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TOP QUARK AND LIGHT HIGGS SCALAR MASS BOUNDS
IN NO-SCALE SUPERGRAVITY

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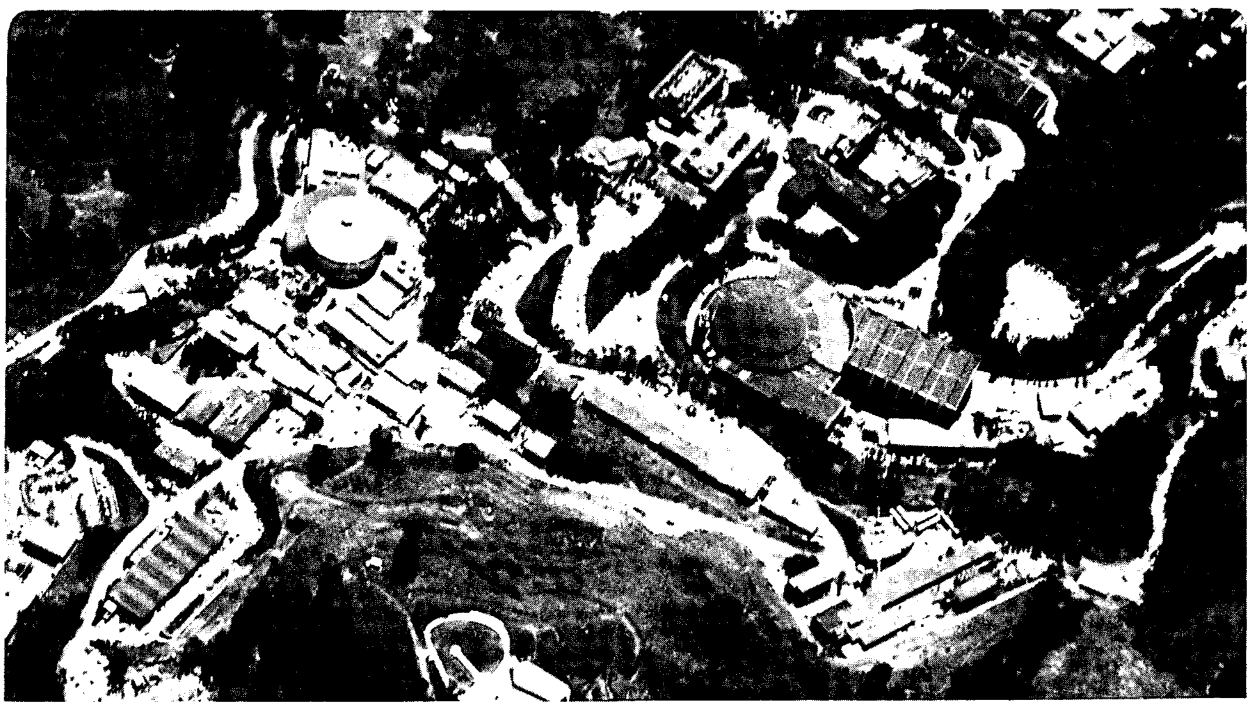

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TOP QUARK AND LIGHT HIGGS SCALAR MASS BOUNDS IN NO-SCALE SUPERGRAVITY [1] [2]

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No-scale supergavity theories with the minimal low-energy particle content are shown to become untenable for a top quark mass m_T much less than 40 GeV. For $m_T < 55$ GeV, a stringent upper bound operates on the mass of the lowest-lying Higgs scalar. Further, the Higgs pseudoscalar is constrained to be nearly a quarter as massive as the gluino.

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No-scale supergravity theories with the minimal low-energy particle content are shown to become untenable for a top quark mass m_T much less than 40 GeV. For $m_T < 55$ GeV, a stringent upper bound operates on the mass of the lowest-lying Higgs scalar. Further , the Higgs pseudoscalar is constrained to be nearly a quarter as massive as the gluino.

No-scale $N = 1$ supergravity theories[1] hold a lot of interest these days. Such a theory is likely to be the effective low-energy limit[2] of the heterotic superstring[3]. The scalar potential in this type of a theory is flat. That is ensured by a noncompact $SU(n,1)/SU(n)\times U(1)$ symmetry in the Kähler sector. Consequently, the Kähler potential is characterized by a particular logarithmic form, specifically $-3\ln[f(z)+f^*(z)+g(\phi^{\dagger\bar{i}},\phi^j)]$, where z is a generic gauge singlet scalar and ϕ^j's are $n-1$ gauge nonsinglet ones, f and g being analytic and real functions, respectively. The gravitino mass gets decoupled from the scale of global supersymmetry breaking at laboratory energies. The latter is seeded by a universal gaugino mass M at the grand unifying scale M_{GUT}. Since such theories have difficulties[4] admitting a fourth generation, I shall consider only no-scale supergravity theories with the minimal low-energy particle content — namely, three fermionic generations, the 3-2-1 gauge bosons, two Higgs doublets and superpartners for all.

An important question concerns the requirements of stability and electroweak symmetry breadkown in the light Higgs sector. These tightly constrain the squared mass parameters μ_i^2 defined through the quadratic part of the Higgs potential in terms of the $Y = \pm 1$ doublets $\phi_\uparrow, \phi_\downarrow$ as

$$V_H^{(2)} = (\phi_\downarrow\ \phi_\uparrow^*)^* \begin{pmatrix} \mu_1^2 & -\mu_3^2 \\ -\mu_3^2 & \mu_2^2 \end{pmatrix} \begin{pmatrix} \phi_\downarrow \\ \phi_\uparrow^* \end{pmatrix}$$

Radiative effects make $\mu_i^2 \equiv \mu_i^2(t)$, t being $\ln(M_{GUT}^2 Q^{-2})$ with Q as the energy scale. These functions of t evolve[5] to $t_W = \ln(M_{GUT}^2 M_W^{-2})$ from their boundary values at $t = 0$. Extrapolations, based on the observed values of the Weinberg angle and the rationalized fine structure constant, imply $M_{GUT} \sim 3.2\times 10^{16}$ GeV and $t_W \simeq 66.95$. The boundary conditions at $t = 0$ are, however, determined from the flavor-independence and the universality of gravitational interactions in the underlying supergravity theory. When the latter is of the no-scale type, all global supersymmetry breaking constants — except M — vanish at $t = 0$. Consequently, very stringent experimentally verifiable restrictions[6] emerge. In effect, these constraints provide laboratory tests of the type of theories considered here.

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Recall that, in the simplest softly broken supersymmetric extension of the standard 3-2-1 theory, the matter fields of the latter are extended into chiral superfields. Thus

$$\begin{pmatrix} u \\ d \end{pmatrix}_{iL} = q_i \to \hat{Q}_i \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_{iL} = \ell_i \to \hat{L}_i$$
$$u_{iL}{}^C \to \hat{U}_i^C \qquad e_{iL}{}^C \to \hat{E}_i^C$$
$$d_{iL}{}^C \to \hat{D}_i^C \qquad \phi_\uparrow \to \hat{\Phi}_\uparrow$$
$$\phi_\downarrow \to \hat{\Phi}_\downarrow$$

The general form of the superpotential is

$$f = \lambda_U^{ij}\hat{Q}_i\hat{U}_j^C\hat{\Phi}_\uparrow + \lambda_D^{ij}\hat{Q}_i\hat{D}_j^C\hat{\Phi}_\downarrow + \lambda_E^{ij}\hat{L}_i\hat{E}_j^C\hat{\Phi}_\downarrow + \mu\hat{\Phi}_\uparrow\cdot\hat{\Phi}_\downarrow \quad (1)$$

where λ's are Yukawa couplings and μ is a Higgs mass-mixing parameter. The soft supersymmetry breaking part of the Lagrangian is characterized by masses M_a of the gaugino fields λ_a, scalar masses m_{s_i} as well as mass parameters μ_3 and $\bar{A}^{ij}$

$$-\tfrac{1}{2}\textstyle\sum_a M_a\bar{\lambda}_a\lambda_a + \mu_3^2(\phi_\uparrow\cdot\phi_\downarrow + h.c.) - \sum_i m_{z_i}^2 \mid z_i \mid^2$$
$$-[(\lambda_U\bar{A}_U)^{ij}Q_iU_i^C\phi_\uparrow + (\lambda_D\bar{A}_D)^{ij}Q_iD_i^C\phi_\downarrow$$
$$+(\lambda_E\bar{A}_E)^{ij}L_iE_j^C\phi_\downarrow] \qquad (2)$$

Returning to the Higgs potential, let me recapitulate the conditions[7] imposed by minimization and stability, i.e.,

$$\cot^{-1}\frac{\langle\phi_\downarrow^0\rangle}{\langle\phi_\uparrow^0\rangle} \equiv \theta = \sin^{-1}\frac{2\mu_{3W}^2}{\mu_{1W}^2+\mu_{2W}^2} \quad real, \quad (3a)$$

$$\frac{1}{2}M_Z^2 = -\mu_{1W}^2 + (\mu_{1W}^2 - \mu_{2W}^2)\frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta} \quad (3b)$$

and that required by electroweak symmetry breadkown, namely

$$\det \mu_{Wij}^2 \equiv \mu_{1W}^2\mu_{2W}^2 - \mu_{3W}^4 < 0 \qquad (4)$$

The four physical Higgs masses $m_{\pm,a,b,c}$ obey the constraints[7]

$$m_c^2 = \mu_{1W}^2 + \mu_{2W}^2 = m_a^2 + m_b^2 - m_Z^2 = m_\pm^2 - M_W^2$$

$$m_b \geq M_Z \mid \cos 2\theta \mid \geq M_Z \mid (m_{1W}^2 - \mu_{2W}^2)(\mu_{1W}^2+\mu_{2W}^2)^{-1} \mid$$

$$m_a \leq \max(M_Z, m_c)$$

The boundary conditions at $t = 0$ are $m_{z_i}(0) = \overline{A}^{ij}(0) = \mu_{30}^2 = 0$, $\mu_{10}^2 = \mu_{20}^2 = \mu_0^2$, $M_a(0) = M$ and $\tilde{\alpha}_{2,3}(0) = 5/3\tilde{\alpha}_1(0) = \tilde{\alpha}(0) \simeq 1/96\pi$. Renormalization group evolution[8] makes μ_1^2 and μ_2^2 increase and decrease with t, respectively, the main driving term being $\lambda_T = \lambda_U^{33}$. Thus (3b) implies $1/2 < \cos\theta < 1$. Moreover, for $m_T < 55\ GeV$, the Alvarez-Gaumé, Polchinski, Wise analysis[9] showing that $\langle\phi_\uparrow^0\rangle \sim \langle\phi_\downarrow^0\rangle$ and $1/2 \sim \cos^2\theta$ holds so that all Yukawa couplings except $Y_T = (\lambda_T/4\pi)^2$ can be safely neglected in the 1-loop evolution equations. The latter are shown in Table 1. These are now analytically integrable. The solutions can best be expressed in terms of certain evolution functions displayed in Table 2.

Recall first that $\tilde{\alpha}_{aW}\tilde{\alpha}_{a0}^{-1} = M_{aW}M^{-1} = (1 + \tilde{\alpha}_{a0}b_a t_W)^{-1}$ where the b_a's are given in Table 1. The other relevant analytic solutions to the 1-loop renormalization group equations at $t = t_W$ may be written in terms of the evolution functions of the table and the dimensionless ratio $3F_W(2\sqrt{2}\pi^2E_W\cos^2\theta)^{-1}G_Fm_T^2 = \beta(\theta)$.

$$\tfrac{d}{dt}(M_a\tilde{\alpha}_a^{-1}) = 0, \ \tfrac{d}{dt}\tilde{\alpha}_a^{-1} = b_a,$$
$$(b_1, b_2, b_3,) = (-3, 1, 11)$$
$$(\tfrac{d}{dt} + 6Y_T - \tfrac{16}{3}\tilde{\alpha}_3 - 3\tilde{\alpha}_2 - \tfrac{13}{9}\tilde{\alpha}_1)Y_T = 0$$
$$(\tfrac{d}{dt} + 3Y_T - 3\tilde{\alpha}_2 - \tilde{\alpha}_1)\mu^2 = 0$$
$$\tfrac{d}{dt}\mu_1^2 = \tfrac{d}{dt}\mu^2 + 3\tilde{\alpha}_2M_2^2 + \tilde{\alpha}_1M_1^2$$
$$(\tfrac{d}{dt} + 6Y_T)(\mu_1^2 - \mu_2^2) = 3Y_T\bar{A}_T^2 + Y_T\cdot$$
$$\cdot\left[\tfrac{16}{3}(M_3^2 - M^2) + 9(M^2 - M_2^2) + \tfrac{13}{33}(M^2 - M_1^2)\right]$$
$$(\tfrac{d}{dt} + \tfrac{3}{2}Y_T - \tfrac{3}{2}\tilde{\alpha}_2 - \tfrac{1}{2}\tilde{\alpha}_1)\mu_3^2$$
$$= \mu(3\bar{A}_T - 3\tilde{\alpha}_2M_2 - \tilde{\alpha}_1M_1)$$
$$(\tfrac{d}{dt} + 6Y_T)\bar{A}_T = \tfrac{16}{3}\tilde{\alpha}_3M_3 + 3\tilde{\alpha}_2M_2 + \tfrac{13}{9}\tilde{\alpha}_1M_1$$

Table 1: 1-loop evolution equations with nontop Yukawa couplings neglected

$$E(t) = \left(1 - \tfrac{t}{32\pi}\right)^{-\frac{16}{9}}\left(1 - \tfrac{t}{96\pi}\right)^3\left(1 + 11\tfrac{t}{160\pi}\right)^{\frac{13}{99}}$$
$$F(t) = \textstyle\int_0^t d\tau E(\tau),$$
$$H(t) = \tfrac{t}{E}\tfrac{dE}{dt}, \ \bar{H}(t) = FH - tE + F$$
$$H'(t) = \tfrac{16}{3}\left[(1 - \tfrac{t}{32\pi})^{-2} - 1\right] + 6\left[1 - (1 + \tfrac{t}{96\pi})^{-2}\right]$$
$$-\tfrac{2}{9}\left[1 + 11\tfrac{t}{160\pi})^{-2}\right]$$
$$\bar{F}(t) = \tfrac{8}{9}\left[(1 - \tfrac{t}{32\pi})^2 - 1\right] + \tfrac{8}{99}\left[1 - (1 + 11\tfrac{t}{160\pi})^2\right]$$
$$g(t) = \tfrac{3}{2}\left[1 - (1 + \tfrac{t}{96\pi})^{-2}\right] + \tfrac{1}{22}\left[1 - (1 + 11\tfrac{t}{160\pi})^{-2}\right]$$
$$G(t) = \textstyle\int_0^t d\tau E(\tau)[\bar{F}(\tau) - \tfrac{1}{6}H'(\tau)]$$
$$-\tfrac{1}{6F}[4FH^2 - 4H\bar{H} - 2\textstyle\int_0^t d\tau E(\tau)H^2(\tau)]$$
$$K(t) = H(H - \tfrac{\bar{H}}{F}) + \tfrac{3}{2}G, \ L(t) = (H - \tfrac{\bar{H}}{F})^2,$$
$$S(t) = \tfrac{t}{32\pi}\left[1 + \tfrac{t}{96\pi})^{-1} + \tfrac{1}{5}(1 + 11\tfrac{t}{160\pi})^{-1}\right]$$

Table 2: Useful evolution functions

$$\mu_W^2 = \mu_0^2\left(1 + \frac{t_W}{96\pi}\right)^2\left(1 + 11\frac{t_W}{160\pi}\right)^{\frac{1}{11}}(1 - \beta(\theta))^{\frac{1}{2}}$$
$$\mu_{1W}^2 = \mu_W^2 + g_WM^2 \qquad (5)$$
$$\mu_{2W}^2 = \mu_{1W}^2 - \beta(\theta)(K_W - \frac{1}{2}L_W\beta(\theta))M^2$$
$$\mu_{3W}^2 = \left\{\frac{1}{2}\beta(\theta)\left(\frac{t_WE_W}{F_W} - 1\right) - S_W\right\}M\mu_W$$

The substitution of μ_{iW}^2 from (5) into inequality (3), together with the numerical evolution of the evolution constants up to the third decimal place, leads to

$$\eta^2 + (0.707 - 0.223\frac{w}{\cos^2\theta} + 0.002\frac{w^2}{\cos^4\theta})\eta$$
$$+0.284 - 0.149\frac{w}{\cos^2\theta} + 0.002\frac{w^2}{\cos^4\theta} < 0 \qquad (6)$$

In (6) $\eta \equiv M^{-2}\mu_W^2$ and $w = (m_T/40\ GeV)^2$. For $1/2 < \cos^2\theta$ and $m_T < 55\ GeV$, the real non-negativity at η turns out to necessarily imply the negativity of the η-independent part in (6). (Note that a vanishing μ_0, and hence μ_W, as suggested by the dimensional reduction[2] of superstring theories, yields this result directly.) The consequences are twofold:

(1) $0.5 < \cos^2\theta < 0.5085w$

$$\text{i.e.,}\quad \frac{m_b}{M_Z} < |\cos 2\theta| < 1.017\left(\frac{m_T}{40GeV}\right)^2 - 1 \qquad (7)$$

$$(2)\quad 0.983 < w,\ \text{ i.e., }\ 39.6\ GeV \lesssim m_T \qquad (8)$$

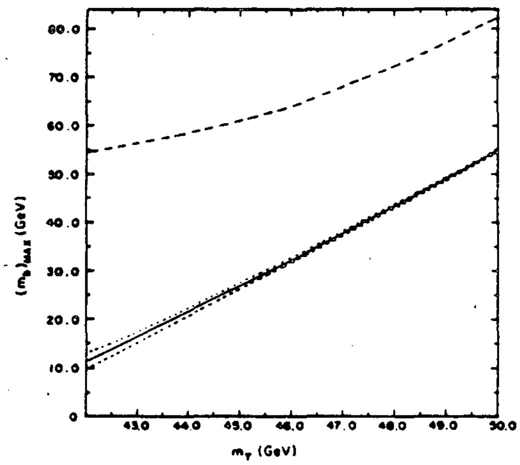

Figure 1: Plot of the upper bound on m_b against m_T in no-scale theories (solid curve, with the dotted band representing the estimated error from radiative corrections). The dashed curve is the bound (Drees et al.[10]) for more general $N = 1$ supergravity theories.

Figure 1 shows the upper bound (7) plotted against m_T in comparison with the earlier weaker result[10] for more general $N = 1$ supergravity theories. For $m_T < 42\ GeV$, the error due to radiative corrections (dotted band) becomes quite large. As m_T goes below 39.6 GeV, the bound becomes imaginary. On the other side, it saturates at unity as m_T exceeds[9] 55 GeV. The pseudoscalar mass is given by

$$\begin{aligned} m_c^2 &= \mu_{1W}^2 + \mu_{2W}^2 \\ &= M^2(2\eta + 1.066 - 0.279w\cos^{-2}\theta \\ &\quad + 0.004w^2\cos^{-4}\theta) \end{aligned}$$

with η varying between 0 and $\eta_+(\cos\theta, w)$ which is the higher root of the quadratic equation corresponding to (6) being an equality. For $m_T < 50\ GeV$, the absolute upper bound on η is $\eta_+(0.15, 1.25) = 0.097$ so that one finds by use of $m^2 \simeq 0.112 m_{\tilde g}^2$ that $0.21 m_{\tilde g} < m_c < 0.28 m_{\tilde g}$, where $m_{\tilde g}$ is the gluino mass.

It is my conclusion that no-scale $N = 1$ supergravity theories with the minimal low-energy particle content face rather critical experimental tests in the near future through (7) and (8).

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