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MULTIPLE SOLUTIONS FOR PROFILES IN FILM BLOWING

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The thin sheet equations describing the blown film process lead to multiple solutions, some of which are unstable, for Newtonian and viscoelastic liquids. Steady axisymmetric solutions do not exist at all for some operating conditions. The observed behavior is consistent with the known sensitivity of the process to small changes in operating parameters.

1. Introduction

The blown film process is used for the production of thin polymer films. Molten polymer is extruded through an annular die, and a biaxial extension is effected by slight internal pressurization and axial drawing. The thin drawn film is cooled and flattened into a double-layered sheet; the sheet is cut along the edge if a single ply film is desired. The process is known to be very sensitive to changes in operating variables.

The blown film process was first modeled by Pearson and Petrie (7,8), who used a "thin film" approximation that averages over the film thickness; the resulting equations describing the steady-state stress and profile development are therefore ordinary differential equations with split boundary conditions, in which the length in the machine direction is the independent variable. Pearson and Petrie considered inelastic fluids and neglected the effect of heat transfer on fluid properties; subsequent studies, most notably the recent work of Luo and Tanner (3), have included heat transfer and viscoelasticity. Process models have generally assumed that the freeze-line height (Z), the initial bubble radius (R_o), and the initial film thickness (w_o) can be specified. Two of the following four dimensionless operating parameters are then independent: inflation pressure (B), axial draw tension (T_z), final radius (R_f), and final film thickness (w_f).

The model equations for both Newtonian and viscoelastic fluids have proved extremely difficult to solve numerically in all studies described in the literature, so results have been reported only over narrow ranges of the relevant process parameters. It has not been possible to use the models to probe the process behavior in order to explore regions of possible sensitivity to changes in operating parameters.

2. Results of This Work

We have overcome the numerical difficulties in solving the boundary value problem through the use of a finite-difference formulation with a banded matrix inversion technique (4,6), and we have been able to explore a wide range of operating parameters for both inelastic and viscoelastic fluid models. The computed results are consistent with the observed operational sensitivity. We find that multiple solutions exist; for a given pressure and draw force, two distinct bubble shapes can be found for even the simplest case of a film formed from a temperature-insensitive Newtonian fluid (Figs. 1 and 2). Indeed, there are combinations of pressure and draw force for which no steady-state solution exists. The vanishing of steady solutions occurs suddenly at parameter values that are close to the values for which regular bubble shapes are computed. The results are qualitatively the same when the effect of heat transfer on temperature-dependent physical properties is included, although there are differences in details of the behavior.

The steady state behavior is even more complex for viscoelastic liquids. Contours of dimensionless internal pressure (B) and draw tension (T_z) for a temperature-independent Maxwell fluid with a Deborah number (dimensionless relaxation time) De = 0.10 are shown in Fig. 3; the results are qualitatively the same when heat transfer and the temperature-dependence of physical properties is included. Closed contours are obtained for sufficiently large pressures, with another branch of the same contour ending along the horizontal line $R_f/R_o = 1$; indeed, all contours contain at least one branch ending along this line, with the stresses growing without bound as $w_f/w_o \to 1 + Z/De$. The bubble radius remains constant at R_o in this limit, and the force and constitutive equations reduce to those for uniaxial fiber spinning (2).

The asymptotic behavior along the line $R_f/R_o = 1$, with consequent "unattainable regions" in the plane of final bubble dimensions, is unique to the Maxwell fluid, and is a manifestation of the asymptotic approach to infinite stresses at a finite extension rate that is contained in the model. Such infinite stresses are not permitted in network models like those developed by Marrucci and coworkers (1,5), though the behavior is otherwise qualitatively like that of the Maxwell model. The line labeled "Marrucci Model" in Fig. 3 is the contour for B = 0.20 for the Marrucci model using a model parameter value of a = 0.002; for this extremely small value of a the rheological behavior is nearly Maxwellian except at very high extension rates. The contours in the plane of final bubble dimensions are close to those of the Maxwell fluid except in this one region; the behavior here more closely approximates that of the Newtonian fluid, except that the asymptotic approach of R_f/R_o to unity is retained as $w_f/w_o \to \infty$.

Some of the steady states are demonstrably unstable from physical arguments based on the necessary response to a virtual displacement. The dynamical response has been studied both through linear stability theory and dynamic simulation; the latter approach demonstrates, for example, that all perturbations about the point marked U in Fig. 1 will take the system rapidly away. Perturbations to the left of the T_z contour continue to the bubble corresponding to point S, which has the same tension and internal pressure, while those to the right cause unbounded growth.

3. Acknowledgment

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4. Notation

a = parameter in Marrucci equation
B = inflation pressure
De = Deborah number
R_f = final bubble radius
R_o = initial bubble radius
T_z = draw tension
w_f = final thickness
w_o = initial thickness
Z = freeze line height

All variables are dimensionless

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Captions

Fig. 1. Contours of constant internal pressure (dashed) and axial tension (solid) for a temperature-independent Newtonian fluid, Z = 5. Intersections represent possible steady states.

Fig. 2. Bubble shapes corresponding to points *U* and *S* in Fig. 1, B = 0.20, T_z = 1.35.

Fig. 3. Contours of constant internal pressure (dashed) and axial tension (solid) for a temperature-independent Maxwell liquid, De = 0.10, Z = 5. The broken line marked "Marrucci Model" is for a Marrucci liquid with *a* = 0.002.

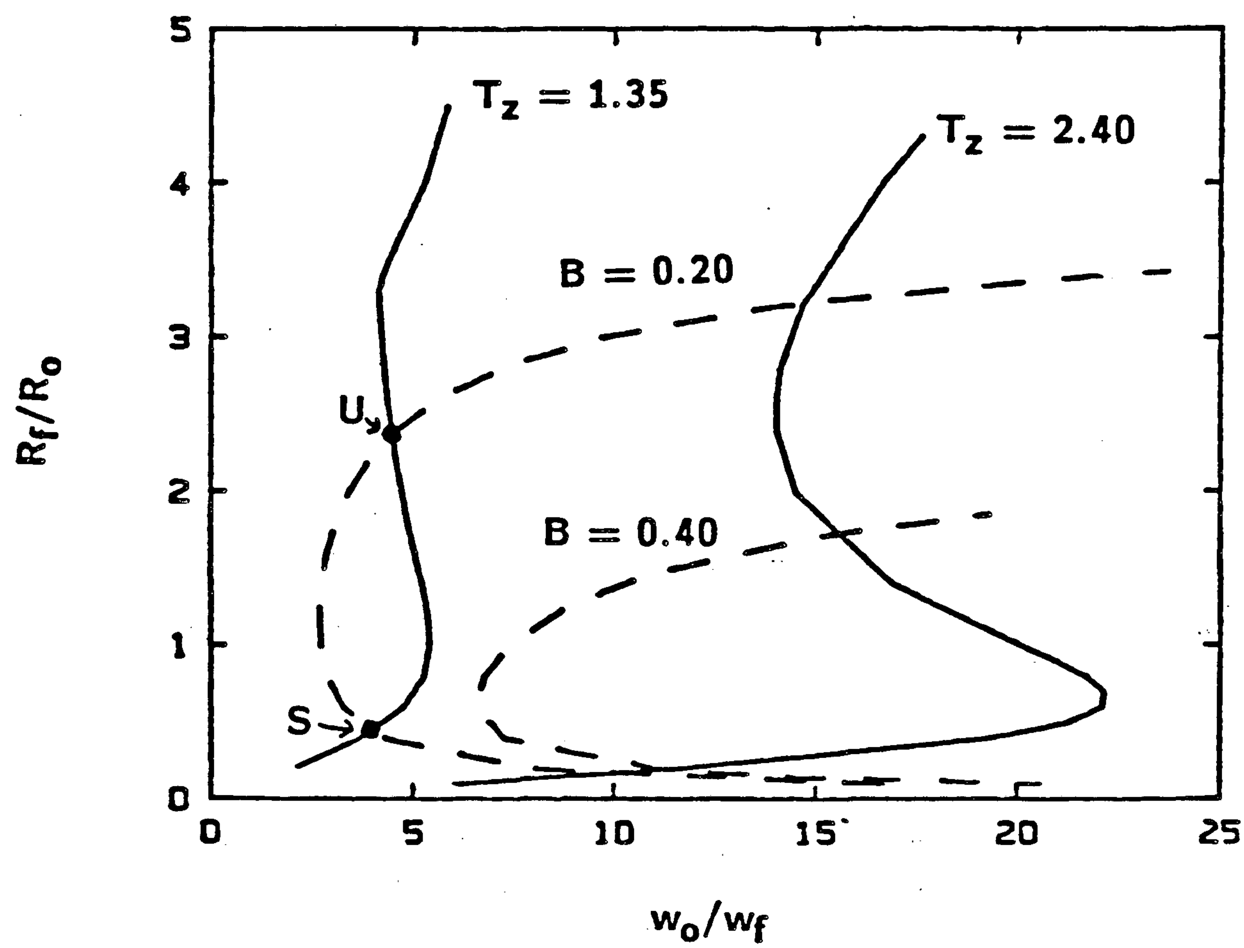


FIGURE 1

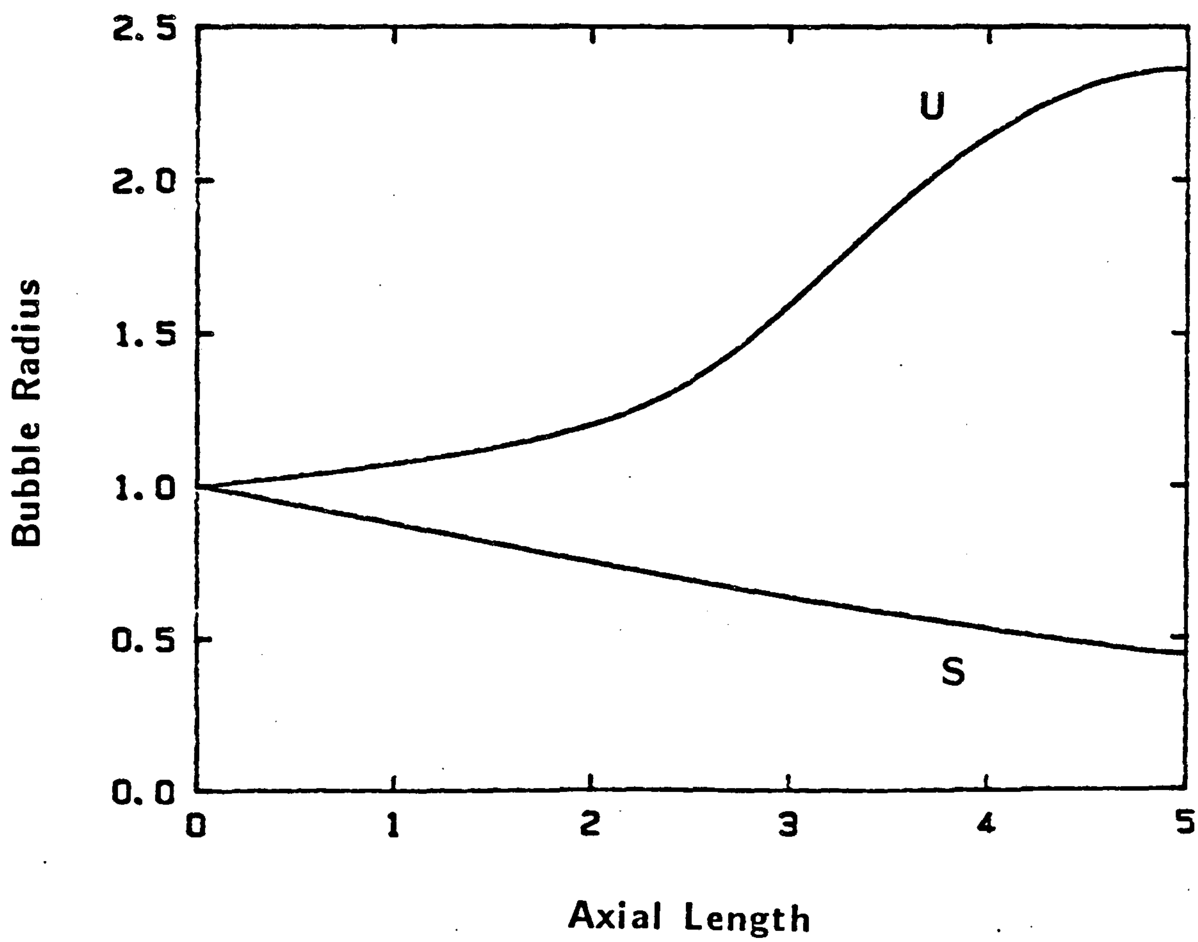
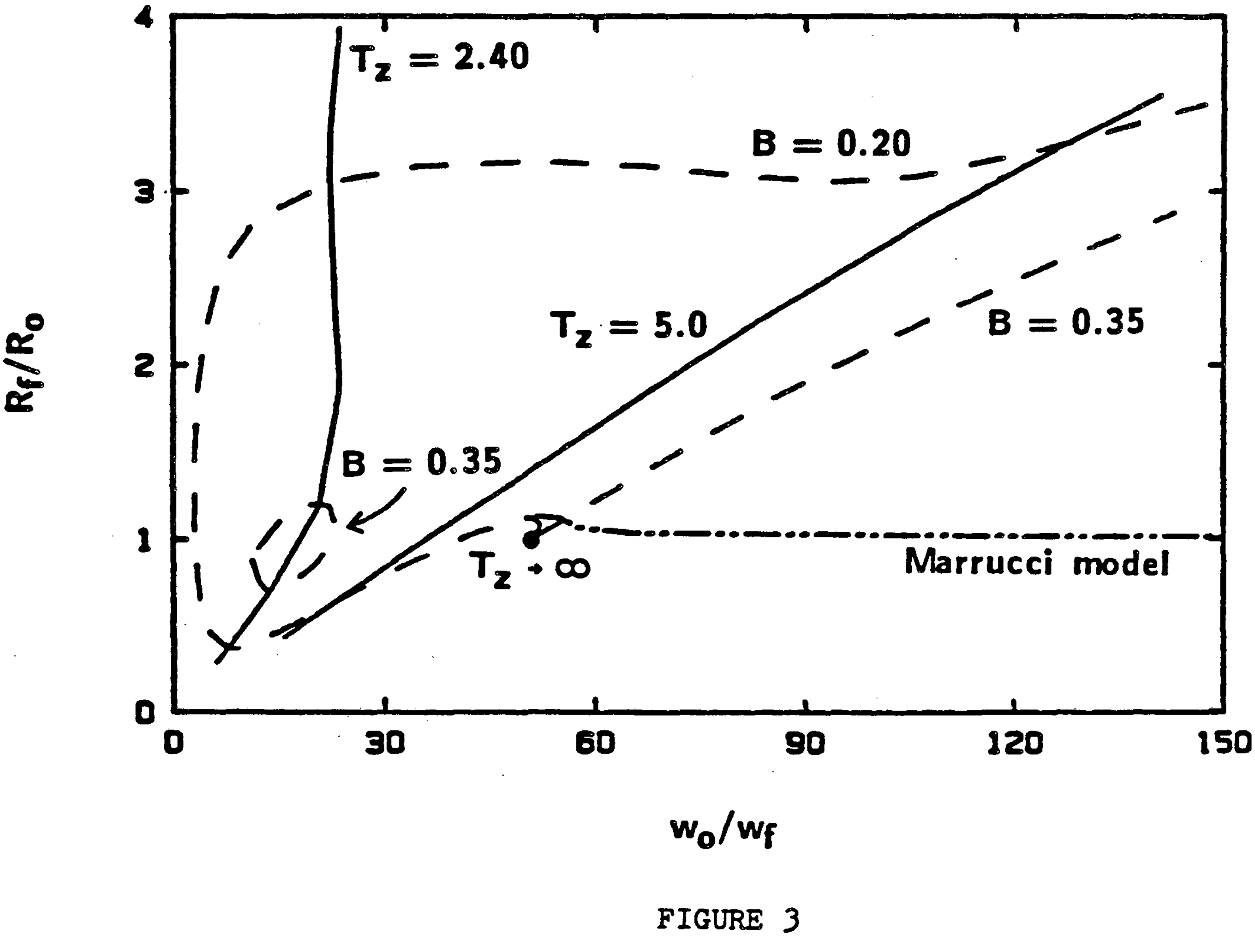


FIGURE 2



4
3
2
1
0
R_f/R_o
$T_z = 2.40$
$B = 0.20$
$T_z = 5.0$
$B = 0.35$
$B = 0.35$
$T_z \to \infty$
Marrucci model
0
30
60
90
120
150
w_o/w_f

FIGURE 3