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Derivation of the Window Formula with Linear Response Theory*

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Linear response theory is employed to derive the rate of energy dissipation in a binary one-body potential well whose two parts are connected by a small "window" and are in slow relative motion. Under suitable randomization assumptions, the "completed wall-and-window formula" is obtained, including the contribution from the change in the mass asymmetry. The perspectives for applying the same method to the transitional shapes encountered in quasi-fission reactions are discussed.

1. Introduction

As was first recognized by Hill and Wheeler, [1] the long nucleonic mean free path has profound consequences for the character of large-scale nuclear dynamics. The first comprehensive study of this "new dynamics", often referred to as *one-body nuclear dynamics* (since the motion of the nucleons is governed by the changing one-body mean field), was carried out about ten years ago within the framework of classical kinetic theory.[2] It led to two remarkably simple formulas for the rate of energy dissipation: the *wall formula* pertaining to a slowly deforming mononucleus, and the *window formula* pertaining to a dinucleus whose two parts are in slow relative motion. The wall and window dissipation formulas have been employed extensively, with a considerable degree of success, to low-energy nuclear dynamical processes as occuring in fusion, fission, and damped reactions.

Quasi-fission reactions, which were discovered only relatively recently, have provided new testing ground for theories of nuclear dynamics (see, for example, ref. 3). These reactions are believed to proceed through shapes which are somewhat intermediate between mononuclei and dinuclei: they are rather compact and yet they possess a well-defined (and slowly evolving) mass asymmetry. As a consequence of this

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more complicated geometry, quasi-fission reactions are harder to treat theoretically and calculations have, so far, employed simple *ad hoc* interpolations between the wall and window formulas. [4] This situation is far from satisfactory. In order to make progress in our understanding of these processes, it is necessary to develop the one-body dissipation theory to encompass also such transitional shapes. It is towards this goal that the present study is oriented.

2. Characterization of the problem

The validity of the simple wall formula has been studied in a variety of more refined formal frameworks. Most relevant for the present study is the work by Koonin and Randrup based on linear response theory. [5] In that work it was shown that the one-body energy dissipation rate can be expressed as

$$\dot{Q} = \lim_{t \to -\infty} \langle \dot{H}_1 \left[\int_t^0 dt' \; \hat{U}_0(t') - \hat{U}_0(t) t \right] \dot{H}_1 \frac{\partial f_0}{\partial H_0} \rangle \; . \tag{1}$$

The instantaneous one-body field is described by the Hamiltonian H_0 and $\hat{U}_0$ is the associated evolution operator. The slow distortion of the nucleus is described by the time-dependent perturbation $H_1(t)$. The expression (1) can be interpreted as follows. At $t=0$ the nucleons have the phase-space distribution $f_0(H_0)$, which is assumed to depend only on the energy $H_0(\mathbf{r},\mathbf{p})$. (Note that the factor $\partial f_0/\partial H_0$ in (1) ensures that only nucleons near the Fermi surface contribute.) Each phase-space point $(\mathbf{r},\mathbf{p})$ is traced back in time from $t=0$ to $t \to -\infty$; the brackets indicate the corresponding phase-space integral. The first term follows the trajectory $(\mathbf{R},\mathbf{P})$ of an individual nucleon as it bounces around in the *unperturbed* field H_0, receiving impulses $\dot{H}_1(\mathbf{R},\mathbf{P})$ along the way due to the perturbation H_1. The second term is a correction which is instrumental in ensuring convergence when regularities are present in the shape or its rate of distortion.

If the perturbation consists of inducing local movements of the surface elements in a leptodermous cavity, the impulses $\dot{H}_1$ are received as the nucleon is reflected from the wall. The dissipation rate then has the form of a double surface integral,

$$\dot{Q} = \int d^2a \int d^2b \;\; u(a)\, \gamma(a,b)\, u(b) \; , \tag{2}$$

where $u(a)$ and $u(b)$ are the (normal) velocities of the surface at the points a and b. The dissipation ker-

nel $\gamma(a,b)$ is non-local, i.e. depends on points far apart on the surface. However, if the nuclear shape and its rate of distortion are sufficiently irregular, only the last reflection at $t \approx 0$ contributes to the time integral in (1) and, consequently, only the local part of γ contributes, $\gamma(a,b) \approx \rho\bar{v}\,\delta^2(a-b)$, and the standard wall formula emerges, $\dot{Q}_{\rm wall} = \rho\bar{v}\int d^2a\ \ u(a)^2$. In ref. 5, the important role of regularities was illustrated for the especially simple cases of slab geometry and nearly spherical shapes. The regularities conspire to induce correlations between impulses received at subsequent wall reflections, in such a manner as to diminish or, for the smallest multipolarities, completely cancel the local contribution stemming from the first reflection. (In particular, if the nucleus is subjected to an overall uniform translation or rotation, the cancellation is complete and there is no associated energy dissipation.)

The work reported in ref. [5] can be seen as providing a "proof" of the simple wall formula, by establishing a formal tool for studying the conditions of its validity and incorporating corrections arising from regularities. The present paper reports an analogous application of (1) to a dinuclear geometry, in which the system consists of two distinct parts, in relative motion, joined by a small "window." This will provide a similar "proof" of the window formula and bring out explicitly the role played by regularities in the nuclear shapes. Moreover, the work brings us in a good position to confront the transitional shapes characteristic of quasi-fission reactions, since we have now a general treatment which gives the proper description in the mononuclear and dinuclear extremes.

The type of system considered is illustrated in fig. 1. The dinuclear potential well has two distinct parts, A and B, joined by a small planar window whose normal direction is chosen as the z-axis. The two parts are subjected to a small uniform translations, $\mathbf{U}_A$ and $\mathbf{U}_B$, but are otherwise not changing in time. This yields the simplest situation for which window friction should arise. Any intrinsic distortions of A or B are expected to contribute separate dissipation terms of the mononuclear type discussed above (and approximately given by the wall formula).

In the usual derivation of window friction [2], the nucleons in part A are assumed to have a velocity distribution which is shifted by the amount $\mathbf{U} = \mathbf{U}_A - \mathbf{U}_B$ relative to those in part B. This corresponds closely to how the distributions would actually develop, provided the two parts are irregular and the window is small. In the linear-response treatment there is only one velocity distribution, namely the one associ-

ated with the unperturbed potential H_0, which in the present case is the dinuclear potential displayed in fig. 1 *without* any relative motion. The effect of the relative dinuclear motion is manifested in the impulses impacted to the particles when they interact with the nuclear boundary, as explained below eq. (1). The analogue of having displaced velocity distributions is then that the unperturbed trajectories be suitably random. This requirement can be expressed more concisely in the form of the following three axioms, on which our treatment will rely:

I. *Randomization of direction.* Let a particle originally located at the window have the direction $\hat{\mathbf{P}}_i$ when it crosses the window the i'th time. It is assumed that the directions $\hat{\mathbf{P}}_i$ and $\hat{\mathbf{P}}_j$ associated with two different crossing are uncorrelated, when an average is performed over the particle's initial location on the window at the i'th crossing.

II. *Randomization of time.* It is assumed that the time between two window crossings i and j is independent of $\hat{\mathbf{P}}_i$, when the average over locations in the window is performed.

III. *Ergodic motion.* Particles originating within the same arbitrarily small phase-space element will eventually cover the corresponding energy shell uniformly. (Note that this requirement does not imply that randomization is achieved between two successive window crossings.)

3. Derivation of the window formula

The time-dependent perturbation describing the relative dinuclear motion of the system shown in fig. 1 is

$$\dot{H}_1 = -\mathbf{u}(\mathbf{r}) \cdot \frac{\partial H_0}{\partial \mathbf{r}} = \mathbf{u}(\mathbf{r}) \cdot \mathbf{p} \ , \tag{3}$$

where the local surface velocity is given by

$$\mathbf{u}(\mathbf{r}) = \begin{cases} \mathbf{u}_A & \text{for } \mathbf{r} \in A \\ \mathbf{u}_B & \text{for } \mathbf{r} \in B \end{cases} . \tag{4}$$

In actual applications of eq. (1), it is convenient[5] to invert the time integration, and the first term then reads

$$<-\dot{H}_1 \int_0^t dt' \, \hat{U}_0(t')\dot{H}_1 \frac{\partial f_0}{\partial H_0}> \tag{5}$$

$$=-\int \mathbf{dr} \int \frac{\mathbf{dp}}{(2\pi)^3} \left[\int_0^t dt' \; \dot{H}_1(\mathbf{R}(\mathbf{r},\mathbf{p};t'), \mathbf{P}(\mathbf{r},\mathbf{p};t');t')\right] \dot{H}_1(\mathbf{r},\mathbf{p},t=0) \frac{\partial f_0}{\partial H_0} .$$

Here $(\mathbf{R},\mathbf{P})$ denotes the phase space coordinates at time t' for the trajectory originating at the point $(\mathbf{r},\mathbf{p})$, at the time $t=0$. With the expression (2) for $\dot{H}_1$, the time integrand is discontinuous each time the trajectory starting in (r,p) crosses the window. As illustrated in fig. 2, the values of $(\mathbf{R},\mathbf{P},t)$ at the time of window crossings are denoted by $(\mathbf{R}_i,\mathbf{P}_i,t_i)$, $i=1,2,\cdots$. for a trajectory starting in A, $\mathbf{r}\in A$, the time integral in (5) then becomes

$$\int_0^t dt' \; \mathbf{u}(\mathbf{R}(\mathbf{r},\mathbf{p};t')) \cdot \dot{\mathbf{P}}(\mathbf{r},\mathbf{p};t') = \mathbf{u}_A \cdot (\mathbf{P}_1-\mathbf{p}) + \mathbf{u}_B \cdot (\mathbf{P}_2-\mathbf{P}_1) + \mathbf{u}_A \cdot (\mathbf{P}_3-\mathbf{P}_2) + \cdots \tag{6}$$

$$= -\mathbf{u}_A \cdot \mathbf{p} + (\mathbf{u}_A - \mathbf{u}_B) \cdot \sum_{n=1}^{N} (-1)^{n+1} \mathbf{P}_n + \mathbf{u}(\mathbf{R}(\mathbf{r},\mathbf{p};t)) \cdot \mathbf{P}(\mathbf{r},\mathbf{p};t) .$$

A similar result holds for $\mathbf{r}\in B$. The number of window crossings, N, is a function of $\mathbf{r}$, $\mathbf{p}$ and t. For $t\to\infty$, the term $\mathbf{u}(\mathbf{R}(\mathbf{r},\mathbf{p};t)) \cdot \mathbf{P}$ will cancel when integrated over a small part of phase space, since values of $\mathbf{P}$ and $-\mathbf{P}$ will be equally probable due to the assumption of ergodic motion. Also, the term $-\mathbf{u}_A \cdot \mathbf{P}$ will cancel when integrated over $\mathbf{p}$, since H_0 is even in $\mathbf{p}$, and $\dot{H}_1$ depends only on $\mathbf{r}$.

A key step in the derivation is the substitution of each phase space element $\mathbf{drdp}$ by one around the first window crossing $\mathbf{dR}_1\mathbf{dP}_1$. Let t'' denote the time it takes to move from $(\mathbf{r},\mathbf{p})$ to $(\mathbf{R}_1,\mathbf{P}_1)$. The time dt'' spent within the phase-space element at $(\mathbf{R}_1,\mathbf{P}_1)$ is determined by its extension dz along the z-axis through the relation $dz = \frac{\mathbf{P}_1\cdot\hat{\mathbf{z}}}{\mu} dt''$, where μ is the nucleon mass. Employing t'' as an integration variable, we obtain

$$\mathbf{drdp} = d\,\sigma_{\text{window}} \frac{\hat{\mathbf{P}}_1\cdot\hat{\mathbf{z}}}{\mu} dt'' \; \mathbf{dP} subC1 . \tag{7}$$

For each $(\mathbf{R}_1,\mathbf{P}_1)$, the upper limit of the t'' integration is found by following the trajectory backwards from $(\mathbf{R}_1,\mathbf{P}_1)$, until the window is finally covered again. It is natural to call that window crossing the zeroth and denote the momentum shape by $\hat{\mathbf{P}}_0$, as shown in the example in fig. 2. When the integral over t'' is performed, each window phase space element substitutes a part of phase space forming a tube around the

trajectory starting in $(\mathbf{R}_1,-\mathbf{P}_1)$. For ergodic motion, all trajectories will eventually cross the window, so the entire phase space is covered by such tubes. After carrying out the substitution (7), we obtain

$$<-\dot{H}_1 \int_0^t dt'\, U_0(t')\dot{H}_1 \frac{\partial f_0}{\partial H_0}>_{\mathbf{r}\in A} \tag{8}$$

$$=-(\mathbf{u}_A-\mathbf{u}_B)\cdot\left(\int \mathbf{dr} \int \frac{\mathbf{dp}}{(2\pi)^3} \sum_{n=1}^{N} (-1)^{n+1}\, \mathbf{P}_n \frac{\partial f_0}{\partial H_0}\, \mathbf{p}\right)\cdot \mathbf{u}_A$$

$$=(\mathbf{u}_A-\mathbf{u}_B)\cdot\left(\int d\sigma_{\text{window}} \int \frac{\mathbf{dP}_1}{(2\pi)^3} \frac{\partial f_0}{\partial H_0} \frac{(\mathbf{P}_1\cdot\hat{\mathbf{z}})}{\mu} \int_0^{t_0} dt'' \left[\sum_{n=1}^{N} (-1)^{n+1}\, \mathbf{P}_n\, \mathbf{P}(t'')\right]\right)\cdot \mathbf{u}_A$$

and equivalently for $\mathbf{r}\in B$.

Axiom II provides a randomization of the time spent on trajectories between window crossings. The sum over the second and further window crossings can then be treated statistically. Although it is not strictly needed for carrying through the proof, we specialize to a leptodermous potential, in which case the magnitude of the momentum, $P_n = |\,\mathbf{P}_n\,|$ is the same at all window crossings, $P_0=P_1=\cdots\equiv P$. For a given value of P, we define the quantity

$$w_{A\to A}(t-t'') \tag{9}$$

as the average probability that a trajectory which left part A at the time t'' will be located in part A at the current time t. The complementary probability is given by $w_{A\to B}=1-w_{A\to A}$. These quantities $w_{A\to A}$ and $w_{A\to B}$ depend only on the time $t-t''$ elapsed since the first window crossing and they satisfy the relations

$$w_{A\to A}(t-t''=0)=0\,, \tag{10}$$
$$w_{A\to A}(t-t''=0)+w_{A\to A}(t-t''=0)=1\,.$$

After a relatively short time (of the order of the time it takes the particle to traverse the nucleus), the probability per unit time of crossing from A to B is given by the flux average

$$\frac{\sigma_{\text{window}}\; 2\pi\int_0^{\frac{\pi}{2}} \frac{\mathrm{P}\cos\theta}{\mu}\, \mathrm{d}\,(-\cos\theta)}{4\pi V_A}=\sigma_{\text{window}}\;\frac{\mathrm{P}}{4\mu}\,\frac{1}{\mathrm{V_A}}\equiv\lambda_{\mathrm{A}}\,, \tag{11}$$

where V_A is the volume of part A. Defining λ_B equivalently, the differential equation for $w_{A\to A}$ reads

$$\frac{d}{d(t-t'')} w_{A\to A} = -\lambda_A\, w_{A\to A} + \lambda_B\, w_{A\to B}$$

$$= -\lambda_A\, w_{A\to A} + \lambda_B (1 - w_{A\to A}) \tag{12}$$

and its solution is

$$w_{A\to A} = \frac{\lambda_B}{\lambda_A + \lambda_B}\left(1 - e^{-(\lambda_A+\lambda_B)(t-t'')}\right) . \tag{13}$$

For $\mathbf{r}\in A$, the z component of each term in the sum $(-1)^{n+1}\,\mathbf{P}_n$ is positive, since the alternating sign compensates for the alternating direction of passage of the window. The time derivative of the sum is given by flux averages of the momentum in combination with the probabilities $w_{A\to A}$ and $w_{A\to B}$,

$$\frac{d}{d(t-t'')} \sum_{n=2}^{N} (-1)^{n+1}\,\mathbf{P}_n = \hat{\mathbf{z}}\,\frac{2}{3}P\,(\lambda_A\, w_{A\to A} + \lambda_B\, w_{A\to B}) , \tag{14}$$

which is integrated to give

$$\sum_{n=2}^{N} (-1)^{n+1}\,\mathbf{P}_n = \hat{\mathbf{z}}\,\frac{2}{3}P\left[2\frac{\lambda_A\,\lambda_B}{\lambda_A+\lambda_B}(t-t'') - \frac{\lambda_B(\lambda_A-\lambda_B)}{(\lambda_A+\lambda_B)^2}\left(1-e^{-(\lambda_A+\lambda_B)(t-t'')}\right)\right] . \tag{15}$$

Inserting this result into the expression (8), and the performing a partial integration over t'' , we obtain

$$<-\dot{H}_1 \int_0^t dt'\; U_0(t')\dot{H}_1 \frac{\partial f_0}{\partial H_0}>_{\mathbf{r}\in A}$$

$$= -(\mathbf{u}_A - \mathbf{u}_B)\cdot \int d\,\sigma_{\text{window}} \int \frac{d\mathbf{P}_1}{(2\pi)^3}\,\frac{\partial f_0}{\partial H_0}\,\frac{(\mathbf{P}_1\cdot\hat{\mathbf{z}})}{\mu} \tag{16}$$

$$\left(\left[\mathbf{P}_1 + \hat{\mathbf{z}}\,\frac{4}{3}P\frac{\lambda_A\,\lambda_B}{\lambda_A+\lambda_B}\,t - \hat{\mathbf{z}}\,\frac{2}{3}P\frac{\lambda_B(\lambda_A-\lambda_B)}{(\lambda_A+\lambda_B)^2}\right](\mathbf{P}_1-\mathbf{P}_0) + \hat{\mathbf{z}}\,\frac{4}{3}P\frac{\lambda_A\,\lambda_B}{\lambda_A+\lambda_B}\left[t_0\mathbf{P}_0 + \int_0^{t_0}\mathbf{P}(t'')\,dt''\right]\right)\cdot \mathbf{u}_A .$$

Here, terms containing the exponential $e^{-(\lambda_A+\lambda_B)\,t}$ have been left out, since they converge to 0 for large t . The remaining integral of $\mathbf{P}(t'')$ in this expression will vanish, because it can be changed back into a phase space integral being odd in the momentum.

Statistical considerations based upon Axiom I are still needed, in order to determine the values of t_0 and $\mathbf{P}_0$ to be inserted in the expression (16). The momentum $\mathbf{P}_0$ is uncorrelated with $\mathbf{P}_1$. Ordering the integrals with increasing t_0, the average of $\mathbf{P}_0$ for integrals with t_0 within a certain interval is just the flux average $-\frac{2}{3}P_1\,\hat{\mathbf{z}}$. Upon integration over the window, t_0 attains its average value λ_A^{-1}.

Now remains only straightforward calculations, starting by carrying out the integral over the magnitude of the momentum P:

$$\int_0^\infty dP \frac{P^2}{(2\pi)^3} \frac{P}{\mu} P^2 \frac{df_0(\frac{P^2}{2\mu})}{dP} \frac{\partial P}{\partial H_0} = -4\int_0^\infty dP \frac{P^3}{(2\pi)^3} f_0(\frac{P^2}{2\mu})$$

$$= -\frac{4}{4\pi} \int d\frac{P}{(2\pi)^3} P f_0(\frac{P^2}{2\mu}) = \frac{\rho\bar{v}}{\pi} . \tag{17}$$

The entire result can be expressed in terms of averages over the directions $\hat{\mathbf{P}}_1$, constrained by the condition $\hat{\mathbf{p}}\cdot\hat{\mathbf{z}}>0$,

$$<-\dot{H}_1 \int_0^t dt' \, U_0(t')\dot{H}_1 \frac{\partial f_0}{\partial H_0}>_{\mathbf{r}\in A}$$

$$=(\mathbf{u}_A-\mathbf{u}_B)\cdot[2\rho\bar{v}\sigma_{\text{window}} \overline{\hat{\mathbf{P}}_1(\hat{\mathbf{P}}_1\cdot\hat{\mathbf{z}})\hat{\mathbf{P}}_1} - \overline{\hat{\mathbf{P}}_1(\hat{\mathbf{P}}_1\cdot\hat{\mathbf{z}})\hat{\mathbf{P}}_0} + \frac{4}{3}\frac{\lambda_A\lambda_B}{\lambda_A+\lambda_B} t \, \overline{\hat{\mathbf{z}}(\hat{\mathbf{P}}_1\cdot\hat{\mathbf{z}})(\hat{\mathbf{P}}_1-\hat{\mathbf{P}}_0)} \tag{18}$$

$$-\frac{2}{3}\frac{\lambda_B(\lambda_A-\lambda_B)}{(\lambda_A+\lambda_B)^2}\overline{\hat{\mathbf{z}}(\hat{\mathbf{P}}_1\cdot\hat{\mathbf{z}})(\hat{\mathbf{P}}_1-\hat{\mathbf{P}}_0)} + \frac{4}{3}\frac{\lambda_B}{\lambda_A+\lambda_B} \overline{\hat{\mathbf{z}}(\hat{\mathbf{P}}_1\cdot\hat{\mathbf{z}})\hat{\mathbf{P}}_0}]\cdot\mathbf{u}_A ,$$

where σ_{window} denotes the area of the window. The directional averages are readily calculated:

$$\overline{\hat{\mathbf{P}}_1(\hat{\mathbf{P}}_1\cdot\hat{\mathbf{z}})\hat{\mathbf{P}}_1} = \frac{1}{8}(\hat{\mathbf{x}}\hat{\mathbf{x}}+\hat{\mathbf{y}}\hat{\mathbf{y}}+2\hat{\mathbf{z}}\hat{\mathbf{z}}) ,$$

$$\overline{\hat{\mathbf{P}}_1(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{z}})\hat{\mathbf{P}}_0} = -\frac{2}{9}\hat{\mathbf{z}}\hat{\mathbf{z}} , \tag{19}$$

$$\overline{\hat{\mathbf{z}}(\hat{\mathbf{P}}_1\cdot\hat{\mathbf{z}})\hat{\mathbf{P}}_1} = -\overline{\hat{\mathbf{z}}(\hat{\mathbf{P}}_1\cdot\hat{\mathbf{z}})\hat{\mathbf{P}}_0} = \frac{1}{3}\hat{\mathbf{z}}\hat{\mathbf{z}} .$$

The expression (18) contains a term, which is proportional to the time t. This term, plus the equivalent term from $\mathbf{r}\in B$, will be cancelled by the second term of expression (7).

For ergodic motion, and for a leptodermous potential, the second term of equation (1) is different from zero, when the motion of the potential boundaries changes the volume of the system[5]. For the motion shown on fig. 1, the change of the volume per time unit is equal to

$$\dot{V} = \sigma_{\text{window}}(\mathbf{u}_A-\mathbf{u}_B)\cdot\hat{\mathbf{z}} , \tag{20}$$

Inserting into the result obtained in ref[5], one obtains for the second term of eq. (1) for large times t:

$$<\dot{H}_1\,\hat{U}_0(t)\,\dot{H}_1\,\frac{\partial f_0}{\partial H_0}\,t>=\frac{10}{9}\,E\,[\frac{\dot{V}}{V}]^2\,t$$

$$=-\frac{8}{9}\,\rho\bar{v}\,\frac{p_F}{2\mu}\sigma^2_{\text{window}}\,\frac{[(\mathbf{u}_A-\mathbf{u}_B)\cdot\hat{\mathbf{z}}]^2}{\mathrm{V}}\,t\ . \tag{21}$$

Here E denotes the kinetic energy content of all particles, V is the volume, ρ is the particle density inside the potential, $\bar{v}$ is the average particle velocity, and p_F is the Fermi momentum.

Inserting the orientation averages, as well as the expression (11) for λ_A with ($p=p_F$) and the analogous one for λ_B, all terms are finally combined with the corresponding terms from $\mathbf{r}\in B$ to yield the result:

$$\dot{Q}=(\mathbf{u}_A-\mathbf{u}_B)\cdot\frac{1}{4}\rho\bar{v}\,\sigma_{\text{window}}\,(\hat{\mathbf{x}}\hat{\mathbf{x}}+\hat{\mathbf{y}}\hat{\mathbf{y}}+2\hat{\mathbf{z}}\hat{\mathbf{z}})\cdot(\mathbf{u}_A-\mathbf{u}_B)$$

$$+(\mathbf{u}_A-\mathbf{u}_B)\cdot\frac{4}{9}\rho\bar{v}\,\sigma_{\text{window}}\left(\frac{\mathrm{V_A}-\mathrm{V_B}}{\mathrm{V_A}+\mathrm{V_B}}\right)^2\hat{\mathbf{z}}\hat{\mathbf{z}}\cdot(\mathbf{u}_A-\mathbf{u}_B)\ . \tag{22}$$

4. Discussion

In the above result (22), the first term is the standard window formula for the energy dissipation rate.[1)] This term arises from the orientation average of $\hat{\mathbf{P}}_1(\hat{\mathbf{P}}_1\cdot\hat{\mathbf{z}})\hat{\mathbf{P}}_1$, and is thus related to the first window crossing only. Since the particles do not receive impulses at the window, the window dissipation stems from correlated impulses in trajectories that are first reflected a couple of times in one part of the dinucleus, then cross the window, and are subsequently reflected a few times in the other part. Preliminary estimates for spherical nuclei indicate that a good convergence of the window formula is obtained after rather few reflections on both sides. Thus, the relaxation time for obtaining the window formula is expected to be of the order of several times the time it takes a particle with the Fermi velocity to cross a nucleus.

The second term of $\dot{Q}$ in the expression (22) arises because the average amounts of time λ_A^{-1} and λ_B^{-1} spent within the two parts of the volume are not equal in the case of an asymmetric volume division. This term can be associated with the change in the relative mass asymmetry

$$a\equiv\frac{V_A-V_B}{V_A+V_B} \tag{23}$$

implied by the motion depicted on fig. 1. Assuming that the change in volume is equally distributed on the two parts, i. e. $\frac{d}{dt}(V_A-V_B)=0$, the second term of expression (22) can be rewritten in terms of the time

derivative of a as

$$\dot{Q}_{\text{asym}} = \frac{4}{9} \frac{\rho \bar{v}}{\sigma_{\text{window}}} (\dot{a}\ V)^2 \tag{24}$$

In the case of the motion considered here, the change in the mass asymmetry is linked to the radial motion. The more realistic and general case of volume conserving motion of the nuclear surface, and with arbitrary changes of the mass asymmetry, has been considered in ref.[6], giving as the result the so-called *completed wall and window formula.* It is noteworthy that the dissipative resistance against changes in the mass asymmetry evaluated in ref.[6] has exactly the form (24) when expressed in terms of the variable a .

Thus we conclude that the present derivation of the expression (22) for the dissipation provides a proof based upon linear response theory of the completed wall and window formula for the special kind of motion shown in fig. 1.

The long mean free path dynamics of nulcei provides a unique dissipation mechanism, depending so much on the symmetries of the motion. In particular, the transition between the wall and window dissipation has posed a key problem of large-amplitude motion of the nuclear surface for the last ten years and so far only schematic interpolations between the two dissipation formulas have been applied. The derivation of the wall formula in ref. [5] together with the present derivation of the window formula establish linear response theory as a reliable starting point for investigating this question. As we view it, such investigations would have to rely on numerical integrations of the expression (1). Furthermore, also other variables, such as the angular momentum, could be studies by means of response theory with the classical phase space trajectories, and dispersions in the quantities could be investigated.

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Figure captions

Figure 1. The dinuclear cavity. In the dinucleus, the individual nucleons move in a leptodermous potential that has two distinct parts, A and B. The two parts are joined by a small planar "window" whose normal is chosen as the z-axis. The two dinuclear parts are endowed with the uniform translational velocities $\mathbf{U}_A$ and $\mathbf{U}_B$.

Figure 2. The window crossings. A nucleon initially located at the position $\mathbf{r}$ and having the momentum $\mathbf{p}$ has the momentum $\mathbf{P}_1$ when it first crosses the window, $\mathbf{P}_2$ at its next crossing, and so forth. Backwards propagation of the path yields the momentum at the most recent window crossing, $\mathbf{P}_0$. The contributing particles originate near the nuclear surface since only there is the effect of the imposed translation felt.

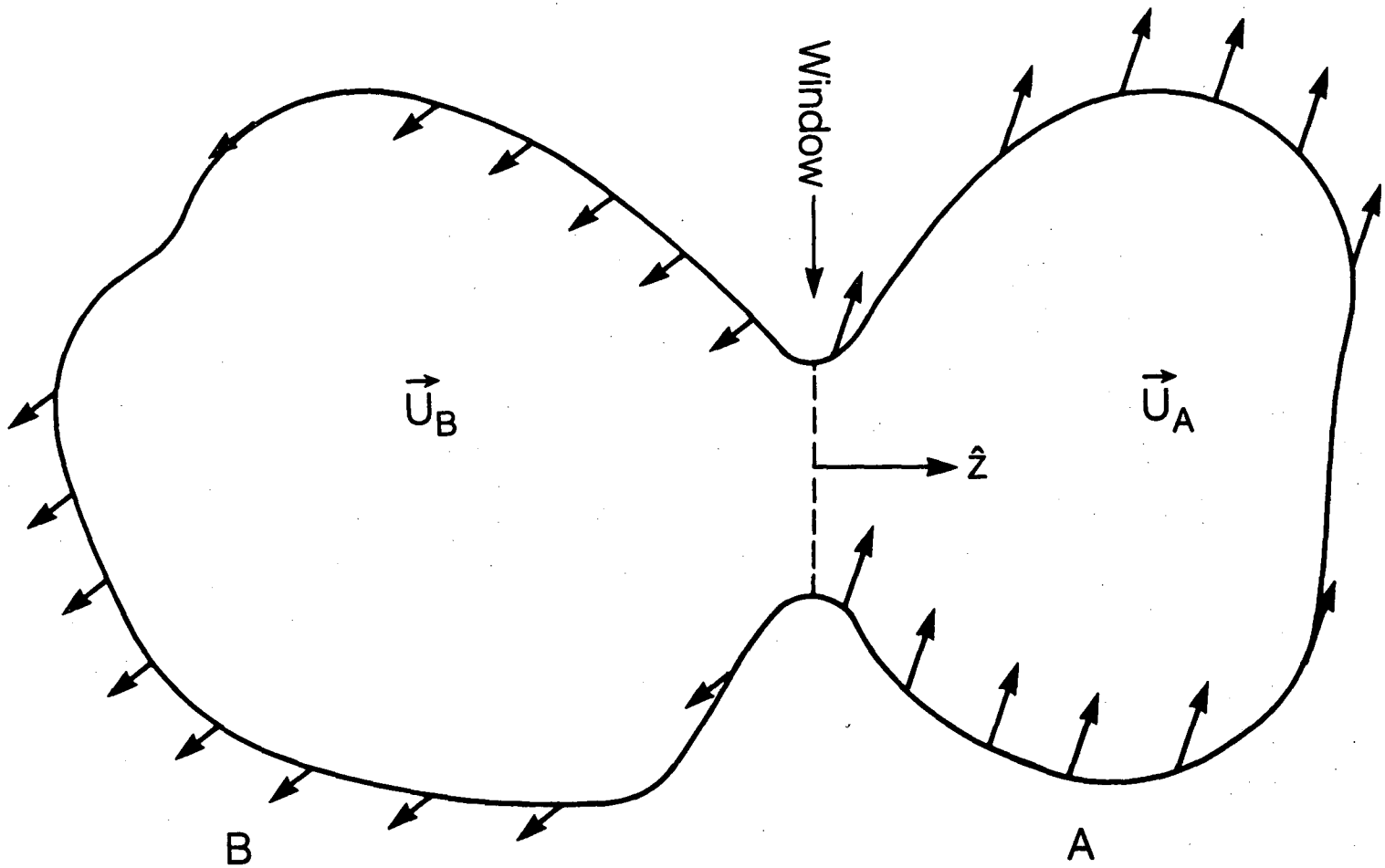

Fig. 1

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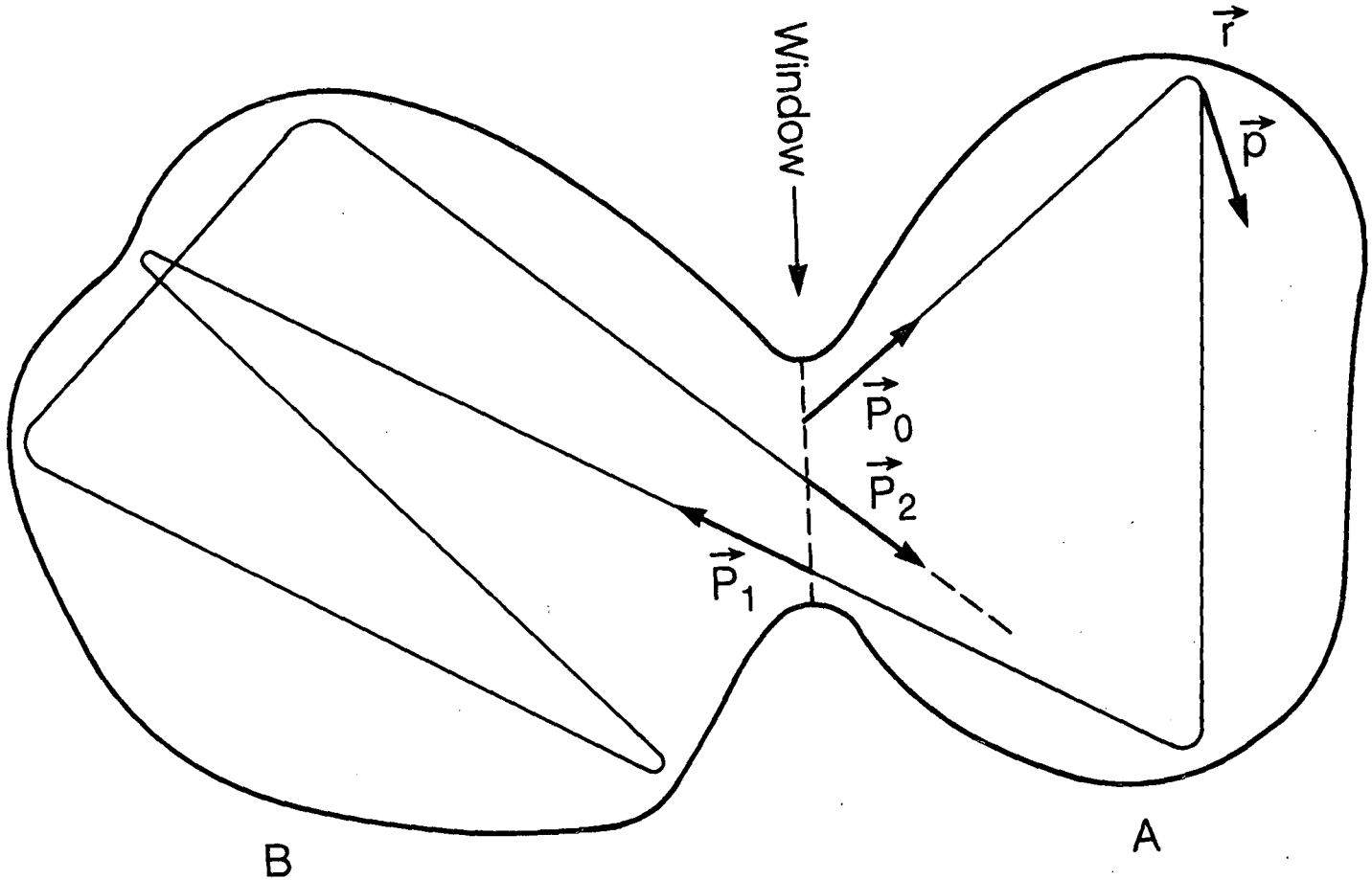

Fig. 2

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