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WITTEN'S STRING THEORY

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Abstract

We formulate Witten's proposal of a covariant open string theory in terms of oscillator modes and show that some basic axioms for the non-commutative geometry are obeyed as algebraic operations, which were defined previously from a geometrical point of view. Our strategy is based on the proper bosonization of the conformal ghost fields.

A main concern in string theory is to investigate covariant descriptions.[1-7] Recently, Witten[4] has proposed an interacting string theory from a geometrical point of view, in which a string interaction[5-7] is uniquely determined as a Chern-Simons term. This theory appears to be quite promising, although it has been formulated with rather intuitive arguments only. Hence, it is desirable to seek a more precise form.

The purpose of this letter is to give a treatment in terms of oscillator modes and to prove the axioms necessary for constructing a gauge invariant formulation of the string. Our main achievements come from the proper bosonization[8] of conformal ghosts, where the zero mode of the boson field plays an important role in realizing properly two degenerate vacua. We treat only open bosonic string theory in 26 dimensional space-time and postpone the treatment to other string theories to a future publication.

To start, we bosonize the conformal ghosts[9] in 2 dimensional space-time (σ,τ). A detailed explanation of the bosonization procedure is needed to clarify the special role of the boson zero mode and to better understand Witten's formulation.

The ghost (C), anti-ghost ($\bar{C}$) and their corresponding canonical conjugates (π_C and $\pi_{\bar{C}}$) are expanded into fermionic creation (C_n^+ and $\bar{C}_n^+$, $n > 0$) and annihilation operators (C_n and $\bar{C}_n$, $n > 0$),

$$C = \frac{1}{\sqrt{\pi}} \sum_{n=1}^{\infty} (C_n e^{-in\tau} + C_n^+ e^{in\tau}) \cos n\sigma + \frac{1}{\sqrt{\pi}} C_o$$

$$\bar{C} = \frac{-i}{\sqrt{\pi}} \sum_{n=1}^{\infty} (\bar{C}_n e^{-in\tau} - \bar{C}_n^+ e^{in\tau}) \sin n\sigma$$

$$i\pi_C = \frac{1}{\sqrt{\pi}} \sum_{n=1}^{\infty} (\bar{C}_n e^{-in\tau} + \bar{C}_n^+ e^{in\tau}) \cos n\sigma + \frac{1}{\sqrt{\pi}} \bar{C}_o$$

$$i\pi_{\bar{C}} = \frac{-i}{\sqrt{\pi}} \sum_{n=1}^{\infty} (C_n e^{-in\tau} - C_n^+ e^{in\tau}) \sin n\sigma \tag{1}$$

with non-vanishing commutators,

$$\{C_n \bar{C}_m^+\} = \{\bar{C}_n C_m^+\} = \delta_{n,m} (n,m > 0) \quad \text{and} \quad \{C_o \bar{C}_o\} = 1 \quad ,$$

where C_o is the Hermitian operator of the zero mode which leads to an unusual normalization of two degenerate ground states[9] $|\bar{0}\rangle_\pm$

$$\bar{C}_n |\bar{0}\rangle_\pm = C_n |\bar{0}\rangle_\pm = 0 \ , \ n > 0 \ , \ \bar{C}_o |\bar{0}\rangle_- = 0 \ , \ C_o |\bar{0}\rangle_- = |\bar{0}\rangle_+$$

and

$${}_\mp\langle\bar{0}|\bar{0}\rangle_\pm = 1 \quad . \tag{2}$$

Bosonization must preserve hermiticity of these ghosts and must have appropriate representation spaces corresponding to the ground states with the above normalization. Such a bosonization turns out to be

$$C_+ = \frac{1}{\sqrt{2\pi}} e^{i\phi_o} \cdot e^{i(\tau+\sigma)(P_o + \frac{1}{2})}$$

$$\cdot \exp\left(\sum_{n=1}^{\infty} e^{in(\tau+\sigma)} a_n^+/\sqrt{n}\right) \cdot \exp\left(\sum_{n=1}^{\infty} e^{-in(\tau+\sigma)} a_n/\sqrt{n}\right)$$

$$b_+ = \frac{1}{\sqrt{2\pi}} e^{-i\phi_o} \cdot e^{-i(\tau-\sigma)(P_o - \frac{1}{2})}$$

$$\cdot \exp\left[- \sum_{n=1}^{\infty} e^{in(\tau-\sigma)} a_n^+/\sqrt{n}\right] \cdot \exp\left[- \sum_{n=1}^{\infty} e^{-in(\tau-\sigma)} a_n/\sqrt{n}\right]$$

$$C_-(\sigma) \equiv C_+(-\sigma) \quad , \quad b_-(\sigma) \equiv b_+(-\sigma) \quad ,$$

$$[a_n a_m^+] = -\delta_{n,m} \ (n > 0) \quad , \quad [\phi_o, P_o] = i \quad , \tag{3}$$

with

$$C \equiv \frac{1}{\sqrt{2}} (C_+ + C_-) \quad , \quad \bar{C} \equiv \frac{1}{\sqrt{2}} (b_- - b_+) \quad , \quad i\pi_c \equiv \frac{1}{\sqrt{2}} (b_+ + b_-)$$

and

$$i\pi_{\bar{C}} \equiv \frac{1}{\sqrt{2}} (C_+ - C_-)$$

where the other commutators of a_n, and a_n^+ vanish. The representation space of operators (a_n, a_n^+, ϕ_o and P_o) are based on the following two vacua corresponding to $|\bar{0}\rangle_\pm$,

$$|0\rangle_\pm \equiv |0\rangle|V\rangle_\pm \ , \ |V\rangle_\pm = \frac{1}{\sqrt{2\pi}} e^{\pm i\phi_o/2} \text{ with } a_n|0\rangle = 0 \quad , \quad n > 0 \ . \tag{4}$$

The normalization of the state $|0\rangle_\pm$ is defined by

$$_\pm\langle 0|0\rangle_\mp = \int_{-\pi}^{\pi} d\phi_o \ f_\pm^*(-\phi_o) \ f_\mp(\phi_o) \cdot \langle 0|0\rangle = 1 \tag{5}$$

with

$$|V\rangle_{\pm} \equiv f_{\pm}(\phi_o) \quad \text{and} \quad \langle 0|0\rangle = 1 \quad .$$

where we have defined an inner product in the zero mode sector such as $\langle\alpha|\beta\rangle = \int_{-\pi}^{\pi} d\phi_o \alpha^*(-\phi_o)\beta(\phi_o)$. The zero modes ϕ_o or P_o play an important role. Actually, P_o is a ghost number operator, which generates transformations $C_{\pm} \to e^{i\theta}C_{\pm}(b_{\pm} \to e^{-i\theta}b_{\pm})$. Because it's eigenvalues for the vacua are $\pm\frac{1}{2}$, respectively, the operators, $C_{\pm}$ and $b_{\pm}$ turn out to be periodic functions of σ with periodicity of 2π on the representation space. Furthermore, we find that the factor, $e^{\pm i(\sigma\pm\tau)/2}$ in $C_{\pm}$ or $b_{\pm}$ are necessary to reproduce correctly the anti-commutation relations. As a check, we see that these factors realize the relations of $C_o|\bar{0}\rangle_- = |\bar{0}\rangle_+$ and $\bar{C}_o|\bar{0}\rangle_- = 0$, such as

$$C_o|0\rangle_- = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{\pi} d\sigma\, C_+(\sigma)|0\rangle_- = e^{i\phi_o}|0\rangle_- = |0\rangle_+$$

$$\bar{C}_o|0\rangle_- = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{\pi} d\sigma\, b_+(\sigma)|0\rangle_- = 0 \quad . \tag{6}$$

It is also important to find that both of $C_{\pm}$ and $b_{\pm}$ are Hermitian because the operators $e^{i\phi_o}\cdot e^{i(\tau\pm\sigma)(P_o+\frac{1}{2})}$ and $e^{-i\phi_o}\cdot e^{-i(\tau\pm\sigma)(P_o-\frac{1}{2})}$ are Hermitian with regard to the inner product, Eq. (5). Anyway, we can easily confirm that the bosonization formula, Eq. (3), reproduces all the two point functions of the conformal ghosts.

For convenience we will write the bosonized form of the ghosts more compactly; at $\tau = 0$ they are given by

$$C_+ = \frac{1}{\sqrt{2\pi}} : \exp\left[\sqrt{\pi}\ \phi + \sqrt{\pi} \int_0^\sigma p_c(\sigma')\ d\sigma'\right] : \quad ,$$

$$b_+ = \frac{1}{\sqrt{2\pi}} : \exp\left[-\sqrt{\pi}\ \phi + \sqrt{\pi} \int_0^\sigma p_b(\sigma')\ d\sigma'\right] :$$

$$\sqrt{\pi}\ \phi = i\phi_o + \sum_{n=1}^{\infty} \frac{\cos n\sigma}{\sqrt{n}} (a_n + a_n^+) \ , \tag{7}$$

$$\sqrt{\pi}\ P_c = i\left(P_o + \frac{1}{2}\right) + i \sum_{n=1}^{\infty} \cos n\sigma (a_n^+ - a_n) \sqrt{n}$$

and

$$\sqrt{\pi}\ P_b = \sqrt{\pi}\ P_c - i \quad .$$

The boson field ϕ satisfies canonical commutation relations,

$$[\phi(\sigma) \quad , \quad P_{b,c}(\sigma')] = -i\delta(\sigma - \sigma') \quad . \tag{8}$$

It should be mentioned that ϕ is not a real field, but Hermitian due to the normalization of the zero mode ϕ_o. If we consider the ghosts in 2 dimensional Euclidean space, ϕ is replaced in Eq. (7) by $i\phi_E$;

$$\phi_E = \phi_o + \sum_{n=1}^{\infty} \frac{\cos\ n\sigma}{\sqrt{n}} (a_n + a_n^+)$$

with $[a_n a_m^+] = \delta_{n,m}$. Likewise $P_{c,b}$ is replaced by $iP_{c,b}^E$;

$$P_c^E = P_o + \frac{1}{2} + \sum_{n=1}^{\infty} \sqrt{n} \cos\ n\sigma (a_n^+ - a_n)$$

and

$$\sqrt{\pi}\ P_b^E = \sqrt{\pi}\ P_c^E - 1$$

(This bosonization in Euclidean space was adopted[4] by Witten, but the hermiticity of the ghosts was not entirely clear because he did not address to the representation space of the boson field.)

Next, we define the wave functional of the string using this bosonization such that

$$W(\mu_x) \equiv W(\{X_n,\phi_n\}) = \langle\mu_x|W(X,C,\bar{C})|0\rangle_- \tag{9}$$

where C and $\bar{C}$ are bosonized ghosts, and X_μ is a string coordinate which is expanded into oscillator modes as usual,

$$X^\mu = \chi^\mu + i\sum_{n=1}^{\infty}\frac{\cos n\sigma}{n}(\alpha_n^\mu - \alpha_{-n}^\mu) = \chi^\mu + 2\sum_{n=1}^{\infty}\frac{\cos n\sigma}{\sqrt{n}}X_n^\mu$$

$$[\alpha_n^\mu \alpha_m^{+\nu}] = n\delta_{n,m}\eta^{\mu\nu} \quad (n,m > 0) \quad \text{and} \quad \eta^{\mu\nu} = (-1,1.....1) \tag{10}$$

with $\alpha_{-n}^\mu = \alpha_n^{+\mu}$ and $\alpha_n^\mu|0\rangle_- = 0$, $n > 0$. Hereafter, we omit space-time indices μ in X_n^μ and write X_n. In Eq. (9) we have adopted the vacuum $|0\rangle_-$ as a physical state according to Ref. 9; physical states must have ghost number, $-\frac{1}{2}$. The state $|\mu_x\rangle$ is an eigenstate of both $\phi_n = (a_n + a_n^+)/\sqrt{2}$ and $X_n = i(\alpha_n - \alpha_{-n})/2\sqrt{n}$, $n > 0$ with eigenvalues $\{\phi_n, X_n\}$.

On this wave functional, Witten has defined two operations, integral ($\int$) and product (x). They are

$$\int A \equiv \int \prod_{n=0}^{\infty} dX_n d\phi_n \prod_{n=odd} \delta(X_n)\delta(\phi_n)\ e^{-3\sqrt{\pi}\phi(\frac{\pi}{2})/2}\ A(\mu_x) \quad , \tag{11.a}$$

and

$$A \times B \equiv \int d\mu_X d\mu_Y \; e^{+3\sqrt{\pi}\phi_Z(\frac{\pi}{2})/2}$$

$$\cdot\; \delta(\mu_Z - \mu_X^L - \mu_Y^R)\; \delta(\mu_X^R - \mu_Y^L)\; A(\mu_X)B(\mu_Y) \quad , \tag{11.b}$$

with

$$d\mu_X \equiv \prod_{n=0}^{\infty} dX_n d\phi_n \quad ,$$

$$\delta(\mu_X^R - \mu_Y^L) = \prod_{\sigma=\frac{\pi}{2}}^{\pi} \delta(X(\sigma) - Y(\pi - \sigma))\; \delta(\phi_X(\sigma) - \phi_Y(\pi - \sigma))$$

and

$$\delta(\mu_Z - \mu_X^L - \mu_Y^R) =$$

$$\prod_{\sigma=0}^{\pi} \delta\left[Z(\sigma) - \theta(\tfrac{\pi}{2} - \sigma)\; X(\sigma) - \theta(\sigma - \tfrac{\pi}{2})\; Y(\sigma)\right]$$

$$\delta\left[\phi_Z(\sigma) - \theta(\tfrac{\pi}{2} - \sigma)\; \phi_X(\sigma) - \theta(\sigma - \tfrac{\pi}{2})\; \phi_Y(\sigma)\right]$$

The δ functions may be expressed in terms of oscillator modes, however their forms do not look simple. In the following discussion an over all factor in the definition of the product, Eq. (11b) will be irrelevant.

These operations along with BRS charge[10] were explained to obey several axioms in non-commutative geometry. Based on these axioms, Witten has constructed a unique gauge invariant interacting string theory.

The theory has action $S = \int (A \times QA + \frac{2}{3} A \times A \times A)$, which is invariant under the gauge transformation, $\delta A = Q\epsilon + A \times \epsilon - \epsilon \times A$ (Q is BRS charge[10]).

Our task will be to examine the axioms in detail with use of our bosonization. The axioms used by Witten are

$$\int A \times B = 0 \quad \text{unless} \quad n_a + n_b = -1 \tag{12.a}$$

$$\int A \times B = (-1)^{n_a \cdot n_b} \int B \times A \tag{12.b}$$

$$(A \times B) \times C = A \times (B \times C) \tag{12.c}$$

$$\int QA = 0 \tag{12.d}$$

$$Q(A \times B) = QA \times B + (-1)^{n_a} A \times QB \tag{12.e}$$

$$\int QA \times B = (-1)^{n_a+1} \int A \times QB \tag{12.e'}$$

where n_a is equal to (ghost number of a state A) $- \frac{1}{2}$. It is easy to understand both Eqs. (12.a) and (12.b), by noting that

$$\int A \times B = \int d\mu_x A(\tilde{\mu}_x) B(\mu_x)$$

and

$$A(B) \propto e^{i(n_a + \frac{1}{2})\phi_o} \left(e^{i(n_b + \frac{1}{2})\phi_o} \right) \tag{13}$$

where $A(\tilde{\mu}_x)$ denotes $A(\{(-1)^n X_n, (-1)^n \phi_n\})$. Equation (12.a) implies

$\int A \times B = \int B \times A; (-1)^{n_a \cdot n_b} = 1$ for $n_a + n_b = -1$ and n_a, n_b integer. Furthermore, from the geometrical interpretation of the operations defined by Witten, we can easily confirm that Eq. (12.c) holds.

Before proceeding further, we give some useful formulae for the BRS charge,

$$Q = \sqrt{\pi} \int_0^{\pi} d\sigma \left\{ - \frac{C_+ + C_-}{\sqrt{2}} \left[\frac{\pi P_x^2 + X'^2/\pi}{2} - i(C'\bar{C} - \pi'_{\bar{c}} \pi_c) \right] - \frac{C_+ - C_-}{\sqrt{2}} (P_x X' + C' \pi_c + \pi'_{\bar{c}} \bar{C}) \right\}$$

$$= \sqrt{\pi} \int_{-\pi}^{\pi} d\sigma \left\{ \frac{-\pi C_+}{2\sqrt{2}} (P_x + X'/\pi)^2 + \sqrt{2} i C_+ C'_+ b_- \right\} \tag{14}$$

where the prime denotes a derivative with respect to σ and P_x is the conjugate momentum of X.

Now, we examine the axiom, $\int QA = 0$. As a detailed discussion of it was already given by Witten, we only sketch its proof using our technique. The formula, $\int QA$ can be rewritten with Eq. (14) so that

$$\int d\bar{\mu}_x \int_{-\pi}^{\pi} d\sigma \; C_+(\sigma) e^{3ik(\sigma)/2} \cdot f(\sigma) e^{-3\sqrt{\pi}\phi(\frac{\pi}{2})/2} \cdot A$$

$$= \int d\bar{\mu}_x \int_{-\pi}^{\pi} d\sigma \; e^{ik(\sigma)} \cdot \prod_{n=e} \exp\left(\sqrt{\frac{2}{n}} (\cos n\sigma \phi_n) \right) \prod_{n=odd}$$

$$\exp\left(i\sqrt{\frac{2}{n}} \sin n\sigma \frac{\partial}{\partial \phi_n} \right) e^{i\phi_0} f(\sigma) \; e^{-3\sqrt{\pi}\phi(\frac{\pi}{2})/2} A \tag{15}$$

with $d\bar{\mu}_x \equiv d\mu_x \prod_{n=odd} \delta(X_n)\delta(\phi_n)$ and $k(\sigma) = \sigma + \sum_{n=e} \frac{(-1)^{n/2}}{n} \sin n\sigma$, where

$\prod_{n=e}$ ($\sum_{n=e}$) denotes a product (summation) with respect to even $n > 0$ and $f(\sigma)$ is $-\sqrt{\pi}\left[\frac{\pi}{2\sqrt{2}}(P_x+X'/\pi)^2 + \sqrt{2}\, i\, (C'_+ b_- + 3ikC_+ b_-/2)\right]$. In deriving Eq. (15) we have used partial integrations of $\frac{\partial}{\partial\phi_n}$ and $\frac{\partial}{\partial X_n}$ for n even, and constraints $X_n = \phi_n = 0$ for n odd. Here, we should note that the factor $e^{ik(\sigma)}$ is odd under $\sigma \to \pi-\sigma$ and that the odd part of the integrand under the transformation vanishes in the integration $\int_{-\pi}^{\pi} d\sigma$. These considerations and a careful analysis of $f(\sigma)$ lead us to the axiom, $\int QA = 0$. For instance, we take the term of P_x^2 in f, which is expanded into oscillator modes as

$$P_{0,x}^2 + 2P_{0,x} \cdot \sum_{n=1}^{\infty} \sqrt{n}\, P_{n,x} \cos n\sigma + \sum_{n,m=1}^{\infty} \sqrt{nm}\, P_{n,x} \cdot P_{m,x} \cos n\sigma \cos m\sigma$$

where $P_{n,x} = -\, i\, \frac{\partial}{\partial X_n}$. The first two terms vanish by partial integration of $P_{0,x}$ in Eq. (15); here we assume that there are no contributions from the boundaries. Terms including $P_{n,x}$, n = even > 0, can also be dropped using partial integrations. In this case we do not have to worry about boundary terms because the modes, X_n are massive. The remaining terms are ones with both n and m odd. Such terms are even under $\sigma \to \pi-\sigma$, so that in the integral, $\int_{-\pi}^{\pi} d\sigma$, they vanish due to the factor $e^{ik(\sigma)}$ which is odd under $\sigma \to \pi-\sigma$. Likewise, the other terms in f vanish.

A point we should be careful about in this derivation is the assumption that boundary terms vanish in partial integration of $\frac{\partial}{\partial X_o}$ (in Witten's paper, the assumption has been taken implicitly). If we compactify extra-space (D = 22), the boundary terms in that space will van-

ish. However, in our space-time (D = 4), the terms of the massless sector will still remain. Actually, we have checked that for a state of a gauge field $\langle \mu_x | A^\mu \alpha^+_{\mu,1} | 0 \rangle_-$, the axiom, $\int QA = 0$, holds without any assumption on such boundary terms.

We wish to mention that the insertion of $e^{-3\sqrt{\pi}\phi(\pi/2)/2}$ in the definition of the integral Eq. (11) is essential for the demonstration of the axiom Eq. (12.d). We see that $e^{-3\sqrt{\pi}\phi(\pi/2)/2}$ induces the factor of $e^{ik(\sigma)}$ in Eq. (15) which plays an important role in actual computations. Furthermore, we mention that the normal ordering[9] in BRS charge does not change the result because the relation $\int C_o A = 0$ also holds.

Next, we proceed to the axiom Eq. (12.e). Although it seems plausible from a geometrical point of view, it is quite difficult to understand it in terms of oscillator modes. The difficulty comes from the fact that we do not have[9] Hilbert space corresponding to half of the string; the functional, $A(X(\sigma))$ with $\frac{\pi}{2} > \sigma > 0$ or $\pi > \sigma > \frac{\pi}{2}$ doesn't correspond to any state vector of the string. Therefore, it is not obvious in Eq. (12.e) that the contribution from half of the string, $\int_{\pi/2}^{\pi} \rho \, d\sigma \, A$ (ρ is the density of the BRS charge) cancels with the contribution from the other half of the string, $\int_0^{\pi/2} \rho \, d\sigma \, B$. For this reason, we have not yet been able to prove the axiom. However, instead of the axiom, we can prove the corrolary Eq. (12.e'), which is obtained by integration of Eq. (12.e). Hence, it appears likely that the axiom Eq. (12.e) holds.

The proof of (12.e') goes as follows,

$$\int QA \times B = \int d\mu_x \langle\tilde{\mu}_x|QA|0\rangle_- \langle\mu_x|B|0\rangle_- =$$

$$\int d\mu_x \tilde{Q} \langle\tilde{\mu}_x|A|0\rangle_- \langle\mu_x|B|0\rangle_- \tag{16}$$

where $\tilde{Q}$ is a differential operator with arguments of $(-1)^n\phi_n$, $(-1)^n X_n$, $(-1)^n \frac{\partial}{\partial X_n}$ and $(-1)^n \frac{\partial}{\partial \phi_n}$. From the formula,

$$C_\pm (\{(-1)^n\phi_n, (-1)^n P_n\}\sigma) = C_\pm (\{\phi_n, P_n\} \sigma - \pi)e^{\pm i\pi(P_o+1/2)}$$

$$b_\pm(\{(-1)^n\phi_n, (-1)^n P_n\} \sigma) = b_\pm(\{\phi_n, P_n\}\sigma-\pi)e^{\pm i\pi(P_o - \frac{1}{2})} \tag{17}$$

with $P_n = i\frac{\partial}{\partial\phi_n}$, $\tilde{Q}$ is found to be $Qe^{i\pi(P_o + \frac{1}{2})}$.

Hence, Eq. (16) becomes

$$\int d\mu_x Q \langle\tilde{\mu}_x|A|0\rangle_-\langle\mu_x|B|0\rangle_-(-1)^{n_a + 1}. \tag{18}$$

To simplify our notation, we write the BRS charge and ghosts as

$$Q = \int_o^\pi d\sigma[(C_+ + C_-)q_+ + (C_+-C_-)q_-] \text{ and } C_\pm = e^{i\phi_o} \cdot e^{i\sigma(P_o + \frac{1}{2})} C_\pm^o \tag{19}$$

where $q_\pm$ and $C_\pm^o$ can be read from Eq. (3) and Eq. (14). Noting that

$$i(C'\bar{C} - \pi_{\bar{c}}'\pi_c) =$$

$$\frac{-1}{2\pi}(2P_o - 1 - 2i \sum_{n=1}^{\infty} \sqrt{2n}\, P_n \cos n\sigma - 2 \sum_{n=1}^{\infty} 1)\exp\left(\sum_{n=1}^{\infty} \frac{1}{n}\right)$$

$$C'\pi_c + \pi'_{\bar{c}}\bar{C} = \frac{-i}{\pi}\left(\sum_{n=1}^{\infty} \sqrt{2n}\ \phi_n \sin n\sigma\right) \exp\left(\sum_{n=1}^{\infty}\frac{1}{n}\right) \tag{20}$$

We find that q_+ is a real operator ($q_+^* = q_+$) and that q_- is a purely imaginary operator ($q_-^* = -q_-$), if P_o may be replaced with a real number; $P_n^* = -i\frac{\partial}{\partial\phi_n} = -P_n$, etc.

We, now, show that

$$\int d\mu_x\ QA(\tilde{\mu}_x)B(\mu_x) = \int d\mu_x A(\tilde{\mu}_x)\ QB(\mu_x).$$

First we rewrite the state $A(\tilde{\mu}_x)$ in terms of $\bar{A}_o(\mu_x)$ such as

$$A(\tilde{\mu}_x) = \langle\tilde{\mu}_x|A|0\rangle_- = \langle\tilde{\mu}_x|A_o|0\rangle\frac{1}{\sqrt{2\pi}}e^{i(n_a + \frac{1}{2})\phi_o}$$

$$= \langle\mu_x|\bar{A}_o|0\rangle^* f(\phi_o)$$

with $A \equiv A_o e^{i(n_a + 1)\phi_o}$ and $f(\phi_o) \equiv e^{i(n_a + \frac{1}{2})\phi_o}/\sqrt{2\pi}$

Then,

$$\int d\mu_x Q\langle\tilde{\mu}_x|A|0\rangle_-\langle\mu_x|B|0\rangle_-$$

$$= \int d\mu_x \int_o^{\pi} d\sigma\left[\left(e^{i\phi_o}\cdot e^{i\sigma(n_a + 1)}\ C_+^o + e^{i\phi_o}\cdot e^{-i\sigma(n_a + 1)}\ C_-^o\right)q_+\right.$$

$$\left. + \left(e^{i\phi_o}\cdot e^{i\sigma(n_a + 1)}\ C_+^o - e^{i\phi_o}\cdot e^{-i\sigma(n_a + 1)}\ C_-^o\right)q_-\right]\cdot$$

$$\langle\mu_x|\bar{A}_o|0\rangle^* f(\phi_o)\langle\mu_x|B|0\rangle_-$$

$$=\int d\mu_x \int_0^\pi d\sigma \Big[e^{i\phi_o} \cdot e^{i\sigma(n_a+1)} \langle \mu_x | C_+^0 q_+ \bar{A}_o | 0 \rangle^*$$

$$+ e^{i\phi_o} \cdot e^{-i\sigma(n_a+1)} \langle \mu_x | C_-^o q_+ \bar{A}_o | 0 \rangle^* +$$

$$e^{i\phi_o} \cdot e^{i\sigma(n_a+1)} \langle \mu_x | C_+^o q_- \bar{A}_o | 0 \rangle^* (-1)$$

$$- e^{i\phi_o} \cdot e^{-i\sigma(n_a+1)} \langle \mu_x | C_-^o q_- \bar{A}_o | 0 \rangle^* (-1) \Big] f(\phi_o) \cdot \langle \mu_x | B | 0 \rangle_- \qquad (21)$$

Using $n_a + n_b = -2$ and the completeness relation, $|\mu_x\rangle \prod_{n=1}^{\infty} dX_n d\phi_n \langle \mu_x| = 1$, we obtain

$$\int d\mu_x \, QA(\tilde{\mu}_x)B(\mu_x) = \int d\mu_x \langle \tilde{\mu}_x | A | 0 \rangle_- \, \langle \mu_x | \int_0^\pi d\sigma \, [(C_+ + C_-)q_+$$

$$+ (C_+ - C_-)q_-]^+ \, B|0\rangle_- = \int d\mu_x \, A(\tilde{\mu}_x) Q B(\mu_x) \, . \qquad (22)$$

Therefore, we find that the corrolary Eq. (12.e') holds. Some comments are in order: The BRS change Q must be defined with normal ordering :Q: and the difference between Q and :Q: is proportional to C_0 (C_0 is a zero mode of $C(\sigma)$). But, C_0 also satisfies Eq. (12.e') and hence :Q: does too.

We have proven in this paper that both operations of integral ($\int A$) and product ($A \times B$) along with the BRS charge obey all the axioms in Eqs. (12) except Eq. (12.e). The difficulty for the proof of Eq. (12.e) is that the Hilbert space for the string can not be decomposed in two parts, each of which corresponds to a Hilbert space for half of the

string.

Our arguments depend on the bosonization of the conformal ghosts with a careful construction of the representation space. Such a bosonization can be used, in principle, for representing string interactions in terms of operators of oscillator modes of the string. Therefore, Witten's proposal of the string theory can be made more precise in terms of these oscillator modes.

Finally we wish to comment on the free string action, $\int A \times QA$. It is easy to show that under the two conditions

$$A(\tilde{\mu}_x) \equiv A_0\,(\tilde{\mu}_x)\; e^{-\frac{1}{2}\phi_0} = A_0^*(\mu_x)\; e^{-\frac{1}{2}\phi_0} \tag{23}$$

and $A^+ = A$ where A is an operator in $\langle \mu_x | A | 0 \rangle_-$, gauge invariant kinetic terms of component fields are produced.[2] These two conditions turns out to be compatible and correspond to a condition of reality of wave functional, $A(x(\sigma), C(\sigma), \bar{C}(\sigma))$.

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