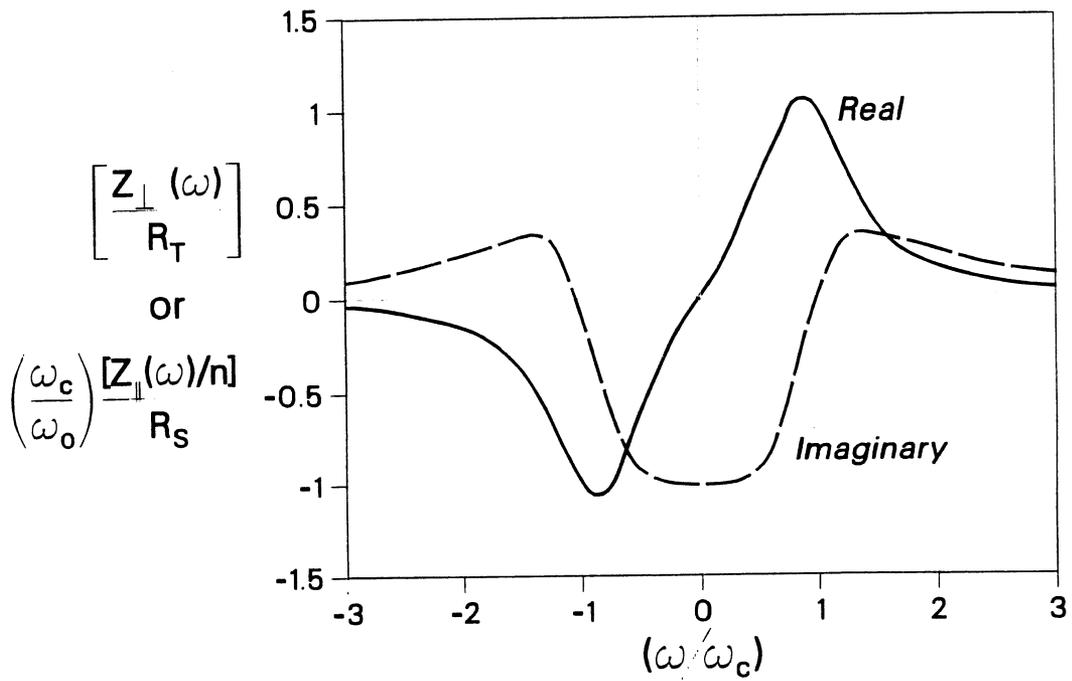


Fig. 2. Normalized longitudinal broadband resonator impedance with  $Q = 1$ .



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Fig. 3. Normalized longitudinal broadband impedance/mode number or transverse broadband impedance ( $Q = 1$  resonator).

that can affect particle motion in the same bunch as it crosses the RF cell. This effect of an RF cell on a single bunch can be described by an RF-equivalent broadband impedance. This equivalent broadband impedance of the RF is meant to duplicate the effect on a single bunch that would be induced by the aggregate action of all the sharp resonances of the RF cavity.

A broadband impedance acting on a long bunch with peak current below the threshold for single-bunch instability does not cause any significant growth rate but rather induces substantial real frequency shifts of the coherent modes.<sup>9,10</sup> We can thus construct an RF-equivalent broadband impedance by comparing the real frequency shift induced in a long bunch by the aggregate of all the RF resonators to that induced by a broadband impedance represented by a single  $Q = 1$  resonator centered at  $\omega = \omega_c$ , as given by Eq. (IV.1.5). For a long bunch, the real coherent frequency shift is governed by the low frequency reactive (i.e., imaginary) part of the impedance  $[Z_{\parallel}(\omega)/n]$ , and is actually proportional to it for constant  $[Z_{\parallel}(\omega)/n]$  at low frequency. Therefore, ZAP uses the following model prescription to construct the RF-equivalent broadband impedance  $(Z_{\parallel}/n)_{RF}^{BB}$ , characterized by shunt impedance  $R_S$  centered at  $\omega_c$ , from the RF resonator input parameters  $(\omega_R^j, Q^j, R_S^j)$ :

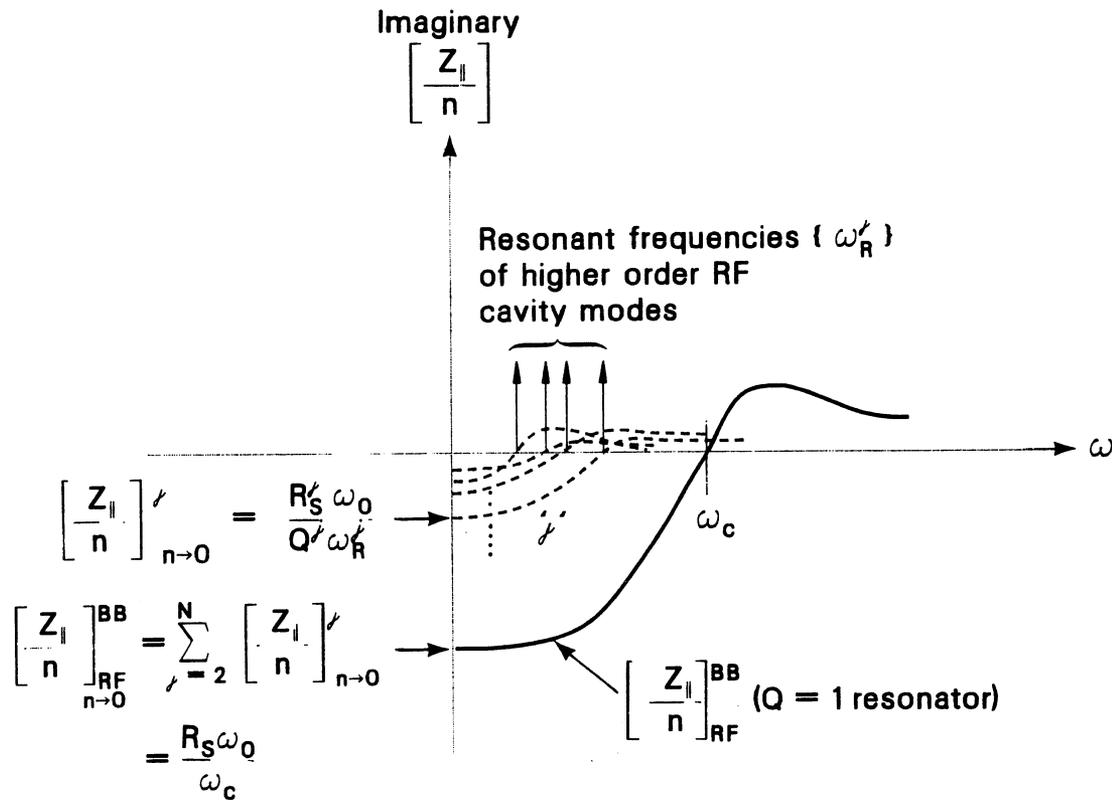
$$\left[ \frac{Z_{\parallel}}{n} \right]_{RF}^{BB} = \sum_{j=2}^N \left[ \frac{Z_{\parallel}}{n} \right]^j \quad (IV.1.7)$$

or, equivalently:

$$\frac{R_S \omega_0}{\omega_c} = \sum_{j=2}^N \frac{R_S^j \omega_0}{Q^j \omega_R^j} \quad (IV.1.8)$$

where  $N$  is the total number of RF resonator modes input by the user. The code presumes that the first longitudinal mode input ( $j = 1$ ) is the cavity fundamental, so

this value is excluded from the summation. Construction of this RF-equivalent broadband impedance by the above prescription is illustrated in Fig. 4.



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Fig. 4. Construction of RF-equivalent broadband impedance.

The total broadband impedance for the ring is given by

$$\left| \frac{Z_{||}}{n} \right|_o^{BB} = \left| \frac{Z_{||}}{n} \right|_{Ring}^{BB} + N_{cell} \left| \frac{Z_{||}}{n} \right|_{RF}^{BB} \quad (IV.1.9)$$

where  $N_{cell}$ , the number of RF cells required to produce the specified RF voltage, is based on the user-supplied value of the maximum number of volts per cell. It is this value that is used in Eq. (IV.1.11) below.

## SPEAR-Scaled Broadband Impedance

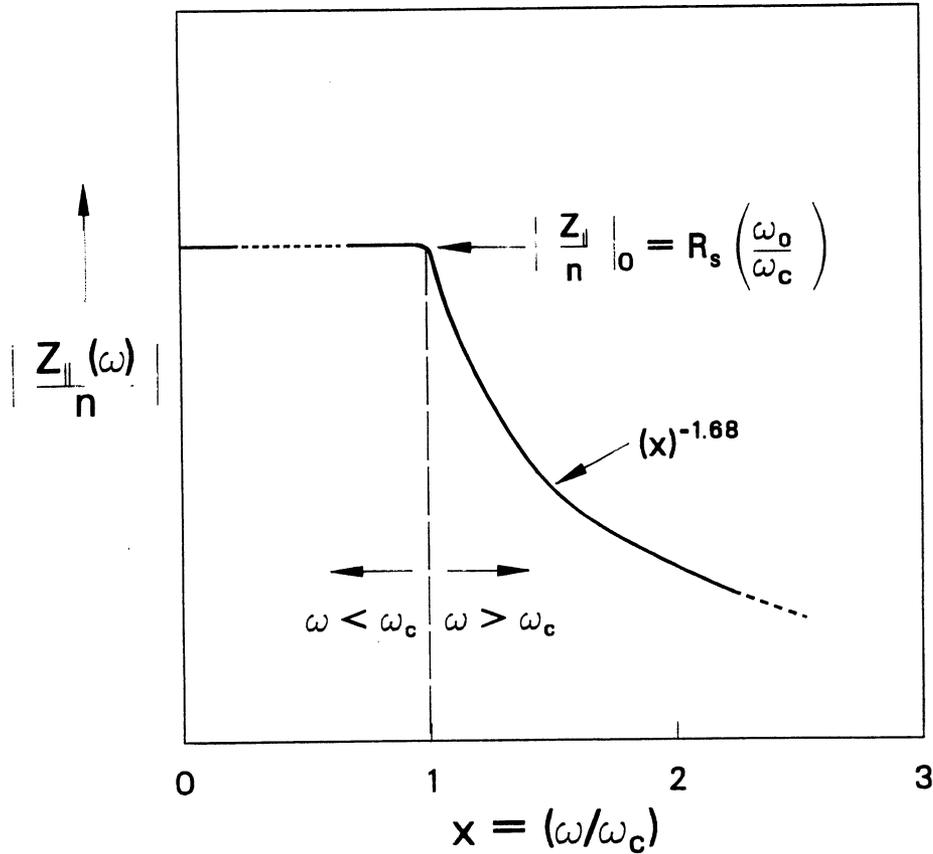
Experimental observations seem to suggest that, for short bunches with rms bunch length smaller than the beam pipe radius ( $\sigma_\ell < b$ ), the effective longitudinal broadband impedance  $|Z_{\parallel}/n|^{\text{BB}}$  seen by the beam at high frequencies (beyond the frequency  $\omega_c = c/b$ ) is reduced substantially from what one might expect at low frequencies, say  $|Z_{\parallel}/n|_0^{\text{BB}}$ . Observations<sup>3,11</sup> on the bunch lengthening phenomenon at the SPEAR storage ring, as a function of bunch current, suggest the following phenomenological power law, which we refer to here as the "SPEAR scaling law," for the longitudinal broadband impedance at high frequencies:

$$\begin{aligned} \left| \frac{Z_{\parallel}}{n} \right|^{\text{BB}} &= \left| \frac{Z_{\parallel}}{n} \right|_0^{\text{BB}} & (\omega \leq \omega_c) \\ &= \left| \frac{Z_{\parallel}}{n} \right|_0^{\text{BB}} \left( \frac{\omega}{\omega_c} \right)^{-1.68} & (\omega > \omega_c) . \end{aligned} \tag{IV.1.10}$$

This scaling law is illustrated in Fig. 5. Since perturbations affecting the single-bunch instability and microwave bunch lengthening must have wavelengths comparable to or less than the rms bunch length, the frequencies relevant in this scaling law are  $\omega \geq (c/\sigma_\ell)$ . For the sake of conservatism (in the sense of assuming the highest possible impedance value), we take  $\omega = (c/\sigma_\ell)$  in this scaled impedance relation (IV.1.10). For  $\sigma_\ell < b$ , ZAP reduces the input  $|Z_{\parallel}/n|_0^{\text{BB}}$  value to its SPEAR-scaled value

$$\left| \frac{Z_{\parallel}}{n} \right|^{\text{BB}} = \left| \frac{Z_{\parallel}}{n} \right|_0^{\text{BB}} \left( \frac{\sigma_\ell}{b} \right)^{1.68} \quad (\sigma_\ell < b) \tag{IV.1.11}$$

and uses this effective broadband impedance to calculate all single-bunch instability



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Fig. 5. SPEAR scaling law.

effects. At the user's discretion, the SPEAR scaling option can be disabled (see Section IV.4 for examples). If the bunches are long, of course, the SPEAR scaling reduction does not apply and ZAP results will be the same whether or not the SPEAR scaling option has been selected.

#### Free-Space Impedance

The SPEAR scaling law is valid in a limited frequency range, starting from  $\omega = (c/\sigma_{\ell})$  up to a few times that value. Extremely short bunches, with dominant

frequencies in the bunch spectrum extending far beyond the characteristic frequency  $\omega_c$ , are in conditions close to those of free space. Instability may be induced by the powerful coherent synchrotron radiation, which serves to generate microdensity modulations within the bunch;<sup>12-15</sup> these, in turn, enhance the coherent radiation, leading to self-sustained spontaneous growth of beam modulations. This coherent radiation is, however, suppressed (screened) to some extent by the presence of the metallic vacuum chamber wall surfaces that surround the beam.<sup>12,16,17</sup> In ZAP, the strength (in ohms) of this "free-space" longitudinal impedance (determined by the synchrotron radiation in the bending magnets), screened by the vacuum chamber walls and dominant at extremely high frequencies, is taken to be<sup>17</sup>

$$\left| \frac{Z_{\parallel}}{n} \right|_{\text{Bend}}^{\text{FS}} = 300 \left( \frac{b}{R} \right). \quad (\text{IV.1.12})$$

If, in addition, there are high-field wigglers in the storage ring, the additional contribution to the free-space impedance from the radiation in the wigglers is taken to be<sup>18</sup>

$$\left| \frac{Z_{\parallel}}{n} \right|_{\text{Wigglers}}^{\text{FS}} = 300 \left( \frac{b_W}{R} \right) \cdot \left( \frac{|\Delta\theta|}{2\pi} \right) \quad (\text{IV.1.13})$$

where  $b_W$  is the wiggler gap and  $|\Delta\theta|$  is the total effective magnetic bend angle of the orbit in the wigglers. For an electron case with  $\sigma_z < b$  and with SPEAR scaling on, ZAP uses either the SPEAR-scaled broadband impedance value, Eq. (IV.1.11), or the free-space impedance value, Eq. (IV.1.12) (and Eq. (IV.1.13) if appropriate), whichever is larger, for single-bunch instability calculations. When SPEAR scaling is not used for electrons, the input broadband impedance value is still compared with the free-space impedance and the larger value is used to estimate the single-bunch thresholds.

### Use of Broadband Impedance

One note regarding the use of broadband impedance in ZAP is in order. When calculating single-bunch instability thresholds in Main Menu options 1 and 2, it is only the overall effective strength  $|Z_{||}/n|^{BB}$  of the sum of all broadband impedances that is actually used. No use is made of the detailed frequency dependence. A thorough evaluation of single-bunch fast instabilities--incorporating the proper frequency dependence of the broadband impedance--is complicated and would call for a separate option in the menu totally devoted to such a calculation. It is envisioned that such an option will be included in a subsequent version of ZAP. In the present version, we have omitted this feature in the interest of keeping the code manageably compact. The frequency dependence of the broadband impedance (given by a  $Q = 1$  resonator centered at  $\omega = \omega_C$ ) is used in the calculation of coupled-bunch instabilities in Main Menu options 3 and 4. Should the user wish not to be restricted by the  $Q = 1$  resonator model in the coupled-bunch calculation, it is possible to attain some amount of flexibility for the frequency dependence by representing the broadband impedance with an arbitrary number of low- $Q$  (not necessarily  $Q = 1$ ) resonators, with arbitrary resonance frequencies (not necessarily  $\omega_R = \omega_C$ ) and shunt impedances. This can be done by explicitly providing these broadband low- $Q$  resonator data as part of the RF resonator data file [CBMLRFi.DAT or CBMTRFi.DAT]. It is important to note that the RF-equivalent broadband impedance is not directly used for the coupled-bunch calculations, since this term is implicitly considered via the RF resonator impedances used in the calculation.

### Space-Charge Impedance

The unneutralized charge and current densities within a beam produce electromagnetic self-fields, modified by the beam-induced current in the vacuum chamber wall. Such self-fields are described by a so-called "space-charge" impedance

of the beam-wall system. For collective space-charge effects, we assume a perfectly conducting pipe wall of radius  $b$ . For a longitudinally excited space-charge wave, the perturbation is effective only if the relativistic factors  $\gamma_W$  and  $\beta_W$  for the phase velocity of the wave are the same as  $\gamma$  and  $\beta$  for the beam particle. The low-frequency longitudinal space-charge impedance, for harmonic numbers  $n \ll \gamma_W(R/b)$ , is capacitive and is given by<sup>1,9,10</sup>

$$\left[ \frac{Z_{\parallel}}{n} \right]_{\text{space-charge}} = i \frac{Z_0}{2\beta\gamma^2} \left[ 1 + 2\ln \left( \frac{b}{a} \right) \right] \quad (\text{IV.1.14})$$

where  $Z_0$  ( $= 377$  ohms) is the impedance of free space and  $a$  is the average rms beam radius. The factor  $\gamma^2$  in the denominator comes from an almost complete cancellation between the electric and the magnetic forces for relativistic particles. At high frequencies, for harmonics  $n \gg \gamma_W(R/b)$ ,  $[Z_{\parallel}/n]_{\text{space-charge}}$  falls off rapidly. In ZAP we use for "a" the following expression:

$$a = \left[ \epsilon_x \langle \beta_x \rangle + \epsilon_y \langle \beta_y \rangle \right]^{1/2} \quad (\text{IV.1.15})$$

where  $\langle \beta_x \rangle$ ,  $\langle \beta_y \rangle$  are the horizontal and vertical beta-functions of the storage ring, averaged over the circumference, and  $\epsilon_x, \epsilon_y$  are the horizontal and vertical beam emittances.

The low-frequency transverse space-charge impedance is given by:<sup>2,9,10</sup>

$$\left[ Z_{\perp} \right]_{x,y}^{\text{space-charge}} = i \frac{RZ_0}{\beta^2 \gamma^2} \left[ \frac{1}{\bar{\sigma}_{x,y}(\bar{\sigma}_x + \bar{\sigma}_y)} - \frac{1}{b^2} \right] \quad (\text{IV.1.16})$$

where  $\bar{\sigma}_x = [\epsilon_x \bar{\beta}_x]^{1/2}$  and  $\bar{\sigma}_y = [\epsilon_y \bar{\beta}_y]^{1/2}$  are the average rms horizontal and vertical beam sizes, respectively. In ZAP, we use the transverse tunes  $\nu_{x,y}$  to calculate  $\bar{\beta}_{x,y}$  for this transverse case as  $\bar{\beta}_{x,y} = (R/\nu_{x,y})$ . The transverse space-charge impedance used in ZAP is taken to be the larger of the two terms:

$$(Z_{\perp})^{\text{space-charge}} = \text{Max} \left( Z_{\perp x}^{\text{sc}}, Z_{\perp y}^{\text{sc}} \right) . \quad (\text{IV.1.17})$$

### Resistive-Wall Impedance

Finally, the non-zero resistivity of the vacuum chamber wall will present a resistive-wall impedance to the beam. In ZAP, we assume a smooth resistive wall of radius  $b$  and conductivity  $\sigma$  around the beam all along the circumference. The longitudinal and transverse resistive-wall impedances are taken to be:<sup>1,2,9,19</sup>

$$\left[ \frac{Z_{\parallel, n}}{n} \right]^{\text{RW}}(\omega) = (1-i) \left( \frac{\beta Z_0}{2b} \right) \sqrt{\frac{2c}{Z_0 \sigma}} \cdot \frac{1}{\sqrt{\omega}} \quad (\text{IV.1.18})$$

$$[Z_{\perp}]^{\text{RW}}(\omega) = (1-i) \frac{R Z_0}{b^3} \sqrt{\frac{2c}{Z_0 \sigma}} \cdot \frac{1}{\sqrt{\omega}} . \quad (\text{IV.1.19})$$

Modifications to these resistive-wall impedances at extremely low frequencies (where the skin depth of the material is greater than the thickness of the resistive wall) are not included in the present version of ZAP. Four types of resistive wall material can be selected in ZAP: aluminum, stainless steel, copper and "cold copper." This last entry is included with an eye towards possible applications to superconducting storage rings. The conductivity of cold copper is taken (somewhat arbitrarily) as thirty times the room temperature value.

## IV.2 Single-Bunch Thresholds

The low-Q broadband impedances in the storage ring excite wake fields that damp out rapidly before subsequent bunches arrive. Thus, they do not drive coupled-bunch instabilities. They do, however, contribute to single-bunch coherent effects, which limit the maximum achievable beam current in one bunch.

The most pronounced longitudinal single-bunch instability is the microwave instability,<sup>20</sup> also known as turbulent bunch lengthening. For a given longitudinal broadband impedance, microwave instability occurs when the peak beam current exceeds a certain threshold value. Beyond the threshold, the bunch lengthens turbulently (with a growth time typically much shorter than the synchrotron period) until the peak current is just below the threshold value. The threshold peak current for longitudinal microwave instability is given by<sup>1,20,21</sup>

$$\left( I_p \right)_{\parallel, \mu W} = \frac{2\pi |n| (E_T/e) (\beta \sigma_p)^2}{|z_{\parallel}/n|_{BB}} . \quad (\text{IV.2.1})$$

There is a transverse analog to the longitudinal microwave instability, the so-called transverse fast blowup instability.<sup>22</sup> For  $\sigma_{\ell} > b$ , the onset of this instability depends on the peak current alone, and there is a corresponding threshold peak current for the transverse fast blowup. For  $\sigma_{\ell} < b$ , the threshold depends not only on the peak current, but also strongly on the bunch length (in addition to the bunch length dependence through the peak current). Written in terms of bunch average currents, the threshold average current for transverse fast blowup is given by<sup>22-24</sup>

$$\left( I_b \right)_{\perp, FB} = \frac{4(E_T/e) v_s}{|z_{\perp}|_{BB} \langle \beta_{\perp} \rangle R} \text{Max} \left( b, \frac{\sigma_{\ell}^2}{b} \right) . \quad (\text{IV.2.2})$$

Written in terms of the peak current, the threshold is given by

$$\left(I_p\right)_{\perp,FB} = \frac{4(E_T/e)v_s}{|Z_{\perp}|_{BB}<\beta_{\perp}>R} \text{Max} \left( b, \frac{\sigma_{\ell}^2}{b} \right) \cdot \frac{F'}{\sigma_{\ell}} \quad (\text{IV.2.3})$$

$$= \frac{4(E_T/e)v_s}{|Z_{\perp}|_{BB}<\beta_{\perp}>R} \text{Max} \left( \frac{b}{\sigma_{\ell}}, \frac{\sigma_{\ell}}{b} \right) \cdot F' \quad (\text{IV.2.4})$$

where

$$F' = \sqrt{2\pi} R \quad \text{for Gaussian bunches} \quad (\text{IV.2.5})$$

and

$$F' = 0.375 \sqrt{2} (2\pi R) \quad \text{for parabolic bunches.}$$

We note that, for a given relative momentum spread,  $\sigma_p$ , in the bunch,  $v_s = |n|R(\sigma_p/\sigma_{\ell})$  and a given  $|Z_{\perp}|_{BB}$  determines a threshold for the peak current  $(I_p)_{\perp}$  alone for  $\sigma_{\ell} > b$  and a threshold for the product  $(I_p)_{\perp} \cdot \sigma_{\ell}^2$  for  $\sigma_{\ell} < b$ .

If  $\sigma_{\ell} > (4\sqrt{\pi} \beta/3)b$ , then another instability, the so-called transverse mode-coupling instability dominates over the transverse fast blowup.<sup>23</sup> This is also known as the fast head-tail instability and occurs even if the chromaticity  $\xi$  is zero, unlike the normal slow head-tail instability. The transverse mode-coupling instability is driven by the imaginary or reactive component of the broadband impedance, and occurs when the average beam current and/or the  $\text{Im}(Z_{\perp})$  is high enough to shift the low-order synchrotron modes of the bunch (typically  $a = 0$  and  $a = -1$ ) sufficiently that their coherent frequencies merge and become degenerate (i.e., confluence). The threshold average current for this transverse mode-coupling instability is given by:<sup>23-25</sup>

$$\left(I_b\right)_{\perp,m-c} = \frac{4(E_T/e)v_s}{[\text{Im}(Z_{\perp})]<\beta_{\perp}>R} \cdot \frac{4\sqrt{\pi} \beta}{3} \sigma_{\ell} \cdot \quad (\text{IV.2.6})$$

In terms of peak current, this threshold is

$$\left(I_p\right)_{\perp, m-c} = \frac{4(E_T/e)v_s}{[\text{Im}(Z_{\perp})] < \beta_{\perp} > R} \cdot \frac{4\sqrt{\pi} \beta}{3} \cdot F' \quad (\text{IV.2.7})$$

where  $F'$  is the same form-factor as in Eq. (IV.2.5). Note that, for a given  $\sigma_p$ , with  $v_s = |n|R(\sigma_p/\sigma_{\ell})$ , the value of  $\text{Im}(Z_{\perp})$  alone determines a threshold in the average beam current ( $I_b$ ), or in the product ( $I_p \cdot \sigma_{\ell}$ ).

Option 1 in the Main Menu calculates the single-bunch threshold current estimates as above for a range of RF voltage values  $V_{RF}$ . With the synchrotron radiation energy loss  $U_0$  as input, the synchronous phase is calculated as

$$\sin \phi_s = (U_0/V_{RF}) \quad (\text{IV.2.8})$$

$$\phi_s = \pi - \sin^{-1}(U_0/V_{RF}) \quad n > 0 \quad (\text{IV.2.9})$$

$$= \sin^{-1}(U_0/V_{RF}) \quad n < 0 \quad (\text{IV.2.10})$$

where  $n$  is the phase-slip factor. The synchrotron tune is then calculated as

$$v_s = \frac{1}{\beta} \left[ \frac{-q_i V_{RF} h n \cdot \cos \phi_s}{2\pi(E_T/e)} \right]^{1/2} \quad (\text{IV.2.11})$$

where  $q_i$  is the charge state ( $\equiv 1$  for electrons and protons) and  $h$  is the harmonic number. For a given relative momentum spread,  $\sigma_p$ , the bunch length is then calculated as

$$\sigma_{\ell} = |\eta| R \frac{\sigma_p}{v_s} . \quad (\text{IV.2.12})$$

With the ring broadband impedance  $[Z_{\parallel}/n]_{\text{Ring}}^{\text{BB}}$  and the RF-equivalent broadband impedance per RF cell  $[Z_{\parallel}/n]_{\text{RF}}^{\text{BB}}$  as input, the total effective broadband impedance is calculated as

$$\left[ \frac{Z_{\parallel}}{n} \right]_o^{\text{BB}} = \left[ \frac{Z_{\parallel}}{n} \right]_{\text{Ring}}^{\text{BB}} + N_{\text{cell}} \left[ \frac{Z_{\parallel}}{n} \right]_{\text{RF}}^{\text{BB}} \quad (\text{IV.2.13})$$

where  $N_{\text{cell}}$  is the number of RF cavity cells. The corresponding transverse broadband impedance is then taken to be:

$$|Z_{\perp}| = \frac{2R}{\beta b^2} \left[ \frac{Z_{\parallel}}{n} \right]_{\text{Ring}}^{\text{BB}} + \frac{2R}{\beta b_{\text{RF}}^2} \cdot N_{\text{cell}} \cdot \left[ \frac{Z_{\parallel}}{n} \right]_{\text{RF}}^{\text{BB}} \quad (\text{IV.2.14})$$

where  $b_{\text{RF}}$  is the pipe radius at the RF cavity noses. (This is the only place in the code where a distinction is made between  $b$  and  $b_{\text{RF}}$ .) The imaginary part of  $Z_{\perp}$  is taken to be

$$\text{Im } Z_{\perp}(\omega = 0) = Z_{\perp}(\omega = \omega_c) \quad (\text{IV.2.15})$$

$$\left| \text{Im } Z_{\perp} \right|_{\omega=0} = |Z_{\perp}| . \quad (\text{IV.2.16})$$

In the longitudinal case, the user can choose the SPEAR scaling option as described in Section IV.1 and comparison with the free-space impedance is always made. The transverse impedance, Eq. (IV.2.14), is always based on the unscaled longitudinal broadband components.

Based on the above impedance values, the code then calculates the longitudinal peak current threshold  $(I_p)_{\parallel, \mu W}$  and the lower of the transverse mode-coupling threshold or the transverse fast blowup threshold for the various requested values of RF voltage  $V_{RF}$ . ZAP compares the peak current thresholds for the longitudinal and transverse instabilities, and the lower (more limiting) one in terms of peak current is used to give the threshold bunch average current,  $I_b$ , the threshold total average current  $I = k_b I_b$ , and the threshold number of particles per bunch

$$N_b = \frac{I_b}{q_i e f_0} \quad (IV.2.17)$$

where  $f_0$  is the beam revolution frequency.

For each value of  $V_{RF}$ , the momentum half-height of the RF bucket is also calculated as:<sup>19</sup>

$$\left(\frac{\Delta p}{p}\right)_{\text{Bucket HH}} = \frac{1}{\beta} \left\{ \frac{-V_{RF} \left[ \frac{2}{\pi} \cos \phi_s - \left(1 - \frac{2}{\pi} \phi_s\right) \sin \phi_s \right]}{h\eta(E_T/e)} \right\}^{1/2} \quad (IV.2.18)$$

This Main Menu option in the program is particularly useful in elucidating trade-offs in terms of bunch length, bucket height, peak current, etc.

### IV.3 Energy Loss to Resistive Wall and Broadband Impedance

As the beam circulates around in the vacuum chamber, it loses energy continuously because of the retarding longitudinal wake fields. This power loss from the beam is related to the real part of the longitudinal impedance of the beam-storage-ring system. A detailed calculation of the power radiated into the machine impedance requires knowledge of the detailed frequency dependence of this

impedance. We consider two components that contribute to this power loss separately: the ohmic (resistive wall) losses in the vacuum chamber wall only, and the parasitic energy loss to the real part of the rest of the machine impedance.

The ohmic energy loss to the resistive wall can be calculated by taking the resistive wall longitudinal impedance as given by Eq. (IV.1.18), folding in the bunch frequency spectrum as a multiplier, and summing the resulting expression over all harmonics of the fundamental revolution frequency,  $p\omega_0$ ,  $p = 0, \pm 1, \pm 2, \dots$  etc. Neglecting skin-depth effects at extremely low frequencies, the resistive-wall impedance does not contain any sharp peaks and the summation over  $p$  can be replaced by an integral. For Gaussian bunches of length  $\sigma_\ell$ , the average resistive-wall energy loss by a single particle per turn, in units of eV, is given by:<sup>3,26</sup>

$$(\Delta E)_{RW}[\text{eV}] = \frac{1.23}{b} \cdot \frac{N_b e c^2 q_i^2}{4\pi^2 f_0} \cdot \left[ \frac{Z_0}{2\sigma} \right]^{1/2} \sigma_\ell^{-3/2} \quad (\text{IV.3.1})$$

In the code, the quantity actually printed is not  $(\Delta E)_{RW}$  but  $(\Delta E)_{RW}/q_i$ . The rate of energy loss to the resistive wall by the whole beam of particles is thus given by the power  $P_{RW}$  in watts, obtained by multiplying  $(\Delta E)_{RW}/q_i$  by  $k_b I_b$ , where  $k_b$  is the number of beam bunches.

For the parasitic energy loss to the longitudinal impedance of the rest of the ring, we use the simple model of a  $Q = 1$  broadband resonator, as described in Section IV.1. Replacing the discrete summation over revolution harmonics  $p$  by an integral again, the average parasitic energy loss per turn by a single particle to the broadband machine impedance (in eV), for Gaussian bunches, is given by:<sup>3,27</sup>

$$(\Delta E)_{BB}[\text{eV}] = \frac{N_b e q_i^2}{4\pi f_0} \left| \frac{Z_{||}}{n} \right|^{BB} \cdot \frac{c^2}{4b^2} \left[ \text{Re}\{w(z)\} - \frac{1}{\sqrt{3}} \text{Im}\{w(z)\} \right] \quad (\text{IV.3.2})$$

where

$$z = (\sqrt{3} + i) \left( \sigma_z / 4b \right) \quad (\text{IV.3.3})$$

and  $w(z)$  is the complex error function.<sup>28</sup> In the code, the quantity actually printed is not  $(\Delta E)_{\text{BB}}$ , but  $(\Delta E)_{\text{BB}}/q_i$ . Again, the rate of parasitic energy loss to the machine broadband impedance by the whole beam of particles is given by the power  $P_{\text{BB}}$  in watts, obtained by multiplying  $(\Delta E)_{\text{BB}}/q_i$  by  $k_b I_b$ .

The parasitic energy losses from the beam become heat in the vacuum chamber pipe. For cold-bore superconducting accelerators, these machine impedance power losses are important, since they have to be absorbed at cryogenic temperatures. Impedances that are located in the cryogenic environment are thus of particular concern since they contribute to the cryogenic load.

The energy losses per turn given by Eq. (IV.3.1) and Eq. (IV.3.2) are computed in Main Menu option 1 and the total loss per turn  $(\Delta E)_{\text{BB+RW}}/q_i = (\Delta E)_{\text{RW}}/q_i + (\Delta E)_{\text{BB}}/q_i$  in MeV is given in the output for each RF voltage and bunch length value. These energy loss values are also used (along with the synchrotron radiation energy loss, for electrons) to determine the synchronous phase angle. Although derived for Gaussian bunches, the above energy loss estimates, Eqs. (IV.3.1) and (IV.3.2), are also used when the parabolic bunch shape is selected.

#### IV.4 Bunch Lengthening by Potential-Well Distortion and/or Microwave Instability

Longitudinal space-charge and reactive wall impedances create longitudinal forces inside the bunch, thus distorting the focusing potential well that confines the bunch azimuthally. In other words, the RF waveform seen by the beam is modified by these self-fields.<sup>1</sup> The space-charge impedance is usually capacitive, whereas the reactive wall impedance is predominantly inductive at low frequency (below the beam pipe cutoff), changing to capacitive at high frequencies (above the cutoff). Above

transition energy,  $\cos \phi_s$  is negative and the space-charge forces increase the effective RF voltage, while the wall inductance at low frequencies (corresponding to long bunches), decreases it. The opposite is true below transition energy.

The self-forces do not have any effect on the rigid longitudinal bunch motions, because the potential of the self-fields moves with the bunch. However, the internal motion of the particles in a bunch is affected. Thus this distortion affects the size and shape of the RF bucket, and hence changes the incoherent synchrotron frequency and the bunch length. Depending on the relative strengths of the space-charge (capacitive) and the reactive wall impedance (mainly inductive for long bunches and capacitive for short bunches), bunches could either lengthen or shorten; the actual effect is also a strong function of the relevant bunch lengths under consideration.

The bunch length is related to the incoherent synchrotron frequency via the momentum spread. This relation is different for light particles such as electrons, and for heavier particles, such as protons. For electrons, there is strong synchrotron radiation emission, and the natural momentum spread is determined by the equilibrium between quantum excitation and radiation damping. In contrast, for protons (with negligible radiation losses) the momentum spread is mainly determined by the almost invariant longitudinal emittance, a consequence of phase-space conservation in the absence of radiation damping. The dynamics of bunch lengthening or shortening under potential-well distortion alone thus differ for electron bunches and proton bunches (or heavy ion bunches).

In addition, above the threshold current for longitudinal microwave instability, as discussed in Section IV.2, the "turbulent" microwave bunch lengthening contributes to an increased rate of overall bunch lengthening (called "anomalous" bunch lengthening), which is related to an increase of the bunch momentum (energy) spread, i.e., "bunch widening."

In any case, the results of bunch lengthening by potential-well distortion and/or microwave instability can be expressed in terms of an effective impedance as in the case of bunched-beam coherent instabilities, discussed in Section IV.5.

For the potential-well distorted bunch length  $\sigma_\ell$  and synchrotron frequency  $\omega_s$ , we define

$$\frac{\sigma_\ell}{\sigma_{\ell 0}} = x \quad (\text{IV.4.1})$$

$$\frac{\omega_s}{\omega_{s0}} = y \quad (\text{IV.4.2})$$

where the subscript zero refers to undistorted values, in the absence of potential-well distortion. Because

$$\sigma_p = \frac{\omega_s \sigma_\ell}{\omega_0 |n| R} \quad (\text{IV.4.3})$$

for small bunches, the radiation damping constraint (i.e.,  $\sigma_p = \text{constant}$ ) for light particles (e.g., electrons) yields the relation

$$\left( \frac{\sigma_\ell}{\sigma_{\ell 0}} \right) = \frac{\omega_{s0}}{\omega_s} \quad \text{or} \quad xy = 1 . \quad (\text{IV.4.4})$$

Similarly, the emittance or phase-space area conservation requirement (i.e.,  $\sigma_p \sigma_\ell = \text{constant}$ ) for heavier particles (e.g., protons or heavy ions) yields the relation

$$\left( \frac{\sigma_\ell}{\sigma_{\ell 0}} \right)^2 = \left( \frac{\omega_{s0}}{\omega_s} \right) \quad \text{or} \quad x^2 y = 1 . \quad (\text{IV.4.5})$$

The incoherent synchrotron frequency in the presence of a reactive component of the longitudinal coupling impedance  $[Z_{\parallel}/n]$  is given by:<sup>1,29,30</sup>

$$\omega_s^2 = \omega_{s0}^2 \left[ 1 - I_b K \left( \frac{\sigma_{\ell 0}}{\sigma_{\ell}} \right)^3 \operatorname{Im} \left\{ \left[ \frac{Z_{\parallel}}{n} \right]_{\text{eff}}^{s=0, a=1} \right\}_{\omega_s=0} \right] \quad (\text{IV.4.6a})$$

or

$$y^2 = \left[ 1 - \frac{I_b K}{x^3} \operatorname{Im} \left\{ \left[ \frac{Z_{\parallel}}{n} \right]_{\text{eff}}^{s=0, a=1} \right\}_{\omega_s=0} \right] \quad (\text{IV.4.6b})$$

where

$$K = - \frac{n (R/\sigma_{\ell 0})^3}{(E_T/e) (v_s \beta)^2} F \quad (\text{IV.4.7})$$

$$F = 1/\sqrt{2\pi} \quad \text{for Gaussian bunches} \quad (\text{IV.4.8})$$

$$F = 3\sqrt{2}/8 \quad \text{for parabolic bunches .}$$

Eliminating  $y$  from Eq. (IV.4.6) and Eq. (IV.4.4) or Eq. (IV.4.5), we obtain the following relations<sup>31</sup> for the pure potential-well bunch lengthening factor  $x$ :

$$x^3 - x - I_b K \cdot \operatorname{Im} \left\{ \left[ \frac{Z_{\parallel}}{n} \right]_{\text{eff}}^{s=0, a=1} \right\}_{\omega_s=0} = 0 \quad (\text{for electrons}) \quad (\text{IV.4.9})$$

$$x^3 - \frac{1}{x} - I_b K \cdot \operatorname{Im} \left\{ \left[ \frac{Z_{\parallel}}{n} \right]_{\text{eff}}^{s=0, a=1} \right\}_{\omega_s=0} = 0 \quad (\text{for protons or heavy ions}) \quad (\text{IV.4.10})$$

Here  $[Z_{\parallel}/n]_{\text{eff}}^{s,a}$  is as defined in Eq. (IV.5.2) later, except that, for the purpose of potential-well bunch lengthening, we evaluate it (at  $\omega_s = 0$ ) for the synchrotron dipole mode,  $a = 1$ , and for a single bunch,  $s = 0$ , and use only its imaginary part. This effective impedance is a function of bunch length. Thus, Eqs. (IV.4.9) or (IV.4.10) for the bunch-lengthening factor,  $x$ , are really transcendental and not simply cubic or quartic equations with constant coefficients.

The final value for the relative momentum spread is given by:

$$\sigma_p = (\sigma_p)_0 \quad (\text{for electrons}) \quad (\text{IV.4.11})$$

or

$$\sigma_p = (\sigma_p)_0 / x \quad (\text{for protons or heavy ions}). \quad (\text{IV.4.12})$$

The final synchrotron tune for small amplitudes is then obtained from  $\sigma_p$  and  $\sigma_\ell$  as:

$$\nu_s = |n| R(\sigma_p / \sigma_\ell) . \quad (\text{IV.4.13})$$

As mentioned earlier, potential-well distortion alone can produce either bunch lengthening or shortening. Extremely short bunches can sample a predominantly capacitive impedance, in spite of impedance roll-off beyond the characteristic frequency  $\omega_c$ , and undergo bunch shortening.

If the bunch current is above the threshold for longitudinal microwave instability, bunch lengthening occurs due to the combined effects of potential-well distortion and the microwave instability. In this instance, bunch lengthening is also accompanied by bunch widening (in momentum spread). The bunch lengthening due to this combined effect above the longitudinal microwave threshold is given by:<sup>31,32</sup>

$$\sigma_{\ell}^3 = \frac{I_b \cdot |n| R^3}{v_s^2 (E_T/e) \beta^2} F \left[ \left| \frac{z_{\parallel}}{n} \right|^{BB} - \text{Im} \left\{ \left[ \frac{z_{\parallel}}{n} \right]_{\text{eff}}^{s=0, a=1} \right\}_{\omega_s=0} \right] \quad (\text{IV.4.14a})$$

$$= I_b K' \left[ \left| \frac{z_{\parallel}}{n} \right|^{BB} - \text{Im} \left\{ \left[ \frac{z_{\parallel}}{n} \right]_{\text{eff}}^{s=0, a=1} \right\}_{\omega_s=0} \right] \quad (\text{IV.4.14b})$$

where  $K' = |K| \sigma_{\ell 0}^3$  and  $K$  is defined in Eq. (IV.4.7). The associated relative momentum spread increase is given by the bunch-widened value:<sup>32</sup>

$$\sigma_p^3 = \frac{I_b \cdot v_s \cdot F}{(E_T/e) \beta^2 |n|^2} \left| \frac{z_{\parallel}}{n} \right|^{BB} \left[ 1 - \left( \frac{\text{Im} \left\{ \left[ \frac{z_{\parallel}}{n} \right]_{\text{eff}}^{s=0, a=1} \right\}_{\omega_s=0}}{\left| \frac{z_{\parallel}}{n} \right|^{BB}} \right) \right]^{1/2} \quad (\text{IV.4.15a})$$

$$= I_b K' \left( \frac{v_s}{|n| R} \right)^3 \left| \frac{z_{\parallel}}{n} \right|^{BB} \left[ 1 - \left( \frac{\text{Im} \left\{ \left[ \frac{z_{\parallel}}{n} \right]_{\text{eff}}^{s=0, a=1} \right\}_{\omega_s=0}}{\left| \frac{z_{\parallel}}{n} \right|^{BB}} \right) \right]^{1/2} \quad (\text{IV.4.15b})$$

The resulting final synchrotron tune is again given by Eq. (IV.4.13).

If potential-well distortion is negligible or otherwise not of interest, the bunch lengthening due to microwave blowup alone above threshold is given (from Eqs. (IV.2.1), (IV.4.8), (IV.4.13) and (IV.4.23)) by:

$$\sigma_{\ell} = \left[ I_b K' \cdot \left| \frac{z_{\parallel}}{n} \right|_0^{BB} \right]^{1/3} \quad (\text{IV.4.16})$$

for  $\sigma_\ell > b$ . For  $\sigma_\ell < b$ , we are in the SPEAR scaling regime, as explained in Section IV.1, and the bunch lengthening is given by:

$$\sigma_\ell = \left[ I_b K' \left( \frac{1}{b^{1.68}} \right) \left| \frac{z_{\parallel}}{n} \right|_{\text{BB}} \right]^{1/1.32} . \quad (\text{IV.4.17})$$

The final relative momentum spread,  $\sigma_p \geq (\sigma_p)_{\text{natural}}$  or  $(\sigma_p)_{\text{initial}}$ , is given by

$$\sigma_p = \frac{v_s}{|\eta|R} \sigma_\ell . \quad (\text{IV.4.18})$$

In Main Menu option 2, for a given bunch length,  $\sigma_{\ell 0}$ , momentum spread,  $\sigma_{p0}$ , and  $|z_{\parallel}/n|_0^{\text{BB}}$  as input, the code calculates the threshold bunch current  $I_b^{\text{th}}$  according to:

$$I_b^{\text{th}} = \frac{1}{|K_0| |z_{\parallel}/n|_{\text{th}}} \cdot \frac{1}{F} \quad (\text{IV.4.19})$$

where  $K_0$  is given by Eq. (IV.4.7) (for  $v_s = v_{s0}$ ), in order to determine whether the beam current is above or below threshold. Here  $v_{s0}$  is calculated from Eq. (IV.4.13) using input  $\sigma_{p0}$  and  $\sigma_{\ell 0}$  values. The  $|z_{\parallel}/n|_{\text{th}}$  is chosen as follows:

SPEAR-scaling "off":

$$\left| \frac{z_{\parallel}}{n} \right|_{\text{th}} = \text{Max} \left( \left| \frac{z_{\parallel}}{n} \right|_0^{\text{BB}}, \left| \frac{z_{\parallel}}{n} \right|^{\text{FS}} \right) . \quad (\text{IV.4.20})$$

SPEAR-scaling "on":

$$\left| \frac{Z_{\parallel}}{n} \right|_{\text{th}} = \text{Max} \left( \left| \frac{Z_{\parallel}}{n} \right|_0^{\text{BB}} \cdot \left( \frac{\sigma_{\ell}}{b} \right)^{1.68}, \left| \frac{Z_{\parallel}}{n} \right|^{\text{FS}} \right) \quad \text{if } \sigma_{\ell} < b \quad (\text{IV.4.21})$$

$$\left| \frac{Z_{\parallel}}{n} \right|_{\text{th}} = \text{Max} \left( \left| \frac{Z_{\parallel}}{n} \right|_0^{\text{BB}}, \left| \frac{Z_{\parallel}}{n} \right|^{\text{FS}} \right) \quad \text{if } \sigma_{\ell} \geq b . \quad (\text{IV.4.22})$$

For protons and heavy ions, there is no free space impedance (assuming energies are low enough that there is no synchrotron radiation), so this value is taken to be  $\left| Z_{\parallel}/n \right|^{\text{FS}} = 0$ .

If the specified beam current is below threshold, or if no microwave calculation is requested, the code then solves the appropriate cubic or quartic equation, (IV.4.9) or (IV.4.10), for the highest positive root and, using Eqs. (IV.4.11) and (IV.4.12), produces the final values of  $\sigma_{\ell} = x\sigma_{\ell 0}$ ,  $\sigma_p$ , and the peak bunch current calculated as:

$$I_p = F \cdot I_b \cdot \frac{2\pi R}{\sigma_{\ell}} \quad (\text{IV.4.23})$$

in the output.

If above threshold, the combined potential-well and microwave bunch lengthening and widening are calculated according to Eqs. (IV.4.14) and (IV.4.15), and the final  $\sigma_{\ell}, \sigma_p$  and peak bunch current  $I_p$  (according to Eq. (IV.4.23)) are given in the output.

If only microwave lengthening is requested above threshold, the code provides the microwave lengthened and widened values of  $\sigma_{\ell}, \sigma_p$  and the peak current  $I_p$  according to Eqs. (IV.4.16) or (IV.4.17), Eq. (IV.4.18) and Eq. (IV.4.23).

In all cases involving potential-well distortion, the code solves the equations iteratively. The local value of the bunch length  $\sigma_{\ell}$  is then checked to see whether the

bunch crosses the boundary for the SPEAR-scaling regime,  $\sigma_\ell < b$ ; if so, the code chooses the effective impedance accordingly.

Sub-option 2 of Main Menu option 2 is a scaling routine that provides values of the synchrotron radiation energy loss, natural emittance, radiation damping rates (transverse and longitudinal) and natural momentum spread as a function of beam energy. These numbers are needed for calculations of electron storage rings as a function of energy. All values at an energy characterized by  $\gamma$  are scaled from a set of starting values (at a specified energy denoted by  $\gamma_0$ ) by the following scaling laws:<sup>33</sup>

$$U = (\gamma/\gamma_0)^4 U_0 \quad (\text{IV.4.24a})$$

$$\epsilon = (\gamma/\gamma_0)^2 \epsilon_0 \quad (\text{IV.4.24b})$$

$$g_{\text{SR}}^{L,T} = (\gamma/\gamma_0)^3 \left( g_{\text{SR}}^{L,T} \right)_0 \quad (\text{IV.4.24c})$$

$$(\sigma_p)_{\text{nat.}} = (\gamma/\gamma_0) (\sigma_p)_{\text{nat.}}^0 \quad (\text{IV.4.24d})$$

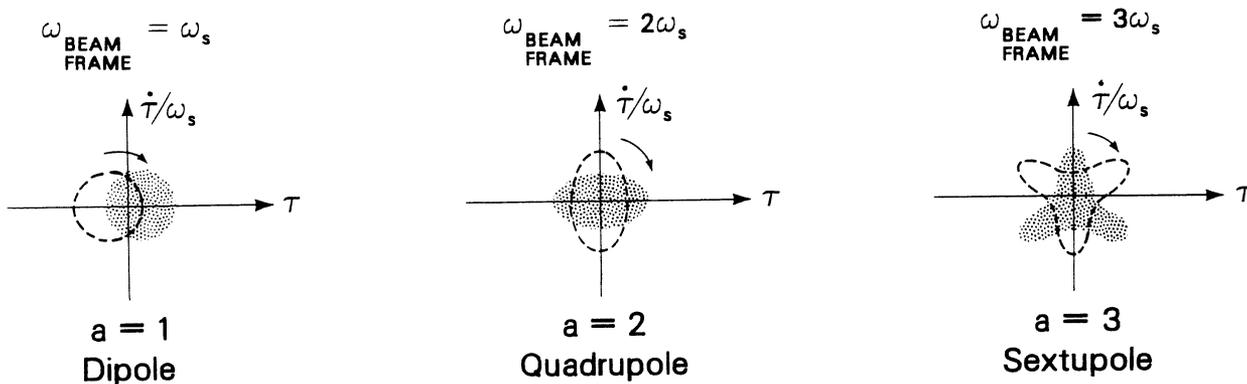
In this energy scaling routine for electrons, the radiation damping rates correspond to emittance damping rates (i.e.,  $g_{\text{SR}}^{L,T} = 2/\tau_{E,X}^{\text{SR}}$ ) and not rms beam size damping rates (i.e.,  $\tau_{E,X}^{-1}$ ).

Finally, a third utility routine, sub-option 3 of Main Menu option 2, provides, as a function of energy, values of the unnormalized emittance (for protons or heavy ions), based on an input normalized emittance value  $\epsilon_N$ , according to

$$\epsilon = \frac{\epsilon_N}{\beta\gamma} \quad (\text{IV.4.25})$$

#### IV.5 Longitudinal Symmetric Coupled-Bunch Instability

The various impedances in the storage ring, especially the long-range ringing wake fields of the narrow-band (high-Q) resonances of the RF cavity parasitic modes, will excite coupled-bunch normal modes in the beam. Two mode numbers are needed to describe a longitudinal coupled-bunch mode. For  $k_b$  bunches in the storage ring, there are  $k_b$  coupled-bunch modes characterized by a longitudinal mode number  $s = 0, 1, \dots, (k_b - 1)$ , specifying the phase-shift  $\Delta\phi = \left[ (2\pi/k_b) \cdot s \right]$  between bunches. In addition, an individual bunch in the  $s^{\text{th}}$  coupled-bunch mode requires an index  $a = 1, 2, \dots$  etc. to describe its motion in synchrotron phase space, e.g., the dipole mode,  $a = 1$ , where the bunches move rigidly as they execute longitudinal synchrotron oscillations, the quadrupole mode,  $a = 2$ , where the bunch head and tail oscillate longitudinally out of phase, etc. The coherent distortions in the longitudinal phase space for bunches oscillating in a few of these synchrotron modes are shown in Fig. 6.



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Fig. 6. Coherent distortions in the bunch phase space, in the beam frame, for the first three longitudinal synchrotron modes. Here  $\tau$  is the time carried along the particle trajectory with respect to the synchronous particle and  $\dot{\tau}$  is the relative angular velocity with respect to the synchronous particle in units of  $\omega_0$ , the angular revolution frequency of the synchronous particle.

The unperturbed modes, when observed in the laboratory at a fixed location in the storage ring, have frequencies given by  $\omega_p'' = (pk_b + s + av_s)\omega_0$ ,  $p = 0, \pm 1, \pm 2, \dots$  etc. In the beam frame, the frequency of mode  $(s, a)$  is simply  $av_s\omega_0 = a\omega_s$ . The perturbed frequency of coherent oscillation is modified from the unperturbed value by a complex coherent frequency shift  $\Delta\omega_{s,a}^{\parallel}$ , caused by the ring longitudinal impedance  $Z_{\parallel}(\omega)$ .

### Sacherer-Zotter Formalism

The coherent frequency shift is given, in the Sacherer-Zotter formalism, by:<sup>9,10,31</sup>

$$\Delta\omega_{s,a}^{\parallel} = i \left( \frac{a}{a+1} \right) \frac{q_i I_b \omega_0^2 n}{3(L/2\pi R)^3 2\pi\beta^2 (E_T/e)\omega_s} \cdot \left[ \frac{Z_{\parallel}}{n} \right]_{\text{eff}}^{s,a} \quad (\text{IV.5.1})$$

where

$$\left[ \frac{Z_{\parallel}}{n} \right]_{\text{eff}}^{s,a} = \frac{\sum_{p=-\infty}^{+\infty} \frac{Z_{\parallel}(\omega_p'')}{(\omega_p''/\omega_0)} h_a(\omega_p'')}{\sum_{p=-\infty}^{+\infty} h_a(\omega_p'')} \quad (\text{IV.5.2})$$

and, as discussed below, we take for the total bunch length  $L$  the value  $L = 2\sqrt{\pi} \sigma_{\ell}$  (Gaussian) or  $L = 2\sqrt{2} \sigma_{\ell}$  (parabolic). In Eq.(IV.5.2),  $h_a(\omega)$  is the bunch mode spectrum characteristic of the synchrotron mode,  $a$ , and depends on the unperturbed bunch shape and length. The real part of  $\Delta\omega_{s,a}^{\parallel}$  gives the real coherent mode (angular) frequency shift and the imaginary part gives the instability growth rate,  $g = \tau_{\text{gr}}^{-1} = \text{Im}(\Delta\omega_{s,a}^{\parallel})$ .

For bunches with a parabolic shape and total length  $L$ , perturbations can be conveniently decomposed into sinusoidal basis functions (the theory is phenomenological for this case and is based on expanding perturbations in terms of sinusoidal modes,

which are experimentally observed but are not the exact eigenmodes of the problem), and the spectral power density of the  $a^{\text{th}}$  synchrotron bunch mode is given by:<sup>31,34</sup>

$$h_a(\omega) = (a+1)^2 \frac{\left[1 + (-1)^a \cos\left(\omega \frac{L}{\beta c}\right)\right]}{\left[\left(\frac{\omega L}{\pi \beta c}\right)^2 - (a+1)^2\right]^2}. \quad (\text{IV.5.3})$$

For bunches with a Gaussian shape characterized by an rms length  $\sigma_\ell$ , perturbations can be decomposed into exact Hermitian eigenmodes and the bunch mode spectrum  $h_a(\omega)$  is given by:<sup>31,35</sup>

$$h_a(\omega) = \left(\frac{\omega \sigma_\ell}{\beta c}\right)^{2a} \exp\left[-\left(\frac{\omega \sigma_\ell}{\beta c}\right)^2\right]. \quad (\text{IV.5.4})$$

For arbitrary impedances, one has to do a numerical sum in Eq. (IV.5.2), with some convergence criterion, in order to evaluate  $\Delta\omega_{s,a}^{\parallel}$  in Eq. (IV.5.1). However, for resonator type impedances of the form given by Eq. (IV.1.3), an exact analytic expression for the summation in Eq. (IV.5.2) exists for the case of sinusoidal modes with a bunch spectrum as in Eq. (IV.5.3). For a resonator with shunt impedance  $R_S$ , quality factor  $Q$  and resonance frequency  $\omega_R$ , the exact analytic expression for the summation in (IV.5.2) is given by:<sup>31,34</sup>

$$\left[\frac{Z_{\parallel}}{n}\right]_{\text{eff}}^{s,a} = -i F_1 (F_2 S_2 - S_1) \quad (\text{IV.5.5})$$

where

$$F_1 = \frac{bR_S}{2k_b Q} \quad (\text{IV.5.6a})$$

$$F_2 = \frac{2}{\pi} \cdot \frac{d^3}{u} (a+1)^2 \quad (\text{IV.5.6b})$$

$$S_1 = \frac{1}{(p_1 - q_1)(p_2 - q_1)} + \frac{1}{(p_1 - q_2)(p_2 - q_2)} \quad (\text{IV.5.6c})$$

$$S_2 = \frac{T_1}{(p_1 - q_1)^2 (p_1 - q_2)^2} - \frac{T_2}{(p_2 - q_1)^2 (p_2 - q_2)^2} \quad (\text{IV.5.6d})$$

$$T_{1,2} = B_{1,2} + (-1)^a \left[ B_{1,2} \cos \pi r_{1,2} + \sin \pi r_{1,2} \right] \quad (\text{IV.5.6e})$$

$$b = \frac{\omega_R}{k_b \omega_0} \quad (\text{IV.5.6f})$$

$$d = \frac{\pi R}{k_b L} \quad \text{for parabolic bunches} \quad (\text{IV.5.6g})$$

$$c = (s + a v_s) / k_b \quad (\text{IV.5.6h})$$

$$u = b \left( 1 - \frac{1}{4Q^2} \right)^{1/2} \quad (\text{IV.5.6i})$$

$$v = \frac{b}{2Q} \quad (\text{IV.5.6j})$$

$$p_{1,2} = -c \pm u - iv \quad (\text{IV.5.6k})$$

$$q_{1,2} = -c \pm (a+1) d \quad (\text{IV.5.6l})$$

$$r_{1,2} = (p_{1,2} + c) / d \quad (\text{IV.5.6m})$$

$$B_{1,2} = \cot(\pi p_{1,2}) \quad (\text{IV.5.6n})$$

For the case of Hermitian modes relevant for Gaussian bunches, as in Eq. (IV.5.4), there also exists an approximate (but very good) analytic expression for the summation in Eq. (IV.5.2) for a resonator type impedance. It is given by:<sup>31,35</sup>

$$\left[ \frac{Z_{\parallel}}{n} \right]_{\text{eff}}^{S,a} = \frac{-iF_1}{du\Gamma\left(a + \frac{1}{2}\right)} (S_2 - S_1) \quad (\text{IV.5.7})$$

where

$$S_{1,2} = (r_{1,2})^{2a} (S_0)_{1,2} + \sum_{\ell=0}^{a-1} \Gamma\left(\ell + \frac{1}{2}\right) (r_{1,2})^{2a-2\ell-1} \quad (\text{IV.5.8})$$

$$(S_0)_{1,2} = \pi \left\{ i \left[ w(r_{1,2}) e^{-r_{1,2}^2} \right] - e^{-r_{1,2}^2} B_{1,2} \right\} \quad (\text{IV.5.9})$$

and

$$d = \frac{R}{k_b \sigma_\ell} \quad \text{for Gaussian bunches.} \quad (\text{IV.5.10})$$

Here too,  $w(z)$  is the complex error function<sup>28</sup> and  $\Gamma(x)$  the gamma function<sup>28</sup> with the recurrence relation:

$$\Gamma\left(\ell + \frac{1}{2}\right) = \left(\ell - \frac{1}{2}\right) \Gamma\left(\ell - \frac{1}{2}\right) \quad (\text{IV.5.11})$$

and

$$\Gamma(1/2) = \sqrt{\pi} . \quad (\text{IV.5.12})$$

All the other symbols are the same as for the sinusoidal case. The approximate nature of this expression comes as a result of neglecting terms of the order of  $\exp(-\pi^2 d^2)$  in the perturbation expansions. The approximation is thus very good for  $d > 1$ . For a small number of bunches in a storage ring,  $d \gg 1$  is typically the case. However,

care must be taken in using this formula for a storage ring with a large number of bunches, each occupying a significant fraction of the RF bucket.

While the bunch length  $L$  is precisely defined for parabolic bunches, it is not so for Gaussian bunches. ZAP always requests the rms bunch length,  $\sigma_\ell$ , as input. For parabolic bunches this value is interpreted to correspond to a total bunch length of  $L = 2\sqrt{2} \sigma_\ell$ , so that " $\sigma_\ell$ " actually refers to the half-length at half-height. For Gaussian bunches,  $\sigma_\ell$  has the obvious meaning of rms bunch length, which is then interpreted to correspond to a total bunch length of  $L = 2\sqrt{\pi} \sigma_\ell$ .

The collective coupled-bunch instabilities are counteracted by Landau damping from the synchrotron frequency spread within the bunch. A spread in synchrotron frequency comes from the nonlinearity of the RF bucket. Landau damping is obtained if the shifted mode frequency lies inside the effective spread of the bunch.

The natural synchrotron frequency spread,  $S$ , in a bunch characterized by an rms length  $\sigma_\ell$  and confined by a nonlinear sinusoidal RF bucket is given (in the octupolar approximation) for small bunches by:<sup>31,36</sup>

$$S = \frac{1}{8} \left( \frac{\sigma_\ell}{R} \right)^2 h^2 \omega_{s0} \quad (\text{IV.5.13})$$

Such a spread will provide some amount of Landau damping for the growing modes. Whether a given mode  $(s, a)$  is Landau damped, unstable even with Landau damping, or stable even without any Landau damping is decided by ZAP based on the following criteria:<sup>9,10,31</sup>

$$\text{Im} \left( \Delta\omega_{s,a}^{\parallel} \right) \leq 0, \quad \text{Stable without Landau damping, "S"} \quad (\text{IV.5.14})$$

$$\text{Im} \left( \Delta\omega_{s,a}^{\parallel} \right) > 0 \quad \text{and} \quad \left| \Delta\omega_{s,a}^{\parallel} \right| \leq \frac{a}{2(a+1)} S, \quad \text{Landau damped, "D"} \quad (\text{IV.5.15})$$

$$\text{Im} \left( \Delta\omega_{s,a}^{\parallel} \right) > 0 \quad \text{and} \quad \left| \Delta\omega_{s,a}^{\parallel} \right| > \frac{a}{2(a+1)} S, \quad \text{Landau unstable, "U" .} \quad (\text{IV.5.16})$$

### Wang Formalism

A more direct approach to the coupled-bunch instability problem, valid for very short bunches and making no use of any particular eigenmode expansion, has been provided by Wang.<sup>36</sup> The "small bunch" approximation in the Wang formalism has to be qualified, since it is dependent on the frequencies of the coherent modes under consideration. The assumption is basically that the bunch be short enough so that

$$\tau_L \equiv \frac{L}{2\beta c} \lesssim \frac{a}{\omega_c} \quad (\text{IV.5.17})$$

where  $\tau_L$  is the maximum phase-oscillation amplitude of particles (in units of time carried along a synchrotron orbit), i.e., the amplitude that bounds the beam in longitudinal synchrotron phase space;  $\omega_c$  is the cutoff frequency beyond which the impedance rolls off very fast; and  $a$  is the synchrotron mode number ( $a = 1, 2, 3, \dots$  etc.) under consideration. The analysis is then valid for short bunches, with angular bunch cutoff frequency  $\omega_B = (\tau_L)^{-1}$  satisfying:

$$\omega_B \gtrsim \frac{\omega_c}{a} . \quad (\text{IV.5.18})$$

Obviously, if this inequality is satisfied by the dipole synchrotron mode ( $a = 1$ ,  $\omega_B \gtrsim \omega_c$ ), it will be satisfied by all higher order synchrotron modes also (but not vice versa).

For bunches filling a very small fraction of the RF bucket, as in electron storage rings, the bunch cutoff frequency is very high. Assuming the ring impedance is

negligible at this high frequency due to pipe cutoff or otherwise, the Wang formalism is a pretty good approximation for electron rings. For proton storage rings, however, the bunches are typically long (filling a third of the RF bucket, for example) and the beam pipe cutoff frequency  $\omega_c$  may be higher than  $\omega_B$ :  $\omega_B \ll \omega_c$ . The approximation of small bunches may thus fail grossly in these circumstances and care must be taken in using the Wang formalism. Within its regime of validity, however, the Wang formalism is more direct and provides a rigorous treatment of Landau damping. We provide the expressions from the Wang formalism below. Note that, if the parabolic bunch shape is chosen (as would usually be the case for protons or heavy ions), only the Sacherer-Zotter formalism can be used in ZAP.

The complex coherent frequency shift  $\Delta\omega_{s,a}^{\parallel}$  for small Gaussian bunches is given, in the Wang formalism, by:<sup>36</sup>

$$\left(\Delta\omega_{s,a}^{\parallel}\right) = i \frac{q_i I_b \omega_0^2 n k_b}{2\pi\beta^2 (E_T/e)\omega_s} \cdot \frac{(\sigma_\ell/R)^{2(a-1)}}{2^a (a-1)!} \left(Z_{\parallel}\right)_{\text{eff}}^{s,a} \quad (\text{IV.5.19})$$

where

$$\left(Z_{\parallel}\right)_{\text{eff}}^{s,a} = \sum_{p=-\infty}^{+\infty} (pk_B + s)^{2a} e^{-(pk_B + s)^2 (\sigma_\ell/R)^2} \left[ \frac{Z_{\parallel}(v_p \omega_0)}{v_p} \right] \quad (\text{IV.5.20})$$

$$v_p = \left( pk_b + s + av_s \right) . \quad (\text{IV.5.21})$$

This  $\left(Z_{\parallel}\right)_{\text{eff}}^{s,a}$  can be rewritten in terms of  $\left[Z_{\parallel}/n\right]_{\text{eff}}^{s,a}$  (as defined in Eq. (IV.5.2)) with  $h_a(\omega)$  for Gaussian bunches given by Eq. (IV.5.4)) as follows:

$$\left(Z_{\parallel}\right)_{\text{eff}}^{s,a} = \frac{d\Gamma\left(a + \frac{1}{2}\right)}{(\sigma_\ell/R)^{2a}} \left[ \frac{Z_{\parallel}}{n} \right]_{\text{eff}}^{s,a} \quad (\text{IV.5.22})$$

where  $d$  is given by Eq. (IV.5.10) for Gaussian bunches. [The factor  $d\Gamma(a+1/2)$  in the above expression arises from summing the Gaussian mode spectrum, Eq. (IV.5.4), over  $p$   $\left(\sum_{p=-\infty}^{+\infty} h_a(\omega_p)\right)$ .] Thus, for resonator type impedances, there exists a very good approximate analytic summation expression for the Wang formula (IV.5.19), similar to Eq. (IV.5.7).

With a Gaussian distribution in longitudinal phase space and the amplitude-dependent synchrotron frequency for small bunches in sinusoidal RF buckets taken to be that given in the octupolar approximation, i.e.,

$$\omega_s(r) = \omega_{s0} \left(1 - \frac{h^2}{16} r^2\right) \quad (\text{IV.5.23})$$

where  $r$  is the amplitude of synchrotron oscillations in actual angle around the ring, the Landau damping effect can be calculated by solving the Dispersion Relation:<sup>36</sup>

$$1 = \frac{(\Delta\omega_{s,a}^{\parallel})}{0.32 a(a!)S} \left[G_a(\omega)\right]^{-1} \quad (\text{IV.5.24})$$

where

$$\left[G_a(\omega)\right]^{-1} = \int_0^{\infty} dx \frac{x^a e^{-x}}{(x-y)} \quad (\text{IV.5.25})$$

$$y = \frac{8}{a\omega_{s0} h^2 (\sigma_{\ell}/R)^2} (a\omega_{s0} - \omega) . \quad (\text{IV.5.26})$$

In Eq. (IV.5.24) we use a "total" frequency spread,  $S$ , defined as the spread at  $r = 2.5(\sigma_{\ell}/R)$ , so that

$$S = \frac{\hbar^2}{16} \omega_{s0} \left( 2.5 \frac{\sigma_l}{R} \right)^2 . \quad (\text{IV.5.27})$$

The threshold curve of the instability is given by the curve traced out by  $G_a(\omega)$  in the complex plane in the limit  $\gamma = \text{Im}(\omega) \rightarrow 0^+$  and  $\omega_r = \text{Re}(\omega)$  ranging from  $-\infty$  to  $+\infty$ . This curve is often referred to as the "stability boundary." The stability boundaries of the first two synchrotron modes,  $a = 1$  and  $2$ , are shown in Fig. 7. The threshold for Landau stability corresponds to the minimum value  $F_a$  of  $|G_a(\omega)|$  along this curve:

$$F_a = \text{Min} \left| G_a(\omega) \right| \quad \begin{array}{l} \omega = \omega_r + i\gamma \\ \gamma \rightarrow 0^+ \end{array} \quad (\text{IV.5.28})$$

In terms of the coherent frequency shift  $\Delta\omega_{s,a}^{\parallel}$  given by (IV.5.19), the Landau stability condition can be written as:

$$\text{Im} \left( \Delta\omega_{s,a}^{\parallel} \right) \leq 0, \quad \text{Stable without frequency spread, "S"} \quad (\text{IV.5.29})$$

$$\text{Im} \left( \Delta\omega_{s,a}^{\parallel} \right) > 0 \quad \text{and} \quad \left| \Delta\omega_{s,a}^{\parallel} \right| \leq 0.32(aS)F_a', \quad \text{Landau damped, "D"} \quad (\text{IV.5.30})$$

$$\text{Im} \left( \Delta\omega_{s,a}^{\parallel} \right) > 0 \quad \text{and} \quad \left| \Delta\omega_{s,a}^{\parallel} \right| > 0.32(aS)F_a', \quad \text{Landau unstable, "U"} . \quad (\text{IV.5.31})$$

The values of  $F_a' = (a!)F_a$  for the first four synchrotron modes have been calculated and are given in Table 7.



### Choice of Bunch Shape

In ZAP, one can use either a parabolic or a Gaussian bunch shape. For protons, the default bunch shape is parabolic, whereas for electrons the default bunch shape is Gaussian. As mentioned, for the parabolic bunch shape, only the Zotter formalism can be used. The actual evaluation of  $(Z_{\parallel}/n)_{\text{eff}}^{S,a}$  can be done either analytically or via explicitly performing the summation in Eq. (IV.5.2). If the analytic method is chosen, numerical summation is still used for the resistive-wall impedance contribution (if a wall material is chosen), because there is no analytic expression for this summation. If the Gaussian bunch shape is selected, the user has the option of using either the Zotter formalism or the Wang formalism for calculating the coupled-bunch longitudinal instability. In both cases the user has the option of choosing either the analytic expression or the (brute force) explicit summation method for calculating  $(Z_{\parallel}/n)_{\text{eff}}^{S,a}$  for resonator type impedances (e.g., the RF cavity modes and the  $Q = 1$  broadband resonator). Again, the resistive wall contribution, if present, is always calculated from an explicit summation.

### General Notes

The inputs required for Main Menu option 3 (and 4) include the storage ring circumference, energy of the beam (negative input required for protons and heavy ions), momentum compaction, rms bunch length, rms momentum spread, beam pipe radius, number of particles per bunch, RF frequency and impedance inputs such as cavity parasitic mode parameters,  $(Z_{\parallel}/n)$  value for  $Q = 1$  broadband impedance, resistive wall choice, etc. The synchrotron tune used in the calculation is derived from the input rms length  $\sigma_{\ell}$  and momentum spread  $\sigma_p$ , as

$$\nu_s = |n| R \frac{\sigma_p}{\sigma_{\ell}} .$$

The code then calculates the real coherent frequency shifts and growth rates, using the appropriate formulae given in this section (depending on parabolic or Gaussian bunch shape, Zotter or Wang formalism, analytic or explicit summation method, etc.). The code is presently limited to calculating any, or all, of the four lowest synchrotron modes ( $a = 1-4$ ). For each of these synchrotron modes, the complex frequency shifts are sorted on the fastest growth rates and also on the largest real frequency shifts. The number of values that can be sorted is limited to 1000; generally it is of interest to look at a much smaller number of values, say 5 or fewer.

Appropriate (Wang or Zotter) Landau damping calculations are also performed; the results are provided as symbols next to the growth rates of the corresponding mode: S (Stable); D (Landau damped); or U (Landau unstable).

For extreme values of input parameters, the explicit summation method may suffer from convergence problems and the analytic formulae may suffer from numerical problems (e.g., in using the analytic expression for the broadband  $Q = 1$  resonator impedance for Gaussian bunches). This latter problem is difficult to pin down exactly. An attempt has been made in the code to flag any numerical problems associated with using the analytic formulae. Especially if a warning message appears, it is safest to compare both the analytic and summation results in order to ensure the absence of numerical problems.

For a large number of RF cells, attention must be given to the de-Qing criterion, as described in Section IV.1. (Experience establishes this procedure to be a good practice.) However, to get some idea about the enhancement of growth rates due to accidental tuning of one of the strong RF resonances exactly on one of the beam spectral lines, ZAP provides the option of using a single dominant RF resonance and placing its resonance frequency exactly at the frequency of the nearest beam spectral line. This can be accomplished by using a negative number for the resonance frequency of the single resonator; only one resonator may be input for this purpose. The growth

rate obtained in this manner provides an estimate of the maximum enhancement factor due to this accidental tuning.

#### IV.6 Transverse Symmetric Coupled-Bunch Instability

For the transverse case, we consider only the transverse dipole motion. As in the longitudinal case, two mode numbers,  $s$  and  $a$ , are again needed to describe a transverse coupled-bunch dipole mode. One difference in the transverse case is that the index  $a$  can assume the value  $a = 0$ , meaning that the bunches move rigidly as they execute the transverse oscillations; we call this mode the transverse rigid dipole mode. The mode  $a = 1$  implies the bunch head and tail oscillate transversely out of phase. The unperturbed modes, when observed in the laboratory at a fixed location in the storage ring, will have frequencies given by  $\omega_p^\perp = (pk_b + s + v_\perp + av_s)\omega_0$ , with  $p = 0, \pm 1, \pm 2, \dots, \text{etc.}$  In the beam frame, the frequency of mode  $(s, a)$  is simply  $(v_\perp + av_s)\omega_0$ . The perturbed frequency of coherent oscillation is modified from the unperturbed value by a complex coherent frequency shift  $\Delta\omega_{s,a}^\perp$ , caused by the ring transverse impedance  $Z_\perp(\omega_p^\perp)$ .

#### Sacherer-Zotter Formalism

The coherent frequency shift is given, in the Sacherer-Zotter<sup>2,9,10</sup> formalism, by:

$$\Delta\omega_{s,a}^\perp = -i \frac{q_i I_b \beta c^2}{2\omega_\beta (E_T/e)L} \cdot \frac{1}{(a+1)} \cdot [Z_\perp]_{\text{eff}}^{s,a} \quad (\text{IV.6.1})$$

where

$$[Z_\perp]_{\text{eff}}^{s,a} = \frac{\sum_{p=-\infty}^{+\infty} Z_\perp(\omega_p^\perp) h_a(\omega_p^\perp - \omega_\xi^\perp)}{\sum_{p=-\infty}^{+\infty} h_a(\omega_p^\perp - \omega_\xi^\perp)}, \quad (\text{IV.6.2})$$

with

$$\omega_{\xi} = \frac{\xi}{n} \omega_0$$

and

$$\xi \equiv dv_{\perp} / (dp/p)$$

$$\omega_{\beta} = v_{\perp} \omega_0 \cdot$$

Here  $h_a(\omega)$  is the same bunch mode spectrum defined for the longitudinal case, except that it is shifted by the chromatic frequency  $\omega_{\xi}$ .

Again, for arbitrary impedances, one has to perform an explicit summation in Eq. (IV.6.2) (with some convergence criterion) in order to evaluate  $\Delta\omega_{S,a}^{\perp}$  in Eq. (IV.6.1). However, for resonator type impedances of the form given by Eq. (IV.1.4), an exact analytic expression for the summation in Eq. (IV.6.2) exists for the case of a sinusoidal mode bunch spectrum, as in Eq. (IV.5.3), and a very good approximate analytic expression for the summation exists for the case of Hermitian modes, as in Eq. (IV.5.4). [In this latter case, the expression is valid within the same type of approximation as for the longitudinal case. In fact, we note that  $Z_{\perp}(\omega)$ , as given by Eq. (IV.1.4), can be rewritten in terms of  $[Z_{\parallel}/n]$ , with  $Z_{\parallel}(\omega)$  given by Eq. (IV.1.3), in the following way:

$$Z_{\perp}(\omega) = \left[ \frac{Z_{\parallel}(\omega)}{n} \right]_{R_S} = R_T \cdot \frac{\omega_R}{\omega_0} \cdot \quad (\text{IV.6.3})$$

Thus, the analytic expressions for the summations in the transverse case become trivially analogous to those for the longitudinal case.] They are given by the same expressions as in Eqs. (IV.5.5) to (IV.5.12) with the following few redefinitions:<sup>31,34,35</sup>

$$A = (s + av_s + v_\perp) / k_b \quad (\text{IV.6.4})$$

$$c = A - \frac{\omega_\xi}{k_b \omega_0} \quad (\text{IV.6.5})$$

$$p_{1,2} = -A \pm u - iv. \quad (\text{IV.6.6})$$

### Wang Formalism

In the Wang formalism, the complex coherent frequency shift  $\Delta\omega_{S,a}^\perp$  for small Gaussian bunches is given by:<sup>37</sup>

$$\Delta\omega_{S,a}^\perp = -i \frac{q_i I_b k_b c}{4\pi(E_T/e)v_\perp} \frac{(\sigma_\ell/R)^{2a}}{2^a a!} [Z_\perp]_{\text{eff}}^{s,a} \quad (\text{IV.6.7})$$

where

$$[Z_\perp]_{\text{eff}}^{s,a} = \sum_{p=-\infty}^{+\infty} \left( pk_b + s + v_\perp - \frac{\omega_\xi}{\eta} \right)^{2a} e^{-\left( pk_b + s + v_\perp - \frac{\omega_\xi}{\eta} \right)^2 (\sigma_\ell/R)^2} Z_\perp \left( \frac{\omega_\xi}{v_p \omega_0} \right) \quad (\text{IV.6.8})$$

and

$$v_p^\perp = pk_b + s + v_\perp + av_s. \quad (\text{IV.6.9})$$

As in the longitudinal case, the summation in Eq. (IV.6.8) is directly related to (indeed essentially the same, except for constant factors, as) the one in Eq. (IV.6.2) when using the Hermitian modes for  $h_a(\omega_p^\perp - a\omega_s - \omega_\xi)$ . Thus, an excellent analytic approximation to Eq. (IV.6.8) and hence Eq. (IV.6.7) exists and follows analogously to that for the Zotter formalism.

### Landau Damping

In the Zotter formalism, Landau damping for modes with  $a > 0$  (arising solely from the synchrotron frequency spread  $S$ , with no assumed betatron tune spread) is

calculated exactly analogously to the longitudinal case, with  $|\Delta\omega_{s,a}^\perp|$  replacing  $|\Delta\omega_{s,a}^\parallel|$  in Eqs. (IV.5.14) to (IV.5.16).

With an octupolar amplitude-dependent synchrotron frequency as in Eq. (IV.5.23), Landau damping in the Wang formalism, for modes with  $a > 0$  (arising solely from the synchrotron frequency spread with no assumed betatron tune spread) is calculated exactly analogously to the longitudinal case, with  $|\Delta\omega_{s,a}^\perp|$  replacing  $|\Delta\omega_{s,a}^\parallel|$  in Eqs. (IV.5.30)-(IV.5.32).<sup>37</sup>

Synchrotron frequency spread alone does not provide any Landau damping for the  $a = 0$  rigid dipole transverse coupled-bunch modes. In order to obtain Landau damping for the  $a = 0$  modes, some amount of betatron tune spread (due to nonlinearity of transverse motion) is needed. Taking a Gaussian distribution of particles in the transverse betatron phase-space variables and an amplitude-dependent nonlinear betatron tune given by

$$v_\perp(I) = v_0 - \frac{\sigma_v}{2} \left( \frac{I}{I_0} \right) = v_0 - \frac{\sigma_v}{2} \left( \frac{A}{A_0} \right)^2. \quad (\text{IV.6.10})$$

Landau damping can be calculated by solving the following dispersion relation for rigid dipole transverse modes in the Wang formalism:<sup>37,38</sup>

$$1 = \frac{\Delta\omega_{s,a=0}^\perp}{\sigma_v \omega_0} G^{-1}(\omega) \quad (\text{IV.6.11})$$

where

$$G^{-1}(\omega) = \int_0^\infty dx \frac{x e^{-x}}{[x-y]} \quad (\text{IV.6.12})$$

and

$$y = \left( \frac{v_0 \omega_0 - \omega}{\sigma_v \omega_0} \right). \quad (\text{IV.6.13})$$

The integral  $G^{-1}(\omega)$  in Eq. (IV.6.12) is the same as the integral  $[G_a(\omega)]^{-1}$  with  $a = 1$  in Eq. (IV.5.25). The stability boundary is given by the curve traced out by  $G(\omega)$  in the complex plane for  $\omega = \omega_r + i\gamma$  with  $\gamma \rightarrow 0^+$  and  $\omega_r$  ranging from  $-\infty$  to  $+\infty$ . Landau stability is determined by the minimum value of  $|G(\omega)|$  along this curve, which we already know from Table 7 to be  $F_1 = 0.77760$ .

The Landau damping condition for the  $a = 0$  transverse rigid dipole mode with nonlinear betatron tune spread characterized by  $\sigma_y$  in Eq. (IV.6.10), is then given in the Wang formalism by

$$|\Delta\omega_{s,a=0}^\perp| \leq 0.77760 (\sigma_y \omega_0) \quad \text{Landau damped, "D"} \quad (\text{IV.6.14})$$

$$|\Delta\omega_{s,a=0}^\perp| > 0.77760 (\sigma_y \omega_0) \quad \text{Landau Unstable, "U"} \quad (\text{IV.6.15})$$

For this calculation, the rms betatron tune spread  $\sigma_y$  is a user-supplied input to the code. There is no option in the present version of the code to calculate the Landau damping condition for the  $a = 0$  transverse rigid dipole mode with nonlinear betatron tune spread in the Zotter formalism.

### General Notes

As in the longitudinal case, ZAP allows a variety of options for the transverse calculation: Wang or Zotter, parabolic or Gaussian shape, analytic or explicit summation, etc. The betatron tunes and chromaticity are the additional inputs required for transverse calculations. The fastest growth rates and largest real frequency shifts are sorted for up to four values of  $a$ , that is,  $a = 0, 1, 2, 3$ . Landau damping results are again given by symbols S, D and U in the output. Possible numerical and convergence problems are flagged whenever possible.

## Laslett Tune Shift

Main Menu option 4 also calculates the incoherent and coherent Laslett betatron tune shifts, in addition to transverse coupled-bunch modes. The incoherent tune-shift is calculated from the formula:<sup>19,31</sup>

$$\Delta\nu_i = - \frac{N_b r_0 R^2}{(\beta\gamma)^2 \gamma L \nu_y \bar{\sigma}_y (\bar{\sigma}_x + \bar{\sigma}_y)} F_i \quad (\text{IV.6.16})$$

where

$$F_i = F_i^{loc} + F_i^{DC} \quad (\text{IV.6.17a})$$

$$F_i^{loc} = \left(1 - \gamma^2 \eta_e\right) + \frac{2\bar{\sigma}_y (\bar{\sigma}_x + \bar{\sigma}_y)}{b^2} (1 - \eta_e) \epsilon_1 \quad (\text{IV.6.17b})$$

$$F_i^{DC} = B \cdot \frac{2\bar{\sigma}_y (\bar{\sigma}_x + \bar{\sigma}_y)}{b^2} (\beta\gamma)^2 \left[ (1 - \eta_e) \epsilon_1 + \frac{\epsilon_2}{(V/2b)^2} \right] \quad (\text{IV.6.17c})$$

$$B = \left(k_b L / 2\pi R\right) \quad (\text{IV.6.17d})$$

$$\bar{\sigma}_x = \sqrt{\epsilon_x \bar{\beta}_x} ; \quad \bar{\sigma}_y = \sqrt{\epsilon_y \bar{\beta}_y} \quad (\text{IV.6.17e})$$

$$\epsilon_1 = -0.156 (2b/W)^2 + 0.21 \quad (\text{IV.6.17f})$$

$$\epsilon_2 = 0.41 (\rho/R) . \quad (\text{IV.6.17g})$$

In the code, the distance between the pole pieces,  $V$ , is taken to be 20% larger than the chamber height  $2b$ , so that  $(V/2b)^2 = 1.44$ . Since the above formula is strictly valid for a rectangular pipe of height  $2b$  and width  $W$  and for  $2b/W \leq 0.7$ , we have

taken  $2b/W = 0.7$  in the calculations. The ratio of the bending radius  $\rho$  to the average radius  $R$  is taken to be 0.5, typical of small electron storage rings. The code calculates the shift for the two extreme cases of: (a) no neutralization by oppositely charged ions from the background gas and walls,  $\eta_e = 0$ ; and (b) full neutralization, i.e., with the neutralization factor  $\eta_e = B$ , the bunching factor. The  $F_i^{loc}$  and  $F_i^{DC}$  correspond to the "local" contribution (independent of the bunching factor  $B$ ) and the "DC" contribution (proportional to  $B$ ).

The coherent Laslett tune-shift is calculated from the formula:<sup>19,31</sup>

$$\Delta\nu_c = - \frac{2N_b R^2 r_0}{(\beta\gamma)^2 \gamma L v_y b^2} F_c \quad (\text{IV.6.18})$$

where

$$F_c = F_c^{loc} + F_c^{DC} \quad (\text{IV.6.19a})$$

$$F_c^{loc} = \xi_1 (1 - \eta_e) \quad (\text{IV.6.19b})$$

$$F_c^{DC} = B(\beta\gamma)^2 \left[ \epsilon_1 (1 - \eta_e) + \frac{\epsilon_2}{(V/2b)^2} \right] \quad (\text{IV.6.19c})$$

$$\xi_1 = -0.10(2b/W)^2 + 0.617 \quad (\text{IV.6.19d})$$

The same parameter choices are made here as for the incoherent case (i.e.,  $\epsilon_1$ ,  $\epsilon_2$ ,  $2b/W$  and  $V/2b$ ).

The Laslett incoherent and coherent tune shifts are printed out in the transverse coupled-bunch output if nonzero emittance values are supplied in the input for space-charge calculations.

## IV.7 Gas Scattering

The beam chamber in a storage ring does not have a perfect vacuum and the particles in the beam can scatter off the atoms of the residual gas. There are essentially two dominant processes in electron beam-gas interactions: the elastic scattering on nuclei and the Bremsstrahlung on nuclei.

Elastic scattering on nuclei of the residual gas leads to an angular kick for the betatron motion. If the induced amplitude exceeds the transverse acceptance  $A_{\perp}$  of the ring, the particle gets lost. The total cross section for this process is:<sup>39</sup>

$$\sigma_{el} = \frac{2\pi r_e^2 Z^2}{\gamma^2} \cdot \frac{\langle \beta_{\perp} \rangle}{A_{\perp}} \quad (\text{IV.7.1})$$

where  $\perp$  means either the horizontal or vertical transverse dimension, whichever is more limiting. For non-smooth storage ring optics, as in strong focusing machines, if we assume that the aperture limit corresponds to a half-chamber aperture of  $b$  located at some azimuth where  $\beta_{\perp} = \beta_0$ , then  $A_{\perp} = b^2/\beta_0$  and is essentially the invariant emittance corresponding to betatron amplitude  $b$  at  $\beta_{\perp} = \beta_0$ . Instead of arising from an actual limiting physical aperture,  $b$  could also arise from dynamical aperture considerations. In any case,  $A_{\perp}$ , the limiting ring betatron acceptance, is a user-supplied input to the code.

Bremsstrahlung on nuclei of the residual gas is an inelastic scattering process leading to an energy loss for the circulating electron. The electron gets lost if its relative momentum deviation  $(\Delta p/p)$  exceeds the limiting longitudinal momentum half-aperture  $(\Delta p/p)_{lim}$  of the ring. As explained in Section IV.10 under Touschek scattering, this limiting momentum half-aperture could be due to the RF bucket momentum half-height or to physical or dynamical aperture considerations. The required acceptance value is a user-supplied input to the code. The total cross section

for the inelastic process is:<sup>39</sup>

$$\sigma_{Br} = \frac{4r_e^2 Z^2}{137} \cdot \frac{4}{3} \cdot \left[ \ln \left( \frac{183}{Z^{1/3}} \right) \right] \left[ \ln \left( \frac{1}{\left( \frac{\Delta p}{p} \right)_{lim}} \right) - \frac{5}{8} \right] \quad (IV.7.2)$$

where  $r_e$  is the classical electron radius, and  $Z$  is the atomic number of the residual gas species. Note that this cross section is practically independent of the particle energy.

The above two effects lead to particle losses and hence reduce the beam lifetime. The gas-scattering lifetime,  $\tau_g$ , is the inverse of the relative rate of loss given by:<sup>39</sup>

$$\tau_g^{-1} = - \frac{1}{N_b} \frac{dN_b}{dt} = \sigma_t \cdot \beta c \cdot n_g \quad (IV.7.3)$$

where

$$\sigma_t = \sigma_{el} + \sigma_{Br} \quad (IV.7.4)$$

and  $n_g$ , the number density of the residual gas atoms, is related to the gas pressure  $P$  and temperature  $T$  via the ideal gas law:

$$n_g = \frac{n_Z}{R_g} \cdot \left( \frac{P}{T} \right) = n_Z G (P/T) . \quad (IV.7.5)$$

Here  $n_Z$  is the number of atoms per gas molecule and  $G = 9.656 \times 10^{24}$  is the inverse of the ideal gas constant,  $R_g$ . For a monatomic gas ( $n_Z = 1$ ) at "room temperature" (taken here to be  $T = 300$  °K), Eq. (IV.7.5) reduces to

$$n_g \text{ [m}^{-3}\text{]} = 3.22 \times 10^{22} P \text{ [Torr]} \cdot \quad (\text{IV.7.6})$$

A negative input for the pressure allows calculation of  $n_g$  at a temperature different from the default value of 300 °K.

The elastic scattering determines the lifetime at low energies, while the Bremsstrahlung becomes dominant at high energies. However, for storage rings with quite small apertures, the elastic scattering (which depends strongly on the aperture as well), may compete with the Bremsstrahlung at high energies too.

For electron storage rings, there are two other processes that can occur:<sup>39</sup> the elastic and inelastic (involving photon emission) scattering of the incident high energy electrons from the electrons of the residual gas. In the present version of ZAP, however, these two processes are neglected.

We note that Eq. (IV.7.1) assumes that the limitation is only in one transverse plane, say the vertical. If horizontal and vertical acceptances are essentially the same, the loss cross section doubles and the corresponding lifetime  $\tau_{e1}$  from elastic scattering alone decreases by a factor of two. Since, for most cases, the horizontal and vertical acceptances are fairly similar, the code assumes this extra factor of two, unless the acceptance is input as a negative value, in which case the code uses Eq. (IV.7.1) exactly, corresponding to limitation in one plane only. If the input for the acceptance is a negative value, a message is printed in the output (see Section III).

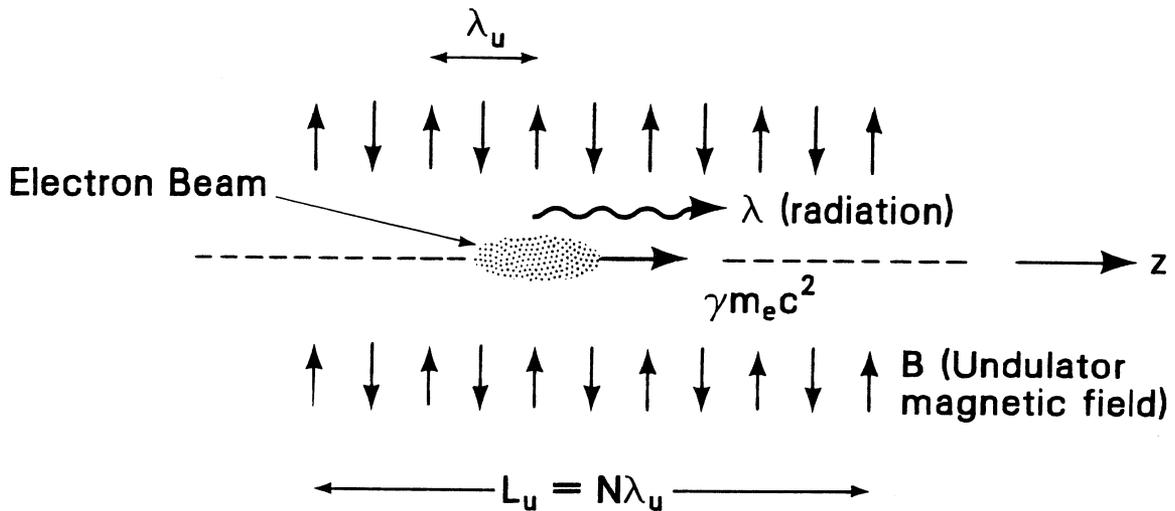
Main Menu option 5 calculates these gas scattering effects and prints out the rates  $\tau_{e1}^{-1}$ ,  $\tau_{Br}^{-1}$  and  $\tau_g^{-1} = \tau_{e1}^{-1} + \tau_{Br}^{-1}$  and the corresponding lifetimes  $\tau_{e1}$ ,  $\tau_{Br}$  and  $\tau_g$  in the output. Note that, in contrast to the Touschek lifetimes, which are half-life values, the gas scattering rates and lifetimes are e-folding times. To get the overall beam half-life from gas and Touschek scattering, the rates should be combined as:

$$\frac{1}{\tau_{1/2}} = \frac{\ln 2}{\tau_g} + \frac{1}{\tau_{\text{Touschek}}} \quad (\text{IV.7.7})$$

#### IV.8 High-Gain Free Electron Laser Calculations

A typical Free Electron Laser (FEL) setup is shown schematically in Fig. 8. An electron beam of energy  $\gamma m_e c^2$  traveling through a magnetic undulator of period  $\lambda_u$  can interchange energy with a transverse laser field of wavelength  $\lambda$  propagating in the same direction.<sup>40,41</sup> The interaction becomes resonant when the following condition is satisfied:

$$\lambda = \frac{\left(1 + \frac{k^2}{2}\right)}{2\gamma^2} \lambda_u \quad (\text{IV.8.1})$$



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Fig. 8. A typical Free Electron Laser setup.

where  $K$  is the deflection parameter, characteristic of the undulator, given by

$$K = \frac{e\lambda_u B}{2\pi m_e c} = 0.934 \lambda_{u[\text{cm}]} B_{[\text{tesla}]} \quad (\text{IV.8.2})$$

An important quantity, which determines the FEL characteristics in the one-dimensional theory, is the dimensionless parameter  $\rho$ , given by<sup>42</sup>

$$\rho = \left( \frac{K^2 [JJ] r_e n_b \lambda_u^2}{32\pi\gamma^3} \right)^{1/3} \quad (\text{IV.8.3})$$

where

$$[JJ] = \left[ J_0(\xi) - J_1(\xi) \right]^2, \quad \xi = \frac{K^2}{4 \left( 1 + \frac{K^2}{2} \right)}. \quad (\text{IV.8.4})$$

The  $J_0$  and  $J_1$  are ordinary Bessel functions of order zero and one, respectively. The quantity  $\rho$  is known as the FEL gain parameter and, for small, high-brightness electron storage rings,<sup>18</sup> is of the order of  $10^{-3}$ .

The characteristics of the power growth of the laser wave, as obtained from the one-dimensional theory, fall into three distinct regimes: the small-signal regime, the exponential-growth regime and the saturation regime. Near the entrance of the undulator, where the small-signal theory applies, the gain  $G$  (defined as the ratio of the laser power at two points separated by a distance  $z$  along the undulator axis) is given by

$$G = 536 \left( \rho z / \lambda_u \right)^3. \quad (\text{IV.8.5})$$

Farther along the undulator, the laser power  $P$  grows exponentially<sup>42-48</sup> with distance from the initial power (in the corresponding polarization state), at a rate proportional to  $\rho$ , according to

$$P = P_0 e^{gZ} \quad (\text{IV.8.6})$$

where

$$g = 4\pi \sqrt{3} (\rho/\lambda_u) . \quad (\text{IV.8.7})$$

The corresponding e-folding length for the growth of the radiation power in the exponential growth regime is

$$\lambda_e = g^{-1} = \frac{1}{4\pi \sqrt{3}} \left( \frac{\lambda_u}{\rho} \right) . \quad (\text{IV.8.8})$$

Eventually, the electrons are captured in the ponderomotive potential well and the growth of radiation power stops. The laser saturates at a distance  $z = z_{\text{sat}}$  with a characteristic saturated peak power  $P_{\text{sat}}$ . The saturation length and the peak power are given approximately, in the one-dimensional theory with zero energy spread, by<sup>42</sup>

$$z_{\text{sat}} \approx (\lambda_u/\rho) \quad (\text{IV.8.9})$$

$$P_{\text{sat}} \approx \rho P_{\text{beam}} \quad (\text{IV.8.10})$$

where

$$P_{\text{beam}} = I_p (E/e) \quad (\text{IV.8.11})$$

is the peak power in the electron beam. The number of undulator periods in the saturation length  $z_{\text{sat}}$  is given by

$$N = (z_{\text{sat}}/\lambda_u) \approx 1/\rho . \quad (\text{IV.8.12})$$

We can relate the parameter  $\rho$  to the electron beam and storage ring characteristics. The volume number density of the bunched electron beam is

$$n_b = \frac{I_p}{ec \, 2\pi \, \sigma_x \, \sigma_y} . \quad (\text{IV.8.13})$$

The rms beam sizes  $\sigma_{x,y}$  can be expressed in terms of the emittances  $\epsilon_{x,y}$  and the amplitude functions  $\beta_{x,y}$ :

$$\sigma_{x,y} = \sqrt{\epsilon_{x,y} \beta_{x,y}} . \quad (\text{IV.8.14})$$

The alternating field of an undulator provides an effective focusing force. For a planar undulator, the focusing is in the vertical direction, with an effective  $\beta$ -function given by<sup>49</sup>

$$\beta_y = \frac{\lambda_u \gamma}{\sqrt{2} K \pi} . \quad (\text{IV.8.15})$$

Horizontal focusing can also be provided by tilting or by shaping the pole surfaces of the undulator.<sup>50</sup> In either case the focusing strength in the vertical plane is thereby reduced. For the purpose of conceptual simplicity, we assume a focusing force of the same magnitude in both directions:

$$\beta_x = \beta_y = \frac{\lambda_u \gamma}{K \pi} . \quad (\text{IV.8.16})$$

By combining Eqs. (IV.8.1), (IV.8.13), (IV.8.14), and (IV.8.16), the FEL parameter given by Eq. (IV.8.3) can be rewritten:

$$\rho^3 = \frac{1}{16\pi} \cdot \frac{r_e}{ec} \cdot \frac{K^3 [JJ]}{2 \left(1 + \frac{K^2}{2}\right)} \cdot \frac{\lambda}{\gamma^2} \cdot \frac{I_p}{\sqrt{\epsilon_x \epsilon_y}} \quad (\text{IV.8.17})$$

The above results apply to the one-dimensional theory with zero energy spread. If a finite non-zero energy spread of the beam is taken into account, the growth rate and the saturation power are modified as follows:<sup>51</sup>

$$g \rightarrow g' = f(\sigma_p, \rho) g \quad (\text{IV.8.18})$$

$$P_{\text{sat}} \rightarrow P'_{\text{sat}} = h(\sigma_p, \rho) P_{\text{sat}} \quad (\text{IV.8.19})$$

The  $f(\sigma_p, \rho)$  and  $h(\sigma_p, \rho)$  are functions of the electron relative momentum spread,  $\sigma_p = (\Delta p/p)_{\text{rms}}$ , and  $\rho$ , and describe form factors that, in general, decrease with increasing values of  $(\sigma_p/\rho)$ . If  $\sigma_p$  is non-zero, the FEL performance is significantly reduced, unless the following condition is satisfied:<sup>52</sup>

$$\sigma_p \lesssim (1/N) \sim \rho \quad (\text{IV.8.20})$$

If the momentum spread of the electron beam is large and the inequality in Eq. (IV.8.20) is violated, both functions  $f$  and  $h$  can become considerably less than unity, thus degrading the FEL performance.

The degradation factors can be obtained by solving, for a quantity  $\mu$ , a Dispersion Relation of the form:<sup>53</sup>

$$D(\mu) \equiv \mu + \frac{\Delta\nu}{2\rho} + \int_{-\infty}^{+\infty} d\delta \frac{\psi'(\delta)}{[\mu+\delta]} = 0 \quad (\text{IV.8.21})$$

where the energy deviation variable,  $\delta$ , and the detuning parameter,  $\Delta\nu$ , are given by:

$$\delta = \frac{1}{\rho} \left( \frac{\gamma - \gamma_0}{\gamma_0} \right) \quad (\text{IV.8.22})$$

and

$$\Delta\nu = \frac{\Delta\omega}{\omega_0} = \left[ \frac{\omega}{\omega_0} - 1 \right], \quad \text{with} \quad \omega_0 = \frac{2\pi}{\lambda_u} \cdot c \cdot \frac{2\gamma^2}{\left(1 + \frac{K^2}{2}\right)}. \quad (\text{IV.8.23})$$

Here  $\gamma_0$  is the central energy gamma factor and  $\psi'(\delta) = d\psi(\delta)/d\delta$  where  $\psi(\delta)$  is the normalized (to unity) relative energy distribution of particles in the beam:

$$\int_{-\infty}^{+\infty} d\delta \psi(\delta) = 1. \quad (\text{IV.8.24})$$

The growth of the laser power is then given by Eqs. (IV.8.6), (IV.8.7) and (IV.8.8) with  $\rho$  and  $(\ell_e)$  replaced by  $\rho_{\text{eff}}$  and  $(\ell_e)_{\text{eff}}$ , given by:<sup>53</sup>

$$\rho_{\text{eff}} = \frac{2}{\sqrt{3}} \left( \mu_I \right)_{\text{max}} \cdot \rho \quad (\text{IV.8.25})$$

$$(\ell_e)_{\text{eff}} = \frac{\sqrt{3}}{2} \frac{\ell_e}{(\mu_I)_{\text{max}}} \quad (\text{IV.8.26})$$

where  $(\mu_I)_{\max}$  is the absolute value of the imaginary part of that complex solution  $\mu = \mu_R + i\mu_I$  to the Dispersion Relation having the largest negative imaginary part.

The Dispersion Relation becomes a simple cubic equation<sup>53</sup> in  $\mu$  for a rectangular or Lorentzian longitudinal momentum distribution of the beam:

Rectangular:

$$\Delta = 2 \left( \frac{\sigma_p}{\rho} \right)$$

$$\psi(\delta) = (1/\Delta), \quad -(\Delta/2) \leq \delta \leq +(\Delta/2) \quad (\text{IV.8.27})$$

$$= 0 \quad \text{otherwise}$$

Dispersion Relation:

$$\mu^3 + \left( \frac{\Delta v}{2\rho} \right) \mu^2 - \left( \frac{\sigma_p}{\rho} \right)^2 \mu + \left[ 1 - \left( \frac{\sigma_p}{\rho} \right)^2 \frac{\Delta v}{2\rho} \right] = 0 \quad (\text{IV.8.28})$$

Lorentzian:

$$\Delta = 2 \left( \frac{\sigma_p}{\rho} \right)$$

$$\psi(\delta) = \left( \frac{\Delta}{2\pi} \right) \frac{1}{[\delta^2 + (\Delta/2)^2]} \quad (\text{IV.8.29})$$

Dispersion Relation:

$$\mu^3 + \left[ \left( \frac{\Delta v}{2\rho} \right) - 2i \left( \frac{\sigma_p}{\rho} \right) \right] \mu^2 - \left[ \left( \frac{\sigma_p}{\rho} \right)^2 + i \left( \frac{\Delta v}{\rho} \right) \left( \frac{\sigma_p}{\rho} \right) \right] \mu + \left[ 1 - \left( \frac{\Delta v}{2\rho} \right) \left( \frac{\sigma_p}{\rho} \right)^2 \right] = 0 . \quad (\text{IV.8.30})$$

For example, for a Lorentzian distribution,  $\rho_{\text{eff}} = \rho$  when  $(\sigma_p/\rho) = 0$  and  $\rho_{\text{eff}} = 0.36 \rho$  when  $(\sigma_p/\rho) = 1$ .

For a normalized Gaussian distribution, the Dispersion Relation has to be solved in the form of an integral, but it can be rewritten as follows:

Gaussian:

$$\sigma = \frac{\sigma_p}{\rho}$$

$$\psi(\delta) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\delta^2/2\sigma^2} \quad (\text{IV.8.31})$$

Dispersion Relation:

$$D(z) \equiv \left(\sqrt{2} \sigma^3\right) z + \left[1 - \left(\frac{\sigma^2}{2\rho}\right) \Delta v\right] + i \sqrt{\pi} z w(z) = 0 \quad (\text{IV.8.32})$$

where  $w(z)$  is the complex error function,<sup>28</sup> given by

$$w(z) = e^{-z^2} \left[1 + \frac{2i}{\sqrt{\pi}} \int_0^z dt e^{t^2}\right] \quad (\text{IV.8.33})$$

and  $z$  is related to  $\mu$  by

$$\mu = -\sqrt{2} \sigma z . \quad (\text{IV.8.34})$$

In ZAP, we make use of the following empirical relation for the field of a hybrid undulator using a steel-permanent-magnet (samarium-cobalt) design:<sup>54</sup>

$$B_{[\text{tesla}]} = 3.338 \exp \left[ - \frac{d}{\lambda_u} \left( 5.47 - 1.8 \frac{d}{\lambda_u} \right) \right] \quad (\text{IV.8.35})$$

where  $d$  is the undulator gap and  $\lambda_u$  is its period length. For a given undulator first-harmonic radiation wavelength,  $\lambda$ , and undulator gap (the default value of  $d = 0.3$  cm can be varied by providing a negative value for the wavelength  $\lambda$ , see Section III), the code iterates Eqs. (IV.8.1), (IV.8.2) and (IV.8.35) to arrive at values for  $\lambda_u$ ,  $B$  and thus,  $K$ , consistent with the given  $\lambda$  and  $d$  inputs. The resulting parameters are checked to ensure that they are consistent with the restriction  $0.07 \leq (d/\lambda_u) \leq 0.7$ , for which Eq. (IV.8.35) is valid. For resultant undulator parameters beyond this range, different inputs are requested. For a given horizontal emittance,  $\epsilon_x$ , emittance ratio,  $(\epsilon_x/\epsilon_y)$ , and peak current,  $I_p$ , as input, the code then calculates the FEL gain and e-folding length parameters,  $\rho$  and  $\ell_e$ , for zero momentum spread. For the specified momentum distribution of the beam (rectangular, Lorentzian or Gaussian), and its momentum spread,  $\sigma_p$ , the code then solves the appropriate Dispersion Relation and finds the root with maximum imaginary part  $(\mu_I)_{\max}$ . The degraded values for  $\rho_{\text{eff}}$  and  $(\ell_e)_{\text{eff}}$  are then obtained from Eqs. (IV.8.25) and (IV.8.26). Normally, the code iterates to find the value of  $\Delta v$  that maximizes  $(\mu_I)_{\max}$ ; this is then used to calculate  $\rho_{\text{eff}}$  and  $(\ell_e)_{\text{eff}}$ . It is possible to fix  $\Delta v$ , however, (or to vary it "manually") by entering the  $I_p$  value as a negative number.

In addition to the natural energy spread, the beam emittance contributes an effective energy spread<sup>55</sup> that is subject to a condition similar to Eq. (IV.8.20). Making use of Eq. (IV.8.16), and under the assumption that the transverse charge distributions are Gaussian, the effective energy spread can be written:

$$\left(\sigma_p\right)_{\text{eff}} = \frac{\sqrt{\epsilon_x^2 + 5\epsilon_y^2} K\pi}{2\sqrt{2} \gamma\lambda} . \quad (\text{IV.8.36})$$

Attention must be paid to not using too high a value of the deflection parameter  $K$  or else the effective energy spread in Eq. (IV.8.36) will become significant and degrade the effective FEL gain  $\rho_{\text{eff}}$  further, beyond the effect of the natural momentum spread  $\sigma_p$ .

#### IV.9 Intrabeam Scattering

Coulomb scattering within a beam leads to excitation of betatron and synchrotron energy oscillations of the particles involved. Multiple small-angle Coulomb scattering causes diffusion in both longitudinal and transverse phase space, resulting in a redistribution of beam emittances in all phase planes.

Typically, the particle motions are nonrelativistic in the beam rest frame. It is also generally true that, in the beam frame, the horizontal and vertical momentum spreads are larger than the longitudinal momentum spread. (The rms momentum spreads in the three dimensions are  $\sigma_x p_0$ ,  $\sigma_y p_0$  and  $\sigma_p p_0/\gamma$ , respectively, in the beam frame.) This implies that Coulomb scattering will predominantly transfer momentum (and hence energy) from the betatron transverse dimensions to the longitudinal synchrotron dimension. On the other hand, the excitation of synchrotron motion will move the closed orbit of a particle due to its changed momentum and will excite horizontal betatron oscillations in the dispersive regions of the lattice. For large enough dispersion, the excitation of betatron oscillations horizontally more than compensates, in terms of beam size, the loss of transverse horizontal momentum in the basic scattering event. In such a situation, common to most storage rings, one therefore expects vertical motion to be very slowly damped, while both horizontal and longitudinal motions grow.<sup>18</sup>

A detailed formalism for intrabeam scattering, taking into account the variation of lattice functions with azimuth, has been developed by Bjorken and Mtingwa.<sup>56</sup> The intrabeam scattering diffusion or growth rates in the three planes are given in their most general form, according to this formulation, by:

$$\begin{aligned}
 (g_{\text{IBS}})_a &\equiv \frac{1}{\tau_a} \\
 &= \frac{\pi^2 r_0^2 c m^3 N_b (\log)}{\gamma \tilde{\Gamma}} \left\langle \int_0^\infty d\lambda \frac{\lambda^{1/2}}{[\det(L+\lambda I)]^{1/2}} \left\{ \text{Tr} L^{(a)} \text{Tr}(L+\lambda I)^{-1} - 3 \text{Tr} L^{(a)} (L+\lambda I)^{-1} \right\} \right\rangle
 \end{aligned}
 \tag{IV.9.1}$$

where

$$L = \sum_{a=h,v,\ell} L^{(a)} = L^{(h)} + L^{(v)} + L^{(\ell)}
 \tag{IV.9.2}$$

$$L^{(h)} = \frac{\beta_x}{\epsilon_x} \begin{pmatrix} 1 & -\gamma\phi & 0 \\ -\gamma\phi & \frac{\gamma^2 D^2}{\beta_x^2} + \gamma^2 \phi^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}
 \tag{IV.9.3}$$

$$L^{(v)} = \frac{\beta_y}{\epsilon_y} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
 \tag{IV.9.4}$$

$$L^{(\ell)} = \frac{\gamma^2}{\sigma_p^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}
 \tag{IV.9.5}$$

$$\phi = D' + \frac{\alpha_x D}{\beta_x} \quad (\text{IV.9.6})$$

$$\begin{aligned} \tilde{\Gamma} &= \Gamma_B \quad (\text{bunched beam}) \\ &= \Gamma_U / \sqrt{2} \quad (\text{unbunched beam}) . \end{aligned} \quad (\text{IV.9.7})$$

Here  $\Gamma_{B,U}$  is the six-dimensional phase-space volume of the beam, defined as,

$$\Gamma_B = (2\pi)^3 (\beta\gamma)^3 m^3 \epsilon_x \epsilon_y \sigma_p \sigma_\ell \quad (\text{IV.9.8})$$

$$\Gamma_U = (2\pi)^{5/2} (\beta\gamma)^3 m^3 \epsilon_x \epsilon_y \sigma_p (2\pi R) \quad (\text{IV.9.9})$$

with  $m$ , the particle mass,  $N_b$  the total number of particles per bunch (or in the beam for the unbunched case),  $r_0$  the classical particle radius, i.e.,  $r_e$  for electrons,  $r_p$  for protons or  $(q_i^2/A_i)r_i$  for ions. The symbol  $\langle \rangle$  denotes an average over positions of the beam around the ring. The factor  $(\log)$  is a Coulomb logarithm, defined as<sup>57</sup>

$$(\log) = \Lambda = \ln \left( \frac{r_{\max}}{r_{\min}} \right) \quad (\text{IV.9.10})$$

where  $r_{\max}$  is taken to be the smaller of the  $\sigma_x$  of the beam or the Debye length  $\lambda_D$ , defined as:<sup>57</sup>

$$\lambda_D[\text{cm}] = \frac{743.4}{q_i} \sqrt{\frac{T_\perp[\text{eV}]}{\rho[\text{cm}^{-3}]}} . \quad (\text{IV.9.11})$$

Here  $\rho = N_b/V$  is the particle volume density where

$$V = \begin{cases} 8\sqrt{\pi^3} \langle \sigma_x \rangle \langle \sigma_y \rangle \sigma_\ell & \text{for bunched beams} \\ 4\pi \langle \sigma_x \rangle \langle \sigma_y \rangle (2\pi R) & \text{for unbunched beams} \end{cases} \quad (\text{IV.9.12})$$

and

$$\langle \sigma_x \rangle = (\epsilon_x \langle \beta_x \rangle)^{1/2}; \quad \langle \sigma_y \rangle = (\epsilon_y \langle \beta_y \rangle)^{1/2}. \quad (\text{IV.9.13})$$

For unbunched beams,  $N_b$  is given by

$$N_b = \frac{I \cdot 2\pi R}{q_i e \beta c} \quad (\text{IV.9.14})$$

where  $I$  is the total beam current. The transverse temperature  $T_\perp$  (in eV) is given by twice the transverse kinetic energy  $E_\perp$  (in eV) in the beam frame:

$$T_\perp = 2E_\perp; \quad E_\perp = \frac{1}{2} (\gamma E - E_0) (\epsilon_x / \langle \beta_x \rangle). \quad (\text{IV.9.15})$$

The  $r_{\min}$  is the larger of  $(r_{\min})_{\text{classical}}$ , the classical distance of closest approach and  $(r_{\min})_{\text{QM}}$ , the quantum mechanical diffraction limit from the nuclear radius where:<sup>57</sup>

$$(r_{\min})_{\text{classical}} = \frac{1.44 \times 10^{-7}}{T_\perp [\text{eV}]} q_i^2 \quad (\text{IV.9.16})$$

[cm]

and

$$(r_{\min})_{\text{QM}} = \frac{1.973 \times 10^{-11}}{2 \sqrt{T_\perp [\text{eV}] E_0 [\text{eV}]}}. \quad (\text{IV.9.17})$$

[cm]

Typical values for  $(\log)$  are 10–20.

Typical inputs for the intrabeam scattering calculations for protons or heavy ion beams are the beam energy, the transverse emittance, momentum spread and bunch length, number of particles per bunch and a complete set of lattice function values stored in the file ZAPLAT.DAT (see Section II). The code then calculates the various complicated integrals in Eq. (IV.9.1) at each lattice point and provides the longitudinal, horizontal and vertical growth rates both at each lattice point and their weighted average. From the output at each lattice point, one can ascertain where the dominant growth occurs, e.g., in the region of high dispersion.

For electron beams, the transverse and longitudinal synchrotron radiation damping rates are additional (optional) inputs. If these rates are entered, the code then iterates to find an equilibrium horizontal emittance that results from a balance among the quantum excitation, intrabeam scattering and radiation damping processes, i.e., it solves:<sup>33</sup>

$$\left[ g_x^{SR} - g_x^{IBS}(\epsilon_x, \epsilon_y, \epsilon_\ell) \right] \epsilon_x - g_x^{SR} \epsilon_x^0 = 0 \quad (IV.9.18)$$

where  $\epsilon_x^0$  is the natural equilibrium emittance of the electron beam in the absence of intrabeam scattering. (The last term in Eq. (IV.9.18) is the quantum excitation term, which must be equal to the radiation damping rate at the natural equilibrium emittance,  $\epsilon_x^0$ ). The value of  $\epsilon_x^0$  is an input to ZAP; it can generally be obtained from any lattice code. ZAP also includes equilibration of the longitudinal dimensions, i.e., it can solve Eq. (IV.9.18) and an analogous equation involving  $\epsilon_\ell$  simultaneously. This provides a full two-dimensional coupled equilibrium solution. The code cannot perform a full three-dimensional equilibration including the vertical dimension, which is its only limitation at present. However for most lattices, the vertical emittance changes so slowly compared with the other two dimensions, one obtains a reasonable equilibrium by this 2-D procedure. The evolution of the vertical emittance is assumed to be

dominated by the coupling terms in the lattice. Thus the ratio of  $\epsilon_x/\epsilon_y$  is held fixed by the code during the iterations to find the equilibrium emittance values.

#### IV.10 Touschek Scattering

For single, large-angle Coulomb scatters within a bunch, a particle's relative momentum deviation ( $\Delta p/p$ ) may exceed the acceptance imposed longitudinally by the RF bucket momentum height or transversely by the storage ring transverse aperture (dynamical or physical), whichever is smaller. When this happens, the particle is lost and the beam lifetime is reduced (Touschek effect). The Touschek scattering half-life for nonrelativistic transverse velocities in the beam frame is given by:<sup>39,58</sup>

$$\left(\tau_{1/2}\right)_T^{-1} = \frac{\sqrt{\pi} r_0^2 c N_b}{\gamma^2 (\Delta p_x)^3 \epsilon_A V_B} F(\epsilon_A) \quad (\text{IV.10.1})$$

where

$$\Delta p_x = \left(\gamma^2 - 1\right)^{1/2} \sigma_x, \quad (\text{IV.10.2})$$

$$V_B = 8\pi^{3/2} \sigma_x \sigma_y \sigma_z \quad (\text{IV.10.3})$$

$$\epsilon_A = \left[ \frac{1}{(\Delta p_x)} \cdot \left(\frac{\Delta p}{p}\right)_{\text{BKT}}^{\text{equiv.}} \right]^2 \quad (\text{IV.10.4})$$

$$F(\epsilon_A) = \int_1^\infty du \left[ 2u - \ln(u) - 2 \right] \frac{e^{-\epsilon_A \cdot u}}{2u^2}. \quad (\text{IV.10.5})$$

In obtaining the bunch volume  $V_B$  in configuration space and the angular divergence  $\sigma_x$ , the effect of the dispersion on the beam dimensions should be

included.<sup>18</sup> In the code, the horizontal size is increased by:<sup>18</sup>

$$\sigma_x = \sigma_{x0} \left[ 1 + \frac{D^2 \sigma_p^2}{\beta_x \epsilon_x} \right]^{1/2} \quad (\text{IV.10.6})$$

and the rms angular divergence is increased by:<sup>18</sup>

$$\sigma_{x'} = \sigma_{x'0} \left[ \frac{\beta_x \epsilon_x + D^2 \sigma_p^2 + \phi^2 \beta_x^2 \sigma_p^2}{\beta_x \epsilon_x + D^2 \sigma_p^2} \right]^{1/2} \quad (\text{IV.10.7})$$

where

$$\sigma_{x0} = \sqrt{\beta_x \epsilon_x} \quad (\text{IV.10.8})$$

$$\sigma_{x'0} = \sqrt{\frac{\epsilon_x}{\beta_x}} \quad (\text{IV.10.9})$$

and  $\phi$  is given in Eq. (IV.9.6).

In Eq. (IV.10.4),  $(\Delta p/p)_{\text{BKT}}^{\text{equiv.}}$  is the limiting longitudinal momentum aperture corresponding to the relevant acceptance limit, transverse or longitudinal. It is equal to the actual RF bucket height if the RF acceptance is the smallest and hence the limiting factor for Touschek scattering. In the code, the limiting physical momentum aperture at a lattice point  $s = s^*$  in the dispersive region is determined by calculating the transverse oscillation amplitude  $\Delta x_i$  resulting at each lattice location  $s = s_i$  from a momentum deviation  $(\Delta p/p)$  induced by a Touschek scattering event at lattice location  $s^*$ , according to the formula:

$$\Delta x_i = \left[ D_i^2 + \beta_i \left( \frac{D^{*2} + (\phi^* \beta^*)^2}{\beta^*} \right) \right]^{1/2} \left( \frac{\Delta p}{p} \right) \quad (\text{IV.10.10})$$

$i = 1, \dots, N$  (lattice point index)

The limiting momentum aperture for the transverse plane for scattering at locations  $s^*$  is then given by the smallest value of  $(\Delta p/p)$  calculated as

$$\left( \frac{\Delta p}{p} \right)_{\perp}^{\text{lim}} (s=s^*) = \text{Min} \left\{ \frac{a_i}{\left[ D_i^2 + \beta_i \left( \frac{D^{*2} + (\phi^* \beta^*)^2}{\beta^*} \right) \right]^{1/2}}, \frac{d_{\hat{D}}}{\sqrt{\beta_{\hat{D}}} \left( \frac{D^{*2} + (\phi^* \beta^*)^2}{\beta^*} \right)^{1/2}} \right\} \quad (\text{IV.10.11})$$

where  $a_i$  is the physical aperture at each lattice point and  $d_{\hat{D}}$  is the dynamic aperture value translated to the lattice location where the dispersion is maximum (location  $\hat{D}$ , with beta function  $\beta_{\hat{D}}$ ). The definition of  $\phi$  is given in Eq. (IV.9.6). This transverse limiting momentum aperture is then compared with the RF bucket height. The final limiting momentum aperture for lattice point  $s^*$  in the dispersive region is then taken to be

$$\left[ \left( \frac{\Delta p}{p} \right)_{\text{BKT}}^{\text{equiv.}} \right]_{s=s^*} = \text{Min} \left\{ \left( \frac{\Delta p}{p} \right)_{\text{BKT}}, \left( \frac{\Delta p}{p} \right)_{\perp}^{\text{lim}} \right\} \quad (\text{IV.10.12})$$

In the non-dispersive regions,  $D^* = 0$  and  $\phi^* = 0$  in Eq. (IV.10.10) and the physical limit is always given by

$$\left( \frac{\Delta p}{p} \right)_{\perp}^{\text{lim}} = \text{Min} \left\{ \frac{a_i}{D_i} \right\} \quad (\text{IV.10.13})$$

For a uniform aperture, of course, the limit must be just  $(a_i/\hat{D})$  where  $\hat{D}$  is the maximum value of dispersion in the lattice. Because no betatron oscillation is induced by a scattering event in the non-dispersive region, there is no dynamic aperture limit in this case. The limiting momentum aperture as defined above is calculated for scattering at each lattice point. (ZAP prints out these momentum limits if Main Menu options 7, 8, or 9 are chosen.) Based on these limits, ZAP calculates the Touschek half-life according to Eq. (IV.10.1) at each lattice point and also prints out the weighted lattice-averaged Touschek half-life.

It is recommended that Main Menu option 2 (and for electron storage rings, also Main Menu option 7) be run prior to running Main Menu option 8 for Touschek scattering calculations. In this way, one can start from bunch-lengthened values of  $\sigma_\ell$  and equilibrium (intrabeam scattering plus radiation damping and quantum fluctuations) emittance values, thus accounting for these effects in a consistent fashion.

#### IV.11 Ion Trapping

The ionization process results from beam interactions with residual gas in the vacuum chamber. The existence of trapped ions, created by these collisions, can be one of the primary beam-current-limiting factors in electron storage rings.

The ions are focused by the passing electron bunches and simply drift during the gap in between the bunches. They execute transverse betatron motion in the potential well of the electron bunches. In a linear theory, these focusing effects are modeled by thin lenses, and it follows from simple optical considerations for single-particle motion that the transverse motion is stable (i.e., an ion is trapped) if the distance between successive equidistant bunches is less than or equal to four times the focal length of the thin lens representing the electron bunch (or, equivalently, if the trace of the cumulative transfer matrix for the transverse phase-space coordinates, consisting of the basic focusing and defocusing units, is less than or equal to two). The focusing

strength of the electron bunches, as experienced by an ion, is inversely proportional to the mass of the ion, and the bunch-to-bunch spacing is inversely proportional to the number of bunches in the storage ring. Heavy ions are easily trapped even with large bunch-to-bunch spacing and, for multibunch operation, the spacing may be small enough to trap light ions also. The focusing strength of an electron bunch is also proportional to the amount of current per bunch and depends on the transverse dimensions of the bunch, which determine the transverse charge density.

It follows that, for any storage ring, there is a "critical mass"  $A_c$  (in atomic mass units), depending on the current per bunch,  $I/k_b$ , the number of bunches  $k_b$ , the bunch transverse dimensions,  $\sigma_{x,y}$ , and the ring circumference, such that ions with mass  $A$  are trapped and stable if  $A \geq A_c$ . The critical mass is given by linear theory as:<sup>59,60</sup>

$$\left(A_c\right)_{x,y} = \frac{1}{2} \left(\frac{2\pi R}{k_b}\right)^2 \cdot \frac{m_e}{m_p} \cdot \left(\frac{I}{I_A}\right) \cdot \frac{1}{\langle\sigma_{x,y}\rangle(\langle\sigma_x\rangle + \langle\sigma_y\rangle)} \quad (\text{IV.11.1})$$

where

$$I_A = \left(r_e/ec\right)^{-1} \cong 17,045 \text{ amperes} \quad (\text{IV.11.2})$$

is the Alfvén current. The effective critical mass is determined by the smaller of the two transverse dimensions.

Given that the ions are trapped and start to accumulate, it is of interest to know their effect on the beam. The accumulation continues until the defocusing force due to ion space-charge in the drift region is strong enough for the stability limit to be reached. The limiting ion density  $\bar{d}_i$  at the center of the bunch in this equilibrium accumulation limit is obtained by replacing the drift length by a defocusing thick lens and is given by:<sup>59,60</sup>

$$\bar{d}_i = \beta \cdot \frac{1}{B} \cdot d_e = \frac{1}{r_e} \cdot \left( \frac{I}{I_A} \right) \frac{1}{2\pi \langle \sigma_x \rangle \langle \sigma_y \rangle} \quad (\text{IV.11.3})$$

where  $d_e = I/(k_b e f_0 V_B)$  is the actual number density of electrons in the electron bunches, and  $B = 2\pi R/(k_b L)$  is the bunching factor for  $k_b$  bunches, each with length  $L$ , in the storage ring. The neutralization factor,  $F_n$ , is then simply

$$F_n = \left( \bar{d}_i / d_e \right) = \frac{\beta}{B} . \quad (\text{IV.11.4})$$

[The neutralization limit as predicted above by the linear theory is only loosely validated by experience at a few low energy electron storage rings at small beam currents. The essential nonlinearity of the ion-electron beam system restricts this linear model to limited applicability. Thus, this model may not apply to a particular storage ring with arbitrary electron currents. The lack of reproducible experiments makes ion-trapping one of the most difficult effects to assess.] The time to reach this equilibrium with neutralization  $F_n$  is given by the neutralization time:<sup>59,60</sup>

$$\tau_n = F_n \cdot B \cdot \tau_i = \beta \cdot \tau_i . \quad (\text{IV.11.5})$$

Here

$$\tau_i = \frac{1}{d_m \cdot (\beta c) \cdot \sigma_i} \quad (\text{IV.11.6})$$

is the ionization time (time taken by one electron to create one ion),  $d_m$  being the residual molecular density and  $\sigma_i$  the ionization cross section. The number density of the residual gas molecules,  $d_m$ , is related to the gas pressure  $P$  and temperature  $T$  via the ideal gas law:

$$d_m = \frac{1}{R_g} \cdot \frac{P_{[\text{Torr}]}}{T_{[^\circ\text{K}]}} = G(P/T) \quad (\text{IV.11.7})$$

where  $G = R_g^{-1} = 9.656 \times 10^{24}$  is the inverse of the ideal gas constant,  $R_g$ .

The equivalent ion gas "pressure," in the limit of full neutralization, is then given by:

$$P_i = \frac{T}{G} \cdot \bar{d}_i \quad (\text{IV.11.8})$$

The accumulated ions will introduce extra transverse focusing forces on the electron bunches and will change the total effective transverse betatron tune of the electrons in the storage ring. The tune shift caused by the accumulated ions in the full-neutralization limit is given by:<sup>59,60</sup>

$$\left(\Delta v_{x,y}\right)_{\text{ions}} = \frac{r_e}{\gamma} \bar{d}_i \int_0^{2\pi R} ds \frac{\beta_{x,y}(s)}{\left[1 + \frac{\sigma_{x,y}(s)}{\sigma_{y,x}(s)}\right]} \quad (\text{IV.11.9})$$

A simpler formula, valid for the case of a round beam with  $\epsilon_x = \epsilon_y$  and  $\beta_x = \beta_y$  averaged over the lattice, is:

$$\left(\Delta v\right)_{\text{ions}}^{\text{round beam}} = \frac{k_b N_b r_e \beta}{4\pi\gamma\epsilon_x} \quad (\text{IV.11.10})$$

In the case where  $\epsilon_x = \epsilon_y$ , tune shift values corresponding to both Eqs. (IV.11.9) and (IV.11.10) are printed by the code; otherwise the second value is given as zero.

The ion trapping calculations done under Main Menu option 9 require lattice data (in the ZAPLAT.DAT file), in order to calculate the various lattice averages for the ion-induced tune shifts, etc. It is always assumed that ion trapping calculations are done only for electron bunches with Gaussian shapes in electron storage rings. The number of species of background gas molecules is presently limited to two, namely, hydrogen ( $H_2$ ) and nitrogen ( $N_2$ ) or equivalently carbon monoxide (CO). The ionization cross sections for these two species of background gas molecules are taken to be<sup>61</sup>

$$\sigma_{H_2^+} = 0.33 \times 10^{-22} \text{ m}^2$$

$$\sigma_{N_2^+} = 1.65 \times 10^{-22} \text{ m}^2 .$$

Given a lattice and other storage ring and beam parameters, the code calculates and provides values of the critical ion masses  $(A_c)_{x,y}$  for trapping, the limiting ion density  $\bar{d}_i$  at full neutralization, the neutralization factor  $F_n$ , the neutralization time  $\tau_n$ , the equivalent ion gas pressure  $P_i$  (in the limit of full neutralization) and the lattice-averaged horizontal and vertical ion-induced tune-shifts, including the simple round-beam case when  $\epsilon_x = \epsilon_y$ .

In using the ion trapping option for a given average beam current,  $I$ , number of bunches,  $k_b$ , and emittance ratio,  $(\epsilon_x/\epsilon_y)$ , the  $\epsilon_{x,y}$  and  $\sigma_\ell$  (or B) should be chosen to correspond to equilibrium beam dimensions determined from Main Menu option 7, i.e., including intrabeam scattering, and Main Menu option 2, i.e., including the blowup due to fast single-bunch instabilities that lead to bunch lengthening.

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## APPENDIX

### List of Symbols Used in the Text

Most symbols are explained in the text as they appear. Here we give a partial list of symbols that appear frequently.

$c$ :	Speed of light in vacuum
$e$ :	Electron charge
$R$ :	Ring average radius
$\beta$ :	Relativistic beta factor ( $\equiv v/c$ )
$\gamma$ :	Relativistic gamma factor ( $\equiv (1-\beta^2)^{-1/2}$ , ratio of energy to rest energy)
$f_0$ :	Revolution frequency of a reference synchronous particle in a bunch
$\omega_0$ :	Angular revolution frequency ( $\equiv 2\pi f_0$ )
$\omega$ :	Angular frequency in general ( $\equiv 2\pi f$ , $f \equiv$ frequency)
$Z_0$ :	Free space impedance ( $\equiv 377$ ohms)
$E_T$ :	Total energy of a particle
$q_i$ :	Charge state of ion
$Z$ :	Atomic number of residual gas species
$r_e$ :	Classical electron radius
$r_0$ :	Classical particle radius
$m_e$ :	Electron rest mass
$m_p$ :	Proton rest mass
$h$ :	Harmonic number of storage ring RF cavity
$n$ :	Phase-slip factor $\left( \equiv \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2} \right)$
$D$ :	Dispersion function of lattice
$\nu_s$ :	Synchrotron tune

$\nu_{\perp}, \nu_{x,y}$ :	Transverse betatron tunes
$\xi$ :	Ring chromaticity ( $\equiv d\nu_{\perp}/(dp/p)$ )
$\omega_{\beta}$ :	Betatron angular frequency ( $\equiv \nu_{\perp}\omega_0$ )
$\omega_s$ :	Angular synchrotron frequency ( $\equiv \nu_s\omega_0$ )
$N_{\text{cell}}$ :	Number of RF cells
$V_{\text{RF}}$ :	Peak RF voltage
$\phi_s$ :	Synchronous RF phase
$\beta_{\perp}, \beta_{x,y}$ :	Transverse beta-function of machine lattice
$\alpha_{x,y}$ :	Transverse alpha-function of machine lattice ( $\equiv -\beta'_{x,y}/2$ )
$\langle\beta_{\perp}\rangle, \langle\beta_{x,y}\rangle$ :	Ring-averaged transverse beta functions
$\sigma$ :	Conductivity of vacuum chamber wall (also used for rms beam energy spread in FEL calculation)
$b$ :	Radius of vacuum chamber pipe
$\omega_c$ :	Characteristic frequency for broadband impedance ( $\equiv c/b$ )
$N_b$ :	Number of particles/bunch
$k_b$ :	Number of bunches
$I_b$ :	Average bunch current
$I_p$ :	Bunch peak current
$a$ :	Average beam radius (also used as a symbol to denote coherent synchrotron modes, $a = 1$ dipole, $a = 2$ quadrupole, etc.)
$\langle\sigma_{x,y}\rangle$ :	Ring-averaged horizontal and vertical rms beam sizes
$\epsilon_{x,y}$ :	Horizontal and vertical rms beam emittances
$\epsilon_N$ :	Normalized transverse rms emittance
$\sigma_p$ :	RMS relative momentum spread ( $= \Delta p/p$ )
$\sigma_{\ell}$ :	RMS bunch length
$L$ :	Full bunch length
$\lambda_u$ :	Undulator wavelength

$\lambda$ :	Resonance radiation wavelength from free electron laser (laser optical wavelength)
B:	Peak magnetic field on-axis of an undulator (also used as bunching factor)
$U_0$ :	Synchrotron radiation loss of a particle per turn
$g_{SR}^{L,T}$ :	Longitudinal, transverse emittance damping rates due to synchrotron radiation ( $\equiv 2/\tau_E$ or $2/\tau_X$ )
$\tau_X$ :	Synchrotron radiation damping rate of horizontal betatron amplitude
$\tau_E$ :	Synchrotron radiation damping rate of longitudinal energy oscillation amplitude
$A_{\perp}$ :	Transverse acceptance of machine
x:	Transverse displacement of beam
$[Z_{\parallel}/n](\omega)$ :	Ratio of the effective longitudinal impedance to the mode number n ( $n \equiv \omega/\omega_0$ ) at frequency $\omega$
$Z_{\perp}(\omega)$ :	Transverse impedance at frequency $\omega$
$R_S(R_T)$ :	Longitudinal (transverse) shunt impedances of resonator-type beam-storage-ring coupling impedance
$b_{RF}$ :	Beam pipe radius at RF cavity noses
$\rho$ :	FEL parameter related to the laser gain
P:	Residual gas pressure (also used as FEL power)
T:	Temperature
$(\log) \equiv \Lambda$ :	Coulomb logarithm
$\lambda_D$ :	Debye length