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FIELD EFFECT AND JOSEPHSON JUNCTION

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Abstract

The value of the Josephson current in an S-M-S system, where M contains low-dimensional electron gas, can be affected noticeably by an applied voltage (field effect). This effect is directly related to the possibility of building a three-terminal device. The dependence of the current on the carrier concentration is studied. The peculiar situation when the state of lowest subband is described by the "dirty" limit whereas the higher subband represent the "clean" limit appears to be realistic. Intersubband scattering results in a non-monotonic behavior of the current.

Introduction

The field effect as a tool allowing one to change the value of the Josephson current is very promising from the point of view of applications. From this perspective, experimental results obtained[1] for the Nb-InAs-Nb system have attracted considerable interest. InAs is characterized by the presence of a natural inversion layer which provides the weak coupling in the Josephson contact.

An externally applied electric field changes the electron concentration n in the semiconductor. It is of interest to study the dependence of the current on n. This question was studied by the author,[2] and it has been shown that the use of low-dimensional systems (an inversion layer is an example of a two-dimensional electron system; the use of a one-dimensional system looks even more promising, see ref. [2]) leads to a noticeable increase of the field effect.

In ref. [2] the situation when only the lowest subband is filled has been studied. For InAs this corresponds to $N_s \lesssim 10^{12}$ cm^{-2} (see Ref. [3]; $N_s = nd$ is the surface concentration, d is the inversion layer thickness). It is interesting to analyze the dependence $j(N_s)$ for greater concentrations. A non-monotonic dependence has been observed.[1]

An increase in N_s results in the appearance of different groups of electrons in the semiconductor. These groups correspond to different electronic levels and are characterized by different values of the coherence length. The interesting situation when one group is "clean", whereas another is "dirty" becomes possible.

Theory

Consider the system S_α-M_β-S_γ (Fig. 1) where S_α and S_γ are superconductors and M_β contains a size-quantized electron gas. An inversion layer or a size-quantizing thin film are examples of such a M_β system. The finiteness of the thickness leads to the appearance of the quantum number i describing the

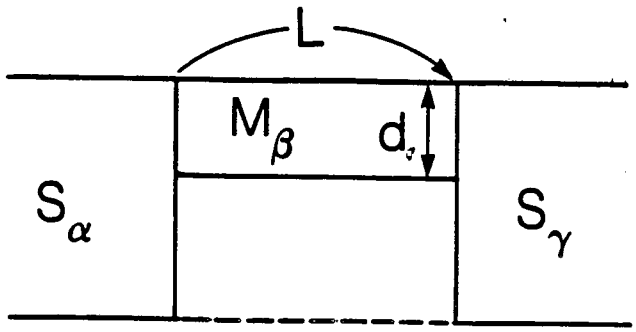

Fig. 1.

subbands of the transverse motion. The length L is assumed to be large (e.g. in ref. 1 L is taken to be ~ (3-5) x10^3 Å), so there is no quantization in the x-direction. If the size in y-direction is large, then the system appears to be two-dimensional in momentum space. If the size in the y-direction is small enough, then we have a one-dimensional system. In this paper, we restrict ourselves to the analysis of a two-dimensional system.

The system of interest differs from a usual Josephson contact. The subsystem M_β is not an insulator but contains degenerate electron gas. The non-dissipative current appears as a result of the proximity effect.

Our analysis is based on the method of thermodynamic Green's functions. Let us introduce the function $G_i(\vec{r},\tau;\vec{r}',\tau')$, where $\vec{r} = \{x,y,z\}$, τ is the imaginary time, and i denotes the transverse quantum number. The current flowing in the system is (in units $\hbar=k=1$)

$$j = ie\,(\nabla-\nabla') \sum_i \frac{1}{m_i^*} G_i(\vec{r},\vec{r}';\,\omega_n)\big|_{\vec{r}'=\vec{r}} \,. \qquad (1)$$

Here $\omega_n = (2n+1)\pi T$. Thanks to the equation of continuity, one can evaluate the current at any point x; usually it is convenient to choose the center of the M film.

The proximity effect implies the existence of transitions $\beta \to \alpha(\gamma)$ with the formation of Cooper pairs (and vice versa). These transitions can be described by the self-energy parts $\Delta_i \equiv \Delta_{ii} = \langle\psi_i\psi_i\rangle$.

The equation for the Green's function in the β film can be written in the form:

$$\begin{aligned} G_{\beta i}(x,x';p_y;\omega_n) &= G_{\beta i}^{\,0}(x,x';p_y;\omega_n) + \\ & G_{\beta i}^{\,0}(x,0;p_y;\omega_n)\,\Delta_i^{\ell}(\omega_n)\,G_{\beta i}^{\,0}(L,0;-p_y;-\omega_n)\,\times \\ & \Delta_i^{r*}(\omega_n)\,G_{\beta i}(L,x';p_y;\omega_n) + G_{\beta i}^{\,0}(x,L;p_y;\omega_n)\,\times \\ & \Delta_i^{r}(\omega_n)\,G_{\beta i}^{0}(0,L;-p_y;-\omega_n)\,\Delta_i^{\ell*}(\omega_n)\,G_{\beta i}(0,x';p_y;\omega_n) \end{aligned} \qquad (2)$$

We made a Fourier transformation with respect to $y-y'$; Δ^{ℓ} and Δ^{r} describe $\beta \to \alpha$ and $\beta \to \gamma$ transitions, respectively. In addition, the self-energy parts are taken in the local approximation.[2,4]

The self-energy parts Δ_i describe pairing of electrons with opposite momenta (and spins). It is important to stress that if $\delta E > \epsilon_0$ (ϵ_0 is the energy gap and δE is the spacing between transverse levels), then the paired electrons belong to the same subband.[5] An increase in thickness and the corresponding decrease in δE results in it becoming necessary to take into account the functions $\Delta_{ik}(i \neq j)$. In this paper we consider the most interesting cases, when one or two subbands are filled and $\delta E > \epsilon_0$.

We are going to analyze various cases which differ in the number of filled subbands and, in addition, in the concentration of impurities. The impurities are playing a double role in the problem of interest. First of all, scattering by impurities leads to the appearance of intersubband transitions. In addition, this scattering affects the value of the current (the usual separation into the "clean" and "dirty" cases is due to this effect).

One Filled Subband

The number of filled subbands is determined by the electron concentration and the spacing δE. If

$$N_s < (m^* \delta E)/\pi, \tag{3}$$

then only the lowest subband is filled (N_s is the surface concentration, m^* is the effective mass). The condition (3) can be easily obtained from the relations:

$$p_f^2/(2m^*) < \delta E, \; p_f = (2\pi N_s)^{1/2} \tag{3'}$$

If only the lowest subband is filled, one can put $i = 1$ in Eq. (2). The inequality $L \gg \xi_N$, where $\xi_N = v_F/kT$, allows one to solve Eq (2), and we obtain:

$$\begin{aligned} G_\beta(x,x';p_y;\omega_n) &= G_\beta^0(x,x';p_y;\omega_n) + \\ & G_\beta^0(x,0;\, p_y;\omega_n)\, \Delta^{\ell}(\omega_n)\, G_\beta^0(L,0;-p_y;-\omega_n) \times \\ & \Delta^{r*}(-\omega_n)\, G_\beta^0(L,x';p_y;\omega_n) + G_\beta^0(x,L;p_y;\omega_n) \times \\ & \Delta^{\ell*}(\omega_n)\, G_\beta^0(0,L;-p_y;-\omega_n)\, \Delta^{r}(\omega_n)\, G_\beta^0(0,x';p_y;\omega_n) \end{aligned} \tag{4}$$

The self-energy parts are equal to

$$\Delta^{\ell,r} = g_{\alpha(\gamma);\beta}\, F_{\alpha(\gamma)};\; F_{\alpha(\gamma)} = |F_{\alpha(\gamma)}| \exp(i\theta_{\alpha(\gamma)})$$

$$|F_\alpha| = \Delta_\alpha(\omega_n^2 + \xi_\alpha^2 + \Delta_\alpha^2)^{-1}\;;\; G_\beta \equiv G_{\beta 1} \tag{5}$$

$F_{\alpha(\gamma)}$ is the anomal Green's function. Based on Eqs. (1), (4), and (5) we arrive at the following expression for the Josephson current:

$$j_x = j_m \sin(\theta_\alpha - \theta_\gamma)\;, \tag{6}$$

where

$$j_m = \bar{\gamma} T \sum_{\omega_n} \frac{\Delta_\alpha \Delta_\gamma}{(\omega_n^2+\Delta_\alpha^2)^{1/2}\,(\omega_n^2+\Delta_\gamma^2)^{1/2}}\, I(L,\omega_n). \tag{7}$$

Here

$$I(L,\omega_n) = \int_0^\infty dp_y\, G_\beta^0(L,0;p_y;-\omega_n)\, S_\beta(L/2;p_y;\omega_n),$$

$$\begin{aligned} S_\beta(x,p_y,\omega_n) &= G_\beta^0(x,0;p_y;\omega_n)(\partial/\partial x)G_\beta^0(L,x;p_y;\omega_n) - \\ & G_\beta^0(L,x;p_y;\omega_n)(\partial/\partial x)G_\beta^0(x,0;p_y;\omega_n), \end{aligned} \tag{7'}$$

G_β^0 is the Green's function of the normal system which can be taken in the form: $G_\beta^0 = \lambda_n^{-1} \times m^* \exp\{i\lambda_n|x-x'|\}$, $\lambda_n = [2m^*_\beta(i\omega_n - \xi_\beta)]^{1/2}$, $\xi_\beta = (p_y^2/2m^*_\beta) - \epsilon_{F\perp}$, $\epsilon_{F\perp} = \epsilon_F - \epsilon_\perp$.

In the "clean" case ($\ell > \xi_N$, ℓ is the mean free path, $\xi_N = v_F/\pi T$), after some calculations we obtain

$$j_m = A \exp\left\{-\rho/(N)^{1/2}\right\} \tag{8}$$

where $\rho = (2\pi)^{1/2}\, Tm^*L$, and the coefficient A depends slowly on $N \equiv N_s$.

One can see directly from Eq. (8) that the current depends exponentially on the electron concentration.

In the "dirty" case ($\ell \ll \xi_N$), it is necessary to average the expression for the current over the impurities positions. Based on the method,[6] we obtain the following dependence $j_m(N_s)$ (we assume $\Delta_\alpha = \Delta_\gamma$):

$$j_m \sim \exp\left\{-\rho_d/N^{1/2}\right\},\; \rho_d = \rho(\hbar/\pi\tau T), \tag{9}$$

or

$$j_m \sim \exp\left\{-r/(N\mu(N))^{1/2}\right\},\; r = (2m^*_\beta Te)^{1/2}L\;. \tag{10}$$

Here $\mu = e\ell/m^*_\beta v_F$ is the mobility.

Two Filled Subbands ("clean" case)

An increase in the electron concentration results in a situation, when $N_s > (m^*\delta E)/\pi$. Then the second subband becomes occupied. If $\delta\epsilon > \epsilon_0$, electrons from the same subband pair up.[5] The evaluation for each subband can be carried out by analogy with the preceding section, and the total current is a sum of the contributions of the subbands, that is

$$j_m = A_1\left\{\exp -L/\xi_{N1}\right\} + A_2\left\{\exp -L/\xi_{N2}\right\}. \tag{11}$$

Here

$$\xi_{N1} = v_{F1}/\pi T,\; \xi_{N2} = v_{F2}/\pi T\;.$$

Despite the fact that the current is a simple sum of two terms, the dependence of j_m upon the electron concentration for one filled subband differs from that in the case of two filled subbands. Indeed, in the latter case one should take into account the following conditions:

$$p_{f1}^2 + p_{f2}^2 = 2\pi N_s,$$

$$p_{f1}^2/2m_1 = p_{f2}^2/2m_2 + \delta E, \qquad (12)$$

and we obtain:

$$v_{F1} = (\pi N_2 + m\delta E)^{1/2}/m,$$

$$v_{F2} = (\pi N_s - m\delta E)^{1/2}/m$$

(we assume $m_1^* = m_2^* = m$).

The change of the dependence $j_m(N_s)$ due to the filling of the second subband becomes apparent if we consider the realistic case when v_{F1} and v_{F2} differ noticeably, i.e., $(\Delta v_F/v_{F2}) \gg 1$. In this case $\xi_{N1} \gg \xi_{N2}$ and the contribution of the second term in Eq. (11) is very small. Hence, the main contribution comes from the lowest subband. However, the dependence $j_m(N)$ is different (cf., Eq. (9) and Eq. (10)). In both cases the dependence $j_m(N_s)$ is determined, mainly, by the dependence $v_F(N_s)$. One can see directly from Eqs. (3') and (12) that these dependences are different. Calculating the derivative dj_m/dN_s, one comes to the conclusion that the second subband being occupied slows down the rate of increase of the current j_m as a function of the concentration N_s.

Intermediate Case

Consider the case of two filled subbands in the presence of a large concentration of impurities. One can see that this case is characterized by some peculiar features.

As is known, the question of a system being "clean" or "dirty" can be resolved depending on the relation between the coherence length ξ_N and the mean free path ℓ. In the case of interest, there are two coherence lengths ξ_{N1} and ξ_{N2} and the situation when $\xi_{N1} \gg \ell \gg \xi_{N2}$ is perfectly realistic. This means that the lowest subband is "dirty" whereas the second one is "clean". Hence, the single sample cannot be assigned to some definite category. It contains two groups of carriers which differ noticeably in their coherence lengths. The amplitude of the current is equal to:

$$j_m = j_{m1} + j_{m2}, \qquad (13)$$

here

$$j_{m1} = A^D \exp\left\{-L/\xi_{N1}^D\right\}, \; j_{m2} = A \exp\left\{-L/\xi_{N2}\right\}, \qquad (13')$$

where

$$\xi_{N1}^D = \frac{1}{2}\,(\xi_{N1}\ell)^{1/2}, \; \xi_{N1} = v_{F1}/\pi T, \; \xi_{N2} = v_{F2}/\pi T. \qquad (13'')$$

In our case, $\xi_{N2} \ll \xi_{N1}^D$ and, therefore, the major contribution comes from the lowest subband, that is $j_m = A^D \exp\left\{-L/\xi_{N1}^D\right\}$. However, the occupation of the second subband leads to an essential change of the dependence $J_m(N_s)$. First of all, the dependence $v_{F1}(N_s)$ differs from the case of one filled subband (see previous section). But more importantly, there is a noticeable change of the relaxation time τ_1. The presence of impurities results in an additional relaxation channel, and, a result, in a noticeable decrease of τ_1. Namely, intersubband transitions due to scattering become possible. The quantity τ_1 is determined by the equation

$$\tau_1^{-1} = \tau_{11}^{-1} + \tau_{12}^{-1}. \qquad (14)$$

τ_{11} and τ_{12} describe intrasubband and intersubband transitions, respectively. τ_{1i} can be evaluated with the use of the equation (see, e.g., ref. 7).

$$\tau_{1i}^{-1} = 2\pi \int V_{1i}^{eff}(\vec{k},\vec{k}')\,(1-\cos\theta_{\vec{k},\vec{k}'})\,\delta(\varepsilon_1(\vec{k}) - \varepsilon_i(\vec{k}'))\,d\vec{k}_i'\,; \qquad (15)$$

$i = 1,2$; $\vec{k}$ is the two-dimensional quasimomentum.

Note that V_{1i} is the matrix element of the $1 \to i$ transition. If the scattering is due to charged impurities then V^{eff} is the screened potential. The electrons of both subbands contribute to the screening. V^{eff} satisfies the equation:

$$V^{eff}(\vec{r},\vec{r}',\omega) = V_0(\vec{r},\vec{r}',\omega) + \int V_0(\vec{r},\vec{r}'\omega)\,\gamma_1\Pi_{11}(\vec{r}_1,\vec{r}_2,\omega)\,\times \gamma_2 V^{eff}(\vec{r}_2,\vec{r}_1)\,d\vec{r}_1 d\vec{r}_2 + \int V_0(\vec{r},\vec{r}_1,\omega)\,\gamma_2\Pi_{22}(\vec{r}_1,\vec{r}_2,\omega)\,\times \gamma_2 V^{eff}(\vec{r}_2,\vec{r}',\omega)\,d\vec{r}_1 d\vec{r}_2\,. \qquad (16)$$

Here $\vec{r} = \{\vec{\rho},z\}$, V_0 is the bare potential. Fourier transforming with respect to $\vec{\rho}-\vec{\rho}'$, one obtains:

$$V_i^{eff}(\vec{q}) = V_{0i}(q)\,[1 - V_{0i}(q)\,\gamma_1\Pi_{11}(\vec{q})\gamma_1 - V_{0i}(q)\gamma_2\Pi_{22}(\vec{q})\,\gamma_2]^{-1}. \qquad (17)$$

Here $V_{0i}(q) = \int V_0(\vec{r})e^{-i\vec{q}\vec{\rho}}\chi_i^*(z)\chi_i(z)d\vec{\rho}dz$; $V_0 = Ze/r$; Π_{11} and Π_{22} are polarization operators for the quantized system (see, e.g., ref. 8); τ_{1i} is determined by Eqs. (15) and (17).

Hence, the occupation of the second subband is accompanied by a decrease of τ_1 and, as a result, by a decrease of ξ_{N1}^D. It is important that the two-dimensional density of states is equal to $4m^*/\pi$ and does not depend on p_{F2}, that is, on N_s. Hence, τ_{12}^{-1} can be large even for small fillings.

Hence, if the concentration $N_s = (m^*\delta\varepsilon)/\pi$ (the second subband becomes occupied; for InAs $N_s = 10^{12}$ cm^{-1}), a sharp decrease of ξ_{N1}^D takes place, accompanied by a decrease of the Josephson current. Therefore, in this region, a peak of j_m should be

observed. Such a peak has been observed experimentally[1] for the Nb-InAs-Nb junction. Similar peaks should be observed if the next subbands become occupied.

Hence, the dependence of j_m on N_s is non-monotonic. The amplitude of j_m oscillates, and these oscillations correspond to fillings of the higher subbands.

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