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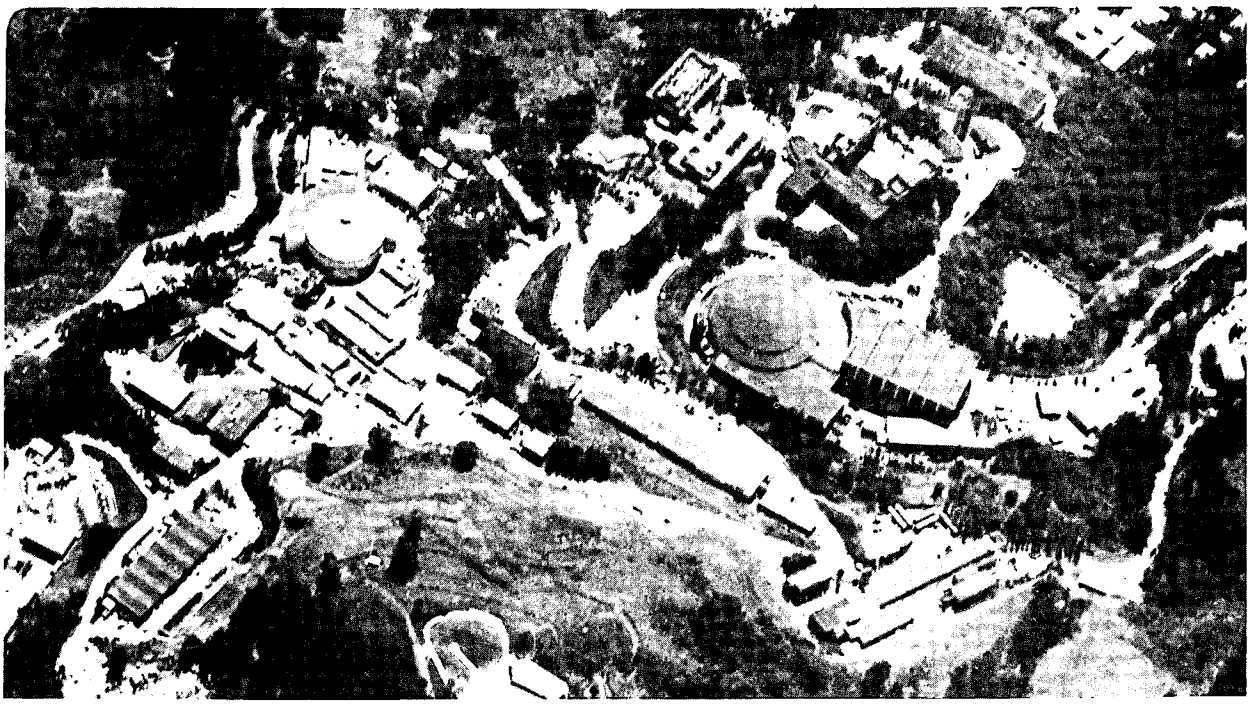

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Topics in Inflationary Cosmologies

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Ph.D. Thesis

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ABSTRACT

In this thesis we discuss several aspects of inflationary cosmologies. The first chapter is an introduction to the standard hot big bang cosmological model. We review the model and some of the problems associated with it.

In the second chapter, a short review of the proposals for solving the cosmological conundrums of the big bang model is presented. We study the old and the new inflationary scenarios and show why they are not acceptable. Some alternative scenarios especially those using supersymmetry are reviewed briefly.

The third chapter is a study of inflationary models where the same set of fields that breaks supersymmetry is also responsible for inflation. In these models, the scale of supersymmetry breaking is related to the slope of the potential near the origin and can thus be kept low. We find that we can get a supersymmetry breaking scale of the order of the weak breaking scale. The cosmology obtained from the simplest of such models is discussed in detail and we find that there are no particular problems except a low reheating temperature and a violation of the thermal constraint. We then present a possible solution to the thermal constraint problem by introducing a second field and discuss the role played by this second field in the scenario.

In the fourth chapter, an alternative mechanism for the generation of baryon number within the framework of supergravity inflationary models is studied. We use the gravitational couplings of the heavy fields with the hidden sector(the sector which breaks supersymmetry). This mechanism is applied to two specific models — one with and one without supersymmetry breaking. We emphasize the complete generality of this mechanism. The baryon to entropy ratio is found to be dependent on parameters which are model dependent. Finally, we remark on the effect of direct couplings between the two sectors on our results.

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I INTRODUCTION

The past few years have seen a tremendous growth in the interaction between cosmology and particle physics. The relationship between these two seemingly different branches of physics is not new. The theory of primordial nucleosynthesis of light elements [1], an application of nuclear physics to cosmology, is the earliest example. What is striking is the way in which progress in both particle physics and cosmology has become dependent upon this interplay [2].

In particle physics, experiments have tended to confirm the validity of the SU(3) × SU(2) × U(1) [3] model as the correct low energy effective theory. But a variety of unanswered questions have led to an investigation of theories which extend this model. Grand Unified Theories (GUTs) [4] unify the gauge fields of strong and electroweak interactions into a single gauge group with a single coupling constant. Apart from their aesthetic appeal, these theories provide us with explanations and predictions of parameters which are undetermined in the SU(3) × SU(2) × U(1) model. Among these are the equality of the electron and proton charge and the value of $\sin^2\theta_W$ (θ_W is the weak mixing angle) which controls the relative strength of the U(1) and SU(2) coupling constants and is an arbitrary parameter in the Glashow-Weinberg-Salam model of electroweak interactions [5]. However, some of the most striking predictions of GUTs like the nonconservation of

baryon number, existence of superheavy magnetic monopoles, occur at energy scales of 10^{14} GeV or higher. These energy scales are accessible only in the early universe which becomes a natural laboratory to study these theories. Furthermore, cosmology provides us with an insight into the nature of new predicted particles by putting very useful bounds on their masses, abundances, coupling strengths etc. As an example, following the evolution of stable or long lived neutrinos, we find that they must be lighter than 100 eV or heavier than 3 GeV [6]. If their mass falls within this forbidden range, then their contribution to the energy density of the universe is too large. Similarly, the big bang nucleosynthesis provides us with a limit on the number of neutrino species [7]. The predicted abundance of light elements is consistent with the inferred primordial abundance if the number of neutrinos is less than or equal to four.

On the other hand, particle physics plays an extremely important role in understanding the very early universe. The standard cosmological scenario of the hot big bang is very successful in explaining phenomena occurring after about 10^{-2} seconds after the big bang (at a temperature of about 10 MeV; we chose units such that $k = c = \hbar = 1$. Temperature is expressed in GeV, 1GeV$\simeq 10^{13}$K.). But in order to understand events happening before this time (i.e. at a higher temperature), it is crucial to take into account the interactions of elementary particles. It is here that particle physics guides cosmology.

Grand Unified Theories have several features which have a profound cosmological importance. Among the most noteworthy are the non-conservation of baryon number and the existence of magnetic monopoles. Non-conservation of baryon number is important in cosmology [8] because it gives us a way to explain the observed baryon-antibaryon asymmetry of the universe, without taking it as an initial condition. It should be stressed that even though experimental evidence for non-conservation of baryon number (nucleon decay) still does not exist [9], it is important to have a mechanism to explain the observed net baryon number of the universe. We will discuss the importance of monopoles later in this introduction.

Attempts have been made to construct models of the very early universe which incorporate Grand Unified Theories in an essential way [10]. These models attempt to solve some of the problems which the big bang model does not address. We will review these models in the next chapter. The rest of this introduction is a brief outline of the big bang model and its shortcomings.

The standard hot big bang scenario [11] assumes a spatially homogenous and isotropic universe which can be described in comoving coordinates by the Friedman-Robertson-Walker metric

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]. \tag{1.1}$$

The expansion of the universe is described by the scale factor $R(t)$ and the

curvature by the parameter k. By adjusting $R(t)$, we can normalize k to $\pm 1, 0$. $k = +1$ corresponds to a closed universe, $k = -1$ to an open universe and $k = 0$ to a spatially flat one.

The energy-momentum tensor is assumed to have a perfect fluid form,

$$T_{\mu\nu} = p g_{\mu\nu} + (p + \rho) U_\mu U_\nu \tag{1.2}$$

where p is the pressure, ρ is the total mass-energy density and $U_\mu = (1, 0, 0, 0)$ is the velocity vector for an isotropic fluid in its rest frame.

Given these assumptions, we can use the Einstein field equation,

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu} \tag{1.3}$$

to derive a first order equation for the evolution of the scale factor. Here $G_{\mu\nu}$ is the Einstein tensor, G is the Newton's constant and Λ is the cosmological constant. Taking the time-time component of (1.3) gives us

$$H^2 \equiv \frac{1}{R^2}\left(\frac{dR}{dt}\right)^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \tag{1.4}$$

where H is the Hubble parameter. Furthermore, energy conservation ($D_\mu T_{\mu\nu}$, D_μ being the covariant derivative) leads to

$$\frac{d(\rho R^3)}{dR} = -3pR^2. \tag{1.5}$$

With these equations, we only need to know the equation of state $p(\rho)$, to study the evolution of the universe. In the big bang model, the equation

of state is taken to be that of a relativistic or non-relativistic ideal gas.

For radiation or a relativistic ideal gas, (i.e. for $T \gg m$)

$$p = \rho/3 \tag{1.6a}$$

$$\rho = (g_b + \frac{7}{8}g_f)\frac{\pi^2}{30}T^4 \tag{1.6b}$$

where g_b and g_f are the number of effectively massless bosonic and fermionic degrees of freedom. Using (1.4) and (1.5), we find that

$$\rho \propto R^{-4} \tag{1.7a}$$

and (with $k = 0, \Lambda = 0$)

$$R \propto t^{1/2}. \tag{1.7b}$$

Similarly for a non-relativistic gas or matter (i.e. for $T \ll m$)

$$p = 0 \tag{1.8a}$$

and

$$\rho = mn \tag{1.8b}$$

where m is the mass and n is the number density of the particles. Once again using (1.4) and (1.5) we get,

$$\rho \propto R^{-3} \tag{1.9a}$$

and (again with $k = 0, \Lambda = 0$)

$$R \propto t^{2/3}. \tag{1.9b}$$

The assumption of thermal equilibrium and the conservation of energy (1.5) implies that the expansion of the universe is adiabatic. This means that the total entropy is constant i.e.,

$$\frac{d(sR^3)}{dt} = 0 \tag{1.10a}$$

where s is the entropy density.

For massless particles,

$$s = (g_b + \frac{7}{8}g_f)\frac{2\pi^2}{45}T^3 \tag{1.10b}$$

and hence (for a fixed number of particle species),

$$RT = \text{constant}. \tag{1.10c}$$

We will see later that this is an important result which will have to be modified in the context of inflationary models of the universe.

The cosmology obtained from this theoretical framework is extremely successful in explaining a variety of observations [12]. Among these are the cosmological redshift, the cosmic microwave background radiation [13] and the origin of light elements in the universe.

The expansion of the universe since the big bang accounts for the cosmological redshift. Radiation from the early universe which decoupled from

matter at a temperature of about $4,000K$ (when the electrons and ions recombined to form atoms) and has been redshifting since, is the microwave background radiation. The big bang model predicts correctly its blackbody nature as well as its temperature ($2.7K$). The light elements were synthesized primarily in the early epoch of primordial nucleosynthesis. Even in the simplest model of nucleosynthesis, we find that the computed abundances of light nuclei like deuterium, ^{3}He, ^{4}He and ^{7}Li, compare very well with the inferred primordial abundances (from observational data) [14].

Successful as it is in explaining these observations, the standard scenario suffers from four problems which motivate our search for alternative models [10]. These are the horizon problem, flatness problem, monopole problem and problem of explaining inhomogenities in the structure of the universe.

Horizon length or the particle horizon is the maximum distance light could have travelled since the big bang. It is the maximum radius of causal contact . It is given by (using 1.7a)

$$l(t) = R(t)\int_0^t R(t')^{-1}dt' = 2t. \tag{1.11}$$

On the other hand the physical radius scales as

$$L(t) = \frac{R(t)}{R(t')}L(t'). \tag{1.12}$$

Using (1.11) and (1.12), we find that at the time of decoupling of the cosmic microwave background radiation ($T \sim 10^4K, t \sim 10^2$ secs) there are 10^5

causally disconnected regions which grew into our present universe.

Observationally, the microwave background radiation is highly isotropic. The current limit on the anisotropy is

$$\Delta T/T \leq 10^{-4}. \tag{1.13}$$

It is difficult to understand how so many causally disconnected regions came to the same temperature at the same time, as suggested by the anisotropy limit (1.13). This is the horizon problem [15] and is not addressed by the standard scenario.

The flatness problem [16] is another one which this scenario does not address. Observations suggest that the ratio of the energy density of the universe to the critical density ρ_c (which gives us a flat universe i.e. $k = 0$ in eq. (1.2)) is close to 1. Since we know that the present value of the cosmological constant is almost 0, we will take $\Lambda = 0$ in what follows. From (1.4),

$$\rho_c = \frac{3H^2}{8\pi G} \tag{1.14a}$$

$$\Omega \equiv \rho/\rho_c \tag{1.14b}$$

$$0.1 < \Omega_{obs} < 4.0. \tag{1.14c}$$

This in itself is not a problem but for the fact that $\Omega = 1$ is an unstable

fixed point under time evolution. To see this we compute the quantity $\frac{|\rho-\rho_c|}{\rho}$. Using (1.4), (1.5) and (1.10c) we have

$$\frac{|\rho-\rho_c|}{\rho} \propto \frac{k}{(R^2T^2)T^2} \propto T^{-2}. \tag{1.15}$$

Thus for Ω to be so close to 1 today, it has to be unnaturally fine tuned in the past. For example, at $t = 1$ second (after the big bang), Ω has to be one to one part in 10^{15}. This curious fact is also taken to be an initial condition in the big bang model.

Many Grand Unified Theories [5] predict the existence of extended topological objects which have a mass of the order of the grand unification scale (10^{14} GeV). These stable topological knots in the Higgs field expectation value behave like magnetic monopoles. Cosmologically these monopoles are formed at the Grand Unified scale M_{GUT} when the grand unified gauge group spontaneously breaks down to a group containing $U(1)$.

The density of monopoles n_m, is estimated by assuming that the correlation length of the Higgs field expectation value is the size of the particle horizon (1.11). Then [17]

$$n_m(T_{GUT}) \sim l(T_{GUT})^{-3}. \tag{1.16}$$

It can be shown that monopole-antimonopole annihilation is negligible at these densities [18].

This density is disastrous because we get an energy density due to monopoles

alone which exceeds the critical energy density, ρ_c, by many orders of magnitude. Observationally, we know that the energy density of the universe is close to the critical density (1.14c). The standard model has no solution for this monopole problem.

Finally, it is difficult to understand the formation of structure in our universe with the big bang model. Although the universe is homogenous on large scales, there is enough evidence for inhomogenities (galaxies, clusters etc.) on the smaller scales. To account for this, the big bang model assumes a spectrum of initial inhomogenities which then evolve to give the observed distribution [19]. This ad-hoc nature of the initial spectrum is further problematic because of the evolution of gravitational instabilities. For the observed galactic evolution, this implies choosing a very unnatural set of initial conditions.

Thus we see that the hot big bang model, inspite of its success in explaining many observations, suffers from some drawbacks. These drawbacks are basically related to the fact that the model assumes a set of very unnatural and arbitrary initial conditions. It is this which has motivated the search for alternative scenarios in cosmology.

The rest of the thesis is organized as follows: In the next chapter, we will present a short review of inflationary cosmologies. We will discuss the various kinds of inflationary models and how they solve the problems of the

big bang scenario. The next two Chapters form the main part of this thesis. In Chapter 3 we discuss the motivations for constructing supersymmetric inflationary models. We then construct, and study in detail, models in which the same sector that is responsible for inflation also breaks supersymmetry. A study of the cosmology based on these models shows that there are no problems apart from a low reheating temperature and a violation of the thermal constraint. We propose a method for solving the thermal constraint. Chapter 4 is a study of baryogenesis in the supersymmetric inflationary models. We first review the standard scenario for baryosynthesis. We then go on to describe an alternative mechanism for generation of baryons in these models. After giving a general framework, we apply this mechanism to models in which supersymmetry is unbroken in the inflaton sector and to the models discussed in Chapter 3. We find that this mechanism could lead to a sufficient amount of baryons under certain model dependent constraints. Finally, we present our conclusions in Chapter 5.

II INFLATION AND NEW INFLATION

In the last chapter, we discussed the hot big bang model and some of its shortcomings. We saw that the standard scenario leads to a set of very unnatural initial conditions when extrapolated back to very early times. In this chapter we will discuss some of the proposals for curing this problem. We will see how particle physics plays an increasingly important role in the formulation of these alternative cosmological models.

The essential idea, in the solution to the cosmological conundrums of the big bang model, is to do away with the assumption of adiabatic expansion of the universe [10]. Recall that in the big bang model, the total entropy of the universe was assumed to be constant (eq.1.10a-c). This need not necessarily be true if the universe went through one or several phase transitions during its evolution.

Our belief in the correctness of the SU(3) $\times$ SU(2) $\times$ U(1) model in describing low energy physics [3], leads us inevitably to expect phase transitions in the universe. At temperatures $\sim 10^2$ GeV, we expect a phase transition associated with the spontaneous breaking of SU(3) $\times$ SU(2) $\times$ U(1) to SU(3) $\times$ U(1). Again at T $\sim$ few hundred MeV, we expect a transition when chiral symmetry, associated with Quantum Chromodynamics (QCD), is spontaneously broken [20]. Finally, at a slightly lower temperature, we might have a confinement transition of quarks in QCD.

Grand Unified Theories, provide us with one or several phase transitions at the GUT scale. If we compute the variation with energy of the gauge coupling constants of SU(3) × SU(2) × U(1), we find that the three are equal at an energy $\sim 10^{15}$ GeV [21]. This is the scale at which the unifying gauge group (eg. SU(5)) is broken spontaneously. This is another phase transition which could have important cosmological consequences. In fact, in the original formulation of the inflationary scenario, Guth [10] suggested using the GUT phase transition to solve the problems of the big bang model.

In the next section we will review Guth's original proposal for the inflationary model, its realization and how it solves the naturalness problems which plague the big bang model. We will also discuss the essential flaws in this scenario and why it is untenable.

1 THE INFLATIONARY SCENARIO

An essential feature of the inflationary model is the description of matter by a quantum field theory. In particular, this means modifying the energy momentum tensor $T_{\mu\nu}$ of (1.2) to include the contribution of the energy momentum operator [22]. If the quantum state does not break the symmetries of the Friedman-Robertson-Walker background, then we add a contribution

$$T_{\mu\nu} = T^M_{\mu\nu} + T^Q_{\mu\nu} \tag{2.1a}$$

$$T^Q_{\mu\nu} = \rho_0 g_{\mu\nu} \tag{2.1b}$$

where $T^M_{\mu\nu}$ is the energy momentum tensor of (1.2) and ρ_0 is the energy density of the state.

To illustrate the working of this scenario, we first discuss a toy model. Consider a field theory which has an effective potential V(ϕ), like the one in Fig 2.1. Here ϕ is a scalar field called the inflaton. The figure shows the effective potential V(ϕ) for $T = 0$ and for $T > T_c$, T_c being the critical temperature. An example would be a pure scalar theory with a quartic potential.

To understand the operation of this model, we have to consider finite temperature effects in quantum field theories [23]. Conventional field theories, describe events in a surrounding heat bath at $T = 0$. In the early universe, this assumption is inapplicable because of the presence of high matter and radiation density. So we have to study the field theory with a background heat bath at temperature $T \neq 0$ [24]. Using the formalism of quantum statistical mechanics with appropriate boundary conditions, we find that the effective scalar potential is [23] (in the high temperature limit)

$$V^T_{eff}(\phi) = V^{T=0}_{eff}(\phi) + CT^2\phi^2 + O(\phi^4) \tag{2.2}$$

where C is a constant depending on the specific model. This temperature dependent correction causes a symmetry restoration at high temperature in

theories with broken symmetry.

The effective potential has, at zero temperature, a global minimum at $\phi = \phi_{true} = \sigma$ and also a local minimum at $\phi = \phi_{false} = 0$. with energy ρ_0. We assume that there is a temperature T_c above which the finite temperature effective potential has a lower value for ϕ_{false} than for ϕ_{true}. T_c is then the critical temperature for a first order phase transition and for $T > T_c$, we have symmetry restoration i.e., the symmetric phase ϕ_{false} is preferred.

We can now trace the evolution of the universe based on this toy model. The evolution starts at $T = T_{Planck} \sim 10^{19} GeV$. Above this temperature quantum gravitational effects are important. Initially the thermal energy dominates the energy momentum tensor and the universe expands like a radiation dominated one (1.6, 1.7). The temperature dependent term in the potential (2.2) causes the state ϕ_{false} to be energetically favored. This will continue till the temperature falls to T_c. Below this, the universe supercools and remains in the false vacuum ϕ_{false}. The phase transition begins by formation of bubbles of the new phase [25]. The time independent energy density of the false vacuum ρ_0, dominates the energy momentum tensor. This acts like a cosmological term in Einstein equation (1.3) with

$$\Lambda = 8\pi G \rho_0. \tag{2.3}$$

Solving for the scale factor $R(t)$ from (1.4), we find an exponential expansion

(or inflation [10]) of $R(t)$.

$$R(t) \sim e^{\chi t} \tag{2.4a}$$

$$\chi = \left(\frac{8\pi}{3} G \rho_0\right)^{1/2} \tag{2.4b}$$

During the phase of exponential expansion, the scale factor soon becomes so large that the metric (1.1) can be approximated locally with the $k = 0$ form

$$ds^2 = -dt^2 + R^2(t) d\vec{x}^2. \tag{2.5}$$

This space is called de Sitter space [26].

The temperature T_m of the universe supercools exponentially

$$T_m(t) \sim T_{GUT} e^{-H(t - t_{GUT})}. \tag{2.6}$$

After a few e-foldings, i.e. the number of times the scale factor R expands by a factor of e, the state $\phi = 0$ is no longer a stable minimum of the effective potential but is a metastable false vacuum. It will decay to the true vacuum ϕ_{true} by tunnelling through the potential barrier. This phase transition is a first order one i.e., it proceeds like the boiling of water by the formation of bubbles of the new, energetically favored phase.

Assuming now that this expansion continues for a time Δt and then the phase transition occurs instantaneously, we can calculate the increase in the

scale factor

$$Z \equiv \frac{R(t)}{R(t_0)} \sim e^{\chi \Delta t}. \tag{2.7}$$

When the phase transition terminates, the energy density ρ_0 is released as latent heat. This energy, if rapidly thermalized, reheats the universe back to $T_R \sim T_{GUT}$. Reheating is a non-adiabatic process and generates entropy. The entropy density is the same as before the exponential expansion (because the temperature is the same (1.10b)), but the scale factor has increased by a factor of Z (2.7). Thus the total entropy increases by a factor of Z^3. This entropy generation is the key to solving the problems we encountered with the standard scenario in the previous chapter. Guth [10] showed that with $Z > 10^{29}$, the horizon, flatness and monopole problems of the big bang model are easily solved.

The horizon problem does not exist in this version of the inflationary model. Recall that the crux of the horizon problem was the existence of many causally disconnected regions which evolved into our present universe. If the scale factor goes through an exponential expansion, the horizon length (1.11) also expands exponentially while the physical radius (1.12) remains constant. Thus the region which grew into our observed universe was well within one horizon length. The isotropy of the microwave background is in a sense due to the immense expansion.

The flatness problem is also easily solved by the inflationary model. The

flatness problem was essentially the problem of understanding the closeness of Ω to one in the present universe. The fine tuning of the initial conditions to give $\Omega = 1$ is also related to the large entropy of our universe. To see this, we consider the total entropy of the universe (a dimensionless quantity).

$$S = R^3 s \tag{2.8a}$$

Using $T = T_\gamma = 2.7K$ (the temperature of the background radiation) and (1.10b), we get

$$s \sim 10^3 cm^{-3}. \tag{2.8b}$$

Using (1.4) and (1.14), we obtain

$$R = H^{-1}|1 - \Omega|^{-1/2}. \tag{2.8c}$$

With $\Omega < 2$, $R > 10^{10}$ years, we get a value of S which is very big [10]

$$S > 10^{87}. \tag{2.8d}$$

Thus the problem is reduced to explaining the enormous magnitude of this dimensionless number which we normally expect to have a value of order 1. In the inflationary model, with $Z > 10^{29}$ the entropy increases by a factor of $Z^3 = 10^{87}$. Therefore we expect Ω to be one to a very high degree of precision. This is one of the firmest predictions of models of the kind which use exponential expansion.

The monopole problem is also avoided in this model. The horizon length

is exponentially bigger, leading to a corresponding decrease in the number density of the monopoles. The exponential expansion dilutes the density of the monopoles to an acceptable value.

The inflationary scenario outlined above does not address the problem of generation of inhomogenities which lead to galaxy formation. For that we have to go to the new inflationary models which we will discuss in the next section.

The toy model that we have discussed above is obviously not very realistic. A more realistic model is the Georgi-Glashow SU(5) model [4]. In this case, the field ϕ is in the adjoint representation of SU(5) i.e., it is a 24 of SU(5). For field configurations which break SU(5) down to SU(3) $\times$ SU(2) $\times$ U(1) i.e., where the expectation value of the Higgs field is of the form

$$\phi = \phi\,\mathrm{diag}[1,1,1,-\frac{3}{2},-\frac{3}{2}] \tag{2.9a}$$

the finite temperature effective potential (to one loop) is given by

$$V_{eff}^{(1)T}(\phi) = A\phi^4\left[\ln\frac{\phi^2}{\sigma^2}-\frac{1}{2}\right]+\frac{1}{2}m^2\phi^2+C\phi^2T^2+\rho_0 \tag{2.9b}$$

where A and C are constants determined by the gauge coupling constant α_{GUT} [27]. Here m is the mass parameter and $m \ll \phi_{true} \equiv \sigma$. This is essential so as to get a temperature independent potential barrier near the origin. It is this barrier which is responsible for the universe being trapped in the metastable false vacuum when the temperature falls below T_c. This

potential looks similar to the one in Fig 2.1 (in one of the Higgs's directions). The previous discussion of the evolution of the universe can also be applied. The false vacuum is the SU(5) symmetric phase while the true vacuum is the SU(3) $\times$ SU(2) $\times$ U(1) symmetric phase.

It was soon realized that the original scenario of Guth could not provide a realistic cosmology [28]. To see this we have to look in detail at the mechanism of phase transition in this model.

In the preceding discussion we have assumed that the phase transition occurs instantaneously. This is actually not the case; the phase transition is a slow first order one taking place in an exponentially expanding space.

The bubbles of the new phase nucleate at a rate λ, given by [29]

$$\lambda = Ae^{-B} \tag{2.10}$$

where A is a quantity which has dimensions of mass^4 and B is the classical action associated with the O(4) invariant solution of the Euclidean field equations (Instanton). Since the details of the parameter A are not important for the following discussion, we will assume A to be T_{GUT}^4 (since T_{GUT} is the characterstic scale of the phase transition).

These bubbles expand at essentially the speed of light, due to the energy released by the conversion of the false vacuum into the true one. Thus the space inside the bubbles is causally disconnected from the region outside,

which is still described by the de-Sitter type metric (2.4 and 2.5). This is a problem because the phase transition never terminates. The space outside is expanding exponentially while the region inside grows only as $t^{1/2}$ (1.7b). Therefore, the bubbles of the new phase never join to form the region which contains our universe. To ensure a large enough expansion to solve the cosmological problems, the barrier has to very high and the tunnelling probability low. But, precisely because of this, the tunnelling probability never catches up with the expansion rate. The universe remains in the de Sitter phase in the SU(5) symmetric phase with some isolated regions of the true SU(3) × SU(2) × U(1) phase. This problem, of the bubbles not percolating [28], has been called the problem of 'graceful exit'.

The inflationary model suffers from another drawback — the problem of large inhomogenities. The bubble walls carry a large fraction of the original vacuum energy. These bubble walls form after the exponential expansion and hence remain within our observed horizon. Not only is the universe very lumpy but is almost empty. This is because the energy stored in the walls is never released since the bubbles never collide. Thus this form of the scenario leads to unacceptably large energy perturbations in the universe [28].

This model, or the 'old' inflationary scenario is untenable. To address the problems associated with it, we must discuss the 'new' inflationary scenario.

2 THE NEW INFLATIONARY SCENARIO

To overcome the problems outlined above, a modified version of this scenario, the new inflationary universe was proposed [30]. The basic idea behind this proposal is to construct a model in which inflation occurs after the bubbles of the new phase have been formed. Recall that in the 'old' inflationary scenario, the problems arose because we had the phase transition (bubble formation) after the exponential expansion. Hence the bubbles did not percolate and there was no graceful end to the inflationary epoch. In the new inflationary model, these problems don't arise because the observed universe lies within a single bubble.

The crucial feature of the new inflationary model is the presence of a long, flat scalar potential. We assume that the effective potential of the inflaton i.e., the scalar field driving inflation, is of the Coleman-Weinberg type [31]. The second derivative of the effective potential vanishes at the origin. In the SU(5) model [4], the one loop effective potential takes the form

$$V_{eff}^{(1)T}(\phi) = A\phi^4\left[\ln\frac{\phi^2}{\sigma^2} - \frac{1}{2}\right] + C\phi^2T^2 + \frac{1}{2}A\sigma^4 \qquad (2.11a)$$

where A and C are coefficients given by [27]

$$A = \frac{5625}{64}\alpha_{GUT}^2 \qquad (2.11b)$$

$$C = \frac{75}{4}\pi\alpha_{GUT}. \qquad (2.11c)$$

Notice that in (2.11a) we don't have the mass term present in (2.9b).

Actually there is a slight subtlety in discussing the mass terms in these models. If the effective potential in de Sitter space is computed, the effects of gravitational curvature for a true Coleman-Weinberg potential must be taken into account. This gives rise to a correction which behaves like an effective mass term [32]. We have not only to set the bare mass term in (2.10) to zero, but also this gravitational correction [33]. The potential is shown in Fig 2.2. There is now no temperature independent barrier near the origin to stabilize the state $\phi = 0$, once the temperature falls below the critical temperature.

Once again we trace the evolution of the universe in this model. We start at $T = T_{Planck}$ when the thermal part of the energy momentum tensor dominates and the universe expands like radiation dominated. As the temperature falls to T_c, the vacuum energy starts dominating the energy momentum tensor and the universe expands like (2.4a) with

$$\rho_0 = \frac{1}{2}A\sigma^4. \tag{2.12}$$

As long as the temperature is non-zero, there exists a bump near the origin which stabilizes the state $\phi = 0$. The height of this bump is of order T^4 and the width of order T [34]. This barrier is needed because otherwise the thermal or quantum fluctuations in the inflaton field will drive the transition too soon, resulting in insufficient inflation.

The quantum fluctuations now drive the inflaton field away from $\phi = 0$ towards $\phi = \sigma$. But as we have already noted, the potential is extremely flat near the origin and the phase transition is the 'slow rollover' type. Therefore, until the field evolves past the flat part of the potential, the universe continues to expand exponentially as it did in the metastable symmetric phase. If the time taken for ϕ to roll over from 0 to $\phi = \phi_{end}$ (Fig 2.2) is sufficiently long, we will get enough inflation to solve the horizon and flatness problems.

The evolution of the scalar field can be described accurately by its semi-classical equations of motion. Initially the quantum effects are dominant but soon these become unimportant and the semi-classical solution is applicable[35]. The semi-classical equation of motion for a scalar field in an expanding universe is

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi}. \tag{2.13}$$

Here the dots imply time derivatives and the second term is the 'friction' term which is due to the redshifting of energy in an expanding universe.

This slow rollover continues till $\phi = \phi_{end}$. The potential is not flat anymore and the field falls rapidly towards $\phi = \sigma$. Around the minimum, the field oscillates with a time scale which is typically the GUT time ($\sim \frac{1}{M_{GUT}}$) and which is very small compared to the expansion rate of the universe. These oscillations are damped quickly and the energy is thermalized [36]. The damping is simply the decay of the inflaton into other lighter particles.

The release of energy reheats the universe back to a temperature of order T_c. From here on, the evolution continues like in the standard model. This reheating is very crucial for the generation of baryon number in the universe. We will discuss the reheating and baryosynthesis in detail in the following chapters.

In this new inflationary model, there are no horizon, flatness or monopole problems for the same reason as they were avoided in the old inflationary scenario. Furthermore, we can now use particle theories in which a discrete symmetry is broken spontaneously. When a theory has a discrete symmetry which is broken spontaneously, we get domain walls separating the phases. The presence of the domain walls in our observed universe is problematic in the same way as the presence of bubble walls — they make the universe too lumpy. In the old inflationary scenario, since we had the inflation before the phase transition, there was no way to get rid of the domain walls once they were created by symmetry breakdown. But in this scenario, the expansion takes place after the inflaton has chosen a direction in group space to roll over. Thus it is possible to inflate away the domain walls so that the typical domain size is much greater than the observed universe.

Another success of the new inflationary scenario is in the explanation of galaxy formation [37]. The universe is homogenous on very large scales but there are inhomogenities which we see on many length scales. In the standard model, it is assumed that these inhomogenities evolved from small per-

turbations about the Friedman-Robertson-Walker background [19]. These perturbations cannot be causally explained within the standard model, since the perturbations on all scales originate outside the effective particle horizon. (The effective particle horizon [38], is the Hubble radius, H^{-1}, and is the maximum distance that microphysics can act coherently. For length scales larger than this, spatial correlations are exponentially suppressed and the cosmological expansion time is larger than the time taken by light to travel this distance.)

Furthermore, a scale invariant spectrum of initial energy density fluctuations explains the experimental constraints [39]. These experimental constraints come from the absence of observed anisotropies in the microwave background [40] and the requirement that the perturbations have enough time to grow to give the structure on the scales observed [41]. The standard scenario, does not give us any theoretical motivation for such a scale invariant or Harrison-Zeldovich spectrum, but postulates it to fit the experimental observations.

The new inflationary scenario is successful in explaining both these puzzles. When the inflaton starts rolling over from the unstable false vacuum, the universe is in the de Sitter phase. Within a fluctuation region (i.e., the region which grows to give us the observed universe), the expectation value of the ϕ field is not spatially uniform. As already noted, there are zero-point or vacuum(quantum) fluctuations initially, which provide the seed for

classical matter perturbations [42]. Moreover, the mechanism of exponential expansion provides a causal explanation of the scales of these perturbations. Perturbations on all scales originate inside the effective particle horizon or Hubble radius during this de Sitter phase.

The Harrison-Zeldovich spectrum emerges naturally within this framework because de Sitter space is time translation invariant [42]. So different scales reach the Hubble radius at different times but with essentially the same amplitude (actually, the time depends only logarithmically on the scales since $R \sim e^{Ht}$). Since microphysics is not operative coherently outside the Hubble radius, the evolution of the perturbations continues unchanged leading to a scale invariant spectrum. This qualitative explanation of the spectrum of perturbations was an impressive success of the new inflationary scenario.

Even with all these spectacular successes, the new inflationary universe is untenable. The Coleman-Weinberg type of potential, which is very desirable because of its flatness, is not completely natural. The mass parameters have to be unnaturally fine tuned in order to obtain enough inflation [43]. The mass terms have to be fine tuned precisely down to 10^9 GeV while radiative corrections tend to push up their value to $O(10^{15}$ GeV). This is undesirable because it was precisely the unnatural fine tuning of parameters in the big bang model, that led us to search for alternate models.

Another serious problem with this model is that the transition from the

false, SU(5) symmetric vacuum might take place into a SU(4) $\times$ U(1) symmetric phase [44]. In the multidimensional configuration space, the SU(5) phase goes to a SU(4) $\times$ U(1) phase which decays to the desired SU(3) $\times$ SU(2) $\times$ U(1) minimum. Thus the scenario cannot be realized in our SU(5) model but it is possible in an SU(5) theory with an enlarged Higgs sector [45].

The fatal blow to this scenario, comes from the density perturbations which it produces. Even though, this model successfully explains the origin and spectrum of inhomogenities, it fails to give a correct estimate for their magnitude. Recall that the limit on anisotropy from the microwave background is given by

$$\Delta T/T \leq 10^{-4}. \tag{1.13}$$

This can be translated into a limit on the size of allowed fluctuations [38, 42]

$$\frac{\delta\rho}{\rho} \leq 10^{-4}. \tag{2.14}$$

In the new inflationary scenario, the calculation of this quantity [38, 42], yields a number which is five orders of magnitude too large

$$\frac{\delta\rho}{\rho} \sim 50. \tag{2.15}$$

There is no way to reconcile this result with the constraint (2.14). This is the most serious failure of the new inflationary universe.

Since the new inflationary model has many successes in explaining the cosmological conundrums, it is fruitful to investigate the requirements for a successful cosmology. Steinhardt and Turner [46], have set down the conditions to be satisfied by the potential of a scalar field to yield a successful cosmological model. We will briefly summarize their results:

1. The potential at the origin must be very flat.
2. The flat portion of the potential i.e., from $\phi = 0$ to $\phi = \phi_{end}$ in Fig 2.2, must be long.
3. The slow rollover transition must last long enough to give enough e-foldings ($Z > 10^{29}$ in (2.7)).
4. The potential should be such so as to give the correct order of magnitude for density fluctuations. It turns out that this puts a lower limit on the curvature near the origin.
5. The curvature at the true minimum $\phi = \sigma$ must be large enough to reheat the universe to a temperature $\geq 10^{10}$ GeV. This is crucial for generating a net baryon number in the universe through GUT interactions.

Any model that we construct, must satisfy these conditions so as to produce a reasonable cosmological scenario — except for No.5 which we will see later can be violated and still enough baryons can be produced.

3 ALTERNATIVE SCENARIOS

As we saw in the last section, the new inflationary scenario based on a reasonable Grand Unified Theory, eg. SU(5), is unacceptable cosmologically. In this section we will look at some alternative proposals to the new inflationary models. These proposals attempt to solve the problems of new inflation in a variety of ways — some natural and some unnatural.

Before we go one to discuss these alternatives, we should recapitulate some of the problems we encountered in the previous section. The quantity which is of importance in solving the horizon, flatness and monopole problems is the number of e-foldings of the scale factor. For the new inflationary models, this is defined as [46]

$$N = \int_{\phi_0}^{\phi_e} H dt \tag{2.16}$$

where ϕ_0 is the initial value of the inflaton field and $\phi_e = \phi_{end}$ (the end of the slow rollover in Fig 2.2.) Recall that to solve the problems of the standard big bang model we needed $N \geq 65$. The other quantity which was crucial in determining the success of the scenario was the magnitude of density fluctuations $\frac{\delta\rho}{\rho}$. In the new inflationary model [22, 38, 42]

$$\frac{\delta\rho}{\rho} = \frac{H\delta\phi(t_i)}{|\dot{\phi}(t_i)|} \tag{2.17}$$

where t_i is the time of horizon crossing in the de Sitter phase and $\delta\phi(t_i)$ the size of the fluctuation at time $t = t_i$. This quantity should be $\sim 10^{-4}$ for a

successful inflationary model (see the discussion surrounding eq. 2.14).

To understand how the cosmology of these models depends on the parameters in the basic particle physics model, let us cut off the logarithm at $\alpha_{GUT}\phi^2 = H^2$ and parameterize the Coleman-Weinberg potential (2.11a) near $\phi = 0$ as (at $T = 0$)

$$V(\phi) = V_0 - \frac{1}{2}\lambda\phi^4 \tag{2.18a}$$

where

$$V_0 = \frac{1}{2}A\sigma^4 \tag{2.18b}$$

and

$$\lambda \sim 4A\left[-\log\frac{H^2}{\alpha_{GUT}\sigma^2} + \frac{1}{2}\right]. \tag{2.18c}$$

In terms of this convenient parameterization it turns out that [33, 47]

$$N \sim \mathcal{O}(1)\lambda^{-1/2} \tag{2.19a}$$

and [22, 38, 42]

$$\frac{\delta\rho}{\rho} \sim \mathcal{O}(10)\lambda^{1/2}. \tag{2.19b}$$

In the previous section, λ was fixed (because σ, α and H were fixed) to be

$$\lambda \sim 4 \tag{2.20}$$

and therefore the constraints from (2.16) and (2.17) could not be satisfied.

Alternative inflationary models try to satisfy these constraints in a variety of ways. One of the approaches taken is to construct a model with a separate

inflaton field [48]. Now since λ is not fixed, we can tune it to a small value ($\lambda \sim 10^{-10}$) and postulate a Coleman-Weinberg type of potential for it. This inflaton has to be a gauge singlet, otherwise the radiative corrections from gauge interactions will induce a self coupling much bigger than λ.

Another approach has been to use supersymmetry. Supersymmetry is a symmetry which transforms bosons into fermions and vice versa [49]. Supersymmetric theories have become very popular in particle physics because they offer very natural solutions to some of the problems which plague gauge theories [50]. These problems include the gauge hierarchy problem which supersymmetry solves because of no-renormalization theorems [51].

The gauge hierarchy problem arises because of the existence of two widely separated scales in the theory — the weak breaking scale ($\sim 10^2$ GeV) and the GUT scale ($\sim 10^{15}$ GeV). The masses of the scalar particles (which are responsible for spontaneous breakdown of the gauge symmetry) are subject to quadratic divergences in perturbation theory which tend to push them up to the GUT scale. To ensure that the scalar masses are $\mathcal{O}(10^2)$ GeV they have to be unnaturally fine tuned. This problem is solved in supersymmetry because the divergences due to the bosons are cancelled by those due to the fermions. Thus the scalar masses are stable against radiative corrections and fixing them at a low scale is technically natural.

If supersymmetry has to do anything with our observed world, it must be

broken.(Otherwise, for example, we should see scalar electrons degenerate in mass with the electron.) This breaking of supersymmetry introduces another scale in the problem, namely the scale of supersymmetry breaking, M_s. In most successful phenomenological models, the scale M_s is related to the weak breaking scale M_W. The no-renormalization theorems then guarantee that these scales are stable in perturbation theory.

Initially, the supersymmetric models proposed employed global supersymmetry which was softly broken [52]. However, it was soon realized that making supersymmetry local automatically introduces gravity in the theory. This incorporation of gravity is significant from the unification point of view. These supergravity models have been used very extensively and successfully in model building [50].

From the point of view of inflation, supersymmetric models have many advantages over non-supersymmetric models [33, 47]. Firstly, a weakly coupled scalar field which is needed for inflation, arises naturally in supersymmetric theories. In exact supersymmetry, the first order corrections of the Coleman-Weinberg type vanish because of the mass degeneracy between the bosons and the fermions. This indicates that λ of (2.18c) is zero. However since supersymmetry must be broken, the mass degeneracy is lifted and we get a non-zero λ. In fact λ is propotional to $M_s^2\epsilon$ where ϵ is some coupling constant. In most models, a value of λ which is consistent with (2.16) and (2.17) is easily obtained. Furthermore, the no-renormalization theorems

guarantee a way out of the fine tuning problem. Recall that the mass terms in the new inflationary scenario had to be fine tuned to very small values to obtain enough inflation. This is no longer a problem because the corrections to the mass^2 terms can now be kept as low as the mass degeneracy between the bosons and the fermions, i.e. $\mathcal{O}(M_s^2\epsilon)$.

These considerations prompt us into constructing inflationary models incorporating supersymmetry [53]. The models which are the most attractive phenomenologically are the supergravity models in which supergravity is broken spontaneously. In these models, M_W is related to M_s in a natural way. We will construct such models with N=1 supergravity in the next chapter and study them in detail.

III SUPERSYMMETRIC INFLATIONARY COSMOLOGIES

The new inflationary scenario, while solving some of the cosmological problems of the standard hot big bang models, suffers from some drawbacks as we saw in the previous chapter. It was these drawbacks which forced us to look at alternative scenarios.

For the new inflationary scenario to be implemented, we need the presence of a very weakly coupled scalar field. Locally supersymmetric or supergravity theories provide such a scalar field [53]. Non-renormalization theorems in supersymmetry solve the problems of fine tuning,and thus such theories are very attractive from the inflationary point of view.

As remarked earlier, supersymmetry has to be broken to give us a realistic phenomenology. In the most popular supergravity theories, the breaking of supersymmetry is accomplished by the hidden sector. In analogy with the spontaneously broken gauge theories, the gravitino (the supersymmetric spin 3/2 partner of the graviton) acquires a mass through the Super Higgs effect [54]. The scale of supersymmetry breaking, M_s, is related to the mass of the gravitino $m_{3/2}$ [55]. Although there is no compelling reason for the hidden sector and the inflaton sector to be the same, it still seems desirable that the sector which drives inflation should also be the one that breaks supersymmetry.

Actually, in the case where this sector consists of one scalar field — called

inflaton — the thermal constraint imposes such a breaking of supersymmetry [56,57]. The thermal constraint is the requirement that at high temperatures, a sufficient amount of energy is stored in the scalar field to give enough inflation – in other words, the inflaton field must start its evolution far away from its global minimum, slowly roll down (causing the universe to inflate) and eventually settle at its global minimum. The problem with this approach is that supersymmetry must be broken at a very large scale: typically [57], the mass of the gravitino $m_{3/2}$ must be greater than μ^2/M. ($M = M_p/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV). Here μ^4 is the energy density of the false vacuum and a typical value for $\frac{\mu}{M}$ of 10^{-3} to 10^{-4} is required to give rise to density fluctuations with the right amplitude; $m_{3/2}$ is then greater than 10^{10} GeV. This has to be reconciled with models describing our low energy world where the breaking of $SU(2) \times U(1)$ gauge invariance is driven by soft terms induced by supergravity — which scale like $m_{3/2}$ [58]. Therefore in these models, the gravitino mass and the mass of the weak gauge boson M_W must be of the same order.

This problem has been addressed recently by Ovrut and Steinhardt who solve it by using two scalar fields in the inflationary sector [59]. They employ a mechanism [60] which sets the symmetry-breaking scale to a much smaller value than the scale μ: typically, the gravitino mass is of order $\frac{\mu^4}{M^3}$ which coincides therefore with the weak interaction scale ($\frac{\mu}{M} \sim 10^{-4}$). This can be worked out into a successful inflationary universe scenario [59] at the price

however of some fine-tuning (at least in the explicit example given in Ref. 59).

In this chapter, we will take a different point of view and relate the smallness of the scale of supersymmetry breaking to the smallness of a parameter which is of basic importance in any inflationary universe scenario: the slope ϵ of the potential near the origin. Actually, since we want the scale of supersymmetry-breaking to be very small compared to the scales of relevance in the inflation sector (of the order of the Planck mass), it seems plausible that the ground state must be obtained by perturbing a supersymmetry conserving ground state. We will see in Sect. 1 that this imposes some constraints on the model. We do not know for the moment what is the nature of the perturbation but it has to be characterized by a parameter which must be very small. A natural (or possible) choice is precisely the slope ϵ: if we want a slow roll-down along the plateau region of the potential, the slope has to be very small at the origin. Actually, in most models, it is taken to be zero. No symmetry argument supports such a choice and hence we have no reason for ϵ to be so small. But we will show that the supersymmetry breaking scale can be related to it for a particular class of potentials. Moreover, even though ϵ is arbitrarily small, the scale of supersymmetry breaking that we obtain is stable under radiative corrections. In other words, in our approach, choosing the gravitino mass of the order of M_W is natural in the technical sense. In Sect. 2, we describe the inflationary scenario that arises

in a model which we consider as a typical example of our approach and we discuss what kind of constraints we obtain for the parameters ϵ and μ. It turns out that the thermal constraint mentioned earlier is violated. In Sect. 3, we show how to circumvent this by introducing a second scalar field in the inflation sector.

1 THE MODELS

We first detail the procedure that we adopt to find a model that fulfills our requirements. The idea is to start with a potential for which $\epsilon = 0$ and the ground state is supersymmetry-conserving, then perturb this potential by taking $\epsilon \neq 0$ and see under which conditions the minimum becomes supersymmetry-breaking.

Let us first prove a result that applies to this situation in general, independently of the nature of the parameter ϵ. Consider a scalar field Φ in a locally supersymmetric theory. Its interactions are described by a superpotential $f(\Phi)$ and the corresponding potential reads (assuming a flat Kähler potential) [61]:

$$V(\Phi) = e^{|\Phi|^2/M^2}[\mid D_\Phi f(\Phi) \mid^2 - \frac{3}{M^2} \mid f(\Phi) \mid^2] \tag{3.1}$$

where

$$D_\Phi f(\Phi) = \frac{\partial f(\Phi)}{\partial \Phi} + \frac{\Phi^*}{M^2} f(\Phi) \tag{3.2}$$

and M is the reduced Planck mass $M = M_p/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$GeV. The variable ϵ parametrizes a perturbation on the coefficients of the superpotential, which is left unspecified for the present.

If the minimum — $\sigma_0 M$— of the potential V (with a zero cosmological constant) is supersymmetry-conserving when $\epsilon = 0$, then a necessary condition in order that the perturbed minimum (with zero cosmological constant) breaks supersymmetry is that:

$$\frac{\partial^2 f}{\partial \Phi^2}(\sigma_0)\Big|_{\epsilon=0} = 0 \tag{3.3}$$

An equivalent formulation involving the potential is that its second derivative (and then automatically its third one) is zero at the minimum:

$$\frac{d^2 V}{d\Phi^2}(\sigma_0)\Big|_{\epsilon=0} = \frac{d^3 V}{d\Phi^3}(\sigma_0)\Big|_{\epsilon=0} = 0 \tag{3.4}$$

The proof is straightforward.

Since, when $\epsilon = 0$, the minimum $\sigma_0 (V(\sigma_0) = V'(\sigma_0) = 0)$ conserves supersymmetry:

$$f(\sigma_0)|_{\epsilon=0} = \frac{\partial}{\partial \Phi} f(\sigma_0)\Big|_{\epsilon=0} = 0 \tag{3.5}$$

On the other hand, since we want a breaking of supersymmetry when we turn ϵ on, we must require that:

$$f(\sigma) \simeq 0(\epsilon) \neq 0 \qquad D_\Phi f(\sigma) \simeq 0(\epsilon) \neq 0 \tag{3.6}$$

The minimum σ is determined by the equations (Φ and f are taken to be real):

$$V(\sigma) = 0 \leftrightarrow | D_\Phi f(\sigma) |^2 = 3 \mid f(\sigma) |^2 \tag{3.7}$$

$$V'(\sigma) = 0 \leftrightarrow [\frac{\partial}{\partial \Phi} D_\Phi f(\sigma)] D_\Phi f(\sigma) = 3[\frac{\partial}{\partial \Phi} f(\sigma)] f(\sigma) \tag{3.8}$$

Therefore combining (3.6) and (3.7), we obtain from (3.8):

$$\frac{\partial}{\partial \Phi} D_\Phi f(\sigma) = \pm\sqrt{3} \frac{\partial}{\partial \Phi} f(\sigma) = 0(\epsilon) \tag{3.9}$$

which in turn gives (3.3). It is immediate to show, using the form of the potential [(3.1) and (3.2)] that (3.3) and (3.4) are equivalent.

The condition (3.3) is only necessary. We now turn to the study of the potential near the perturbed minimum to determine the sufficient conditions. It will prove to be crucial to study the potential in the complex plane. We therefore write (we take $M = 1$)

$$\Phi \equiv \phi + i\chi \equiv \sigma_0 + \tilde{\phi} + i\chi. \tag{3.10}$$

The superpotential can be written as a power series in ϵ:

$$f(\Phi) = f_0(\Phi) + \epsilon f_1(\Phi) + \epsilon^2 f_2(\Phi) \ldots \tag{3.11}$$

Using these notations, (3.3) and (3.5) now read

$$f_0(\sigma_0) = f_0'(\sigma_0) = f_0''(\sigma_0) = 0. \tag{3.12}$$

The non-linear nature of the relation between potential and superpotential causes $\tilde{\phi}$ and χ to get a vacuum expectation value of order $\epsilon^{1/2}$. To determine under what conditions this happens, we have to keep terms in the potential up to order ϵ^2 (where we consider $\tilde{\phi}$ and χ to be of order $\epsilon^{1/2}$). Using (3.12) we obtain from (3.1)[1]

$$
\begin{aligned}
V(\tilde{\phi},\chi)\Big|_{\epsilon^2} &= e^{\sigma_0^2}\left\{\left|\epsilon f_1'(\sigma_0)+\frac{1}{2}(\tilde{\phi}+i\chi)^2 f_0'''(\sigma_0)+\sigma_0\epsilon f_1(\sigma_0)\right|^2\right. \\
&\qquad \left. -3|\,\epsilon f_1(\sigma_0)\,|^2\right\} \\
&= e^{\sigma_0^2}\left\{\epsilon^2\left[(f_1'(\sigma_0)+\sigma_0 f_1(\sigma_0))^2-3f_1(\sigma_0)^2\right]\right. \\
&\qquad +\epsilon f_0'''(\sigma_0)(f_1'(\sigma_0)+\sigma_0 f_1(\sigma_0))(\tilde{\phi}^2-\chi^2) \\
&\qquad \left. +\frac{1}{4}f_0'''(\sigma_0)^2(\tilde{\phi}+\chi^2)^2\right\}
\end{aligned}
\tag{3.13}
$$

We therefore have three possible extrema:

$$
\begin{aligned}
&\text{a)} \quad \tilde{\phi}_0=\chi_0\simeq 0(\epsilon) \\
&\text{b)} \quad \tilde{\phi}_0=0(\epsilon), \quad \chi_0^2=2\epsilon\frac{f_1'(\sigma_0)+\sigma_0 f_1(\sigma_0)}{f_0'''(\sigma_0)} \\
&\text{c)} \quad \tilde{\phi}_0^2=-2\epsilon\frac{f_1'(\sigma_0)+\sigma_0 f_1(\sigma_0)}{f_0'''(\sigma_0)}, \quad \chi_0\simeq 0(\epsilon)
\end{aligned}
\tag{3.14}
$$

where we have supposed that $f_0'''(\sigma_0)\neq 0$. In all three cases, the requirement that there is no cosmological constant at the new minimum gives the

[1] If $\tilde{\phi}$ and χ were of order $\epsilon^{1/n}(n>2)$, the leading term in the potential would be $e^{\sigma_0^2}|\,\frac{1}{2}(\tilde{\phi}+i\chi)^2 f_0'''(\sigma_0)\,|^2$ which is of order $\epsilon^{4/n}>\epsilon^{1+2/n}$ (next to leading terms). This has clearly a minimum only at the origin. Therefore $\tilde{\phi}$ and χ are zero at the order $\epsilon^{1/n}, n>2:\tilde{\phi},\chi\simeq 0(\epsilon^{1/2})$.

condition:

$$f_1'(\sigma_0)=0. \tag{3.15}$$

Moreover the second derivatives at the new minimum are non-negative at the condition that, respectively:

$$
\begin{aligned}
&\text{a)}\, f_1'(\sigma_0)=0 \\
&\text{b)}\, f_1'(\sigma_0)>0 \\
&\text{c)}\, f_1'(\sigma_0)<0
\end{aligned}
\tag{3.16}
$$

To conclude, a sufficient condition for having a perturbed minimum which is supersymmetry-breaking and corresponds to a zero cosmological constant is:

$$f_1(\sigma_0)=0 \qquad f_1'(\sigma_0)\neq 0 \tag{3.17}$$

along with the constraint (3.3) on the unperturbed superpotential. Let us compute, for example, the gravitino mass corresponding to case (3.16c) ($f_1'(\sigma_0)<0$). It is given by [61]

$$m_{3/2}=e^{|\Phi_0|^2/2}\,|\,f(\Phi_0)\,|\,, \quad \Phi_0=\sigma_0+\tilde{\phi}_0+\chi_0 \tag{3.18}$$

where, to the lowest order in the expansion parameter ϵ,

$$f(\tilde{\phi}_0,\chi_0)=\tilde{\phi}\epsilon f_1'(\sigma_0)+\frac{1}{6}\tilde{\phi}_0^3 f_0'''(\sigma_0). \tag{3.19}$$

Therefore, using (3.14c)

$$m_{3/2} = \frac{2}{3}\epsilon^{3/2}e^{\sigma_0^2/2}\left[\left|2\frac{f_1'(\sigma_0)^3}{f_0'''(\sigma_0)}\right|\right]^{1/2}. \tag{3.20}$$

A similar result is valid in case (3.16b) ($f_1'(\sigma_0) > 0$).

In case (3.16a) ($f_1'(\sigma_0) = 0$), one has to push the analysis to the next order in ϵ to see if supersymmetry is broken at the new minimum.

Before turning to a specific example, let us summarize at this point our analysis of the general case. In order to have a global minimum of the perturbed potential which is supersymmetry-breaking, we have to impose on the unperturbed superpotential the conditions (3.3) and (3.5)

$$f_0(\sigma_0) = f_0'(\sigma_0) = f_0''(\sigma_0) = 0 \tag{3.12}$$

and on the unperturbed superpotential (to first order in the perturbation) the condition (3.17)

$$f_1(\sigma_0) = 0 \qquad f_1'(\sigma_0) \neq 0. \tag{3.17}$$

In that case, the gravitino mass which determines the scale of supersymmetry-breaking is of order $\epsilon^{3/2}$ (3.20).

We now apply this result to a specific example where the small parameter ϵ is related to the slope of the potential near the origin. Let us consider the superpotential

$$f(\Phi) = \mu^2 M(a_0 + a_1\frac{\Phi}{M} + a_2\frac{\Phi^2}{M^2} + a_4\frac{\Phi^4}{M^4}). \tag{3.21}$$

The corresponding potential reads, near the origin on the real axis,

$$V(\phi) = \mu^4 e^{\phi^2/M^2}[(1 - 3a_0^2) + 4(a_2 - a_0)\frac{\phi}{M} + \ldots] \tag{3.22}$$

As a first step, we require that the potential is flat near the origin and that the minimum $\sigma_0 M$ is supersymmetry-conserving. The first condition ($V'(0) = 0$) gives:

$$a_2 = a_0 \equiv \beta_0 \tag{3.23}$$

and the second (3.5) yields

$$\beta_0 = \frac{-3\sigma_0}{2(2+\sigma_0^2)}, \qquad \sigma_0^2 = \frac{-\beta_0 \pm \sqrt{\beta_0(\beta_0 + 12a_4)}}{6a_4} \tag{3.24}$$

It has been noticed already [56] that in such a family of superpotentials, where β_0 and σ_0 are of opposite signs there is a violation of the thermal constraint [56,57]. Condition (3.3) reads

$$6a_4\sigma_0^2 + \beta_0 = 0 \tag{3.25}$$

in which case we obtain from (3.24)

$$a_4 = -\frac{1}{12}\beta\,, \sigma_0 = \sqrt{2}\,, \beta_0 = -\frac{3}{8}\sigma_0. \tag{3.26}$$

Hence (3.21) takes the form

$$f_0(\Phi) = \mu^2 M\sqrt{2}[-\frac{3}{8} + \frac{\Phi}{M\sqrt{2}} - \frac{3}{4}(\frac{\Phi}{M\sqrt{2}})^2 + \frac{1}{8}(\frac{\Phi}{M\sqrt{2}})^4] \tag{3.27}$$

which has a supersymmetry conserving minimum at $\sigma_0 = \sqrt{2}$.

The corresponding potential $V_0(\Phi)$ is very flat near its minimum σ_0 since its first three derivatives are zero (3.14):

$$V_0(\Phi) = \mu^4 e^2 \frac{9}{16}(\tilde{\phi}^2 + \chi^2)^2 + 0(\tilde{\phi}^5, \chi^2\tilde{\phi}^3, \chi^4\tilde{\phi}) \tag{3.28}$$

One can easily show that the real axis is a valley of the potential V_0. If initial conditions (high temperatures?) force the inflaton field to start at the origin, it will evolve along the real axis until it reaches the minimum σ_0. The behavior of V_0 is therefore shown in Fig. 3.1 on the real axis, together with its shape at temperature $T = M$, as computed from the results of Ref. 57. As stressed earlier, the temperature corrections do not stabilize the field at the origin, at high temperatures.[2] We will return to that point in Sec. 3.

We then relax condition (3.23) by allowing a small slope near the origin and we write instead:

$$a_2 = a_0 - \epsilon \equiv \beta. \tag{3.29}$$

The superpotential now reads:

$$\begin{aligned} f(\Phi) &= \mu^2 M \left[\beta + \epsilon + \frac{\Phi}{M} + \beta(\frac{\Phi}{M})^2 - \frac{1}{12}\beta(\frac{\Phi}{M})^4\right] \\ \beta &= \beta_0 + \beta_1\epsilon + 0(\epsilon^2) \end{aligned} \tag{3.30}$$

Condition (3.17) gives

$$\beta_1 = -\frac{3}{8} \qquad f_1'(\sigma_0) = \frac{4}{3}\mu^2\sqrt{2}\beta_1 < 0. \tag{3.31}$$

[2]One can show that this is so even if we include a cubic term in (3.21).

which shows that we are in case (3.16c). The minimum is therefore at

$$\tilde{\phi}_0 = \epsilon^{1/2}\left(\frac{2\sqrt{2}}{3}\right)^{1/2} \qquad \chi_0 \simeq 0(\epsilon) \tag{3.32}$$

and the gravitino mass which determines the scale of supersymmetry breaking is given by (3.20):

$$m_{3/2} = \frac{2}{9}\frac{\mu^2}{M}\,\epsilon^{3/2}\,e\sqrt{3\sqrt{2}} \tag{3.33}$$

The first terms of the corresponding potential expanded around the origin are, to order ϵ, on the real axis:

$$\begin{aligned} \frac{V(\phi)}{\mu^4} &= \frac{5}{32}(1 + 9\sqrt{2}\epsilon) - 4\epsilon\frac{\phi}{M} + \epsilon\frac{9}{4}\sqrt{2}\frac{\phi^2}{M^2} \\ &- \frac{1}{2}(\sqrt{2} + 5\epsilon)\frac{\phi^3}{M^3} + 0\left(\epsilon^2, \frac{\phi^4}{M^4}\right) \end{aligned} \tag{3.34}$$

and around the minimum $\sigma M = \sigma_0 M + \tilde{\phi}_0$, to order ϵ:

$$\begin{aligned} \frac{V(\phi)}{\mu^4 e^2} &= \left(\frac{\phi}{M} - \sigma\right)^2 \epsilon\frac{3}{2}\sqrt{2} + \left(\frac{\phi}{M} - \sigma\right)^3 \left[\epsilon^{1/2}\frac{3}{4}(6\sqrt{2})^{1/2} + \epsilon\frac{69}{4}\right] \\ &+ \left(\frac{\phi}{M} - \sigma\right)^4 \left[\frac{9}{16} + \epsilon^{1/2}\frac{45}{16}(6\sqrt{2})^{1/2} + \epsilon\frac{95}{2}\sqrt{2}\right] + 0(\epsilon^2) + 0((\frac{\phi}{M} - \sigma)^5) \end{aligned} \tag{3.35}$$

If this were plotted on Fig. 3.1, it will be indistinguishable from potential V_0.

An objection that could raised to our linking the scale of supersymmetry breaking to the parameter ϵ is that we need to choose an arbitrarily small value for ϵ. This seems unnatural (in the technical sense); the radiative

corrections could induce large corrections to the scale of supersymmetry, thus putting an end to our hopes of bringing that scale down to M_W. But, as we will now see, one has to take into account the very special properties of renormalization in supersymmetric theories and the unique features of the superpotential that we consider [(3.3)]. The one-loop radiative corrections to the potential V of (3.1) are given by [62]

$$\delta V_\Lambda = \kappa \left(V + e^{|\Phi|^2/M^2} \frac{|\, f(\Phi)\, |^2}{M^2} \right) \tag{3.36}$$

where

$$\kappa = \frac{1}{16\pi^2} \frac{\Lambda^2}{M^2} (N-1). \tag{3.37}$$

In this formula, Λ is the cut-off which is of $O(M)$ and N is the total number of chiral fields in the theory.

We first note that, since $f(\sigma_0)|_{\epsilon=0} = f'(\sigma_0)|_{\epsilon=0} = 0$, the ground state remains unchanged at the zeroth order in ϵ when we include the radiative corrections. Moreover it is straightforward to prove that if V satisfies (3.4) then $V + \delta V_\Lambda$ satisfies also (3.4) (using (3.3) for f). Therefore our necessary condition is stable under radiative corrections. Moreover, the second term in (3.36) does not contribute to the development of the potential up to order ϵ^2, when we study the region $\tilde{\phi}, \chi \simeq 0(\epsilon^{1/2})$, since, in this region,

$$f(\Phi) = \frac{1}{6} f_0'''(\sigma_0)(\tilde{\phi} + i\chi)^3 + \epsilon(\tilde{\phi} + i\chi) f_1'(\sigma_0) + \ldots \simeq 0(\epsilon^{3/2}) \tag{3.38}$$

Let us note that this is true only because we required $f_1(\sigma_0) = 0$ (3.17). Thus the expression (3.13) for the potential up to order ϵ^2 is only multiplied by an overall factor $(1+\kappa)$ when we include the one-loop corrections and our analysis of the minimum is unchanged. Therefore the mass of the gravitino which determines the scale of supersymmetry breaking is still of order $\frac{\mu^2}{M}\epsilon^{3/2}$ as in (3.20) or (3.33). It is precisely this fact that justifies our approach a posteriori. Had corrections of order $\mu^2\kappa$, for example, appeared in the gravitino mass, the smallness of the scale of supersymmetry breaking would have been a unnatural feature of our model.

2 COSMOLOGICAL CONSTRAINTS

We now review the set of constraints that the models that we consider must satisfy in order to give rise to a successful cosmological scenario [see for example Ref. (46)]. We will do that for the explicit example of (3.30) but, its salient features being a consequence of (3.3) and therefore shared by more general potentials, we believe that this analysis is applicable to any such potential. The time evolution of the inflaton field is summarized in Table 1.

The inflationary period starts when the energy density becomes dominated by the energy stored in the vacuum:

$$\rho_0 = V_0 = \frac{5}{32}\mu^4 \tag{3.39}$$

We assume that the inflaton field is initially located near the origin; then its value when inflation starts is of the order of the Hubble parameter:

$$\phi_0 \simeq H_0 = (\frac{\rho_0}{3M^2})^{1/2} = \sqrt{\frac{5}{96}}\frac{\mu^2}{M} \tag{3.40}$$

As long as radiation can be neglected, the classical evolution of the inflaton field is governed by the equations

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} &= -\frac{\partial V}{\partial \phi} \\ H^2 &= \frac{1}{3M^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \end{aligned} \tag{3.41}$$

During the slow rollover — *i.e.* the inflationary period — the motion of ϕ is friction dominated and the $\ddot{\phi}$ term is negligible. In terms of the potential, this can be expressed as [46]

$$\begin{aligned} V''(\phi) &\leq \frac{3}{M^2} \mid V(\phi) \mid \\ V'(\phi) &\leq \frac{\sqrt{6}}{M} \mid V(\phi) \mid \end{aligned} \tag{3.42}$$

In the class of potentials that we consider, it is the second of these equations that breaks down first, at a value ϕ_e that is almost independent of ϵ:

$$\phi_e \simeq 0.71M \ , \quad V(\phi_e) \simeq 8 \times 10^{-2}\mu^4. \tag{3.43}$$

The number of e-foldings that the scale factor undergoes during inflation is given by

$$N = -\int_{\phi_0}^{\phi_e} \frac{3H^2}{V'(\phi)} d\phi. \tag{3.44}$$

We can approximate the Hubble parameter in the numerator by its value at the origin [(3.39)] and the potential in the denominator by the first terms of its expansion [(3.34)]. When $\epsilon \ll \frac{\mu^4}{M^4}$ (in particular $\epsilon = 0$), the main contribution comes from the lower bound ϕ_0 (where the field spends most of the inflation epoch):

$$N = \sqrt{\frac{5}{48}}\frac{M^2}{\mu^2} + 0(\frac{M}{\mu}) \tag{3.45a}$$

On the other hand, when $\epsilon \gg \frac{\mu^4}{M^4}$ (non-negligible linear term in the potential), the upper bound ϕ_e gives the leading contribution which happens to be independent of μ:

$$N = \frac{5\pi}{32}\frac{1}{(24\sqrt{2})^{1/2}}\epsilon^{-1/2} \tag{3.45b}$$

We checked numerically that these approximate formulas are very accurate and computed N in the intermediate region ($\epsilon \simeq \mu^4/M^4$). A value of N typically greater than 60 is required [10] if we want our present observable universe to have emerged from a single causally-connected patch. Using (eq. 3.45a,b) and our numerical computation, we can use the condition $N > 60$ to constrain our parameters ϵ and μ/M. To be more accurate, we must take into account that some time elapses between the end of inflation and reheating. During this period, the cosmic scale factor R grows by a factor

$$\frac{R(\Gamma^{-1})}{R(t_e)} \sim \left(\frac{\rho_\phi(t_e)}{M^4}\right)^{1/4}\left(\frac{\rho_\phi(t_t)}{M^4}\right)^{1/12}\left(\frac{\Gamma}{M}\right)^{-2/3}$$

(c.f. (3.49) and (3.53); t_t is defined in (3.50), (3.51) and Γ is the inflaton

decay rate). Expressing all these quantities in terms of μ and ϵ, we obtain the condition on the number of e-foldings [46]

$$N > 66.5 + \frac{5}{3}\ln\frac{\mu}{M} - \frac{1}{12}\ln\epsilon. \tag{3.46}$$

We draw the corresponding curve (labelled "$N = 60$") in the ϵ, μ/M plane of Fig. 3.2.

The inflation field reaches the value ϕ_e at time $t_e \simeq \frac{N}{H_0}$. This value corresponds also to the point where the curvature of the potential changes sign. The field therefore starts oscillating around the minimum σ. At first, it does not feel the details of order ϵ of the potential near the minimum [(3.35)] and therefore oscillates in the ϕ^4 potential of (3.28). We show in Fig. (3.3) the first few oscillations of the inflation field ($t_e < t < 1.006t_e$). It is straightforward to compute the frequency of these oscillations:

$$\frac{\omega}{M} \simeq 0.2\frac{\mu^2}{M^2} \tag{3.47}$$

We have

$$\frac{\omega}{M} \gtrsim \frac{H_e}{M} \equiv \left(\frac{V(\phi_e)}{3M^4}\right)^{1/2} \simeq 0.13\frac{\mu^2}{M^2} \tag{3.48}$$

and, after a few oscillation, $\omega \gg H$. Following Turner [63], we average over an oscillation period, and note that the energy associated with the coherent field oscillations behaves like relativistic matter. Therefore the cosmic scalar

factor R and the coherent energy density ρ_ϕ scale with time as:

$$\frac{R(t)}{R(t_e)} = [1 + 2H_e(t - t_e)]^{1/2} \quad \frac{\rho_\phi(t)}{\rho_\phi(t_e)} = \left(\frac{R(t)}{R(t_e)}\right)^{-4} \tag{3.49}$$

This will continue until time t_t when the field oscillations take place only in the close vicinity of the minimum where the potential can be approximated by the first term of its expansion in (3.35). This will happen approximately for:

$$\left|\frac{\phi_t}{M} - \sigma\right| \simeq \epsilon^{1/2}\frac{2}{3}\sqrt{6\sqrt{2}}\,, \quad V(\phi_t) \simeq 32\epsilon^2\mu^4 e^2 \tag{3.50}$$

which gives, from (3.49)

$$t_t \simeq 6 \times 10^{-2}\epsilon^{-1}\frac{M}{\mu^2}. \tag{3.51}$$

From t_t onward, we can consider that the field oscillates in a ϕ^2 potential, with a frequency equal to the mass of the inflaton field:

$$m_\phi \simeq \frac{\mu^2}{M}\epsilon^{1/2} e\sqrt{3\sqrt{2}} \tag{3.52}$$

and (since $m_\phi > H(\phi_t) \equiv H_t$), according to Ref. [63], the coherent energy density behaves like non-relativistic matter. Therefore, for $t > t_t$,

$$\frac{R(t)}{R(t_t)} = \left[1 + \frac{3}{2}H_t(t - t_t)\right]^{2/3} \quad \frac{\rho_\phi(t)}{\rho_\phi(t_t)} = \left(\frac{R(t)}{R(t_t)}\right)^{-3}. \tag{3.53}$$

This will last until $t \sim \Gamma^{-1}$ when reheating takes place through the decay

of the inflaton field. The decay rate is, following (3.52),

$$\Gamma \sim \frac{m_\phi^3}{M^2} \sim \frac{\mu^6}{M^5}\,\epsilon^{3/2}. \tag{3.54}$$

The photon density at $t = \Gamma^{-1}$ reads (assuming that the inflaton decays mostly into photons)

$$\rho_\gamma\left(\Gamma^{-1}\right) = \rho_\gamma(t_t)\left(\frac{R(\Gamma^{-1})}{R(t_t)}\right)^{-4} + \rho_\phi(t_t)\left(\frac{R(\Gamma^{-1})}{R(t_t)}\right)^{-3}. \tag{3.55}$$

One can check that $\frac{R(\Gamma^{-1})}{R(t_t)} \gg 1$ and $\rho_\phi(t_t) \gg \rho_\gamma(t_t)$, in which case the second term is dominant. Using (3.53), we thus obtain

$$\rho_\gamma(\Gamma^{-1}) \simeq \frac{4}{3}(\Gamma M)^2 \tag{3.56}$$

and the universe is reheated to a temperature

$$T_{RH} = \left(\frac{40}{\pi^2 g^*}\right)^{1/4}(\Gamma M)^{1/2} \tag{3.57}$$

where g^* is the number of effective spin degrees of freedom ($g^* \sim 10^2$). We see that, although our potential has some peculiar features, the result for the reheating temperature agrees with the standard one (in the so-called poor reheating case)[46,63,64]. Using (3.54) we find that

$$T_{RH} \sim \frac{\mu^3}{M^2}\epsilon^{3/4}. \tag{3.58}$$

We wish to emphasize at this point the complete generality of this result. Because all the potentials that we consider must satisfy (3.4), the mass of

the inflaton field must of of order ϵ and is therefore given by (3.52) [in the general case, μ is an overall scale defined as in (3.10)]. This in turn gives (3.54) for Γ and (3.58) for T_{RH}.

Before discussing the consequences of such a low reheating temperature, we have to further constrain the parameter μ/M by studying the amplitude of the density fluctuations. It is well known [22,38,44] that inflationary models yield a scale independent spectrum (the so-called Harrison-Zel'dovich spectrum [39]) with an amplitude at time t_f when the fluctuations reenter the horizon in the FRW phase given by:

$$\frac{\delta\rho}{\rho}(t_f) = 0(1)\frac{\delta\dot{\phi}(t_i)}{\dot{\phi}(t_i)} \tag{3.59}$$

where t_i is the time when the perturbations leave the horizon in the de Sitter phase, and $\delta\phi(t)$ is the space-averaged perturbation of the scalar field.

One can show that the number of e-foldings that take place between t_i and the end of inflation t_e is given, for a scale ℓ, by [46]

$$N_\ell = \int_{t_i}^{t_e} H\,dt = \frac{1}{3}\ln\left(\frac{M_\ell}{M_\odot}\,\frac{T_{RH}}{MeV}\,\frac{V(\phi_e)^{1/2}}{MeV^2}\right) \tag{3.60}$$

where M_ℓ is the corresponding mass scale. Considering the typical scale of a large galaxy ($M_\ell \simeq 10^{15}M_\odot$) and using Eqs. (3.43) and (3.58), this gives:

$$N_\ell = 60 + \frac{5}{3}\ln\frac{M}{\mu} + \frac{1}{4}\ln\epsilon \tag{3.61}$$

It is easy to show that the corresponding value for the scalar field $\phi(t_i)$

($\gg \epsilon^{1/2}$) is given by

$$\frac{\phi(t_i)}{M} \simeq \frac{5}{48\sqrt{2}} \frac{1}{N_\ell} \simeq 1.7 \times 10^{-3}. \tag{3.62}$$

Since we are in the slow-rollover period of the evolution of the ϕ field, its motion is friction-dominated and, linearizing the equation of motion for $\delta\phi$, we can write (3.59) as:

$$\frac{\delta\rho}{\rho}(t_f) = 0(1) \left| \frac{V(''\phi(t_i))}{V'(\phi(t_i))} \right| \delta\phi(t_i) \simeq 0(1) \frac{\delta\phi(t_i)}{\phi(t_i)} \tag{3.63}$$

Taking $\delta\phi(t_i) \simeq \frac{H_0}{2\pi}$ [22,33,38,42], we obtain from (3.40) and (3.62)

$$\frac{\delta\rho}{\rho}(t_f) \simeq 20 \frac{\mu^2}{M^2}. \tag{3.64}$$

Let us note that the uncertainty on the numerical factor (20) is at least of one order of magnitude. If we consider that the amplification factor due to the evolution of the fluctuations subsequent to t_f is not larger than 10^5 [see e.g. Ref. (41)], galaxy formation ($\frac{\delta\rho}{\rho} \sim 0(1)$) requires

$$\frac{\delta\rho}{\rho}(t_f) > 10^{-5} \quad \text{and} \quad \frac{\mu}{M} > 7 \times 10^{-4}. \tag{3.65}$$

On the other hand, the scales relevant to the cosmic microwave background reenter the horizon when the universe is matter-dominated, which decreases the amplitude of the density fluctuations [(3.61)] by a factor $\frac{1}{10}$ [38]. Following Sachs and Wolfe [40], this gives an anisotropy in the cosmic microwave

background

$$\frac{\delta T}{T} \simeq \frac{1}{2} \frac{\delta\rho}{\rho}(t_f) \simeq \frac{\mu^2}{M^2}. \tag{3.66}$$

Allowing for an observed temperature anisotropy on large angular scales smaller than 10^{-4} puts a limit

$$\frac{\mu}{M} < 10^{-2}. \tag{3.67}$$

Therefore the study of the amplitude of the density fluctuations restrict the parameter μ/M to the region $10^{-4} < \mu/M < 10^{-2}$, as shown in Fig. 3.2. On the same figure, we have also drawn the curve $m_{3/2} = M_W$ where $m_{3/2}$ is given by (3.33). If we restrict ourselves to such values of the supersymmetry-breaking scale, then ϵ is typically of order $10^{-(7\pm1.5)}$.

We now turn to the problem of baryon number generation. The limits on $\frac{\mu}{M}$ [(3.67)] and ϵ (see Fig. 3.2) put a bound on the reheating temperature:

$$T_{RH} < 10^{+8}\, GeV \tag{3.68}$$

Similarly, because the mass of the ϕ field is related to the gravitino mass according to [compare (3.33) and (3.52)]

$$m_\phi \simeq \frac{9}{2} \epsilon^{-1} m_{3/2} \tag{3.69}$$

the ϕ field is too light (taking $m_{3/2} = M_W$ gives $m_\phi < 10^9 M_W = 10^{11}$ GeV) to decay into the color triplet isosinglet superheavy Higgs bosons whose decays

can lead to baryon number generation. Therefore, such superheavy Higgs bosons cannot be produced directly by the decay of the coherent inflaton field oscillations, as in the standard poor reheating scenarios [36]. We thus have to resort to models of cosmological baryon generation at low temperature [65][3] We will also see in Sect. (3) that in trying to fulfill the thermal constraint, we gain another possible solution to baryogenesis. We will discuss the issue of baryosynthesis in these models in great detail in the next chapter.

We finally consider the so-called gravitino problem. Light gravitinos such as the ones that we consider have a very long lifetime:

$$\Gamma_{3/2} \sim \frac{m_{3/2}^3}{M^2} \sim \frac{\mu^6}{M^5}\epsilon^{9/2} \tag{3.70}$$

It is therefore quite plausible that they will become non relativistic and dominate the energy density of the universe before they decay, which would dramatically perturb the successes of the standard big bang scenario. Gravitinos produced before inflation are diluted away [66] and we need not consider them. But they can be produced by thermal equilibrium processes after reheating [67,68] or directly through the decay of the inflaton field [69].

[3]Let us note however that in the case of interest to us ($m_{3/2} = M_W$), the reheating temperature is in the 10^2 GeV, 10^6 GeV region ($T_{RH} = (m_{3/2}/M)^{3/2}M\epsilon^{-3/2} \simeq 10^{-6}$ GeV $\epsilon^{-3/2}$). We therefore need to adapt the models of Ref. [65] to such a low reheating temperature. This is possible because the mass of the fields responsible for baryon number generation is only limited by the mass of the inflaton field which lies in the 10^8 to 10^{11} GeV region [see (3.69)].

In the first case, the density of gravitinos produced after reheating has been shown to be proportional to T_{RH} [68] and a low value for T_{RH} can solve the problem. It turns out that the most stringent bound comes from the analysis of deuterium dissociation caused by the photons resulting from gravitino decays; this gives [68]

$$T_{RH} < \left(\frac{m_{3/2}}{100GeV}\right)^{-1} \times 10^{10}GeV. \tag{3.71}$$

It is easy to check from (3.33) and (3.58) that this does not give any further constraint on the parameters ϵ and μ/M.

The second source of gravitinos is the decay of the inflaton itself. Using an argument due to Ovrut and Steinhardt [69], one can show that, because the mass of the inflaton field is much bigger than the reheating temperature, the gravitinos that it produces will remain relativistic for a long period and will decay before they dominate the energy density of the universe.

3 A SOLUTION TO THE THERMAL CONSTRAINT

We stressed earlier that the temperature corrections to the potential V_0 or V do not have an absolute minimum at the origin [see Fig. 3.1]. Therefore the thermal constraint is not satisfied. In this section, we wish to study in detail a remedy to this problem which has been recently suggested [64]. The idea is to introduce a second chiral field in the inflaton sector of the theory. We will denote its scalar component by Ψ. The superpotential is chosen to

be:

$$W(\Phi,\Psi) = f(\Phi) + \Psi^2 g(\Phi) \tag{3.72}$$

where $f(\Phi)$ is given by (3.30) and g is a function of the Φ field only. Actually, we will only be interested here in the first terms of $g(\Phi)$ and write

$$g(\Phi) = \mu^2 M \left[b_0 + b_1 \frac{\Phi}{M}\right]. \tag{3.73}$$

The corresponding potential $\tilde{V}(\Phi,\Psi)$ is given by the standard formula, generalizing (3.1) to the case of two fields:

$$\tilde{V}(\Phi,\Psi) = e^{\frac{|\Phi|^2}{M^2}+\frac{|\Psi|^2}{M^2}} \left[\left|\frac{\partial W}{\partial \Phi} + \frac{\Phi^*}{M^2}W\right|^2 + \left|\frac{\partial W}{\partial \Psi} + \frac{\Psi^*}{M^2}W\right|^2 - \frac{3}{M^2}|W|^2\right] \tag{3.74}$$

From the results of Ref. [57], it is easy to compute the temperature corrections to that potential. For a moment, we will restrict ourselves to the $\Psi = 0$ direction, where the potential at temperature T reads:

$$\tilde{V}_T(\Phi,\Psi=0) = V_T(\Phi) + \frac{T^2}{24M^2}\, 12\mu^4 e^{|\Phi|^2/M^2}\left|b_0 + b_1\frac{\Phi}{M}\right|^2 \tag{3.75}$$

$V_T(\Phi)$ is the non-zero temperature version of the potential $V(\Phi)$ studied in the previous sections. Its first terms in a Φ expansion are:

$$\begin{aligned} V_T(\Phi) =& V(\Phi) + \frac{T^2}{24M^2}\mu^4 e^{|\Phi|^2/M^2}\left\{\left(\frac{7}{8}N + \frac{13}{16}\right)\right. \\ & \left. -\frac{3}{4}\sqrt{2}(N+2)\left(\frac{\Phi^*}{M} + \frac{\Phi}{M}\right) + 0(\epsilon) + \ldots\right\} \end{aligned} \tag{3.76}$$

where N is the total number of chiral fields in the theory [57,70]. Typically,

N is of order 10^2. It is clear from (3.76) that the potential V alone does not satisfy the thermal constraint since already the linear term in Φ tends to destabilize the inflaton field towards the minimum σ. But if we allow the parameters of $g(\Phi)$ — b_0 and b_1 — to satisfy the relation:

$$b_0 b_1 > \frac{1}{16}\sqrt{2}(N+2) \tag{3.77}$$

the extra terms in (3.75) will thwart this effect and stabilize the field Φ near the origin (at least along $\Psi = 0$). Similarly, the coefficients of higher order terms in $g(\Phi)$ can be arranged in order to cancel destabilizing effects of higher order terms in $V_T(\Phi)$.

Of course, if we consider the superpotential $W(\Phi,\Psi)$ as a whole, the constraint (3.77) which imposes that certain parameters (b_0, b_1) are of order N ($\simeq 10^2$) compared with the others, is extremely artificial. This could be a sufficient reason for rejecting the solution of introducing a second field in the inflation sector, and advocating some new mechanism to explain why the scalar field Φ starts its evolution near the origin.[4] We will however pursue that solution to see what we can gain from it. In fact, we will take b_0 of order N and b_1 of order 1 [satisfying (3.77)] and show that this is enough to obtain an absolute minimum at high temperature near the origin and a valley of the potential (at $T = 0$ and $T \neq 0$) in the $\Psi = 0$ direction.

It is easy to see that, because $W(\Phi,\Psi)$ has only terms independent of Ψ

[4]The chaotic inflation scenario of Linde [71] could actually provide an answer.

or quadratic in Ψ, $\frac{d\tilde{V}}{d\Psi}$ is of order Ψ^*. Keeping only terms of order b_0^2 which are leading in $N(0(N^2))$, we have

$$\frac{d\tilde{V}}{d\Psi} = \Psi^* \quad f\left(|\Phi|^2, |\Psi|^2\right)$$

$$f\left(|\Phi|^2, |\Psi|^2\right) = \mu^4 b_0^2 \exp\left(\frac{|\Phi|^2}{M^2} + \frac{|\Psi|^2}{M^2}\right)$$

$$\left[\frac{|\Phi|^2}{M^2}\frac{|\Psi|^2}{M^2}\left(2 + \frac{|\Psi|^2}{M^2}\right) + 4 + 6\frac{|\Psi|^2}{M^2} + 4\frac{|\Psi|^4}{M^4} + \frac{|\Psi|^6}{M^6}\right] > 0 \tag{3.78}$$

or equivalently,

$$\begin{aligned} \frac{d\tilde{V}}{d(\Re e\Psi)} &= 2\Re e\Psi f\left(|\Phi|^2, |\Psi|^2\right) \\ \frac{d\tilde{V}}{d(Im\psi)} &= 2Im\Psi f\left(|\Phi|^2, |\Psi|^2\right) \end{aligned} \tag{3.79}$$

Since $f(|\Phi|^2, |\Psi|^2)$ is strictly positive, this shows that $\Re e\Psi = Im\Psi = 0$ is a (global) minimum. A similar analysis can be performed on the temperature corrections which shows that $\Re e\Psi = Im\Psi = 0$ remains the minimum at non-zero temperature. Therefore, at high temperature, the Φ field is stabilized around the origin and as the temperature decreases it evolves with Ψ fixed at the origin, in precisely the way studied in the previous section since $\tilde{V}(\Phi, \Psi = 0) = V(\Phi)$. The only apparent effect of the Ψ field is to give the right behavior at high temperatures.

But what happens to the Ψ field subsequently? To answer this question, it is interesting to note the following point. From (3.72), (3.73), (3.74), we

find the Ψ field mass:

$$m_\Psi = 2\sqrt{2}e\frac{\mu^2}{M}(b_0 + b_1\sigma) \simeq 0\left(\frac{\mu^2}{M}N\right) \tag{3.80}$$

Therefore in the region $\frac{\mu}{M} \sim 10^{-2}$, the Ψ field is heavy enough to decay into the superheavy color triplet Higgs field of GUTS, which can lead to the standard scenario of baryogenesis. Of course the Ψ fields that we consider here are not the primordial ones since those have been diluted away by inflation in the $\Re e\Phi$ direction. But, in most cases, the behavior near the global minimum ($\Re e\Phi = \sigma M, Im\Phi = \Psi = 0(\epsilon)$) of the terms coupling Ψ and Φ will induce oscillations in the Ψ direction. This scenario requires a detailed analysis of the coupled terms near the minimum, including non-leading terms in W (we have just shown that there are no such oscillations in the leading N approximation: (3.78) and (3.79)).

We conclude therefore that there exist viable cosmological scenarios which allow a mass for the gravitino as low as M_W (see also Refs. [59] and [69]). This is because we related the scale of supersymmetry breaking to a small parameter, the slope of the potential at the origin. This scale can be as low as the mass M_W of the weak gauge boson. These models share in common a low reheating temperature which helps in solving some of the problems (e.g. the gravitino problem) that inflationary models usually face but is also somewhat undesirable from the point of view of baryosynthesis. In the next chapter, we will discuss baryosynthesis in detail and study an alternative mechanism

which might be operative in these models. This mechanism could lead to a production of enough baryons and hence circumvent the problems associated with a low reheating temperature.

IV BARYOGENESIS IN SUPERSYMMETRIC INFLATIONARY COSMOLOGIES

In the last chapter we saw that inflationary scenarios employing local supersymmetry seem to be very attractive for providing "natural" solutions to many cosmological conundrums . The success of these models is somewhat marred by one potentially serious problem – a low reheating temperature after the exit from the inflationary era. A low reheating temperature is undesirable because it is a potential blow to one of the most important achievements of the application of Grand Unified Theories to cosmology – the generation of baryon-antibaryon asymmetry from symmetric initial conditions [8]. This is so because in the standard scenario, in order to generate a baryon asymmetry after the de–Sitter expansion has diluted any primordial asymmetry, one needs to reheat the universe to at least a temperature of $O(10^9 - 10^{10} GeV)$ [46]. It could be argued that the standard out of equilibrium decay of the color-triplet Higgs is not the mechanism responsible for the generation of the asymmetry, but alternative mechanisms: decay of coherent Higgs field oscillations which are very far from equilibrium [36], low temperature baryon generation scenarios [65] etc. could be operative. While this may be reasonable, it still seems fruitful to us to investigate alternate origins for baryon number generation , since this feature is potentially the most restrictive on model building.

In this chapter, we will investigate the possibility of generating a satisfactory baryon excess within the framework of locally supersymmetric inflationary models. More specifically, we will use the hidden sector models [72], since they seem to be the most attractive phenomenologically. ("no–scale " models [73] will not be considered here.)

These models have a very weakly coupled scalar field, the inflaton which is responsible for the de–Sitter expansion and the subsequent reheating. The very weak interactions of the inflaton imply the reheating temperature is low because the lifetime is large and there is a significant redshifting of energy [46,63,64]. This causes problems for baryosynthesis.

We investigate the possibility of remedying this situation by using other heavy fields in the theory (e.g. the adjoint Higgs in SU(5)). Due to the gravitational couplings between these heavy fields and the hidden sector, energy is transferred from the inflaton to these fields. Since these fields have gauge interactions and hence a short lifetime, their decays occur before any significant redshifting has taken place, giving rise to a significant baryon excess.

After giving a brief review of baryogenesis, we establish a general framework in Section II. We then investigate two representative models in Section III and IV. Supersymmetry is unbroken in the first model, which is simpler to analyze while in the second model it is broken. We compute the baryon to

entropy ratio in both these models and show that with reasonable values of various model–dependent parameters we obtain a satisfactory baryon excess. Both the models, in spite of giving a satisfactory cosmology, do not however, satisfy the thermal constraint. We find that even with the incorporation of heavy fields, the situation does not change. Finally, we comment on the finite temperature corrections and the use of direct couplings between the heavy fields and the inflaton in solving the thermal constraint and its effect on our results.

1 REVIEW OF BARYOGENESIS

The observed universe seems to be dominated by matter and not by anti-matter. There are a variety of experimental observations which support this claim. At galactic scales, energetic cosmic rays (i.e., with energies $> .1$ GeV and which are supposed to originate outside of our solar system) have many more particles than anti-particles. This is true for both protons and helium-4 nuclei [74]. At the scales of galactic clusters, absence of a significant γ ray flux indicates that matter and anti-matter galaxies do not coexist. For if they did, the π^0's from the collisions of nucleons and anti-nucleons will decay to give a significant γ ray flux [75]. Thus we can be reasonably certain that even if there is an equal quantity of matter and anti-matter in the universe, it is separated on scales greater than (1-100) L_{galaxy}.

This asymmetry in the universe is quantified by a dimensionless number,

B, which is the ratio of the average baryon number density to the entropy density. Since the most abundant particles in the universe are the 3 K microwave photons, the asymmetry is also characterized by η, the ratio of the baryon density to the photon density. Observationally, the number density of photons, n_γ, is well determined

$$n_\gamma = 399\left(\frac{T}{2.7K}\right)^3 \mathrm{cm}^{-3} \tag{4.1}$$

where T is the temperature of the microwave background. The number density of baryons, n_B, is not constrained very much by direct observations [76]. However, from big bang nucleosynthesis, we know that the abundances of light elements depends strongly on η. To produce quantities of these elements which are consistent with observations, we must have [7,77]

$$\eta \equiv \frac{n_B}{n_\gamma} \simeq (2-8) \times 10^{-10} \tag{4.2}$$

From this value of η, and the fact that the present entropy is divided equally between the photon and the neutrino backgrounds, we can get a value for B. Assuming the constancy of entropy (1.10a), we obtain

$$B \equiv \frac{n_B}{s} \simeq \frac{\eta}{7} \sim (3-10) \times 10^{-11}. \tag{4.3}$$

It is this small number which has to be explained. It should be noted that an initially baryon symmetric universe (with no baryon number non-conservation) will lead to a value of η that is many orders of magnitude

smaller than (4.3) [75,78]. Before the advent of GUTs, there was no theoretical motivation for having baryon number non-conservation. Thus this small value of η had to be chosen as an initial condition.

To generate a non-zero η from a universe which is initially symmetric, three conditions have to be satisfied [79]:

1. Baryon number violating interactions.
2. C and CP violation.
3. Departure from thermal equilibrium.

It is necessary to have B number violation or otherwise a baryon symmetric universe will stay symmetric. GUTs provide us with exactly these interactions but at very high energy scales.

There must exist particle-antiparticle asymmetry, i.e., charge conjugation (C) symmetry and charge conjugation with a parity inversion (CP) symmetry must be violated. If C and CP are not violated, then an initial state which is symmetric (and hence C and CP invariant) evolves into a symmetric state. C and CP violations are needed to provide an arrow and insure that excess baryons are produced. Weak interactions violate C but there is no system apart from the K^o-$\bar{K}^o$ where CP violation is observed [80]. It seems highly unlikely that this is the only system in nature which violates CP. A detailed understanding of CP violation is still unavailable, neverthe-

less, GUTs can incorporate CP violation and hence can provide two of the ingredients necessary for the evolution of an asymmetry.

The necessity of departure from thermal equilibrium is slightly more subtle. Firstly, we have to define the meaning of thermal equilibrium. Although a rigorous definition of thermal equilibrium is not possible in an expanding universe, it can be defined operationally. When the interaction rates of the important processes are much greater than the expansion rate H of the universe, then we say that there is thermal equilibrium. In equilibrium we always get the same number of baryons as anti-baryons. This is so because of unitarity and CPT [12,81]. Unitarity implies that the chemical potential is vanishing and CPT ensures that the masses of baryons and anti-baryons are the same. Thus the equilibrium distributions for them are identical

$$f(p) = \left[\exp\left(\frac{\mu + E}{T}\right) \pm 1\right]^{-1} \tag{4.4}$$

where the + sign is for Fermi-Dirac statistics and the - sign is for Bose-Einstein statistics. Also μ is the chemical potential and $E^2 = p^2 + m^2$ for the particles. In standard cosmology, the universe has gone through several epochs when the reaction rates have not been able to keep up with the expansion rate (or vice-versa).

The essential ideas of baryogenesis can be incorporated into the standard out-of-equilibrium scenario [82]. We will briefly discuss this scenario in a semi-qualitative way to fix our ideas for the following sections.

Let X be a superheavy boson (either Higgs or gauge) of mass M whose interactions violate baryon number. If its coupling strength to fermions is α, then dimensionally its decay rate, Γ_D, is given by

$$\Gamma_D \sim \alpha^2 M. \tag{4.5}$$

We start at $T \sim T_{Pl}$ with a baryon symmetric universe. From $T \sim T_{Pl}$ to $T \sim M$, the X and $\overline{X}$ are in equilibrium and as abundant as the photons. When the temperature falls below M, the equilibrium abundance of the bosons relative to the photons is given by

$$X_{eq} \equiv \frac{n_X}{n_\gamma} \sim \left(\frac{M}{T}\right)^{3/2} \exp\left(\frac{-M}{T}\right). \tag{4.6}$$

If the decay rate $\Gamma_D > H$ then the X's can decay fast enough and adjust their abundance to this equilibrium value. (Decay is the dominant process since all other processes are higher order in α.) There is then no departure from thermal equilibrium and no asymmetry develops.

If however $\Gamma_D \gg H$ (for $T \sim M$) then the X's cannot decay rapidly enough and are overabundant. There is then a departure from thermal equilibrium which is needed for the generation of an asymmetry.

Consider a pair of X and $\overline{X}$ decaying into two channels with baryon number B_1 and B_2 and branching ratios r and $(1-r)$ [for $\overline{X}$, the quantities are $-B_1$, $-B_2$, $\bar{r}$ and $(1-\bar{r})$] Thus the decay of a pair of X and $\overline{X}$ will produce

on average a baryon number ϵ

$$\epsilon = (r - \bar{r})(B_1 - B_2). \tag{4.7}$$

It can be shown that if C and CP are not violated then $r = \bar{r}$ and hence $\epsilon = 0$.

Now when the temperature falls to $T \ll M$, the decay rate catches up with the expansion rate (because H is decreasing) and the X bosons decay freely. At this time, $n_X \sim n_\gamma$ and thus the net baryon number density produced is $n_B \sim \epsilon n_\gamma$. Since $s \sim g_* n_\gamma$ (where g_* is the number of degrees of freedom), we get

$$\frac{n_B}{s} \sim \frac{\epsilon}{g_*} \sim 10^{-2}\epsilon. \tag{4.8}$$

It turns out the favored candidates for producing the asymmetry are the Higgs bosons. Recall that the condition for falling out of equilibrium is $\Gamma_D < H$ (at $T = M$) which means

$$M > g_*^{-1/2}\alpha^2 M_{Pl}. \tag{4.9}$$

For gauge bosons, α is fixed to be the gauge coupling constant $\sim 1/45$, but it is essentially arbitrary for the Higgs boson. Thus we can have a fairly light Higgs and still satisfy the condition for being out of equilibrium. Furthermore, the CP violation for a Higgs is expected to be more than that for a gauge boson [83]. This is because we get CP violation (and hence a non-zero ϵ) at a lower number of loops for a Higgs than for a gauge boson.

For example, in minimal SU(5), we get a non-zero ϵ at three loops for the color triplet Higgs, H_3, and at four loops for the superheavy X and Y gauge bosons [83].

This completes our brief review of baryogenesis. We now go on to give a general framework within which we will present alternative mechanisms for baryosynthesis in supersymmetric inflationary models.

2 GENERAL FRAMEWORK

Consider a set of scalar fields ϕ_i in a locally supersymmetric theory with a superpotential $W(\phi_i)$ (This is the quantity we called $f(\phi_i)$ in the last chapter). Then the corresponding scalar potential is given by (assuming a flat Kähler metric)[61]

$$V(\phi_i) = \exp\left(\sum_i |\phi_i|^2/M^2\right)\left[\sum_i |D_{\phi_i}W(\phi_i)|^2 - \frac{3}{M^2}|W(\phi_i)|^2\right] \tag{4.10}$$

where $D_{\phi_i}W(\phi_i)$ is the Kähler covariant derivative

$$D_{\phi_i}W(\phi_i) = \frac{\partial W}{\partial \phi_i} + \frac{\phi_i^* W(\phi_i)}{M^2} \tag{4.11}$$

and $M = \frac{M_P}{\sqrt{8\pi}} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck Mass.

We consider the superpotential W to be a function of two fields ϕ and Σ. ϕ is the field which causes inflation, the inflaton and Σ is some heavy field in the theory. Throughout we assume that ϕ is a gauge singlet while Σ can have non-trivial transformation properties under the gauge group. We will

for our purposes take Σ to be the adjoint Higgs of SU(5) but most of the results will be independent of this choice.

As a first step, we assume that the superpotential $W(\phi, \Sigma)$ be written as the sum of two superpotentials $f(\phi)$ and $g(\Sigma)$. This implies that the two fields only interact gravitationally (we will comment on the effect of direct coupling later). Then,

$$W(\phi, \Sigma) = f(\phi) + g(\Sigma). \tag{4.12}$$

Next we demand that at the true minimum, ϕ_0, Σ_0, the cosmological constant is zero and supersymmetry is unbroken. It is easy to show that these conditions imply

$$\left.\frac{\partial f}{\partial \phi}\right|_{\phi_0} = 0 \tag{4.13a}$$

$$f(\phi_0) + g(\Sigma_0) = \left.\frac{\partial g}{\partial \Sigma}\right|_{\Sigma_0} = 0 \tag{4.13b}$$

The most general gauge invariant and renormalizable superpotential for Σ is given by

$$g(\Sigma) = \frac{b_1}{2} Tr \Sigma^2 + \frac{b_2}{3} Tr \Sigma^3 + b_0 \tag{4.14}$$

where the constants b_0, b_1, b_2 will be fixed by condition (4.13b). It is convenient to work with dimensionless variables x and y defined as

$$x \equiv \phi/M \qquad y \equiv \Sigma/M. \tag{4.15}$$

Then

$$g(y) = \frac{b_1 M^2}{2} Tr\ y^2 + \frac{b_2 M^3}{3} Tr\ y^3 + b_0. \tag{4.16}$$

Furthermore, we want the true minimum in the Σ direction to break SU(5) $\rightarrow$ SU(3) $\times$ SU(2) $\times$ U(1) which implies that

$$y_0 = \frac{\Sigma_0}{M} = \frac{\Delta}{M} \begin{pmatrix} 2 & & & & 0 \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ 0 & & & & -3 \end{pmatrix} \tag{4.17}$$

where Δ is a scale characteristic of Σ (typically M_{GUT}). Now the condition $g(y_0) = 0$ implies

$$15 b_1 \Delta^2 - 10 b_2 \Delta^3 + b_0 = 0 \tag{4.18a}$$

and

$$\left.\frac{\partial g}{\partial y_{ab}}\right|_{y=y_0} = 0 (\text{with the constraint Tr y} = 0)$$

implies

$$b_0 = -5\Delta^3 \tag{4.18b}$$

$$b_1 = \Delta b_2. \tag{4.18c}$$

With the choice $b_2 = 1$, we have

$$g(y) = \frac{\Delta M^2}{2} Tr(y^2) + \frac{M^3}{3} Tr(y^3) - 5\Delta^3. \tag{4.18d}$$

For our case

$$W(x,y) = f(x) + g(y)$$

and

$$V(x,y) = \frac{e^{x^2+y^2}}{M^2}\left[\left(\frac{\partial W}{\partial x} + xW\right)^2 + \left(\frac{\partial W}{\partial y_{ab}} + y_{ab}W\right)^2 - 3W^2\right] \tag{4.19}$$

assuming x and y to be real.

From this expression, it is straightforward but tedious to compute the derivatives of the potential in the two directions. We only display $\frac{\partial V}{\partial x}$ since the others are messy and not particularly illuminating

$$\begin{aligned}\frac{\partial V}{\partial x} &= 2xV + \frac{2e^{x^2+y^2}}{M^2}[(f' + xW)(f'' + W + xf') \\ &+ Wf'Tr\ y^2 + \Delta M^2 f'Tr\ y^2 \\ &+ M^3 f'Tr\ y^3 - 3Wf']\end{aligned} \tag{4.20}$$

where primes denote $\frac{\partial}{\partial x}$. Using these expressions, one can determine what the value of the Σ field is when $\phi = 0$ i.e. at the beginning of inflation.

In the Appendix we show that it is impossible to simultaneously satisfy $\frac{\partial V}{\partial x} = \frac{\partial^2 V}{\partial x^2} = \frac{\partial V}{\partial y} = 0, V > 0$ and V $\sim 0(\mu^4)$ at $\phi = 0$ if the Σ field is at its true minimum i.e. in the 3-2-1 phase. Since all the above conditions are necessary for a successful inflationary model, the Σ field must start its evolution away from the true minimum. If the Σ field is at its true minimum when $\phi = 0$ then it will be less likely that Σ oscillations will be generated as ϕ evolves

from $\phi = 0$ to $\phi = \phi_0$.

We now estimate the baryon to entropy ratio in two representative models.

3 MODEL I

The superpotential for the inflaton field is [64]

$$f(x) = \mu^2 M(x-1)^2 \qquad x \equiv \phi/M \tag{4.21}$$

where the scale μ is fixed at $(10^{-3} - 10^{-4})M$ by demanding that the model gives the correct order of magnitude of density fluctuations which lead to galaxy formation. [41,64] This superpotential leads to an absolute minimum at $x = 1$ with zero cosmological constant and unbroken supersymmetry.

The evolution equations for x and y can be solved numerically and the energy stored in the Σ field can be determined. We find that a more transparent strategy is to solve the evolution equations analytically using various physically reasonable approximations. This is the approach we choose in the following analysis.

There are two natural scales in this model: the scale μ associated with the inflation sector $\left(\frac{\mu}{M} \sim 0(10^{-3} - 10^{-4})\right)$ and the scale Δ associated with the Σ sector which has a typical value $\sim 10^{-2}M$ [84]. Thus a reasonable parameter to use is μ/Δ. We will throughout keep only the lowest order

terms in μ/Δ.

At $\phi = 0$, we need to determine the value of the Σ field. Assuming that the value at $\phi = 0$ is a small perturbation from the true minimum, we write

$$y = \frac{\Delta}{M}\begin{pmatrix} 2 + a\mu/\Delta & & & & \\ & 2 + a\mu/\Delta & & & \\ & & 2 + a\mu/\Delta & & \\ & & & -3 - \frac{3}{2}a\mu/\Delta & \\ & & & & -3 - \frac{3}{2}a\mu/\Delta \end{pmatrix} \quad (4.22)$$

Using the derivatives $\frac{\partial V}{\partial y}$ we can solve for a to get

$$a = \frac{5}{21}\frac{\mu}{M} \sim 10^{-5}$$

which confirms our expectations of keeping only the lowest order terms in μ/Δ.

Next we need to trace the evolution of the ϕ and Σ system in the $\phi - \Sigma$ plane as ϕ evolves from $\phi = 0$ to $\phi = \phi_0 = M$. Once again we need to solve the evolution equations numerically, but we can simplify matters. Since the position of $< y >$ at $\phi = 0$ is not very different from that at $\phi = \phi_0$, it is reasonable to assume that the evolution of ϕ is unaltered.

With these assumptions, we now obtain the position of the Σ field at the end of inflation. The inflationary epoch is characterized by a slow rollover

in the ϕ direction and in terms of the potential this implies,[46]

$$V''(\phi) \leq \frac{3}{M^2}\,|V(\phi)| \quad (3.42)$$

$$V'(\phi) \leq \frac{\sqrt{6}}{M}\,|V(\phi)|\,. \quad (3.42)$$

For the potential we consider, the first equation breaks down first at a value

$$x_e \sim 0.2425. \quad (4.23)$$

Using this value of x_e, we once again solve $\frac{\partial V}{\partial y}$ to get the value of Σ at this point (to lowest order in μ/Δ). Assuming the form of y to be as in (4.22) we get

$$y(x = x_e) = \frac{\Delta}{M}\begin{pmatrix} 2 + 1.15\frac{\mu^2}{\Delta M} & & & & \\ & 2 + 1.15\frac{\mu^2}{\Delta M} & & & \\ & & 2 + 1.15\frac{\mu^2}{\Delta M} & & \\ & & & -3 - 1.725\frac{\mu^2}{\Delta M} & \\ & & & & -3 - 1.725\frac{\mu^2}{\Delta M} \end{pmatrix} \quad (4.24)$$

The evolution of the ϕ and Σ fields is governed by the evolution equations

which are [36]

$$\ddot{x} + 3H\dot{x} + \Gamma_x \dot{x} = -\frac{1}{M^2}\frac{\partial V}{\partial x}$$
$$\ddot{y}_{ab} + 3H\dot{y}_{ab} + \Gamma_y \dot{y}_{ab} = -\frac{1}{M^2}\frac{\partial V}{\partial y_{ab}} \tag{4.25}$$

where

$$H^2 = \frac{1}{3M^2}[V(\phi,\Sigma) + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\Sigma}^2 + \rho_\gamma]. \tag{4.26}$$

Here Γ_x and Γ_y are the decay rates of the ϕ and Σ fields respectively and ρ_γ is the energy density in radiation. The equation for y can be rewritten as an equation for a using (4.22)

$$\ddot{a} + 3H\dot{a} + \Gamma_y \dot{a} = -\frac{1}{\mu M}\frac{\partial V}{\partial y}. \tag{4.27}$$

We can get a sensible approximation scheme for these quantities by comparing the orders of magnitude. Since the ϕ field has only gravitational couplings, its decay rate is

$$\Gamma_\phi \sim \frac{m_\phi^3}{M^2}. \tag{4.28}$$

On the other hand, Σ is a gauge nonsinglet and its decay rate is

$$\Gamma_\Sigma \sim \alpha m_\Sigma \sim \alpha\Delta (\text{assuming } m_\Sigma \sim \Delta) \tag{4.29}$$

where α is the GUT gauge coupling constant. At the origin in the ϕ direction, the value of the Hubble parameter H is $\sim \frac{\mu^2}{M}$. Assuming $m_\phi \sim \frac{\mu^2}{M}$ [64] and and $\alpha \sim \frac{1}{20}$ [85], we obtain

$$\Gamma_\phi << 3H << \alpha\Delta. \tag{4.30}$$

Furthermore, the time taken for slow rollover, t_e, is given by [64]

$$t_e \sim \frac{M}{\mu^2} >> \Gamma_\Sigma^{-1}. \tag{4.31}$$

The physical picture which emerges from this is as follows: at $t = 0$, the ϕ field is at its origin while the Σ field is displaced from its true minimum at a value given by (4.22). From $t = 0$ to $t = t_e$, the ϕ field evolves slowly from $\phi = 0$ to $\phi = \phi_e$, giving rise to the de-Sitter expansion of the scale factor. Since this time is much longer than the lifetime of the $\Sigma's$, all the primordial $\Sigma's$ decay and the density of the decay products is exponentially diluted. However, at $t = t_e$, Σ is not at its true minimum but is displaced to a value given by (4.24).

Taking into account the inequalities given by (4.30), we can approximately solve the evolution equations for ϕ and Σ. These equations give us essentially the same result as if the Σ field was moving in a pure quadratic potential around the true minimum. Thus for our purposes, we take the motion in the Σ direction to be governed by

$$V = \frac{1}{2}M^2 m_\Sigma^2 (y - y_0)^2$$
$$= \frac{1}{2}M^2\Delta^2 a^2 \frac{\mu}{\Delta} \tag{4.32}$$

At time $t = t_e$, the value of Σ is given by (4.24) and the total energy in the Σ direction is at least

$$\rho_\Sigma(t = t_e) \sim \Delta^2 \frac{\mu^4}{M^2}. \tag{4.33}$$

The field is oscillating in a pure quadratic potential with a frequency given by its mass. Since this frequency is comparable to the decay rate of Σ, this energy rapidly goes into decay products before redshifting decreases it significantly. On the other hand, the ϕ field has a very long lifetime and it continues to oscillate near $\phi = \phi_0$ for a long time, with its energy redshifting significantly before decay into radiation. So we need to study the evolution of the energies associated with the ϕ and Σ directions from time $t = t_e$ to $t = t_\phi \equiv \Gamma_\phi^{-1}$ and compute the ratio $\frac{n_B}{s}$ at $t = t_\phi$.

To study the evolution, note that the energy associated with the oscillations in the ϕ direction is $0(\mu^4)$ and that in the Σ oscillations is $0(\mu^4 \frac{\Delta^2}{M^2})$. Since $\Delta \sim 10^{-2}M$, we can safely ignore the contribution of ρ_Σ to the evolution of the scale factors.

We assume that the dominant mechanism for the production of baryon asymmetry is the decay of color triplet Higgs which are produced in the decay of Σ. This will give us a lower limit on the magnitude of $\frac{n_B}{s}$.

Let n_H be the number density of the Higgs triplets of mass m_H produced by the decay of the $\Sigma's$. Then the energy density ρ_H is given by, since the Higgs' are non relativistic,

$$n_H = \rho_H / m_H. \tag{4.34}$$

Further let a fraction f of the Σ energy before decay go into the triplets

and for simplicity the rest into photons. Then

$$\rho_H = f \rho_\Sigma \tag{4.35}$$

and the reheat temperature is

$$T_{RH}^{(1)} = \left[\frac{30}{\pi^2 g_*}(1-f)\rho_\Sigma\right]^{1/4} \tag{4.36}$$

where g_* are the effective relativistic degrees of freedom.

The potential in the ϕ direction is given by

$$V = e^{x^2}\mu^4[x^6 - 4x^5 + 7x^4 - 4x^3 - x^2 + 1] \tag{4.37}$$

and near $x = x_0$ by

$$V = \mu^4 e^{x^2}[4(x-x_0)^2 + 12(x-x_0)^3 + \cdots]. \tag{4.38}$$

Thus near $x = x_0$, the dominant term is the quadratic term and the expansion is matter dominated [63]. The energies at $t = t_e$ and $t = t_\phi$ are related by

$$\rho_H(t = t_\phi) = \rho_H(t = t_e)\left[\frac{R(t = t_\phi)}{R(t = t_e)}\right]^{-3} \tag{4.39}$$

where R is the cosmic scale factor. But

$$\frac{R(t_\phi)}{R(t_e)} = \left[1 + \frac{3}{2}H_{t=t_e}(t_\phi - t_e)\right]^{2/3} \tag{4.40}$$

where $H_{t=t_e}$ is the Hubble parameter at $t = t_e$.

From (4.30), (4.31) we obtain

$$\frac{R(t_\phi)}{R(te)} \sim \left(1 + \frac{3}{2} H_{t=t_e} \Gamma_\phi^{-1}\right)^{2/3}. \tag{4.41}$$

Also from (4.26) and $\rho_\phi(t_e) \sim \mu^4$ we get

$$\rho_H(t_\phi) = \frac{4}{3} \rho_H(t_e) \frac{\mu^8}{M^8}. \tag{4.42}$$

Using (4.42) and $\rho_\Sigma(t_e) \sim \frac{\Delta^2}{M^2}\mu^4$ we obtain the number density of the triplets at the time of ϕ decay as

$$n_H(t_\phi) = \frac{\rho_H}{m_H} \sim \frac{f}{m_H} \Delta^2 \frac{\mu^{12}}{M^{10}}. \tag{4.43}$$

Assuming that ϵ_B is the baryon excess produced per triplet decay we obtain the number density of excess baryons as

$$n_B \sim \frac{\epsilon_B f}{m_H} \Delta^2 \frac{\mu^{12}}{M^{10}}. \tag{4.44}$$

From Ref. 64, we know the reheat temperature for this model,

$$T_{RH} \sim \sqrt{M\Gamma_\phi} \sim \mu^3/M^2. \tag{4.45}$$

Note that this is the final reheat temperature, produced by the decay of the inflaton. There might be some intermediate reheating associated with the decay of other particles, for eg. $T_{RH}^{(1)}$ associated with the decay of $\Sigma's$. This produces a negligible amount of entropy because the small amount of energy

gets redshifted significantly between t_e and t_ϕ. Thus the baryon to entropy ratio at $t = t_\phi$ is given by

$$\frac{n_B}{s} \sim \frac{45}{2\pi^2 g_*} \frac{\epsilon_B f}{m_H} \frac{\Delta^2 \mu^3}{M^4}. \tag{4.46}$$

Using (4.46), we can estimate the numerical value of $\frac{n_B}{s}$ and compare it to the observed value of $\sim 10^{-10}$. There are however, ambiguities in the values of the parameters entering (4.46). The values of $\frac{\mu}{M}$ and $\frac{\Delta}{M}$ can be fixed, as already indicated at $10^{-3} - 10^{-4}$ and 10^{-2} respectively [64, 85]. g_* can be assumed to be 0(2 x 10^2) at these scales. ϵ_B, f and m_H are more uncertain and model dependent.

It is known [86], that in supersymmetric models, apart from the usual dimension 6 operators responsible for proton decay, there can also exist dimension 5 operators which could give a disastrously small proton lifetime. If these operators are present, we have a lower bound on the mass of the superpartners of the triplets given by [87].

$$m_{\tilde{H}_3} \geq 10^{16} GeV. \tag{4.47}$$

However, one can invoke certain symmetries, for example a Peccei Quinn symmetry [88] or a discrete symmetry, which forbid proton decay by dimension 5 operators. In these cases the limit is much smaller. For example Ref. 88 shows that it is possible to reconcile a low mass Higgs triplet with the experimental bounds on proton lifetime. The lower bound is considerably

reduced to

$$m_H \geq 2.85 \text{ x } 10^{10}\text{GeV}. \tag{4.48}$$

The value of ϵ_B, or the net baryon number produced by the decay of a particle–antiparticle pair is also very model dependent. At tree level, $\epsilon_B = 0$ and $\epsilon_B \neq 0$ comes from loop diagrams. For supersymmetric GUTs, no "surprising" cancellations occur at one loop level and so $\epsilon_B \leq 0(\alpha/4\pi)$[84]. The quantity f is to be determined by looking at the decay modes of the $\Sigma's$. The $\Sigma's$ can decay into anything lighter–triplet, doublet Higgs, gluons etc. A value of 1/10 is not an unreasonable value for this parameter. Using $\epsilon_B \sim 10^{-3}$ [88], we obtain from (4.46)

$$\frac{n_B}{s} \sim 10^{-20.5\pm1.5}\frac{M}{m_H}. \tag{4.49}$$

If we use $m_H \sim 10^{+10}GeV$ and $\mu \sim 10^{-3}M$, we obtain a value of $\frac{n_B}{s}$ which almost agrees with that observed. However, if the higher bound on m_H is taken from models where dimension 5 operators are not suppressed by some symmetry, then this mechanism gives us a much smaller value of $\frac{n_B}{s}$ in disagreement with observations.

4 MODEL II

Having computed $\frac{n_B}{s}$ for this simple model with no supersymmetry breaking, we go on to consider a model with supersymmetry breaking in the in-

flaton sector. We will use the simple model discussed in Chapter 3.

To recapitulate what we found in the previous chapter, consider the inflaton superpotential

$$f(x) = \mu^2 M[\beta + \epsilon + x + \beta x^2 - \frac{1}{12}\beta x^4] \tag{3.27}$$

where $\beta = -\frac{3}{8}\sqrt{2} - \frac{3}{8}\epsilon + 0(\epsilon^2)$. The minimum is supersymmetry breaking and is at

$$x = \sqrt{2} + \left(\frac{2\sqrt{2}}{3}\epsilon\right)^{1/2} \tag{3.32}$$

and the gravitino mass is

$$m_{3/2} = \frac{2}{9}\sqrt{3\sqrt{2}}e\frac{\mu}{M}^2\epsilon^{3/2}. \tag{3.33}$$

In this model, supersymmetry breaking is associated with a non-zero value of ϵ. However, for the first part of our analysis we will assume $\epsilon = 0$ since this does not change our conclusions. We start with a superpotential

$$f(x) = \sqrt{2}\mu^2 M\left[-\frac{3}{8} + \frac{x}{\sqrt{2}} - \frac{3}{8}x^2 + \frac{x^4}{32}\right]. \tag{4.50}$$

Coupling the Σ field to ϕ and carrying out the same analysis as for model I, we obtain the value of Σ at the end of inflation. The slow rollover or the inflationary epoch ends at a time $t = t_e$ when the inequalities in (3.42) are no longer satisfied. We found in the last Chapter that the second inequality

breaks first when the ϕ field is at ϕ_e given by ,

$$\phi_e \sim \cdot 71M. \tag{3.43}$$

Using this value of ϕ_e, we solve $\frac{\partial V}{\partial y} = 0$ to obtain

$$y(x = x_e) = \frac{\Delta}{M}$$

$$\begin{pmatrix} 2 + \cdot 41\frac{\mu^2}{\Delta M} & & & & \\ & 2 + \cdot 41\frac{\mu^2}{\Delta M} & & & \\ & & 2 + \cdot 42\frac{\mu^2}{\Delta M} & & \\ & & & -3 - \cdot 62\frac{\mu^2}{\Delta M} & \\ & & & & -3 - \cdot 62\frac{\mu^2}{\Delta M} \end{pmatrix} \tag{4.51}$$

Once again, as for Model I, we use these initial conditions to solve approximately the evolution equations for Σ and ϕ. Not surprisingly, we find again that the motion in the Σ direction is governed by a pure quadratic potential. At time t_e, the Σ field sits away from its true minimum and has energy $\rho_\Sigma \sim \mu^4 \frac{\Delta^2}{M^2}$ which rapidly goes into its decay products. In computing $\frac{n_B}{s}$, we need to trace the evolution of ρ_ϕ and ρ_Σ from t_e to t_ϕ. It is here that the difference from Model I occurs.

Recall that for Model I, the potential was predominantly quadratic in the ϕ direction and hence the universe expanded like a matter dominated one. In Model II however, there are two stages of expansion (once again

$\rho_\phi >> \rho_\Sigma$ and the evolution is governed by ρ_ϕ). From time t_e to a time $t = t_t \sim 6 \text{ x } 10^{-2}\epsilon^{-1}\frac{M}{\mu^2}$ (3.51), the ϕ^4 term dominates and the universe expands like radiation dominated .

Thus for $t_e \leq t \leq t_t$

$$\frac{R(t)}{R(te)} = [1 + 2H_{t=te}(t - te)]^{1/2}. \tag{3.49}$$

From time t_t to $t_\phi \equiv \Gamma_\phi^{-1}$, the dominant term is quadratic and expansion is matter dominated.

$$\begin{aligned} &t_t \leq t \leq t_\phi \\ &\frac{R(t)}{R(t_t)} = \left[1 + \frac{3}{2}H_{t=t_t}(t - t_t)\right]^{2/3} \end{aligned} \tag{3.53}$$

Now following the same steps as in Model I with the same notation, we find that

$$n_B = \frac{\epsilon_B \rho_H(t_\phi)}{m_H} = \frac{\epsilon_B f}{m_H}\frac{\Delta^2 \mu}{M^{1/2}} t_t^{1/2} \Gamma_\phi^2. \tag{4.52}$$

Since the energy density in Σ is much smaller than that in $\phi's$, one can easily check that the reheating temperature is the same as obtained in the previous chapter.

$$T_{RH} \sim \frac{\mu^3 \epsilon^{3/4}}{M^2} \tag{3.58}$$

Using (4.52), (3.58) and $\Gamma_\phi \sim \frac{m^3_\phi}{M^2} \sim \frac{\mu^6 \epsilon^{3/2}}{M^5}$ (3.54) we obtain

$$\frac{n_B}{s} \sim \frac{\epsilon_B f}{g_*}\frac{\mu^3 \epsilon^{1/4} \Delta^2}{M^4 m_H}. \tag{4.53}$$

From the last chapter, we have $\mu/M \sim 10^{-3} - 10^{-4}$ and $\epsilon \sim 10^{-7\pm1.5}$ (Fig. 3.2). Taking $g_* \sim 2\text{x}10^2$, $\frac{\Delta}{M} \sim 10^{-2}$, $m_H \sim 10^{10} GeV$, $\epsilon_B \sim 10^{-3}$ and $f \sim 10^{-1}$ we obtain

$$\frac{n_B}{s} \sim 10^{-11.5} \tag{4.54}$$

which is similar to that obtained in Model I apart from a factor of $\epsilon^{1/4}$. In fact the reheating temperature in this model is smaller by $\epsilon^{3/4}$ than that of Model I, and so one expects a larger $\frac{n_B}{s}$. However, because the inflaton field has a longer lifetime in Model II we do not get a larger $\frac{n_B}{s}$. The energy in the triplets is redshifted more and the enhancement due to a lower reheating temperature is more than cancelled to give us $\frac{n_B}{s}$ in (4.54).

The two models we have considered suffer from the same disease; they both violate the requirement that at high temperatures, a sufficient amount of energy is stored in the scalar field ϕ to give enough inflation – the thermal constraint. In other words, the inflaton must start its evolution far away from its global minimum, slowly roll down and eventually settle in its global minimum. This is not surprising however, because of a general result given in Ref. 57. In a hidden sector with a single field and a flat Kähler metric, the temperature corrections do not stabilize the field at the origin.

A possible solution to this problem suggested in Ref. 64 and discussed in Chapter 3 is to allow for direct couplings between ϕ and another field ψ. For our case, we have until now, only considered the situation where the GUT

sector and the inflaton sector are separate, i.e., only coupled gravitationally. If direct couplings between the two sectors are allowed, the situation in the two models is somewhat different.

In Model I, the inflaton sector does not break supersymmetry and hence direct coupling of ϕ and Σ, will not be in danger of changing the supersymmetry breaking scale. In Model II however, the inflaton sector is also responsible for the breaking of supersymmetry (with $\epsilon \neq 0$). In this case we need to be careful because there is a danger that the supersymmetry breaking scale will be pushed up to m_{GUT} since the $\Sigma's$ now couple directly to the ϕ.

Thus in both cases we see that if we include direct coupling of ϕ and Σ, then the thermal constraint can be satisfied. Furthermore, it is possible that with direct couplings, the value of $\frac{n_B}{s}$ will improve because more energy can be transferred now from the inflaton to the Σ. However, the direct couplings make the analysis very complicated. This is because firstly, one has to be careful that gauge radiative corrections do not spoil the nice features of the inflationary potential. Secondly, both the fields are now responsible for inflation and reheating [for an exception see Ref. 59]. We do not carry out this analysis since it is beyond the scope of the present work.

We conclude then that there exists another possible mechanism for baryon number generation (beyond the standard scenario) within the framework of

supersymmetric inflationary cosmologies. This is significant because in supersymmetric models, the reheating temperature is usually low and hence the generation of enough baryons could be a problem.

V CONCLUSIONS

In this thesis, we first outlined briefly the standard hot big bang cosmological scenario. We discussed the successes of this model and some of the problems which it does not address.

In the next chapter, we presented a brief review of the various attempts to solve the problems associated with the hot big bang model. These included the original idea of 'old' inflation and the 'new' inflationary scenario. We saw how, even though these scenarios are successful in explaining some of the cosmological conundrums of the big bang model, they suffer from some drawbacks. We outlined some alternative proposals, especially those incorporating supersymmetry.

In Chapter 3 we discussed some of the motivations to construct inflationary models using supergravity. We then studied inflationary models where the scale of supersymmetry breaking is proportional to a small parameter which we chose to relate to the slope of the potential at the origin. This scale can therefore be as low as the mass M_W of the weak gauge boson. The study of the simplest of these models showed that no particular problem arises except for a violation of the thermal constraint. We showed however that one can deal with this problem by introducing a second field in the inflaton sector, whose sole effect is to modify the temperature corrections. Anyway, whether or not we introduce this second field, the only field which plays a

dynamical role as far as inflation is concerned is the original inflaton.

In the fourth chapter, a brief review of the mechanism for baryosynthesis was given. We then outlined the general framework to study an alternative mechanism for the generation of baryon asymmetry which involves the use of the couplings of heavy fields with the hidden sector. This mechanism seems to be a very general one since in any model with an inflationary sector and a GUT sector which has heavy fields, there will exist the possibility of the transfer of energy from the inflaton to the heavy fields. We obtained the value of $\frac{n_B}{s}$ in the case of two inflaton superpotentials (one with and one without supersymmetry breaking). The numerical value of $\frac{n_B}{s}$ however was seen to be dependent upon parameters which are model dependent. We saw that if we use the bound on m_H from supersymmetric GUTs where some symmetry prohibits dimension 5 operators for baryon decay, then a value of $\frac{n_B}{s}$ which is almost in agreement with the observations is obtained. In both models the thermal constraint is violated unless one includes direct couplings between the inflaton and the Σ fields.

We have seen how supersymmetric inflationary models offer solutions to some of the problems which exist in non-supersymmetric models. However, there are two problems which plague these models: the thermal constraint and the generation of an appreciable baryon asymmetry. Baryon generation is a problem because of a low reheat temperature which is needed to solve the gravitino problem. Although we have studied a mechanism to generate

enough baryons in models with a low reheat temperature, there still is the question of direct couplings between the inflaton and the heavy fields in the theory. The effect of these couplings might very well be to increase the baryon asymmetry. A careful study of the case with direct couplings is needed before this issue is settled. A similar analysis should also be carried out in the context of no-scale models. These models are attractive because they might emerge naturally from superstring theories. A detailed analysis of the mechanism of energy transfer needs to be carried out in such models to understand the issue of baryon generation.

VI APPENDIX

In this appendix we show that under very general conditions it is impossible for the Σ field to sit at its absolute minimum when $\phi = 0$. The notation is that of the text. Let f(x) and g(y) be the superpotentials in the 2 sectors. Let

$$f(x) = \mu^2 M f_1(x) \tag{A1}$$

$$g(y) = \Delta^3 g_1(y) \tag{A2}$$

where $f_1(x)$ and $g_1(y)$ are dimensionless. Further assume that there is no direct coupling between the 2 fields. Then

$$W(x,y) = f(x) + g(y). \tag{A3}$$

Now we impose the following conditions: at $x = x_0$, $y = y_0$ (the true minimum) we must have unbroken supersymmetry and zero cosmological constant. This implies

$$f_1(x_0) = f_1'(x_0) = 0 \tag{A4}$$

$$g_1(y_0) = g_1'(y_0) = 0 \tag{A5}$$

Assume that when $x = 0$, $y = y_0$ i.e. the field y starts off at its absolute minimum. Then demanding that the potential be flat means

$$\frac{\partial V}{\partial x} = \frac{\partial^2 V}{\partial x^2} = \frac{\partial V}{\partial y} = 0 \text{ at x} = 0 \text{ , y} = \text{y}_0. \tag{A6}$$

These conditions imply

$$\mu^4 M^2 \left\{ f_1'[f_1'' + (y_0^2 - 2) f_1] \right\} = 0 \tag{A7}$$

$$\mu^4 M^2 \left\{ f_1^2 (y_0^2 - 2) + f''^2_1 + f_1' f_1'' + y_0^2 f'^2_1 + (y_0^2 - 1) f_1 f_1'' \right\} = 0 \tag{A8}$$

$$\mu^4 M^2 y_0 \left[f'^2_1 + (y_0^2 - 2) f_1^2 + f_1 g_1'' \frac{\Delta^3}{\mu^2 M} \right] = 0. \tag{A9}$$

Futhermore at $x = 0$, $y = y_0$

$$V(0, y_0) = \frac{\mu^4 e^{y_0^2}}{M^2} [f'^2 + (y_0^2 - 3) f_1^2]. \tag{A10}$$

Using (A9) gives us

$$\begin{aligned} V(0, y_0) &= \mu^4 \frac{e^{y_0^2}}{M^2} \left[-f_1^2 - f_1 g_1'' \frac{\Delta^3}{\mu^2 M} \right] \\ &= -\frac{\mu^4 e^{y_0^2}}{M^2} f_1 \left[f_1 + g_1'' \frac{\Delta^3}{\mu^2 M} \right] \end{aligned} \tag{A11}$$

But $g_1''(y_0) \sim 0(\frac{M^2}{\Delta^2})$ since $y_0 \sim 0(\frac{\Delta}{M})$ for the example in text which is quite general. Then (A11) immediately tells us that

$$V(0, y_0^2) \sim 0(\Delta^2 m^2).$$

This is unacceptable because we know that the potential at $\phi = 0$ must scale like μ^4 with $\mu \sim 0(10^{-4})$ to give us the correct density fluctuations! If the field Σ at $\phi = 0$ sits at its absolute minimum then the scale μ drops out of the potential.

Thus we assume that the field Σ starts at some other value at $\phi = 0$, i.e. we solve for $\frac{\partial V}{\partial y} = 0$ at $\phi = 0$ as in the text.

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Figure Captions

Figure 2.1 Schematic sketch of the scalar potential for the old inflationary scenario [for $T = 0$ and $T > T_c$].

Figure 2.2 Schematic sketch of the scalar potential for the new inflationary scenario.

Figure 3.1 Potential V/μ^4 corresponding to the superpotential $f(\phi)$ given by Eq. (3.30) (or Eq. (3.27) since, on this scale, they are indistinguishable). The dashed curve gives the shape of the potential at $T = M$ (taking $N = 50$ chiral superfields in the theory [57]).

Figure 3.2 Cosmological constraints on the parameters ϵ and μ/M. The curve $N = 60$ limits the region where enough inflation takes place [see the condition given by Eq. (3.46)]. The study of the amplitude of density fluctuations gives limits on μ/M only [Eqs. (3.65 - 3.67)]. Finally, we have drawn the line $m_{3/2} = M_W$ [Eq. (3.33)] which corresponds to the successful low energy models [58].

Figure 3.3 Oscillations of the ϕ field around the minimum σM of potential V (Fig. 3.1) immediately after the end of inflation ($t_e < t < 1.006 t_e$).

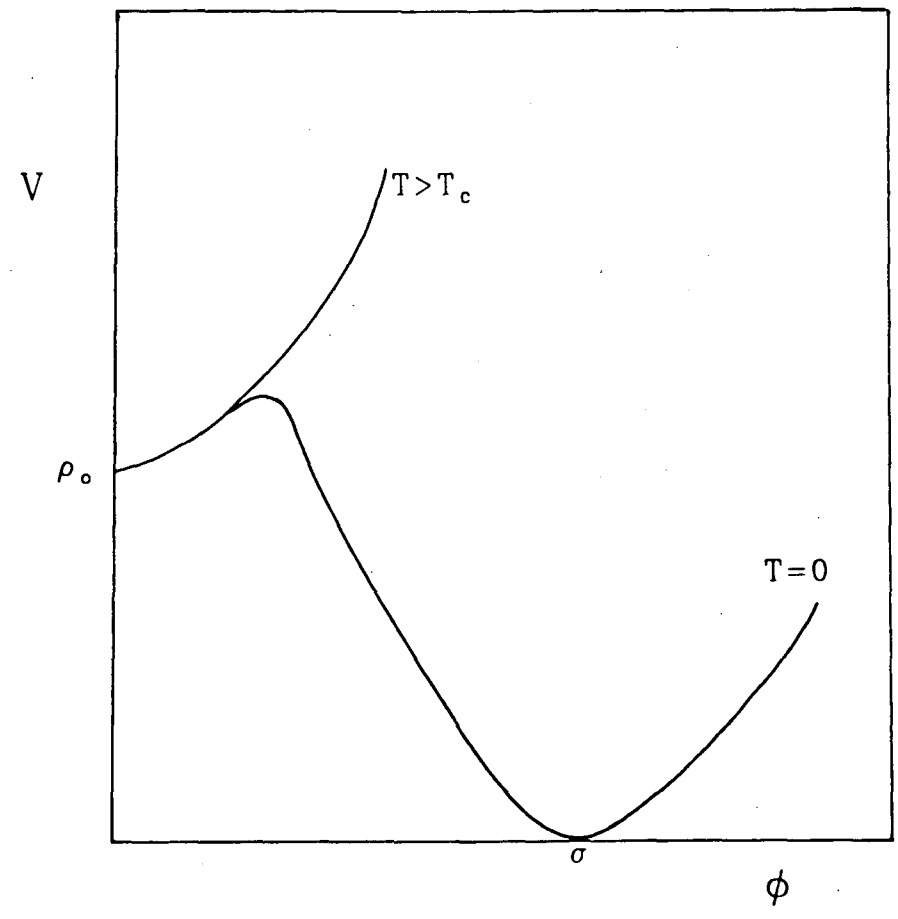

FIG 2.1

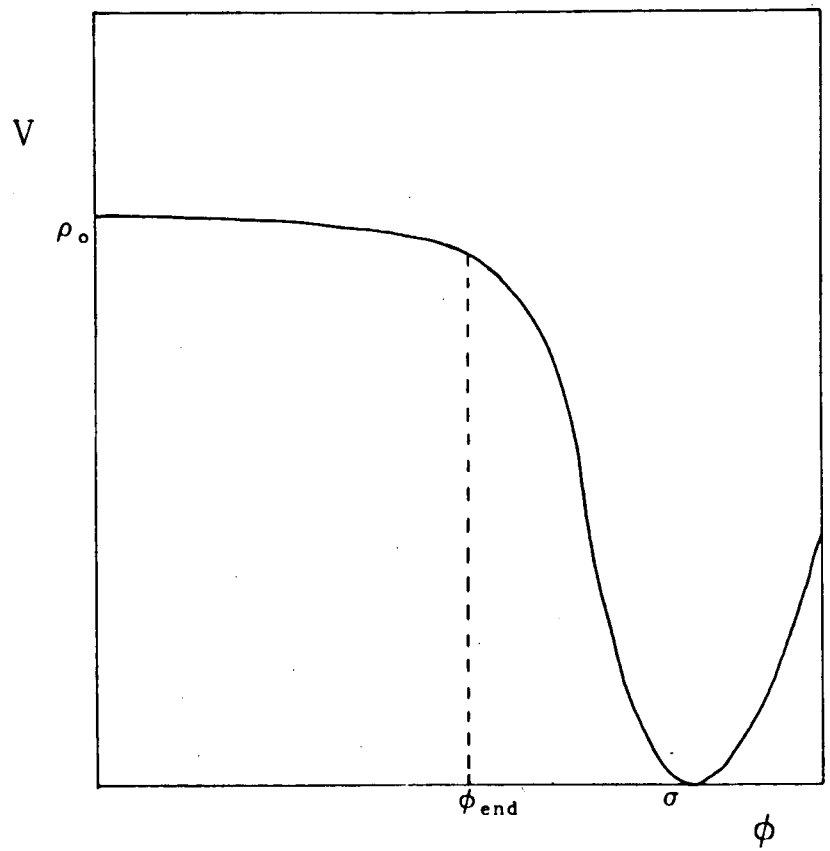
V
ρ_o
ϕ_{end}
σ
ϕ

FIG 2.2

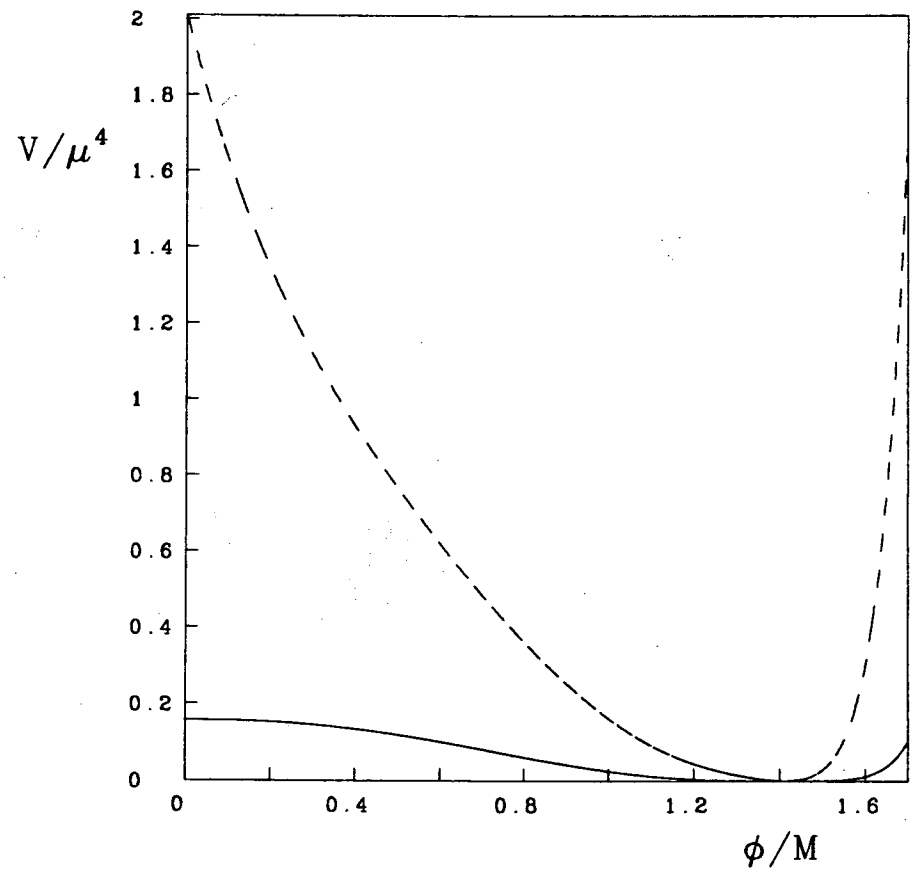
V/μ^4
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.4
0.8
1.2
1.6
ϕ/M

FIG 3.1

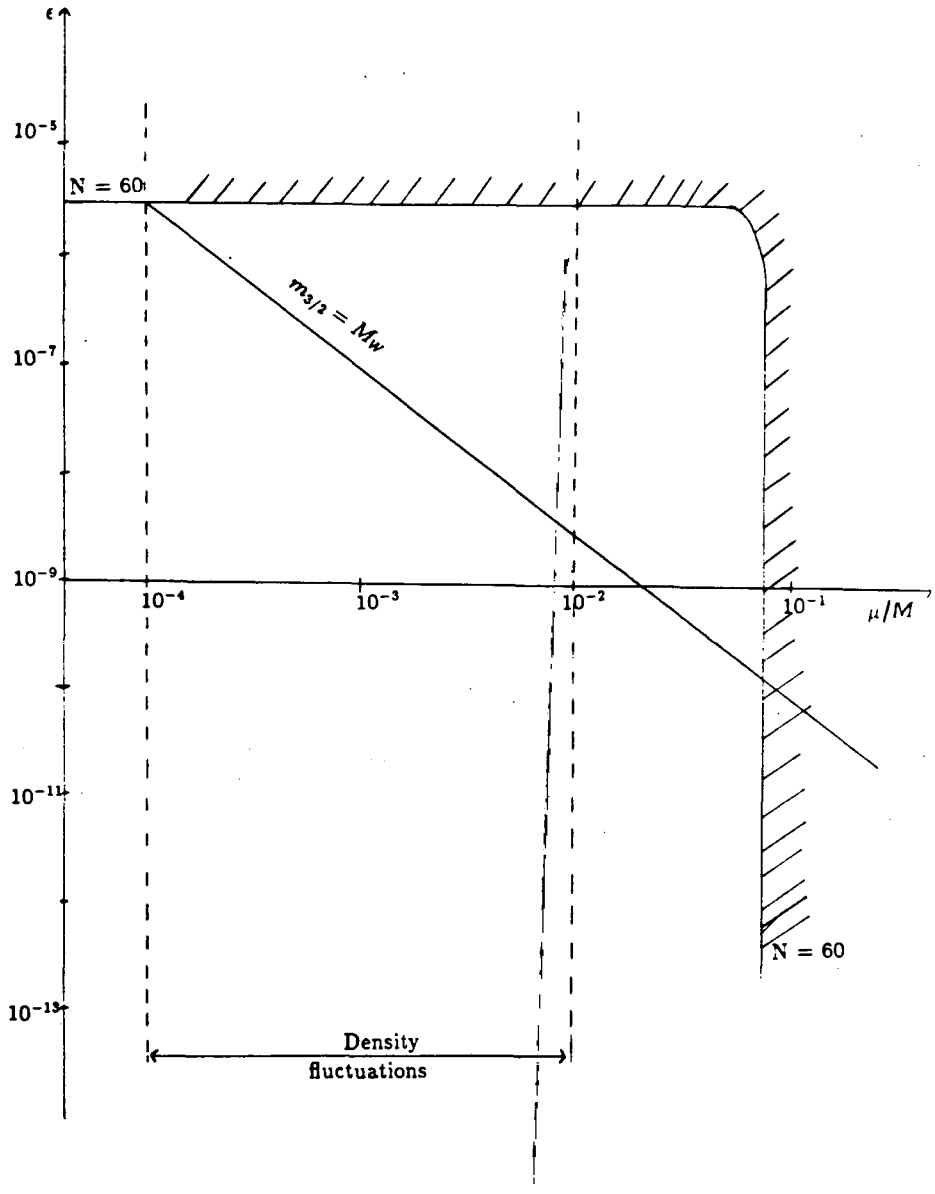
ε
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
N = 60
$m_{3/2} = M_W$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
μ/M
N = 60
Density
fluctuations

FIG 3.2

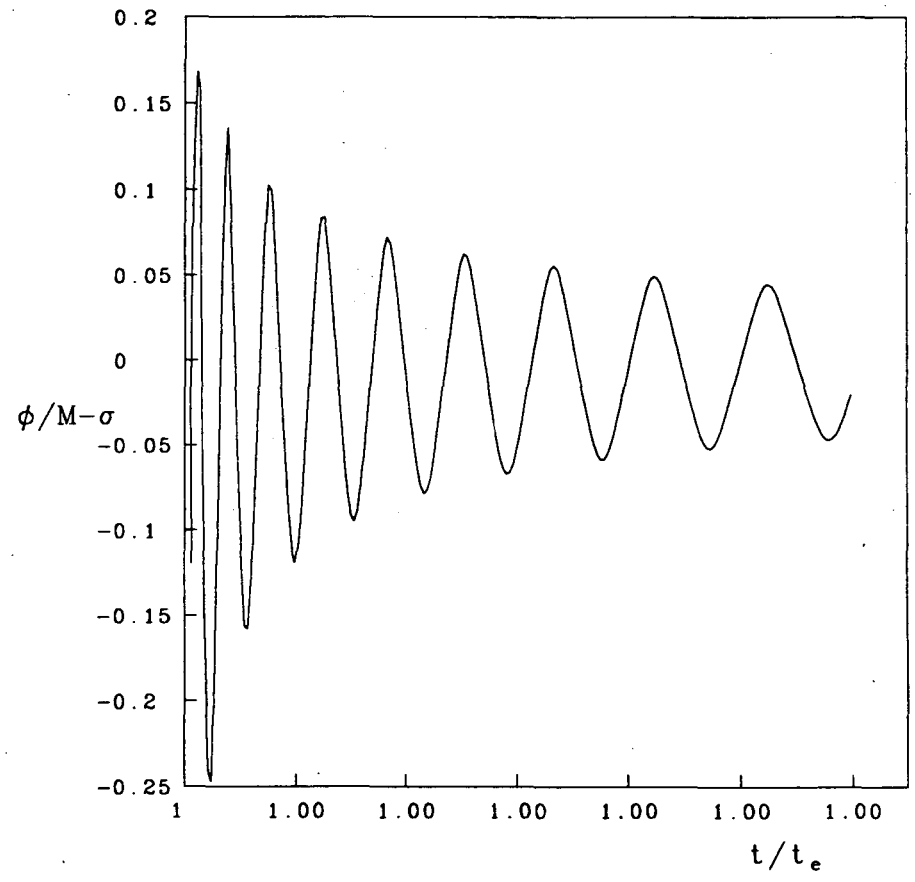
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−0.25
φ/M−σ
1
1.00
1.00
1.00
1.00
1.00
1.00
t/t_e

FIG 3.3

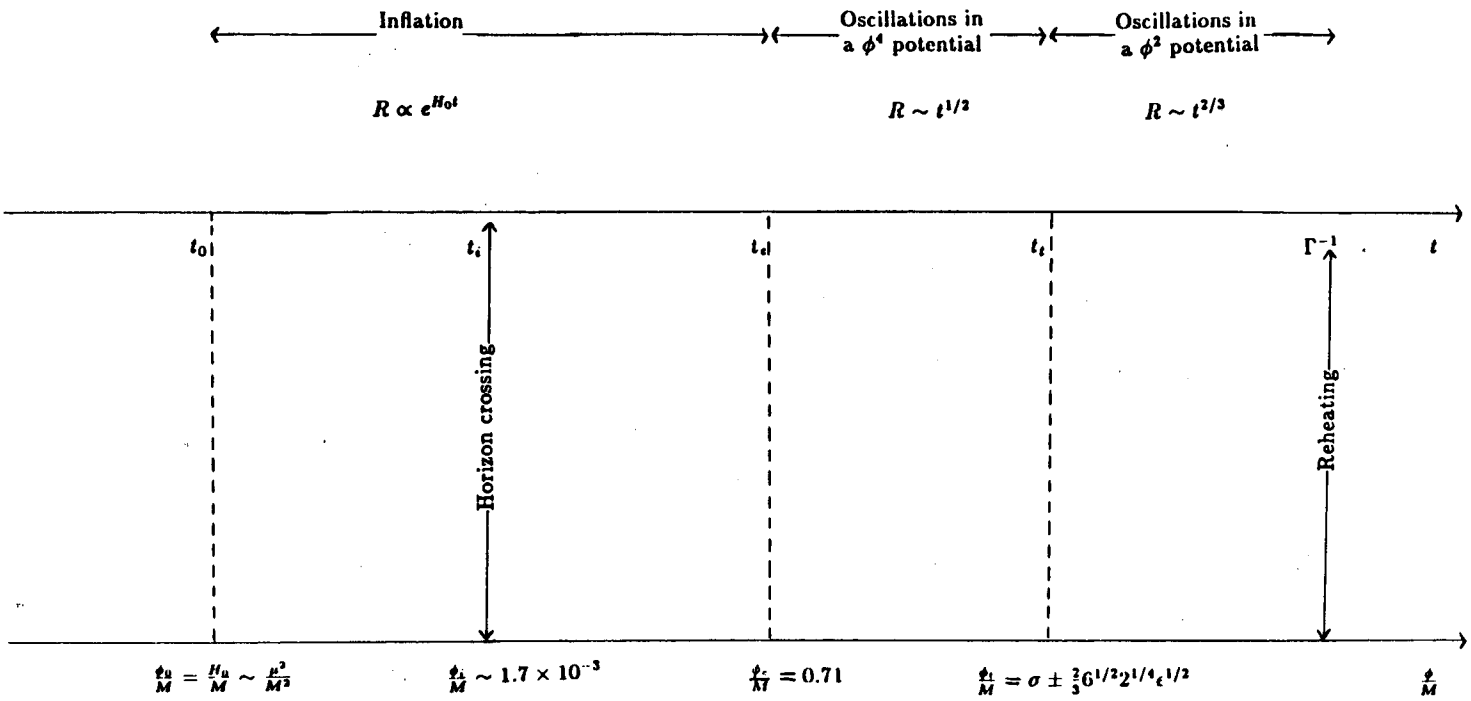
Inflation
Oscillations in a ϕ^4 potential
Oscillations in a ϕ^2 potential
$R \propto e^{H_0 t}$
$R \sim t^{1/2}$
$R \sim t^{2/3}$
t_0
t_i
t_d
t_t
Γ^{-1}
t
Horizon crossing
Reheating
$\frac{\phi_0}{M} = \frac{H_0}{M} \sim \frac{\mu^2}{M^3}$
$\frac{\phi_i}{M} \sim 1.7 \times 10^{-3}$
$\frac{\phi_c}{M} = 0.71$
$\frac{\phi_t}{M} = \sigma \pm \frac{2}{3} 6^{1/2} 2^{1/4} \epsilon^{1/2}$
$\frac{\phi}{M}$

Table I

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

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